Course on Structural Analysis 1
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Lecture 26
Module 5
Deflection of Beams and Frames (Continued)

Hello everyone, this is a last lecture of this week what we have been doing is we are exploring different methods to find out deflection in beams and frames statically determinate beams and frames. What we will do today is today we will see how the principle of virtual force can be used to find out deflection at any particular point in beams and frame this method is also known as unit load method if you remember we have already discussed the method of virtual work and its application to determination for determination of deflection for thrust we have already discussed that.

So we are not going to discuss the principle of virtual once again here, we will just quickly review means one slide to quickly review what is the method what are the different steps and then see some examples, okay.

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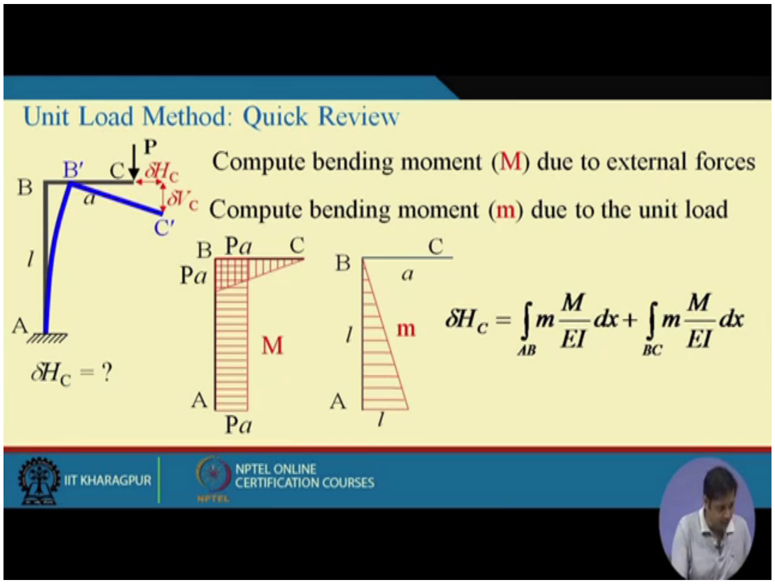

So let us quickly review what is unit load method suppose we have a frame here if you remember we already obtained the bending moment and shear force for this frame and we have also seen the deflected shape for this frame in the last class, okay.

So the this is the deflected shape so B point B comes to B dash and point C comes to C dash so let us see what is the deflection at point C, so this delta AC comes to C dash so delta AC is the horizontal deflection at C and delta VC is the vertical displacement at C. So suppose we want to find out what is the horizontal distance for at point C, okay. Now what are the different steps? Similar steps can be obtained for vertical displacement as well or for that matter for displacement at any point.

The first step is compute the bending moment due to the external forces in this case external force is vertical load P at point C and if you remember we already obtained the bending moment diagram for this beam and this was the bending moment diagrams, so this is M so first we need to find out what is the bending moment at any section due to the external load, so this is the first step and this is the bending moment diagram.

Then what is the next step? Next step is apply unit load or moment in the direction of desire deflection or rotation means now take the same beam same structure same boundary condition now at point C we want to find out the deflection so at point C apply unit load. And what would be the direction of this unit load? Direction of the unit load will be the direction in which we want to determine the deflection. Now if it is deflection then a unit load needs to be applied, but if it is rotation then unit moment has to be applied, okay.

Now once we have this, then next is compute the bending moment due to the unit load, now this is the structure same structure subjected to unit load at point C horizontal load at C compute the bending moment at any given location for the structure. Suppose this is small m this that bending moment is denoted by small m, so what is the bending moment? For this the bending moment diagram will be this so it is for segment BC bending moment is 0 and for segment Bl it linearly varies at point l the bending moment is l which is the length of AB.

