Structural Analysis 1
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Lecture 25
Deflection of Beams and Frames (Contd.)

Hello everyone. Welcome to the lecture 25. You see today what we will do is we will review the internal forces in plane frame, okay.

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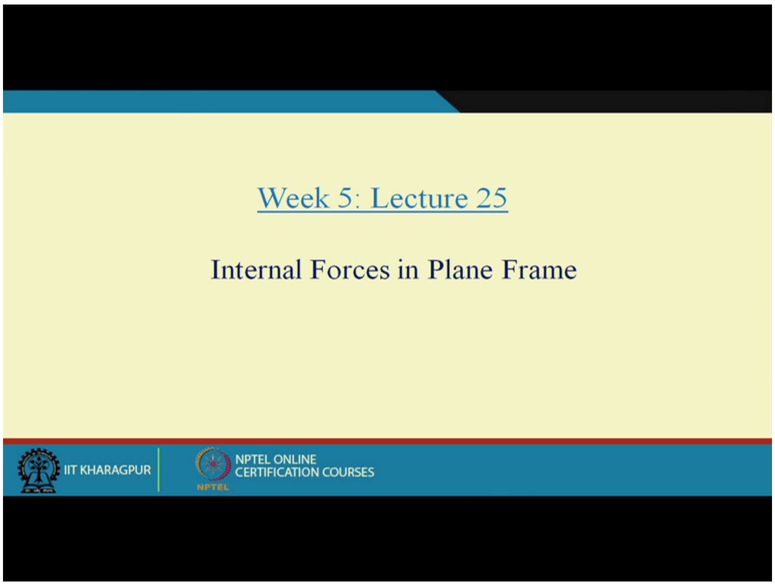

Now see some examples of frames. These are some examples of frames, okay. No not example of frames it is some structure, okay. I mean we do not know whether they are frame or different other structures like relation. But these are some structure. This is a bridge Howrah bridge, this is Eiffel Tower, this is this is a building, the assembly of beams and columns in a building, this is a cycle frame, this is a chair, the folding chair and this is also loading frame. This is very common in any gym. This is human skeleton then again this is a drafter that we are using in engineering drawing. There could be many other such examples.

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Now in the first class I showed you some examples of structures. So here the purpose of showing all these figures is not to tell you what the structure is. The purpose is different. The purpose is, can we find out some similarity between all these structures that are shown here? And if you look at them very carefully what are the common things in all these structures?

The common thing is if you look at them they are all, it is a structural system which consists of skeletal members.

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For instance if you see this skeleton, this is a collection of all these skeletal members. This structure is also a collection of all these members. This structure is also collection of this, this, this and this frame. This human skeleton is also collection of these ribs, then these bones. This is also collection of these rods and then this angle. This is also collection of these rods.

The cycle is also collection of this rod and this. All are individually slender members so the common thing between all these structures are there essentially a collection of several skeletal members.

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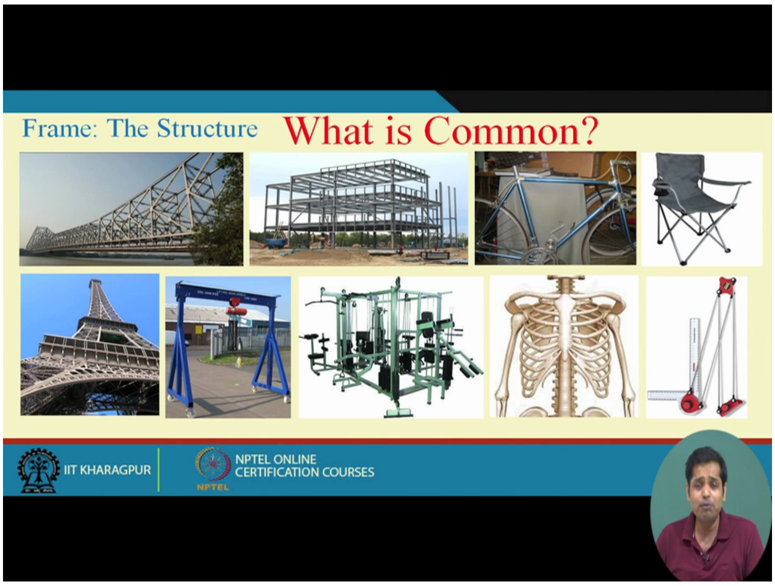

Now how the skeletal members are connected? Depending on that we have different structural idealization, right? So if you remember when we introduced truss we said that truss is also a collection of skeletal members, slender members. But in truss slender members are connected at the end through pins. Now so this was truss, right?

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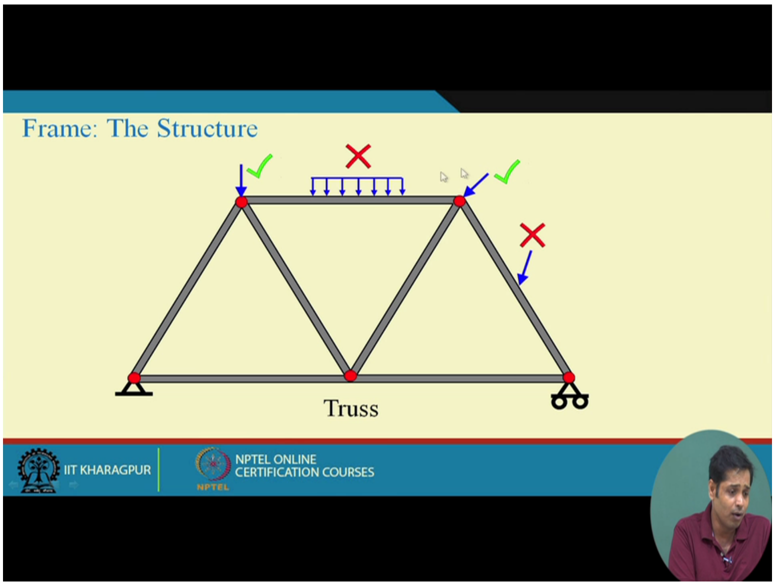

We already introduced truss. These are all slender members. Individually they are connected through pins at the end and also another important part is in truss we cannot have force on the member.

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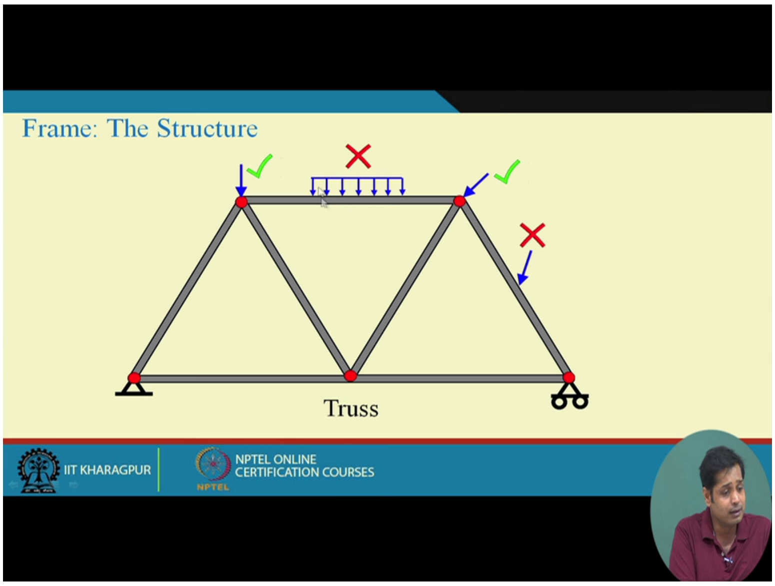

The force always has to be on the joints, right? And of course we have supports. Now consider a system again a collection of skeletal member where these conditions that we had for trusses, these conditions are released. Then what we have? This is first you release the condition where the members are not necessarily be connected through pins. So remove these pins, okay.

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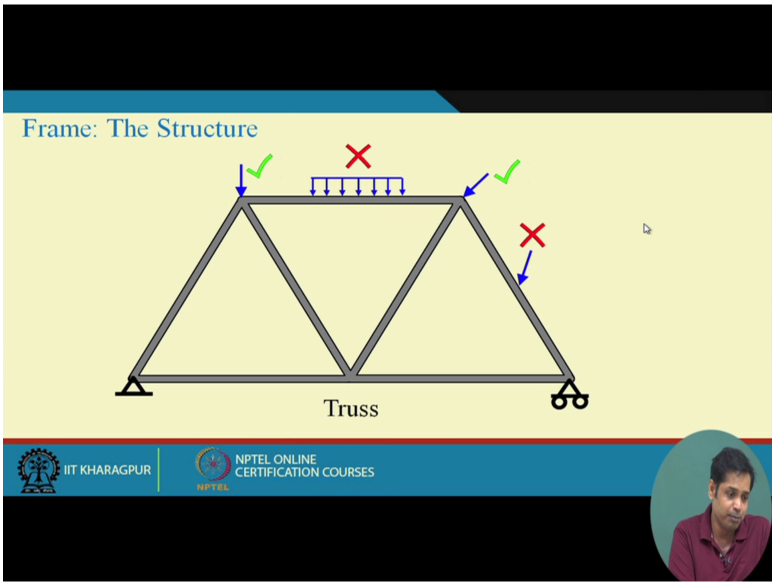

Now the next restriction is also released where in this structure suppose the forces can have on the members as well, okay. Now this structure what you get is no longer truss, right? Though the shape remains same but the structural requirement for a structure to be truss is not satisfying here. Now though this is no longer a truss so this is a frame, okay.

