Structural Analysis 1
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Lecture 21
Deflection of Beams and Frames (Contd.)

Welcome, this is the last lecture of this week. If you remember what you have been doing is, last class we introduced moment area method and also demonstrated through one example. What we will do today is we will see few more examples so that the concept of moment area method becomes clear. Sotoday this class agenda isdeflection of beams, moment area method and of coursesome examples.

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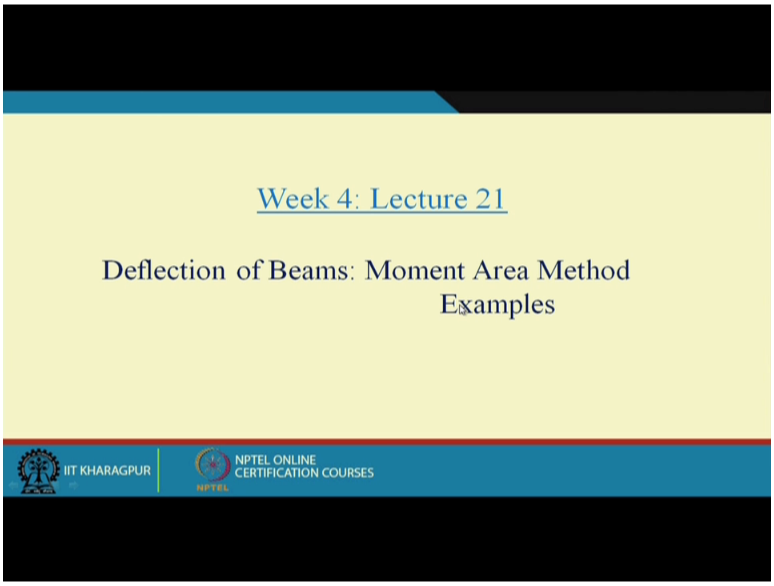

So let's see the first example. The first example again is a very straightforward example. It's a cantilever beam subjected to tip load like this. Length of the cantilever is l. A, B and this is fixed.

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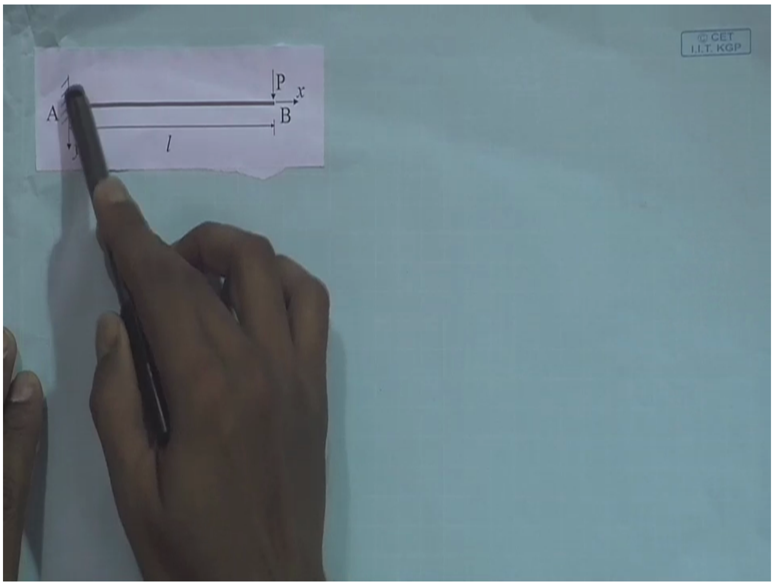

Now if wedraw the deflected shape of this beam, then the deflected shape will be. This is the beam in its original configuration. You see at point A slope is zero and this is point B. So this is the slope. If you draw a slope at point A the slope will be this. This is the slope at point A. Similarly this is the point B and if we draw slope at point B and this will be the slope at point B.

Now if this is Bthen this point will be B dash which is the projection of B. So this is actually your B and this becomes B dash. Because this is deflected shape and this is the original position of B. And this is B dash. This Bdash essentially is a projection of B onto B dash.

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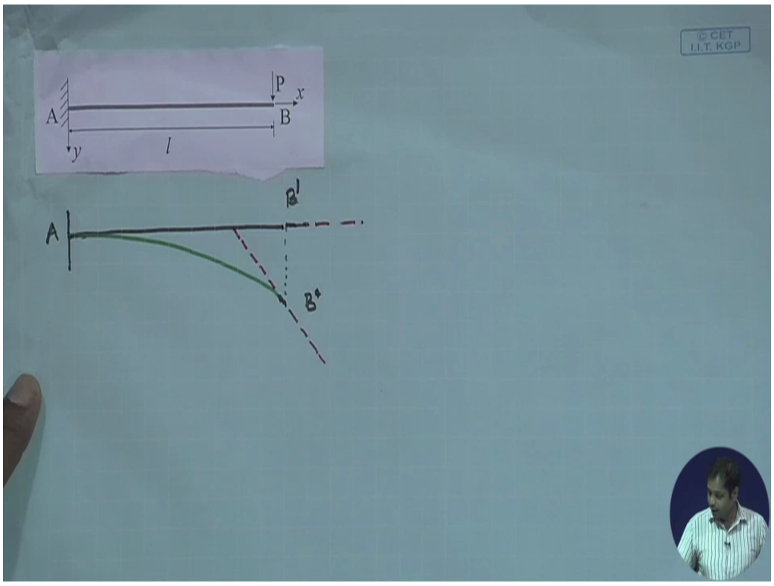

Now if we draw the bending moment diagram for this beam, this will be P into L. This is B, this is A. This is the bending moment diagram, right? Nowwe know how to draw bending moment diagram for this beam, right?

