Structural Analysis 1
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Lecture 20
Deflection of Beams and Frames (Contd.)

Well, welcome to lecture 20 of unit 4. What we are going to do today is, we will see another method for determination of deflection for statically determinate beam and the method is moment area method. As you can see this method was originally formulated by Otto Mohr in 1868 which is almost 150 years ago. Now may ask question that a method which was derived in 150 years ago and still we are using it in spite of having lot of limitations, we will see those limitations anyways.

Why still we are using this method or learning this method in this course and similar many other methods in this course which probably has many limitations as well? You leave that discussion for the last class of this entire course. We will see what was the motivation of learning all these methods in spite of their many limitations. Okay.

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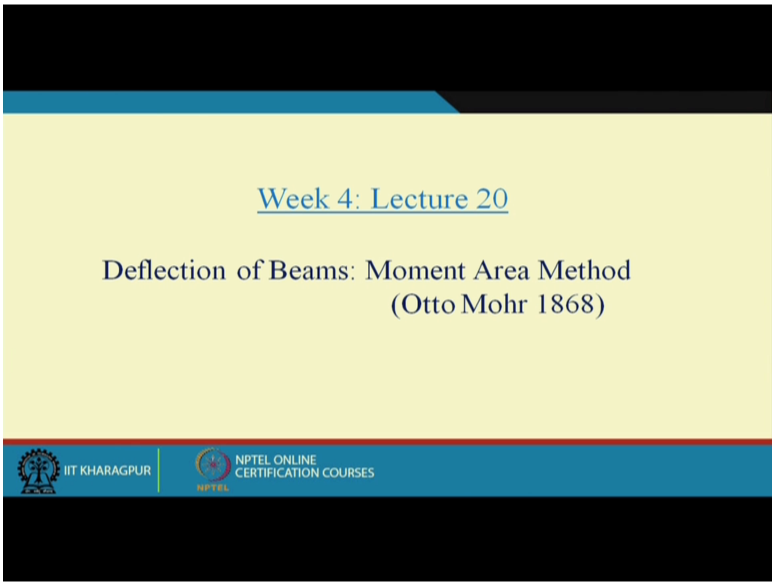

So let us first understand the concept behind moment area method, but remember as I said earlier that premise of this method is the equation of elastic curve, okay? If you have understood the equation of elastic curve and the bending moment shear force diagram, probably understanding of this concept will be very trivial. Now you see again let us demonstrate this through simply supported beam but that similar concept can be extended to any other boundary conditions, any other loading conditions as well.

Now take a simply supported beam AB and as per our sign convention this is x positive and we are since assuming sagging is positive. So sagging means deflection is downward positive. So this is our positive direction y downward. And suppose this is the deflected shape of the beam AB, okay?

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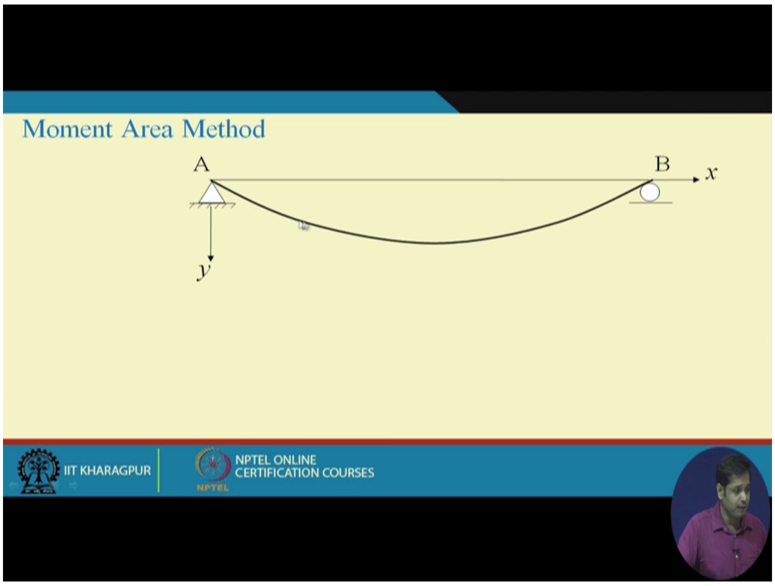

Now suppose take a point here, any point which is at a distance x from A and suppose at that point deflection is y. So we already discussed what is the equation that represent the (def) deflected shape of this beam. And that was equation of elastic curve. Okay.

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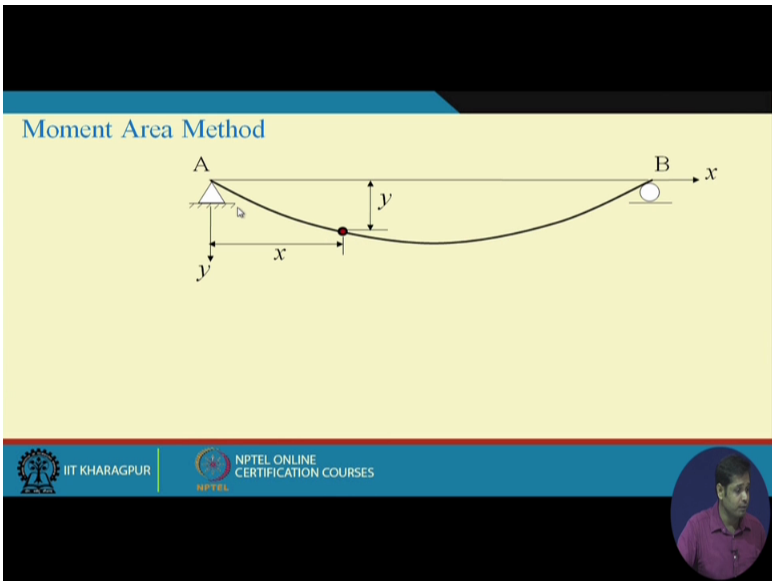

Now suppose draw a tangent at this point. Okay. And then this angle will be theta which is the slope at this point. Theta is equal to dy dx at this point.

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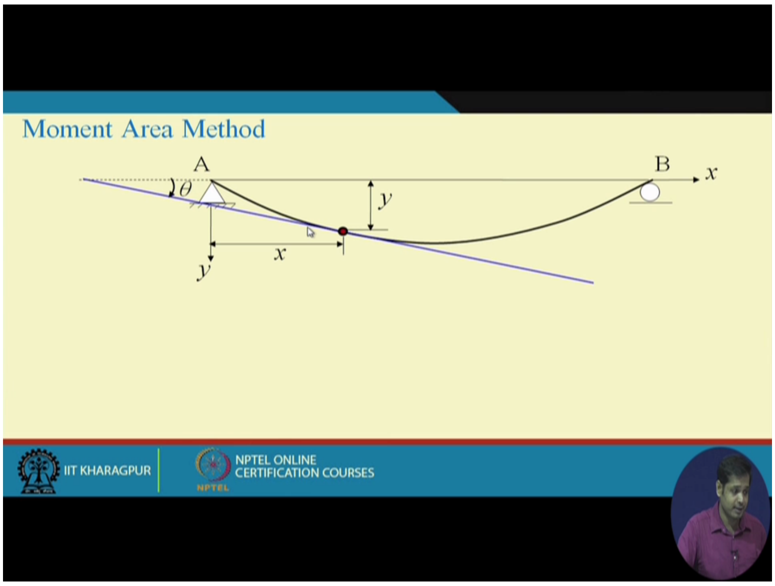

Now please remember the sign convention. We are assuming theta which is measured clockwise direction as positive and anticlockwise direction as negative. And if we see book, many books users, many other sign convention, again whatever sign convention you use please be consistent with the sign convention. Now so this is the angle, this is theta. Okay. Now you take any point.

If any point you take and at that point draw a slope and the slope intersect at AB and the angle made by the slope and this line AB gives you the slope at that particular point.

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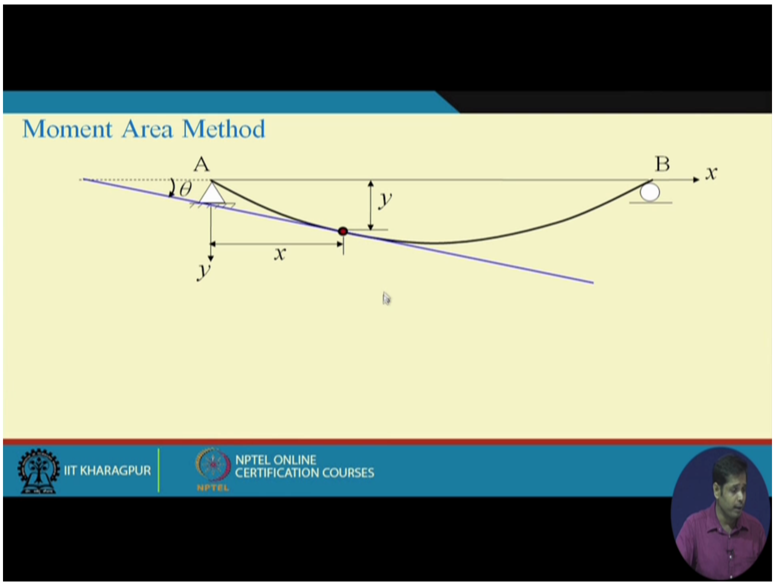

Well, now again this is theta, so we know that theta is equal to dy dx right? And you remember the assumption theta equals to dy dx was the deflection of the slope small. Therefore the nonlinear term, the curvature can be neglected. Okay. And we also discussed the Euler Bernoulli equation which is equation of elastic curve as this. Okay.