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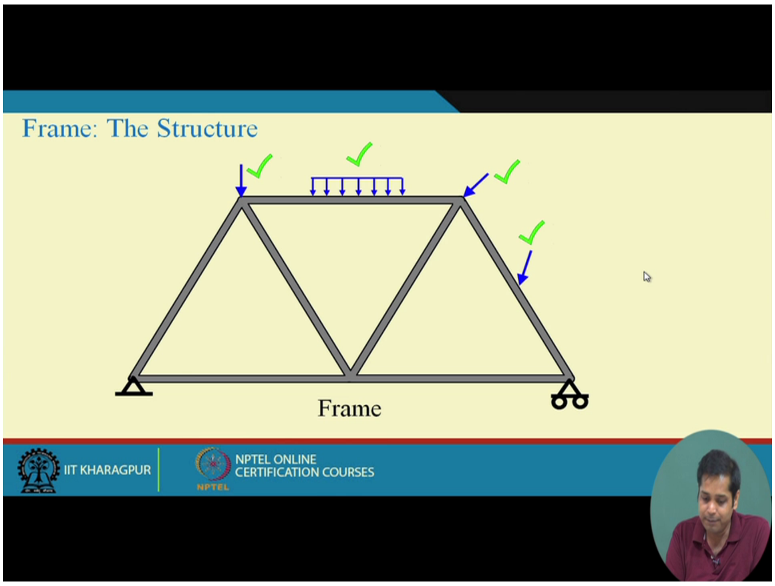

Now so let us see what are the differences? So this is a truss and this is a frame. Now if you look at the joints we said that in truss all the joints are pin connected. But in frames those joints could be pin connect, some of the joints are pin connected, some of the joints are rigid but when in a frame all the joints become pin connected and the loading condition also satisfied then this can be idealized as truss, okay.

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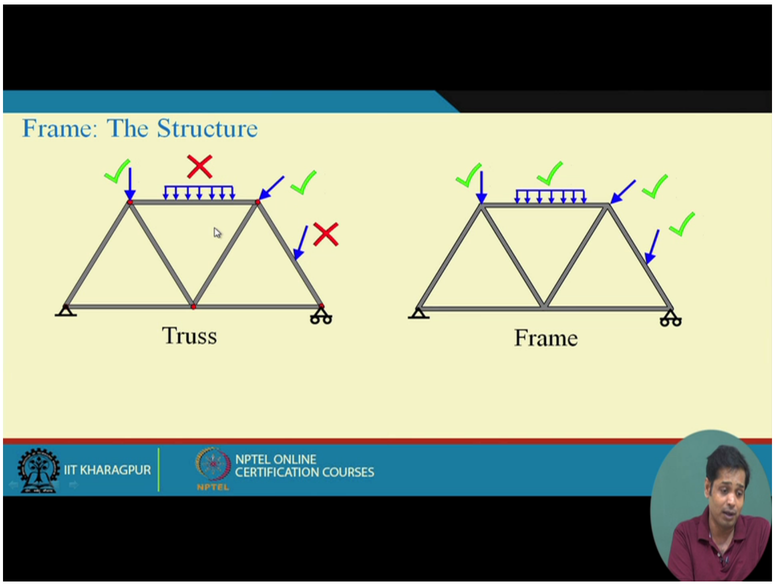

Now from the analysis point of view what is the consequence of releasing this restriction? The major consequences is when we studied structural analysis of truss then what makes our life easier is all the members in truss are two force members, okay. So if you take a member out from the truss, only forces you have is the two forces at the two ends and that will be two collinear forces at the two ends. And they are (oppo) equal and opposite and that makes our analysis easier.

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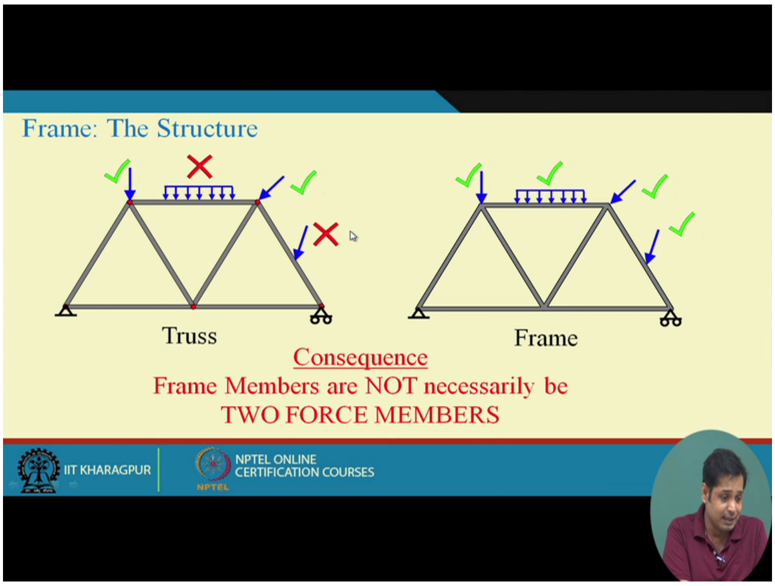

But now in frame since those restrictions are released the members are not necessarily be two force members, okay. Then what is the characteristic of those members? Now so before you do that let us define frame. Then what is frame? Frame is a structural system that consists of skeletal members.

Just now we have seen that it is an assembly of several skeletal members which are capable of resisting externally applied load in case of truss by developing axial force but in case of frame it is by developing axial force, bending moment and shear forces as well. There could be twisting moment also but since we are talking about plane frame these are the major three internal forces that we need to consider, okay.

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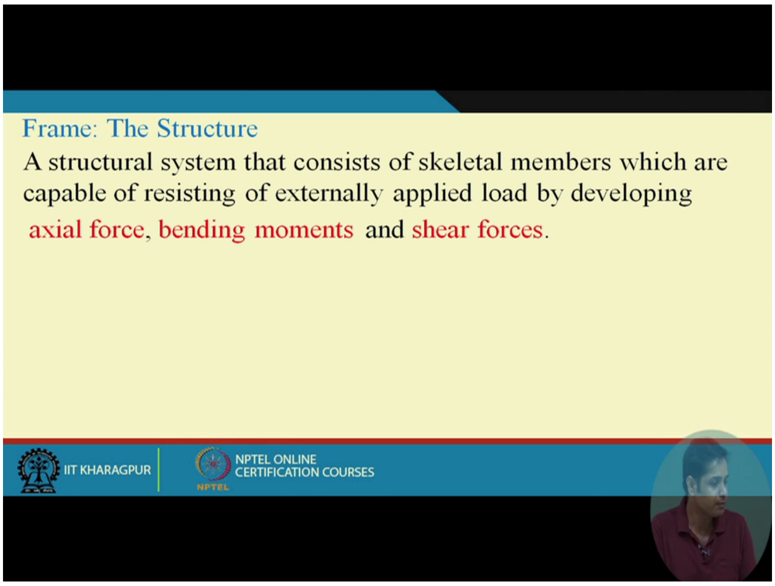

But in case of truss there was no bending moment and shear force, it was only axial force. Now so what is the consequence of that? Suppose this is a truss and this is a frame.

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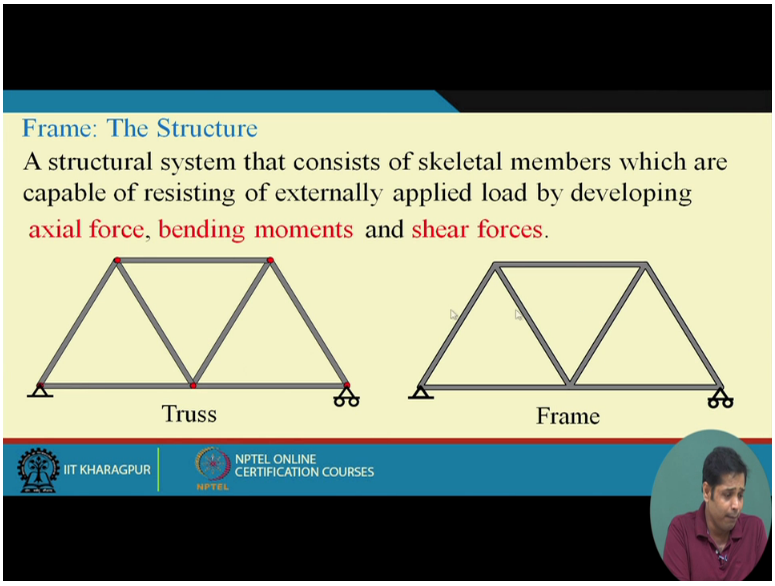

If we remove a member from here or if you break a member in a truss then the internal forces are only the member forces, okay. Because all members are two force members.

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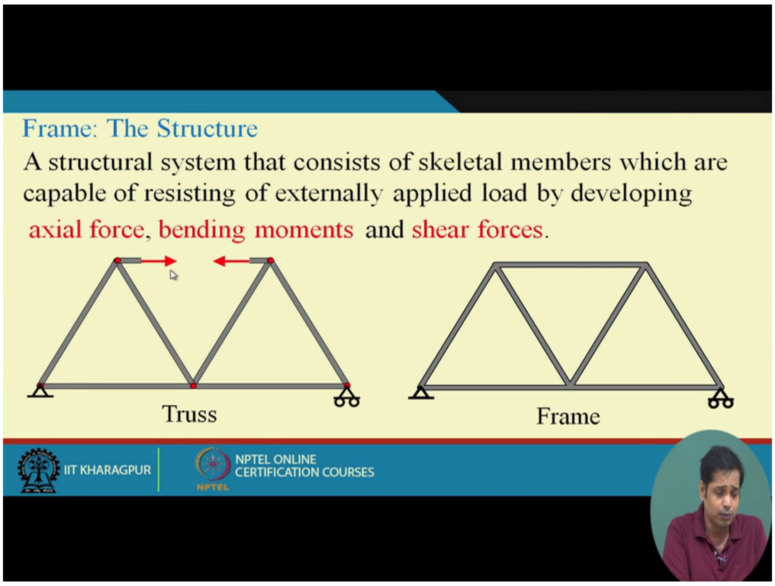

Whereas in the case of frame the situation is different. We do have this force in frame but in addition to that the effect of bending moment and shear force also need to be considered. So if we break a frame member and then draw the free body diagram of that then in the free body diagram we have all the forces like beams. We have this axial force, we have shear force and we have bending moment as well.

