Structural Analysis 1
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Lecture 29: Influence Line Diagram
And Moving Loads (Continued)

Hello everyone! This is lecture number 29, last class we just introduced the concept of influence line diagram and demonstrated through one example simply supported beam and also a cantilever beam, what we'll be doing today is we'll demonstrate the influence line diagram with few more example ok.

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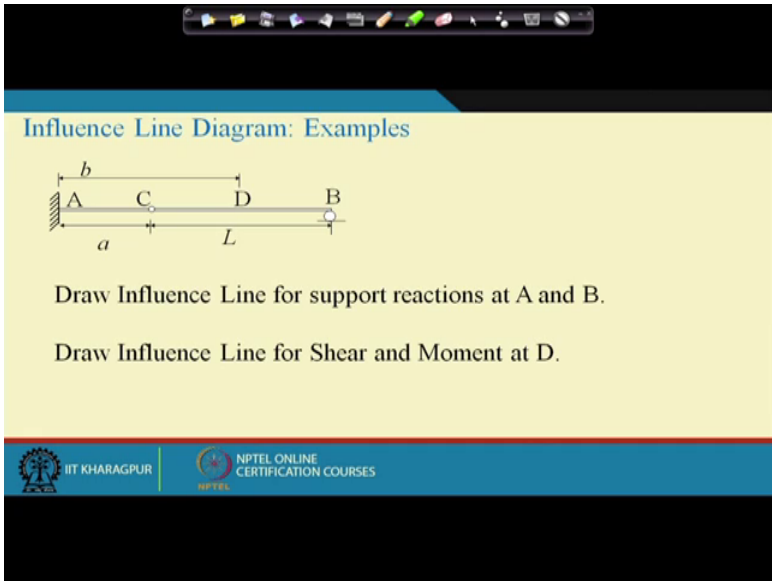

Now the first example we'll consider is this ok, this problem you remember we have considered this problem in some of the previous classes. This is a proft cantilever B, but we have an internal inch at C, now what we have to find out, we have to find out here what is the influence line of support reactions at A and B and influence line for shear and moment at D which is at a distance B from, at any arbitrary distance B from point A ok, so this is the problem,

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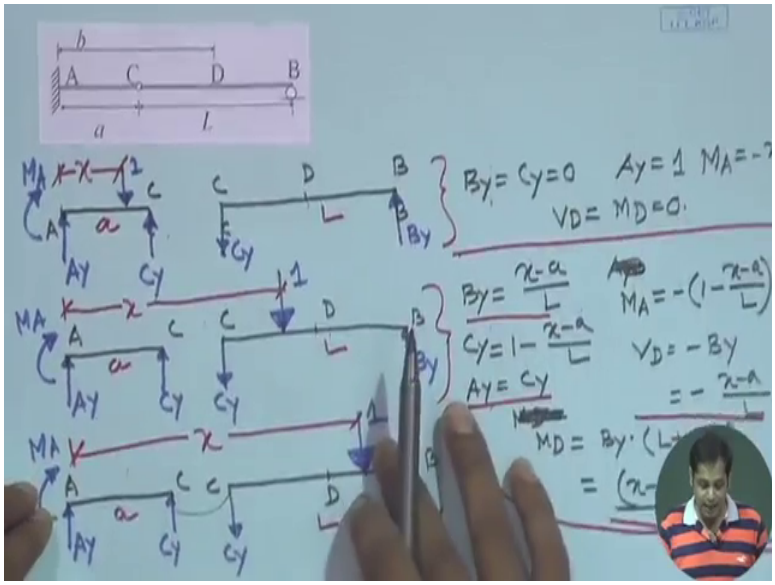

Now we can see that there are three segments AC, CD and DB, we need to consider three cases, one is influence line when the unit load is between A and C.

And then C and D and in third case is between D and B, C is the internal hinge so we know that moment at C is equal to zero ok, we also know that this is statically determinate problem ok, let us draw three separate cases ok, now first is the unit load is on AC, so we divide this problem for each case, we divide the inter beam into two parts, one is AC and one is CB, so first this is AC and this is CB ok, so what are the forces.

A unit load we have on AC which is at a distance x, x from A right and what are the support reactions, we have A Y and the moment at A, M A or another reactions we are not writing and then this is C Y and since it is hinge and C Y is shown when belongs to this is shown downward, it will be shown upward here, so that hinge remains in equilibrium and that's all, so this distance, this is A, this is C, some are it is D and it is B, at B we have another reaction force that is B Y ok.

So this is the, figure diagram of the when the unit load is between A and C, let's we'll apply the equilibrium condition on figure diagram, but let us consider other two cases as well, so again this is the case, now we have this is A Y, this is M A, this is C Y, B Y, this is B, this is D, this is C and

this is A, ok and suppose the unit load is between C and D, so this is the unit load which is at a distance, x is always measured from A ok.

So that we use the same origin, same reference code net, now let's consider the third case when the unit load is between D and B, consider this and then third case again draw the, this is reactions A Y then moment M A, this is C Y, C Y here B Y and this is B, this is D, this is C, C, this is A and the influence unit load is here which is at a distance A X from x ok, so we considered three cases.

What we have done is we divided the beam and the beam is broken at point C since we have additional condition at point C is moment at C is equal to zero it is internal hinge, so this is part A C and this is part C D B and then we can have three cases, three cases are unit load is between A and C, then C D and then D B and these are corresponding figure diagrams, now what we'll do is we apply equilibrium conditions on this.

Now by applying equilibrium condition on this we can see that there is no load in between C and B, so C Y, B Y both at zero right, so we can say B Y is equal to C Y is equal to zero from this figure diagram, now let us see this figure diagram, now C Y is equal to zero, so what are the forces, only vertical forces we have A Y and one, A Y acting upward, one is downward, so naturally A Y will be one ok and if we take moment at A.

C Y is equal to zero, so moment will be one into x, this is clockwise direction, this is sagging moment and if we equate them so we'll get M X, M A is equal to minus x ok, so this is support reactions, now let us find out what is the shear force and bending moment at D, you see if you consider this figure diagram, the shear force at D will be zero and similarly because there is no load in between so V D is equal to zero, so this is for this part.

