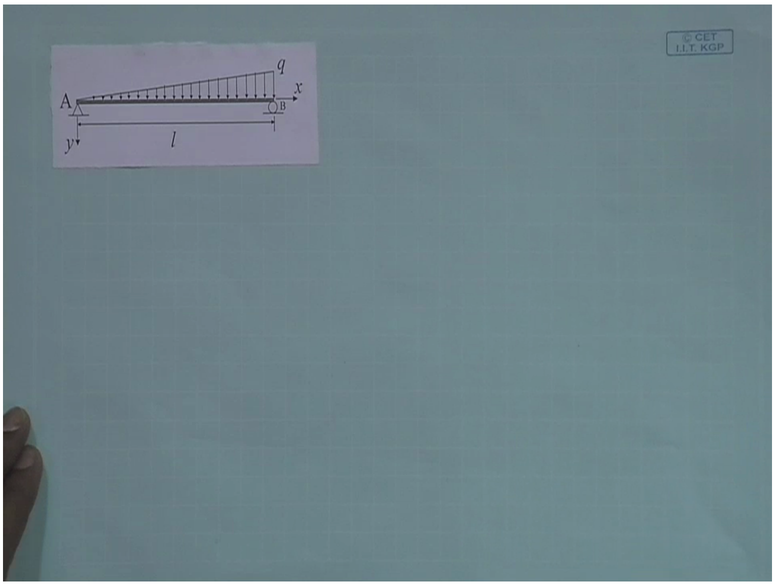
Structural Analysis 1
Professor Amit Shaw
Department of Civil Engineering
Indian Institute of Technology Kharagpur
Lecture 19
Deflection of Beams and Frames (Contd.)

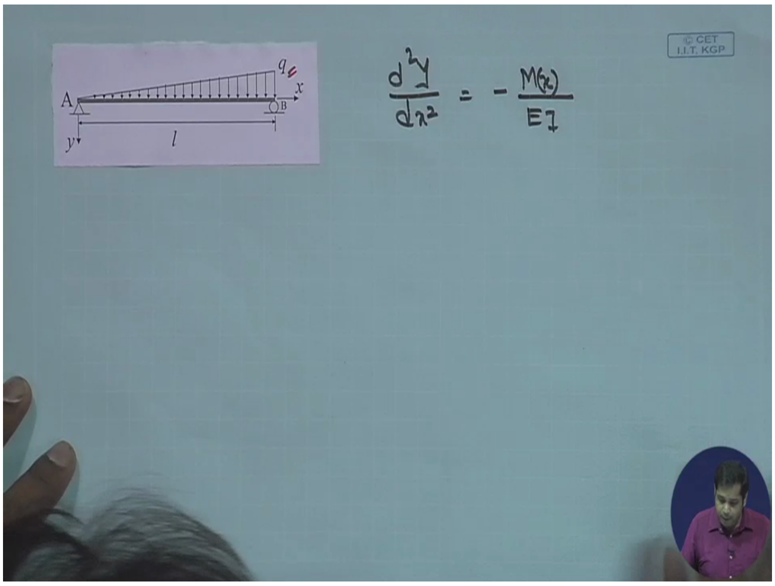
Hello everyone welcome to the lecture number 19 of week 4. We were supposed to start moment area method today but then again I thought since the equations of elastic line that we derived in first lecture of this week is very important and we will come across this equation many times not only in structures mechanics in other engineering courses as well. So let us spend some more time on that equation and explore that to other problems using direct integration method, okay. So today we will at least two-three problems using the equation of elastic line and direct integration method, okay. The first problem that we do is this one.

(Refer Slide Time: 01:20)



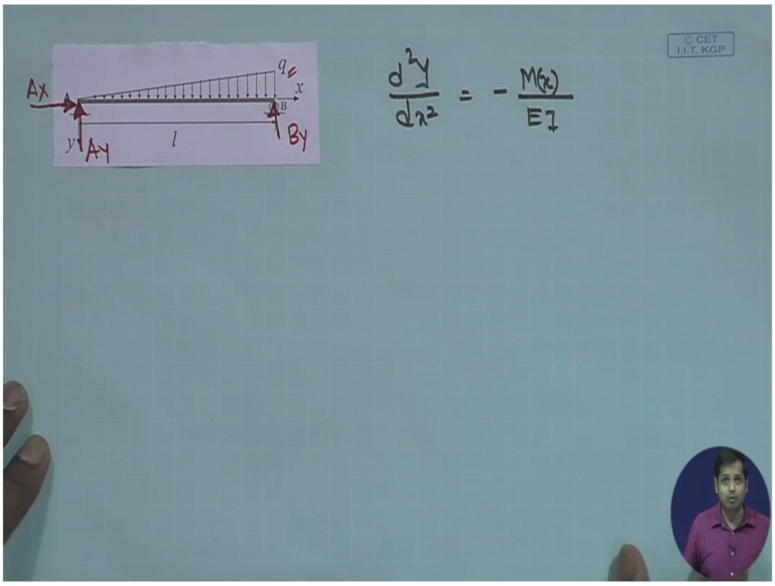
It is again a simply supported beam. But now the distribution of load is not uniform it is triangular distributed load. And the intensity of this load q means at this point it is q and it is linearly varying and at point A it is zero. So what we have to do is we need to find out the slope and deflection at point A and B and what is the profile of deflection using direct integration technique? Now if you remember the equation that we have is d2y dx2 that was the equation for elastic line, M by EI. Mx is function of x, okay.

(Refer Slide Time: 02:10)



So first we need to find out what is the expression of the bending moment Mx. Let us assume for this beam E and I both are constant, both are uniform across the length, okay. Now so first we need to find out the support reactions. We already discussed how to find out support reactions by drawing free body diagram of the entire structure. So I am not going to find out the support reaction once again. And if we draw the free body diagram like this, it is By and then Ay and horizontal reaction Ax, horizontal reaction will be zero in this case.

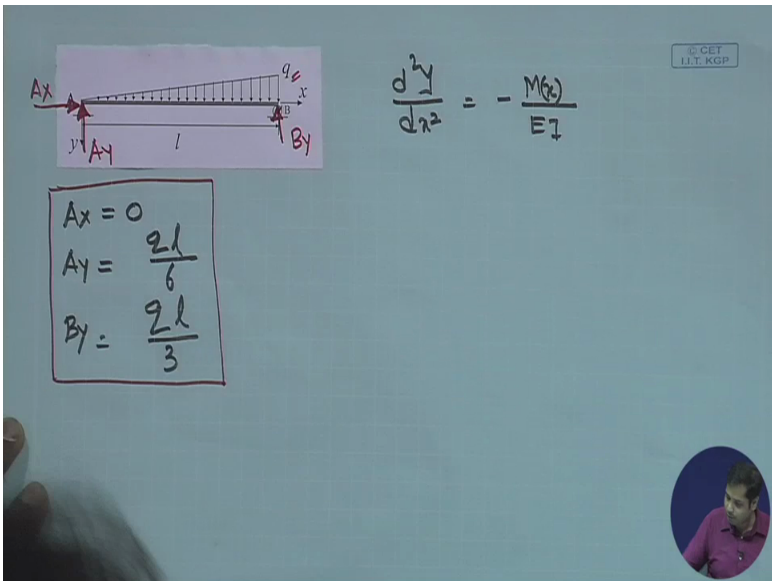
(Refer Slide Time: 02:53)



So we get Ax is equal to zero, Ay is equal to qL by 6 and By is equal to qL by 3. If you sum them Ay plus By will give you the total load. Total load is half into this length into intensity

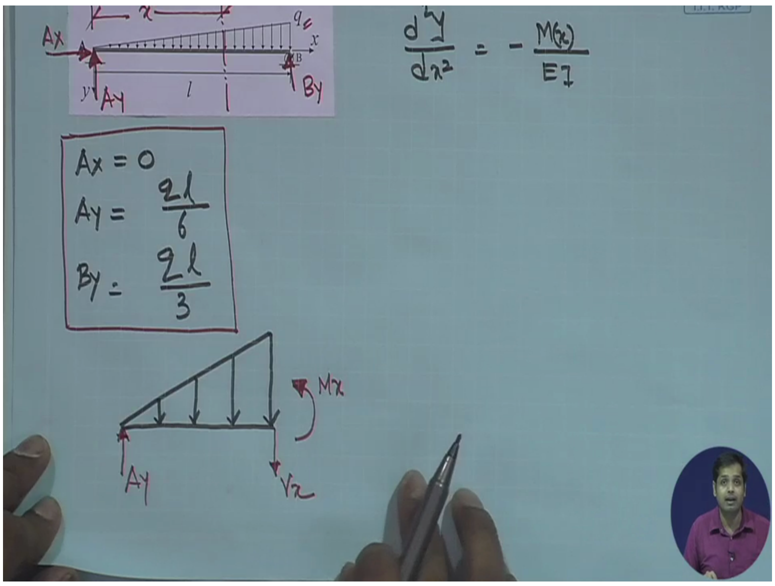
q. So these are the support reactions. We know how to determine these support reactions, right? Okay.