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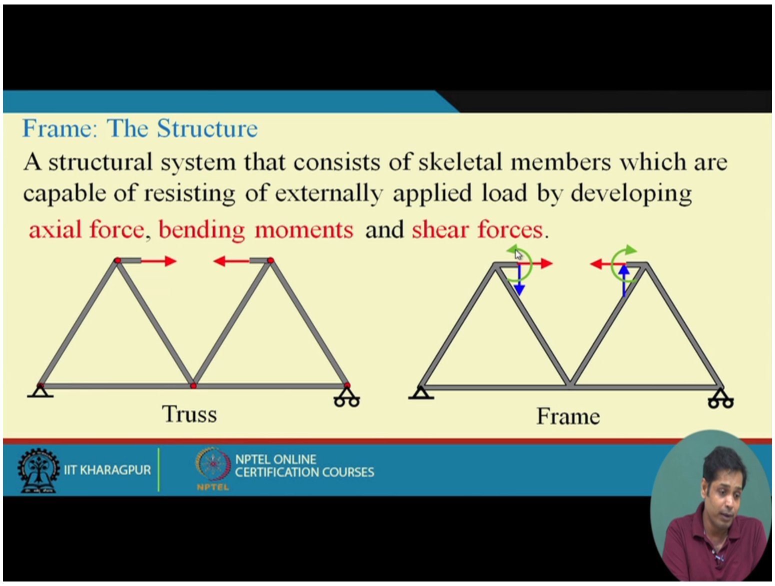

Now there is one important point to be noted. In beam also ideally we have a bending moment, axial force and shear force. But when we analyse beam if you remember we said that axial force was neglected. It was assumed that there is no axial force. At least the loading condition we have considered so far there was no axial force developed in the beam. That is why we did not consider axial force and even if you consider the axial forces were zero.

But in case of frame we will see that the axial force also plays a very important role. And along with bending moment and shear force diagram that we did for beam we also need to draw axial force diagram, okay. Now let us make one important point. The truss is one extreme case where the all joints are pin joints. There is no compromise with that requirement, right? But in case of frames we can have different kinds of joints.

Some of the joint are rigid joints, means those joints which are not allowed any rotations. Some of the joints are pin connected. And now just give you an example. Suppose if we take a tripod of a camera, many of you might have seen this. This is a frame. Now but if you see all these joints are pin joints.

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This member can rotate about this joint, this member can rotate about this joint. These three members also pin joints. These members also can rotate here, can rotate here or rotate here.

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There is no rotational constraint in all these joints. But still it is not a truss, it is a frame. Why it is a frame? Because another important part in case of truss is all these members are separate members, okay. They are connected only at the end. Now here this is one member and this member is connected to this member at some intermediate point, not at the end points, okay.

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So members cannot be treated as two force members. Because the reaction from two force members is we have forces only at the two ends and those two forces are collinear forces. But in this case because of the reaction from this member we can have a force on the member itself, in addition to the ends we have force in member itself.

So these members are not two force members so that cannot be idealized a truss. So this is a frame. But in this case the frames are pin jointed. But in this case you see this is a factory shed.

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This is a frame but this frame is not pin jointed. This joint is rigid joint. So s no rotation is allowed at this joint. So this member and this member, joint between these two members are rigid joints.

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If you take a frame of a cycle, this is a frame and these joints are also rigid joints. There is no rotation allowed in these joints. This joint, this joint, all are rigid joints.

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Now on the other hand this is a collapsible gate. You all have seen it. Collapsible gate is a very interesting structure. You see the name the collapsible gate, right? The gate which can be collapsed. Ideally we want our structure not to collapse, right? That was the safety requirement. But it is one example where we make a structure such that if we apply load, the structure is collapsed, okay. That is why it is called collapsible gate, okay.

Now this is also a frame. These are all different members. They are all connected through pins here but this member is a continuous member.

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This is not that the different members are connected at the end, so this is not a truss. So this is a frame. But in this case the joints are pin joints, okay. So in a nutshell that in frame the restriction for truss is (idea) released but the joints of a frame we can have a rigid frame where all joints are the rigid joints, where the rotation is not allowed, we can have frame which is pin jointed, which can have a (joi) frame where some of the joints are rigid and some of the joints are pins, okay.

Now let me show you some example. Suppose take a frame, okay. This is called portal frame and which is supported at point A and supported at point D. And we have a joint B and joint C, okay. And this is subjected to some horizontal load, okay.

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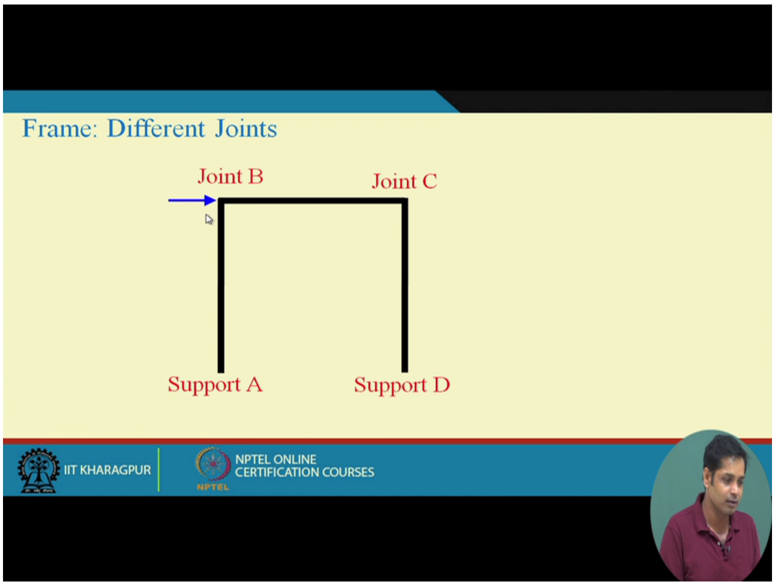

This is a problem. What we will see in next few slides is in this problem we have not defined what we say that point A and point D are supported but which kinds of support? We have not specified. We say that joint B and joint C they are joints but which kinds of joints we have not specified yet. What you will see next few slides is we will see different kinds of supports and different kinds of joints and for the same frame.

Same frame means same geometric shape but different connections between the members and when they are applied to same horizontal loads then how all these frames behave. So with that probably you can have a better understanding of the effect of different joints and supports, okay. Now the first case is this end is fixed end and this end is also fixed end.

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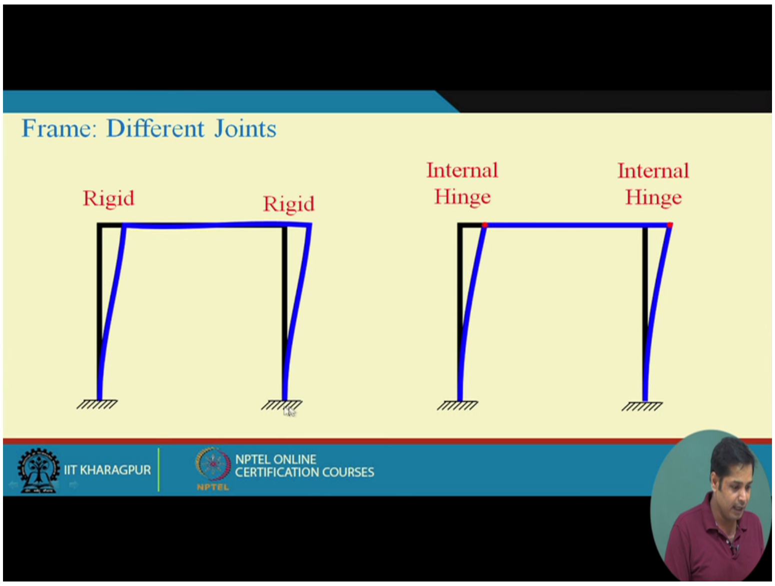

And this joint is rigid joint and this joint is also rigid joint. Rigid joint means now between this member and this member if the initial angle is 90 degree that angle is always preserved. There is no rotation allowed between this member and this member.

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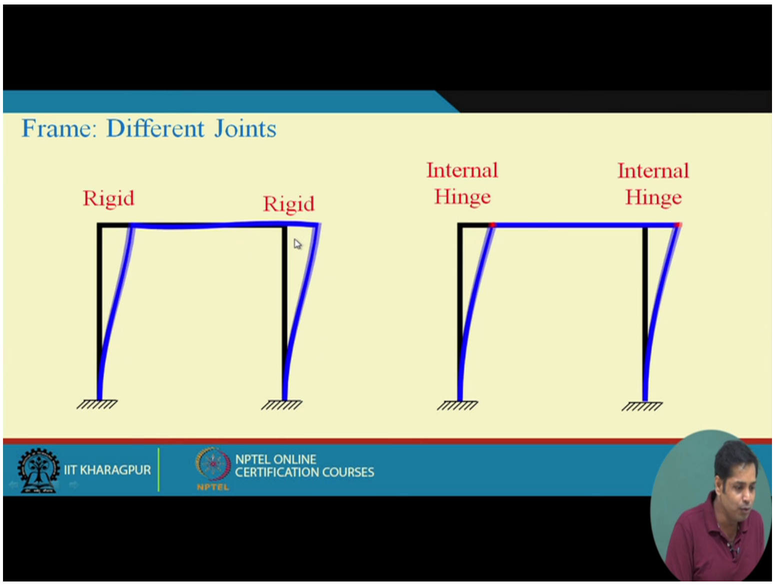

And if you look at this deflected shape you see this angle and this angle if you draw slope at the deformed shape, slope at this member and slope at this member you will see the angle between these two slopes remain 90 degree. Same here, angle between these two slopes remains 90 degree.

