Structural Analysis 1
Professor Amit Shaw
Department of Civil Engineering
Indian Institute of Technology Kharagpur
Lecture 22
Deflection of Beams and Frames (Contd.)

Hello everyone welcome to week 5. So what we will do this week is we will learn one more method for finding deflection of statically determinate beams which is conjugate beam method. Then we will briefly review the concept of bending moment and shear force diagram in frames. We have already done that for beams. And then finally we will determine the deflection for beams and frames via virtual work method.

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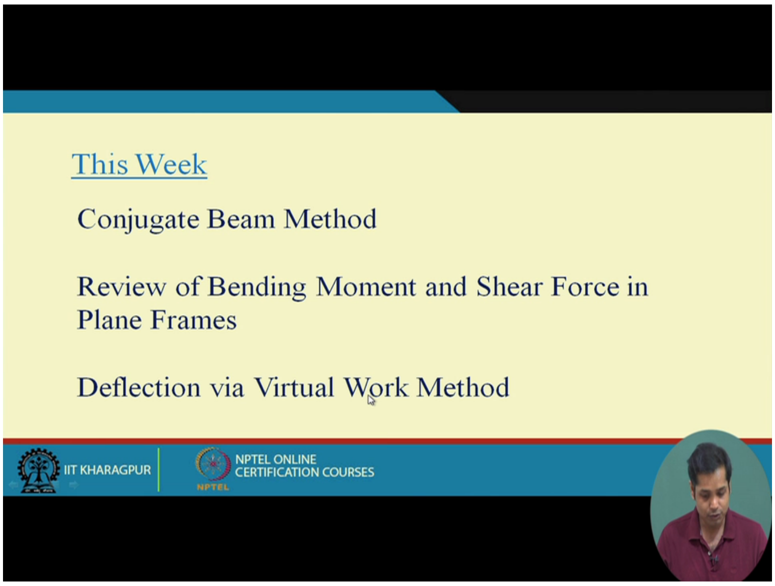

So today we will start with deflection of beam via conjugate beam method, okay.

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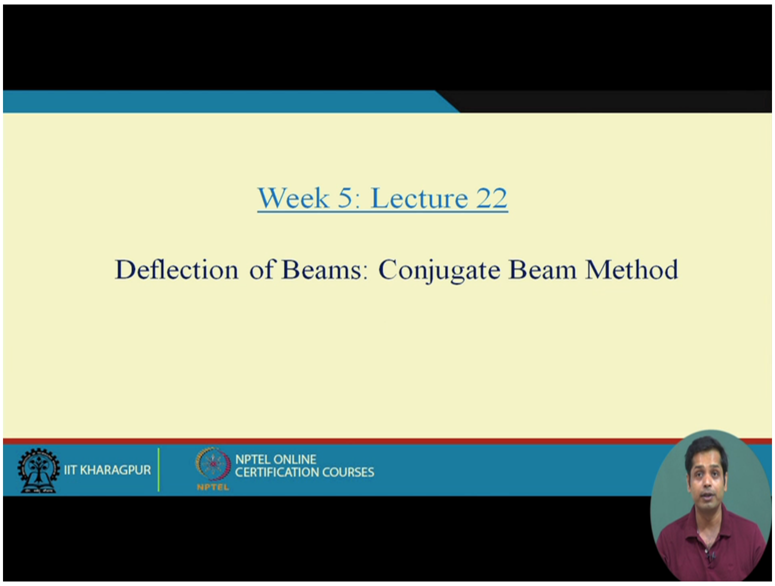

Okay let us first give you the basic or let us first set the platform to derive the concept for conjugate beam method. You see suppose take any beam, in this case it is simply supported subjected to some loading. But that we will see this concept is same for any boundary condition and any loading pattern. So suppose this is a beam which is subjected to distributed load like this.

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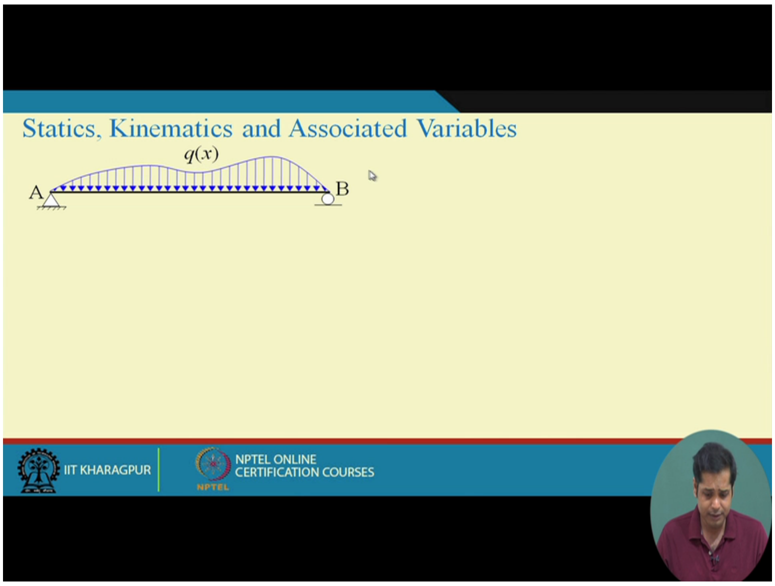

Then what we have done so far? We know that suppose this is a bending moment diagram of this beam.

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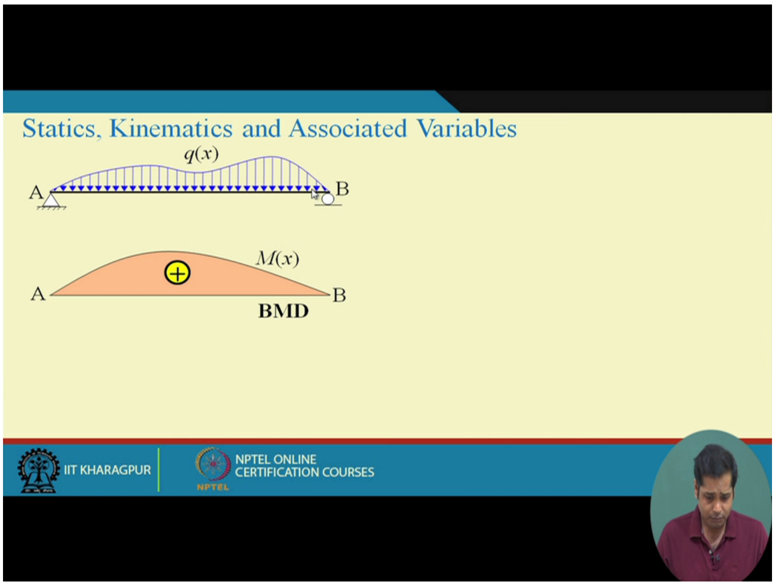

Now as far this loading condition is concerned the beam will deflect downward. So the beam will sag, which is according to our sign convention positive. So this is positive bending moment. We also know how to (deter) draw the shear force diagram for this. So this is the shear force diagram for this. This is the typical representative bending moment and shear force diagram for this kind of loading condition, okay.

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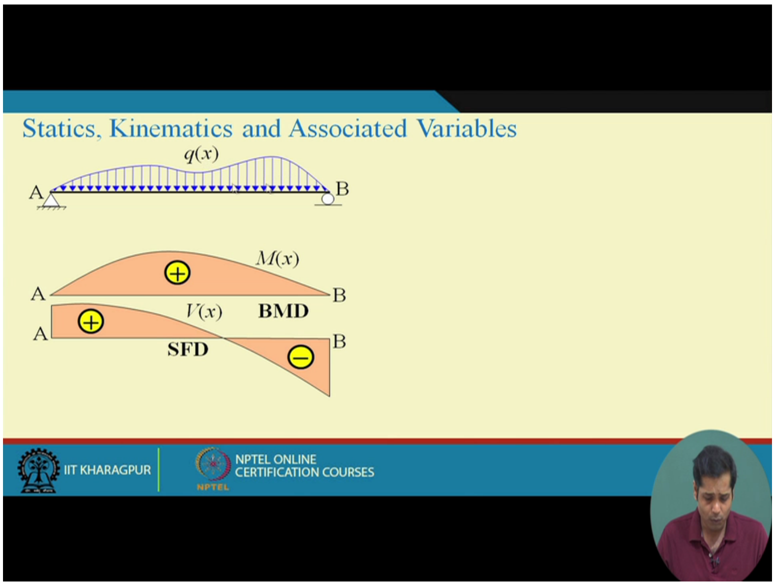

Now so we know how to do that. For a given problem we know how to draw the bending moment and shear force diagram and this step is called statics, okay. For a given problem determine the internal forces.

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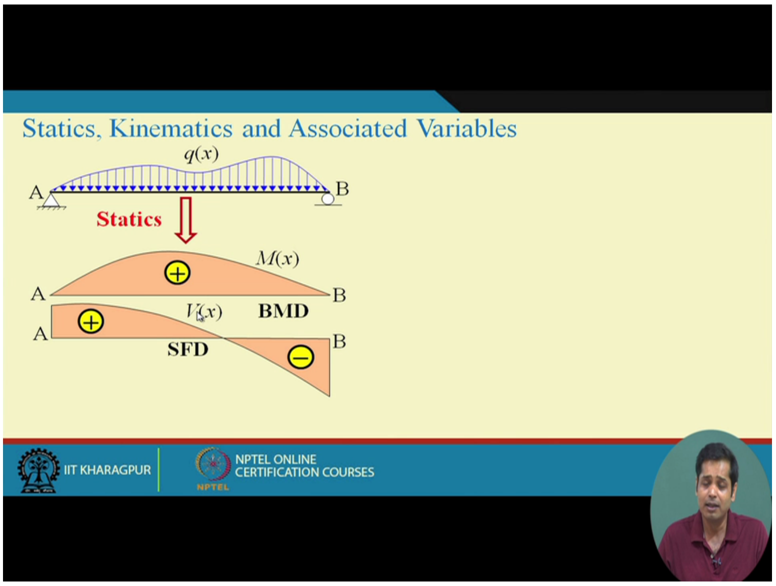

Now from this to get the internal forces what plays a major role is the equilibrium equation. So we use the concept of static equilibrium to get these internal forces, right? Now in this what are the variables we have? The static variables are the applied intensity of the load q, bending moment M and V shear force. The bending moment and shear force may also change with length of the beam but for a given length these are the parameters which can influence the bending moment and internal forces.

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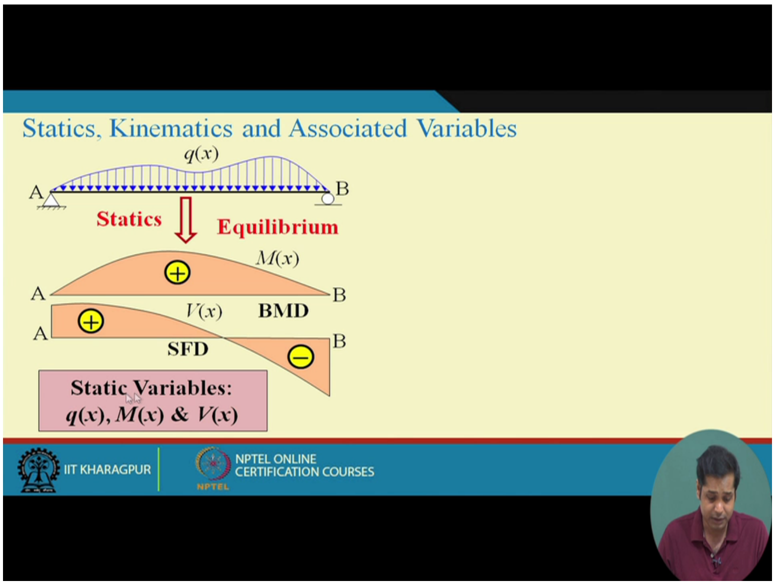

So in this entire step our static variables are intensity of the load, bending moment and shear force, right? Now this is not the only thing we want. What we want in addition to internal forces are deflection of the beam. Suppose this beam deflects like this, okay.

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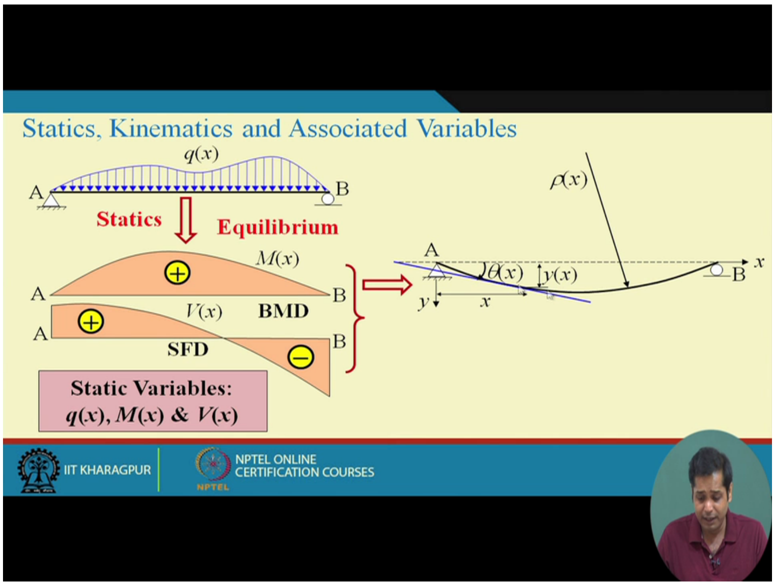

And what we want in addition to the internal forces? We want what is the deflected shape? How the (deflect) deflection changes with this? Then e how the slope changes with x? So this deflected shape also we want to determine, okay. So if you remember this is from the internal forces from safety point of view and this is for the deflection from serviceability point of view.

