Course on Structural Analysis 1
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Lecture 26
Module 5
Deflection of Beams and Frames (Continued)

Hello, this is lecture 26 and what we will do here is in the last lecture we saw the concept of frame and then just demonstrated in one example how the bending moment and shear force and axial force diagram for a frame can be drawn. This is a very short lecture where I quickly show you two examples not go through the entire process of arriving the bending moment and shear force not doing the entire calculations but we show you the examples and what are the final diagrams and how they are deflected ships are.

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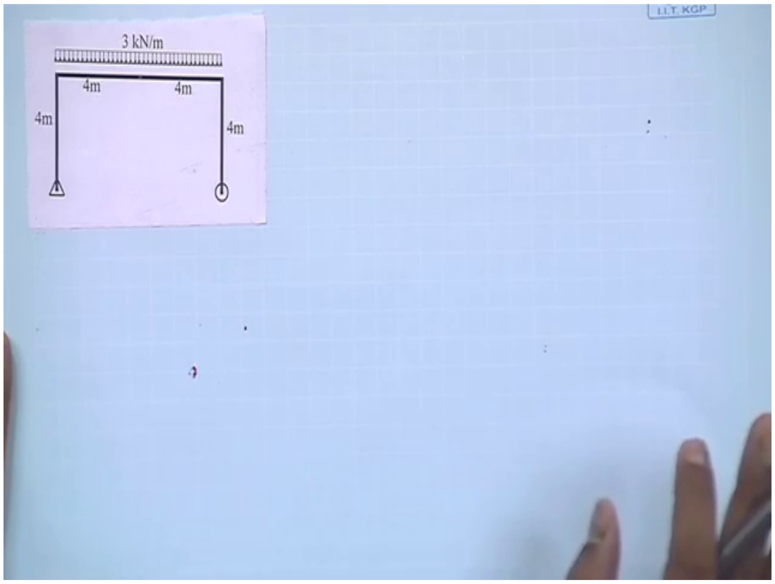

The first example is it is again a portal frame, now there is one important part is here, we have one internal hinge here this a hinge here, okay. So this is a frame subjected to uniformly distributed load three kilo Newton kilo per meter 4 meter, 4 meter, 4 meter, 4 meter and in addition to that there is an internal hinge, so this is A, this is B, this is C, D and E there is an internal hinge at C. What you need to find out, we need to draw the bending moment and shear force diagram for this.

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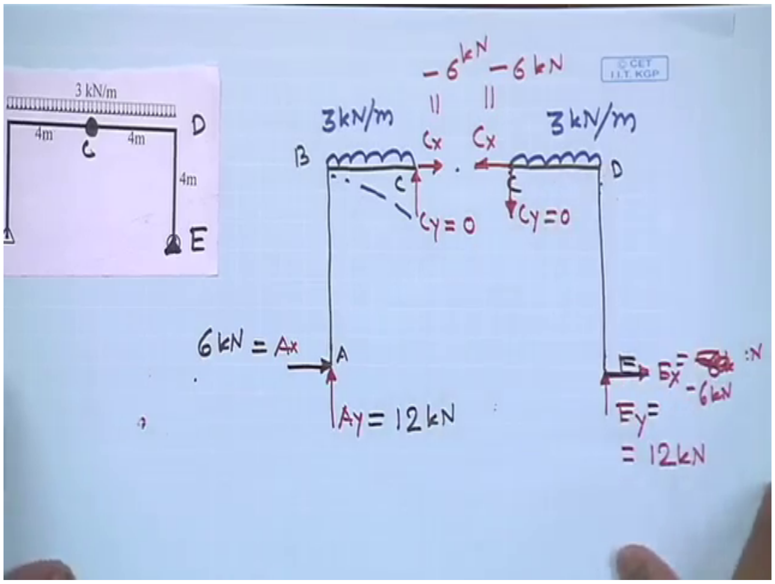

Now you if you break the entire part into two entire structure into two part and draw the their free body diagrams then free body diagram that will be something like this, so this is hinge support this will be Ax let us use different pen Ax and then this will be Ay support reaction here is Ey then we have internally externally applied load this is 3 kilo Newton per meter, okay. Now the bending moment draw the other, now this is A, B, C this is A, B, C, C, D, E. okay.