(Refer Slide Time: 03:27)



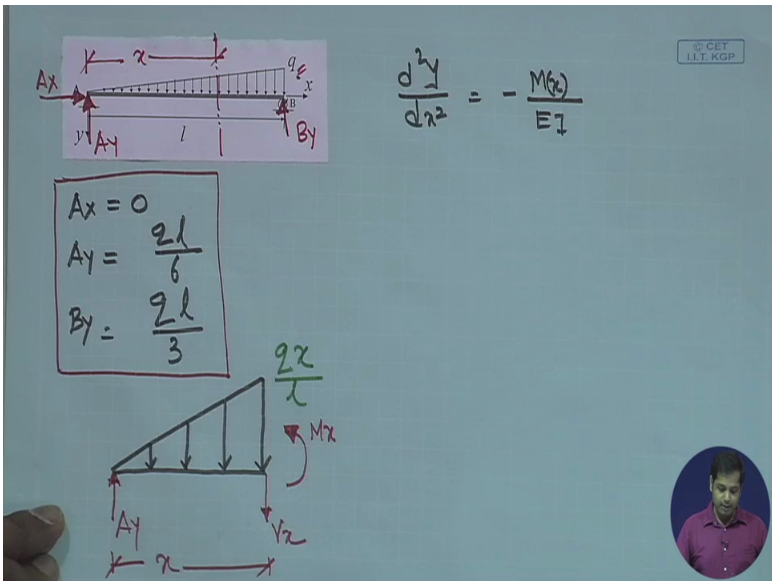
Now let us find out the expression for bending moment. In order to do that let us take a section at a distance x from A. So draw the free body diagram of that section. So free body diagram will be the intensity of load will be Ay here and then externally applied load which is linearly varying and then support shear force Vx and then moment Mx, okay. Ax and Fx here are not shown because they are zero here.

(Refer Slide Time: 04:24)



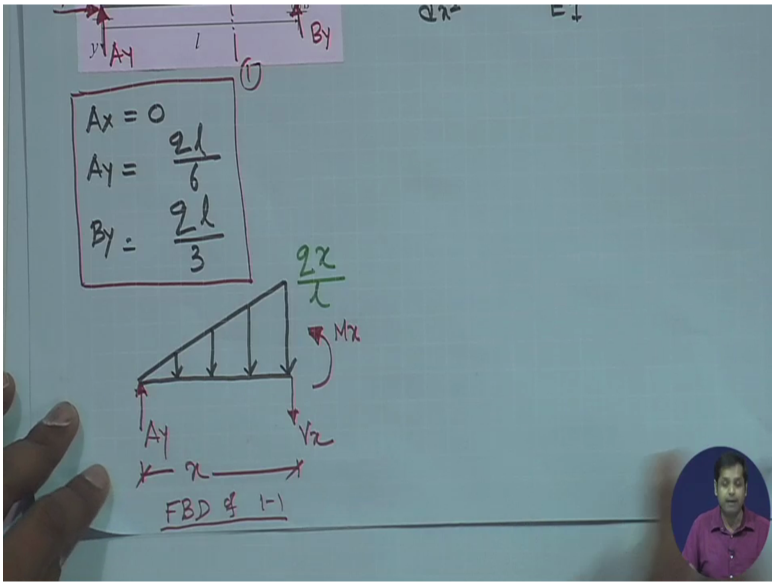
Now distance is x. So if this is q then at a distance L intensity is q then at a distance x intensity of this load will be qx by L, okay, qx by L.

(Refer Slide Time: 04:51)



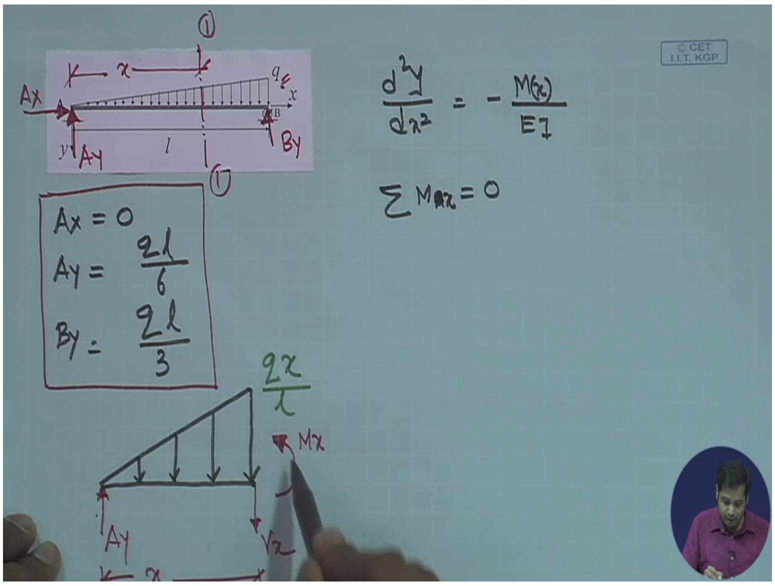
Now this is the free body diagram of section, say it is section 1-1. If this is section 1-1, then it is free body diagram of section 1-1, okay.

(Refer Slide Time: 05:09)



Now from this free body diagram if we take moment about this point is equal to zero, summation of moment at x is equal to zero, then the forces will contribute is Ay and the (momex) Mx and the externally applied load here.

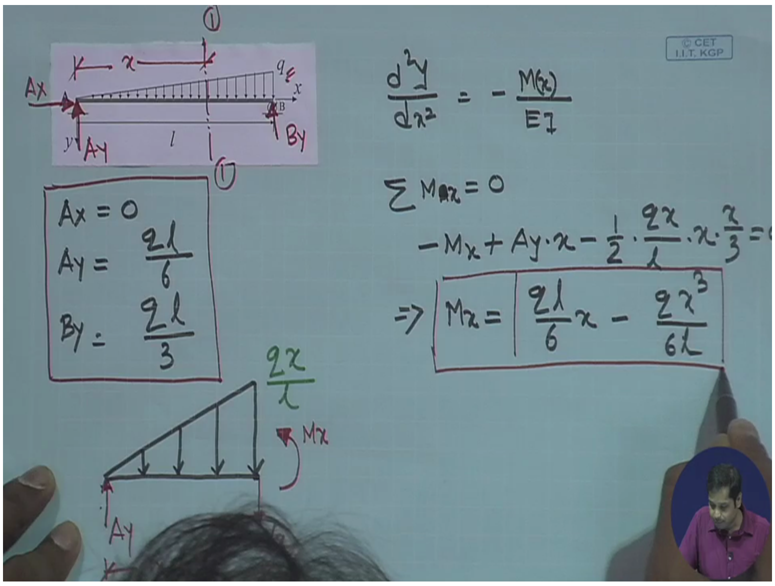
(Refer Slide Time: 05:34)



And the expression will be now Mx is anticlockwise so it is negative Mx. And then the moment due to Ay will be clockwise. This is Ay into x and then moment due to externally applied load will be again anticlockwise and this will be minus the area is half into qx by L into x. This is the area of the triangle and this centroid of this triangle will be at a distance x by 3 from this, so into x by 3.

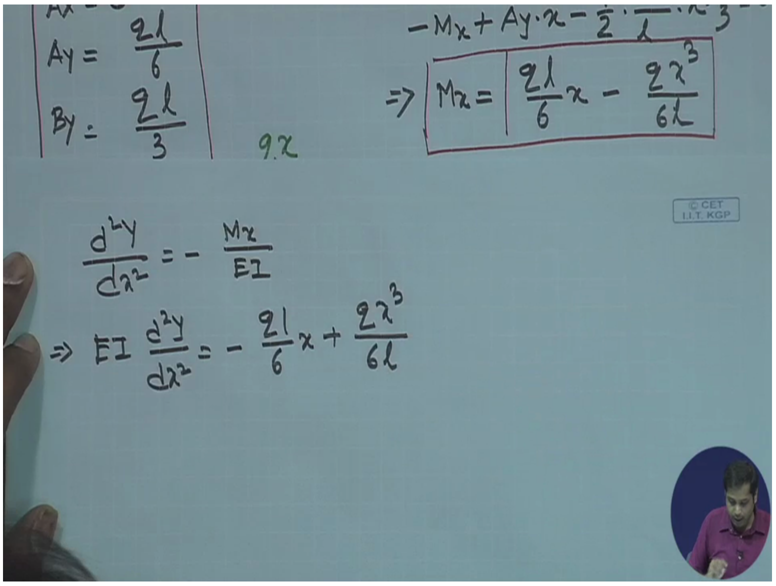
This will be the moment due to this load. So that is equal to zero and this will gives us Mx is equal to qL by 6 x, Ay is equal to already determined qL by 6, minus qx cube by 6 L. So this will be the expression for Mx, okay.

(Refer Slide Time: 06:59)



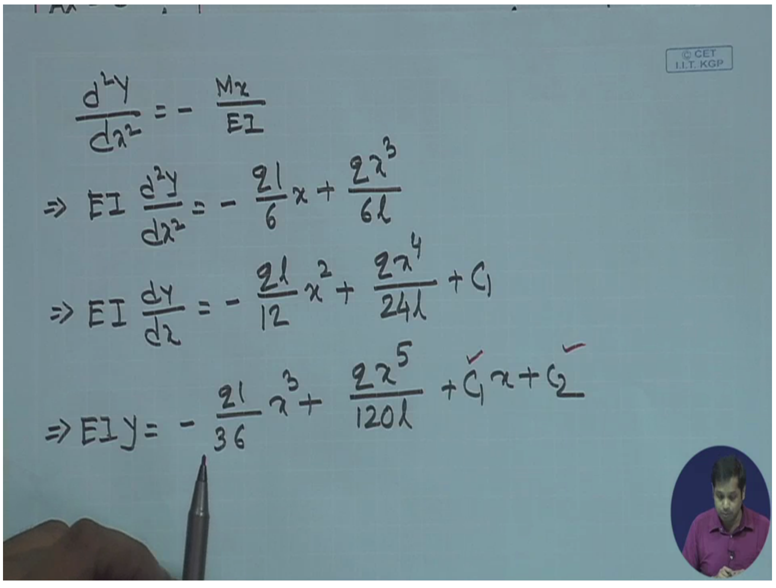
Now this expression is to be substituted in this equation and then integrate it. Let us do that, if we substitute this expression in this equation, what we have is this. So we have d2y dx2 is equal to minus Mx by EI. So this will give us EI d2y dx2 is equal to minus of this expression and this will be minus qL by 6 x plus qx cube by 6L. Now this needs to be integrated.

