Structural Analysis Part 1
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Lecture 28: Influence Line diagram
And moving loads

Hello everyone! Welcome to week 6, by now I believe you are comfortable with the concept of structural analysis of statically determinate structure means how to determine the internal forces and deflections of beams, then plain traces and plain frames, now all the problem that we have addressed so far, what you assume the structure is at rest condition, that's why it is statically equally terms conditions and in addition to that we also assumed implicitly.

The applied load on the structure that is also at the same position, they do not change their position with time right, but there are many cases in many structure the primary load they are subjected to the loads are not fixed to the structure, the loads change their position with time, those are called moving loads, now what we'll be doing in this week is we'll see how to deal with this moving loads.

So this week what we do is the influence line diagram and the moving loads, what is influence line diagram I am not going to tell you right now, we'll understand as we go through, ok first let me show you some structures which are subjected to moving loads.

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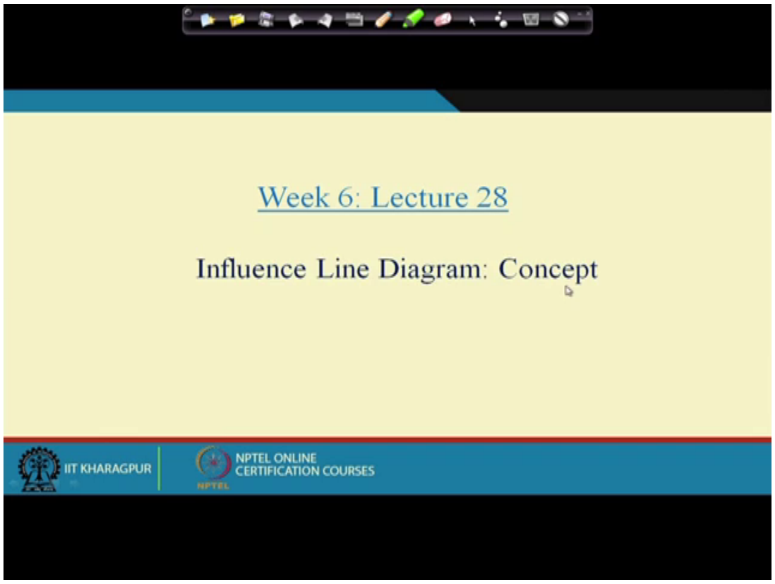

So today we'll just introduce the concept of influence line diagram and the subsequent lectures we'll see different examples and other different aspects of the influence line diagram, ok you see these are some structure there are many.

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These are some structures they are subjected to dead load anywhere, dead load is always there but the primary load, they are subjected to easy in this case is a railway bridge, when the train moves then changes its position then the flyover where the vehicles are running and the position of the vehicles continuously changing same here it is a rope way and if you see the rope way this is moving and the supporting structure of the roof is subjected to a load which is not fixed.

In its position, we changes this position with time, in fact when we climb a stair, when we climb a ladder that is also moving load because if we assume that, if we consider the ladder is a structure then our weight is the load applied on the structure then as we move, as we climb the position of this load is also changing, these are typical example many of you have seen this, so when the person walks through, walking on the rope.

When walks on this rope and depending on the, these are the supporting structure, these tripod kind of thing that you can see is very similar to the tripod, the concept wise at least very similar to the tripod that you can see in camera, so the loads subjected to these structures are not constant, the position of the load is changing, now there are many other examples like this, so if you look around and see different kinds of structures which are subjected to these moving loads.

What we'll be doing here is we'll see how this moving load can be considered while in analysis of structure, now remember one thing we'll be still doing static analysis when, you may see that if load is moving then probably the dynamic effect is important.

If we find that inertia of the structure is important to be considered then of course we need to go for dynamic analysis and for that you'll be having a separate course called structure dynamics. But what we'll be doing is the same problem but dealing with, we'll be still stick to the concept of static equally beam conditions, ok now let us first describe what exactly the problem we are to address.