Now so this is suppose this is hinge so these there is no moment here at this point, so suppose this is Cx and this is Cy and when we draw when we show Cx and Cy at C it will C belongs to other part it will in opposite direction, so this will be Cx and this will be Cy, this is the complete free body diagram. Now if apply equilibrium equations then what we have is Ay is equal to 12 kilo Newton Ax is equal to 6 kilo Newton and Ey is equal to this is again this is also hinge there is this is also hinge so there will be Ex as well, Ex is equal to minus 6 kilo Newton, okay and minus 6 kilo Newton Ey is equal to 12 kilo Newton, okay.

And Cy is equal to 0, Cy is equal to 0, Cx is equal to minus 6 and Cx is equal to minus 6, okay minus 6 kilo Newton, okay. Now this is the complete support reaction and hinge reaction that we can we know how to determine it using the equilibrium concept or equilibrium. Now let us draw quickly what is bending moment and shear force diagram, okay. Now you see again let us draw the bending moment and shear force diagram by looking at the frame itself without taking a section and then see and then representing (())(4:55) moment and shear through a function.

Now between this, this is a symmetric part and so and this point at point C bending moment will be 0. Now and between at point C bending moment is 0 and since between C and B it is uniformly distributed load we know the bending moment varies quadratically second order and how they what would be the bending moment diagram hogging and sagging this part if we say this is C is equal to 0 so this part will cause hogging in the beam, right? Hogging in the beam, so this will be negative bending moment same as here.

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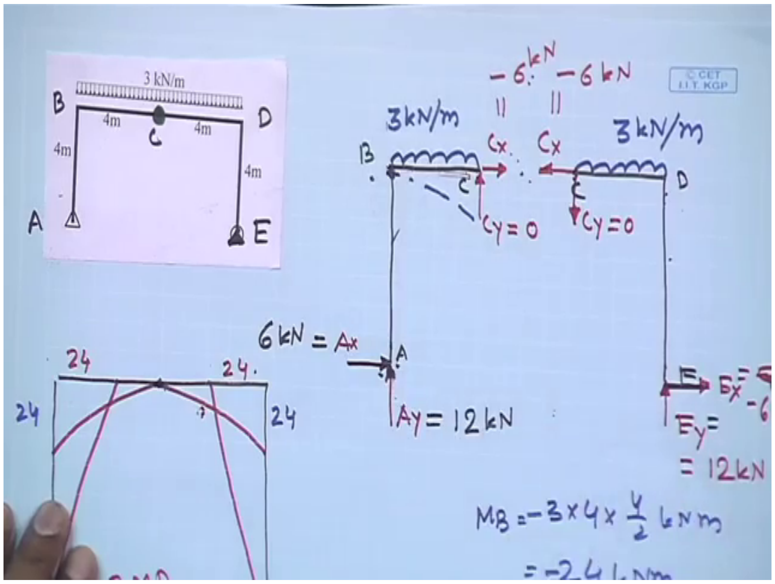