Now so this is the bending moment for unit load, so we have two bending moments one is capital M one small m this is for the externally applied load and this is for the unit load applied in the direction in which we want to determine the deflection or rotation. Now once we have this M and small m and then the expression for horizontal displacement for this problem will be small m into capital M by EI, EI is the property of this E is the (())(4:23) I is the second moment of area and now since in this frame we have two component, one is AB and BC so that integration has to be done on actually on the entire structure but that integration can be divided into several parts means that integration can be done over each member and then sum them.

So this is very brief review of the unit load method we have already discussed detail of this unit load method, okay. Now let us see some examples, before we do that let us find out let us continue with this example and find out what is the horizontal displacement for this, okay.

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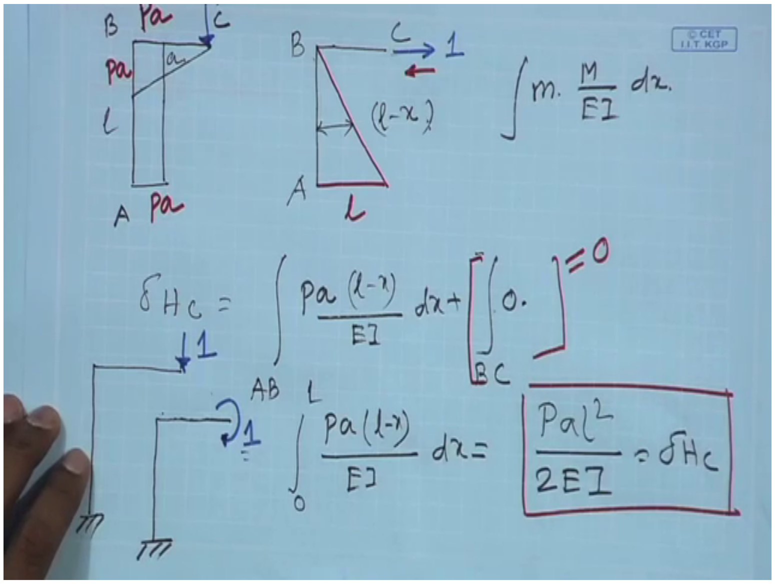

So the problem was this, okay this is A, B and then C so this length is l this length is a so and then delta AC will be and then for unit load this is the bending moment diagram for actual load is this this is this value is Pa, this is Pa, this is also Pa, okay and for unit load this is the bending moment diagram this distance is l this is A, B, C, okay.

Now what will be delta AC? Then delta AC will be first integration over AB integration over AB and that will be we know the this is small m into capital M by EI into dx integration over the entire structure, okay. Now this integration has to be done over AB and then BC and then sum them. So AB means it is small m is this the at any location this distance is l minus x, okay. Now x varies from 0 to l, okay and then this continue this is constant Pa so this becomes Pa into l minus x this divided by EI this dx plus integration over BC, now PC this part is 0 there is no bending moment due to the unit load on BC so this part becomes 0 into and this part so this part anyway becomes 0.

So this part becomes 0, okay. So this is 0 to l and then integrate it Pa l minus x EI dx and this become Pa l square by 2 EI. So this is delta AC so this is the horizontal displacement at C. Now if we have to find out vertical displacement the same thing this remains same this capital M remains same, only thing is instead of applying unit load in (horiza) in this case the unit load is was applied like this and this is the actual load and this is P in this case unit load was applied because we wanted to find out horizontal displacement.

Now if we have to find out what is the vertical displacement, then unit load needs to be applied like this, okay. And if we want to find out what is the rotation here, then instead of unit load it will be unit moment, okay. Now if we get this value positive means the deflection is in the direction of unit load, but if we get you can apply unit load in these direction as well if we want you can apply unit load in this direction as well but in there if you do that, then instead of 1 it will be minus 1 and then this deflection you will get negative.

So negative means your deflection is in the opposite direction of the unit load, similarly for rotation if you get the rotation is negative then your deflection rotation will be in opposite direction of the applied moment things will be clear if we do some more example.