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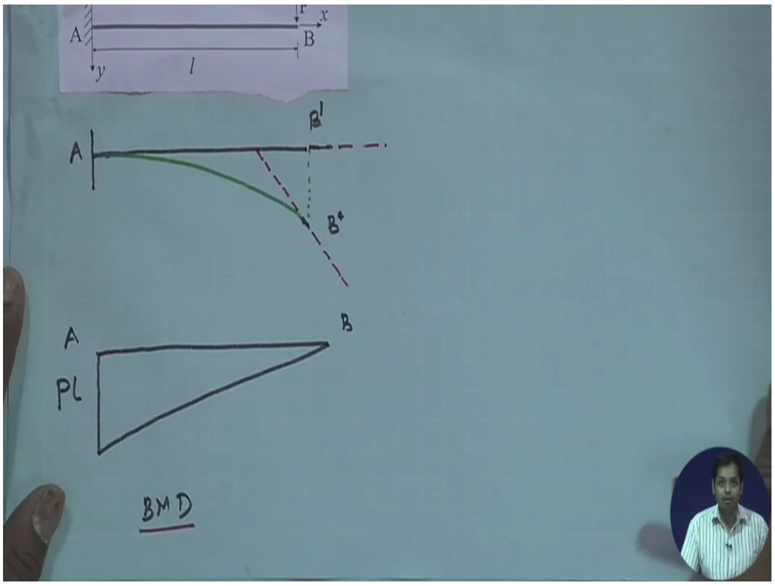

Now please check this bending moment. This isfor a cantilever beam subjected to tip load. The bending moment generated at fixed and due to hogging movement, that's why it is shown below. Now what is the central support? This is the centroid of this triangular area and this length is L. So we know that this is 2L by 3. This distance is 2L by 3.

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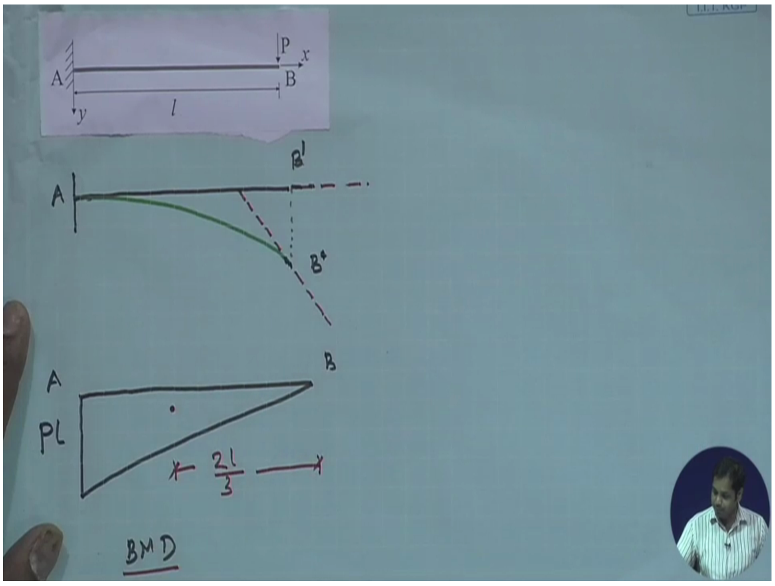

So as per moment area method we know that theta AB which is the angle between the slopes drawn at A and B. So slopes drawn at A is this and slope drawn at B is this. So angle between this angle is theta AB, right? And we know that theta AB will be the area of bending moment diagram between A and B. So area of bending moment diagram means this.

So this is half into PL into L. This is PL by EI diagram. Lost bending moment diagram because in moment area methodwe will take thePL by EI diagram. So this become PL square by 2EI. So angle between the slope at A and B is PL square by 2EI.

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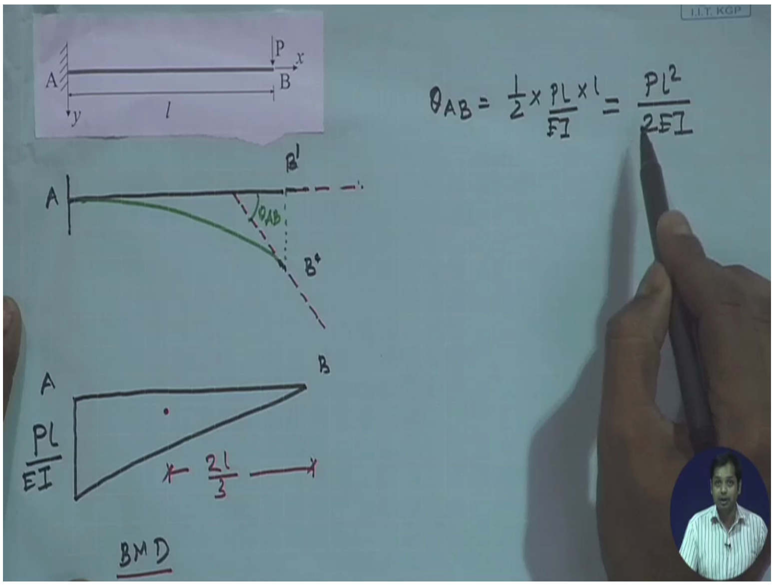

Now what is angle between A and B? Theta A minus theta B, right? Theta A minus theta B is equal to PL square by 2EI. Now slope at A is equal to zero because it is fixed end. So this is equal to zero. So this gives us theta B is equal to minus PL square by 2EI. So this is slope at free end. Theta AB is equal to theta B minus theta B. So this is the slope.