(Refer Slide Time: 07:59)



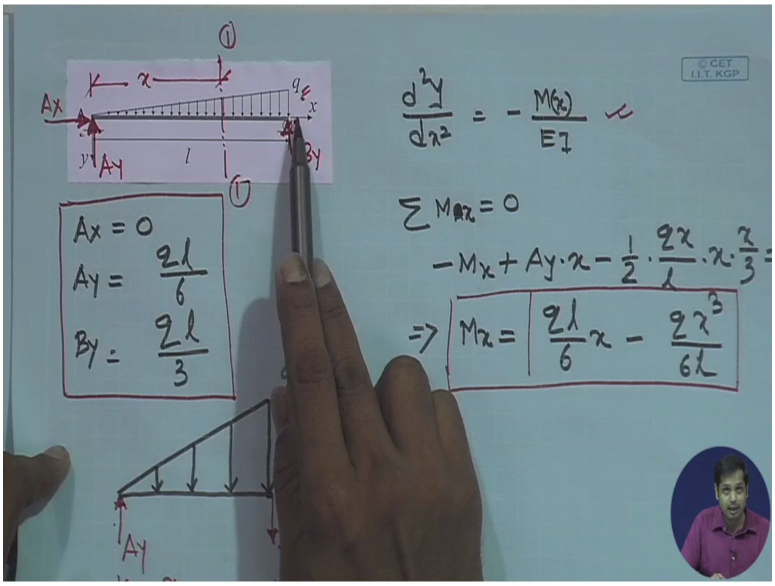
So if we integrate first time dy dx becomes minus qL by 12 x square plus qx to the power 4 by 24L plus C1, okay. And then again integrate it second time this will become minus qL by 36 x cube plus qx to the power 5 by 120L plus C1 x plus C2. Now we have two constants. C1 and C2, these two constants need to be determined. So we need two boundary conditions.

(Refer Slide Time: 08:59)



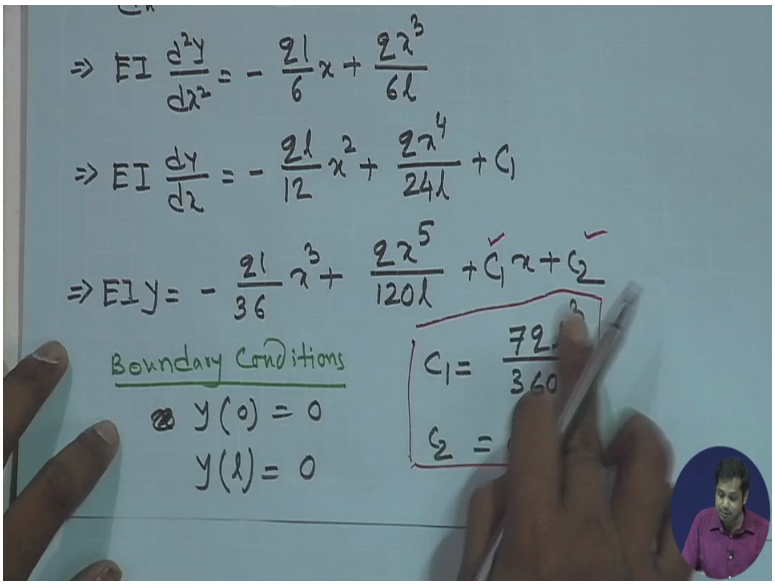
What are the boundary conditions we have? The boundary conditions are at x is equal to zero means at A, y is equal to zero and x is equal to b at x is equal to L means at B again y is equal to zero.

(Refer Slide Time: 09:12)



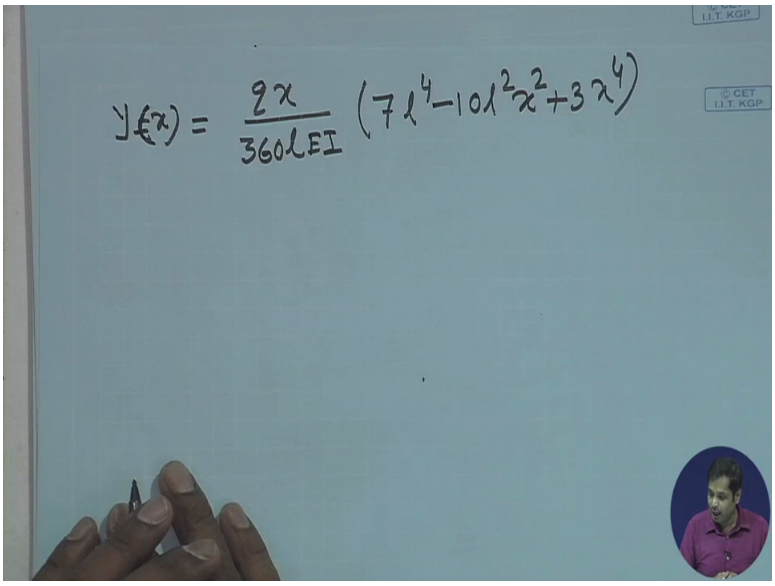
So boundary conditions are y at x is equal to zero, is equal to zero and y at x is equal to L, is equal to zero. And if we substitute these two boundary conditions and evaluate C1 and C2 what we get is C1 is equal to 7 qL cube by 360. I leave it to you please check that. C2 is equal to zero. So this is C1 and this is C2.

(Refer Slide Time: 10:15)



So if you substitute this C1 and C2 in this expression so final expression of y we get as this. Y let us write as a function of x is equal to qx by 360LEI 7L to the power 4 minus 10L square x square plus 3x to the power 4. So this is expression of y as a function of x.

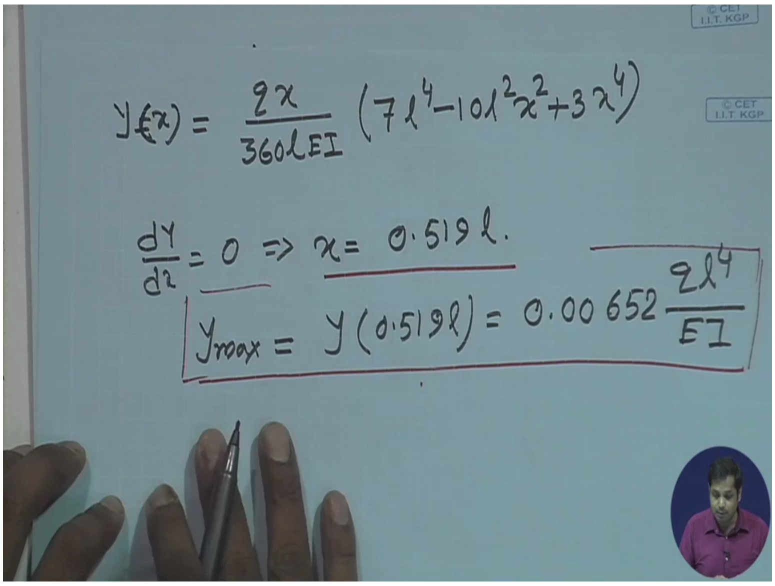
(Refer Slide Time: 10:57)



Now let us find out where y is equal to maximum. How do we get? Dy dx is equal to zero, slope will be zero and if you (sub) get dy dx is equal to zero this give us x is equal to 0 point 519L. Means at x is equal to this value, point 519L, y will be maximum. And if you evaluate entire y at x is equal to this, then we get y max or delta max is equal to y at 0 point 519L is equal to 0 point 00652. Please check these values.

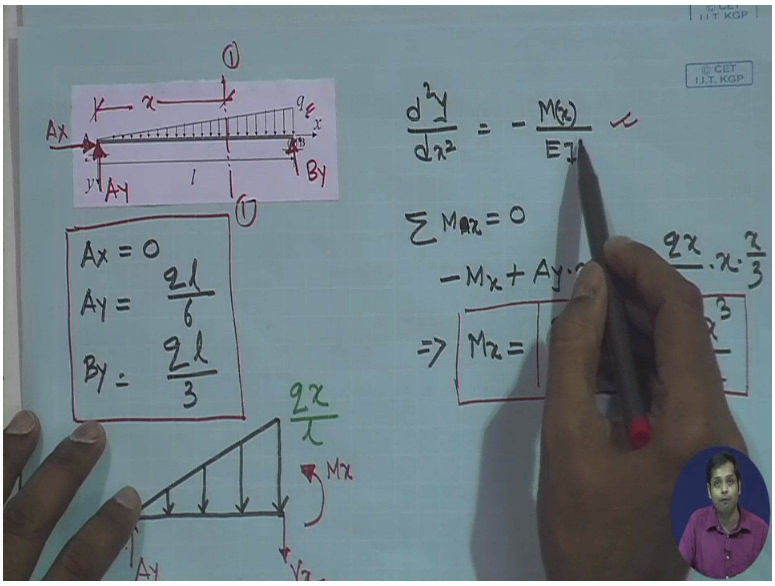
Do not take whatever results I am writing here. Do not take for granted. Please check yourself whether these results are correct or not. So this is the (min) maximum y value for this problem, okay.