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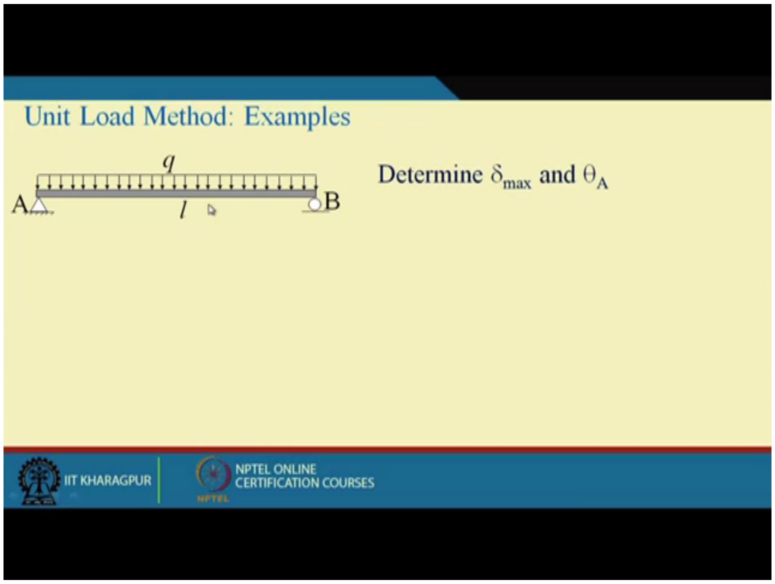

Now next example is a simply supported beam which is subjected to uniformly distributed load and we want to find out what is the delta max and theta A means what is the rotation at theta rotation at A and what will be delta max since it is symmetric we know the delta max will be at the (())(10:10) so essentially in this problem what we have to find out what is the rotation at A and what is the deflection transverse deflection at the (())(10:19).

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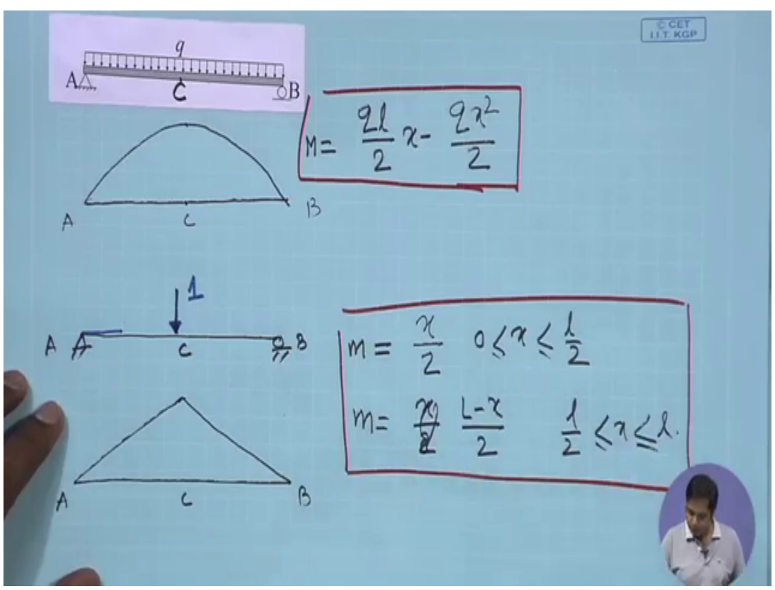

So this is the problem, okay now we know that for this problem bending moment diagram is this, right? This is A, B, C and the expression for bending moment we know that M which is a function of x is equal to ql by 2 x minus q x square by 2 we have done it many times and at the middle at this point your bending moment will be wl square by 8.

So this is the this is capital M, right? Now we need to find out what is small m. Now first let us find out small m for transverse deflection at C, so what would be the problem we need to consider same beam same support condition at point C we want to find out the vertical displacement so we need to apply an unit load at point C this is A, this is B and this is C, okay. Now we know that for this problem we know what will be the bending moment diagram, bending moment diagram will be like this, right?