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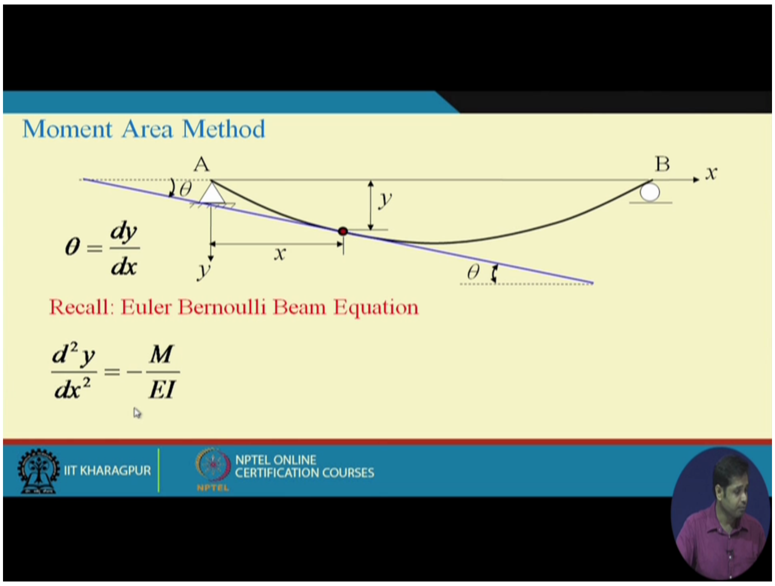

So this is essentially the mathematical representation of this process. Beam which is reflected like this. Okay. Now this equation can also be written as like this.

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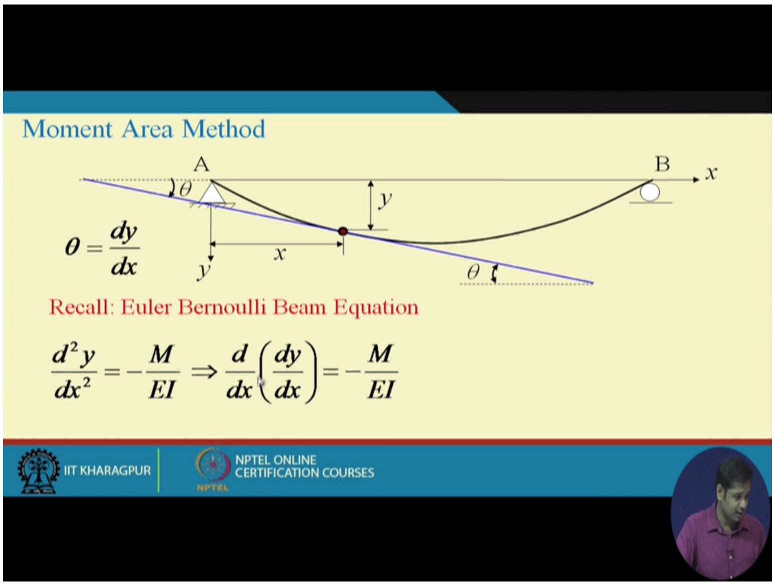

Where d dx taken out and this is dy dx and since dy dx is equal to theta so we can say that d theta dx is equal to minus M by EI. Say this M by EI is phi x because M is function of x. That is why it is written as phi x. So wherever we refer phi it means M by EI. Okay.

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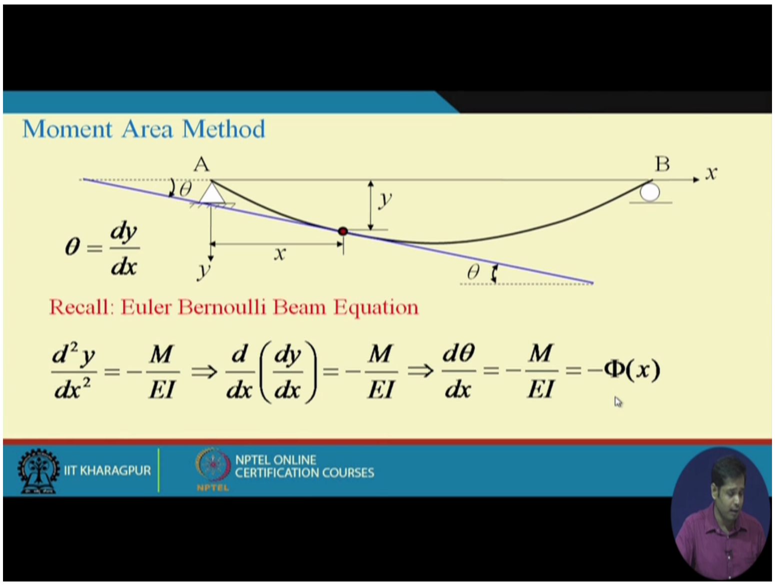

And now M is anyway function of x but EI can also be function of x. So together it is written as phi as a function of x. Okay. Now phi x, Mx by EI, okay?

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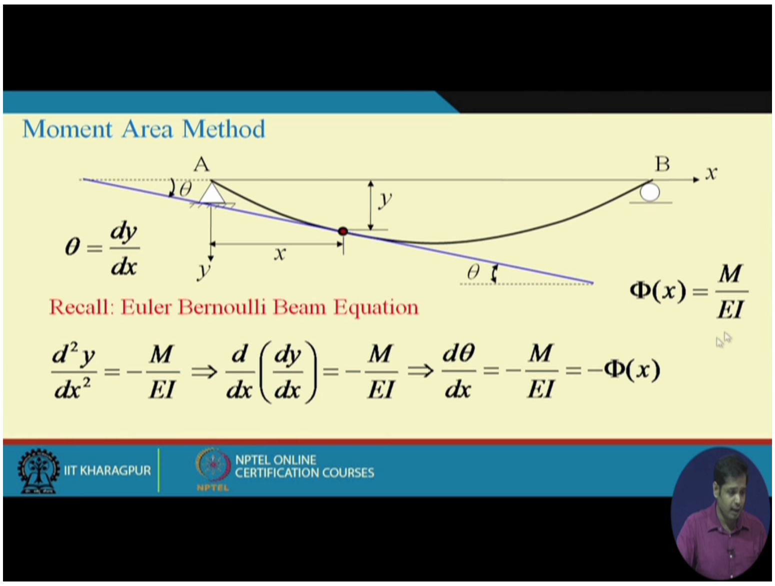

Now suppose this is the deflected shape of any beam. Okay. Now similarly if we take two point at A and B and draw tangent at A and again draw tangent at B. Now tangent at A makes an angle theta A with the initial profile or with the horizontal and tangent at B makes an angle theta B. So theta A is this direction that is why it is positive and this is written as minus theta B because it is measured anti clock direction. Okay. So theta A and theta B are essentially the slope at point A and (ss) at point B respectively, right?

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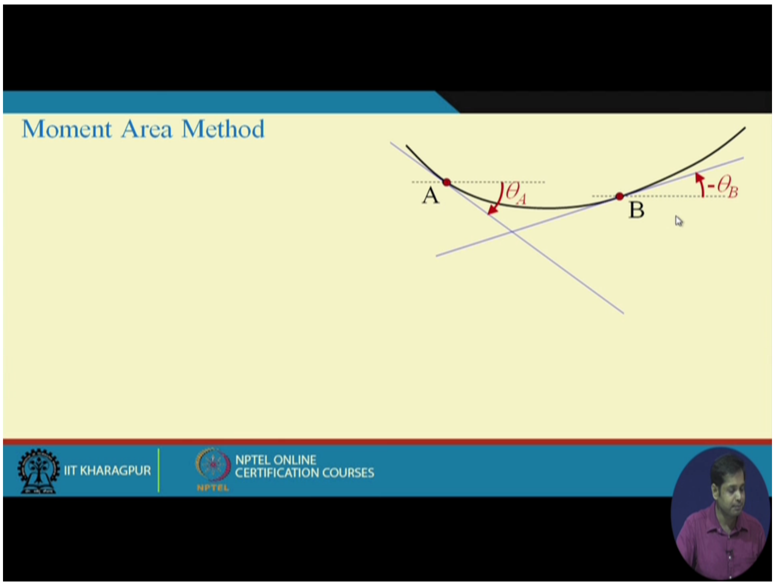

Now so this angle is theta AB. So theta AB is the angle between the slope at A and at B. So by just simple geometry we can say that theta B is essentially this angle plus this angle. So theta A minus theta B. Okay.

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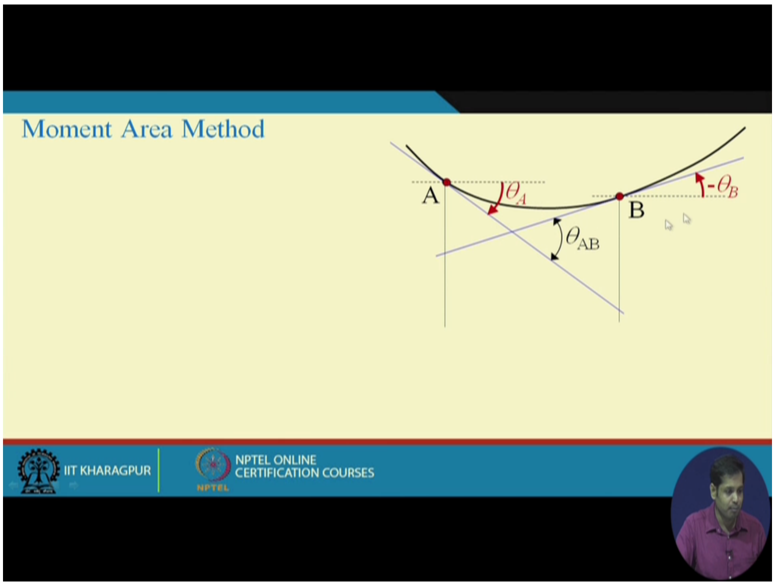

Now again this is the projection of A on slope at B and this is the projection of B on slope drawn at A.