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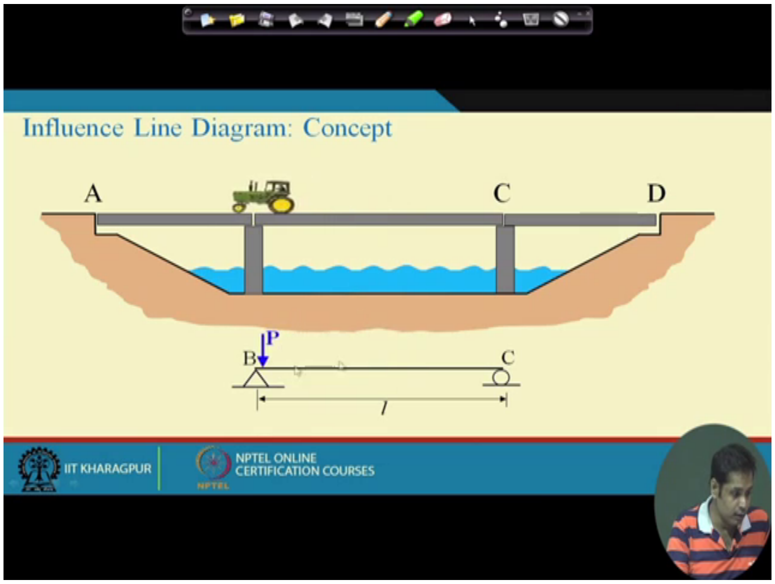

You see, take any example this is to for any problem but just for demonstration, I am taking example like this, this is a bridge and there is a three span, Both sides supported an apartment and the central span which is supported on two periods, now look at this structure carefully, you can see some gaps between these two guarder and these two guarders.

Means this guarder is not continuous, so they are simply supported guarder, they are just supported on this pear, this is on abatement in a pair and this is on two pair and this is on pear and abatement, so each every guarder can be idealized as a simply supported beam ok, now suppose there is a vehicle standing on the central guarder ok, now, there is a self weight of the vehicle will cause if we assume that length of the vehicle is small.

As compare to the length of the guarder we can assume that this will apply a concentrated load at this point right, and that we know how to analyze the structure but now if the load is not constant, now if the load moves right, now consider central guarder as the vehicle changes its position then the internal forces in the central guarder or for that matter any guarder internal forces they keep on changing right.

We'll see how to deal with that but let's take a snapshot for a given timing stand suppose the position of this vehicle is this ok, suppose these are the numbering, now as I said these force, if we assume this length is small as compared to the length of the guarder it can be idealized as a concentrated, look we’ll see other examples where load is not concentrated it is a distributed load or a train of concentrated load, those examples we’ll see in subsequent classes.

Now, self weight of these structure is now idealized as a concentrated load at that point at a given instant, so this is a snapshot, now this single guarder BC is idealized as if this is a real system, the physical system and this is an idealized system, idealized model of central guarded BC as I said, these guarders are not continuous it is simply supported so these ends are simply supported, length is L and it is subjected to concentrated load right.

So this is are the snapshot at a given time instant, at a given position of the load but now what is happening, so for this problem we can find out what are the reactions supports, what are the bending moment shear force at any cross section of the beam, so these whatever knowledge, whatever understanding we have based on that we can determine that right, now what is happening this load P is not constant, this is an idealized model of this vehicle.

Now as the vehicle moves this load is not constant, so what is happening this vehicle is moving between B and C and this load is also corresponding load, now what we will do is now as this load moves, in the previous slide what we saw is for a given instant of time, for a given particular location of the load we can determine what are the internal forces reactions in the beam, now at the load changes its position, internal forces.

And support reactions also change their value, what we are interested now is, internal forces and support reactions how their values changed with change in the location of, that is the problem, so problem is once again let me tell you as this load move in, as this load moves on this beam, how the internal forces and support reaction change for this movement of this load,

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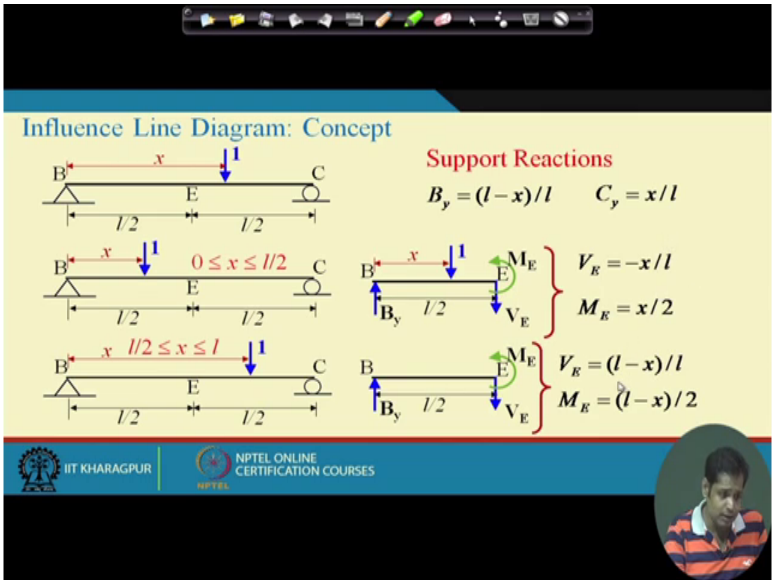

