Structural Analysis 1
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Lecture 23
Deflection of Beams and Frames (Contd.)

Okay, welcome to the lecture 23. What we have been discussing from the last class is the conjugate beam method for determination of deflection of statically determinate beams, okay.

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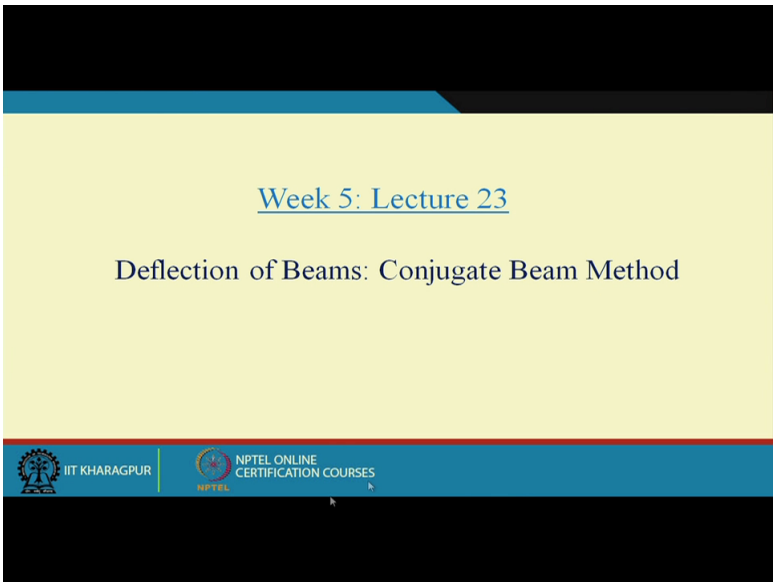

Now if you remember this was the slide where we actually introduced the concept of conjugate beam.

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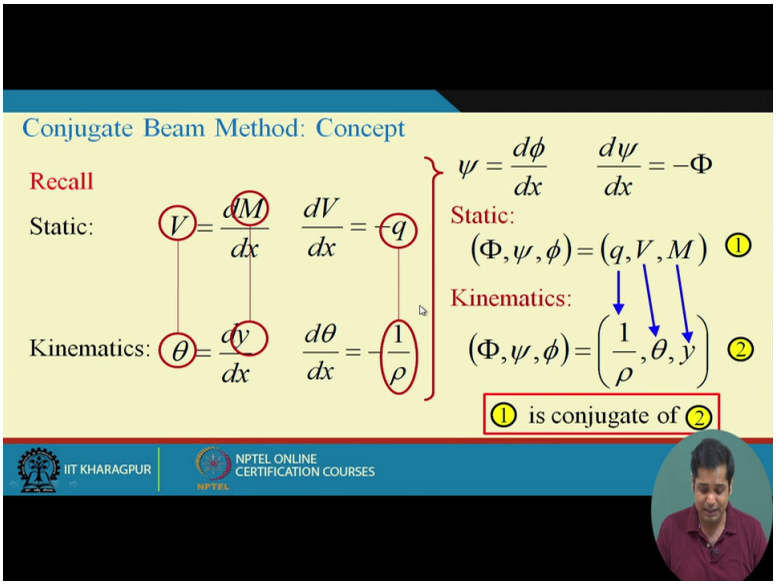

Before we go further let us once again review this. The idea has been, we have two problems. One is the problem of statics, one is a problem of kinematics. Problem of statics tells you that we have a structure subjected to some external load then the static problem is determine the internal forces in the structure and the static equilibrium is the premise of that process.

Now the kinematics is, once we know the internal forces or the external then use the relation between internal force and deflection or relation between stress and strain to determine the deflection or the deformation in the structure, okay. And in this process the stress strain relation plays the important role, okay. That is the cornerstone of this process. Now conjugate beam method is essentially problem of kinematics. in conjugate beam method what you do is a problem of kinematics.

Now formulate it as problem of statics. And how it is done? It is done by exploiting the similarity between the way these static variables are related to each other and kinematics variables are related to each other. And that similarity is shown in this slide. You see the static variables are shear force, bending moment and intensity loading q and kinematics variables are slope, deflection and curvature, okay.

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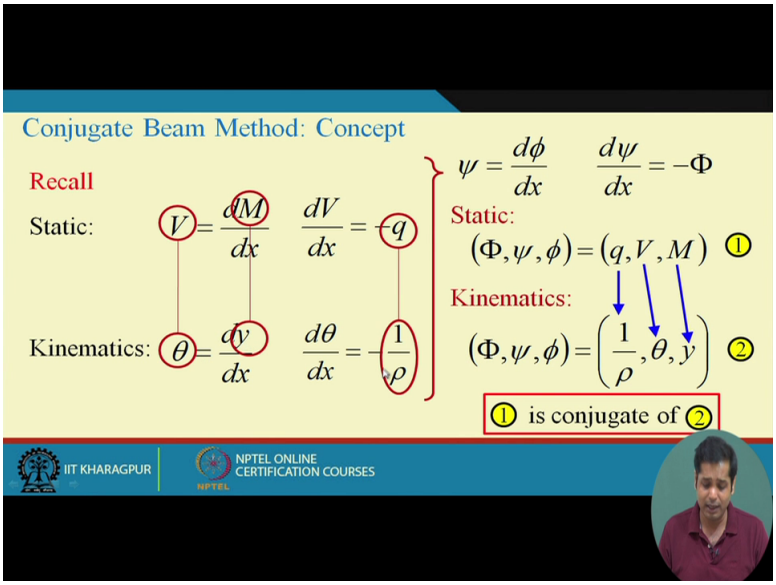

Now we discussed in the last class that the q curvature is conjugate to q or deflection is conjugate to M or theta is conjugate to V. Now you have real system where we need to find out the curvature, slope and deflection and then what we have to do is we have to construct a conjugate system where the intensity of the load is replaced by the curvature of the real system means M by EI diagram of the real system.

And if we do that then shear force of this conjugate system will give us the slope of the real system and the bending moment of the conjugate system will give us the deflection of the real system. So this was the problem of kinematics originally we had. Now instead of directly trying to solve the problem of kinematics what we do is this kinematics problem is translated to a problem of statics and then we solve this.