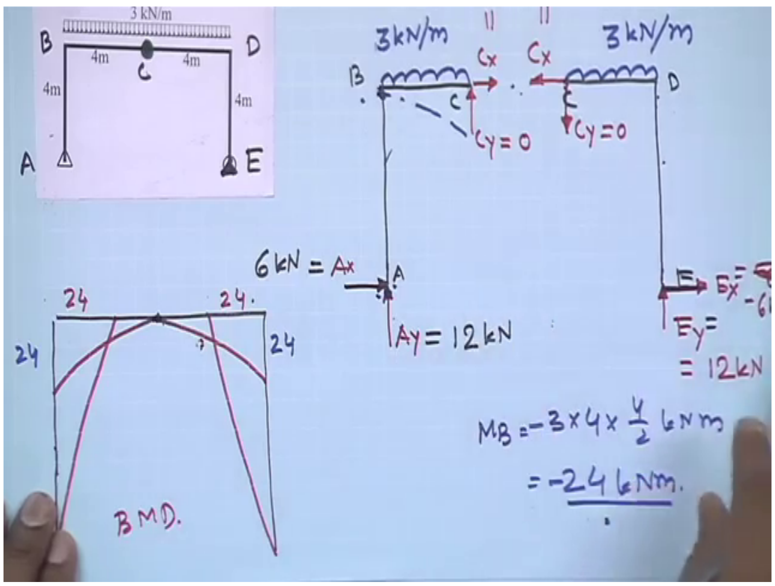

So let us draw the bending moment diagram here, so this part this is part C where bending moment is 0, okay. Now between C to this as I as it can be seen that it is a hogging moment and they varies quarditically and this will be the bending moment between part C and B, similarly it is symmetric so bending moment between other part will be like this, and what will be the this value? This value will be the if you take this value will be the moment at B because of this entire loading on this entire length and this value will be MB, MB will be if you take this length is 4 meter so 3 into 4 into 4 by 2 so this become 24 kilo Newton meter, okay 24 kilo Newton meter.

So this is 24 kilo Newton meter so this value will be 24, this value will be 24 now at B if we draw the free body diagram part AB and apply the equilibrium condition we will see that MB is equal to 24 kilo Newton (())(7:15) we have obtained minus 24 kilo Newton we already obtained and between this to this there is no external load, so at this point bending moment is minus 24 and at point A bending moment 0 and between B to A bending moment varies linearly.

So the bending moment between this to this will be varies linearly again as per our sign convention we are bending moment are drawn bending moment are drawn on the compressions site, okay. Now so this is bending moment diagram, now if I have to draw the shear force diagram again find out the shear force, shear force will be maximum here this will behave as it simply like simply supported beam this shear force will be maximum here, then since bending moment is quadratic shear force will be linearly varying and then shear force become this.

Now and what would be shear force here? The shear force here will be the axial force in this member, what will be the what is the axial force Cx is equal to minus 6, Cx is equal to minus 6 so axial force in this segment will be the shear force for B.

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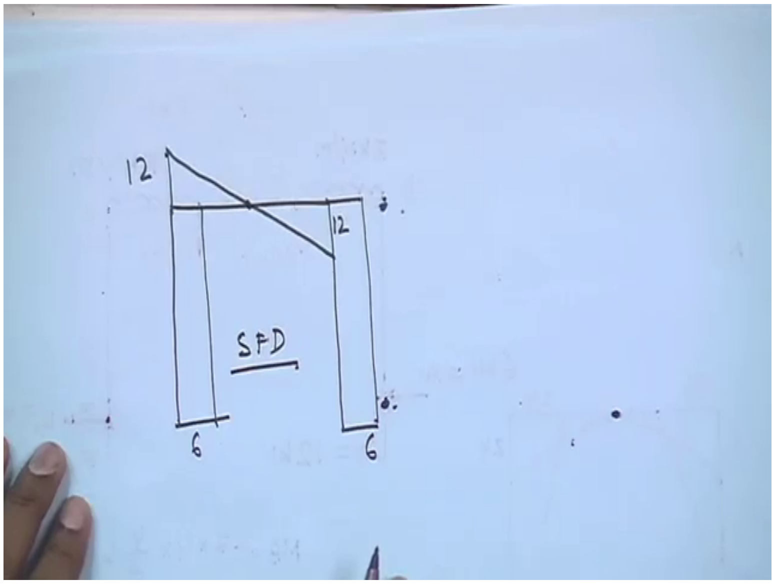

So the shear force for AB will be minus 6 so the shear force diagram finally becomes again this is internal hinge this becomes and this becomes borrow this become 12 this 6 this 6 this is shear force diagram, okay.