So take the problem. Let us find out how they move, take the problem it is at a given position of this load. We know that we are linear, linearity is one of the important assumptions in our analysis, whatever concepts we learned, the load deflection is linear and then we also know that the internal forces are linear function of the external forces right, so instead of though in this problem the actual load applying on the structure is P, it is a standard practice instead of assuming load P, let us assume the value of the load is unit.

So the structure is subjected to an unit load, now whatever response we get that response if you multiply, if the actual load is P, then response will be multiplied by P so you'll get the actual response ok, now so it is subjected to a unit load, suppose this load is added distance x from P, now this x is continuously changing for different position of the load, we have the position of x with the different values of x ok, let us find out for a given value of x.

For a given position of x what will be the internal forces and support reactions ok, now these are static problem, so we can apply the static equilibrium condition we can draw the figure diagram of the entire structure and final the reaction, this problem we have attempted many times in last several lectures, so we can determine what are the support reactions, support reactions are B Y is this and C Y is this, B X will be zero here, because there is no horizontal force.

So therefore it is not explicitly return, so we know the support reaction, so this and at B and C at this, now once we know the support reaction let us find out, suppose the pending moment and shear force at point E reaches at the mid span of BC, ok so this distance is L by 2, this distance is L by 2, let us find out what is the shear force and pending moment at point E, now we need to consider two situations, first when x is such that the load is applied between B and E.

This is a case one, so in this case your x varies from zero to L by two ok, now the next case is when the load is acting on EC, so the next case is load is acting on EC, so x is between L by 2 to L, so these are two cases ok, now we can take the figure diagram of part BE and we can take the part BE and draw the figure diagram, so this is B this is the unit load acting at a distance x, this distance is L by 2 and the internal forces at E will be shear force.

So please check, be consisting with the sign convention, this is our sign convention V E and then sagging moment is positive, this is M E, the horizontal force as well that is not explicitly shown here because anyway horizontal forces at zero, so this is the figure diagram of BC, now from this figure diagram we can apply the equilibrium condition, if we apply equilibrium conditions summation of forces is equal to zero and summation moment is equal to zero.

Then what we get is V E is equal to the shear force is equal to this and M E is equal to X by 2, this exercise we have done many times so I believe you can do this, now similarly take the same part from this case and draw the figure diagram of this, this is BE there is no unit load here because this load is acting on EC, so there will be no load here, same V E and M E at the internal forces apply equilibrium conditions and if we do that.

Then V E will be this and M E will be this ok, so what we have done so far it is still a static problem for a given position of the load, we determine what is the reactions and what are the shear force and bending moments at any particular location and for demonstration purpose we consider the midpoint but it can be done, similar exercise can be done for any intermediate points right.

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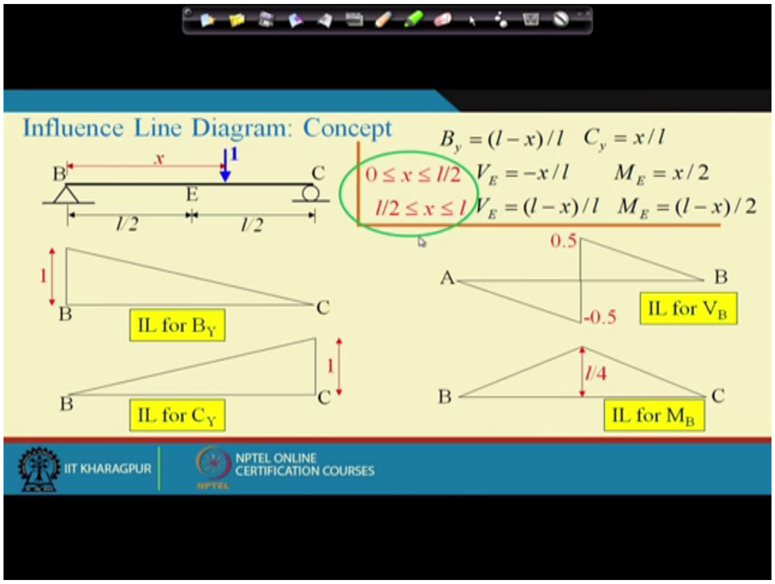