From this figure diagram we have this expressions ok, now let's consider this case if we again consider this figure diagram, apply the equilibrium conditions summation of vertical forces is equal to zero, summation of moment at C or moment at B is equal to zero though what expression we get is you please verify that B Y is equal to x minus, we know how to do this right, and then B Y is obtained.

And similarly we can calculate C Y is equal to one minus x minus A Y L ok, this we can obtain by applying the figure diagram on this part ok, now once we know C Y, let's go to this figure diagram, and what we can say is, we can say that A Y is equal to C Y and if we take moment about A, then the moment A will be C Y into this distance, this distance is A, intra distance is L ok, so if we do that then we get A Y is equal to C Y we can right ok.

C Y is equal to this, so let's find out moment and moment will be minus A Y into A and the value of moment will be then M A will be minus one minus x minus A by L into A ok, this is momentum ok, now once we have the moment, now let us find out what is the V D and M D L again we can draw figure diagram of D B and calculate, if we apply the equilibrium condition and get the expression for V D and M D and the expression will get is V D is equal to minus B Y

And which is equal to minus x minus A Y L and M D will be, I am just writing the final expression M D will be B Y into this distance, this distance will be L plus A minus B, so this value will be x minus A, L plus A minus B by L, so this is M D, so this we obtained this figure diagram what we have is we have B Y is this and A Y is this, we know V D is this and M D is this, this is the final thing we need ok, so these are all function of x, now let us consider this.

Now if we look at this two figure diagrams then expression for B Y, for this case and this case will remain same and therefore expression for C Y for this case and this case will remain same, now since C Y is same, the expression of A Y and M Y for this case and this case, they will also same, so what is the expression for support reactions for when the unit load is between C and D and the unit load between D and B in both the cases. Expression for support reactions they will remain same only change will be the moment at D.

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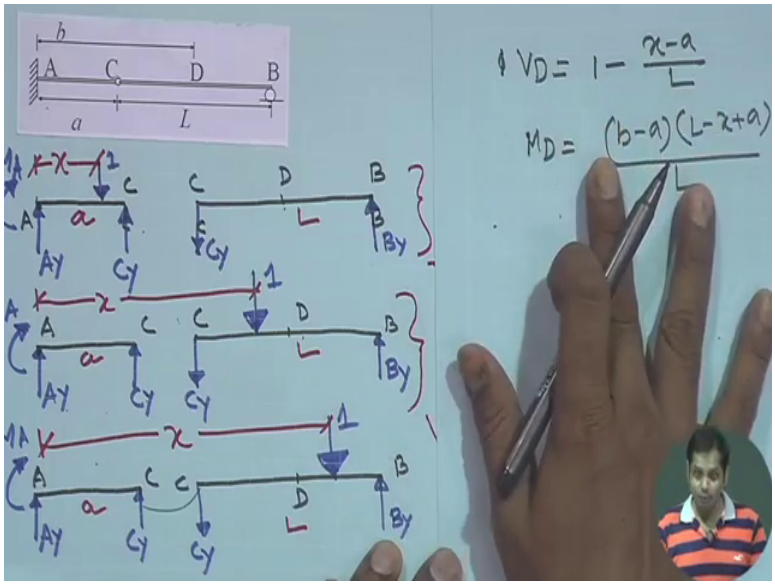

Now how to determine moment at D again we apply the same concept and find out moment, the expression for moment will be first V D will be one minus x minus A by L, I am not doing this calculation this is the final result I am writing because we have done this exercise many times in previous classes, now M D is equal to we'll get B minus A into L minus x plus A by L. So this is expression for shear force and bending moment at point when the load is between D and B, now let us draw the influence line diagram ok.

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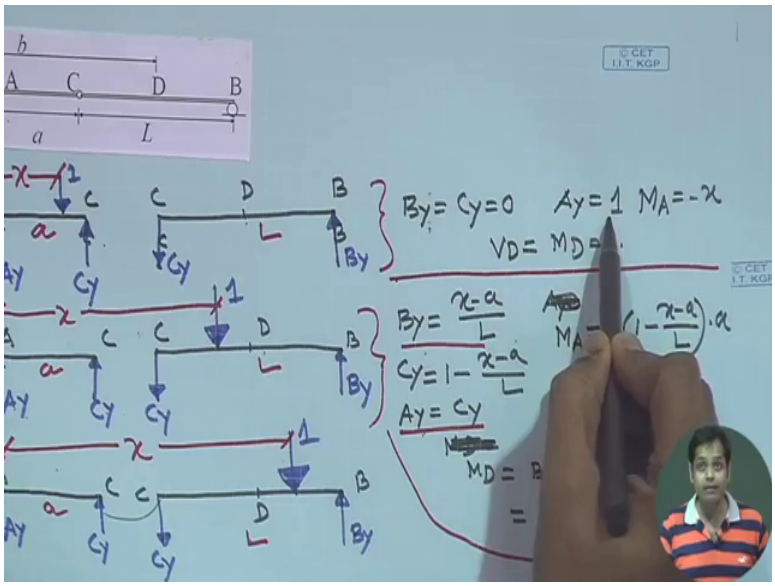

Now first draw the influence line diagram for say A Y ok, now what is expression of A Y, A Y is equal to one when for the first case when load is between A and C and then all other cases A Y is equal to this it is linearly varying ok and at x is equal to, if we substitute x is equal to (13:18 L plus A) means at B this A Y becomes zero.