(Refer Slide Time: 12:02)



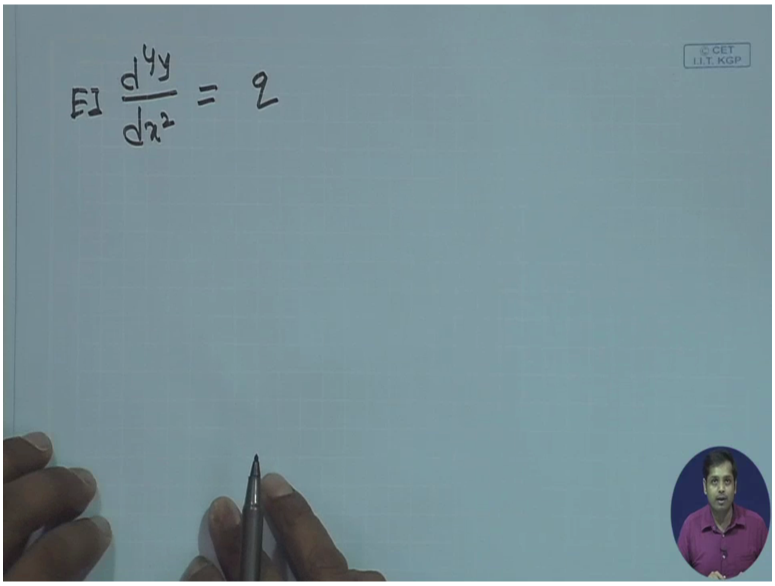
Now let us do the same problem again using the direct integration method but using a different form of this equation.

(Refer Slide Time: 12:16)



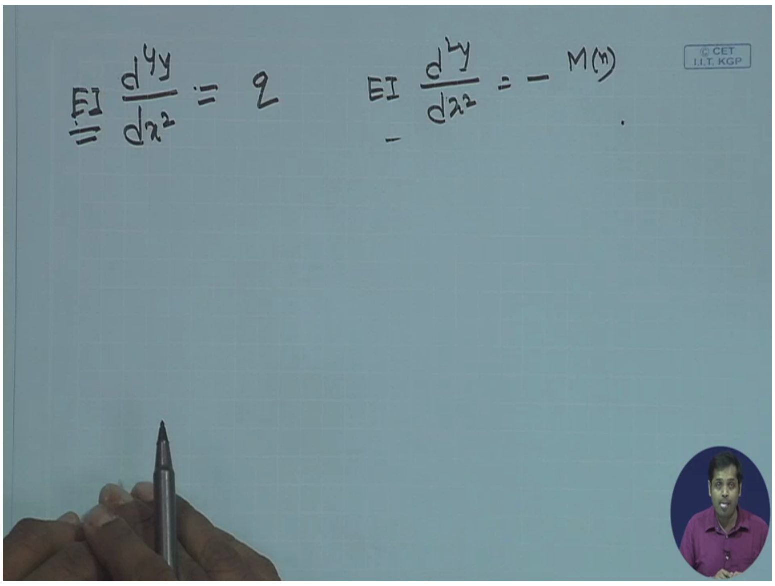
If you remember (elas) there are two other forms of the same equation. One form was d4y dx4 EI that is equal to q, okay, where q is the intensity of the load at that particular point, okay.

(Refer Slide Time: 12:45)



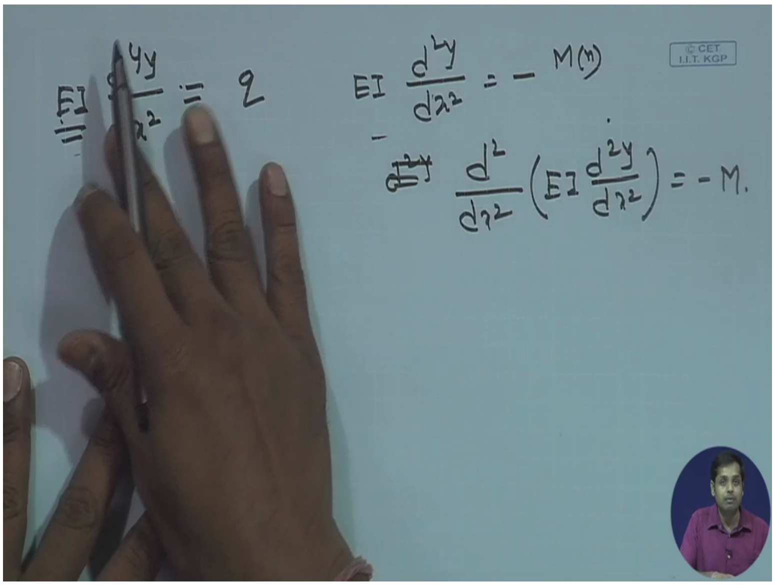
Now when you write this expression, actual expression was this, right? D2y dx2 is equal to minus Mx. Actual expression was this. From this expression we derived this. When we write this expression it is assumed that E and I are constant uniform across the length, they are not function of x.

(Refer Slide Time: 13:14)



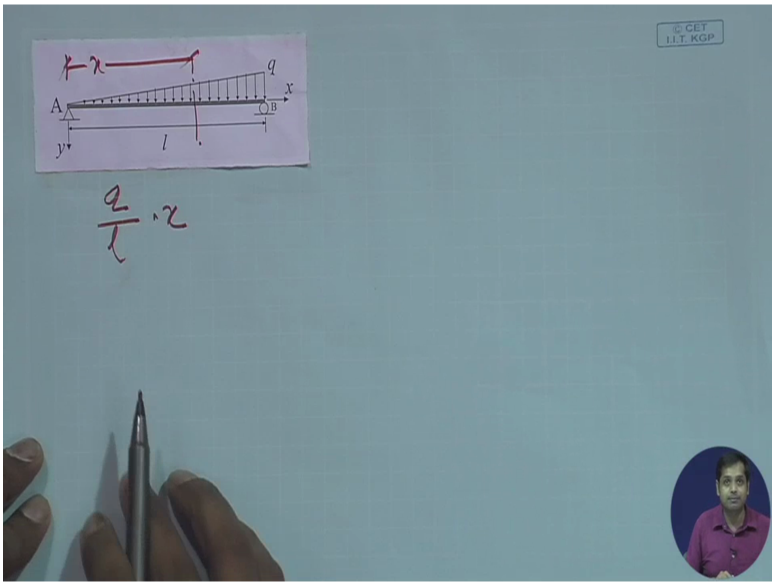
But if they are function of x then you cannot write this expression like this. Then what you have to write is then that expression becomes d2 dx2 EI of d2y dx2 is equal to minus M, okay. The expression becomes because if EI are the function of x. But if they are constant uniform throughout the length you can take it out and finally you will get it. But at as far as this problem is concerned EI are constant so we will be using this.

(Refer Slide Time: 13:50)



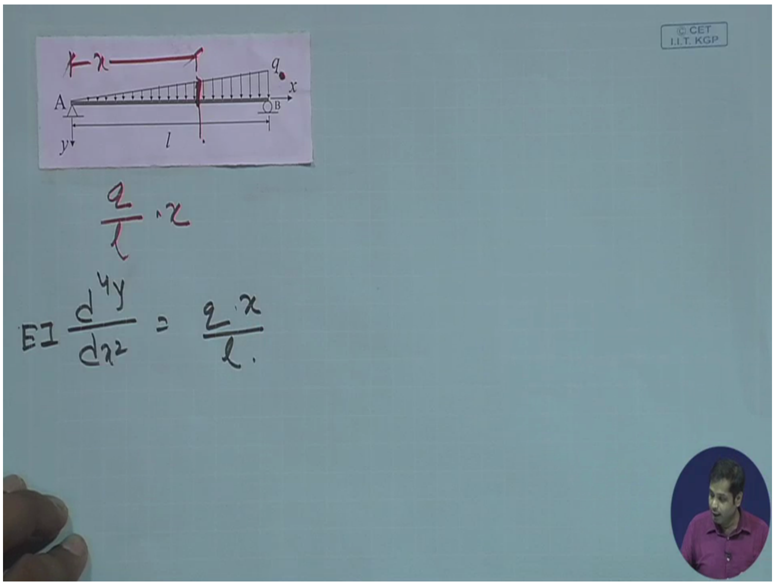
But make sure whenever any problem you address by using direct integration method or any other method check what are the information given about the material properties, okay. Material in geometric properties. So let us do the same example once again by using the other form of the equation of elastic curve, okay. Great now so again if we take at L this is q at any point x. If we take any section x here at a distance x at any point x, the intensity of the load will be q by L into x, okay.

(Refer Slide Time: 14:40)



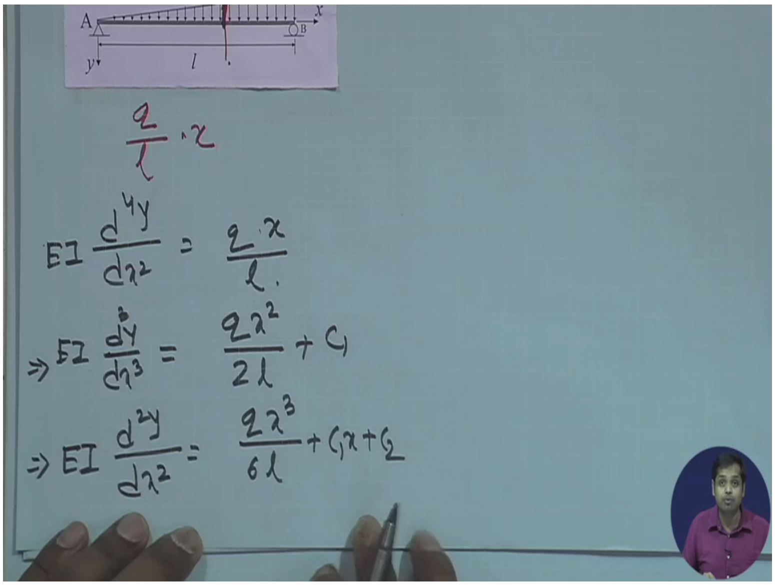
So when I write this expression d4y dx2 is equal to q EI, this q is actually the intensity of the load at that particular point. So in this case it will be qx by L. So this is the equation, right? So this is the intensity of the load at this point. So intensity of the load also changes as per this equation. Q is this value but at that particular point intensity of the load will be qx by L. So this is varying across the length. But if q is for (uni) uniformly distributed load this will be constant, right?