Now so what we have so far, we have these as the support reactions and these are the internal forces and (14:56shear force and moment) ok, this equation is valid between this part and this equation is valid between this part , you put a star mark here because we will see for shear force this equality is not applicable, because we'll see that shear is not continuous, we'll come to that point, so what this expression gives us, this expression gives us how shear force, pending moment at a particular location and the support reactions.

They change with change in location of this unit load ok, because x is the location of the unit load, now if we graphically represent these equations, say for instance first represent this reaction B Y, then what would be the expression, expression would be at X is equal to zero, this will be one and X is equal to L, this will be zero and in between B and C, it is linearly vary so this is linearly vary, so what this expression gives you, this graphical representation gives you.

They tell you how the reaction at B is influenced by the location of the semi slope ok, or in other way how the location of this concentrated load influences the reaction at P, for instance if we applied the load at P then what it says, x is equal to zero then what it says the reaction will be one, if we apply this load at C then reaction at B will be zero and if we apply the load any intermediate point will between B and C, the load on reaction B Y will be this ok.

This will be B Y ok, so one if we have this diagram then really we do not have to solve the problem again and again, what we have to do is ok, we know the location of the load, what is the value of y in this diagram and that odd net in this diagram will give us the reaction at BI, this graphical representation is called influence line diagram, so this is called influence line diagram for B Y, means it is a line diagram which tells you the influence of the unit load.

On support reaction at B, now similarly if we have to draw this influence line diagram for C Y now it will be like this at x is equal to zero, it will be zero and x is equal to L, this is one and in between it is linearly vary, similarly so at when the load is acting at C, reaction at C will be one, when the load is acting at B, reaction at C and for any intermediate position of the load, reaction at C will be this value, this is the influence line diagram for C Y.

Now let us draw the influence line diagram for shear force and bending moment, now this is the influence line diagram for shear force at this will be at E, so this gives you the influence line diagram so what is the X is equal to zero and influence is zero, shear V is zero and X is equal to L, this equation is not valid, we have to use this equation, we substitute this is again zero and in between this two value, between these A and B this is linearly vary and this is linearly vary.

And (19:25 this is 9 minus and L X positive), now remember just now I said this equality is not hold for shear force because at point E there is a certain change, it is discontinuous so for E this expression is only valid between zero to L by 2 excluding the point L by 2 and again this expression is valid between L by 2 to B excluding the point L by 2, so this equality this and this that is not valid for shear force ok.

Now again let us draw the, similarly if we have to find out what is the given location of the unit load what is the shear force, this value will give us the what is the shear force, if the load is here this is the shear force, if the load is here this will be the shear force ok, now similarly let’s find out what is the influence line for bending moment and at again please correct this will be E ok, so this is the influence line diagram for bending moment at E.

Now what it says that when the load is acting at B, bending moment is zero, load is acting at C bending moment is zero, and the load is acting at E then the bending moment is maximum which is L by 4 and for any other intermediate value the bending moment can be determined, so if these values will give us the bending moment for any particular location of the unit load, please please please note that this influence lines and the bending moment.

And shear force diagrams are different, this is not the bending moment diagram of BC, what it, it is the influence line of bending moment at E, it gives you how the bending moment at E changes with different location of the unit load ok, so this is influence line diagram ok, now as I said this equality is not valid for shear force, for bending moment it is fine because it is continuous,