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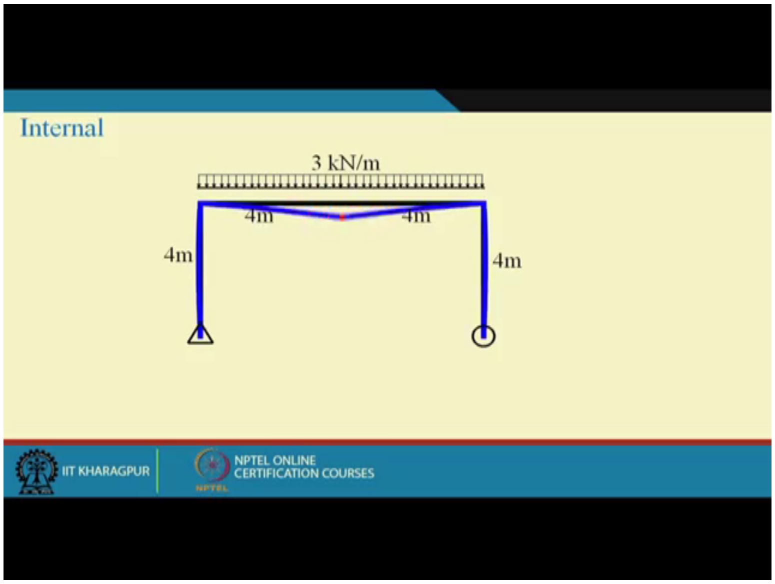

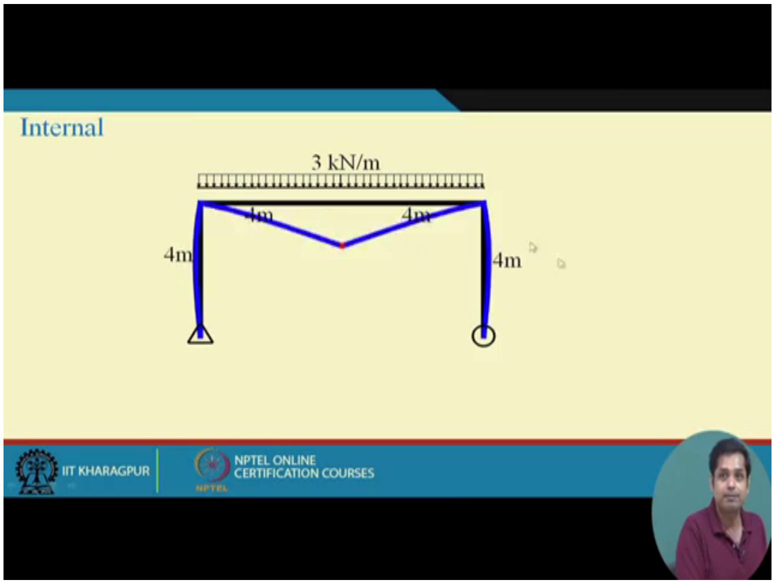

Now you can take the section and draw the free body diagram and gave the expression and draw the and draw the free body diagram you can do that, by here just to show how the bending moment and shear force may look like for this member just quickly see the deflected shape of this, the deflected shape of this will be this, okay.

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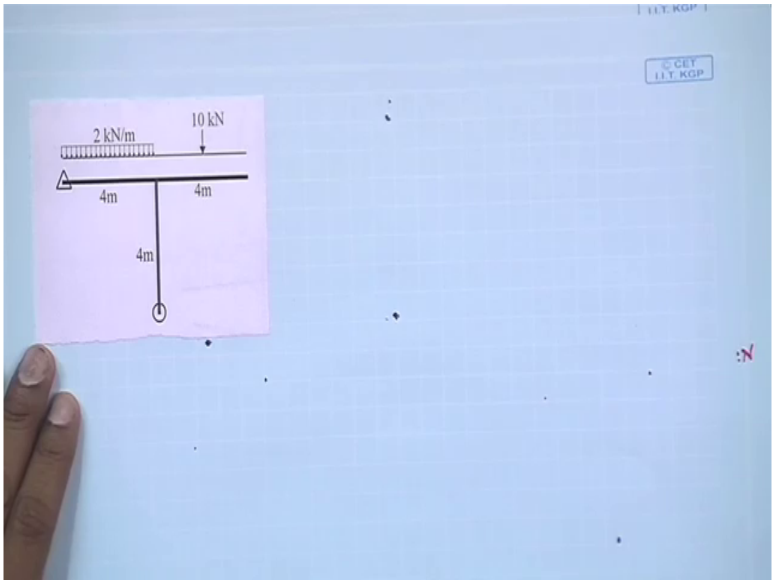