(Refer Slide Time: 15:22)



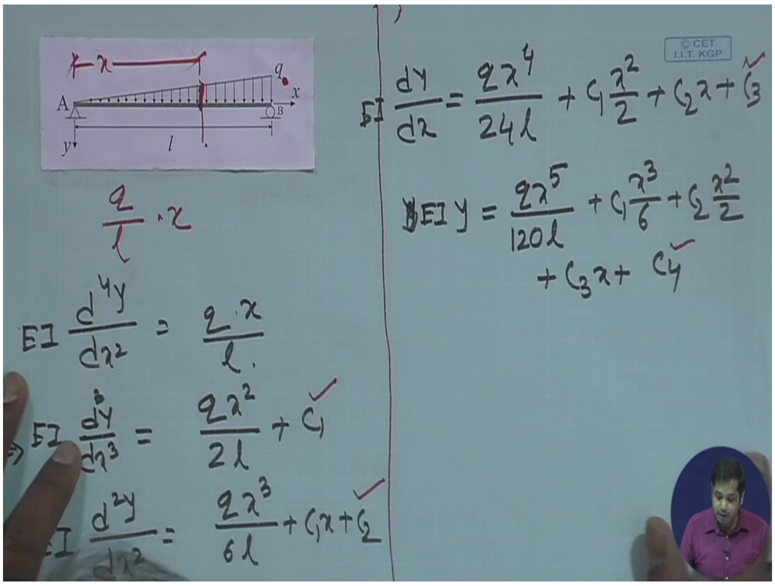
Now so once you have this expression let us differentiate it. Now if we differentiate it first time then it becomes EI d cube dx3 is equal to qx square by 2L plus C1. And then if you do it next time second time EI d2y dx2 this becomes qx cube by 6L plus C1 x plus C2, one more constant.

(Refer Slide Time: 16:03)



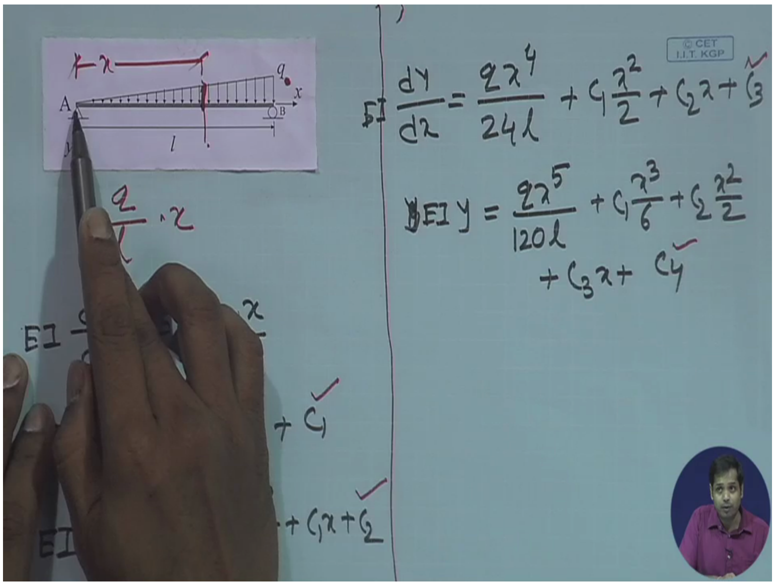
Then do it third time then this becomes dy dx is equal to 1x to the power 4 24L plus C1 x square by 2 plus C2 x plus C3, one more constant. And then finally do it from slope get the final deflection and becomes, this will be EI. This becomes EI of y, y is a function of x. This becomes qx to the power 5 by 120L plus C1 x cube by 6, then plus C2 x square by 2 plus C3 x plus C4. So we have now one constant C1, then C2, C3 and C4. Four constant which is justified because the equation was fourth order equation. So therefore four constants.

(Refer Slide Time: 17:23)



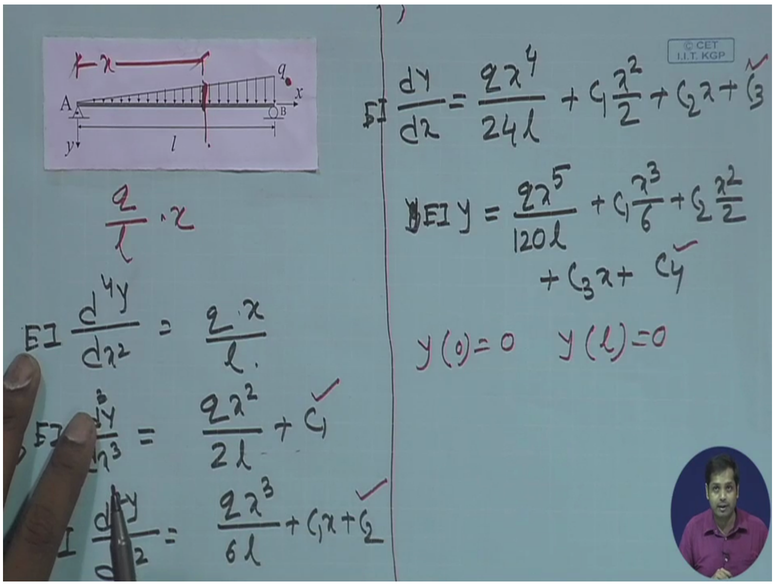
Then let us see how many boundary conditions we have? One of the two boundary conditions is obvious. One is here, deflection is zero here and deflection is zero here.

(Refer Slide Time: 17:33)



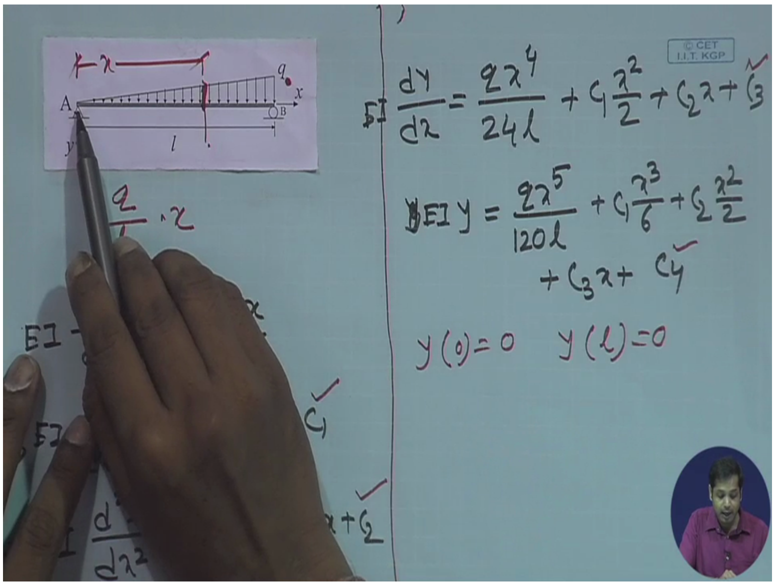
So two boundary conditions, one is y at x is equal to zero, is equal to zero and then y at x is equal to L that is equal to zero. These two are obvious boundary conditions.

(Refer Slide Time: 17:49)



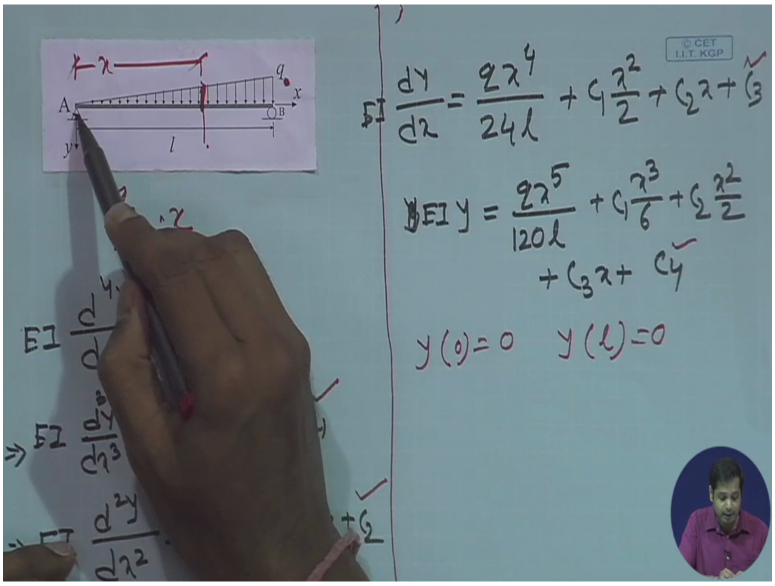
Now let us see any other boundary conditions we have. We do have other two boundary conditions. You see these are simply supported, these are hinge.