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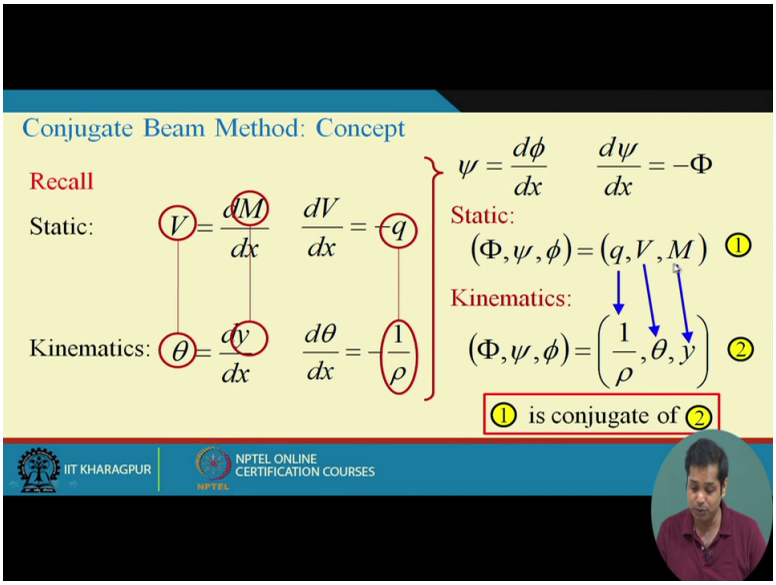

So the problem is reduced to determine determination of shear force and bending moment of the conjugate beam. And once we do that directly shear force and bending moment will give us the other kinematic variables of the real system. And this is the concept behind conjugate beam method. We demonstrated one small example where a simply supported beam subjected to uniformly distributed load. Now so that was the problem we discussed in the last class.

So this is the real beam. This is the bending moment diagram of the real beam and this is the conjugate beam where the loading on the conjugate beam is replaced by curvature or in this case M by EI diagram of the real beam, okay.

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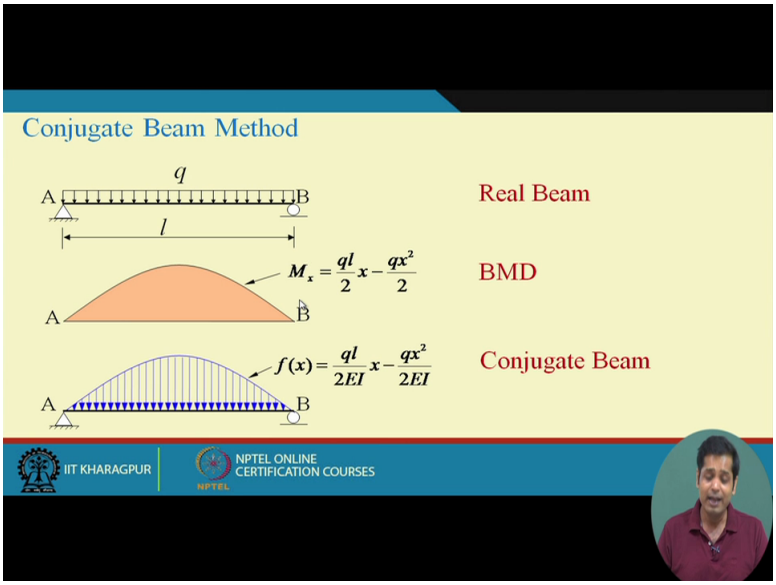

Then in the last class also we said this is the first step of forming a conjugate beam. But there are few other things that needs to be looked into while formulating a conjugate beam. Let us see what are the things that also important. Now so one aspect is loading on the conjugate beam will be the M by EI diagram of the real beam, okay.

Now in order to appreciate next requirement for a conjugate beam let us see one example. Suppose this is a cantilever beam subjected to tip load, okay.

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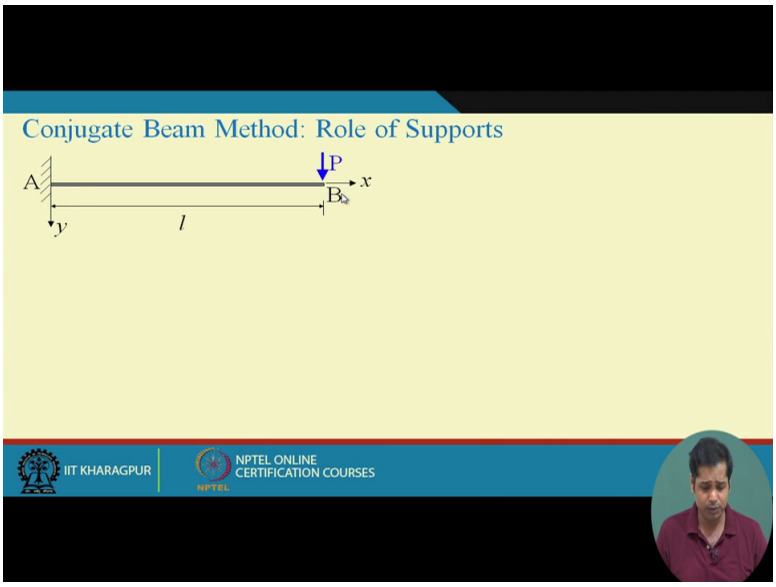

Now what is the bending moment diagram for this real beam? And what we need to determine? We need to determine what is the slope at point B and what is the deflection at point B? We already know how to determine that using direct integration method and moment area method. Now let us apply the conjugate beam method. In this case whatever understanding of the conjugate beam method we have so far let us apply that.

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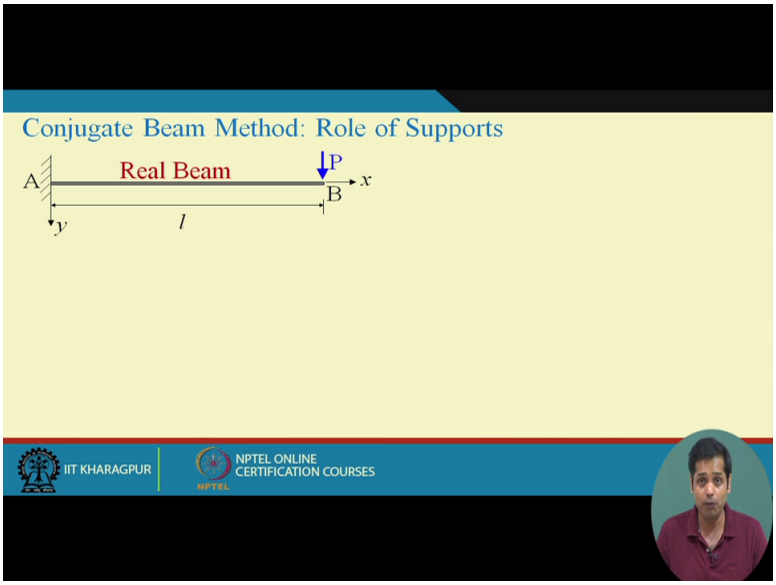

So this is the bending moment diagram of this beam. We know how to do this and this is negative. You remember our sign convention is sagging is positive. But in this case this load will cause hogging in the beam. So the bending moment is negative.