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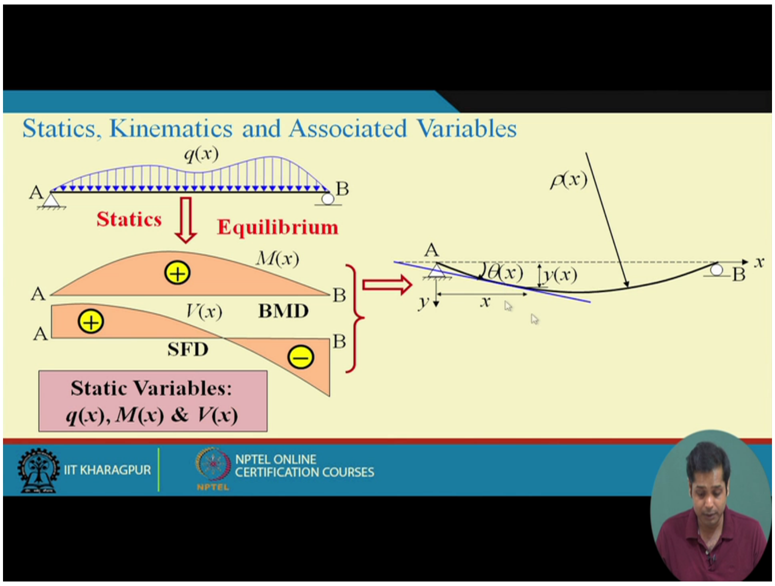

So these are the two responses we want from structural analysis. Now from this step once we know the internal forces on a structure and from that internal forces (app) determine the deflection. This step is called kinematics. Kinematics essentially is the branch where we determine the motion of an object as a relation with the forces that causes the motion. In this case motion is the deformation or deflection of the beam, okay. So this step is called kinematics.

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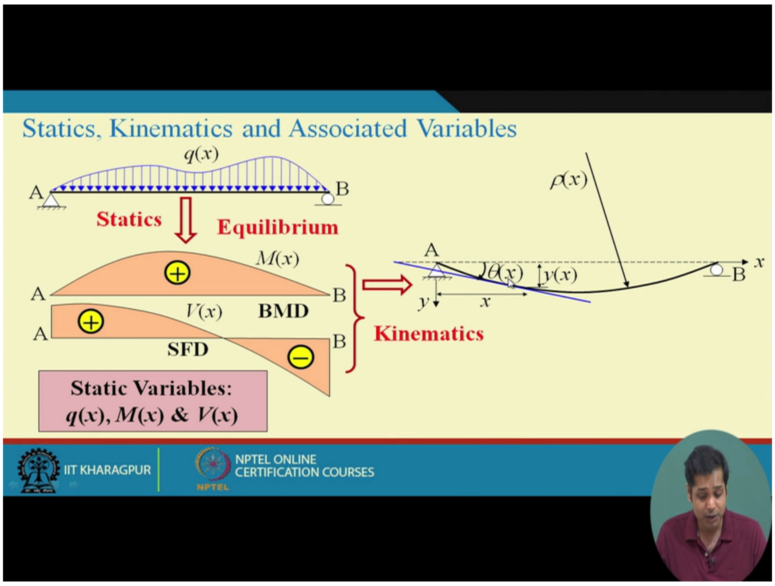

And what plays the major role in kinematics is the stress strain relation or load deflection relation. We know how these internal forces are related to deflection. And (gi) in this course we assume it is linear elastic stress strain relation and (04:57). Now what are the variables in kinematics? The variables are, if you look at this figure what are the things we want? At a given particular point we want what is deflection of that point?

What would be the slope at that point and what would be the curvature of the point? Curvature is 1 by rho. Rho is the radius of curvature. So we want the curvature, slope and deflection of the point, okay.

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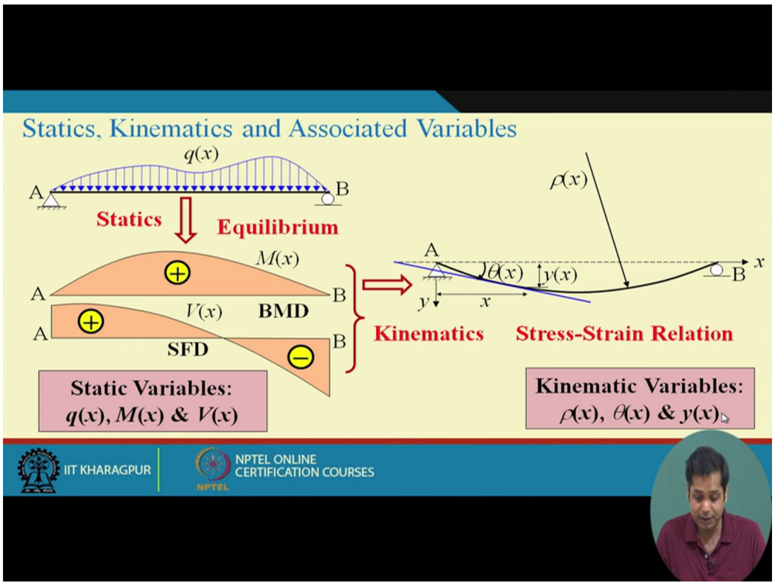

So these are static variables and these are kinematics variables. Is there any relation between static and (varia) kinematics variables and have we studied that relation? Yes we have. You see that relation is the elastic curve, okay. So elastic curve gives you how the internal forces varies with the deflection or in other way how the static variables related to kinematics variables.

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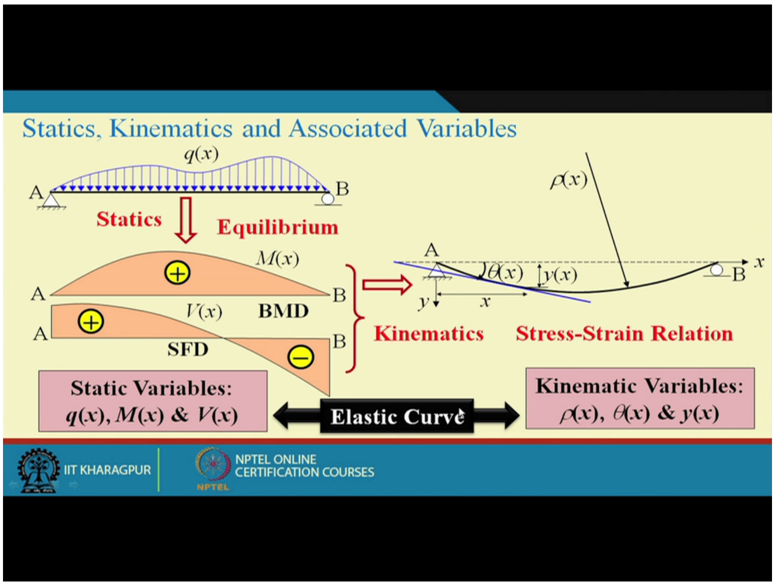

And if you observe from the last week whatever method we studied in determination of deflection of beams, the premise of all the method is elastic curve. They use the equation of elastic curve in a different way. Now so this was we have done. We have done directly integrating the elastic curve.

We have also done the moment area method which is another interpretation of elastic curve and the method that we are going to study today is conjugate beam method. Now (intr) before I introduce conjugate beam method you see there is a similarity, the static variables are related to each other and the kinematics variables are also related to each other. Means rho is related to theta and then theta is related to y. Similarly q is related to moment and moment is related to shear and all these variables are related to each other.

Now what we can see there is a very interesting similarity between the way static variables are related to each other and the way kinematics variables are related to each other. And conjugate method essentially makes use of that similarity. Now just to make it clear suppose let us recall the equations for static. We already derived these equations that how shear force is related to bending moment.

Then if you remember that all the bending moment diagram we observed, the bending moment is one order higher than the shear force. So this we derived already.

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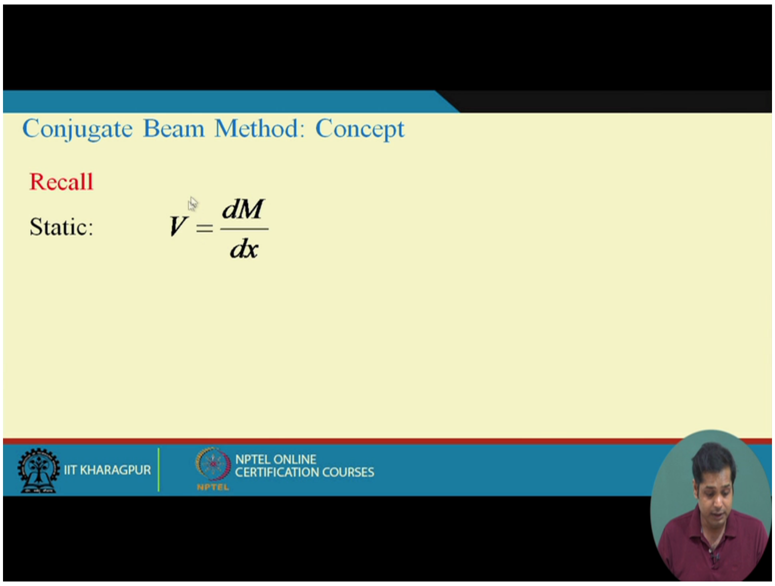

And we also have another relation that dV dx is equal to minus q. Q is the intensity of the load of the beam. So this is how shear force related to intensity of the load. These are all static, V, M and q these are all static variables and these are the way the static variables are related to each other.

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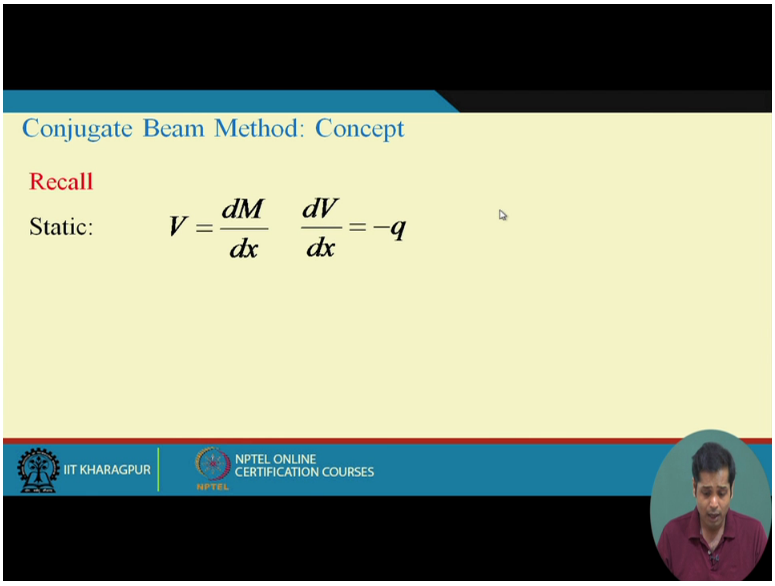

Now let us see what happens to kinematics variable. We know theta is equal to dy dx. That is the definition of slope and then we also derived that how the d theta dx is equal to curvature. Remember this minus sign is because if the curvature increases the moment decreases or the moment increases curvature decreases.

That is how our sign convention is. That is why it is negative sign. Now so this is theta y and 1 by rho, they are the kinematics variables and these kinematics variables are related to each other like this.

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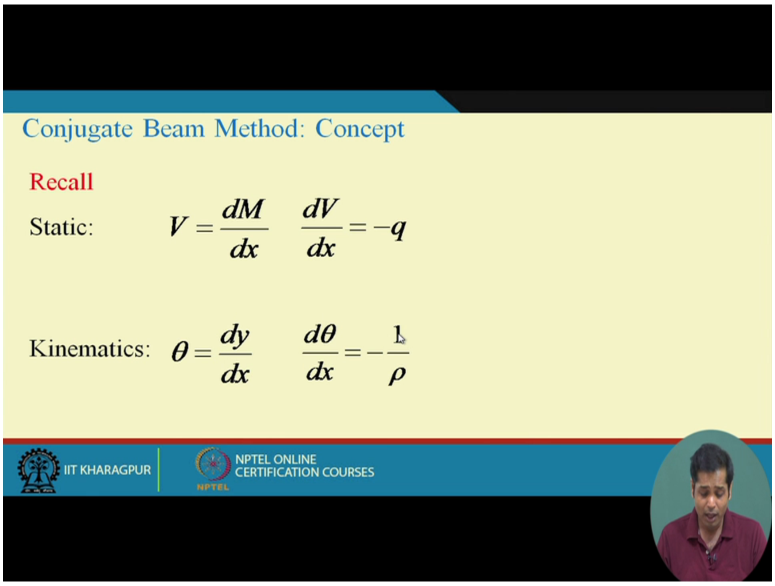

Now let us see what are the similarities? Now if we say V is equal to dM dx. Suppose see how V is related to M and how theta is related to y. Now V is equal to dM dx and theta is equal to dy dx and if we take V as static variable and it is related to static variable M, the way V related to M, theta is related to y in the same way, is not it?