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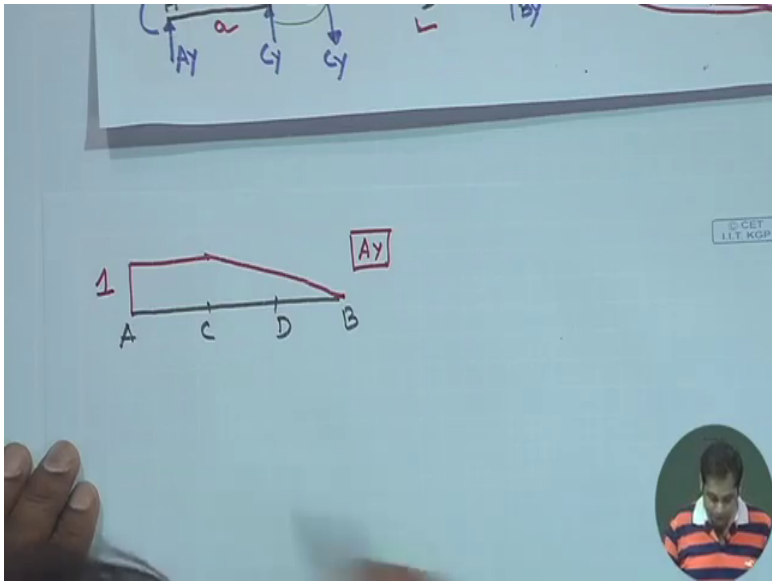

So up to this is A then C, D, B influence line becomes up to C it is constant which is value is one and then zero and this value is one, this is for A Y, this is influence line diagram for A Y, similarly let us draw influence line diagram for B Y.

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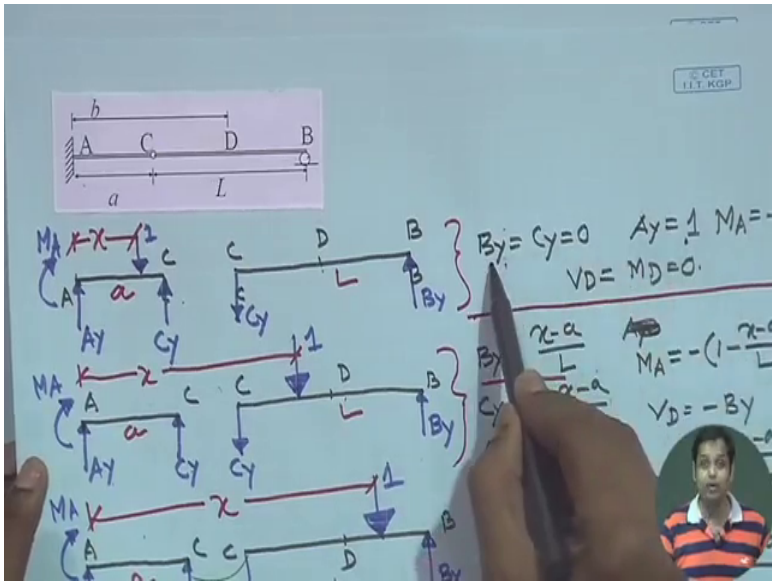

What is the value of B Y if you check, B Y is equal to zero when the load is between A and C and B Y is equal to x minus A by L when the load is between C and D and D and B both means between C and B the B Y varies. As x minus A by L, so if you substitute at x is equal to L plus A means at B, so this becomes one.

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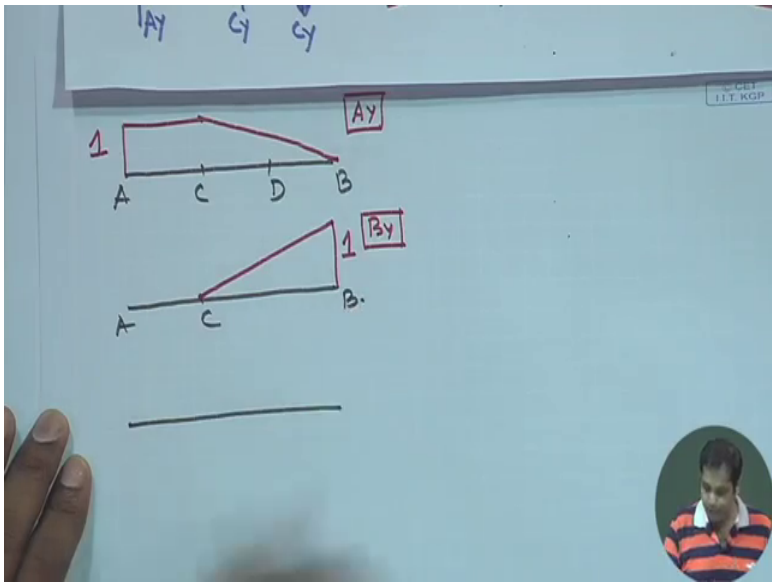

So B Y is equal to zero up to C and then at B it becomes one and between this it varies linearly, so this is one, so this is for B Y, so this is the influence line diagram for B Y, let us draw it for another support reactions means M A, this is again A C B, ok.

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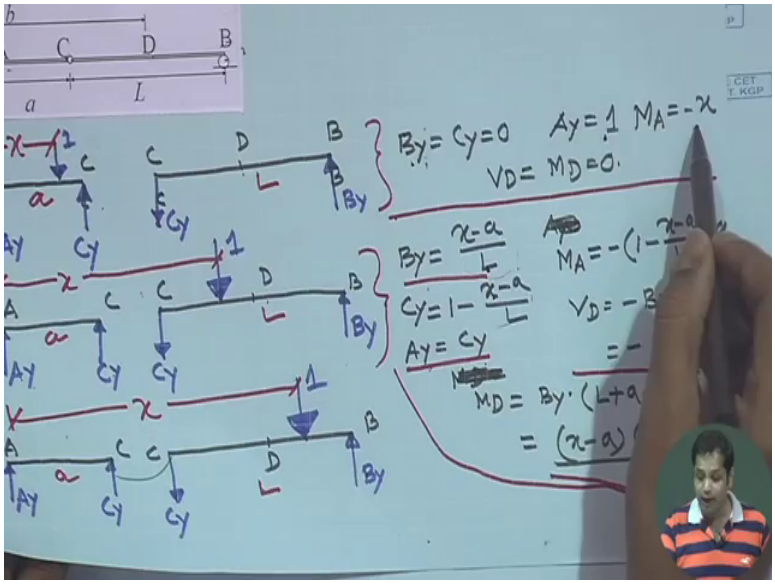

Now what is M A, M A is minus x between A and C, so at C this value becomes minus A.

And then unit load between C and D, M A varies with this expression ok, so if you substitute x is equal to L plus A means at B, M A become zero and if you substitute x is equal to A here then M A becomes minus A, so this is the continuity of M A is maintain, if you substitute x is equal to here minus A, here also it is minus A.