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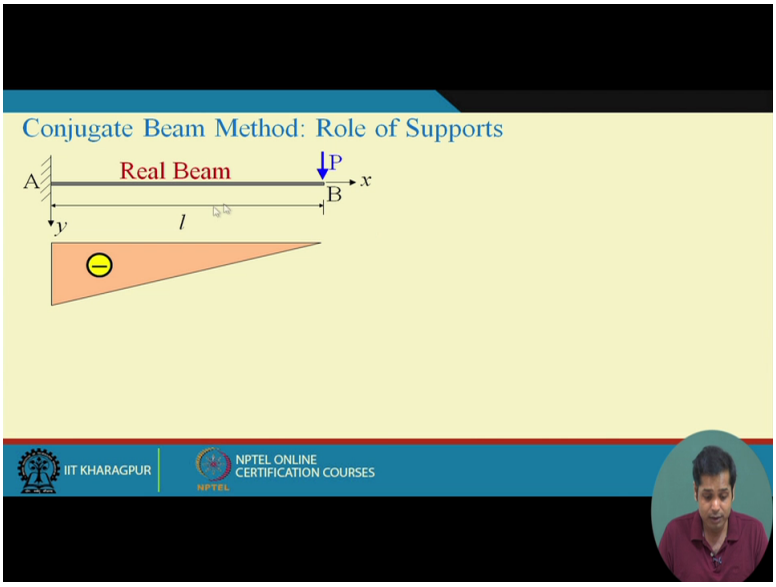

And this expression is P into L minus x where x is measured from this. So at x is equal to L bending moment is zero. X is equal to zero bending moment is PL. This bending moment is PL.

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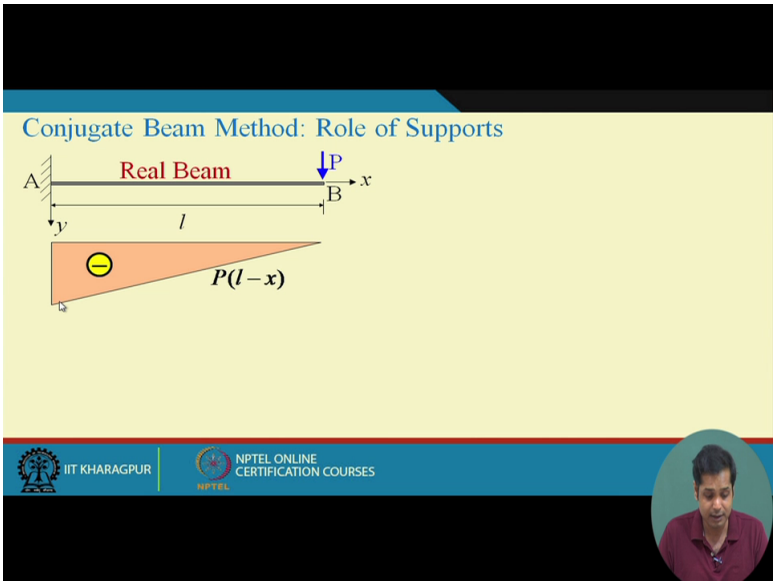

So this is the bending moment diagram and then from the conjugate beam, conjugate beam will be the let us take the same beam and then replace the load by the M curvature diagram of the real beam. Curvature diagram means the M by EI diagram of the real beam. So M by EI diagram is this. So since it is negative, the load will be upward direction in this case. So what is the distribution load? Distribution of load is this divided by EI. Now this is conjugate beam.

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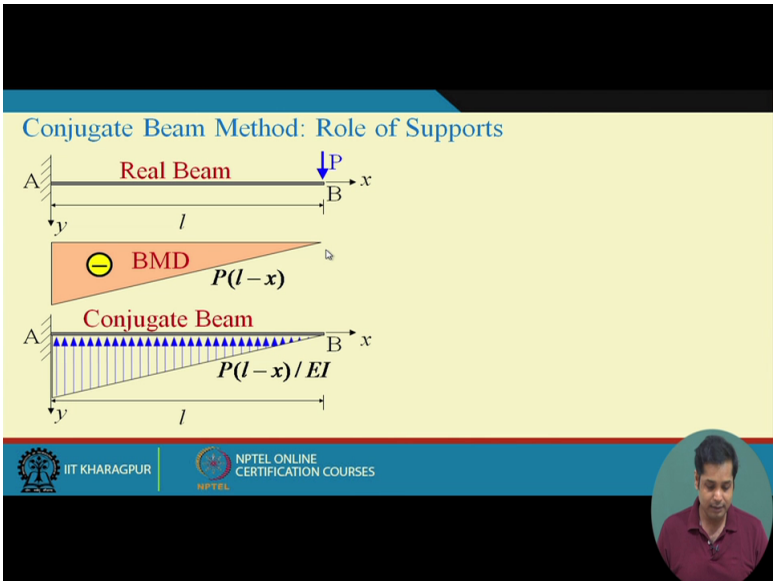

Now the problem is instead of determining directly the slope and deflection of this point what we have to do is we have to determine what is the shear force and bending moment at B of

the conjugate beam. So this is now a problem of statics. We can apply the equilibrium equation and determine the bending moment and shear force at a particular point.

Let us do that. So this theta B will be the bending slope shear force at B and delta B will be the bending moment at B of conjugate beam. Now if this beam deflects like this, this is the delta B and this is the theta B, okay.