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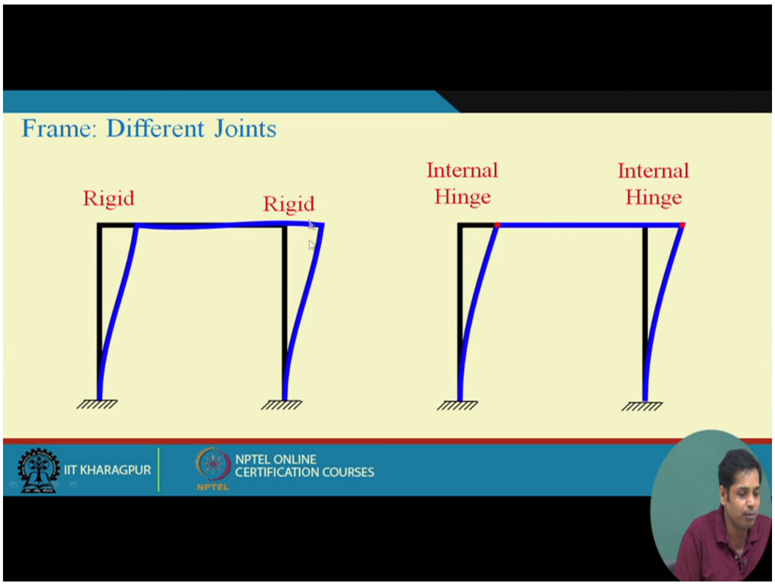

But whereas in this case the support condition is same but now the joints are not rigid joints. They have internal hinge here, okay. So hinge means it cannot give constraint against rotation. So naturally this angle between this member and this member changes which is evident from this animation.

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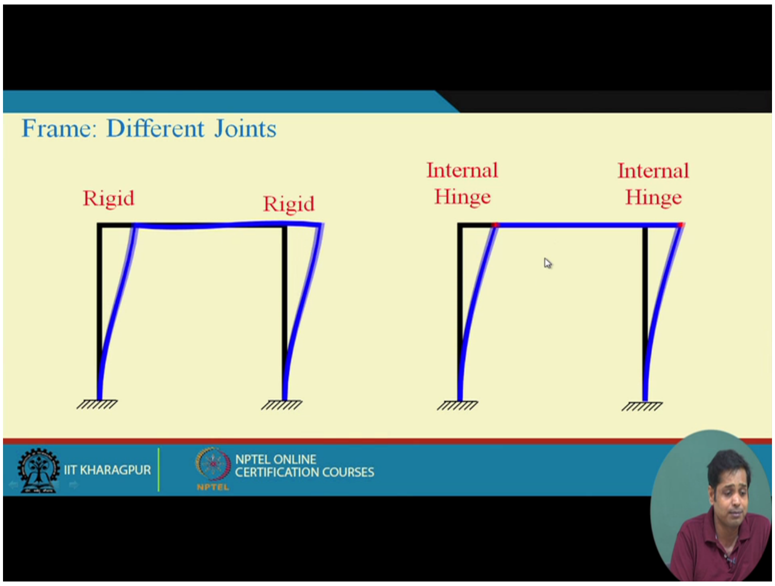

So if you draw the slope then you will see this angle not same, okay. Now in this case angle (incr) increases and in this case angle decreases. Now in the next case again the first one is fixed and then in this case this joint is not rigid, it is pin joint but other joint is rigid. Now you see this joint the angle remains 90 degree whereas this joint the angle does not remain 90 degree because it allows rotation because we have an internal hinge here, okay.

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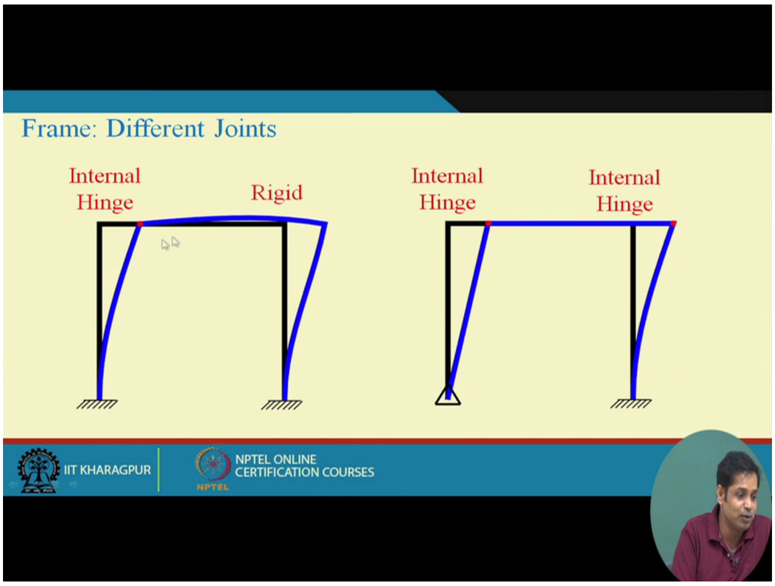

Now again in this problem this fixed support is replaced by hinge support and we have both hinge at this joint B and joint C. Now when it was fixed support see the slope at this point is zero but in this case the slope is not zero and that is the characteristic of hinge support, it does not give any constraint against rotation, okay.

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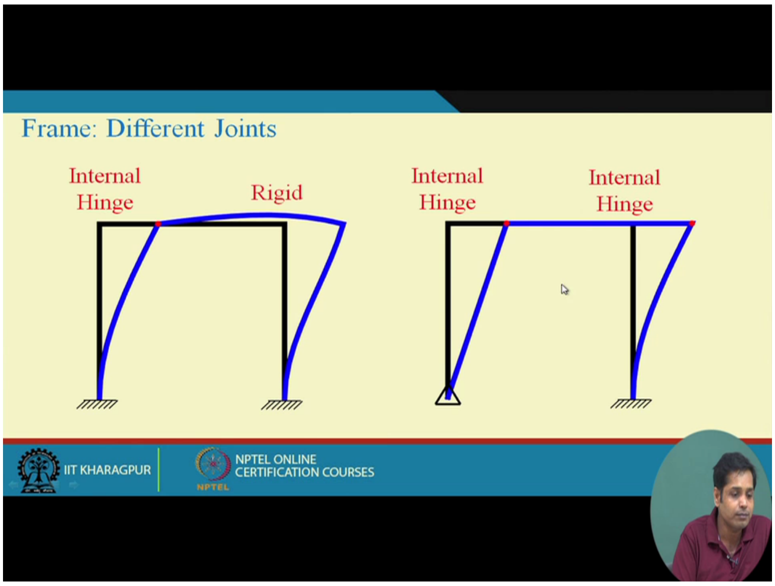

So you see the same problem and the same geometry but the different support and different joint conditions they behave differently under the same load, okay. Now this is another example you see. It is a cantilever frame kind of thing. One end is fixed and the other end there is no support here and how they behave. Now these joints are rigid joints and you see the 90 degree angle is preserved.

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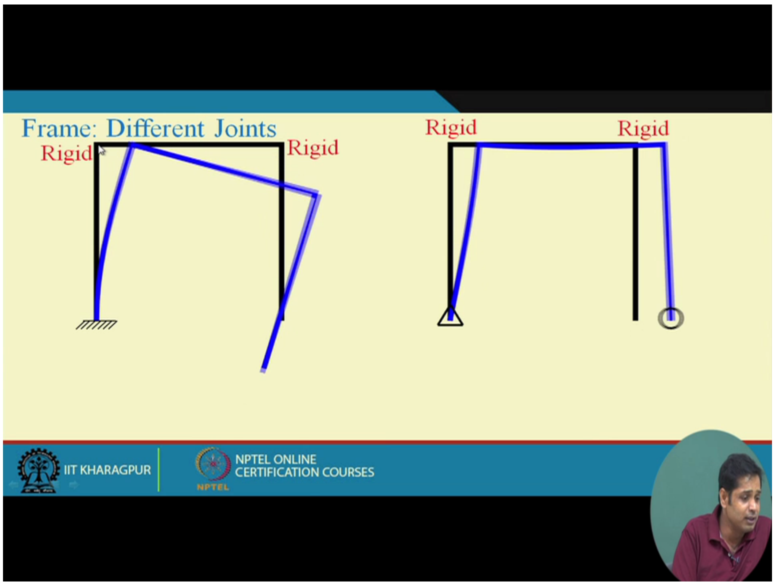

In this case also these joints are rigid joints but this is hinge and this is roller and this 90 degree angle is preserved. If you draw a slope, angle between these two slopes will be 90 degree. Here also the same thing, okay.

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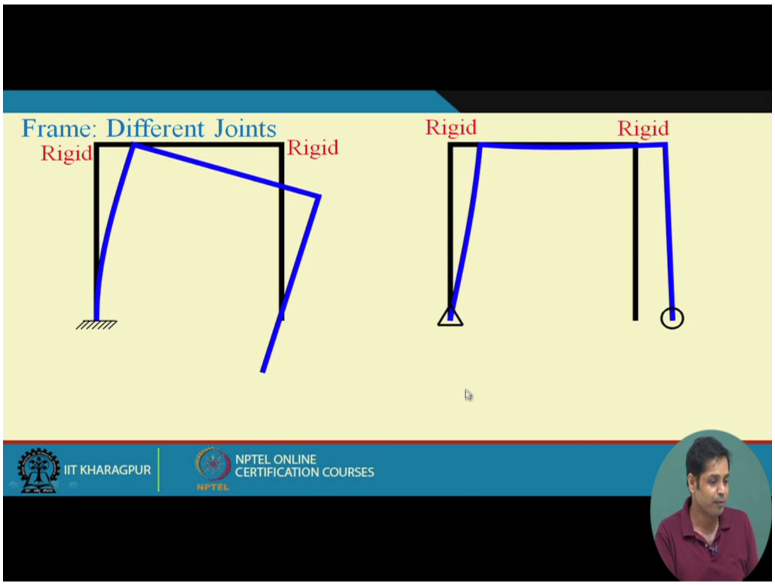

Now as we have defined what is frame and how in a frame we can have different kinds of supports and different kinds of joints. Again when you look at any frame when you see any frame then please look at their joints, look at their support condition and then you decide yourself that whether if you look at any structure then you decide yourself then (whe) whether the structure can be idealized as a truss or as a frame, okay.