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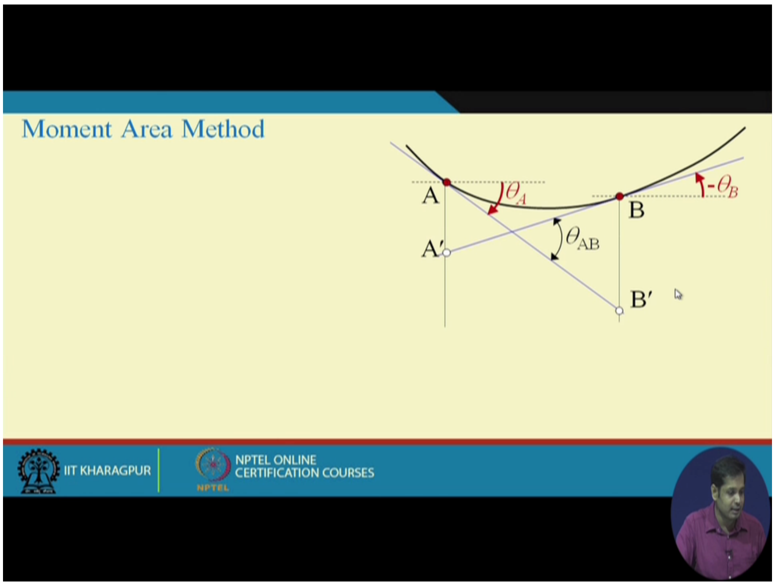

Suppose then they are A dash and B dash. So A dash is essentially also called that it is the deviation of A on slope drawn at B and this is also of point B on slope drawn at A. Okay. Now what is the physical significance of these two points? It will be cleared shortly. Suppose we have such two points A dash and B dash, right? Now this is denoted as delta BA and similarly so this is denoted at delta AB.

So this is essentially what? This is the distance between the point B and its projection on slope drawn at A and this is this delta B is the distance between point A and its projection on slope drawn at B. Okay.

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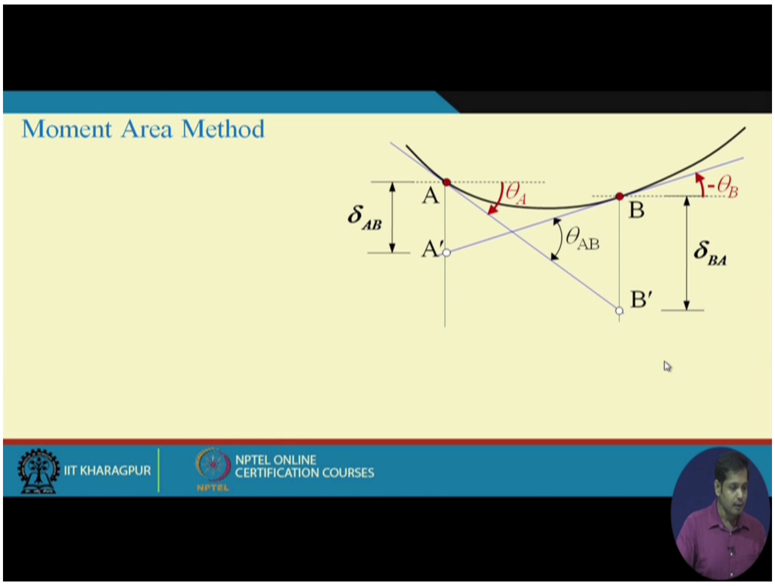

Now we know that d theta dx is equal to minus phi x. Just now we denoted this. Okay. Now if we integrate the entire thing between point A and B so what we get is integration of d theta between A and B is equal to minus integration of phi x between A to B, right?

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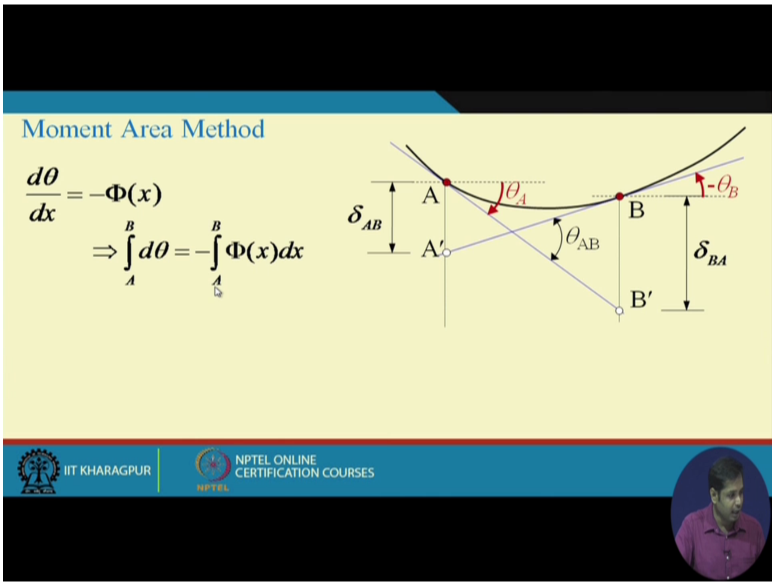

Now what is this integration of d theta? Integration of d theta between point A to B is equal to theta B minus theta A. But since we have a minus sign here so it can be written as theta A minus theta B. And the integration between A to B is phi x dx. So what is theta A minus theta B? Theta A minus theta B is essentially theta AB. So this is theta AB is equal to integration of phi x between A and B.

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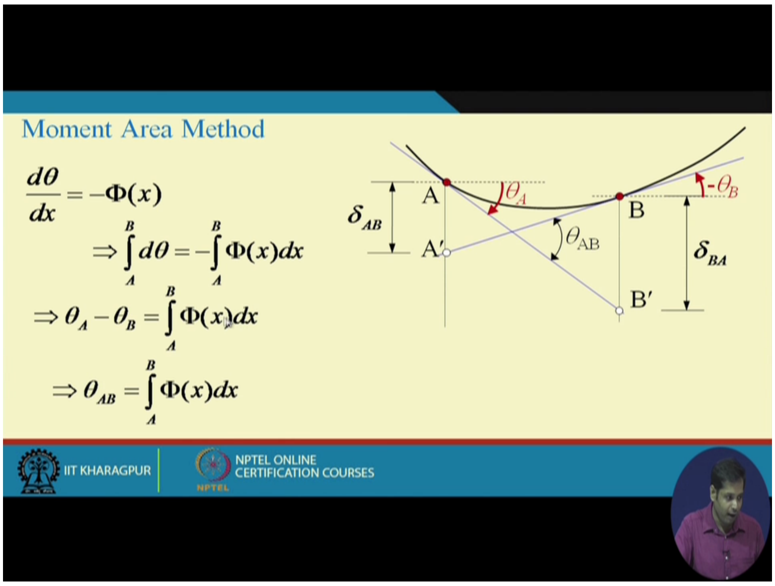

So we can see that the angle between tangents drawn at A and B which is theta B is equal to this. So what is the physical significance of this equation? Now suppose this is the bending moment diagram divided by EI. Okay.

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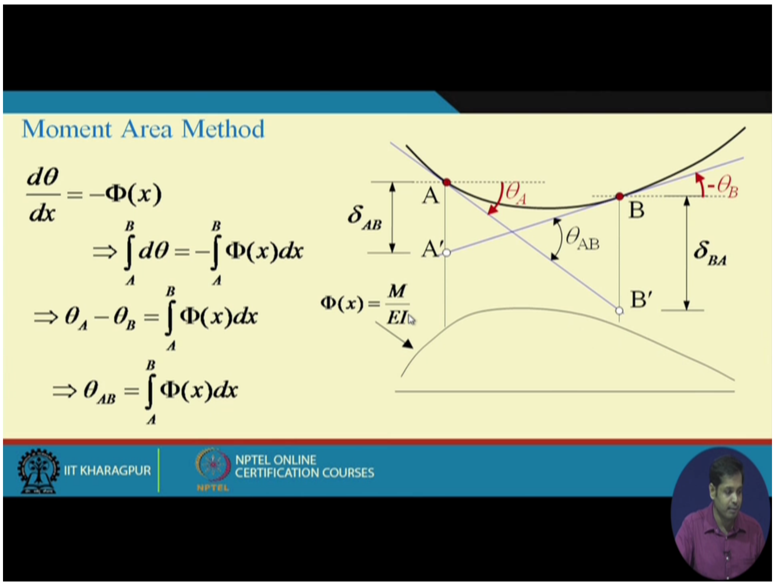

So this is phi x. Okay. And then area of this curve is this, between point A and B. So integration of phi x dx between point A and B essentially this shaded area, right?

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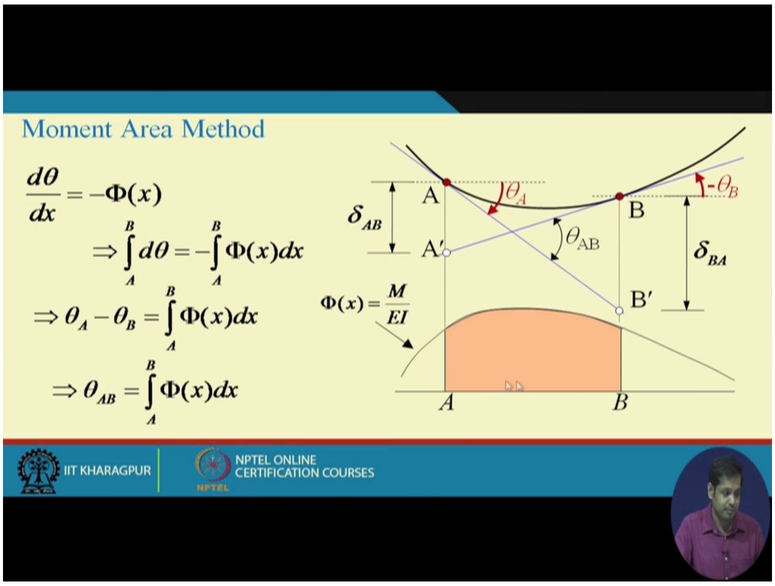

So what it says? It says that if we take two points A and B and draw tangent at those two point A and B then the angle between those tangents will be the area of M by EI diagram between these two points A and B, right? So here we have the first theorem of moment area method. What it says? Just now what I said that the change in slope between the (chanjent) tangents drawn to the elastic curve at any two points A and B, so any two points A and B.

And the change in slope between this is theta AB is equal to area of bending moment diagram between A and B and divided by EI. Okay.