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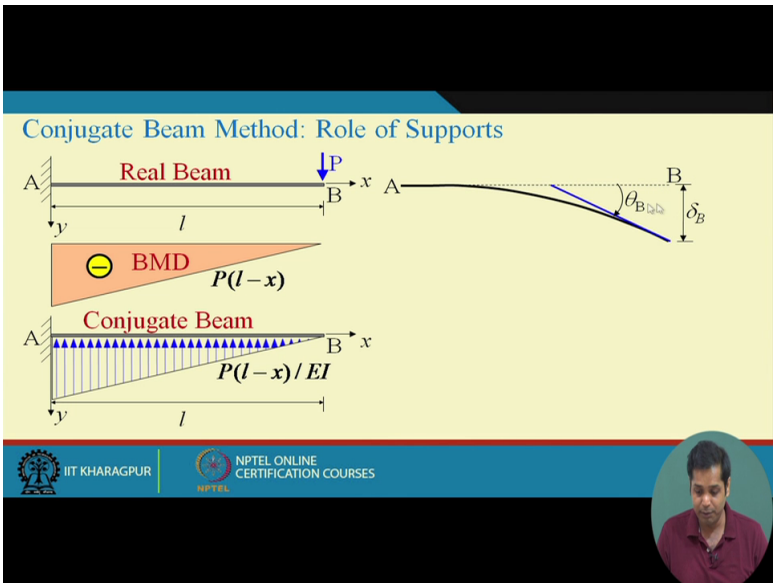

Now what we know is that theta B will be the shear force of the conjugate beam at B. So shear force of conjugate beam at B which is VB and we also know conjugate beam theory. The method tells us that delta B will be the bending moment (con) of conjugate beam at B, okay.

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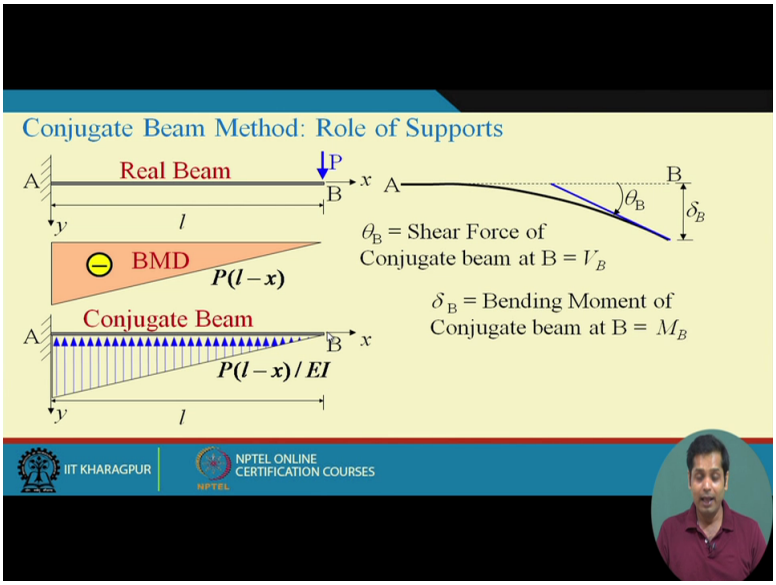

Now let us visually inspect the beam. It is a cantilever beam subjected to a triangular loading distribution and the load is zero here and this linearly varies and maximum at the fixed end, okay.

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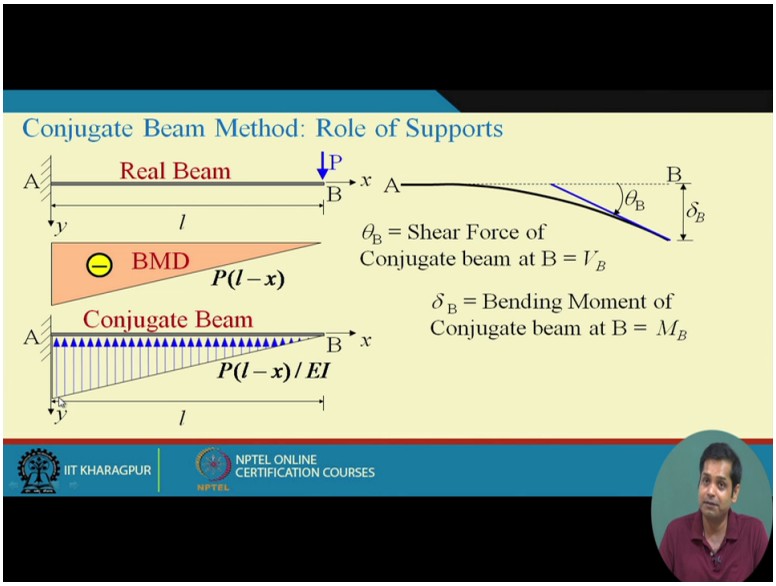

By just looking at this diagram we really do not have to do any calculation. By just looking at the diagram and applying our engineering sense that we have developed, we can say that shear force at B will be zero, okay. And we also can say that bending moment at B will be zero, okay. Now if shear force at B is zero and bending moment at B is zero but just now we

have seen that shear VB will give us theta B and MB will give us delta B, does that mean that at this point slope and deflection is zero?

Naturally not because the beam deflects like this, slope and deflection they both are non zero, okay.

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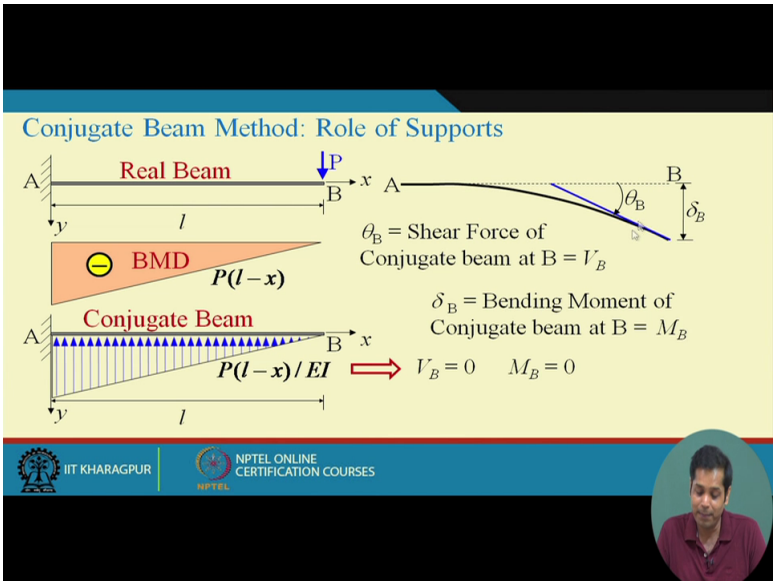

Then what went wrong in this. Because we know that theta B and delta B are non zero of real beam. Then what went wrong in this? Means as of the concept of conjugate beam what we have seen so far based on that and same form if we apply then we can see that, no that this is not applicable for this beam. So this means this cannot be the conjugate beam of this, okay.