(Refer Slide Time: 17:56)



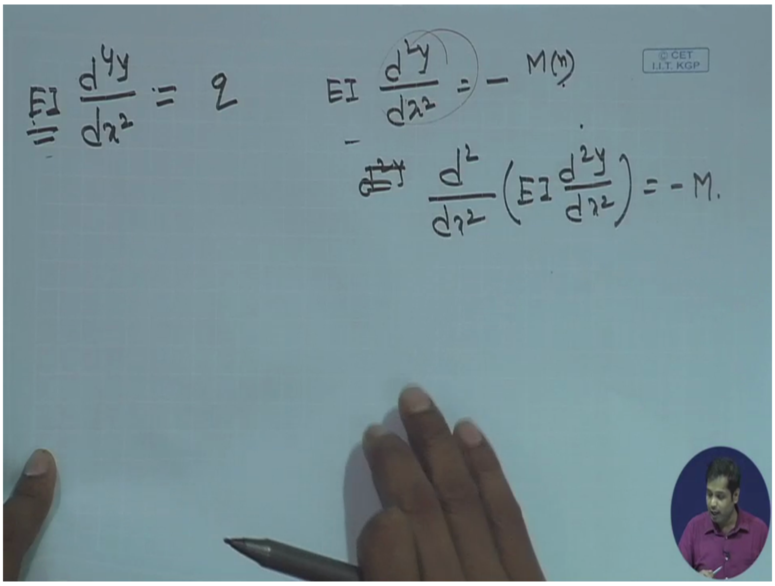
Here what is pinned support and the roller support they do not have any constraint against (rotai) rotation. So moment at this point and this point will be zero.

(Refer Slide Time: 18:04)



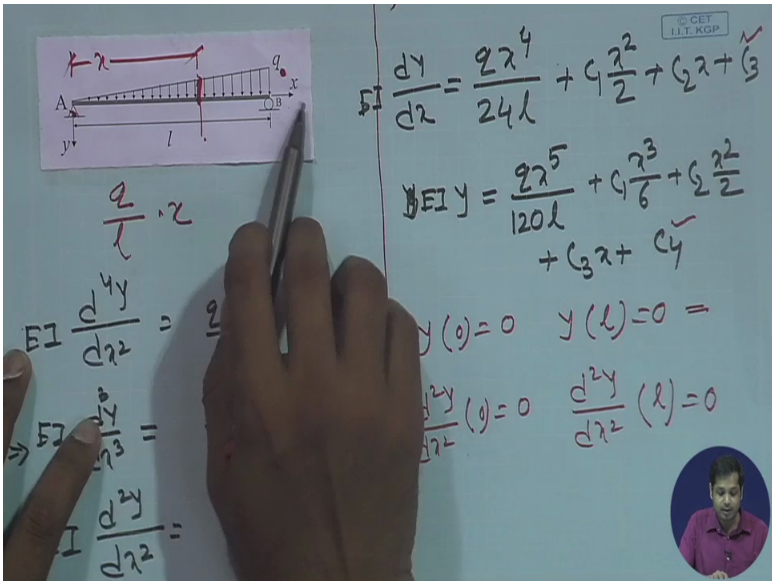
Now if you remember this equation if the moment is zero at any point then curvature has to be zero, right? So d2y dx2 has to be zero.

(Refer Slide Time: 18:17)



So what other two boundary conditions we have? D2y dx2 at x is equal to zero, is equal to zero. And then d2y dx2 at x is equal to L, is equal to zero. So these two come from the fact that deflection at this point and this point is zero.

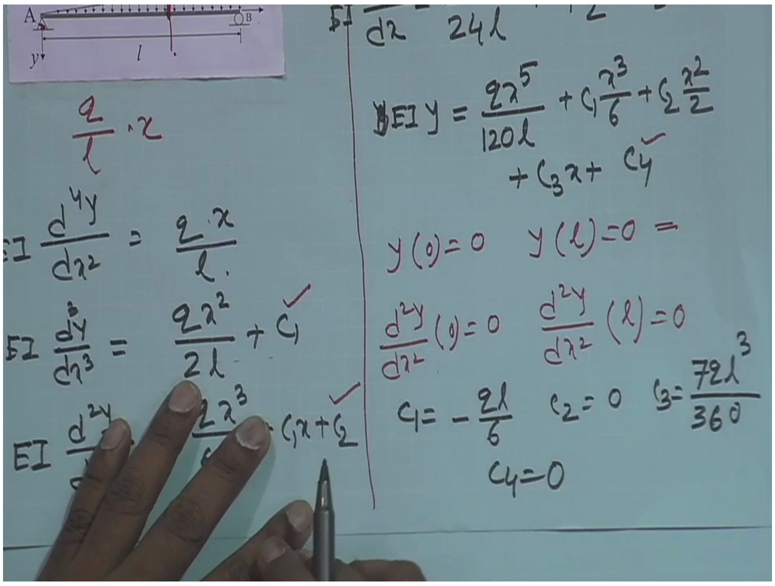
(Refer Slide Time: 18:41)



Now you will see in any boundary value problem, boundary value problem is where the governing equations are given and along with some boundary conditions are given. There are two kinds of boundary conditions we can have. One boundary condition is directly applied on the primary variable. For instance here the variable is deflection. Those boundary conditions are called essential boundary condition or Dirichlet boundary condition.

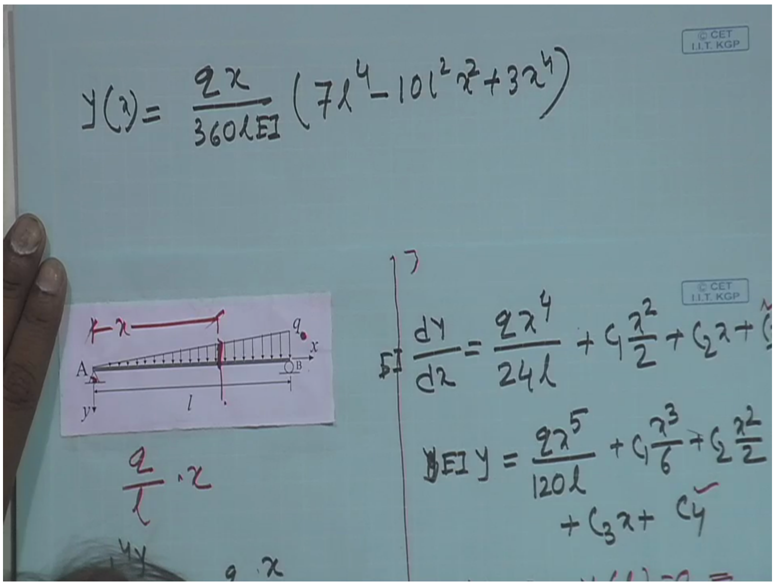
And another boundary conditions are specified on forces. For instance moments at zero at a given point or moments specified at a certain point, those boundary conditions are called Neumann boundary condition or natural boundary conditions. So here two different kinds of boundary conditions are given, right? Now if you use these for boundary conditions and compute the four constants C1, C2, C3, C4, then final value will be C1 is equal to minus qL by 6. C2 is equal to zero. C3 is equal to 7qL cube by 360. And C4 is equal to zero.

(Refer Slide Time: 19:57)



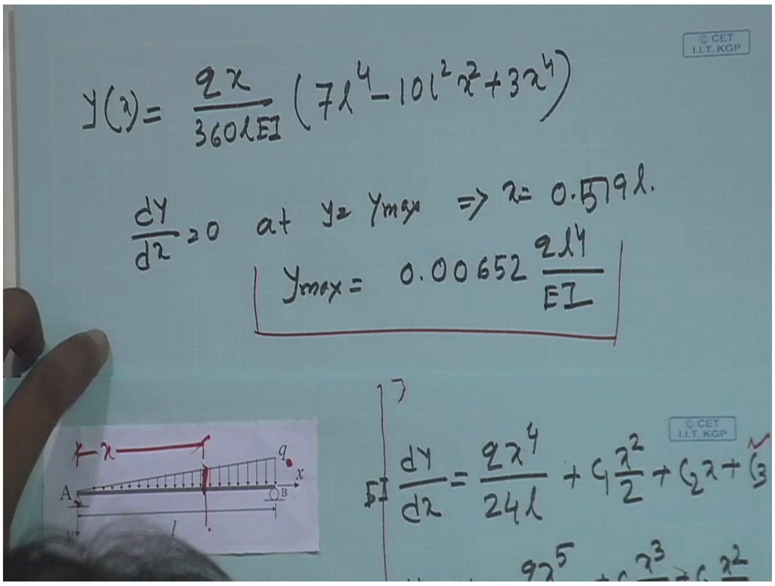
So again if you substitute C1, C2, C3, C4, in this expression then the expression of y will be, y again let us write as a function of x, qx by 360LEI 7L to the power 4 minus 10L square x square plus 3x to the power 4. This expression is same as the expression we obtained just now.

(Refer Slide Time: 20:30)



So again you take dy dx then rest of the things is same dy dx is equal to zero where y is equal to y max and this will give you x is equal to 0 point 519, just like the previous case and final y max is equal to 0 point 00652 qL to the power 4 by EI. Again please do not take these results for granted. Please verify yourself, okay.

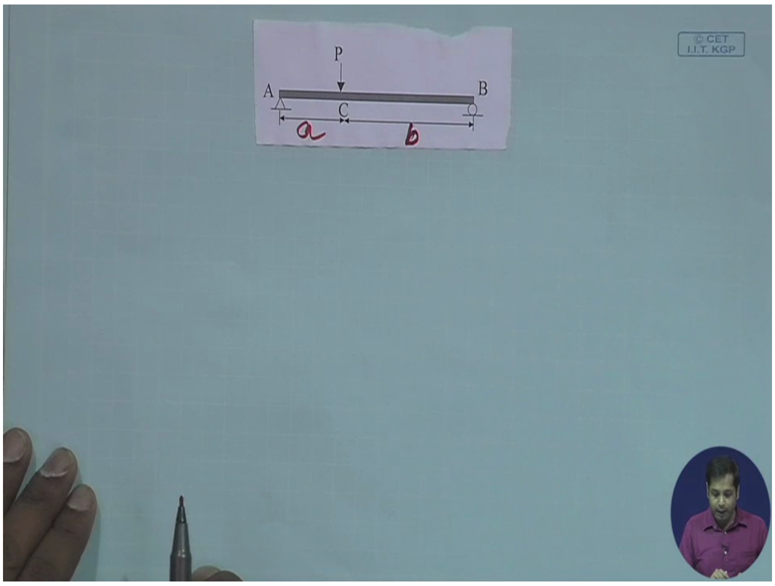
(Refer Slide Time: 21:02)



So the point is there are three forms available of the equation of elastic curve. Depending on the equation you have to decide which form to be used, okay. But any form you use essentially you will end up with same solution, okay. Now let us do one more example. in the last class I briefly talked about this example but it will be better if we solve it here itself.