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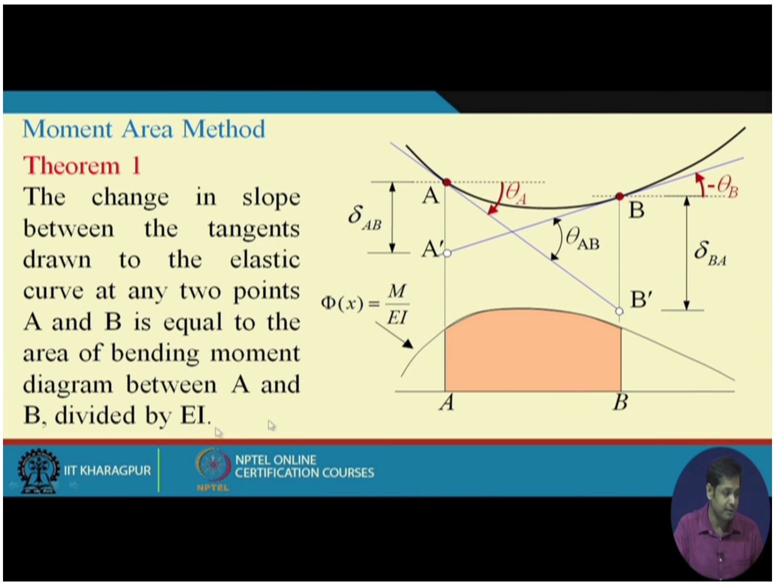

So this is the first theorem of moment area method. When we see the application of this probably it will be more clear. Okay. So the first theorem essentially gives you some information about the slope, right? So theta AB. Let us see how to get some information about these two deflections which is not in a deflection in true sense but some measure of deflection delta BA and delta AB. Okay. Now this is this theorem says, right? Now so take just first point.

Let us find out what is delta B and the similar concept can be extended to delta AB as well. Now so B dash, this point is essentially projection of B on slope drawn at A. So we need to find out what will be the delta BA, right? And this is the bending moment diagram by EI. Okay. And this is the area between two points A and B.

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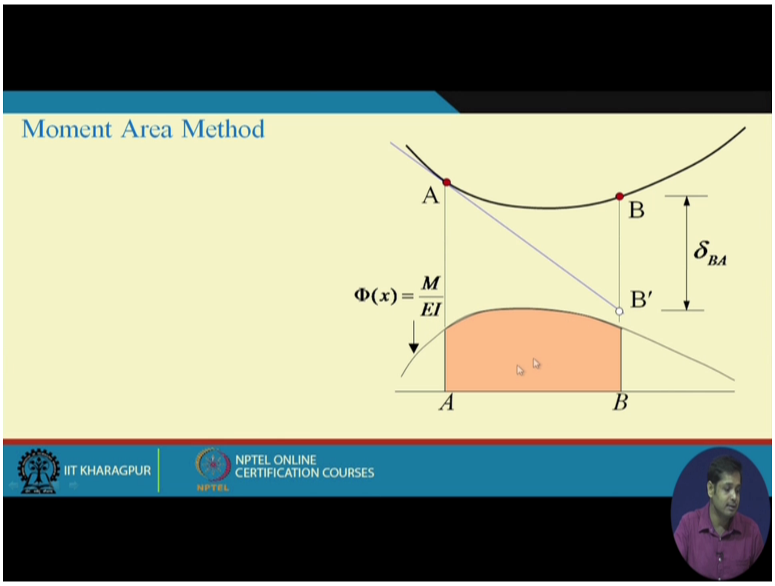

So now take any arbitrary point between A and B. Say that point is at a distance xB from B. So it could be any point between A and B. Now take another small segment which is of length delta x. So this is delta S and but as I said deflection is small, delta S can be this curvilinear length, this arc length. This length can be represented by delta x if the slope and deflection is small. So this small length of this small segment is delta x. Okay.

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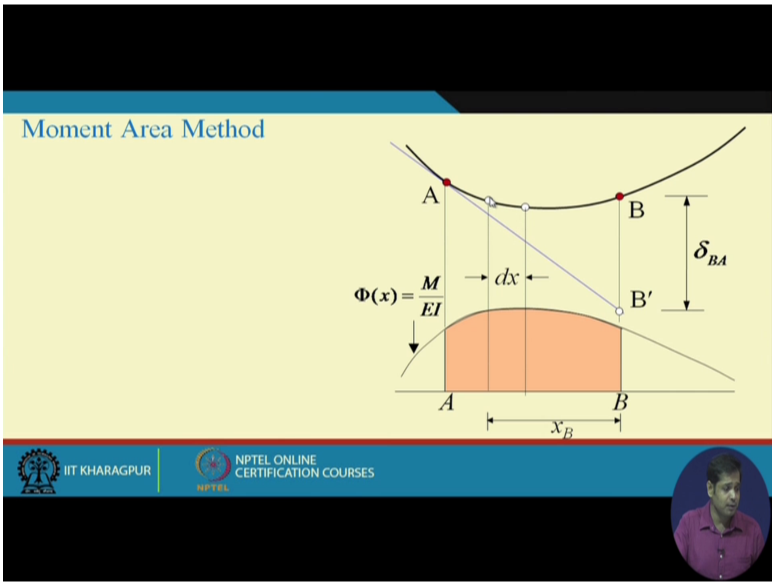

So the angle due to this length is d theta. Okay. So if this is d theta then if we draw slope at point this and at a distance delta x then the angle between these slopes will also be d theta.

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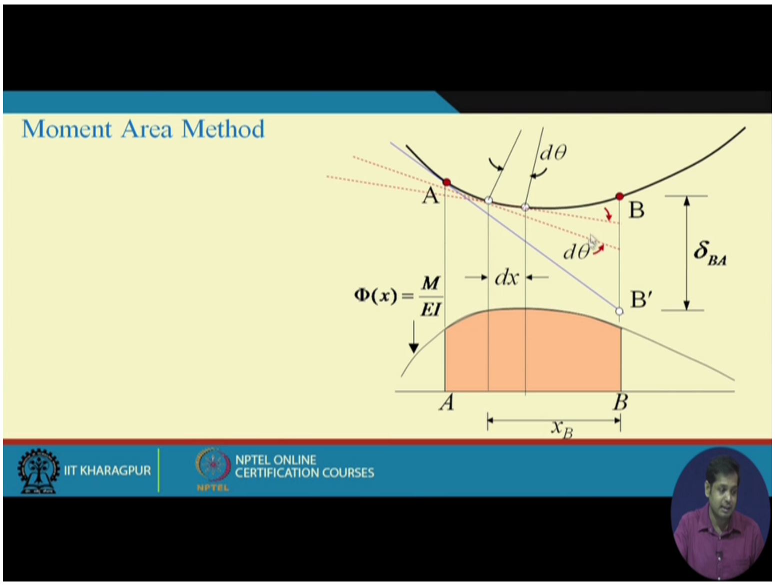

That we have seen while deriving the equation of elastic curve, right? Now so this angle is also d theta. If so, then what (expressi) now and suppose this distance is (del) d of d delta BA. Now if this point becomes A and this point becomes B so naturally this point becomes delta BA, right?

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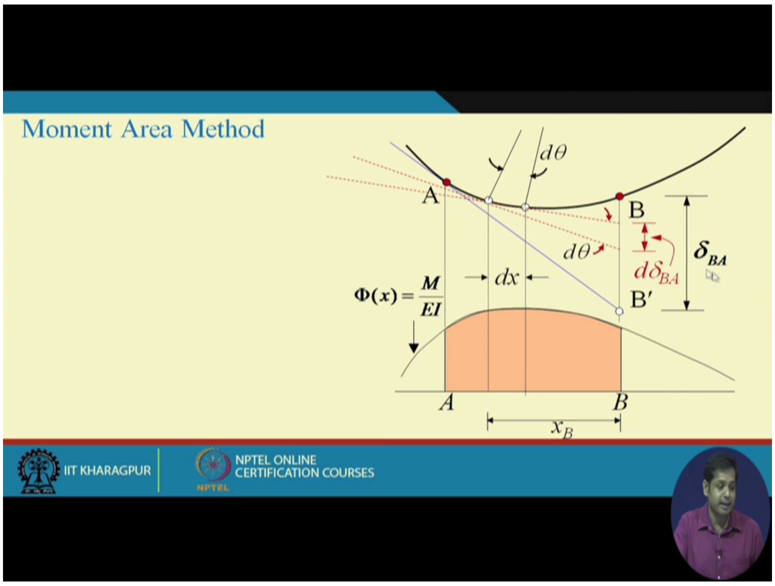

Now we know that this length will be this xB into d theta. Okay. And we can write this if theta is very small. Okay. So this we can write.

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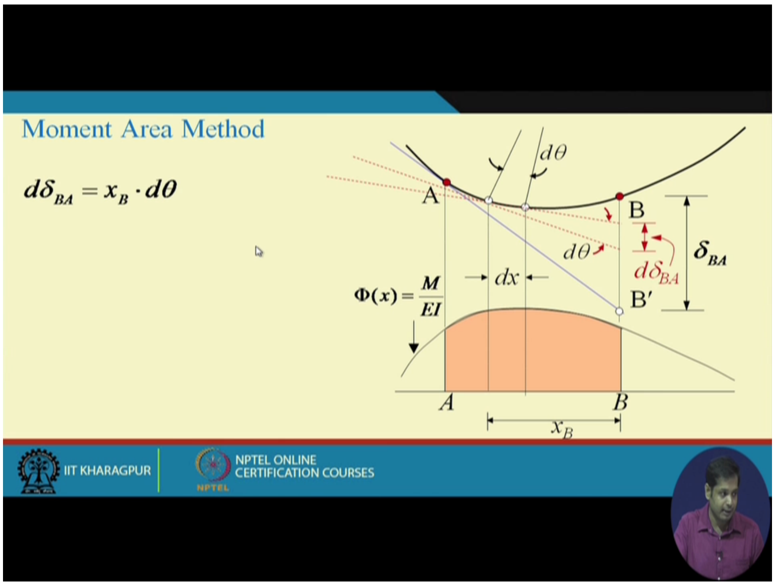

Now so if we integrate it between A to B, what we get is delta BA. And this integration will be xB d theta between B to A because xB is measured from B. Okay. Now so what this is exactly?