Now this lecture is on determination of internal forces in frame. Means (detr) for drawing bending moment, shear force and axial force diagram for a frame. Let us just demonstrate through one example. Now this is a cantilever frame which is one end is fixed and another one is applied load is P.

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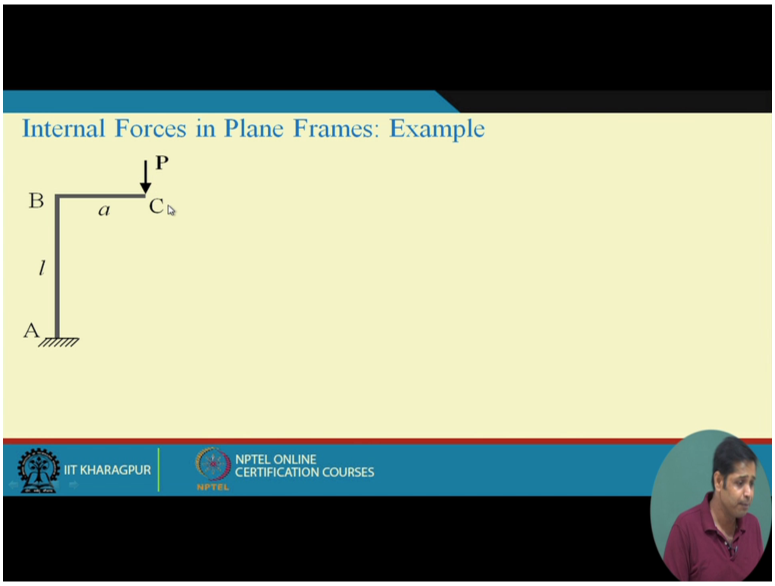

We need to find out what is the bending moment, shear force and axial force diagram for this frame, okay. This joint is rigid joint. So first is take these two parts separate, AB and BC because we have two members. Now when we draw the bending moment diagram and shear force diagram or axial force diagram we have to draw it for BC, we have to do it for AB. Now let us separate them. This is BC and AB, and draw the free body diagram of both the parts separately.

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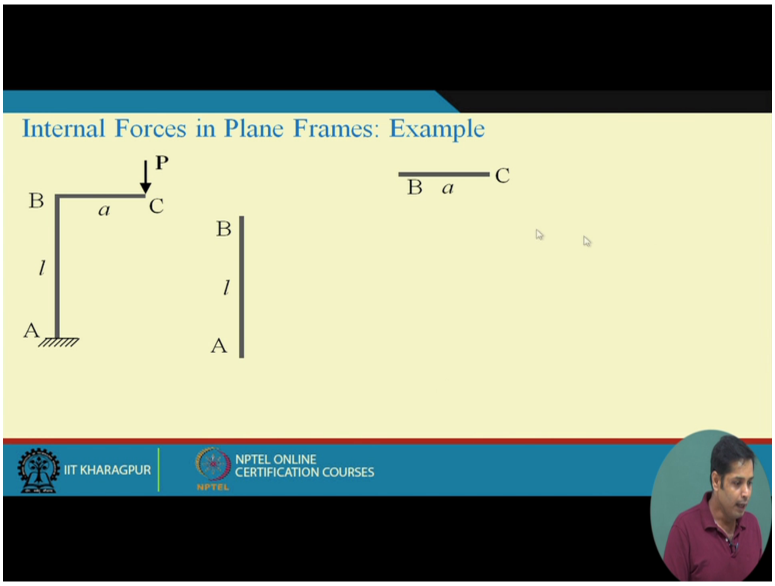

Now draw the free body diagram for C. This is the (ex) externally applied load P at C then since it is frames we have axial load FB, then shear force VB and then bending moment MB. So this is a free body diagram of part BC.

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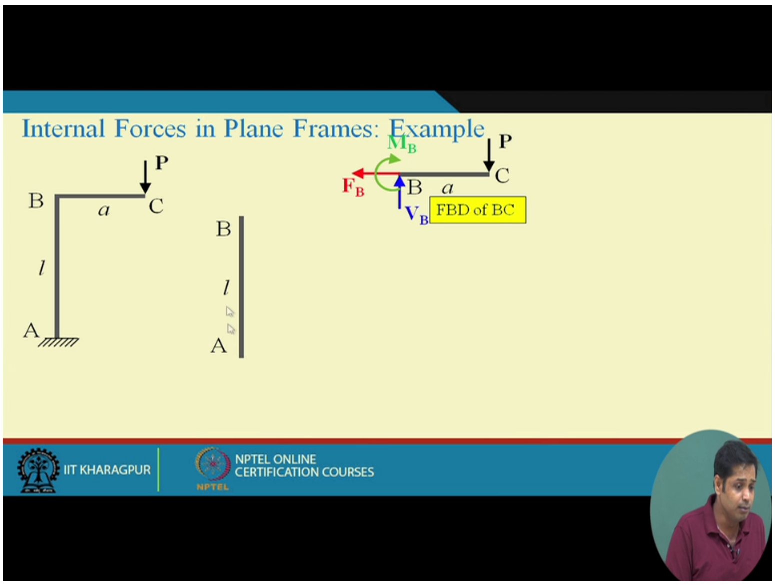

Similarly let us draw the free body diagram of part AB. First we have support reaction Ax and then Ay and then Ma. This is fixed support so all three forces we have, okay. Now at B also we have FB and then VB and then MB. And look at joint B, when joint B belongs to part BC the sign we use and the joint B belongs to part BA the sign we use, they are opposite.

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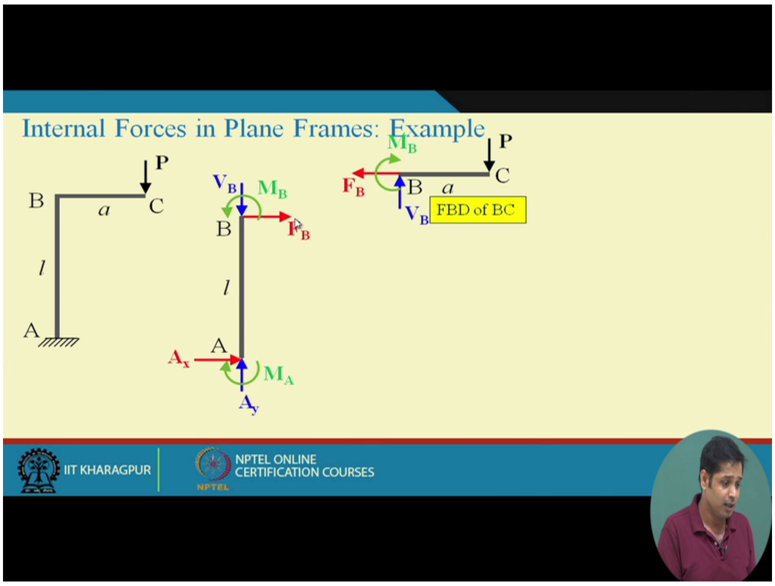

So therefore when they are connected the net internal forces balance each other. They cancel each other. So this is the free body diagram of joint AB. Now you put a star and make one comment here is, joint B have just only two members, BC and BA. We will see a situation where at the same joint we have three members or four members and in those cases how the free body diagram of the joints to be shown, that we will discuss. But in this case it is slightly simpler case because at B it is connected only to two members.

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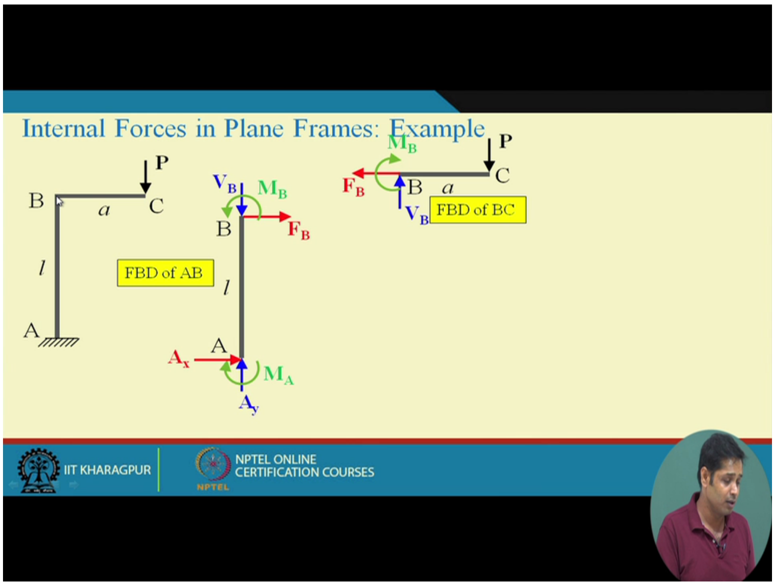

Now apply equilibrium equations summation of Fx is equal to zero. This will give you FB is equal to zero. Fy is equal to zero it will give you VB is equal to P. And MB is equal to zero this will give you MB is equal to minus Pa. This is straight forward we have done it many times.