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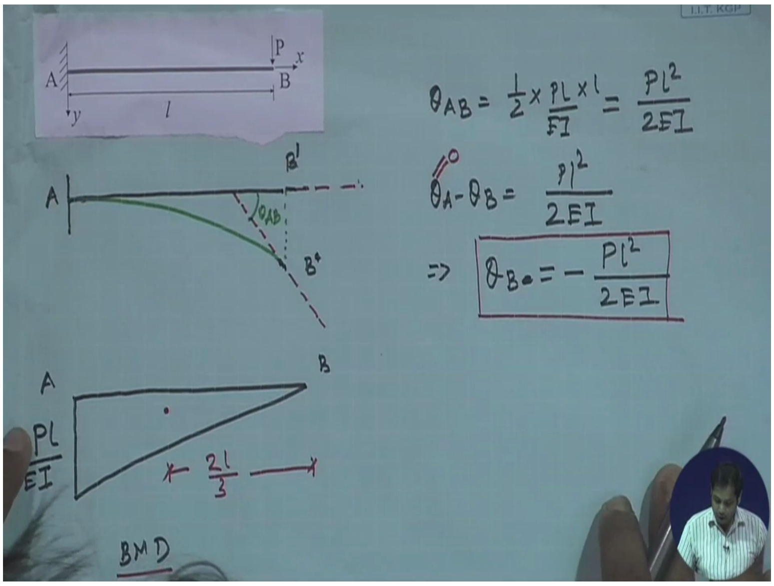

Now what will be the deflection? Now moment area method doesn't tells you directly about the deflection. What it tells you? That the deviation of or the projection of any point on a slope drawn at another pointwill be the moment of the M by EI diagram about the concerned point at which the deflection needs to be measured. So in thiscase what we need to do is. This is delta, right? This is the projection of B onto the slope drawn at A.

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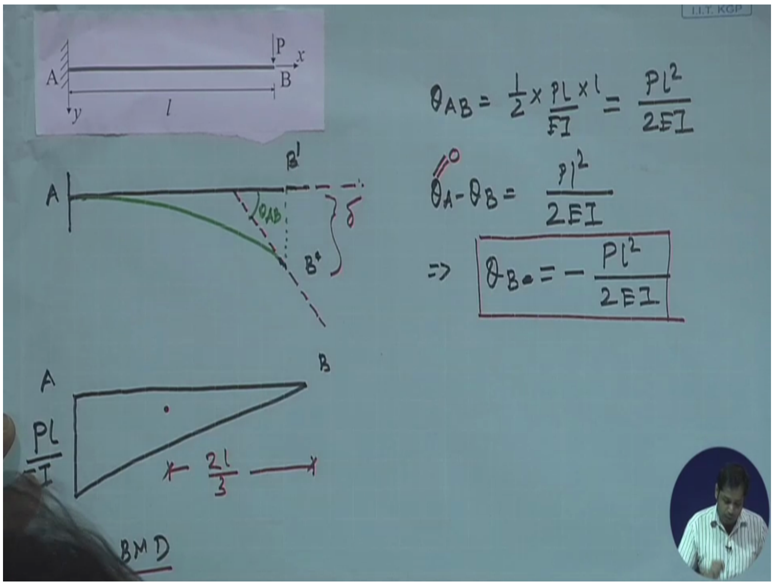

Now since A slope is zero, so this projection itself gives you the deflection at point B. So what will be the deflection at point B? So Yb, or you can write delta B as well. Delta B is equal to moment of this M by EI diagram about point B. So this area is PL square by 2EI. PL square by 2EI is the area into, this distance is 2L by 3, 2L by 3. So this gives us PL cube by 3EI, delta B.

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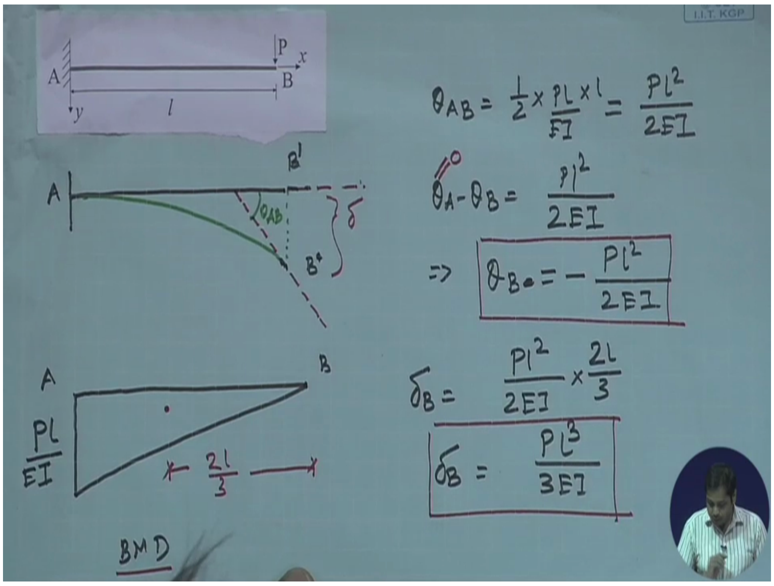

So this is again a very standard result.A cantilever beam subjected to tip load. The deflection at the free end is PL cube by 3EI. So this is again a very simple problem. Now you verify this results with the results that you obtained usingdirect integration method. Now let's see one more example. This example is a simply supported beam subjected toconcentrated load at any arbitrary point between A and B.

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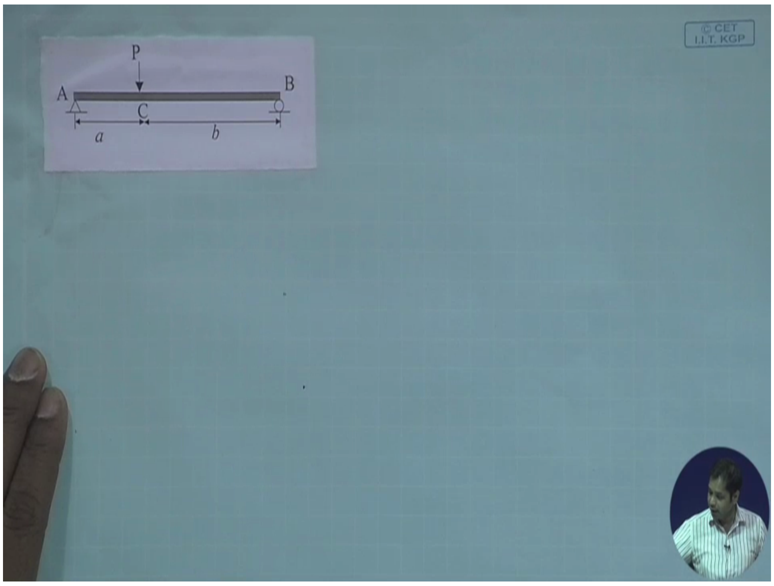

Now if you remember when we use direct integration method, then the method was one expression for bending moment here and another expression for bending moment here. Then integrate those two expression, get 4 constant, apply boundary condition and continuity condition at point C and determine those 4 constants. It was a big lengthy process. Now let's see how things become easier if using momentarea method.