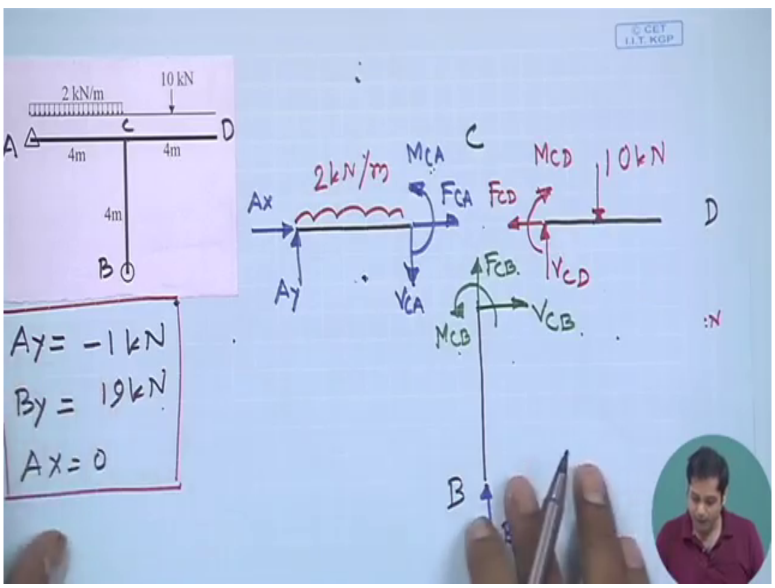

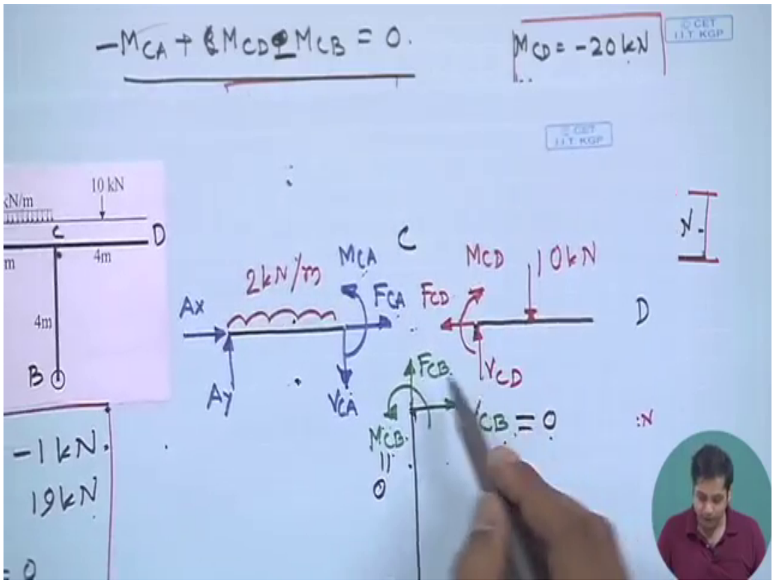

Now just quickly one last example this is the last example we need to draw the bending moment and shear force diagram for this.

Now remember in the previous class I showed you at the same joint you can have several members not only two we can have 2 members, 3 members this is a one example where you have 3 members, this is joint A, this is B, this is C and this is D though idea is draw the bending moment and shear force diagram. Now let us first divided the entire take all the members separately and draw there free body diagram. Now this is member AB, this is member say member CD and say this is member CB, and then what are the forces we have here? This is we have AX and then AY this is then we have BY, okay.

And then externally applied load say this on this this is externally applied load here this is 10 kilo Newton anything else all the externally load are shown now let us draw the internal forces now internal forces are now this is A, this is C, this is D and this is B, okay. Now the member C belongs to this part, this part and both this part, okay. Now let us this force is FCD means it is elongational force in members CD this force is VCD and this MCD, okay this is the free body diagram of this and then suppose for this case this is FCA and then VCA and then moment MCA and if we draw for this part, this part will be VCB and then FCB and then moment MCB, okay.

So these are the free body diagram or the same joint but when belongs to different part, okay. Now why it is important? First is you can if you can draw the free body diagram of the entire structure and calculate the support reactions, right? That can be done if we calculate the support reactions then the support reactions are like this AY is equal to minus 1 and BY is equal to 19 and AX is equal to 0. So this you know how to determine just by applying the free body drawing the free body diagram the entire structure and apply the equilibrium condition.