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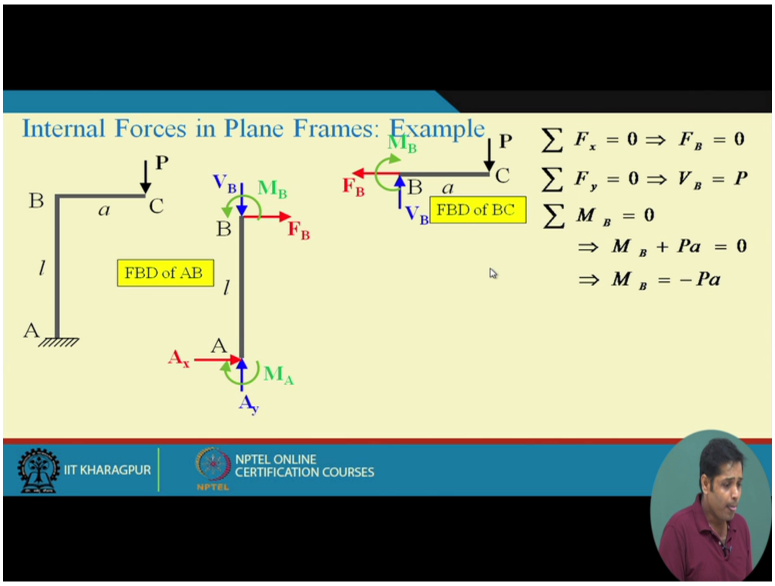

Now similarly apply equilibrium condition for this part. Again Fx is equal to zero this gives us Ax plus FB is equal to zero. FB is equal to zero we already obtained here so naturally Ax is equal to zero.

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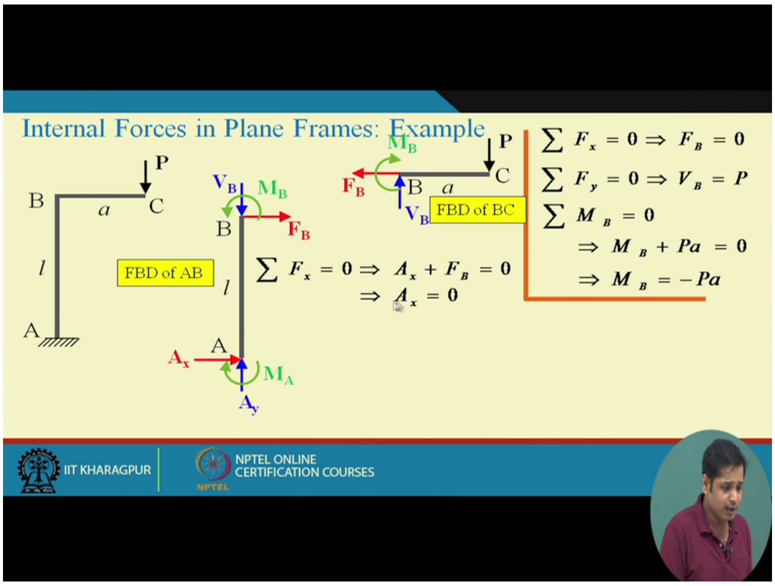

Fy is equal to zero so y direction forces are Ay positive, VB in this negative downward minus and Ay is equal to P. VB is equal to P we have already obtained. Similarly MB is equal to take moment about B. If you take MB is equal to zero so this force will not contribute, this force will not contribute because it is passing through B. Ax is equal to zero anyways. So contribution will be from MA and MB and this gives us MA minus MB is equal to zero. MA is equal to MB is equal to minus Pa. MB we have already obtained.

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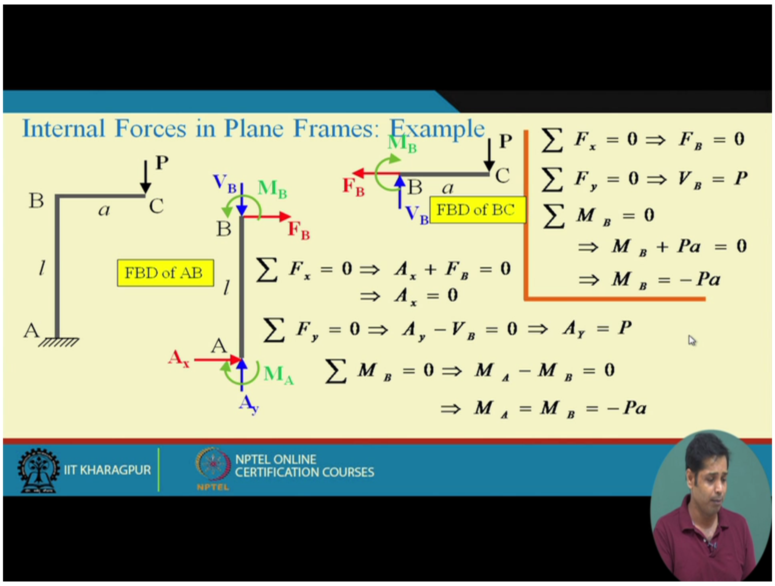

So now we have already computed the support reactions and what are the internal forces at the joint. Let us draw the bending moment diagram. One way bending moment diagram, shear force diagram or axial force diagram is essentially the graphical representation of the bending moment, shear force and axial force, right?

And one way of drawing bending moment of this diagram is to take a section at any particular given distance x and calculate the bending moment, shear force and axial force as a function of x. And then bending moment, shear force diagram will be the graphical representation of those functions, okay. This is one approach and this is the fundamental approach. But at least for this problem we really do not need that.

Just by looking at the problem and by looking at the joint condition and the internal forces of the joint we can draw the bending moment and shear force diagram without actually doing that exercise. By taking a section, representing bending moment internal forces as a function of x and then represent them graphically. We can avoid that step here, okay. How can we do that?

So suppose these were all the forces. Now some of the forces are zero. So let us remove those forces and just keep all the forces which are a non zero. So non zero forces are this forces, okay.

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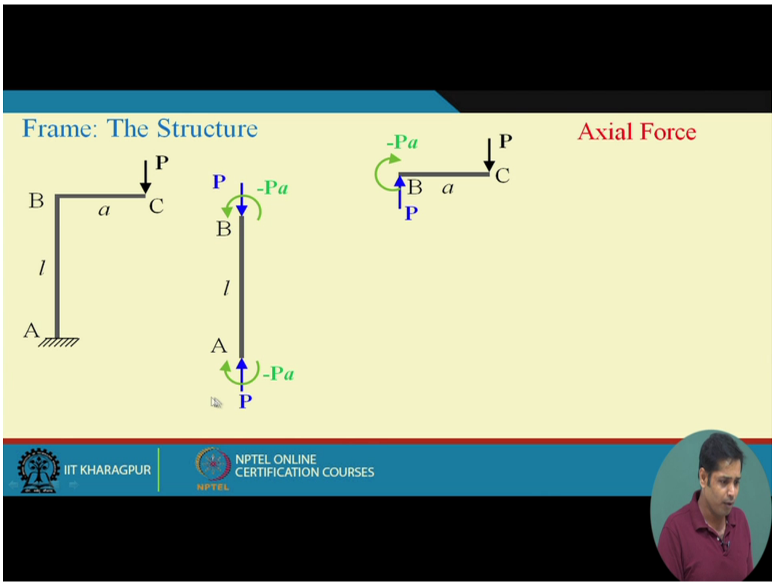

Now let us first draw the axial force. Now it is very important you see this VB was shear force for BC but same VB for AB it becomes (ax) axial force. Now FB which was axial force in BC but the same FB in AB becomes shear force. But bending moment remains same. So because their directions are different that is why when you draw shear force and (dia) and axial force diagram please check that.

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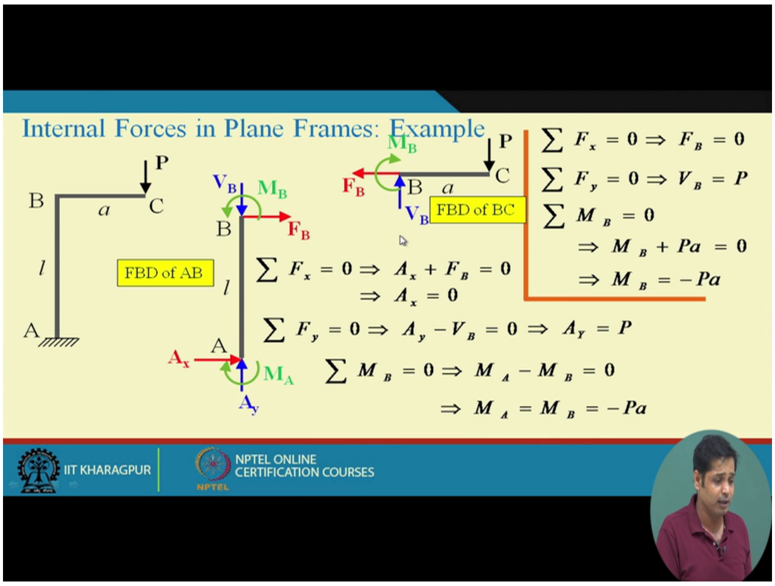

Now so the shear force P was the shear force for BC but the same P becomes (ax) axial force for AB, okay. And the horizontal direction of all the forces is zero anyway in this case.