So firstthing is, draw the bending moment diagram of this. At this point and at this point bending moment will be zero.And at this point bending moment will be maximum. So this is the bending moment diagram,right? This is point A, this is point B. This is BMD, bending moment diagram. And this value is Pab. This is P. This value is, let’s write, Pabby L and now since we need to use theM by EI diagram and this divided by 1 by EI. So this is the M by EI diagram.

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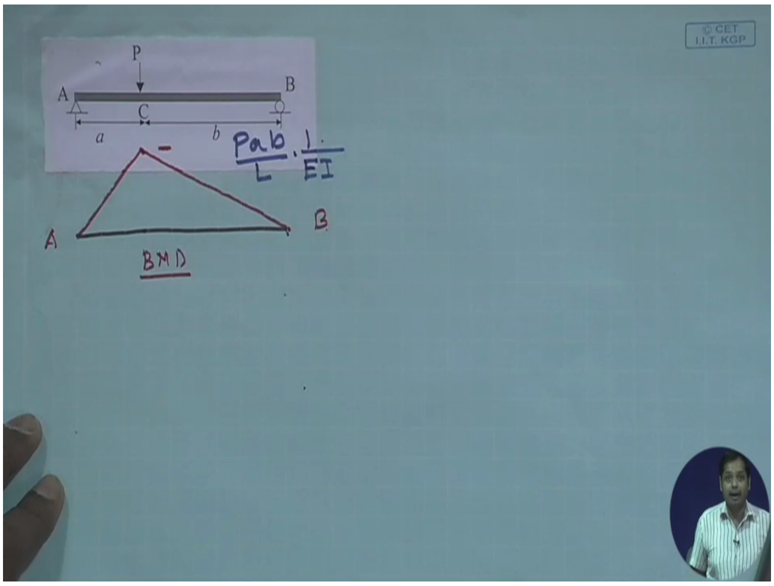

In this case please note EI is constant. That's why we can directly divide bending moment by EI. So this distance is this distance. Now first draw the deflected shape of the beam. So this was the original configuration of the beam and the deflected shape of the beam will be something like this.

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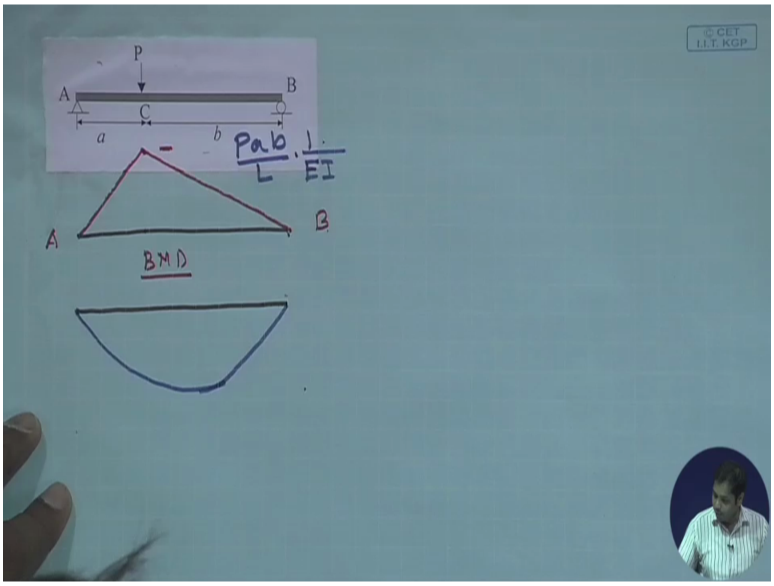

You know before actually you solve the problems, you should be able to, by just looking at the problem and by looking at the boundary conditions and the applied loads, you should be able to, by intuition, you should be able to understand what would be the deflected shape of this beam. Now this is the deflected shape of this beam. Now let's draw, this point is A and this point is B and this point is C.The question is, determine delta C. What is the deflection at point C?

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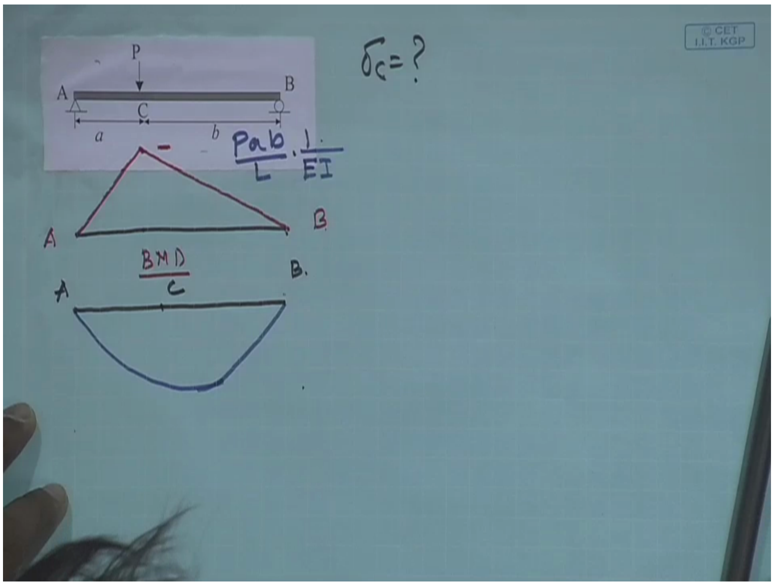

Now so first draw a slope at point A. So this is slope at point A. Now, so this is theta A, right? Now if this is point B, then projection of point B on the slope drawn at A, this point is B dash.