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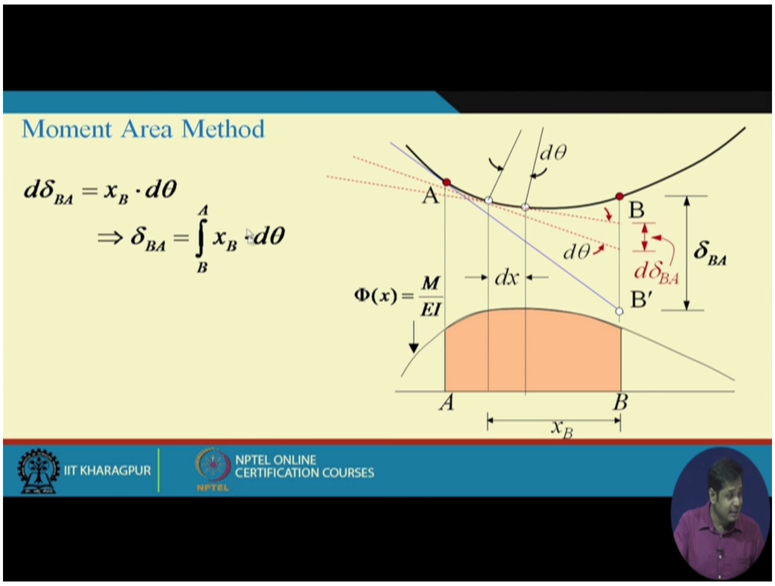

Now we know that d theta dx is equal to minus phi x. We have defined phi x like this. Let us substitute this in this expression. If you do that then what we have is delta BA which is the distance between point B and its projection B dash which is drawn on slope at A. So projection of B on slope at A, delta BA, that is equal to this.

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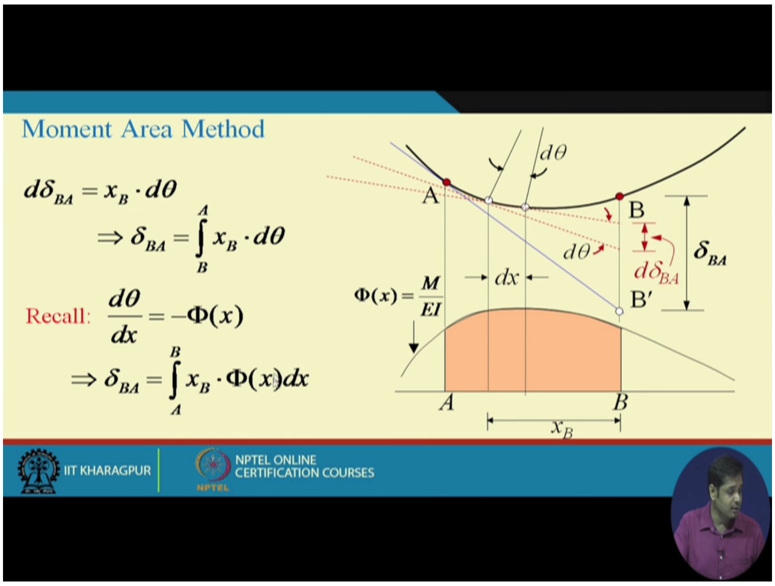

Now what this expression? This expression is essentially the moment of this area about point A. If we do not have xB if had been integration A to B phi x dx, this was the area between A and B. But if we have xB into phi x dx then this is essentially the moment of this area about. You see xB varies from A to B. Okay. Now it was B to A initially but it become A to B because of this minus sign.

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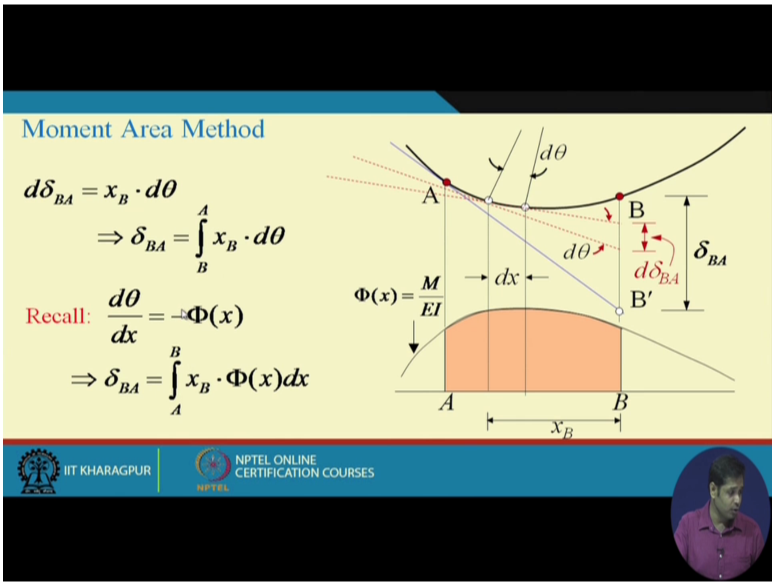

Now if this is the moment of this area about B then what we can write, delta BA is equal to that moment of this area about point B.

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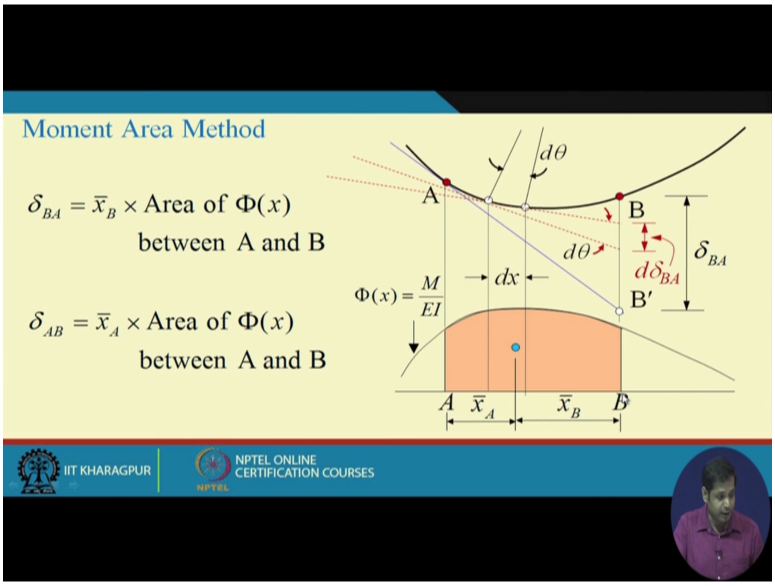

Now which is again can be written as xB bar cross area of this area into the distance of centroid from B. That is what would be the moment of this area from B.

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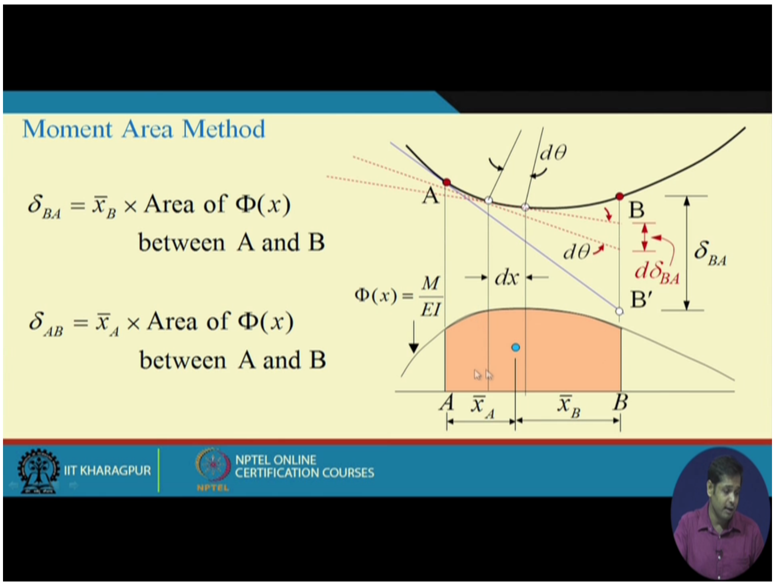

Similarly the moment of this area from A will be the area itself multiplied by the distance of centroid from A. So delta BA will be xB bar which is the distance of centroid from B into area of this area between A and B. So similarly we can also determine delta AB is equal to xA bar which is the centroidal distance from A into area of this. And for different area I believe you know how to determine the centroidal distances. Okay.

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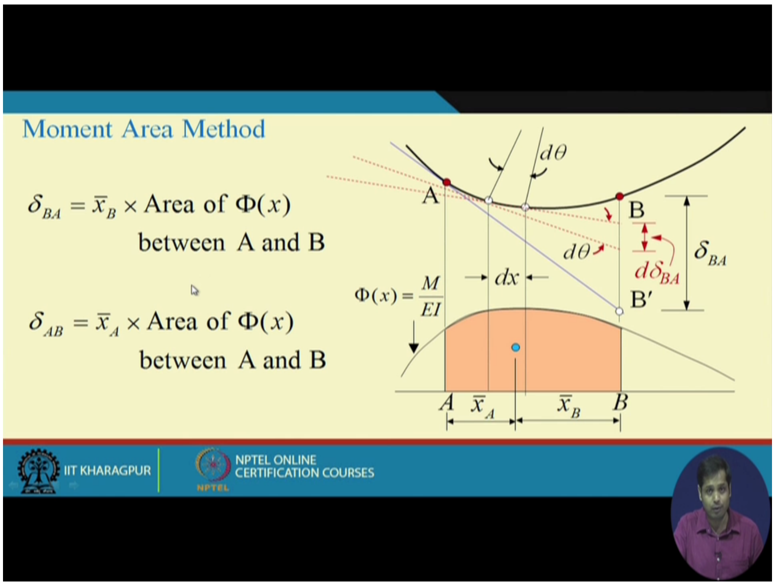

So if it is then what it says? It says that the projection of the distance between point B or the deviation of point B on slope drawn at A that is equal to the moment of M by EI diagram about point B. Okay. This gives you theorem two.