We have already done it A, B and C and these values will be the bending moment values will be this that small m will be it small m it is unit load so it is half half will be the reaction half will be the reaction here it will be x by 2 x between x is between 0 to l by 2 and m is x by 2 minus or we can write l minus x by 2 the final expression where x is between l by 2 to x to l, okay. So this equality also valid so this is the bending moment for the unit load, okay. Now what we need to do?

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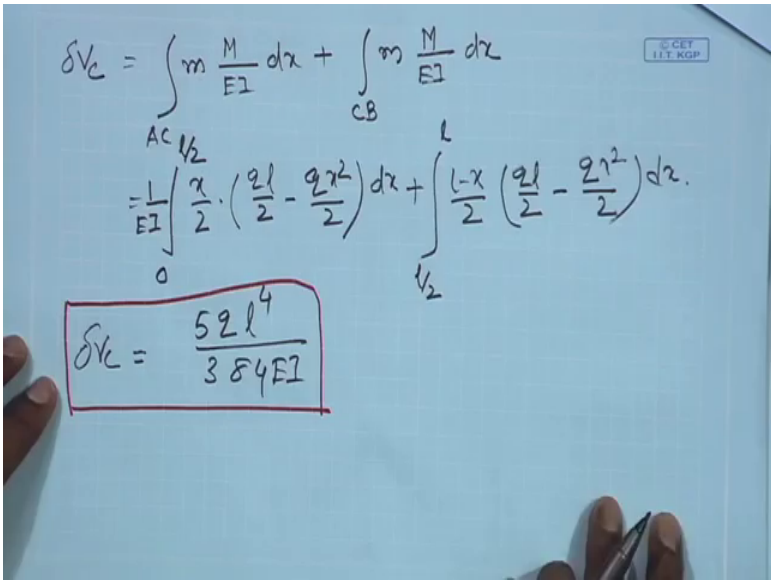

We need to find out the vertical reaction so vertical reaction will be vertical reaction we know that this will be delta say Vc will be integration of we need integrate over AC and then CB though the expression for M is same for the entire part but the expression for small m are different for AC and CB.

So this will be AC small m capital M by EI dx plus integration of CB small m capital M by EI dx, right? Now write the expression for small m and capital M expression will be this is x by 2 into this will be ql by 2 minus qx square by 2 dx you can take 1 by EI outside because it is constant and this becomes 0 to l by 2 plus integration of l by 2 to l and this become l minus x by 2 small m is l minus x and capital M remains same this is ql by 2 minus qx square by 2 and then dx, okay.

So now if we do the integration then this becomes we will get 5 ql to the power 4 by 384 EI if you remember this is the value we obtained we have we determined this deflection using direct integration method using conjugate beam theory, using moment area method and so you can verify this results with the results you obtained through other methods, okay this was for vertical displacement, now we need to find out rotation at theta, rotation at A so for vertical displacement we applied the unit load but now for rotation we need apply unit moment, right?

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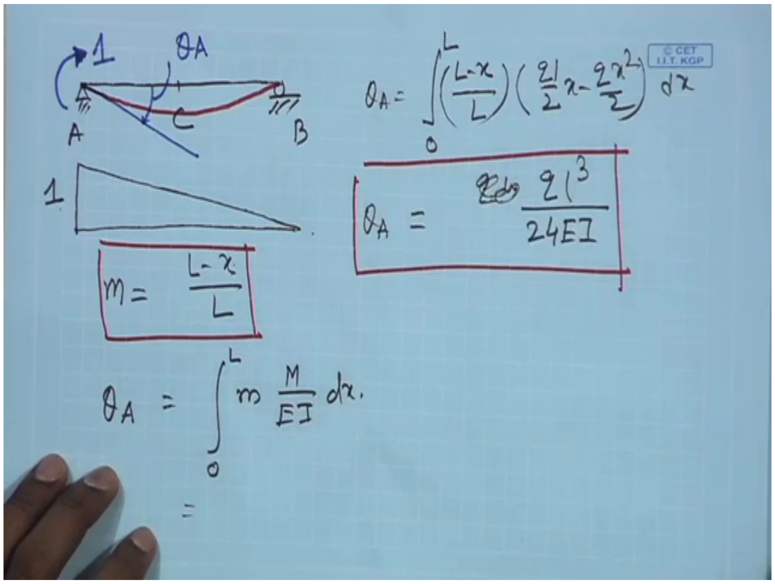