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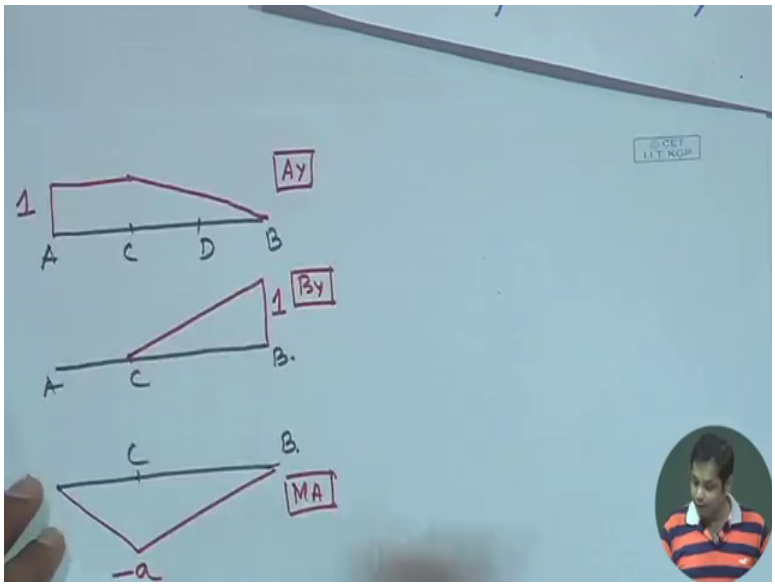

So influence line for this becomes, for C it is this and then linearly vary and this value is minus A and this is A then C B. So this is influence line for M A, similarly let us draw influence line for shear force at D and bending moment at D.

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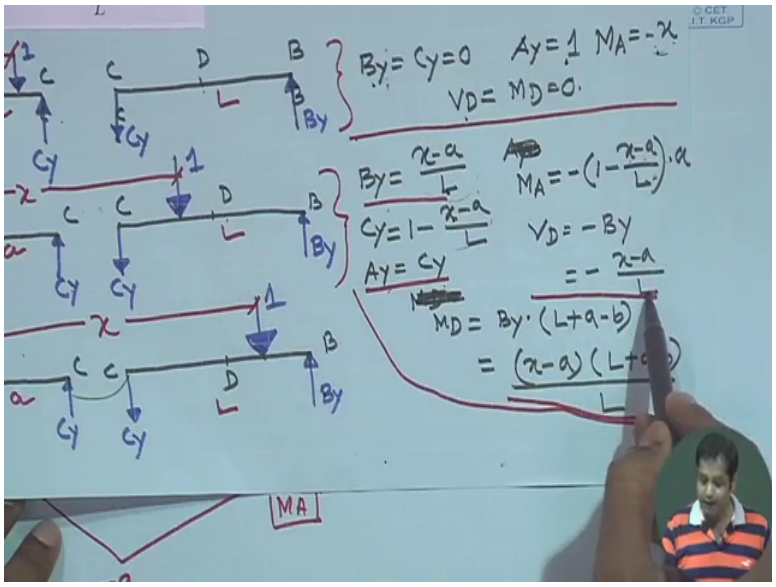

Now shear force at D when load is between A and C is equal to zero. Shear force at D varies like this when load is between C and D.

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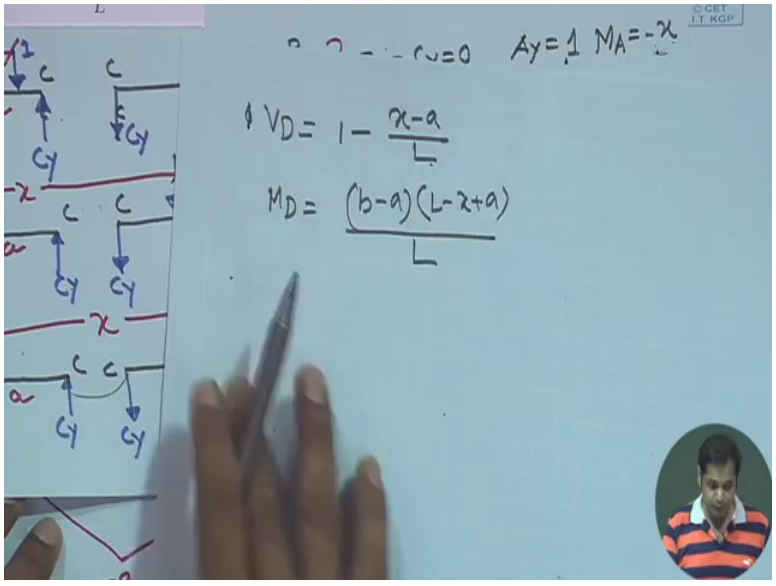

And shear force at D varies like this when the load between D and B and if you draw them then the expression what you get is,

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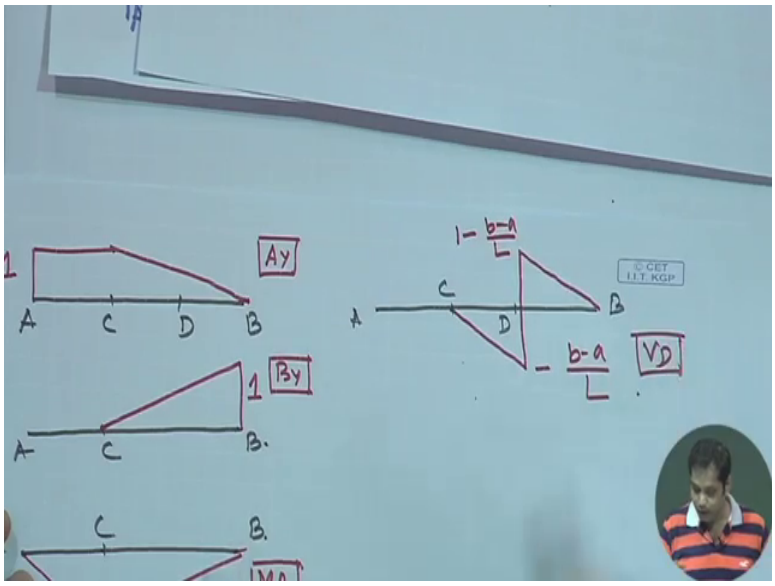

This is A, this is C, this is D, this is B. The expression you will get at this and this value is minus A minus A by L and this value is one minus B minus A by L, another important check is the absolute value, if you add them their absolute value will be one in this case because only applying load downward is one ok, so this is for V D.