So now look at this, now we need to find out what is MCD. FCD, VCD and all these internal forces, but what we know you see there is no external there is no externally applied moment here, so what happens is the summation of MCA, so what is the total moment at C? Total moment at C is MCA, MCD and MCB there sum.

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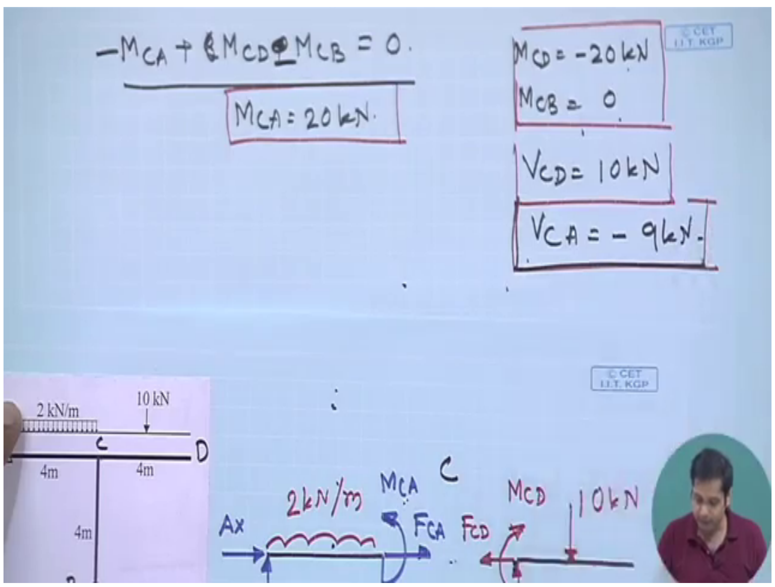

So there sum should be equal to 0 so what we have is MCA plus MCD plus MCB they all equal to 0 that is the equilibrium condition for point C, okay.

Now let us draw the free body diagram of this part if we take moment from here then we get MCD is equal to minus 20 kilo Newton MCD is equal to minus 20 kilo Newton take moment from here, this force will contribute this distance is 20 so it should be minus 20 kilo Newton, okay. Now you take free body diagram of this point calculate the apply the equilibrium condition with summation of moment at this point these two force will not contribute, this force will not contribute only force will contribute MCB and then MCB is equal to 0.

So if MCB is equal to 0 and MCD is equal to minus 20 you need to maintain the support maintain the equilibrium condition here and what equilibrium condition says is the algebraic sum is 0. So this are these are not all plus MCA is anti-clock wise so this is negative MCD is clock wise positive and MCB is anti-clock wise so this is negative. So this has to be equal to 0 we know MCD is equal to minus 20 MCB is equal to 0 this gives us MCA is equal to 20 kilo Newton, okay.

Now similarly you can calculate from this free body diagram, from this free body diagram if you see there is no horizontal load so VCB will be equal to 0, okay MCB is equal to 0. Now if you take these free body diagram and then apply summation of FY is equal to 0 so only forces are VCD and 10 so VCD will be 10 kilo Newton so VCD is 10 kilo Newton, okay. And let us find out VCA, now in if we draw the free body diagram of this part and leaves the free body diagram of this part apply summation of forces are is equal to 0 AY plus minus VCA and this force will contribute this force is known AY is already known and if you do that then VCA will be I am not explicitly calculating it but how to calculate I am just telling you that please verify (())(16:29).

So all these forces now we have determined, okay all these internal forces we have now determined. One other important point see FCD is the axial force for this member, FCA is the axial force for this member but the same FCB will be in the same, yes but in this direction your shear force is VCB, okay. Now similarly the way this moment at point the equilibrium condition is satisfied through moment similar equilibrium condition can be satisfied through these forces as well what is says that summation of all horizontal forces at point C will be 0, what are the horizontal forces you have? You have FCA, FCD and VCB, now from this free body diagram we have CD is equal to 0, VCB is equal to 0 naturally FCA is equal to 0.