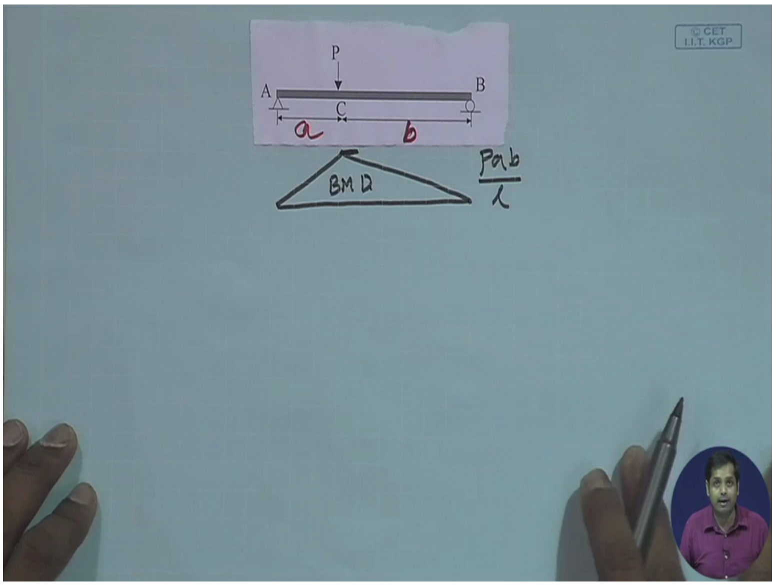
The example is this, again a simply supported beam which is concentrated load acting at a distance A from point at a distance this A and B. This distance is A and this distance is B. If it is not properly visible, okay.

(Refer Slide Time: 21:56)



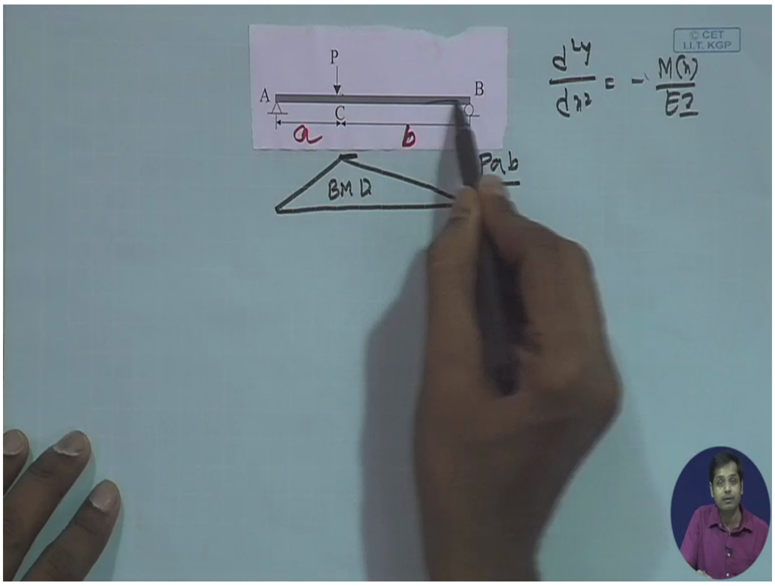
What we need to find out? We need to find out the expression for y, okay. Now you see this problem, what would be the bending moment and shear force diagram? If you remember the bending moment diagram for this problem was like this. Let us not write the shear force diagram right now because we do not need it. And this expression was pab by L, this value was pab by L. This was the bending moment diagram. If you remember we drew this bending moment diagram.

(Refer Slide Time: 22:30)



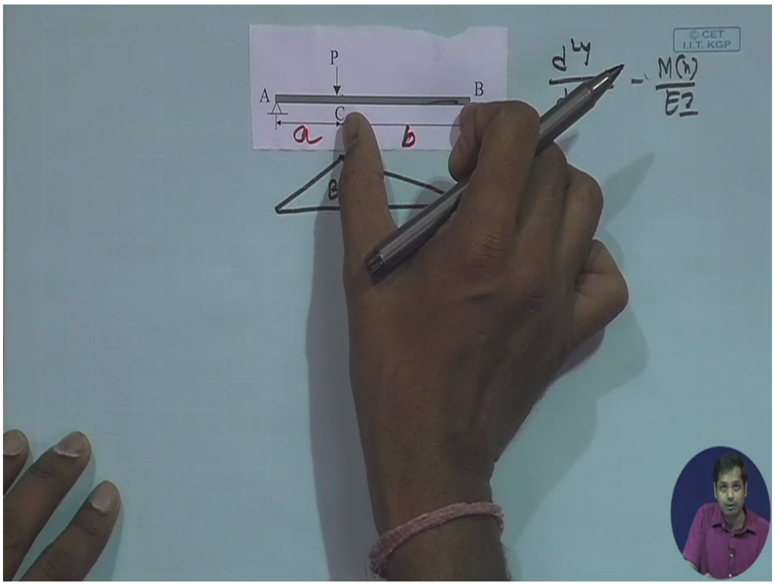
So expression for bending moment between A to C and between C to B, they are different. So when we use this expression that d2y dx2 is equal to minus M by EI, so expression of M is different for this part and (differ) different for this part.

(Refer Slide Time: 22:57)



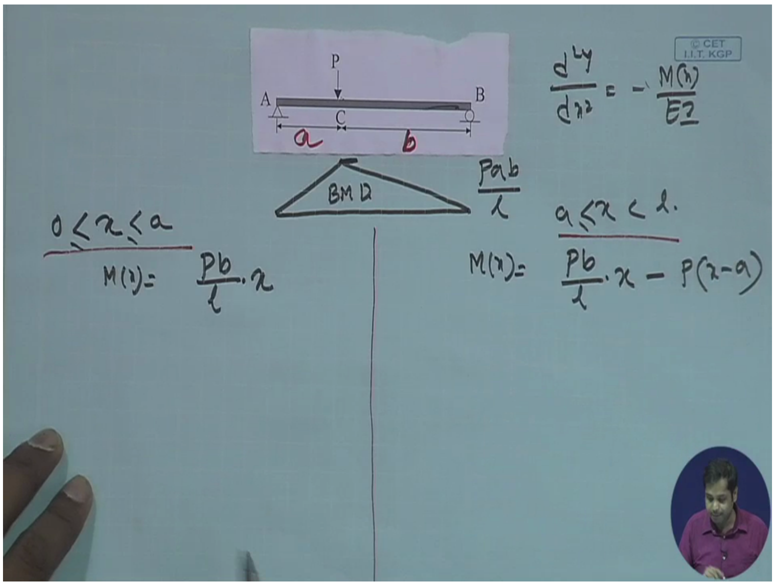
So we need to divide the problem into two sub problems, one for this and another for this with a condition at point C you have sufficient level of continuity.

(Refer Slide Time: 23:04)



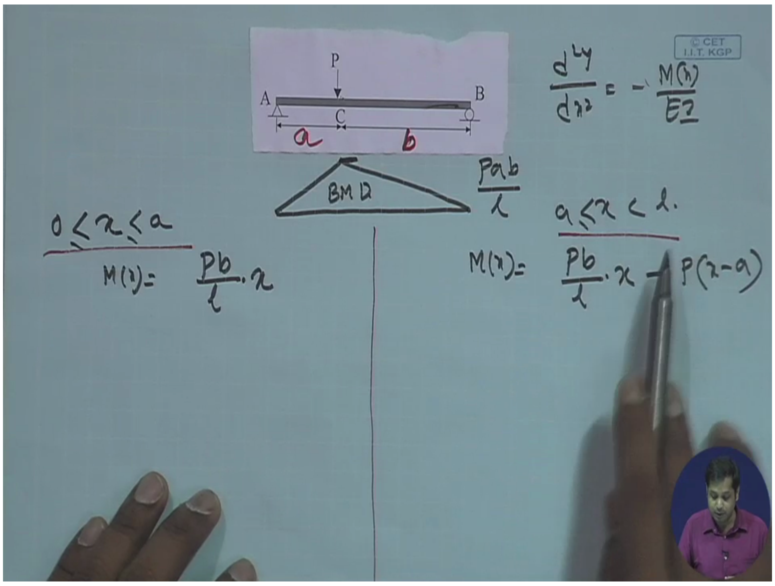
So let us do that. So take in the first case this is for x which is between zero to A and then it is x between A to L, okay. And expression for bending moment here is Mx is P into b by L x. Please verify this yourself. And for this the expression for bending moment is Pb by L into x minus P into x minus A, okay.