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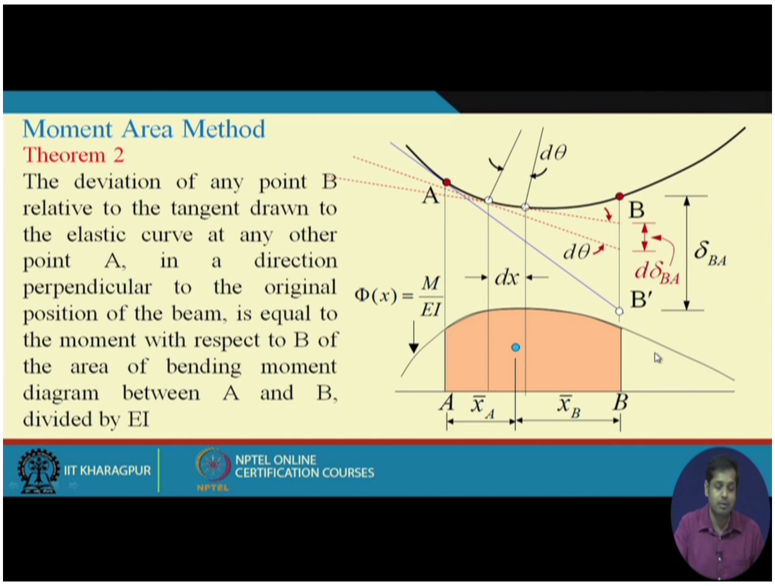

What it says that the deviation of any point B relative to the tangent drawn to the elastic curve at any other point A. So deviation of point B relative to the tangent drawn at A and that deviation is delta BA here in a direction perpendicular to the original position of the beam which is anyways true because we are calculating perpendicular direction, okay, is equal to the moment with respect to B of the area of bending moment diagram divided by EI between A and B.

So this is the second theorem. So moment area method is essentially. So if we look at this theorem they do not say anything new about the behaviour of the beam because this behaviour can be expressed through equation of elastic curve itself. It is a different interpretation of equation of elastic curve and use that interpretation to determine slopes and deflection. Now moment area method is essentially application of these two theorems, just now we stated and proved. Okay.

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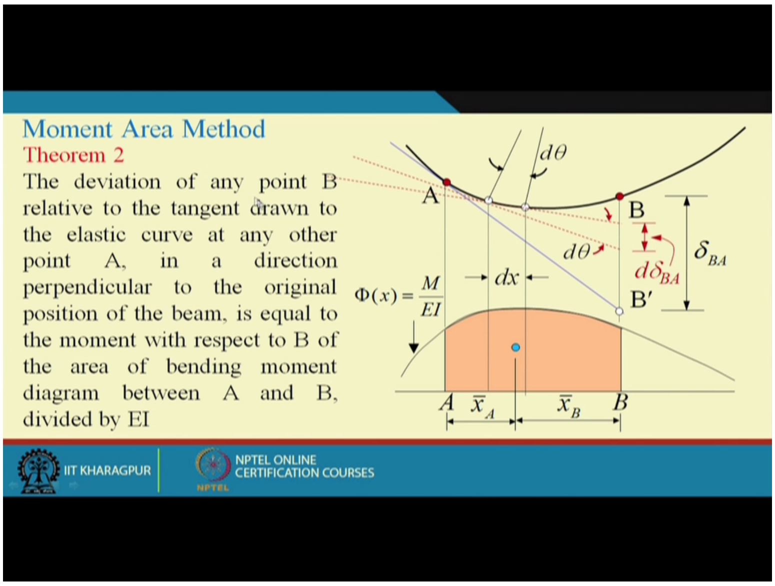

Let us see one example. Probably then it will be clear. Take a simply supported beam. We have already determined slope and deflection of this beam using direct integration method, right? Now let us find out the slope and deflection using moment area method. So expression of MX is this anyway. You have to determine any method you use. This is the how the bending moment varies.

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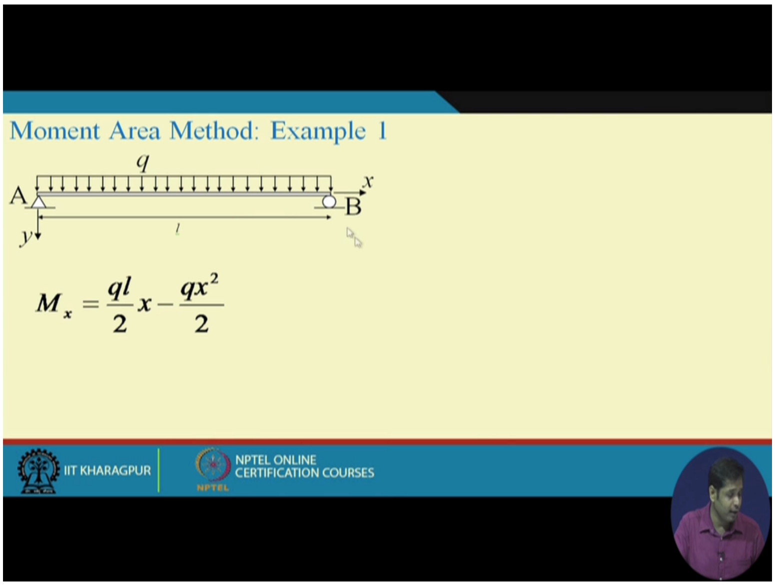

Now suppose this is the deflected shape of this beam. Okay. And the reaction will be qL by 2 and qL by 2 it is symmetric and suppose C is the midpoint.

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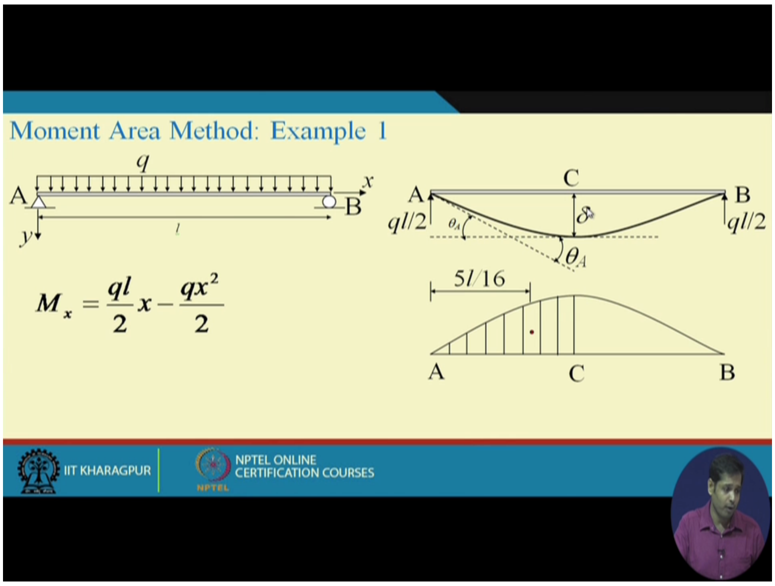

If C is the midpoint then we know due to symmetry the deflection will be maximum at the midpoint and if we will draw slope at point C then this slope will be zero. Okay. Because how do we calculate deflection? What is the position of maximum deflection where dy dx is equal to zero? Means dy dx is essentially this slope.

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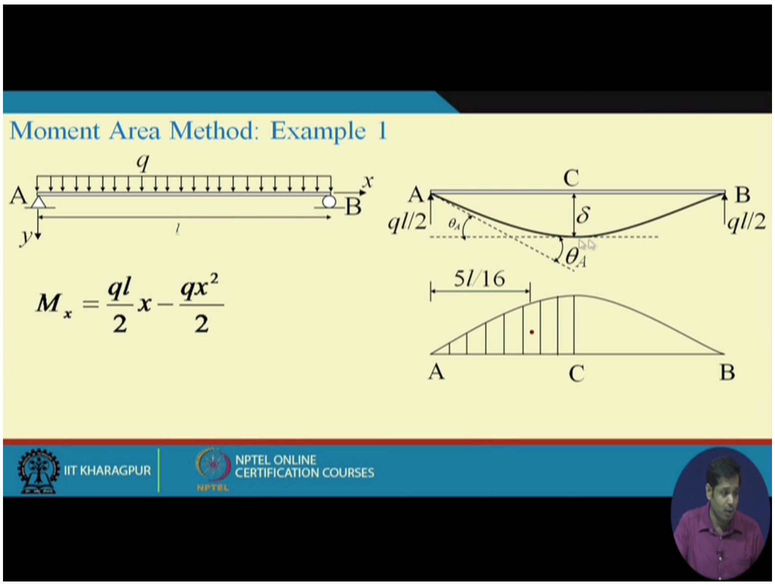

So slope is zero at this point. Okay. So let us draw one tangent at A and one tangent at C. So this will be essentially the angle between tangents at A and C. So this is essentially theta AC, right? But since we know that theta AB was theta A minus theta B, but since in this case theta C is equal to zero so this angle is essentially theta A. Okay. So this angle is theta A. So this angle is essentially slope at point A. Okay.

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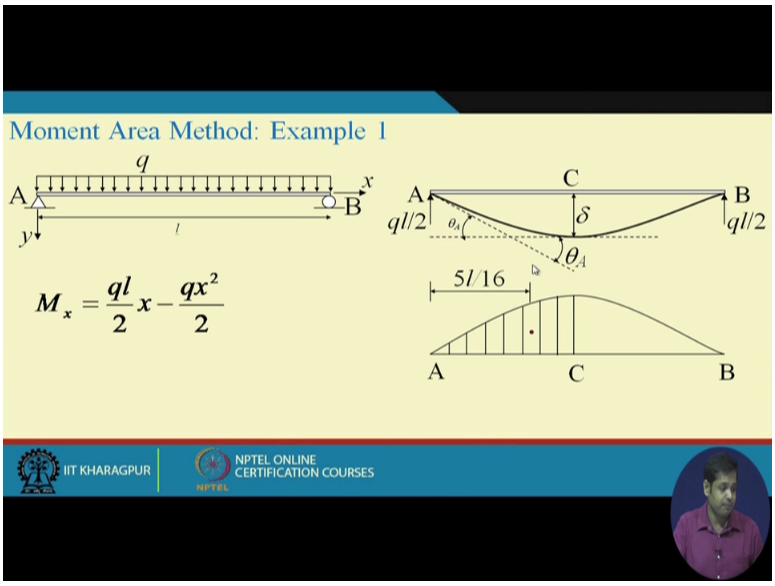

Now what the first theorem says? The first theorem says that if we draw tangent at two points the (ta) angle between those two tangents will be the area of bending moment diagram M by EI diagram between those two points. And this is the bending moment diagram.