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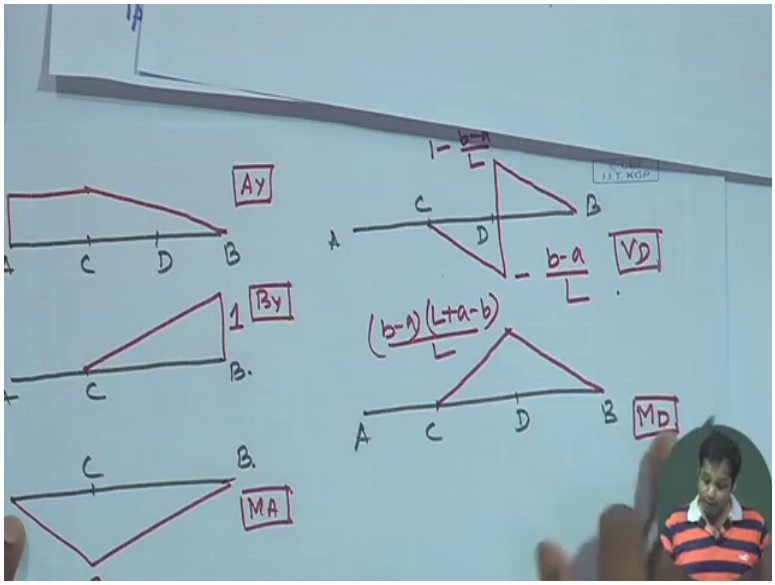

Now similarly if you draw the shear force bending moment diagram between A and C it is zero, just now we check and then it becomes this value. And this is A C D and B this value is B minus A into L plus A minus B by L, this is for bending moment at D,

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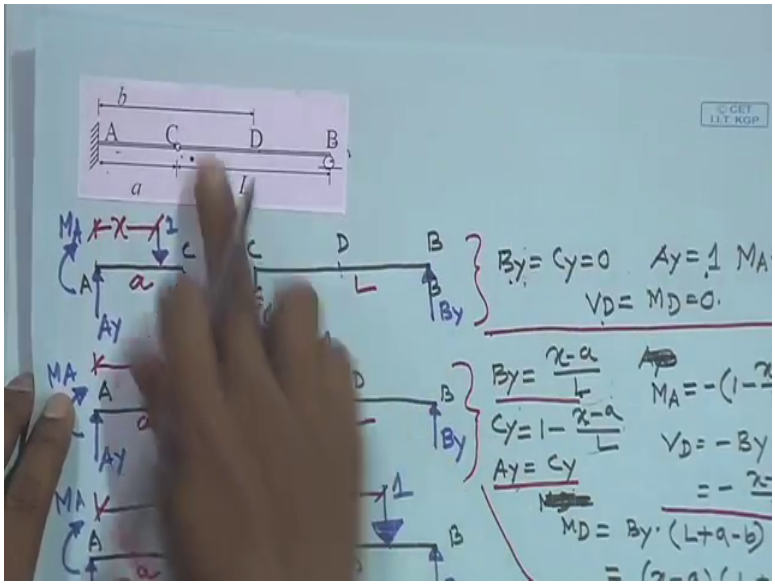

So these are the influence lines for support reactions and bending moment at D ok, for this problem ok, let us once again please, I have not done all the calculations because this similar calculations we have done many times so please verify this results yourself.

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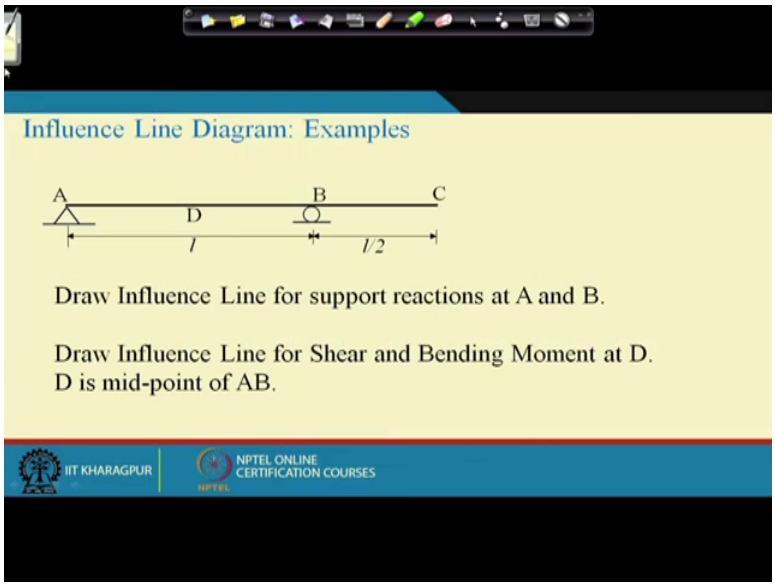

Now let us quickly go to our second example, the second example is this we have again. A simply supported beam with an overhang like this.