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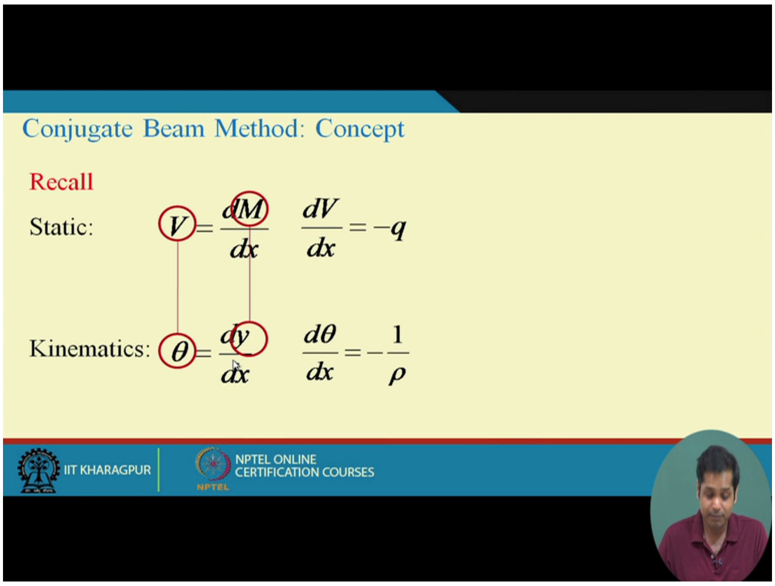

Now how V is related to q and how theta is related to curvature that is also very similar. So what we can say the shear force is the first derivative of moment and here theta is first derivative of deflection. First derivative of shear force is intensity of the loading, first derivative of slope is equal to the curvature.

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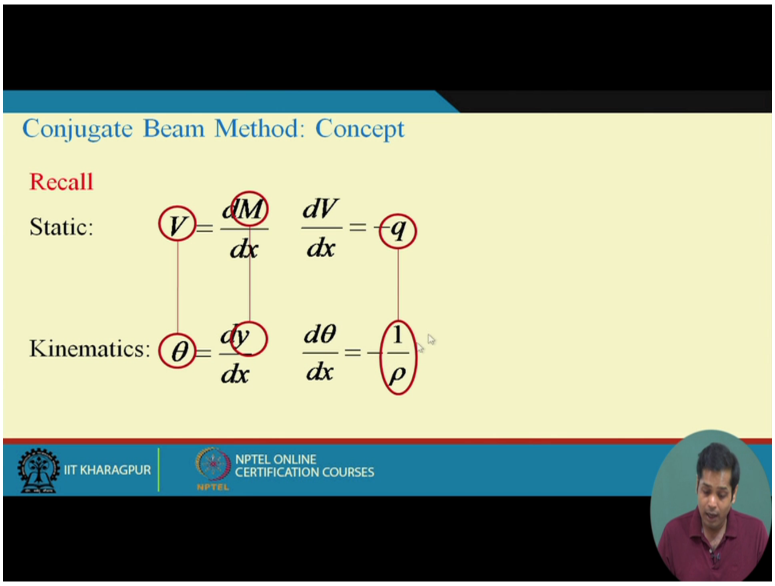

Now if we combine these static equations and kinematics equations and define a general equation like this. Suppose if we define an equation like this where three variables psi, phi and capital Phi. Psi is equal to d phi dx and d psi dx is equal to minus Phi.

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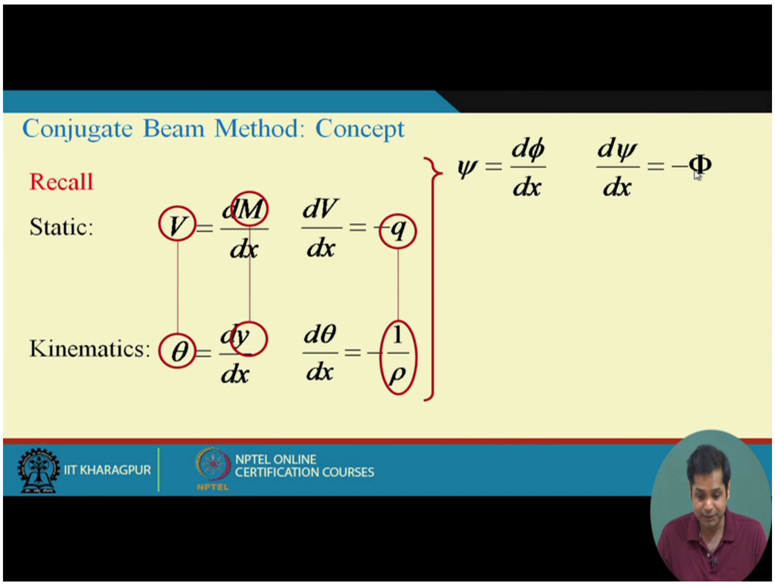

Now you see this is a general equation which can give you both relations. For instance for static case if Phi, psi and small phi if that is replaced by q, V, M the static variables then this equation is static equation.

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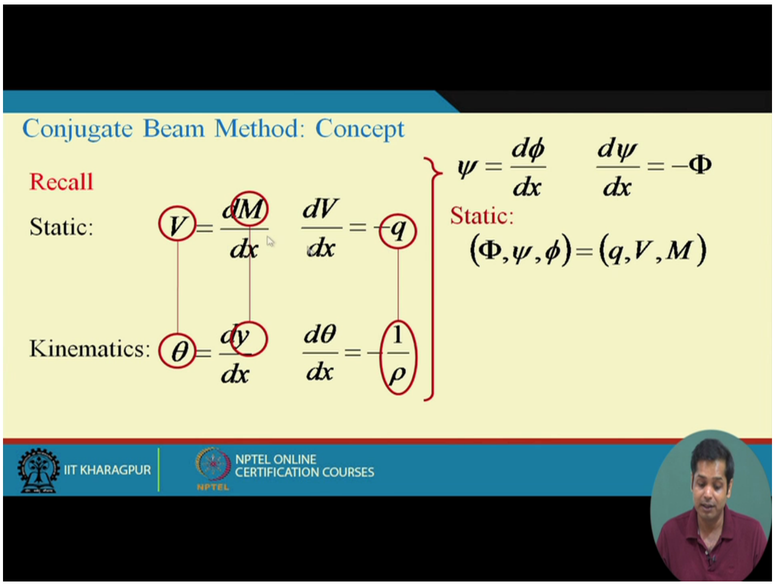

Similarly if this Phi, psi and small phi they are replaced by this static variables then this equation gives you this equation.

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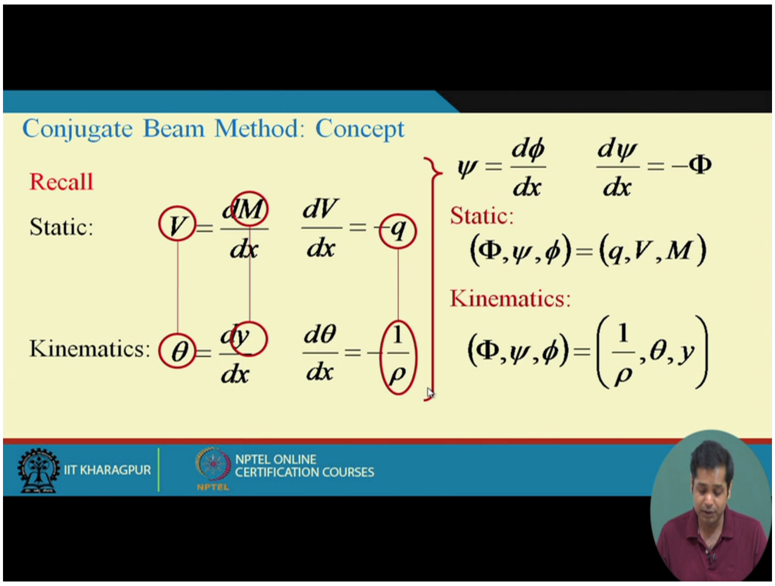

So this is the general equation which gives you static and kinematic equation depending on what psi, phi and capital Phi are chosen. Now what point I want to make here? You see if q is replaced by 1 by rho then V, the shear force will be replaced by theta and similarly moment will be replaced by y.

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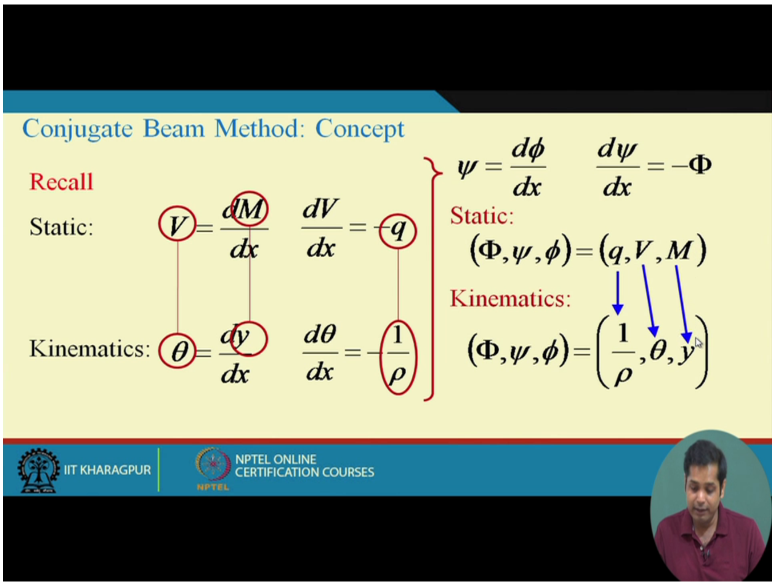

Or in other way suppose if we have two systems. It is system 1 and this is system 2, okay. Now in system 1 our variables are q, V and M and in system 2 variables are curvature, slope and deflection. And we also know that how these variables are related to each other and the same way these variables are also related to each other, right?

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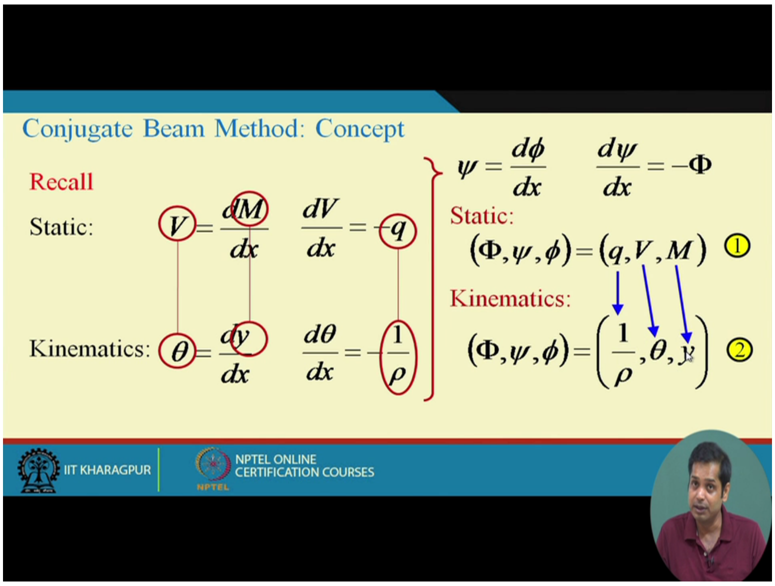

Now since in system 1 suppose if q is replaced by curvature, okay, if q becomes curvature then curvature of system 2 then (natu) automatically the shear force of system 1 becomes slope of system 2 and the bending moment of system 1 becomes deflection of system 2.

Means I repeat once again if intensity of the load in system 1 is replaced by the curvature of system 2 then the shear force of system 1 gives maybe slope of system 2 and bending moment of system 1 will give us the deflection of system 2.