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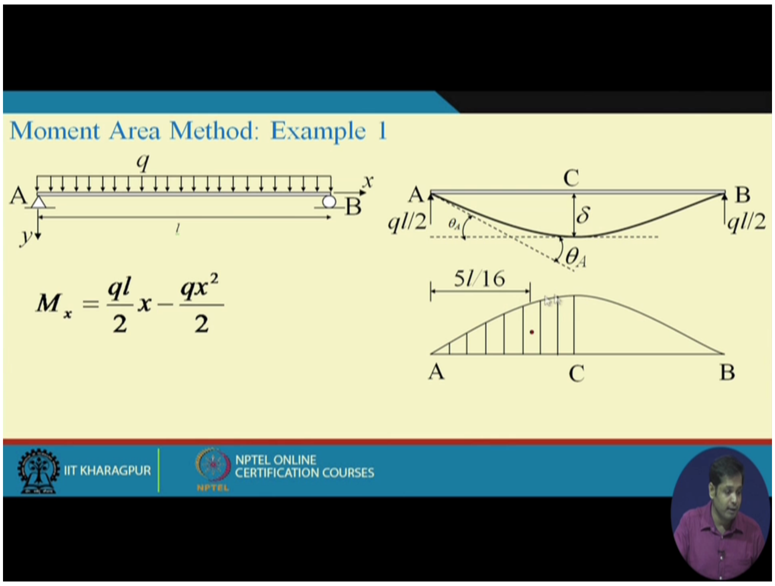

We have also seen how to draw bending moment diagram for this case and this is the area of bending moment diagram between A and C. And the centroid of this parabolic curve is at a distance 5L by 16 from point A because this distance is L by 2. Okay.

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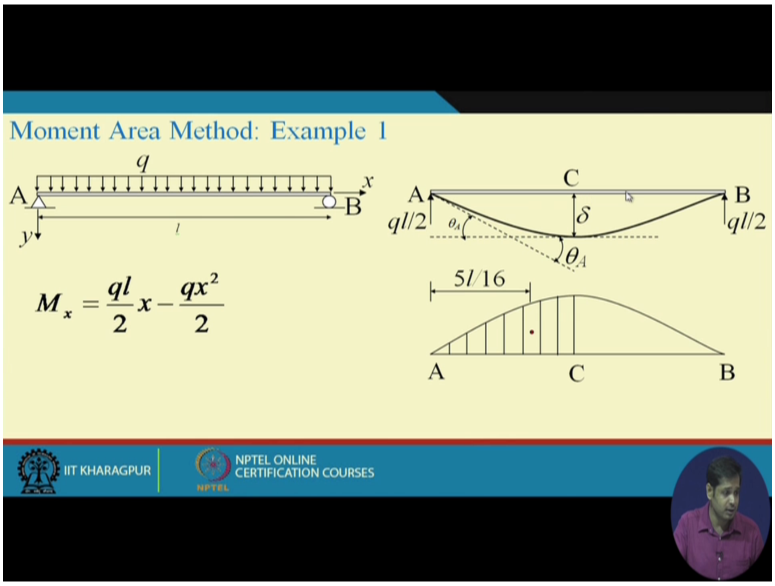

So essentially if distance is L then this is 5L by 8 but this is L by 2 so this is 5L by 16. Okay. So what will be theta AC? Theta AC which is again in this case theta A because theta C is equal to zero will be the integration of xA between A to C of area of bending moment diagram.

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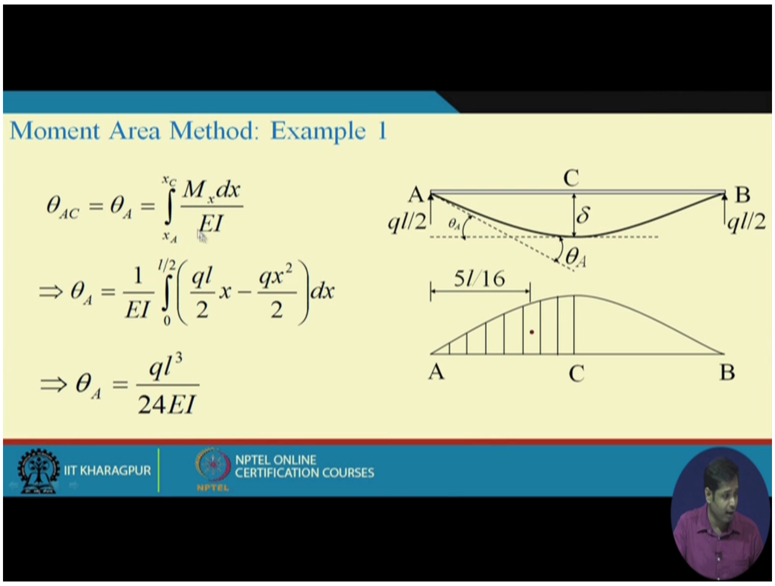

And if we substitute bending moment diagram just now we have (ex) determined what is bending moment diagram.

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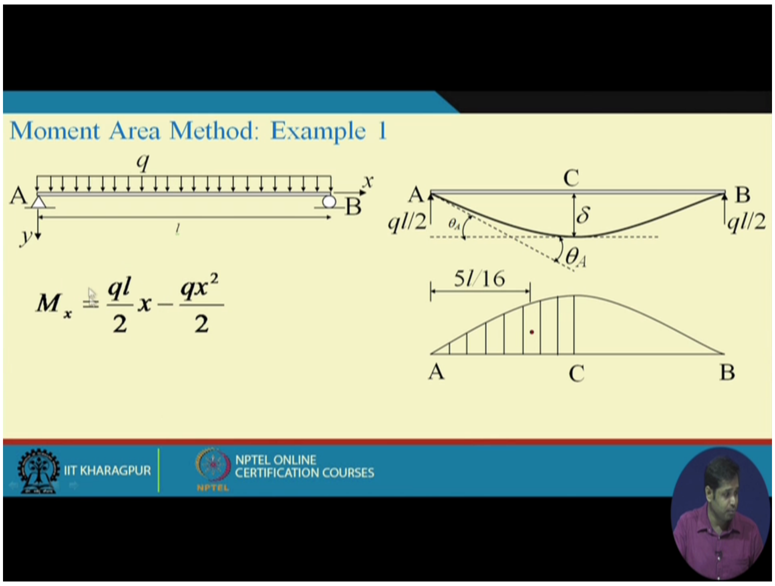

So if you substitute bending moment expression here and integrate between zero to L by 2 we get expression of theta A is equal to this.

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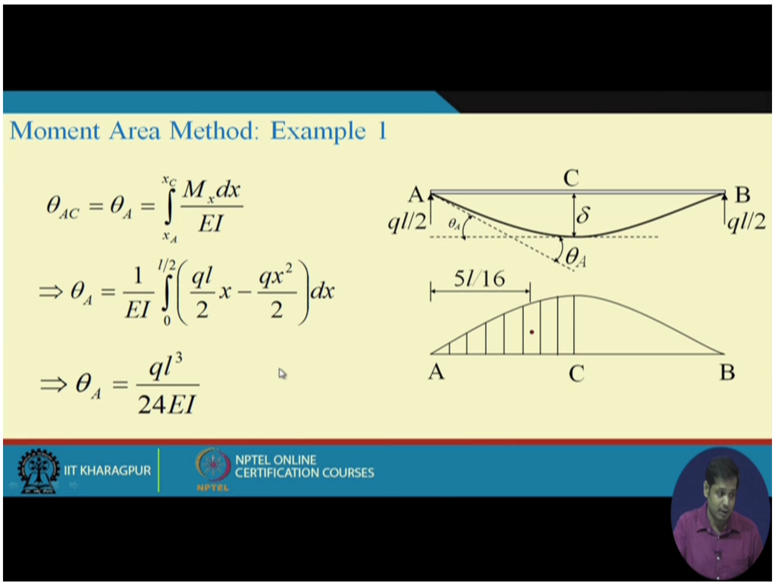

And then what will be the delta? Again you see in this case what that theorem says? The theorem says is deviation of any point with respect to this slope at any other point will be the moment of the bending moment M by EI diagram with respect to the about that point. Now so this is the slope at C this is a slope at A.

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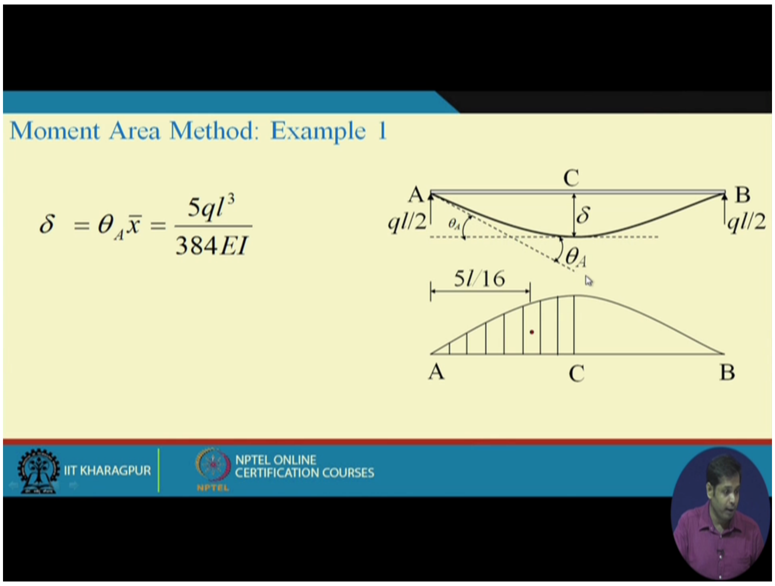

So what would be the deviation of point C with respect to the slope at A? These deviations will be this distance, right? So this is the projection of point C on slope drawn at A so this is the deviation.