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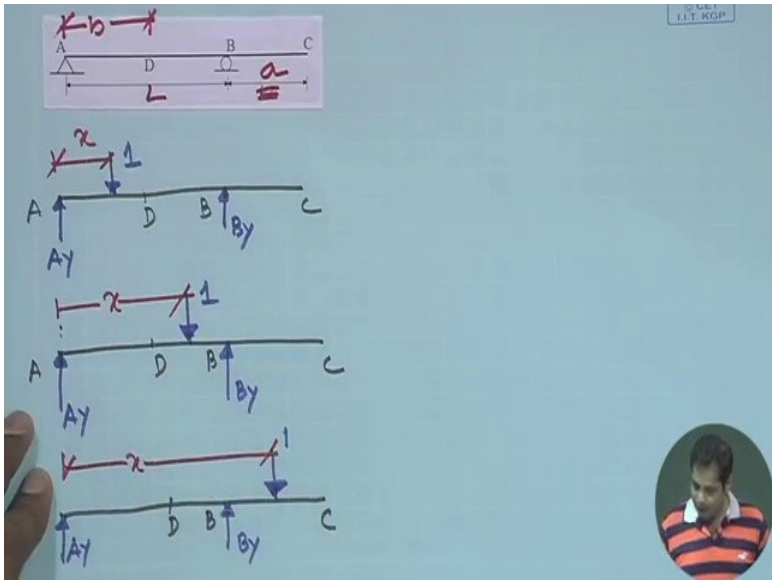

Now what we need to find out, we need to find out what is the influence line for support reactions and then what is the any intermediate point D what is the bending moment and influence line for bending moment and shear force, let us slightly change as this value is not L,

make it more general, say this is A and this is L fine and let's take this D is at a arbitrary distance B so that the influence line what we get.

Is more general ok, now again we need to consider three cases when influence line is here, the unit load is between A D then D B then B C ok, so quickly draw the figure diagram of each cases, this is the first case where influence line is, this is A Y and then this is B Y, A D, B C and influence line, unit load is one here which is at a distance, this distance is x, so this is case one, the second case is this where you, this is A Y, this is B Y.

B D A C and the unit load is between D and B and this distance is one x ok, and quickly take the third case, third case is this draw the figure diagram, this is A Y, B Y, C B D A and unit load is applied on this which is at a distance x from this ok, now this is a statically determinate problem so individually we can find out what is the reactions by applying equilibrium condition we can determine what is the reactions in this case A Y B Y.

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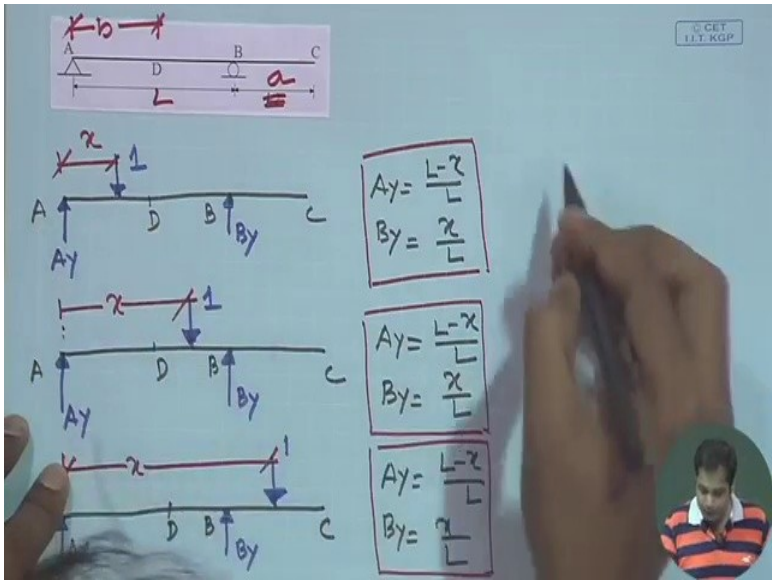

And the reactions will be in this case A Y is equal to L minus x by L and B Y will be x by L, in this case A Y will be L minus x by L and B Y will x by L and the third case it is A Y will be L minus x by L and B Y is x by L, so what we can see is, in all the cases support reactions are same ok, please verify this ok, these are all support reactions ok, now we need to find out what is the bending moment and shear force at point D.

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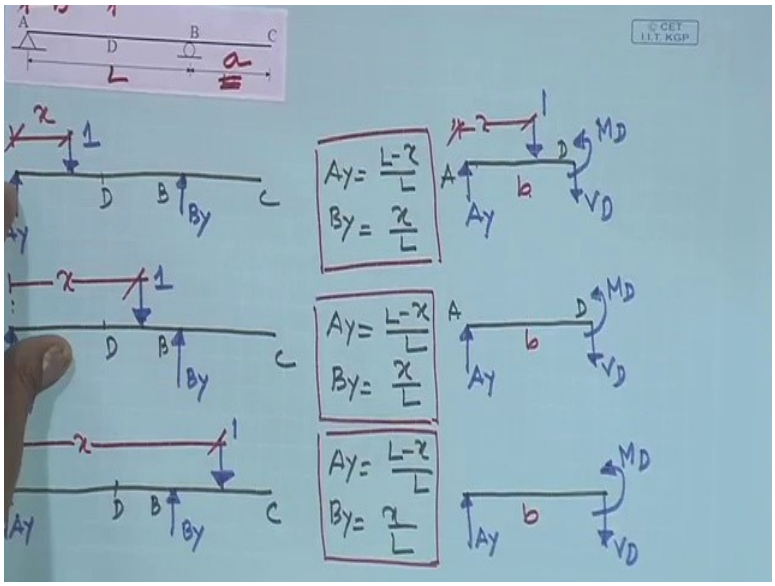

So if we take the free body diagram of this part so we have A Y and at D we have B D and moment M D and unit load is one at a distance x ok, and this distance is B ok, now similarly if we for this case take the free body diagram of same part this, the free body diagram will be, there is no unit load here it is just only A Y and then V D, M D there will be no load because load is now D B, this is A and D, A D A D ok and third case also same thing.

This is A Y, this is B D, this is M D and there is no load in between, this distance is B, this distance is B ok, now what we need to do is, we need to apply equilibrium conditions here, and equilibrium conditions here and if we do that then what are the expressions we have.