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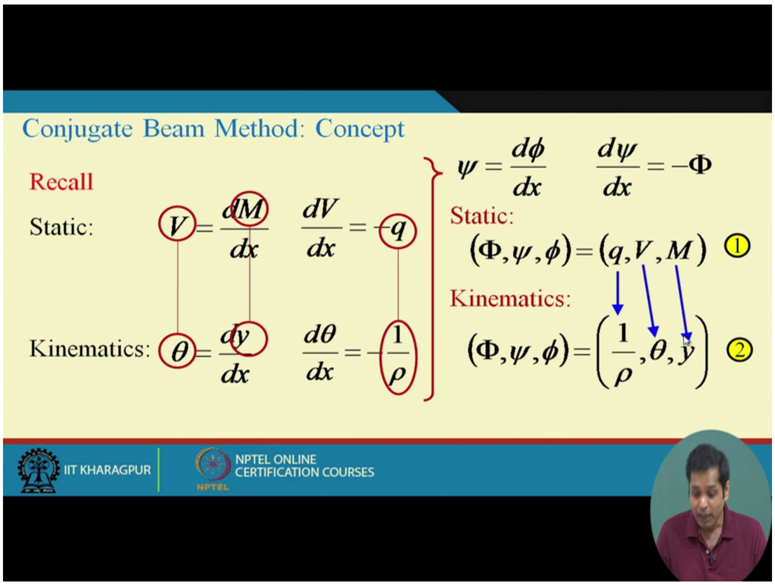

Now in this case it is called that system 1 is conjugate of system 2. So how this conjugate is obtained? Conjugate is obtained by replacing q by curvature of system 2. So only thing we have to do is in order to get the conjugate system 1 we have a system 2. Now in order to get the conjugate system 1 what we have to do is we have to replace the intensity of intensity q by 1 by (ra) rho in the curvature of system 2. Now if we do that then what you have to do?

We determine the internal forces in system 1. Internal forces are V and M. So since the 1 by q is replaced by 1 by rho, internal forces in system 1 will give us other two kinematics variables of system 2, theta and y. And this is conjugate beam theory.

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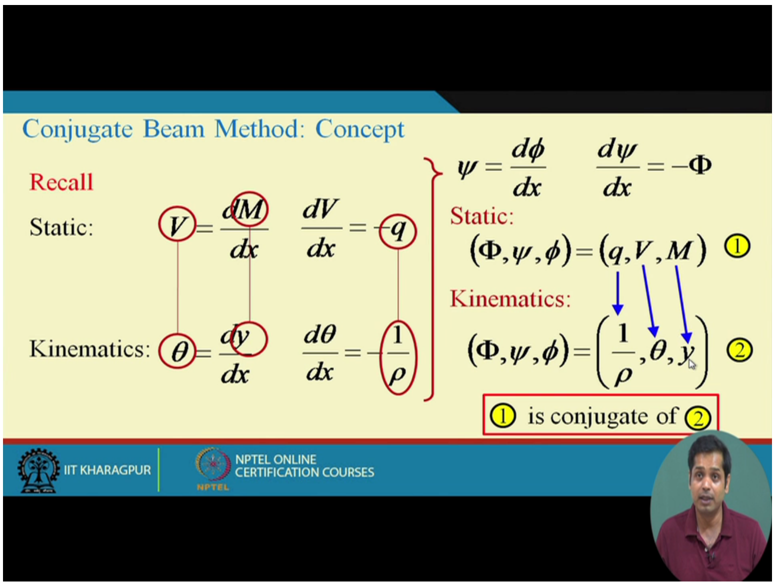

Let us demonstrate this through an example. Now take a simply supported beam which is subjected to uniformly distributed load. We have seen this problem several times. Now the bending moment diagram of this beam is this and we know how the bending moment changes with x is this, okay.

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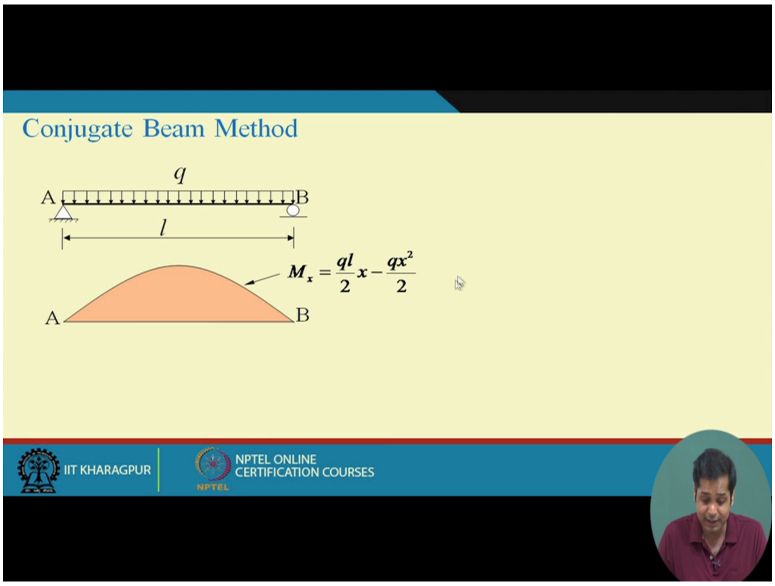

We have already seen it in various places. Then so (re) now consider a system. So this is the bending moment diagram for this system. Now construct a system where this same length of the beam, same dimensions, everything will be same but the load on this beam is replaced by M by EI diagram of the real system, okay.

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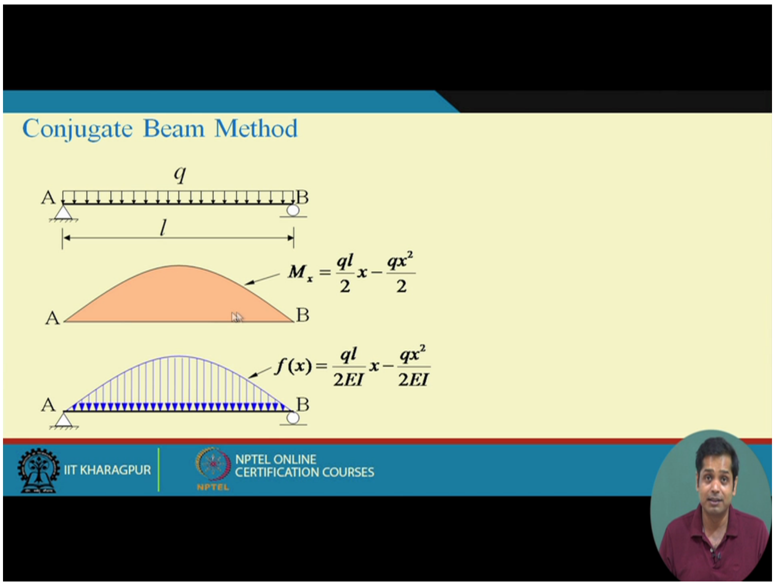

And if you do that so this is the real beam, this is a bending (mom) moment diagram of the real beam and this is the conjugate beam.

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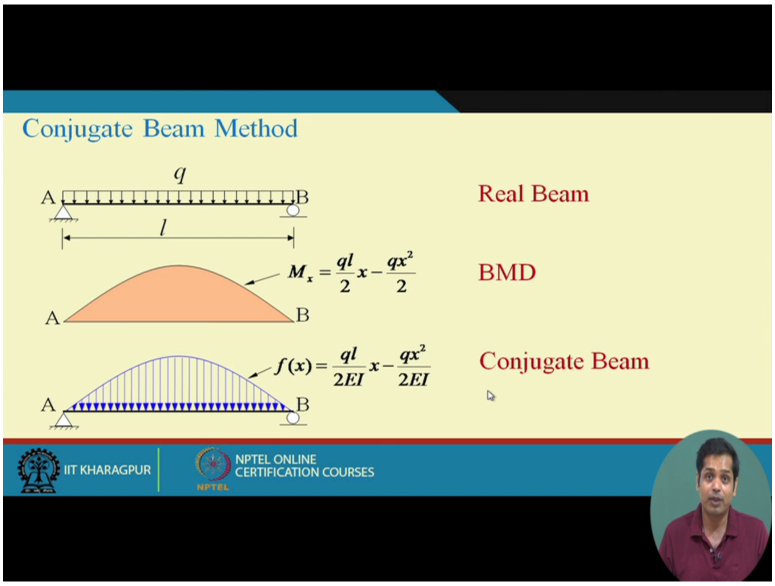

Now what the problem reduced to? Now initial problem was determine the deflection of this beam, okay. Now instead of directly computing the deflection of this beam what we have done is we constructed a conjugate beam where the length is kept same, only the load on the beam is replaced by the bending moment diagram divided by EI.

And then what we have to do is we have to find out the shear force and bending moment (diag) of this system because shear force of the system will give us slope of the system and bending moment of the system at any point will give me the deflection of this system at that particular point, okay.

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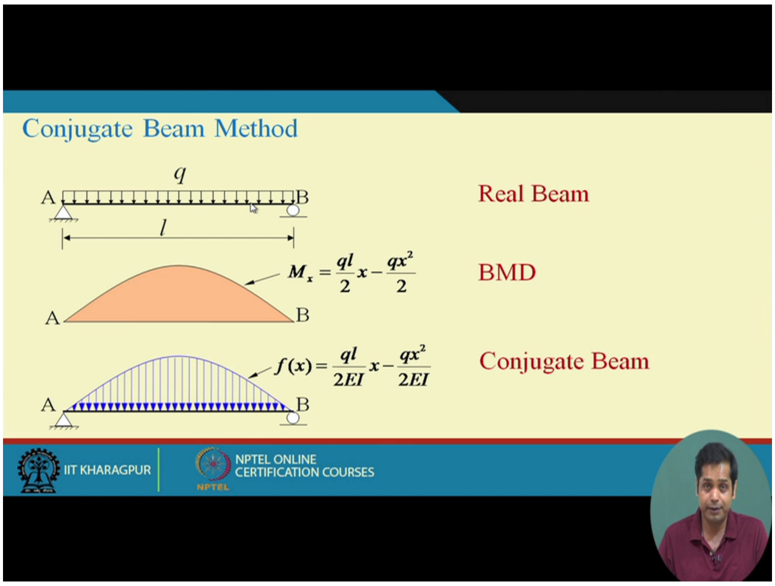

Now this is conjugate beam method. Now this is the conjugate beam where in real beam it was uniformly distributed load. Now in conjugate beam the load is replaced by the bending moment by EI. This was the bending moment M by EI. How the M by EI changes with the x and we also know that the maximum moment is qL square by 8. So this point becomes qL square by 8 EI.

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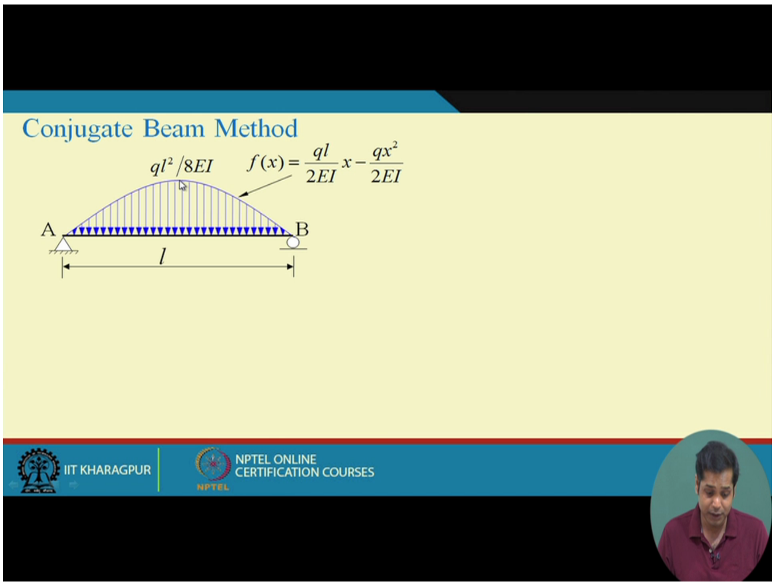

Now let us draw the free body diagram. Now only thing is we need to find out the internal forces of this. So a problem of kinematics now is formulated as problem of statics, okay. Now determination of internal force is probably easier than finding out deflection and slope in a structure. Now so draw the free body diagram of the entire structure. So this is hinge support so automatically these are the two forces and then the roller support is one force. This is a free body diagram.

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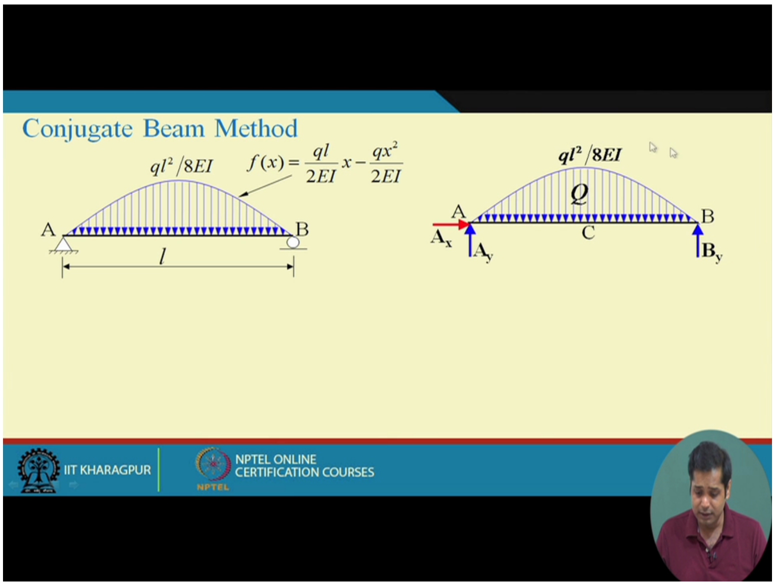

Now so this is the total Q. Total Q will be the area of the entire parabolic distribution and the Q will be integration of Fx between zero to L. And so either you can directly do the integration or we know the expression for parabolic area. So this is two third of this length into length. This will give qL cube by 12 EI. So this is the total load on this, okay.