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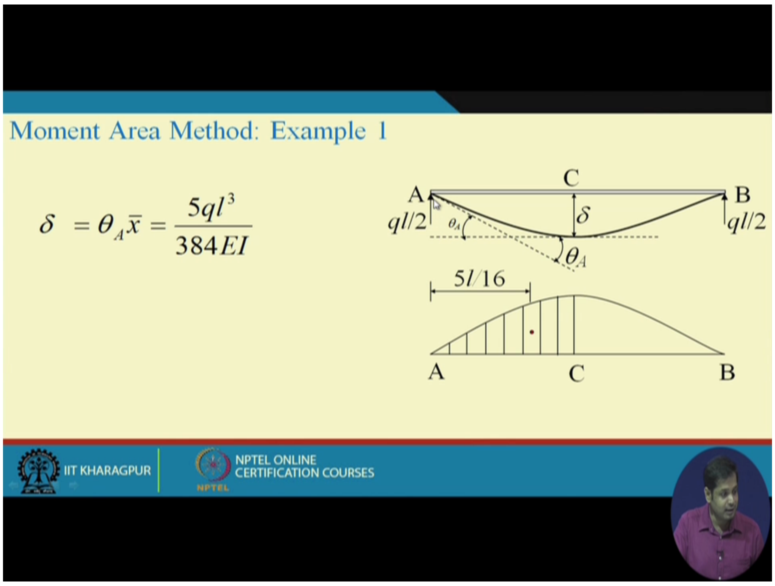

But in this case since slope at C is equal to zero, this deviation itself is the deflection at C delta. So delta will be the moment of this diagram about point A.

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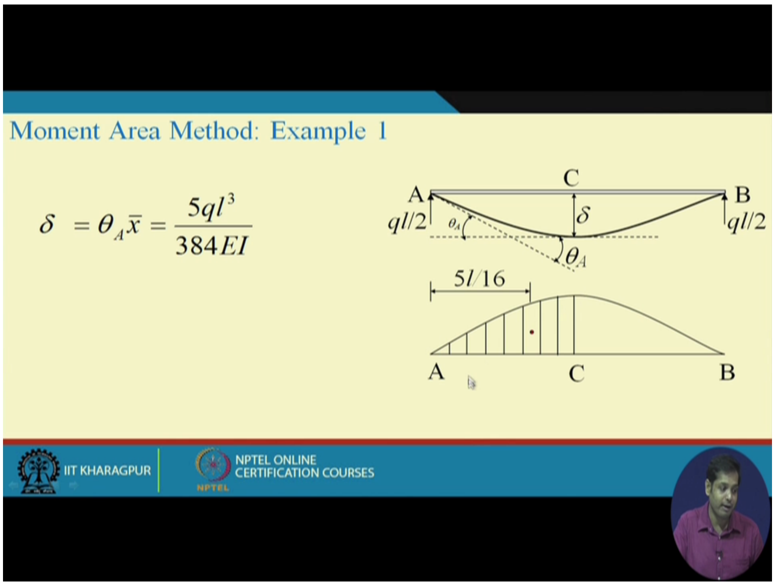

Please note that for calculating slope it is very straight forward because it is the area, right? But when you calculate the deflection since it is moment about what point the moment has to be taken that is very important. And that you need to decide based on what exactly you are calculating. In this case you see though it is deflection at point C but we are looking this at in a different way.

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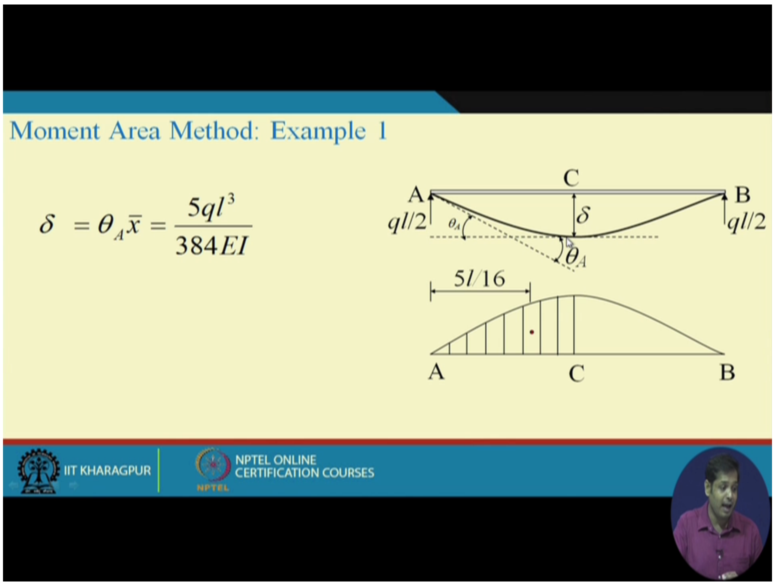

(Wa) How we are looking it? We are looking it as if it is the projection of point A on the slope at C. So this distance will be the moment of about point A.

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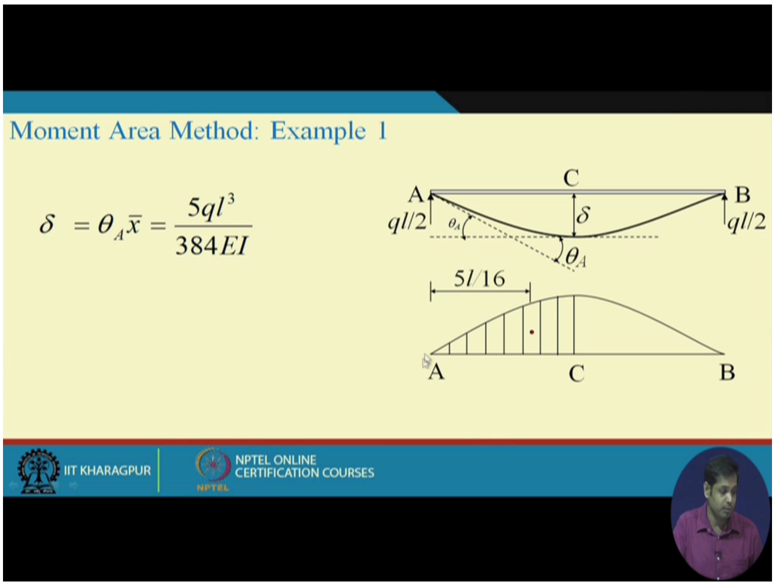

So if you multiply that so this will be the delta for this and if you remember for simply supported beam we determined using direct integration technique the deflection at point B at the midpoint is 5qL cube by 384 EI. Okay.

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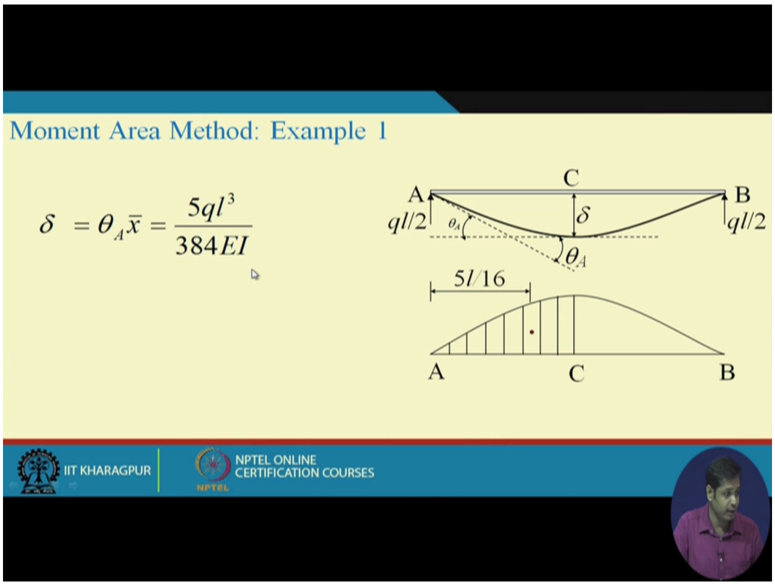

Now (deter) what is important here is. Please there is a correction this will be 5ql to the power 4 by 380 EI. It will not be cube, it is 4. Okay.

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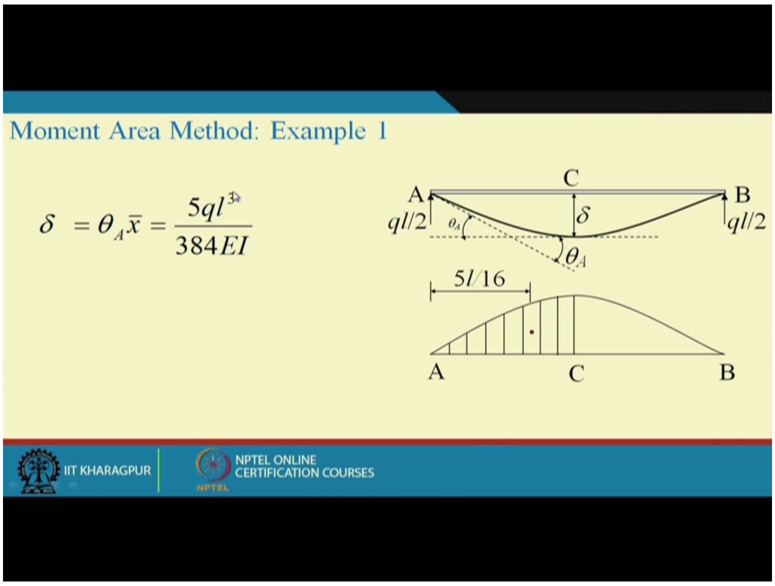

So this is moment area method. So what exactly moment area method is? It is the same equation of elastic line, same differential equation but it is interpreted in a different way. Instead of directly integrating the equation, the integration is still there but (integra) since interpretation is different, the step for integration is also different. But essentially the premise of moment area method is the same as elastic equation of elastic curve.

So today what we have done is we have just introduced the moment area method and demonstrated it through one simple example. In the next class we will probably give you few more examples so that you can understand the moment area method in a better way. Okay here we stop today. So next class we will see some more example on moment area method. Okay. Thank you.