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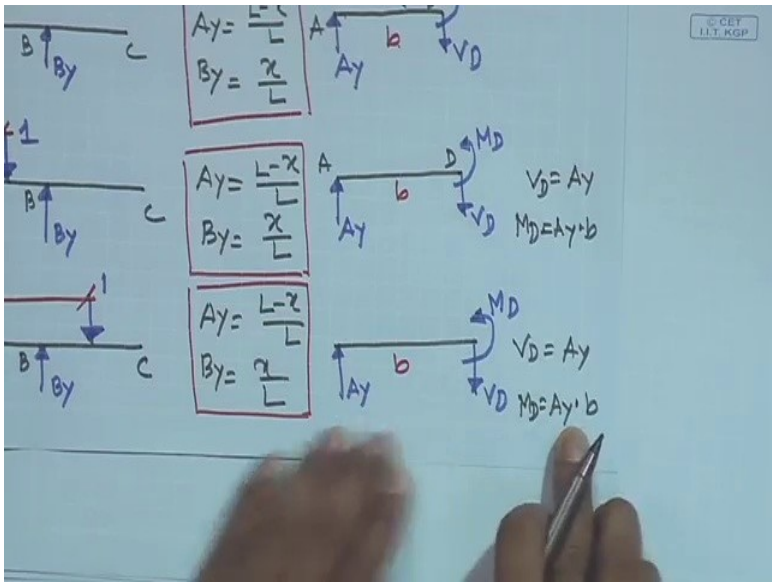

we can find out what is the expressions for M D and in this case what will be the expression for V D, in this case V D will be A Y, in this case also V D will be A Y. A Y already we obtained here, in this case also A Y we already obtained and what will be M D, M D will be A Y into B ok.

And here also M D will be A Y into B, A Y already we have determined ok and we can also apply in this case M D and V D so what is expression M D and what is expression for V D, I leave it to you find out the expression because it is very simple we need to apply the equilibrium conditions only. Now once we do that we have the expressions of all these required forces right.

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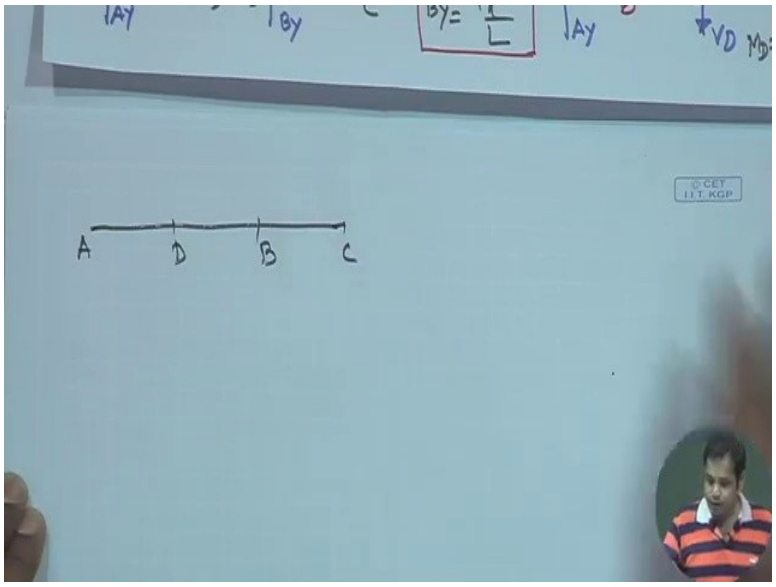

Now let us draw the influence line for shear force, let's first draw the influence line for A Y, so this is A, then D, B C.

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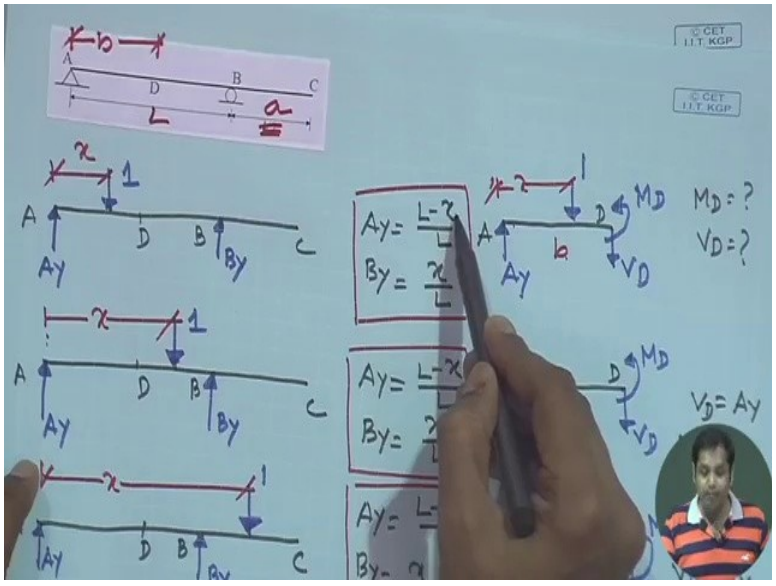

So what is A Y, A Y is L minus x by L and that is too for all so if we substitute x is equal to zero, so it becomes one at x is equal to L, it becomes zero and then x is equal to L plus A, it becomes minus A by L.

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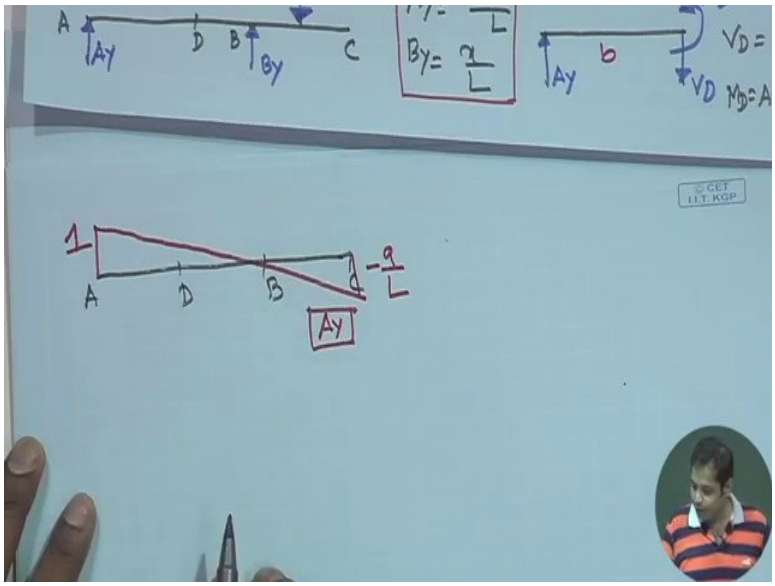

So influence line becomes at x is equal to this becomes one at B it is zero and it is linearly varying, so this becomes one and this becomes minus A by L ok, so this is the influence line for A Y, this is for A Y, ok.