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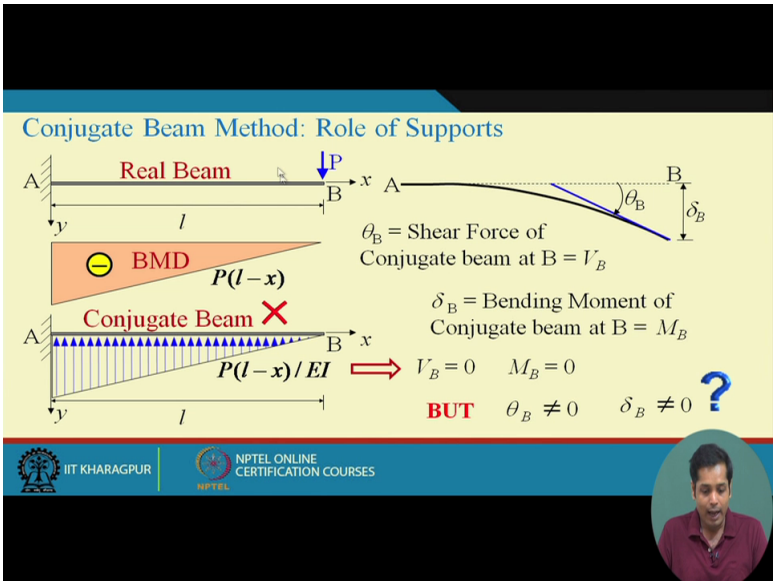

Now why it cannot be the conjugate beam? Because one aspect of conjugate beam is the loading. That loading on the conjugate beam is M by EI diagram of the real beam and another aspect of the conjugate beam is the support condition. Is it that whatever support we have in the real beam, is it the same support the conjugate beam also have or the conjugate beam has different kinds of support? Okay. So now next you will see that in order to form a conjugate beam, two steps are required.

The first step is replace the load by M by EI diagram and the second step is provide the support accordingly, okay. It is not that the same support that we have in the real beam, the same support we need to apply for the conjugate beam, okay. Now so this next part is what is the role of support? Okay. Now role of support, let me explain this through this.

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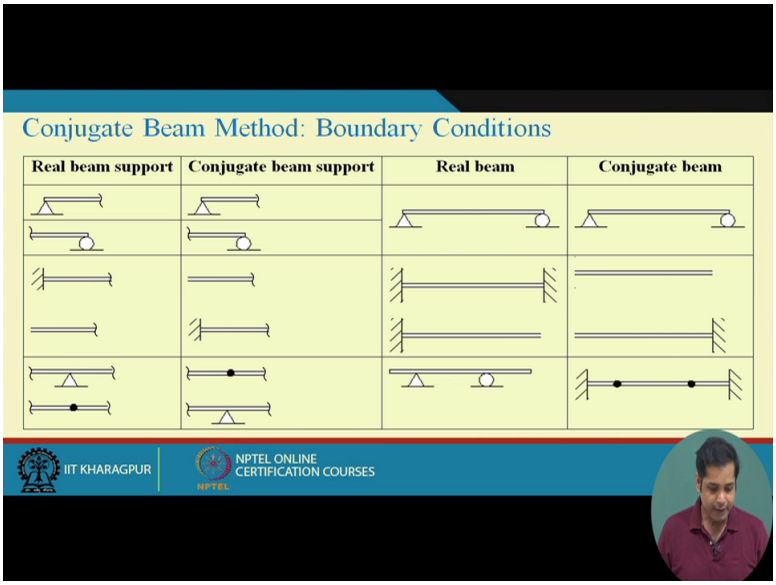

Suppose these are different kinds of support, okay. This is hinge support, roller support, fixed support and this is without any support means free end, this is an internal support and this is an internal hinge, okay. These are the possible different kinds of support that we can have in a structure, okay.

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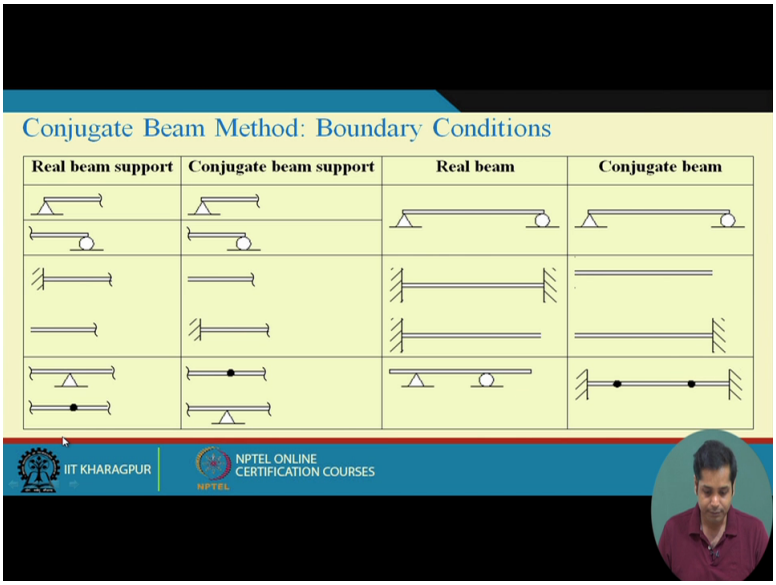

Now what this support says? This support says that since it is hinge support deflection at mid point is zero and slope is non zero, right? Now deflection is related to bending moment in conjugate beam and slope is related to shear force in conjugate beam, right? Now since in this case the deflection is zero means in the conjugate beam the bending moment at this end should be zero.