Another free body diagram another equilibrium condition is all the forces which is vertical direction are 0, vertical direction 0 forces are VCD, VCA and FCA we have already (())(17:33) VCD, VCA and then FCB will be we can calculate what is the calculate of FCB.

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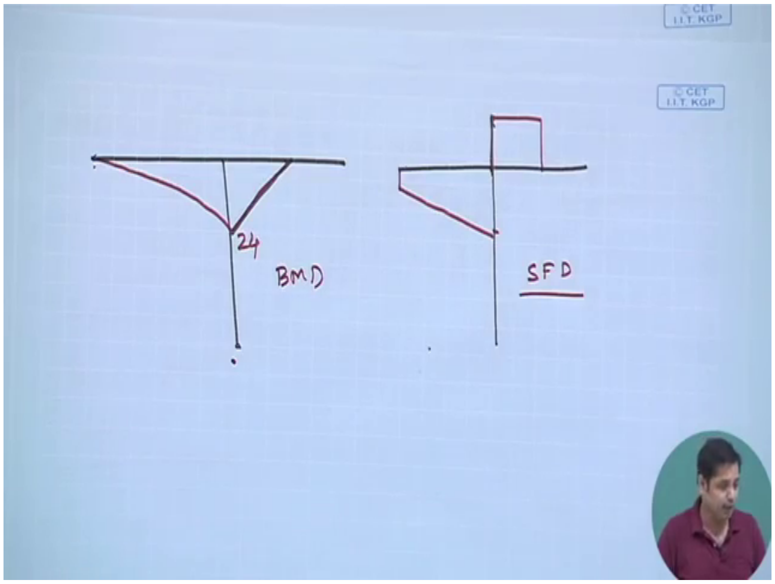

Now from this if we have to draw the bending moment diagram, okay just draw this so bending moment will be 0 here, no not this I need another paper draw the bending moment diagram so bending moment will be 0 here at this point so between this point to this point bending moment will be 0 between this point to this point bending moment linearly varies and the bending moment will be hogging moment so it will be hogging moment.

And then hogging moment and for this there will be no bending moment and for this bending moment is 0 here and between this point to this point bending moment varies quarditically and again it will be for this load actually this is hogging moment, okay. So this will be your bending moment diagram this is bending moment diagram and this is bending moment diagram this is 24 and similarly if I have to draw the shear force diagram, shear force diagram will be you check yourself at this point between this point to this point shear force is 0 and between this point to this point shear force is constant and that shear force will be 10 kilo Newton.

So shear force diagram will be here it is 10 kilo Newton at this point your shear force will be support reaction which is minus 1 at this point your shear force will be VC which is minus 9 that we have already calculated shear force will be this shear force diagram is this, so this is shear force diagram, okay. Now like this I have not done all the calculation I have the calculation with me I am just going showing you the final result please verify this results and then see whether you are getting this similar shear force diagram or not.

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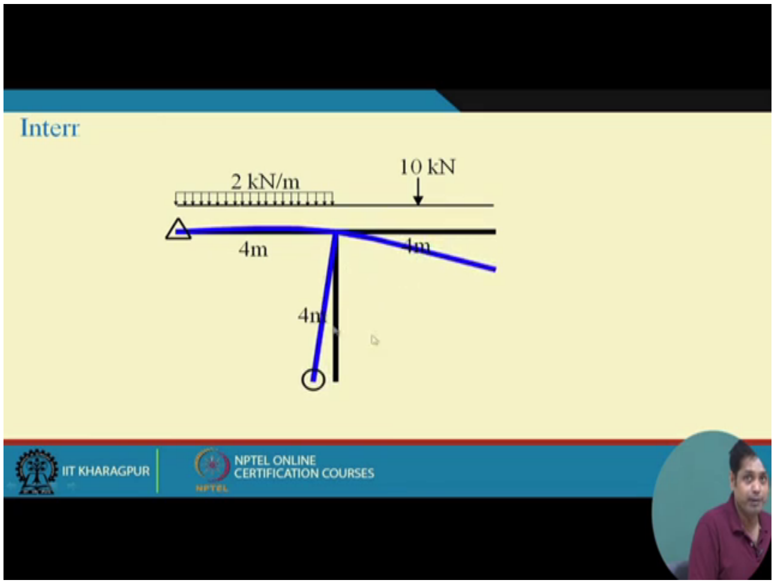