And this distance is delta BA. So this means that this is deviation of B with respect to the slope drawn at A. So this is delta BA, right?

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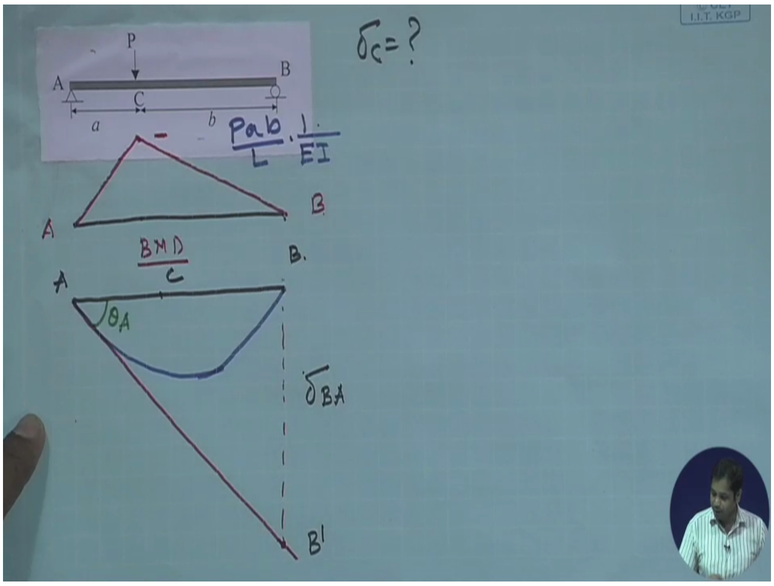

Now momentarea method says that this deviations delta BA will be the moment of the M by EI diagram about this point B. Now moment of this diagram will be, area of this diagram multiplied by the centroid distance of the diagram from B. Now you can verify yourself that if this is a centroid of this entire triangular area and this distance is XBof that. XB bar, that’s what we use.

Then you can show XB bar is equal to L plus B by 3. I leave it to you to verify this. You know how to determine centroid of any arbitrary area. So you can determine and check. So this iscentroid of this area about from point B.

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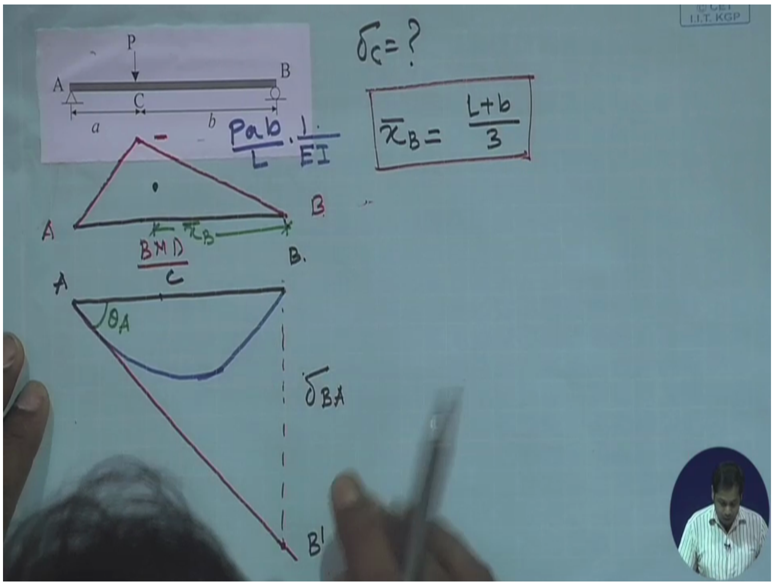

So what would be deviation delta BA? Delta BA will be the area multiplied by this distance. So delta BAwill be, the area of this diagram is half into Pab by L into L. This is the total area multiplied by this distance XB which is L plus B by 3. So this becomes Pabby 6EI into L plus B, delta BA.

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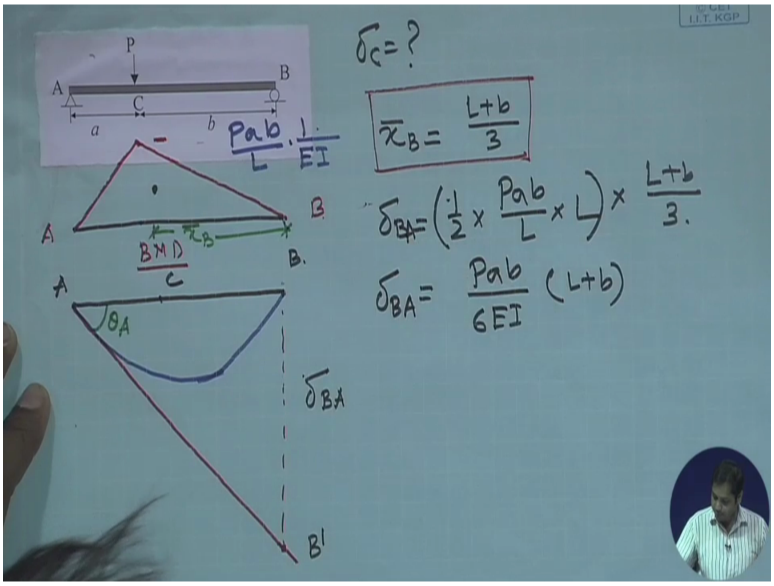

So we have obtained the deviation of B with respect to the slope at A, delta BA. What is the question? Question is we need to determine this distance. If this is delta C, we have to determine this distance.