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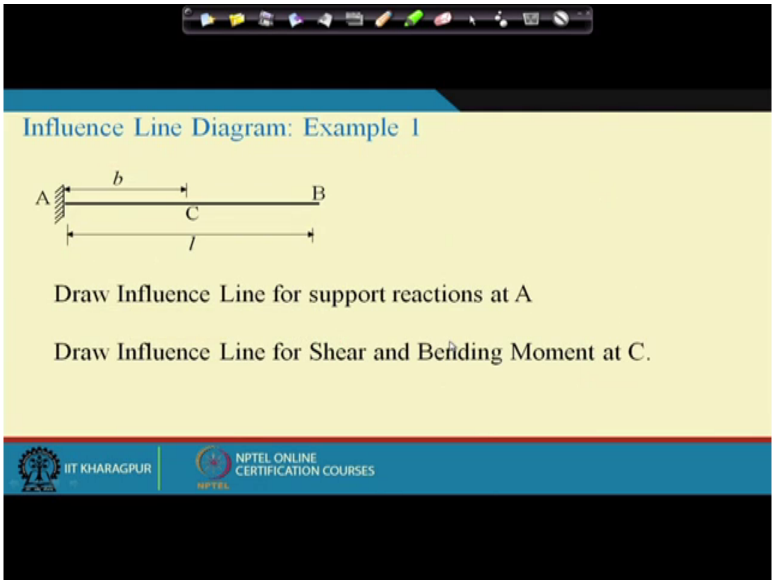

Now let us take one more example. Suppose this is the example where this is a cantilever beam.

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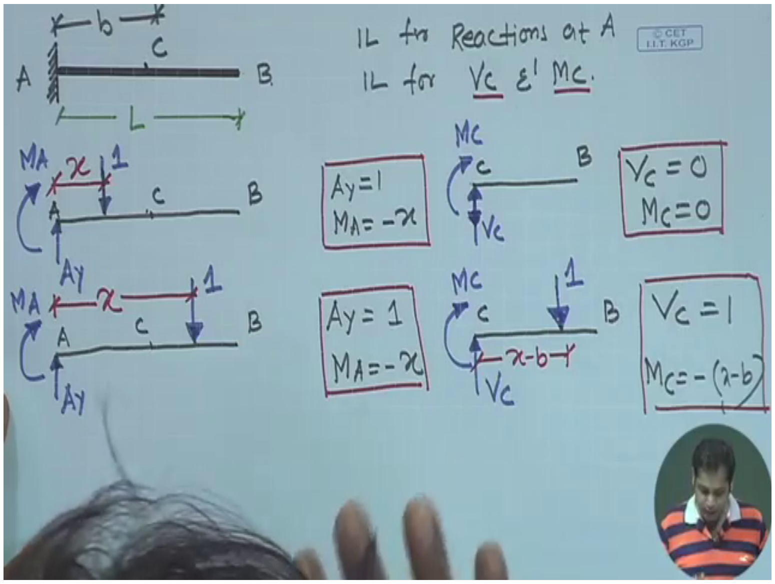

Suppose this is A, this is B and this length is L suppose I want there is a point C here which is at a distance B from A ok, a distance from A, now what we want to find out, draw the influence line for support reactions means reactions at A and also so influence line IL for reactions at A and IL for V C and M C, so this is the problem right, we need to find out what is the influence line diagram.

For the reactions and shear force and bending moment at any (23:28 other point C) which is located at a distance B from support A ok, now let us consider, now two situations, the first situation is where the unit load is between A and C, and the second case will be where the unit load between C and B, so suppose the unit load is some I here, the unit load which is at a distance x from A ok, now what are the forces we have so this is A.

This point is C and this point is B, now what are the support reactions we have, we have A Y and then we have M A ok, A X will be zero that's why because there is no horizontal load that's why it is not wrong explicitly, so this value of this unit load is one ok, not the value it is assumed that is value is one, so this is case one right, while the second case is when the unit load is between C and B, so take another figure diagram between C and B.

This is A, then C, B, support reactions are again same A Y and then bending moment is M A, and the unit load is acting, this is the unit load which is at a distance x ok, now in this case x is between A and C, and in the second case x is between C and B so this is valid for this portion and this is valid for this portion, now let us find out the expression for bending moment so expression for bending moment and shear force at C and the support reactions at A ok.

Now consider this figure diagram, just by looking at the figure diagram we can say that A Y is equal to one right, and again if we take moment from A we can say that M A is equal to minus x, so this is we can say right, now again if we apply summation of F Y vertical force is equal to zero, we can say that A Y is equal to one and similarly if we take moment at A we'll get M A is equal to, this exercise we have done many times, A is equal to minus x, right.

So this is the support reactions when the unit load is between A and C this is the support reaction when an unit load between C and B, now let's find out shear force and bending moment at C, now consider this case, draw the figure diagram of C B, if we draw the figure diagram of C B, what are the forces we have, we have shear force which is, sorry as per our sign convention it will be up, this is V C and this is M C ok and this is C, this is B ok.