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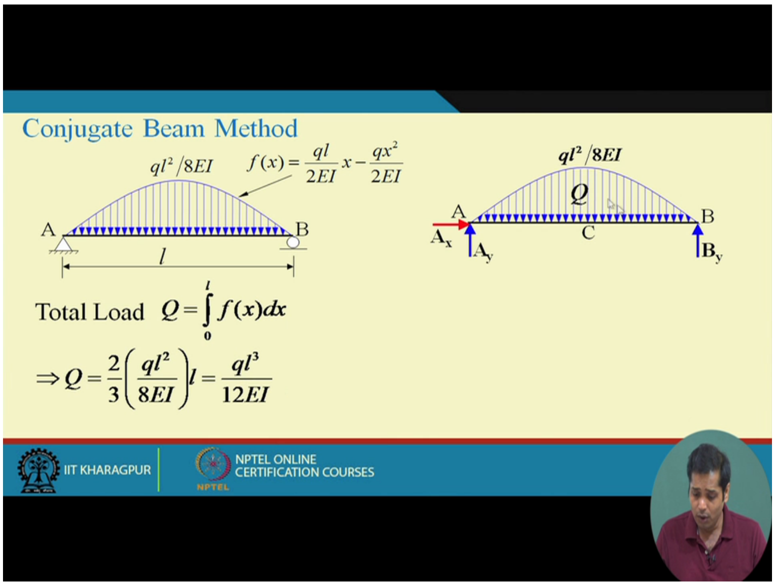

Now if we apply the equilibrium condition on the free body diagram of this so summation of Fx is equal to zero which automatically gives us Ax is equal to zero. Summation of Fy is equal to zero since the symmetric beam, so Ay will be equal to By and Ay equal to By will be Q by 2, total load divided by 2. It will be shared by both the support and this will give us qL cube by 24 EI. This is the total Q we already obtained, okay. So these are the support reactions, okay.

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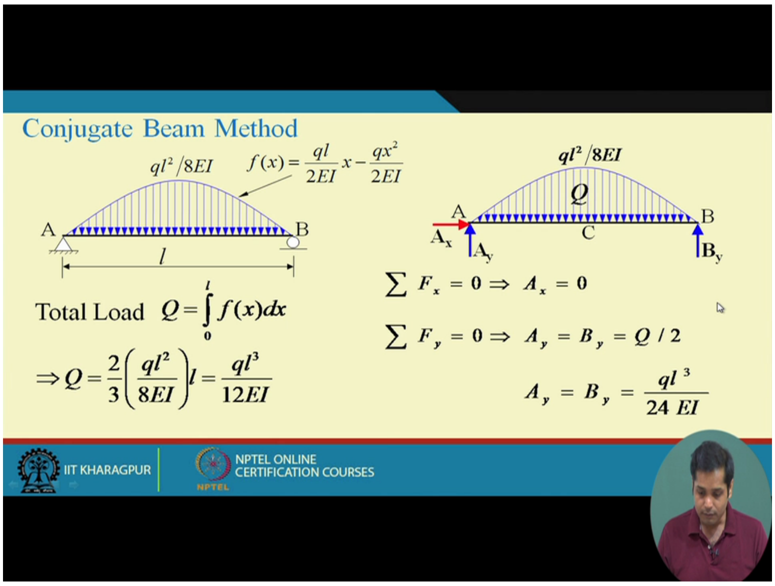

Now so these are the support reactions just now we obtained, right? Now this is the total Q. Now suppose I want to determine what is the bending moment and shear force at the midpoint, okay. How to do that? We take a section at midpoint C and draw the free body diagram of this part.

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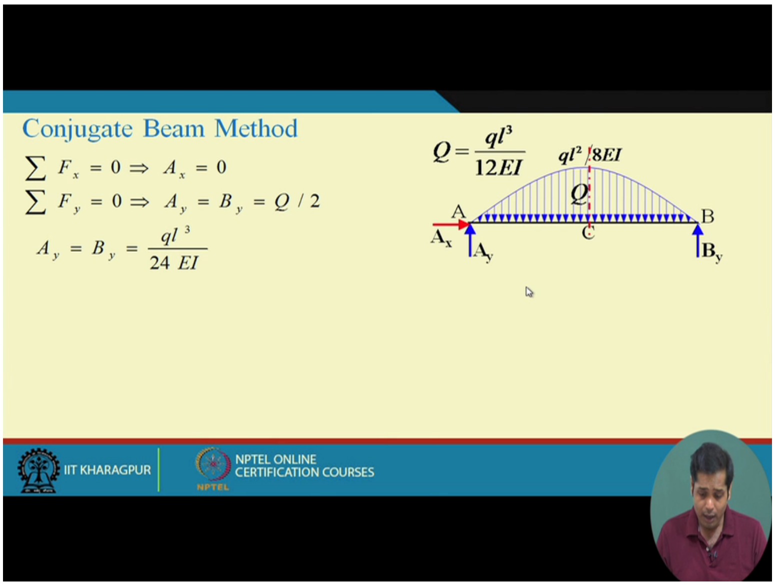

Now if you draw the free diagram of this part so this length becomes L by 2, this is qL square by 8 EI and this is the moment and this is the shear force.

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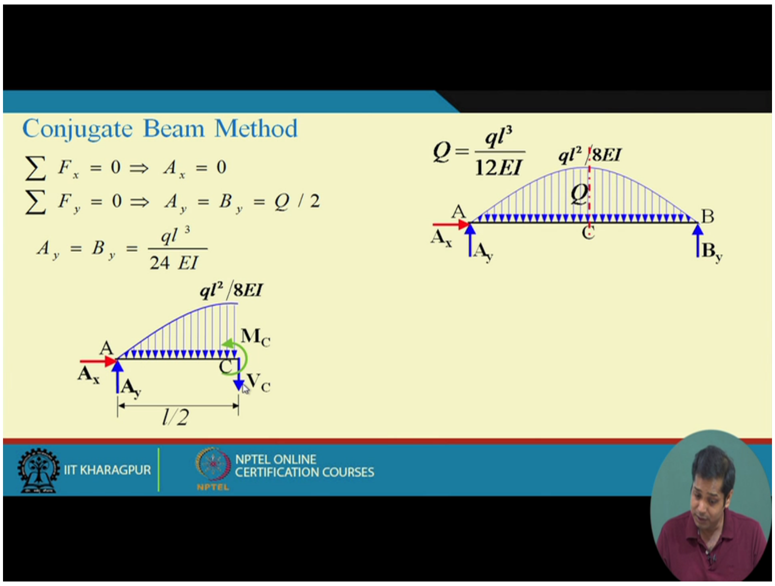

Please note the sign convention what we are using for moment and shear force. Moment is sagging moment positive and clockwise couple, shear is positive. And ideally there will be a horizontal force as well but since Ax is equal to zero that is why it is not explicitly shown here. So it will be anyways zero, axial force here.

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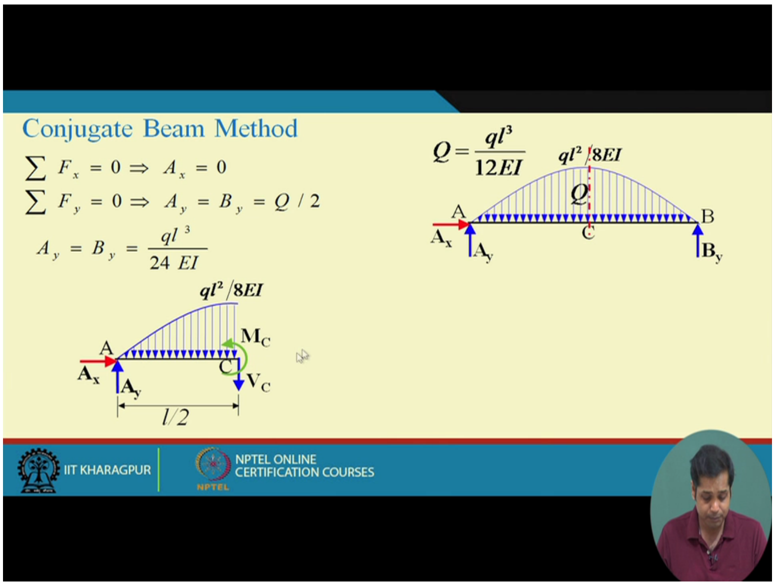

Now let us apply the equilibrium condition on this beam. Take summation of moment at C is equal to zero. So what it gives us? What are the contributions? Contribution will be from MC which is (an) anticlockwise, then contribution from Ay which is clockwise and contribution from the external applied load which is again anticlockwise.

So MC is anticlockwise minus, contribution of Ay will be clockwise that is why it is positive and then contribution from externally applied load again is anticlockwise that is why it is negative. So this gives us MC is equal to this, okay.

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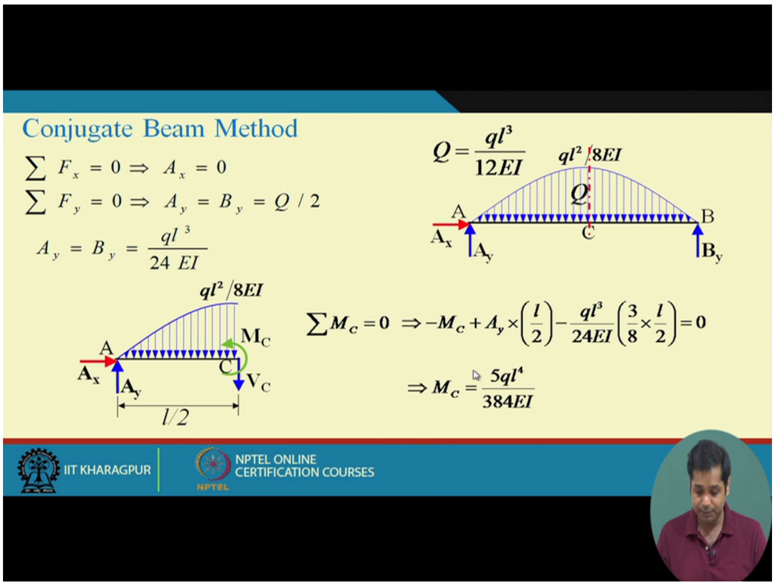

Now what we have? Shear at A is equal to the support reaction at A which is equal to qL cube by 24 EI that we already obtained what is the support reaction. What will be the shear at B? Shear at B will be minus By not AB, okay.

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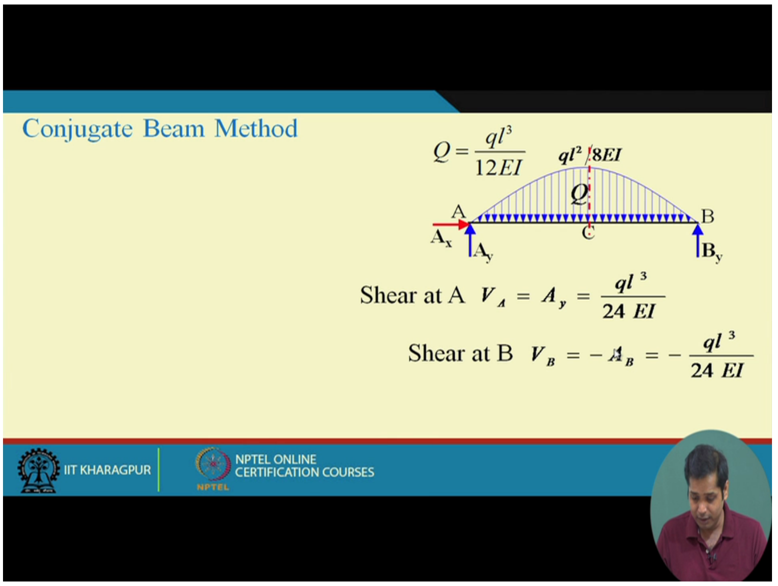

Shear at B will be minus By, why it is minus? Because that is what our sign convention is because our sign convention is shear which produces clockwise couple is positive. Now but By is taken upward positive here that is why shear will be minus By. And this is equal to minus qL cube by 24 EI.