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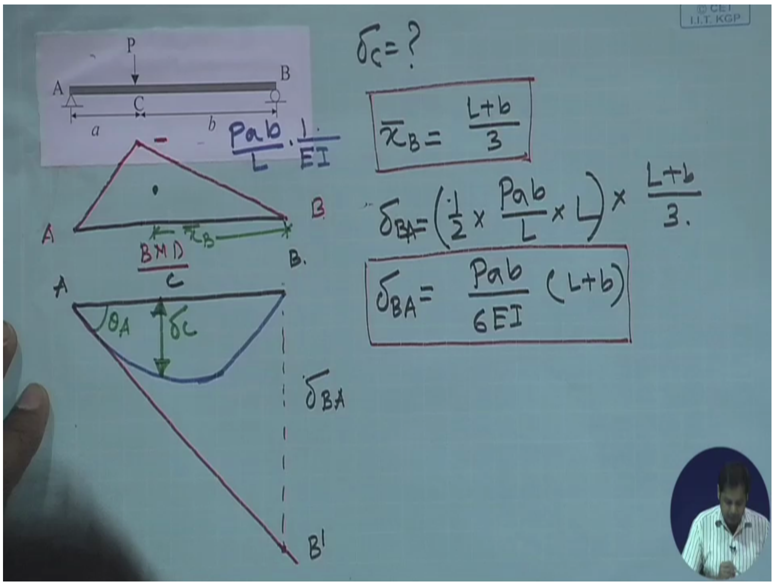

Now suppose this distance is delta.

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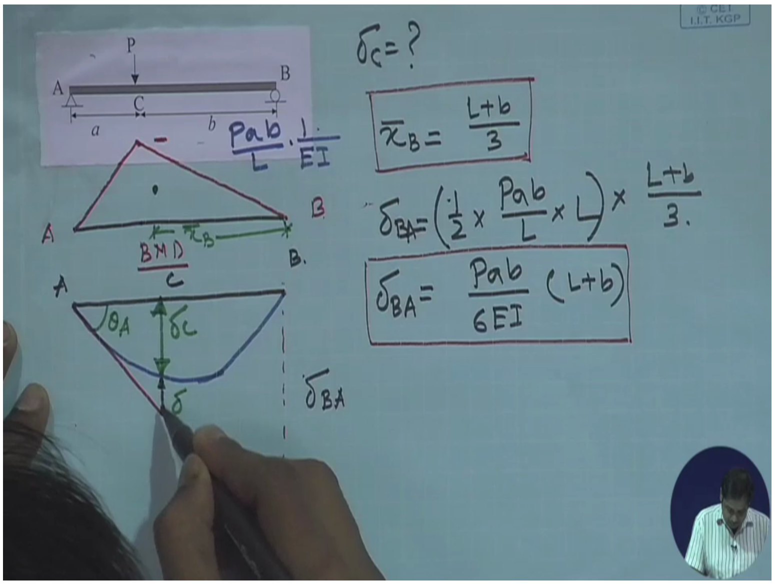

Now, then what is delta? Delta is essentially the deviation of point C.On deflected shape this is point C. So it is, delta is equal to deviation of point C with respect to slope at A. So it is essentially delta CA, right?

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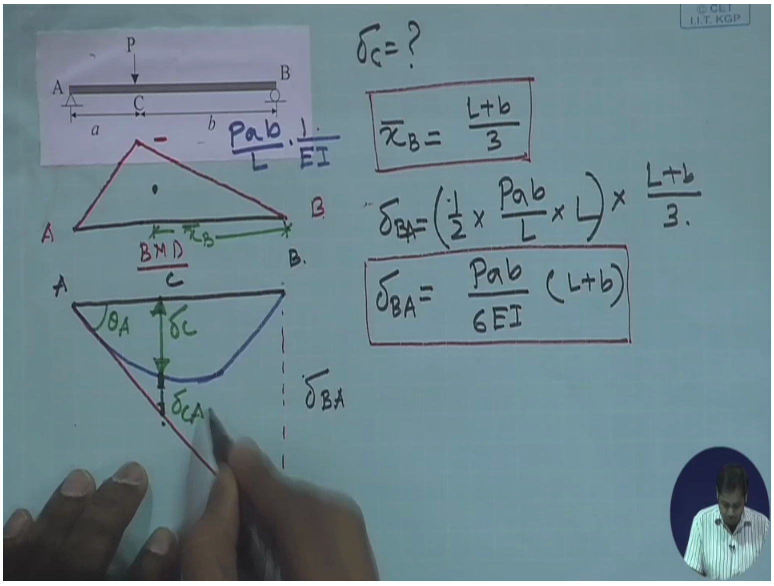

Now again applying moment area method and what would be delta CA? Delta CA will be the moment of M by EI diagram between A and C aboutfrom C. So the delta CA will be moment of this diagram. Moment of the bending moment diagram between A and C is this, this triangular portion.Moment of this diagram about point C here.

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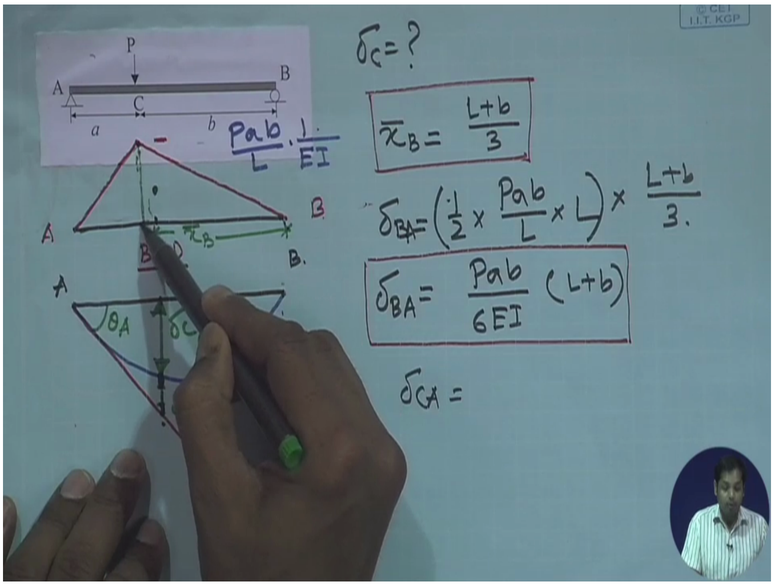

So this will be then, half Pab by EIL. Here also it should be EI.And into A. This distance is A, into A by 3. The centroid of this triangular area from point C is A by 3. So this becomes Pa cube b divided by 6EIL, delta CA. So delta CA is not the deflection at C. Delta CA is the deviation of point C with respect to slope drawn at point A. And remember one thing, when we are talking the deviation, it is the deviation is measured from the deflected shape of the beam, not the original un-deformed shape of the beam. So we obtained this and we obtained this.