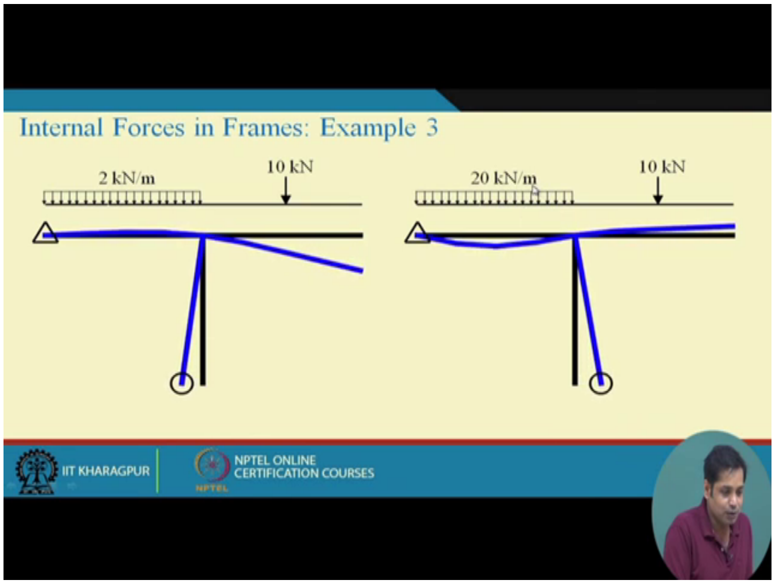

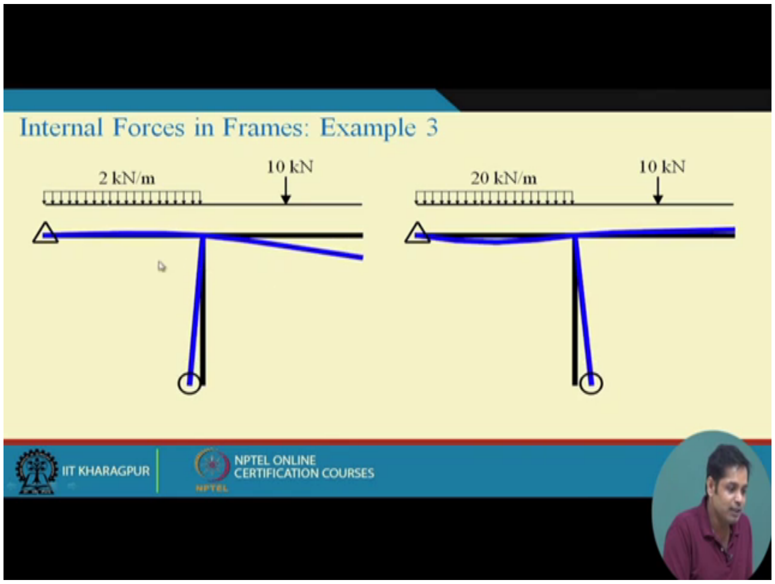

Quickly check the deflected shear this is the deflected shape of the beam, okay. Now one important part is for the same beam supports if I change this load this load is 2 kilo Newton per meter, but if I change this load to 2 20 kilo Newton per meter say 20 kilo Newton per meter but the same structure same boundary conditions but if this load is different this load is 2 kilo Newton and this is 20 kilo Newton you see the deflected behavior is completely different I mean there deflecting in a different direction, okay because now this load is more so that see this load is produce some hogging and this hogging will be balanced by this load.

But if this load is small this hogging will not be balanced, not be balanced means this hogging will dominate and this will deflect like this, but in this case this will produce hogging and this will produce sagging in the beam but this sagging load is this load is more entire deflection is dominated by that this part sagging of this beam, so this is the deflected (())(20:54), okay. So please go do some examples from the books and please make yourself comfortable with bending moment shear force diagram for frame throughout this lecture we will be using extensively shear force and bending moment and axial force diagram for beams and frames, okay.

With this I stop here today thank you next class we will see how to find out deflection of frames using method of virtual force, thank you.