Now there is no load acting on it, so automatically we can say that V C will be zero, V C is equal to zero and M C is equal to zero, so this is how shear force and bending moment they vary when the unit load is between A and C, now let's for this case draw the figure diagram, if we draw the figure diagram of BC once again, this is B, this is C and this is V C, shear force at C and this is M C, pending moment at C and then we have the unit load and this distance is x.

Suppose this distance is x so this will be x minus V ok, this distance will be x minus V ok, so if we apply the summation of forces, summation of forces in white reaction is equal to zero, we'll get V C is equal to one and then if we take moment about point C, we get M C is equal to minus x minus B ok, this we know how to, we have done it many times, so this is the expression for this, now let us draw the influence line diagram for a support.

So these expressions actually give us how these values, the internal forces and support reactions they change with x and x is the location of the unit load, so essentially we give us how these internal forces and support reaction they change with the location of the unit load, ok?

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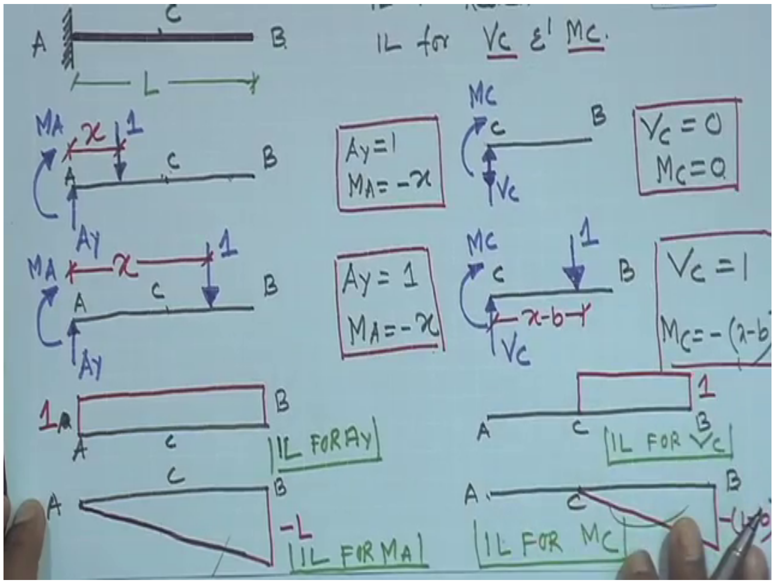

Let us first find out and draw the influence line diagram for support reactions ok, so this is A, this is B, now let us see support reaction A Y is equal to one between.

Take C. A Y is equal to one between A and C, and between C and B still A Y is equal to one, so the influence line will be this and this value is one, so this is influence line for, this is I L for A Y ok, now draw the influence line for moment, now the influence line for moment will be at it is minus x when A to C and when C to B still it is minus x, so x is equal to zero, it is zero, x is equal to L, it becomes minus L, so and in between it varies linearly.

Let us use different color and this value is minus L, this is A B C, so this is influence line for IL for M A, influence line for moment A ok, now what it says that you applied load anywhere in between and to get the support reactions you just get the corresponding alternate from this diagram, now let us draw the influence line diagram for moment and shear, now this is the expression for moment and shear, now let us draw for shear.

So this is again A, this is C, this is B, what it tells BC is equal to zero between A and C, so between A and C it is zero and between C and B, BC is equal to one, so this is the influence line for BC so this value is one, this is influence line for I L for V C ok, now draw the influence line for bending moment or M C, what it say this is A, this is C, this is B, M C is equal to zero between A and C, so it is zero between C and B it varies with this.

So at zero here and x is equal to L, it becomes minus L minus B and in between it varies linearly and this is influence line for M C ok, so these four diagrams are the influence lines, these are for reactions, these are for internal forces and similar exercise can be done for any intermediate points ok, now what is the use of influence line diagram as I said, now once you have this diagram it is essentially graphical representation of these equations.

So instead of having these equations separately because these equations are not, for one part another equation, for another part another equation, so instead of having these equations what we have is, if we have this representation then from this representation we can find out what is the internal forces and support reactions for a particular location of the influence line, influence line diagram gives you some more information that will understand as we proceed ok.

We stop it today, next class we'll have, we'll do few more example on influence line diagram, Thank you.