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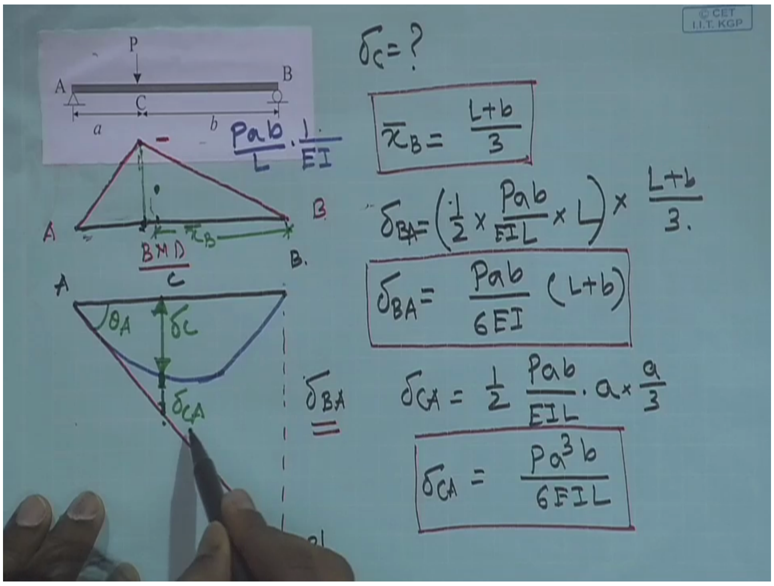

We need to find out this. Now you see, from the similar triangle what we can see is, we know this length and we know this length as well. So we know we can find out this entire distance. Delta C plus delta CA, we can find out from similar triangle. So what will be the delta C plus delta CA? Delta C plus delta CA, this distance will be delta BAby L into A. It is just simply using the similar triangle.

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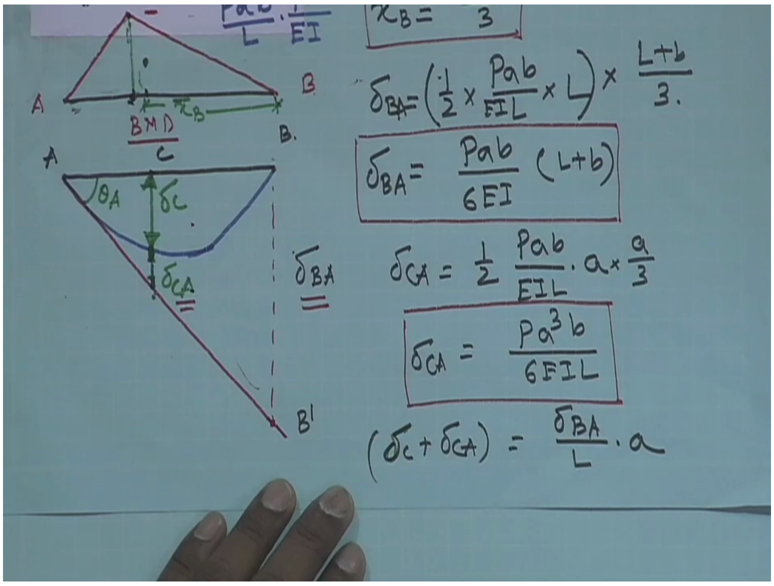

So delta BA is this we know. Let's substitute delta and delta CA also we know from this. Delta CA we know from this and delta BA we know from this.

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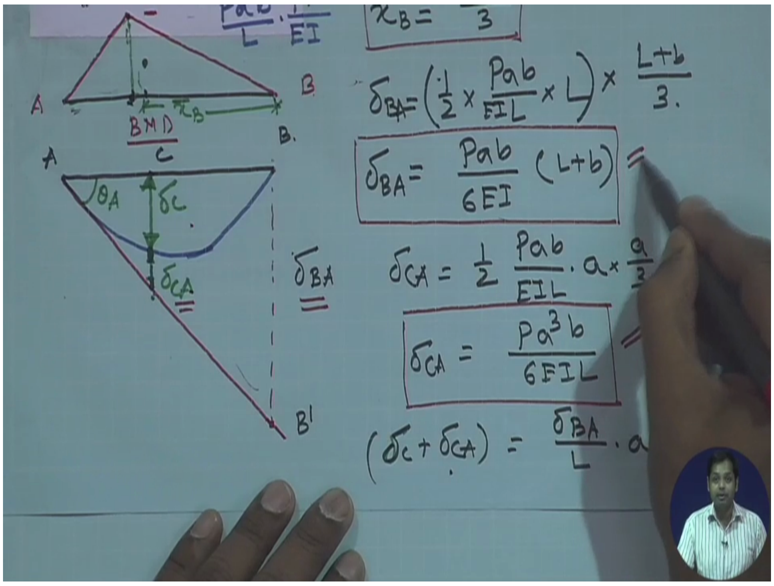

Let's substitute delta CA and delta BA in this expression. And what we get is, delta C is equal to delta BAby L into A minus delta CA. So this becomes Pa square by 6EIL into L plus b. This is delta B we have obtained. Minus Pa cube b by 6EIL. And you do some simplification and finally we will get delta C is equal to Pa square b square by 3EIL. This is the final expression for delta C.