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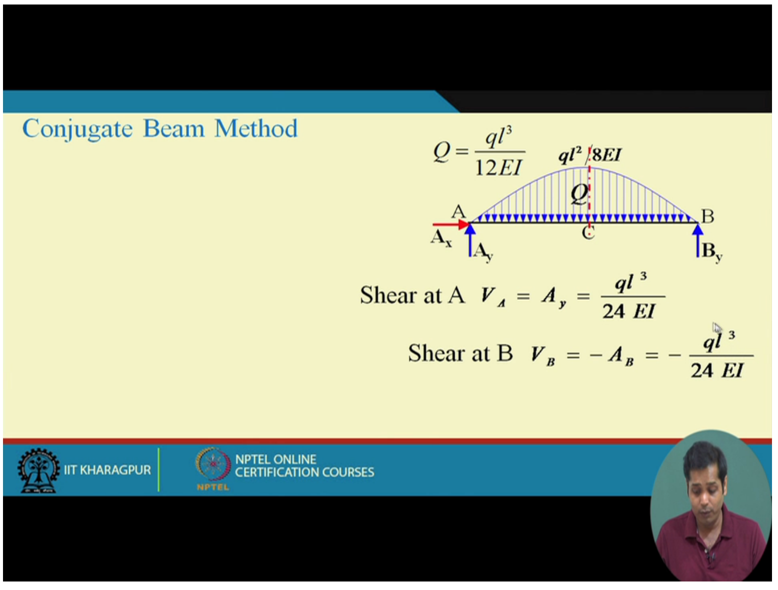

Please again and again I am telling you please be careful about the sign convention, okay. And also we have just now obtained that moment at C is equal to 5 qL cube by 384 EI. This will be again 5 qL to the power 4, not cube, L to the power 4 by 384 EI. Please correct it, okay.

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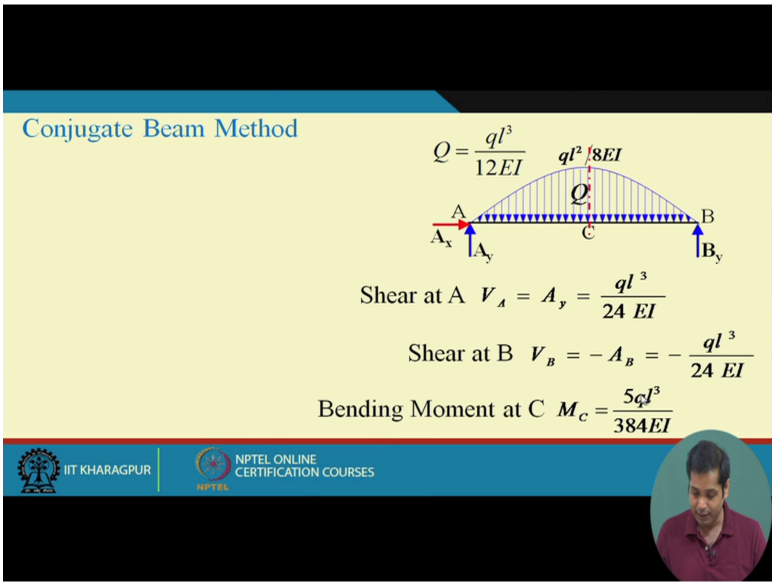

Now this is the real beam and what results we have if the real beam deflects like this and this is theta B and this (ang) distance is delta C, deflection at midpoint. Then we have already seen in this examples through other methods and we have these results that theta A is equal to minus theta B is equal to qL cube by 24 EI.

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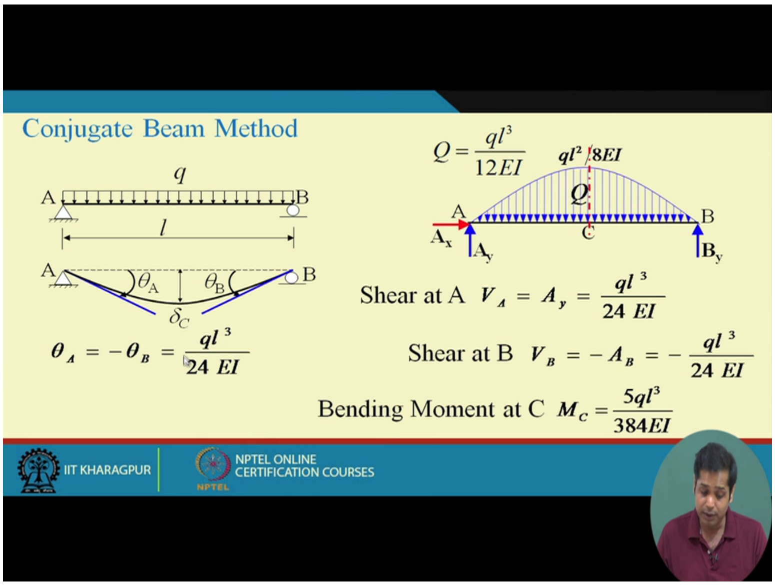

Theta A is positive because it is clockwise taken and theta B is taken anticlockwise that is why negative. This is consistent with the sign convention that we use for bending moment and deflection. And what is delta C? Delta C will be 5 qL to the power 4. Again please correct it. It will be L to the power 4 by 384 EI.

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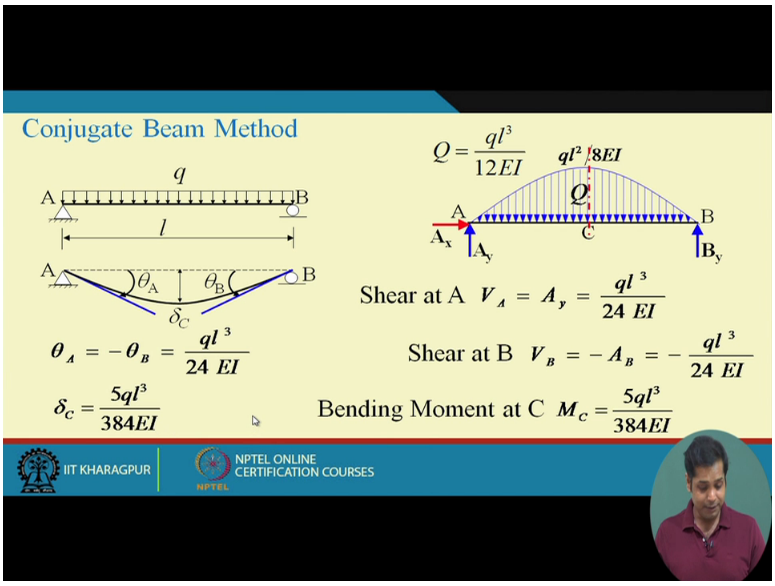

Now can you see the similarity? Can you see this is the real beam and this was the conjugate beam? Now you see the shear force at A is equal to theta A, shear force at A of conjugate beam gives you slope at A of real beam. And shear force at B of conjugate beam gives you slope at B of real beam. Please check this.

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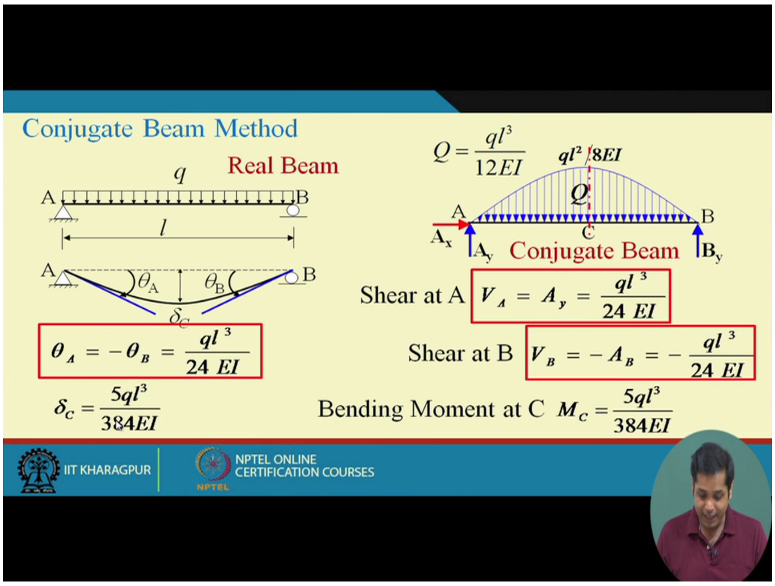

And then another thing is bending moment at C of conjugate beam gives you the deflection of real beam at C. So method is essentially suppose for a beam we need to determine the slope and deflection. What we need to do is first we need to construct a conjugate beam and first step of constructing a conjugate beam is replace the load on the conjugate beam by the (bend) M by EI diagram of the real beam, okay.

We will see other aspects of making conjugate beam as well but the definition of conjugate beam what we have as of now, replace the load on the conjugate beam by M by EI diagram of the real beam and then instead of finding the deflection and slope directly of the real beam, determine the bending moment and shear force diagram of the conjugate beam and bending moment is conjugate to the deflection of the real beam and shear force of conjugate beam is conjugate to slope of the real beam.

The problem of kinematics is now represented as or formulated as problem of statics. Now before we go further you see we know how to draw bending moment and shear force diagram of a beam, right? My advice is the first is suppose you are asked to draw a bending moment diagram of a given beam, okay.

Now before we actually compute the bending moment diagram, before you actually use your pen and calculator, try to understand for that particular support condition how the beam may deflect. And in your mind try to draw the bending moment and shear force diagram before you actually compute using calculator using the concept of structural analysis, okay.

Now if you do that, that exercise will help you to understand or to have a better sense of (beh) behaviour of structure under different loading. Now reverse is also true. Reverse is, suppose a bending moment diagram is given to you or a shear force diagram is given to you, right? Now just by looking at the bending moment diagram and shear force diagram we should be able to say something about the loading condition of the beam, support condition of the beam, okay.

And that exercise is also important because it will develop the sense through which we can understand the structural behaviour without actually doing any computation. Now here what I am going to do is we have already discussed the part of conjugate beam. Again we will go back to that. This is important for the concept of conjugate beam. Suppose this is a bending moment diagram given to us, okay.

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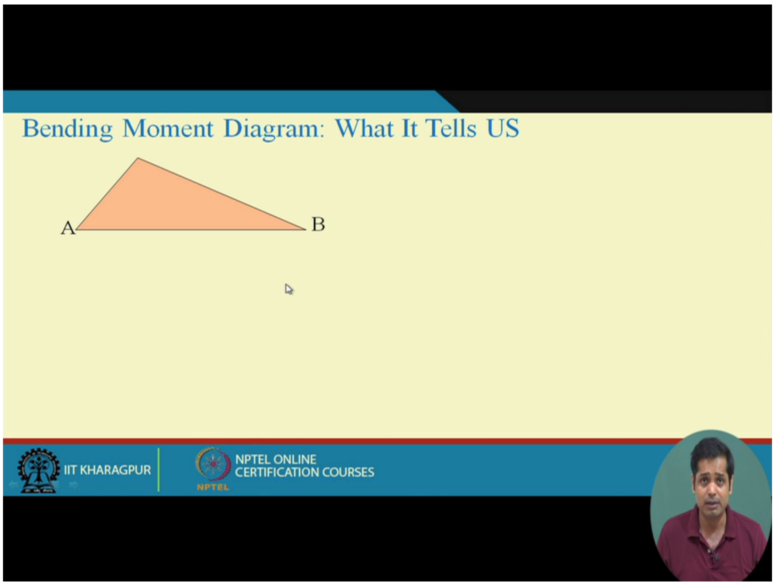

Now what this bending moment diagram tells you? That suppose it is also known that we have a support at A and support B. Now bending moment is zero at support A and bending moment is zero at support B. So the support condition on the real beam will be there will be no constraint against rotation. So naturally it will be hinge support and it will be roller support or this is hinge support, this is a roller support, okay.

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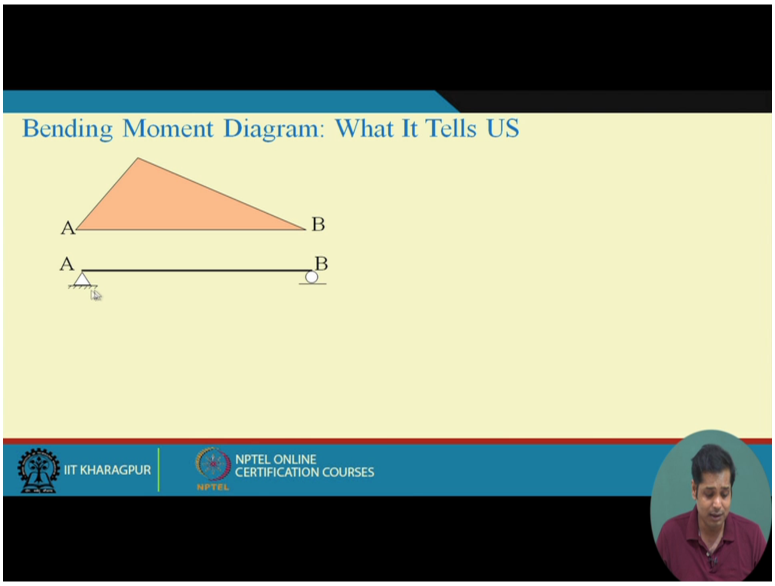

Now what is the loading condition? Loading condition will be since the bending moment is linearly varying between these two points, bending moment linearly varying means the shear force between this point to this point will be zero because that is what shear and bending moment they are related to each other. Shear is one order less than bending moment.