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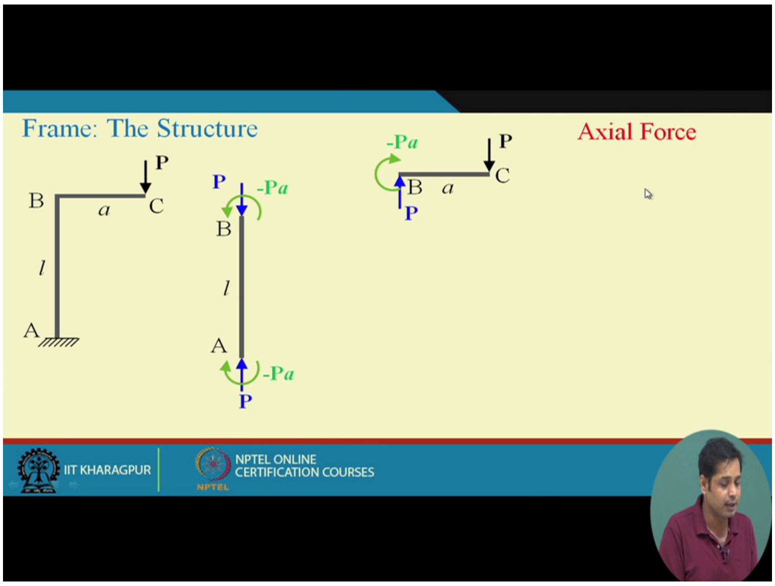

Let us draw the axial force diagram. So what would be the axial force for BC? There is no axial force for BC and what is the axial force for AB? Axial force for AB is P and which is compressive. The axial force of this is compressive. This is negative sign. So this is the axial force for the entire frame.

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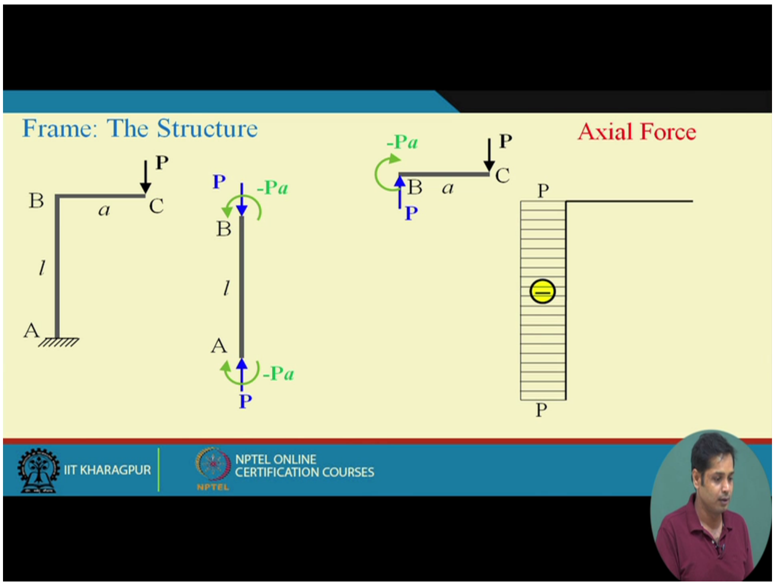

Now let us draw the shear force diagram. What is the shear force diagram? Now let us first considered part BC. Now in part BC what happens, the shear force is positive here and there is no other load between BC so shear force remains constant.

And the shear force will be positive because this is acting downward and if you take a section here, shear force will be upward. Clockwise couple will be positive and therefore shear force is positive here. So shear force is positive for BC.

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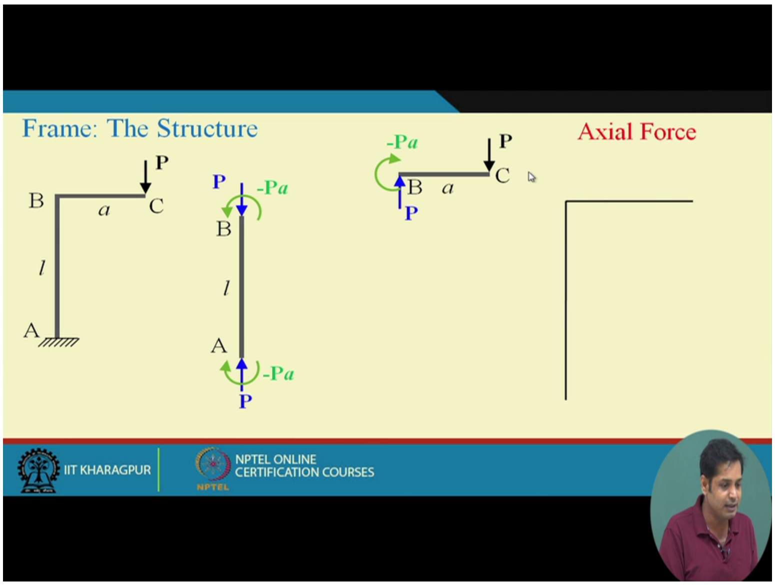

Now what is the shear force for AB? There is no shear force for AB because the horizontal forces all are zero. So this is the shear force diagram for the entire structure. Now let us draw the bending moment diagram. Bending moment diagram is at point C, it is tip of the cantilever so bending moment will be zero. And between C and B bending moment should vary linearly. It will be hogging moment or sagging moment?

It will be hogging moment because BC will behave as a cantilever. So between B and C bending moment will be negative linearly varying. And at point B the bending moment value will be minus Pa. So BC the bending moment is this. That is what our sign convention is.

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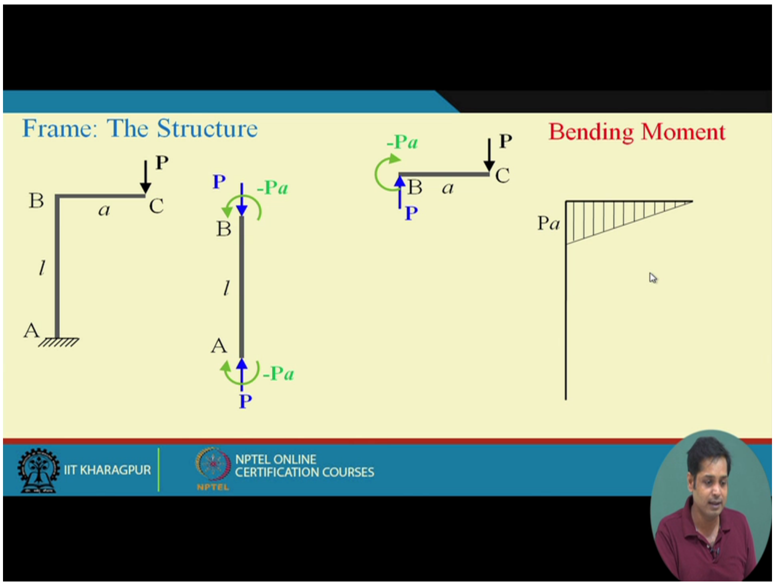

In many places we will see this bending moment is drawn on the other side. So there sign convention taken different, okay. Let us see what happens to this. In this case remember the sign. If you see in our case if a beam is like this we assume this is our positive bending moment. And the beam deflects like this.

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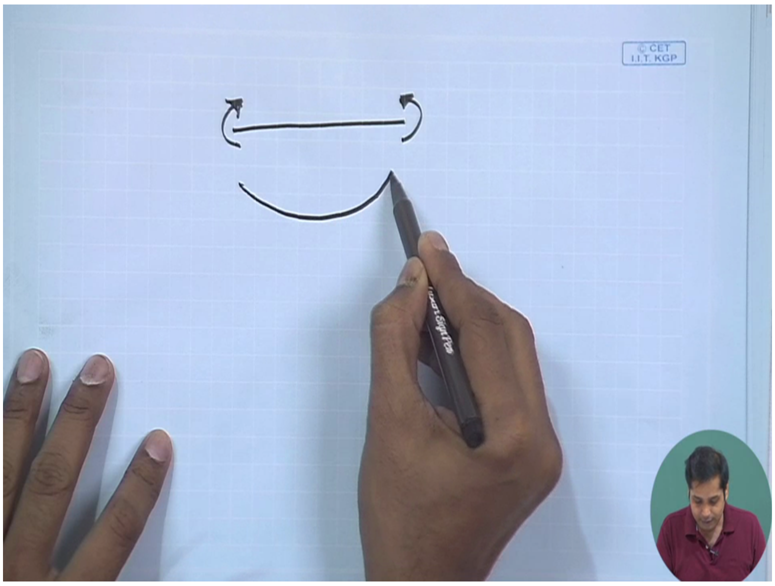

If the beam deflects like this then this is compression side and this is tension side, okay. And how we draw this bending moment diagram? The bending moment diagram is drawn as this, okay. So this is positive.

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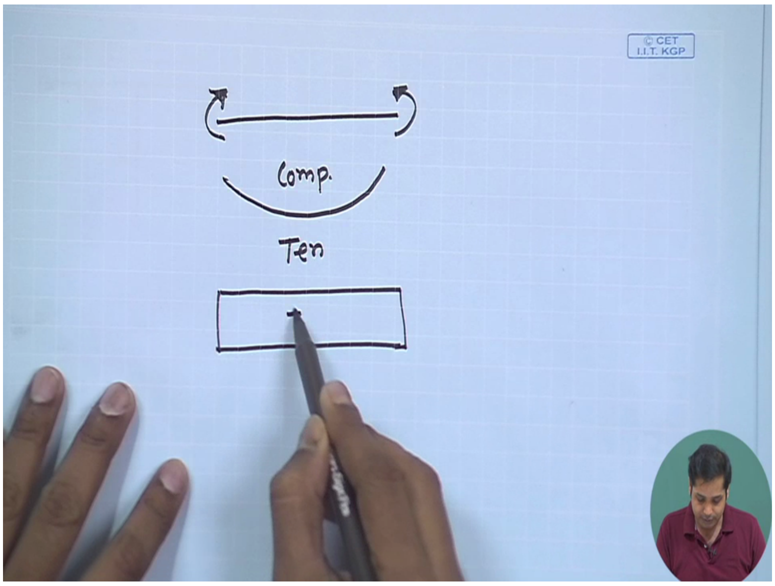

So in this case bending moment is positive. Now if we take a cantilever beam, cantilever beam is subjected to tip load so it deflects like this. So this is tension and this is compression. And how we draw the bending moment as per our sign convention, this is okay. So this is AB, so this is AB and this is AB, this is AB, okay.