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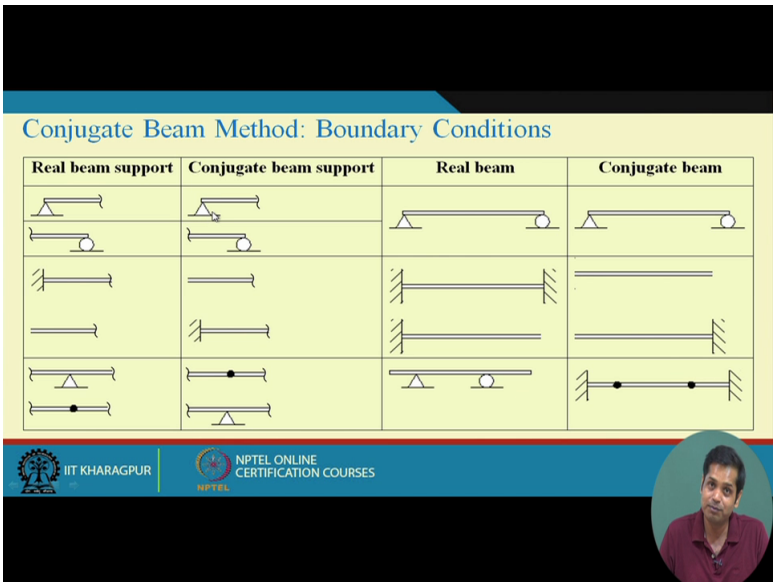

Since at this real beam support slope is non zero then the conjugate beam shear force has to be non zero. Now you think a support condition where your bending moment is zero but shear force is not zero. In the previous class I showed you some bending moment diagram

and I asked you to just looking at the bending moment diagram try to understand the loading condition and the support condition. That exercise will help us now to do this.

Now we need a support condition where bending moment is zero but shear force is not zero. Naturally that support condition will be hinge support, is not it? Now similarly suppose this is a roller support and what we have in the roller support? Roller support is this deflection in y direction is zero here and slope is non zero, okay.

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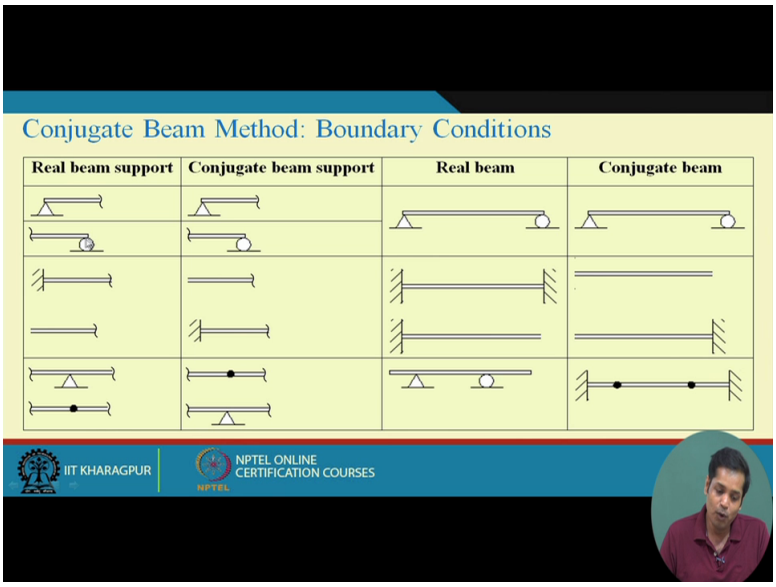

Now deflection is (deflect) related to moment since deflection in y direction is zero. So the support will be such that does not have any constraint against moment. And since the slope is non zero here so the support condition should be such that the support has non zero shear. So we need a support where shear is again non zero but moment is zero. So this is a roller support. Now let us see this beam, this is a fixed support.

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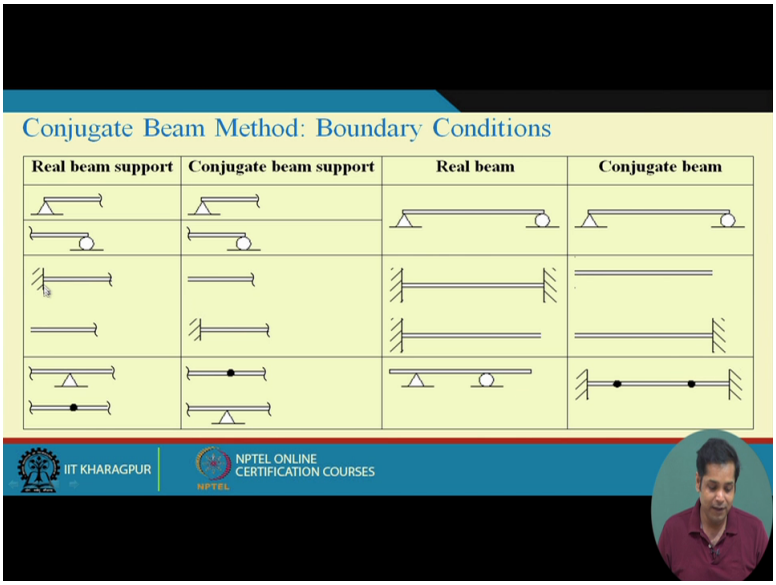

Now this is the fixed support in the real beam. And what are the characteristic of fixed support? Characteristics of fixed support is all deflections and rotations are zero. So in this case theta is zero and delta is zero. So naturally in a conjugate beam since both are zero, deflection and slope in a conjugate beam moment and shear both have to be zero. Now what are the supports that we have? Moment and shear is equal to zero. That is free end, okay.

You remember a cantilever beam and the tip of the cantilever beam is free end. It is the tip of the cantilever beam. This is the free end, okay.