So let us do that, take the same beam A, B midpoint C now we apply unit rotation like this, okay. Now what would be the bending moment diagram for this? The bending moment diagram for this will be you check this, this is the bending moment diagram and what would be the expression for bending moment diagram? Expression for bending moment diagram will be expression for capital M will remain same because it is due to the actual load, now only thing is expression for small m will change and the expression for small m will be you check this l minus x by l and this is true for, okay. Now this is the expression for you can check when x is equal to 0 this become 1 when x is equal to l this become 0. So this is the expression for small m, okay.

Now what will be theta A then? Theta A is integration of 0 to l small m capital M by EI into dx now this becomes let us say this become theta A is equal to small m is l minus x by l and then capital x capital M is this ql by 2 x minus qx square by 2 ql by 2 x minus qx square by 2 dx and this is 0 to l this integration and then theta A becomes you can try this w ql cube by 24 EI so this is theta A, okay. Now since delta c is positive means it is the deflection is in the direction of applied unit load.

Now direction of applied unit load is downward means your deflection is downward. Now similarly theta A is positive, if theta A is positive then it means that your (def) your rotation at theta A is in the direction of applied unit moment. Now this is the unit moment is applied in this direction so your rotation will be and if we know that for the applied the deflection of the beam will be like this and if the deflection is like this, then this will be theta A this will be theta A and as per our sign convention this is positive. So theta A is positive, okay.

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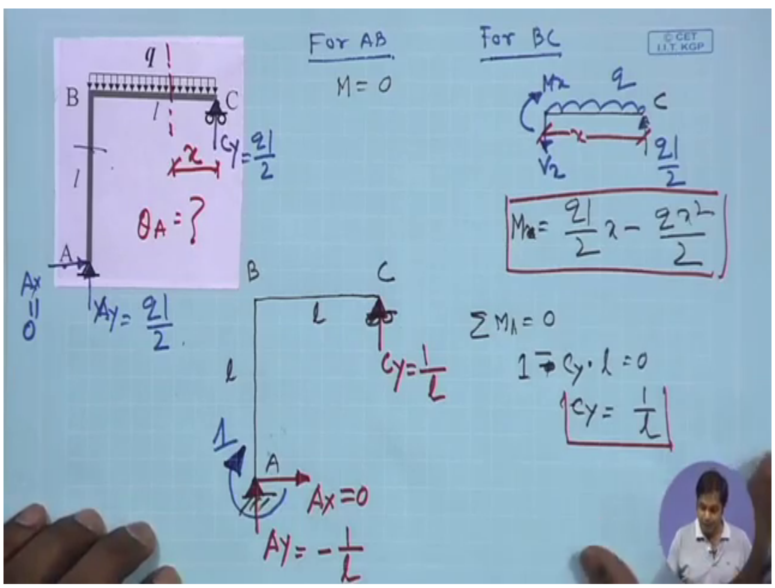