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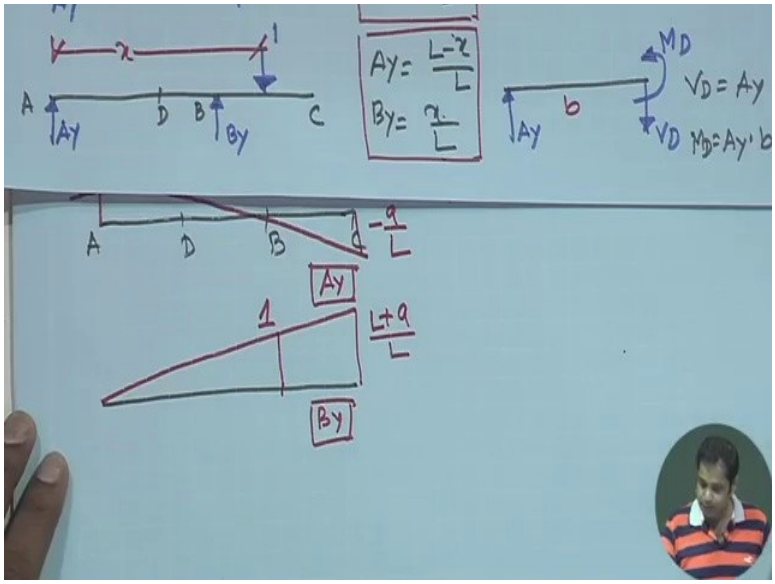

Now let us draw influence for B Y, what is B Y, B Y is x by L here, x by L here, x by L here, x is equal to zero, then B Y is equal to zero, and after that this linearly varying so it varies linearly

and x is equal to L it becomes one. So this value becomes one and this value becomes L plus A by L, so this is B Y ok.

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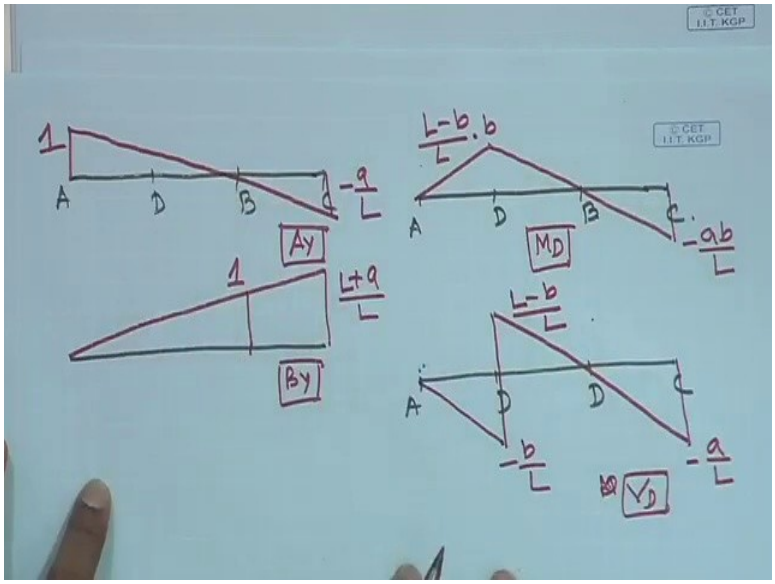

Now let's draw the influence line diagram for bending moment at D we have the expression for bending moment for D now this is A D B and C, now just plot those expression that you have obtained and what will get is this is for bending moment and this values will be, this will be minus AB by L and this will be L minus B by L into B ok, this is for M D and finally we get shear force.

Just plot the expression, one expression we have not determined please find out that and check this influence line diagrams, this becomes this and then there is a sudden jump here and then again this becomes this ok, now these values are minus B by L, this is minus B by L and this is minus A by L and this is L minus B by L, so these are influence line, this is for V D, so this is the influence line diagram for support reactions.

And shear force and bending moment at any particular location D for this problem ok, now one thing you might have noticed that the influence lines are essentially, it is just a concentrated unit load moving along the length of the beam, the influence lines are always piece wise linear means that the influence line always a collection of some lines ok, so therefore what you need to, the variation is always linear that's what I wanted to mean.

So what you need to find out if you know the point between the value of the influence at two points in between that you can just join them and get the influence line ok, so these are some examples but again if you go through books and there are many examples given, some solved examples, some exercise problems, please do that and make use of comfortable with influence line ok, now what we do next, next class is what we have done so far is we discussed.

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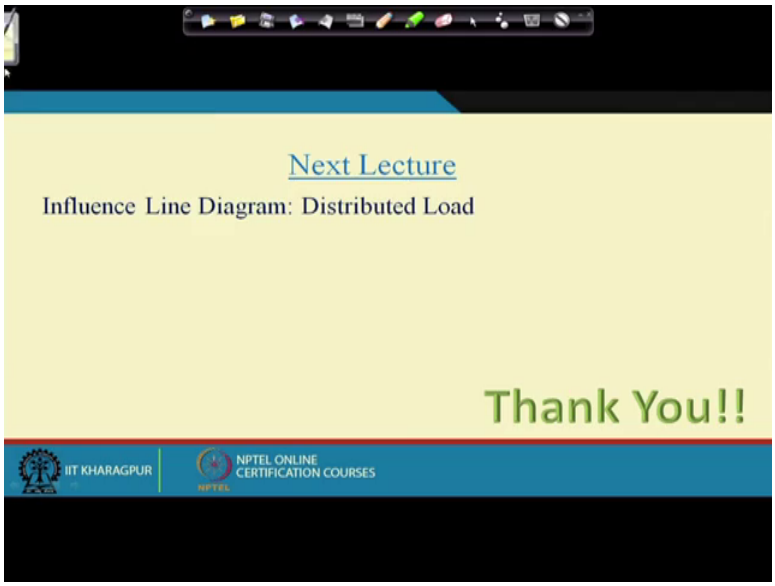

If a unit, a concentrated load is moving through the length of the beam, now instead of concentrated load if the load is distributed load, a distributed load moves so then how that to be considered and what are the important aspects that we need to consider, that we'll discuss in the next class ok, thank you, see you in next class.