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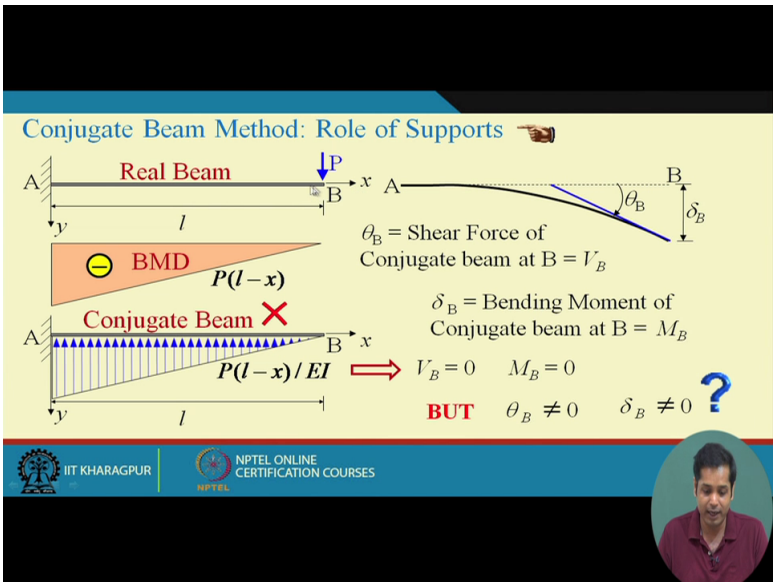

Now so this is the real support and this is the support in conjugate beam. Similarly if this support is free end. Suppose in this case this support is free, okay. Now if this support is free what would be the support in conjugate beam? The free means it can have slope, it can have deflection both. So both are non zero. Similarly in the conjugate beam support shear force and bending moment both have to be non zero and both non zero in where it is fixed.

If the support is fixed support then bending moment and shear force both are zero. Now if it is a hinge support at intermediate location what it tells you? It tells you that this real beam support in the conjugate beam becomes this.

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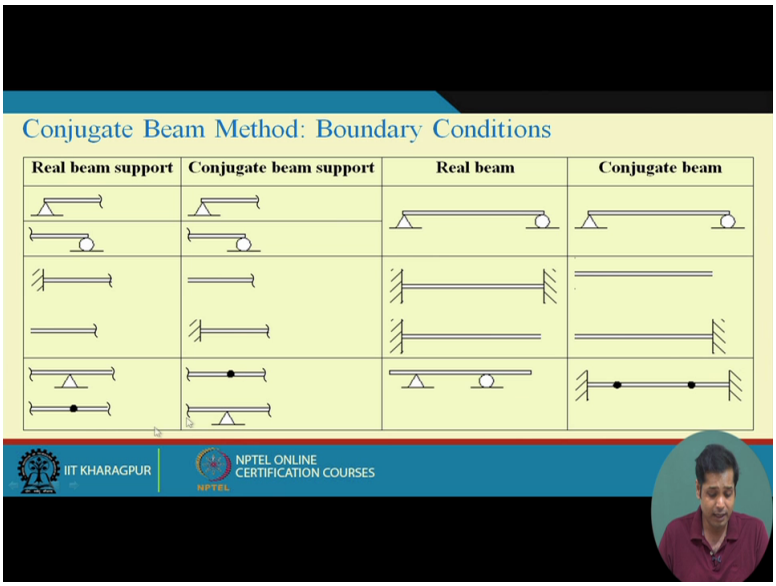

And similarly if the real beam is an internal hinge, what is the internal hinge in the real beam? In real hinge rotation is possible but in this case now rotation is possible means what? Rotation corresponds to shear force. So your beam will be such that shear force is non zero.

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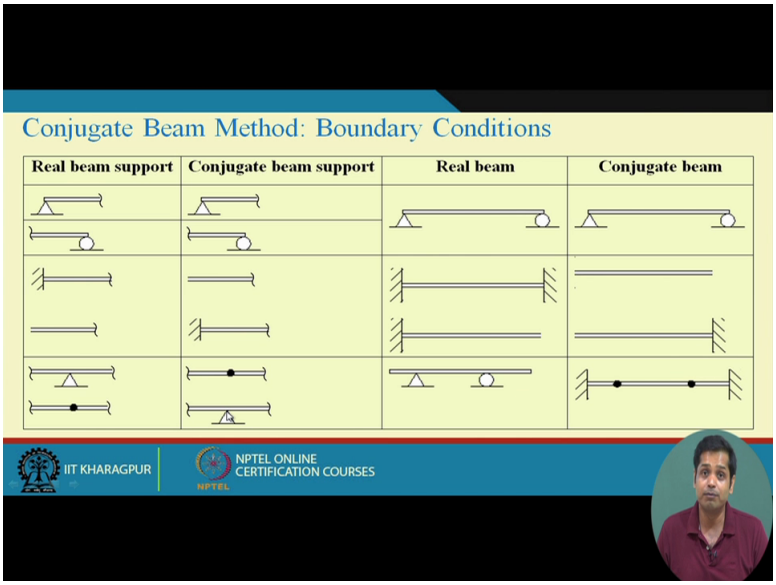

So these are the supports in the real beam and this is the corresponding support in the conjugate beam. Now some example if this is a simply supported beam is a real beam and the conjugate beam also become simply supported because hinge support in the real beam, conjugate beam also it is hinge support. And roller support in a real beam, just now we have seen the conjugate beam also increments on roller support.

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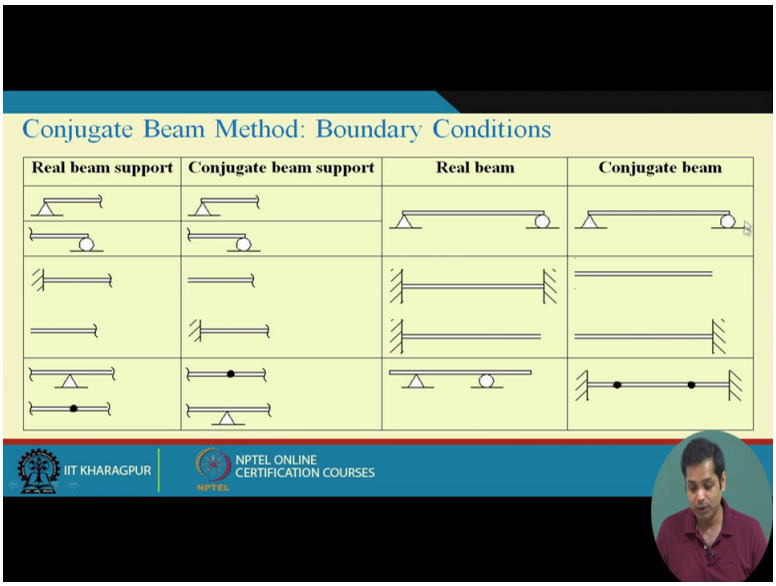

So if the beam is simply supported real beam and the conjugate will be also simply supported and remember in the last class we first applied the conjugate beam concept on a simply

supported beam subjected to UDL and we checked that the concept is working fine because if the beam is simply supported then the conjugate beam also remains simply supported.