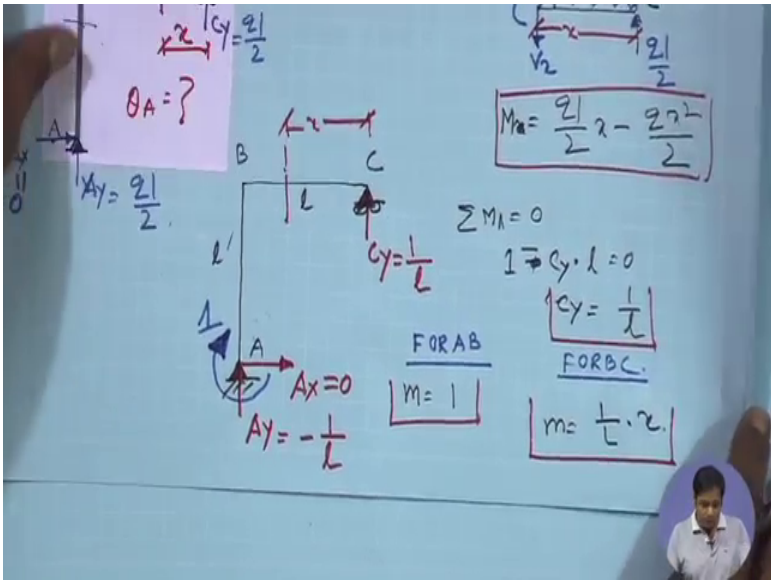

So that is how we can determine the deflection and rotation at any particular point. Let us do one more example, so this is a frame now again it has two member AB and BC member BC is subject to uniformly distributed load both the members are having same length and see the support condition this is hinge here this is roller here so this is a statically determinate structure, what we need to find out? We need to find out what is theta A means what is the rotation at point theta A, okay.

Now so let see this problem, the problem is this, okay. Now since we need to find out, what we need to find out? We need to find out theta A, now first let us find out what is the bending moment for this, we have two point one is AB and BC for AB m will be you check that m is 0 and which is obvious because okay because if you take any point here there will no horizontal load here because this is roller support therefore there is no horizontal reaction at A, only reaction will be vertical reaction let us draw that, if this is the if this is Ay and this is Ax and this is Cy, Ax will be 0 because there is no horizontal other horizontal load here there is no horizontal constraint also.

Now Cy will be ql by 2 and Ay will be ql by 2, okay. Now if we take any section here, then at this section Ax is equal to 0 if we take moment here Ay will not contribute because Ay is passing through that point Ax is anyway 0 so the moment will be 0 so between A and B the moment is 0, okay. Now, and what is the moment between for BC, for BC is if we take any section say if we take any section at a distance x same measured from C and draw the free body diagram of this,

this is ql by 2, this is q and then this is Vx and this is Mx and this distance is x. So apply a equilibrium condition we get Mx is equal to or M is equal to since we are using M here, M will be is equal to ql by 2 x minus q x square by 2 please note in this case x is measured from C this point is C, okay x is measured from C. So for real structure for the real actual load this is the bending moment bending moment is 0 between AB and bending moment is expression is this between B and C.

What we need to find out? We need to find out rotation at A so we need to apply a unit moment at A, so let us apply so this is the beam this was the support condition this is roller support and then this is hinge support this is A, B, C we need to apply an unit say this is the unit moment over here, okay. Now you can as I said you can you can give the unit moment in other direction as well, then in that case you will get rotation is negative, okay. Whatever value of rotation you get with sign that tells you what would be the direction of that rotation, okay.

Now let us find out what is the bending moment for what is the bending moment for this, this length is l, this length is l. Now for this what would be the reaction? This will be Cy, this will be Ay and this will be Ax, Ax is anyway 0 because there is no horizontal load and then if you take moment about A and then if you take moment about A that is equal to 0, so this is clock wise 1 and then anti clock plus Cy minus Cy into l is equal to 0 this gives you Cy is equal to 1 by l, okay Cy is equal to 1 by l, okay.

Now then the Ay will be minus 1 by l because it has to satisfy equilibrium condition, okay. Now find out what is the moment for BC and what is the moment for AB, for AB you take for AB if you take any section here and Ax will not contribute because Ax is 0, A will not contribute because Ay is passing through this section only moment will contribute the moment at this section and the externally applied moment unit load unit moment here. So what you get is please check small m is equal to 1 for AB.

And similarly for BC what you get is m is equal to small m is equal to you take a section here you take a section here at a distance x from C draw the free body diagram of this section and then we get m small m is equal to x by 1 by l is the Cy load into x, so x by l. So this is My and this is m for AB and this is m for BC. Now then what would be the rotation?