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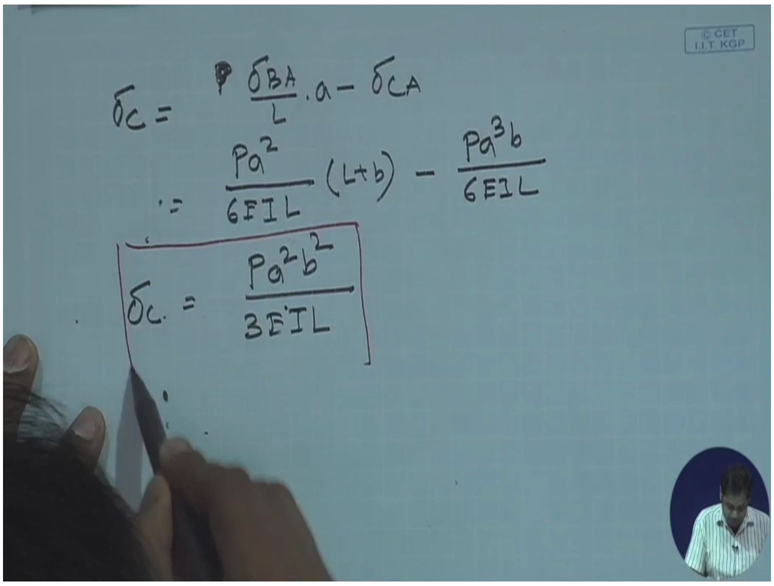

Please check, already we have determined the expression for delta for this problem using direct integration method and there you substitute x is equal to M and check whether you are getting this value or not. So this is moment area method. Similarly you can apply similar (con) concept to any other beam with any other boundary conditions. Now two-three points quickly let me tell you.

You see when we draw the bending moment diagram, then for instance t now for a cantilever beam. A cantilever beam which is subjected to load P and we know the bending moment diagram is PL here.

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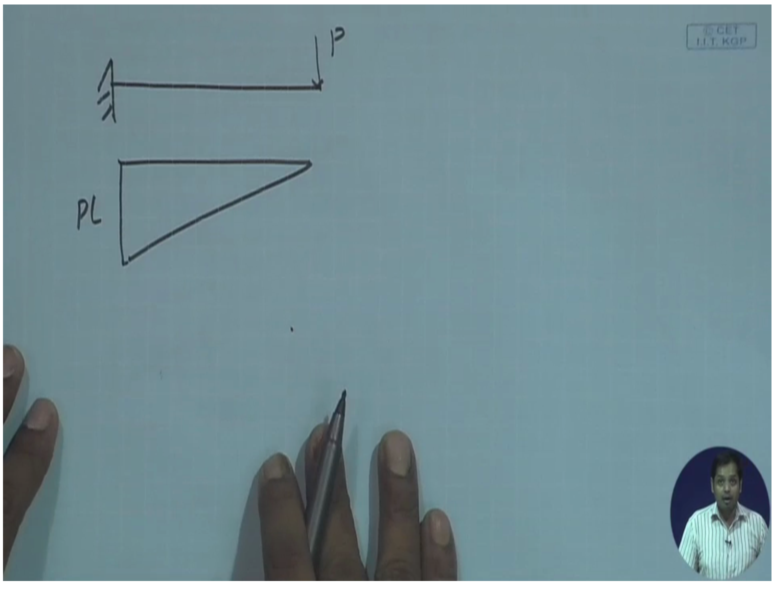

And similarly we can draw bending moment diagram for any problem. Now this bending moment diagram depends only on what? Only on this applied load. For given length of the beam, this bending moment depends on the applied load. If P is more, then bending moment is more. If P is less, then bending moment is less. It does not depend on what material the beam is made? What would be the cross section of the beam? Right?

So if you take any material, any cross section, your bending moment for a given length of the beam and is subjected to tip load, your bending moment will remain same. But then does it mean that the material has no role in to play? Yes it has. It plays role in calculating of deflection. So deflection is a function of material property. Because the Young's modulus comes there. So the bending moment internal forces in member, we will see later as well. Internal forces as such, they do not depend on the material properties.

But when we calculate deflection using internal forces then the material property come into picture. This was just few examples on moment area method. There are many examples given in books. So again I suggest you please go through to the books and attempt some more examples to understand the concept behind the moment area method. We will stop today.

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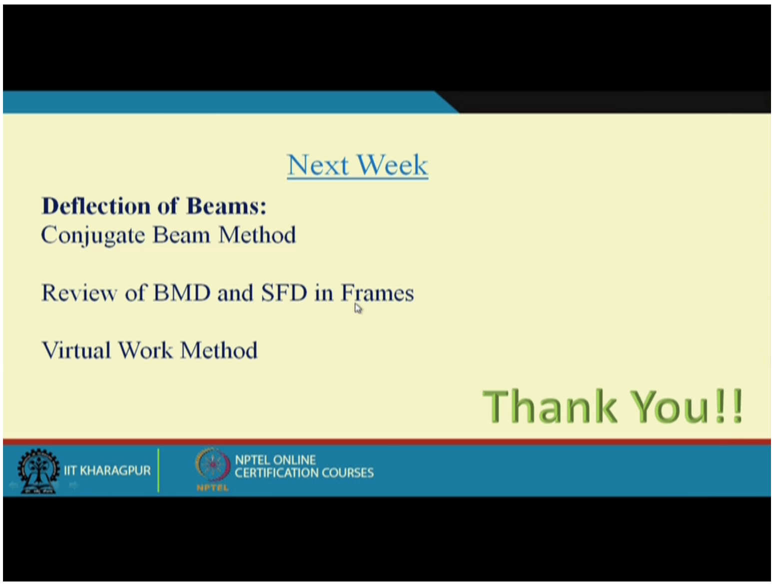

Then,the next weeks agenda is, we will start with conjugate beam method. We will spend 2(lec) lectures on conjugate beam method. Then quickly review the bending moment and shear force diagrams in frames. We have already reviewed for beam for one lecture. And then use virtual work method to determine bending moment time to determine deflection in beams and frames. Virtual work method, if you remember it was introduced in the previous week. That's all for the day. Thank you. See you next week.