But when we apply this same concept for a cantilever beam it did not work because of this reason. If it is a cantilever beam in a real cantilever real beam so the fixed support becomes free and this free support becomes fixed, okay.

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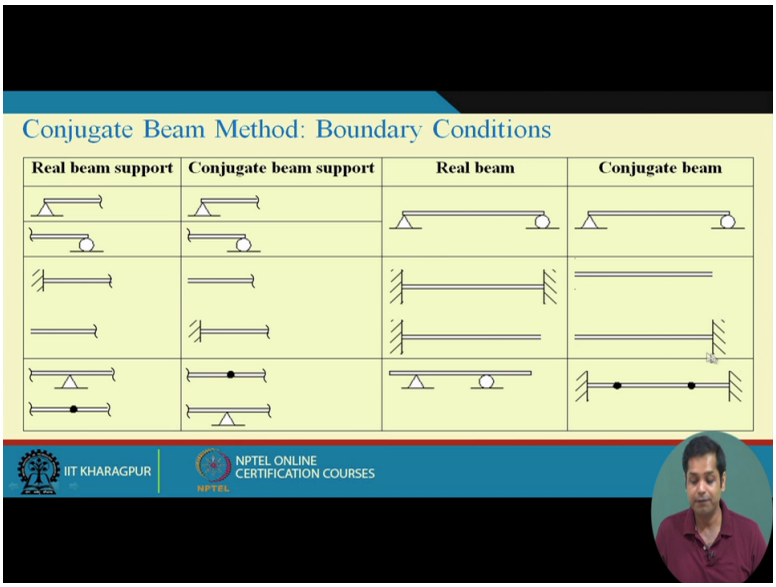

So if this is the real beam, conjugate beam will be of this. Just the mirror image of this. But remember in this example what we did is if this is fixed that also need to fixed, if this is free we took free here.

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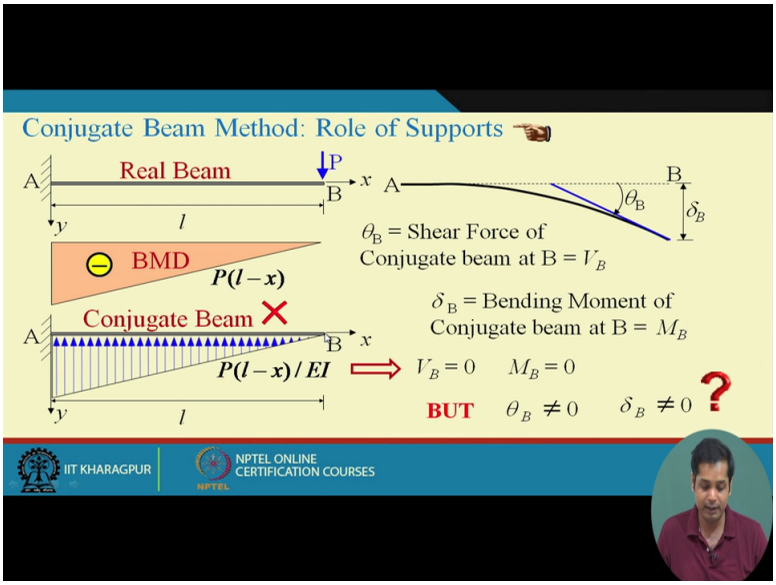

So the loading is according to the conjugate beam concept but the support condition was not. That is why the result did not come. So if this is the real beam then this is the conjugate beam, okay.

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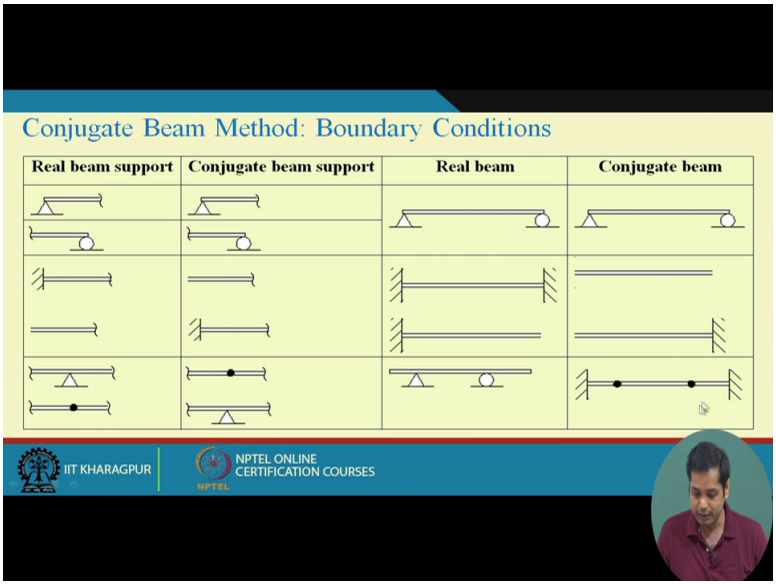

Now so this is very important because many places we make a mistake to identify what supposed to be given in a conjugate beam. So please be careful on that. Now just quickly summarise the concept behind conjugate beam method. The next class we will have some example on conjugate beam method. Now probably this is the repetition but let me repeat what the concept behind conjugate beam method is.

It is the process where the problem of kinematics is formulated as problem of statics and how it is? Instead of solving the real beam, we determine the internal forces in the conjugate beam and these internal forces are conjugate to the kinematics variable of the real beam. Shear force is conjugate to rotation and slope and the moment is conjugate to deflection. Now how to form a conjugate beam?

There are two important steps for forming a conjugate beam. The first step is to take the length of the beam as it is then draw the bending moment diagram of the beam and then the M by EI diagram will be the distributed force acting on the conjugate beam. The first step in conjugate beam is apply the load on the conjugate beam and that load will be the M by EI curvature diagram of the real beam. This is the first step. And the second step is the support condition of the conjugate beam.

You check the support condition of the real beam and check what are the kinematics constant you have at those support condition and (bas) based on that you decide what will be the support condition on the conjugate beam. Now once this conjugate beam is formed then rest of the process is just determination of internal forces in statically determinate beams, okay. Now what you do next class is next class quickly apply this concept to some examples, okay.

And if you see any book this concept of conjugate beam probably is given in a very elaborated way so please go through that and make yourself comfortable with the concept, okay. See you next class. Thank you.