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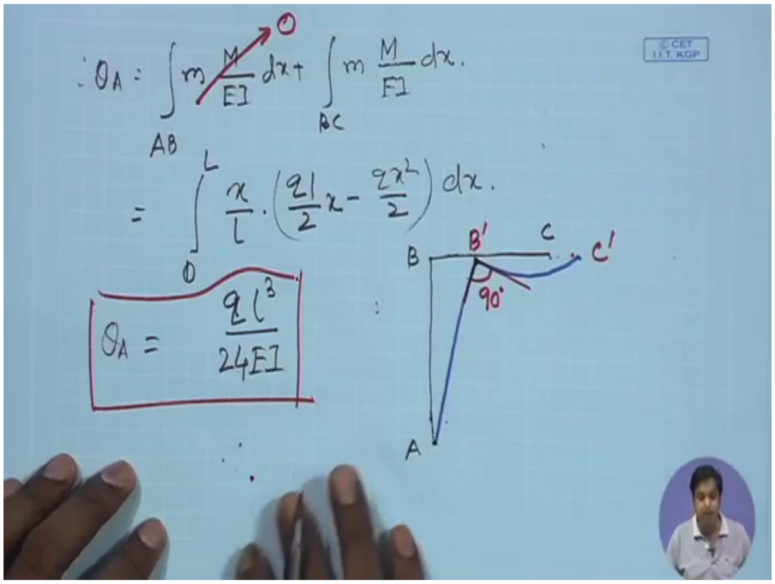

Rotation will be theta A is integration over AB small m capital M by EI dx plus again small m capital M by EI dx, okay. And then small m is now for AB capital M is 0, small m is 1 so for contribution from AB will be 0 and an only non 0 contribution will be from BC where capital M is this and small m is this, okay.

Now if we so this becomes this contribution is 0, so this become integration of integration over BC and BC is means 0 to l this length is 0 to l m is x by l into capital M is ql by 2 x minus q x square by 2 dx, okay. Now if you do this integration what you get is this is ql cube by 24 EI, so this is theta A. Now theta A is positive it means that it will be your rotation will be in the direction of applied unit load so rotation will be like this, okay.

Now just let us find out what could be the probable deflected shape for this structure? It is something like this and this point is hinge, this point is roller so this point can move in this direction and since this point is hinge so your and also we obtained that theta is positive means the rotation will be in this direction at least that is the applied unit load so it will be continuous like this and probably something like this, okay and this is this angle remained 90 degree because this joint is rigid joint, okay.

And this will sag because of this uniformly distributed load, this uniformly distributed load will cause sagging in this beam, but if you look at the similar problem, but when the load is concentrated load this was the deflected shape, so this is almost a straight line, okay it is not sagging like this, okay. But in this case it will sag because it is supported here this beam BC is supported at C and supported at B and it is subjected to uniformly distributed so. So this could be the probable I am saying because you need to find out actual deflected shape based on the information that what we have, we can say that that could be the deflected shape of this structure but that you have to verify.

So there are many examples many problems like this can be we can find out what is the deflection and rotation at any particular point using method of virtual force or unit load method it is just few examples I showed you here if you take any book there are many examples many such examples for different kinds of beams, beams with internal hinges then different kinds of frames, different support conditions again frame with internal hinges please go through those examples go through exercise problems and then probably the you can better appreciate the method, okay.

With this thank you very much, next week what we do is next week we will start influence line diagram and moving loads you see as of now whatever problem we have addressed so far we assumed that the load is acting at a point and the load is also a static condition it is fixed at a point, but in many particular cases load is not fixed, load is moving load, load moves on the structure, okay now so as the load moves on the structure the internal forces support reaction in the structure continuously changes depending on the location of the load.

So we need to find out what could be the at a given location of the load, what was the internal forces and support reaction or we need to find out what could be the critical location of the load where the internal force and support reactions are maximum we will discuss all this things in the next class influence line diagram and moving loads, okay thank you.