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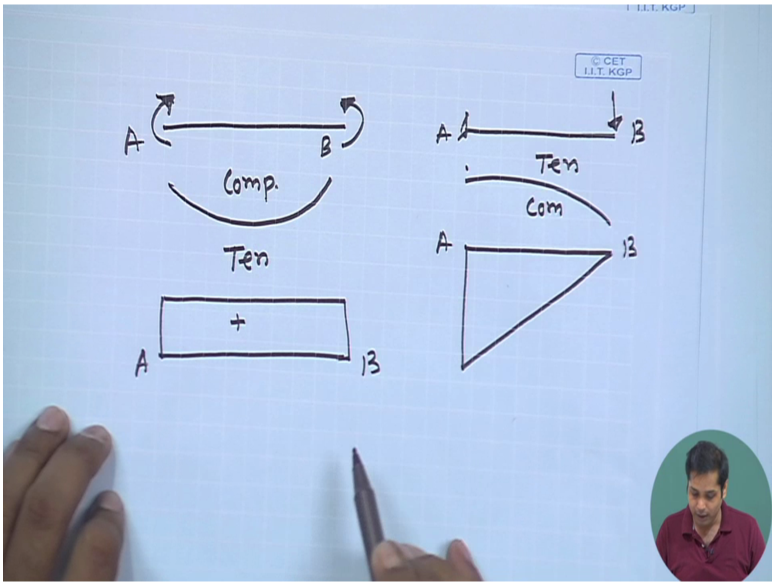

So this is bending moment diagram. So you see in our case always we draw the bending moment diagram on the compression side, okay. Bottom side is the compression, bending moment diagram is drawn in the compression side. Compression side is top side, the bending moment diagram drawn is at the top side. But in many books and many other places you see the sign convention is taken just opposite.

The bending moment diagram is drawn on the tension side. But both are correct. But whatever sign convention we use we should be consistent with that, okay. Now this is our sign convention. Now so this is minus Pa.

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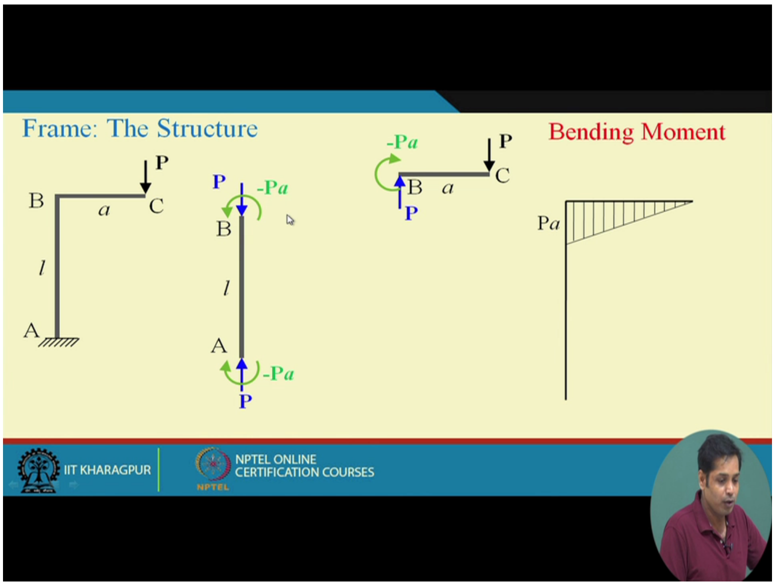

So what would be the compression side? Compression side will be this side because it is minus Pa. So essentially it is in this direction. The compression side will be this side, tension side will be this side.

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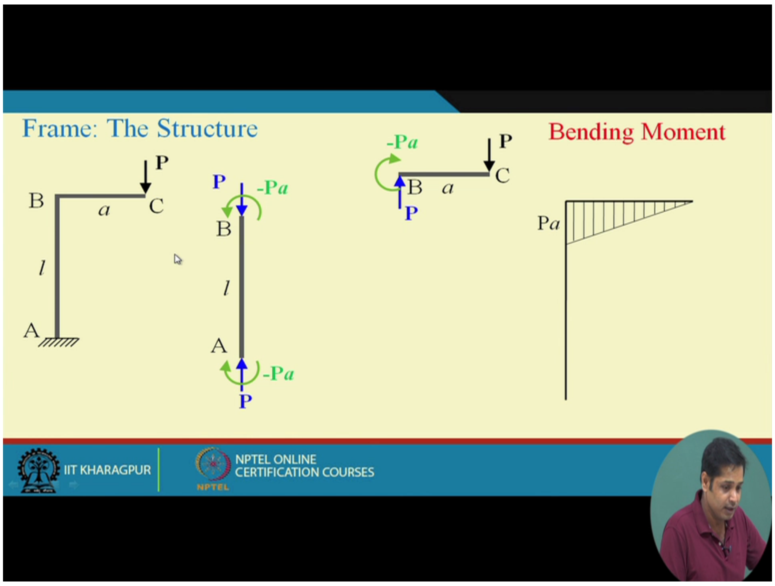

In this case compression side is this side and tension side is this side.

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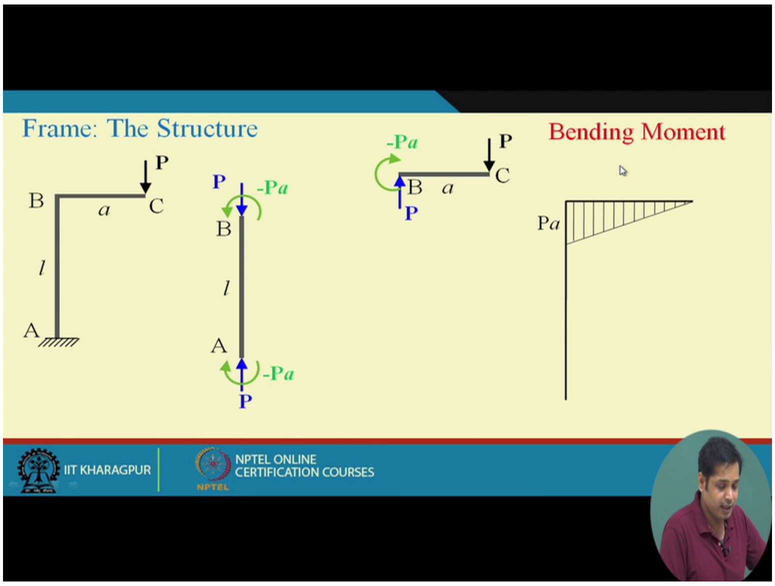

So for this part bending moment will be this and this bending moment is constant throughout. So this is the bending moment diagram of this frame, okay.

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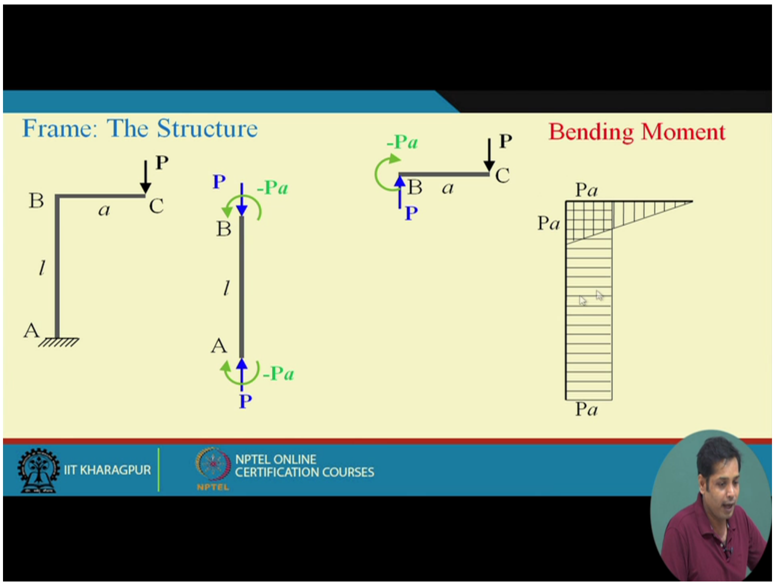

Now if we see the deflected shape of this frame, deflected shape is something like this. You see just from the deflected shape you can understand that this side is compression and this side is top, this side is compression and this side is top.

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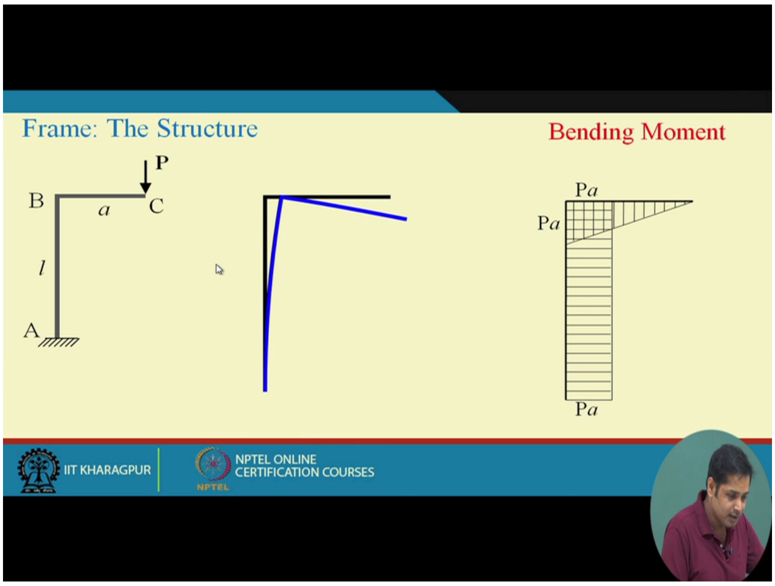

So this is how the bending moment and shear force, okay. Now we will stop here. Next class we will see just one or two quick examples on bending moment, shear force and axial force diagram of frame. Thank you.