Bending moment linear means shear is zero, okay. Now then between these two points to this point there will be no external load. And then again bending moment decreases like this.

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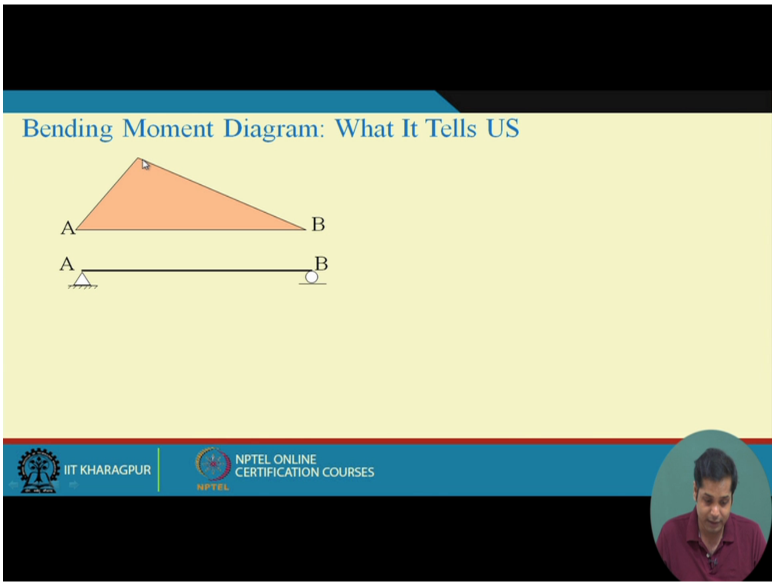

So again we have some load here and between this to this point there is no other load. So just by looking at this bending moment diagram we can say that this would be the real structure, okay.

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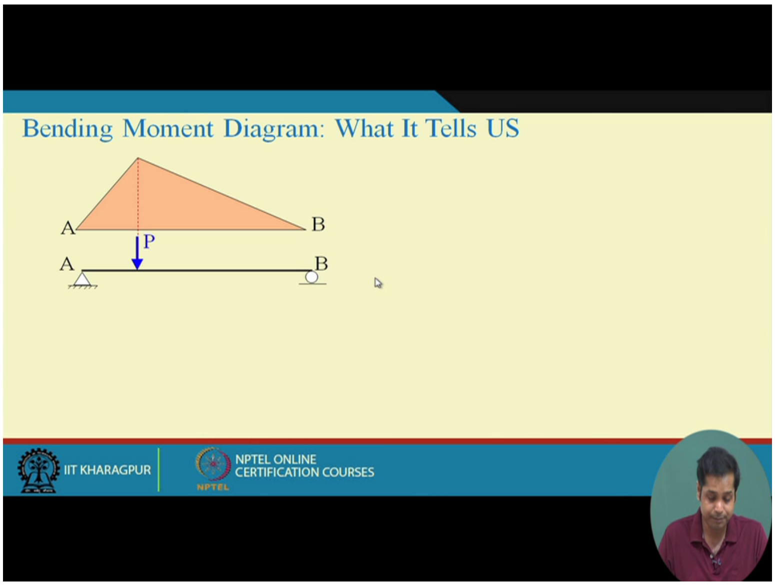

As of now what we have done is this structure is given and we determined the bending moment and shear force diagram. What I am telling you to do is the inverse thing. You know the bending moment diagram and try to identify what the problem or what the structure is. Now let us take another bending moment diagram. It is again similar to this but then this is simply supported beam. We have one load here and that should be another load here.

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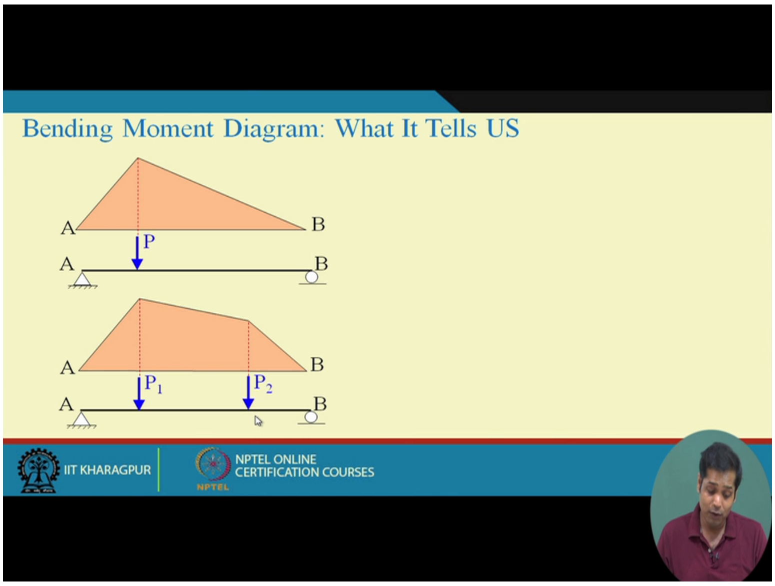

And not only that, after reaching this point bending moment decreases, this value is lesser than this value. We can also say that P1 should be greater than P2 or P2 should be less than P1.

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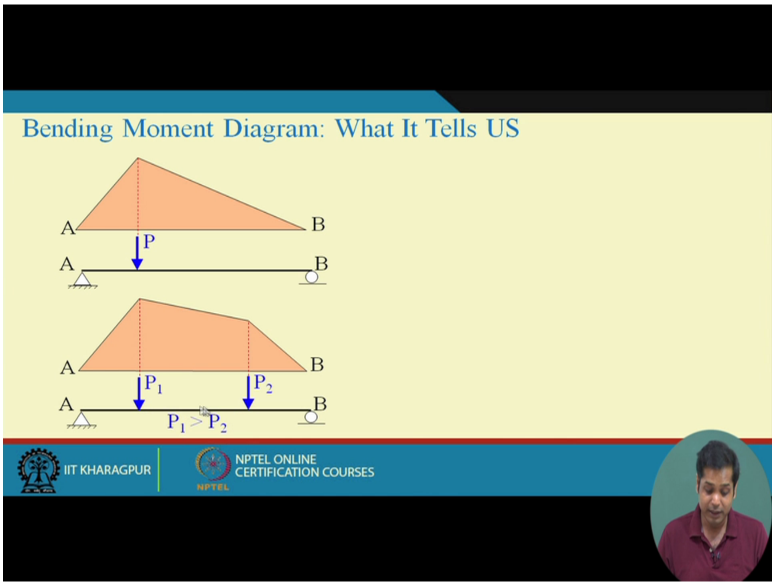

So again by looking at the bending moment diagram we can comment on the structure. Now let us take this problem where bending moment is linearly varying and then becomes constant. Now since here bending moment is zero so this support will be either hinge support or either free support.

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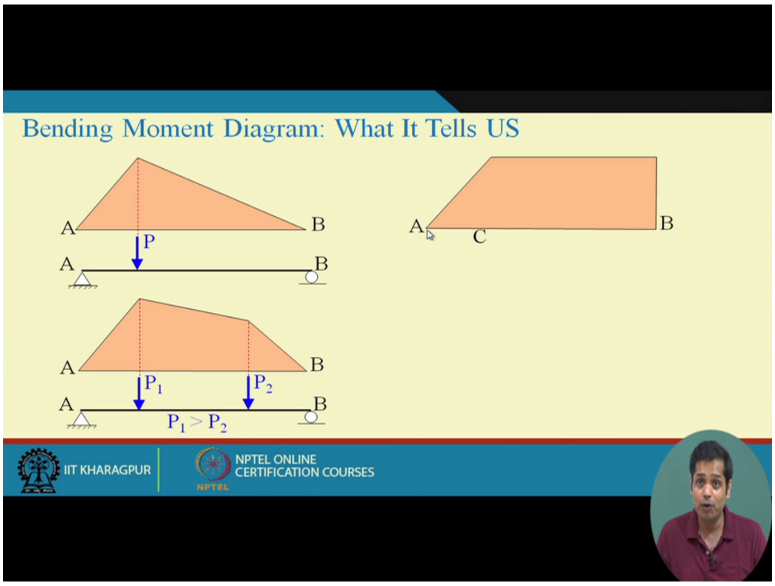

Since we have a (pos) non zero bending moment here means either we have externally applied moment or the support is fixed support. So this could be the beam and the loading condition could be like this because this is constant. So since this is constant so this is the loading condition we will have.

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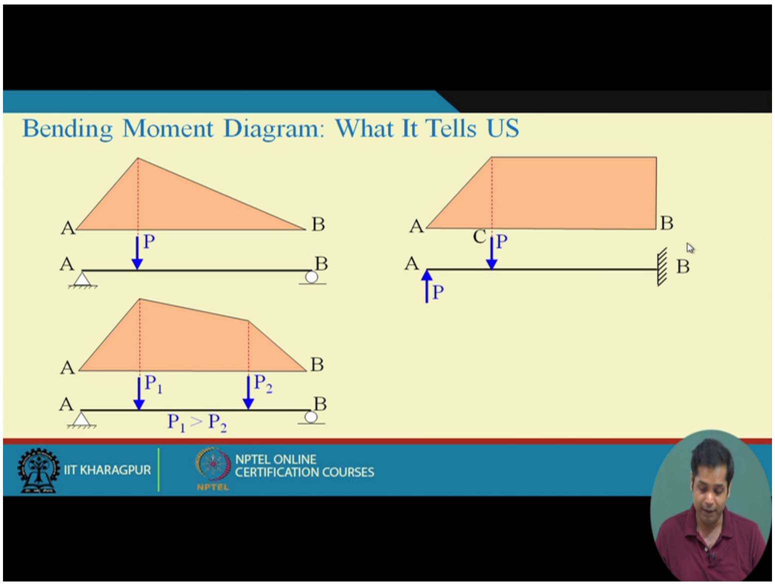

Similarly for this if your bending moment diagram is this then probably the loading will be something like this, okay.

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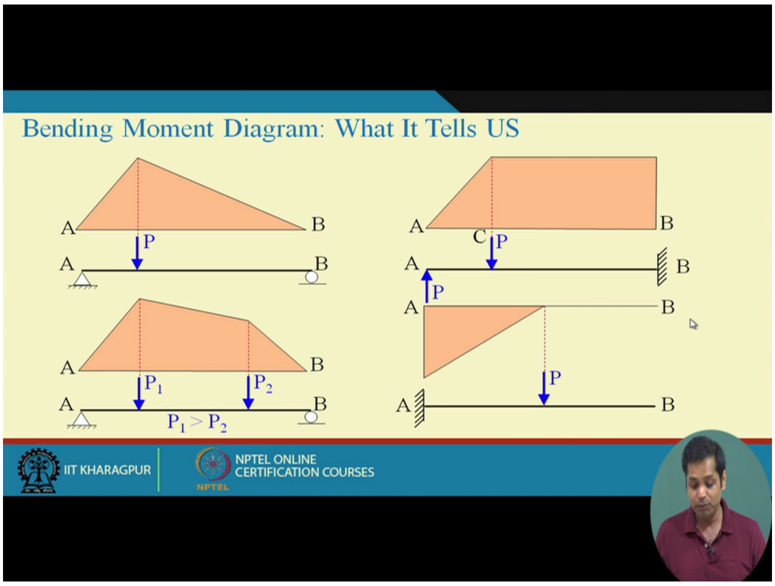

Now similarly if we take another case, a bending moment diagram is like this and then probably the loading condition of the beam will be like this, okay. The bending is symmetric that is why it is uniformly distributed load.

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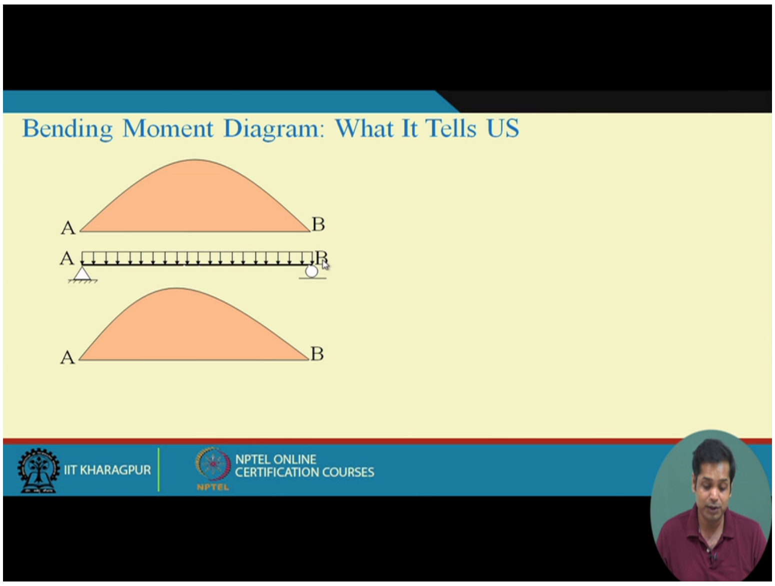

Now if the bending moment diagram is parabolic but not symmetric probably our bending moment loading condition will be like this, okay.

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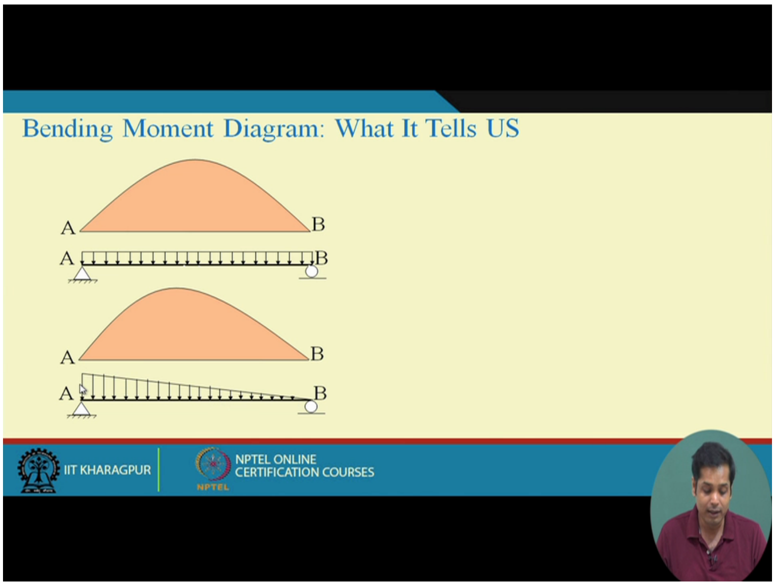

Or any other distribution but not uniform distribution. Now this is interesting. Suppose our bending moment diagram is this. What it tells you? The bending moment is zero here, bending moment is zero here and the bending moment is zero here, bending moment is zero here, okay.

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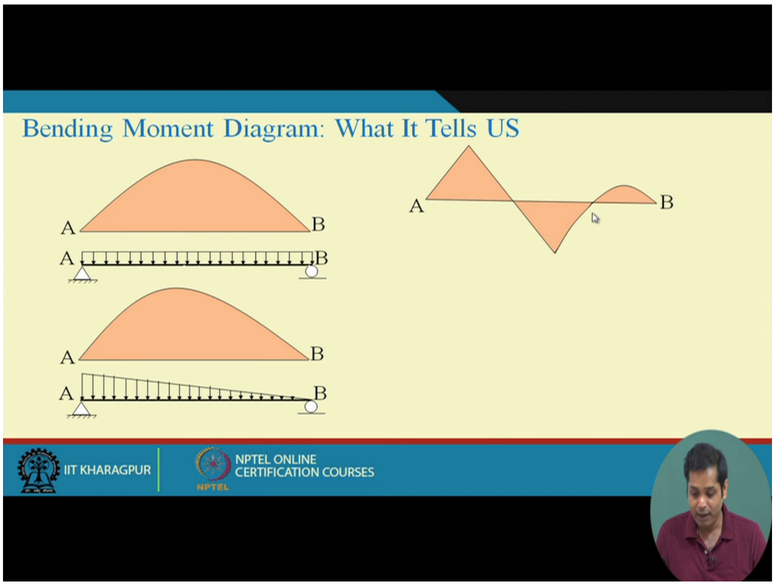

So suppose it is known that point A and point B supported. So at point A and at point B there is no bending moment. So it will be hinge support, okay.

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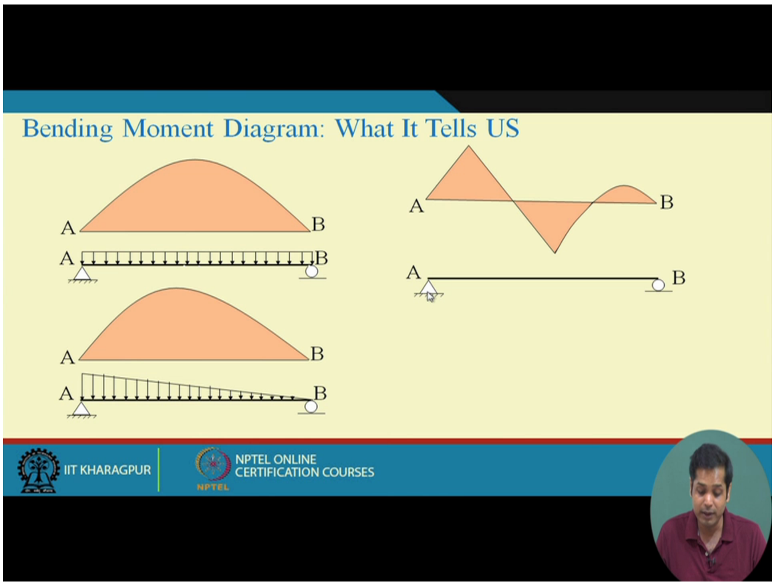

Now then since the bending moment is zero here what it tells you? That we may have a hinge here, okay.

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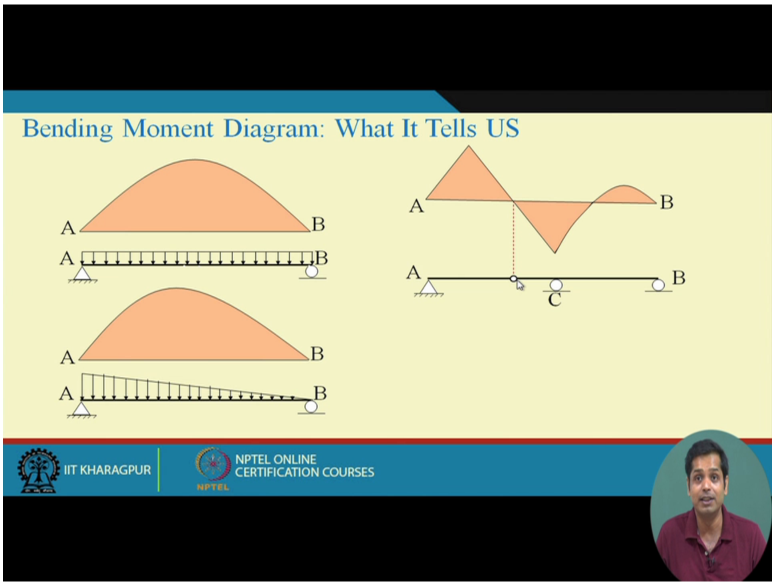

Because of the hinge location bending moment is zero. And since bending moment reaches the value and then decreases, we have either some externally applied load or support at this point. Suppose you have a support here.

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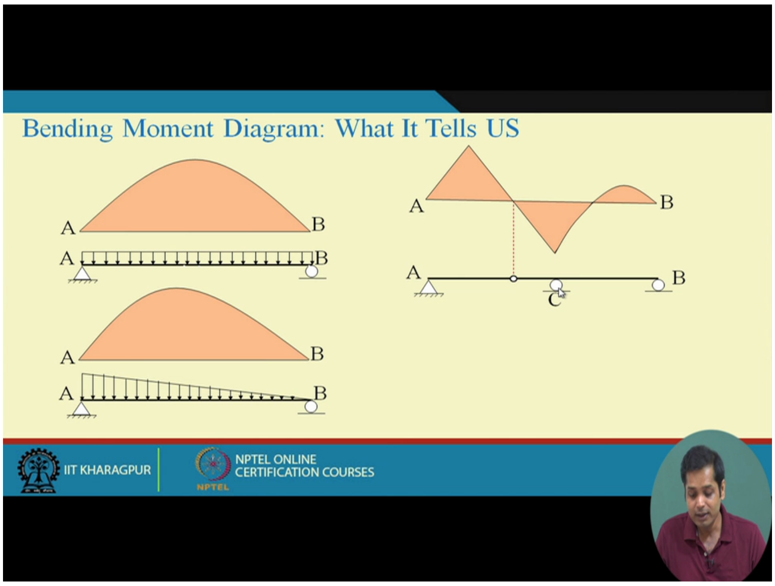

Now bending moment is parabolic distribution between this point to this point and then linearly varying between this point to this point. What it tells us? We have an applied (ax) (ax) load like this concentrated load. And since it is a parabolic distribution probably we have a uniformly distributed load like this. So this is probably the real structure which corresponds to this bending moment diagram.

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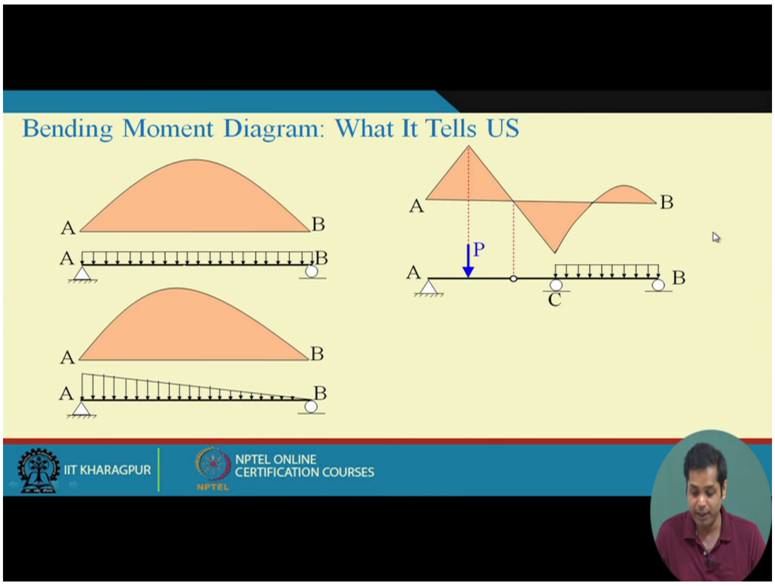

Similarly if the bending moment diagram is this where the bending moment is constant positive constant negative then probably the real structure would be constant bending moment applied here and then another bending moment applied here. Since it changes sign from positive to negative. Now and also we know that in this case M2 is greater than M1, okay.

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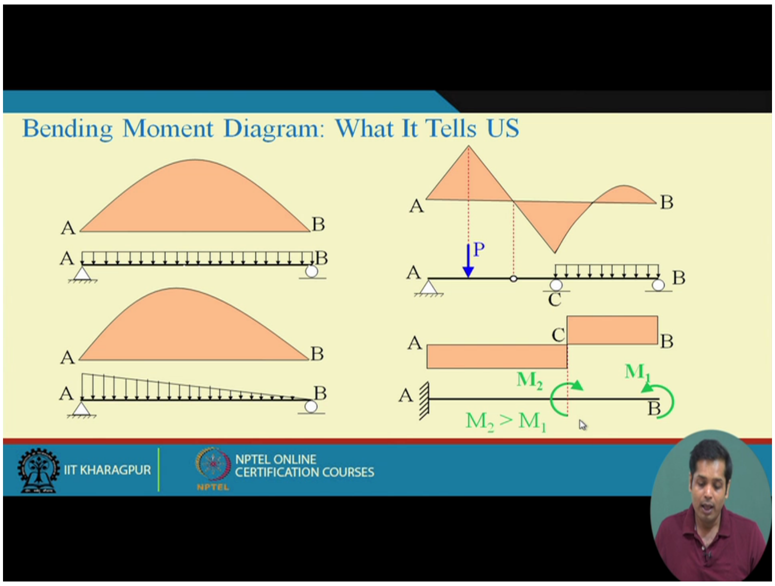

Now so if we have bending moment diagram and that is true for shear force diagram as well. If we have bending moment diagram and shear force diagram then we can get some information from those diagrams. We can have some idea about the support conditions of the structure and the loading conditions of the structure. Now but whether these loading conditions are unique or not that I leave it to you. That would be for all the bending moment I (sho) showed here.

That could be for each bending moment. That could be other structures or other support conditions or other loading conditions that may give you similar bending moment, okay. So whether the (ex) real structure that we derived from the bending moment diagram are unique or that derivation is unique or not that I leave it to you. But the essence is do not just draw the bending moment diagram and shear force diagram. You should be able to read those diagrams as well.

You should be able to interpret those bending moment and shear force diagrams as well. Even in moment area method as well. In this method bending moment plays a very important role. In moment area method we saw that we have the bending moment diagram and the moment of that bending moment diagram and the area of that bending moment diagram will give you slope and deflection.

In this case also the bending moment plays a very important role. So in addition to just drawing bending moment diagram and shear force diagram you do the inverse as well. Given bending moment diagram try to identify the loading condition and the support conditions. Now we will stop here today and now next class we have started the definition of conjugate beam. The definition of conjugate beam is not yet complete.

One aspect of conjugate beam is loading condition, the loading on the conjugate beam will be same as the M by EI diagram of the real beam. Now in the next class we will see some other aspects of conjugate beam and find the complete information that we required to make a conjugate beam and then see some examples how this conjugate beam concept can be can be used to determine deflections and slope, okay. Thank you.