(Refer Slide Time: 23:55)



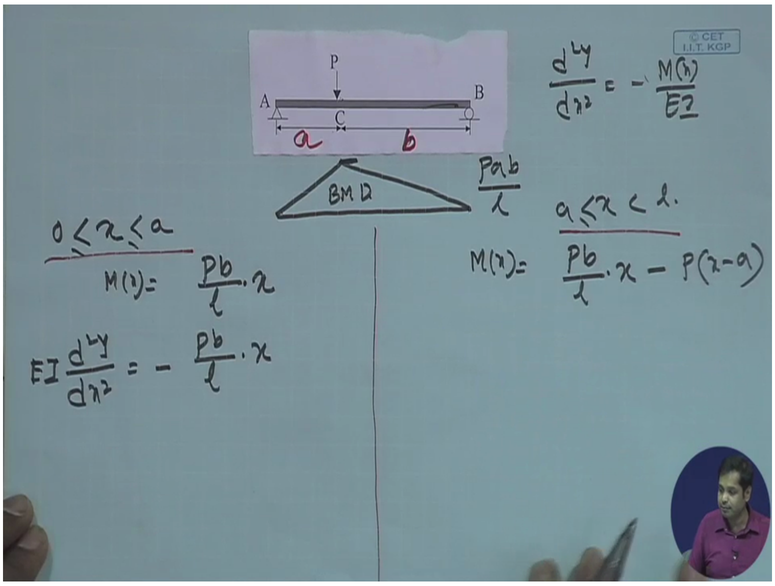
So let us see whether they are consistent or not. At x is equal to zero bending moment will be zero here, this is consistent. At x is equal to B bending moment should be (con) zero here. Please substitute that at x is equal to L bending moment should be zero. So x is equal to L if you substitute then you will get this value is also zero.

(Refer Slide Time: 24:21)



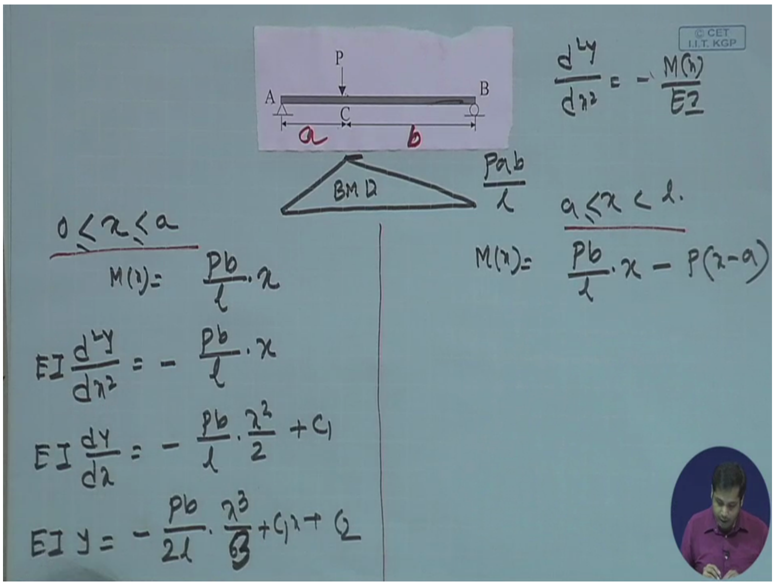
At x is equal to A they should have the same value. X is equal to A it is pab by L, x is equal to A this is also pab by L, okay. So then apply this expression d2y dx2 minus EI. So EI d2y dx2 is equal to minus pb by L into x, okay.

(Refer Slide Time: 24:42)



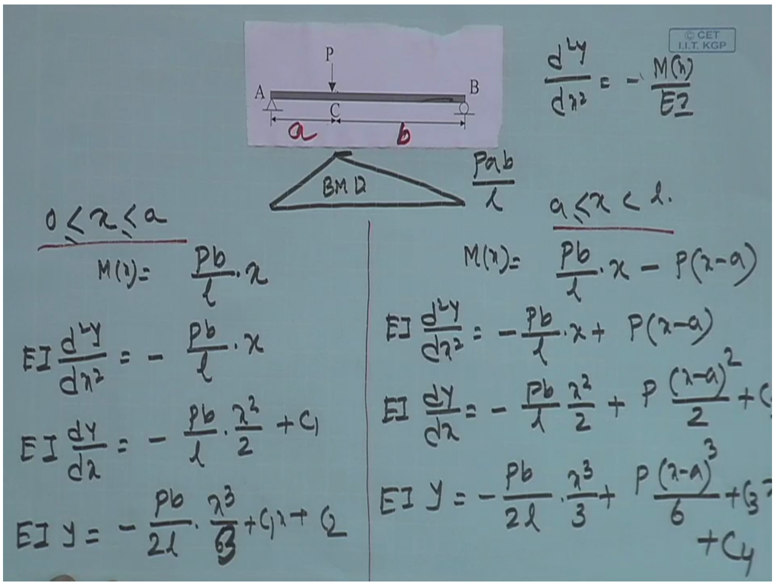
So integrate it first time dy dx is equal to minus pb by L into x square by 2 plus C1. And then second time y is equal to minus pb by 2L into x cube by 6 plus C1 x plus C2, okay. This is 3 sorry this is 3, okay.

(Refer Slide Time: 25:22)



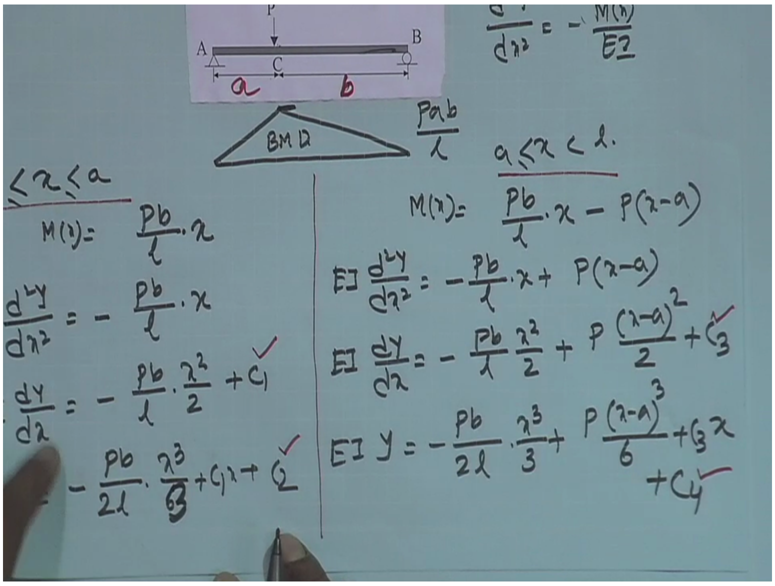
Now let us do the same thing for this part. So this is EI d2y dx2 is equal to minus of this, minus pb by L into x plus P into x minus A. So EI integrate it first time dy dx becomes minus pb by L x square by 2 plus P into x minus A square by 2 plus C3. Let us write C3 here because C1, C2 we have used for the other parts. EI y is equal to minus pb by 2L x cube by 3 plus P into x minus A cube by 6 plus C3 x plus C4.

(Refer Slide Time: 26:27)



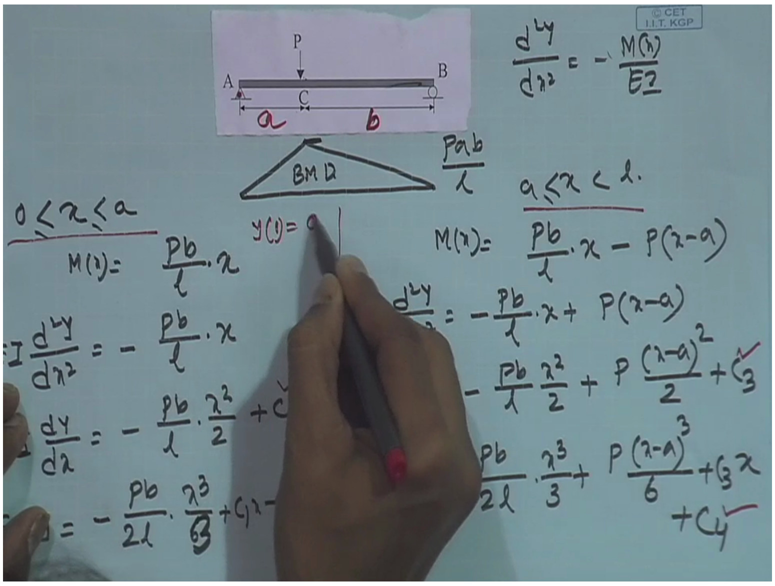
So we have one constant, two constant, three and C4. We have four constants. These four constants need to be determined, okay.

(Refer Slide Time: 26:36)



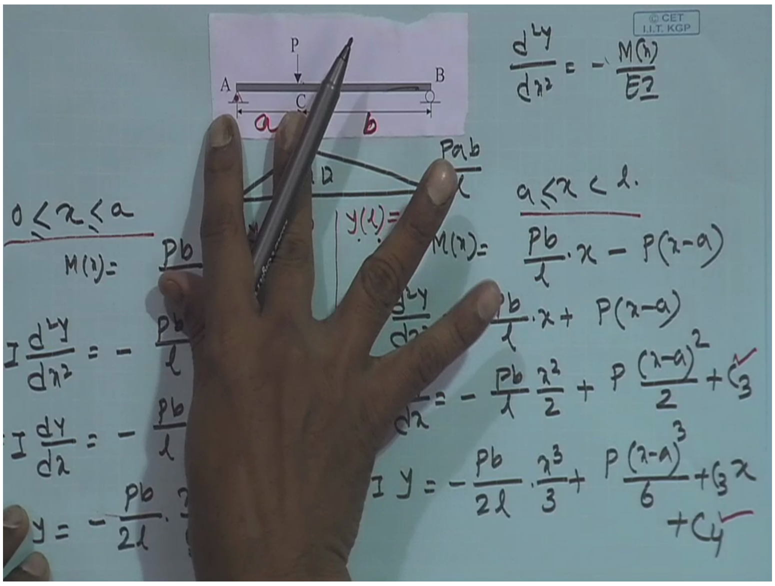
Now what are the boundary conditions we have? Now two obvious boundary condition at x is equal to A, y is equal to zero, at x is equal to B, y is equal to zero. So for this part the boundary conditions are x is equal to x, y is equal to zero, is equal to zero, okay.

(Refer Slide Time: 26:57)



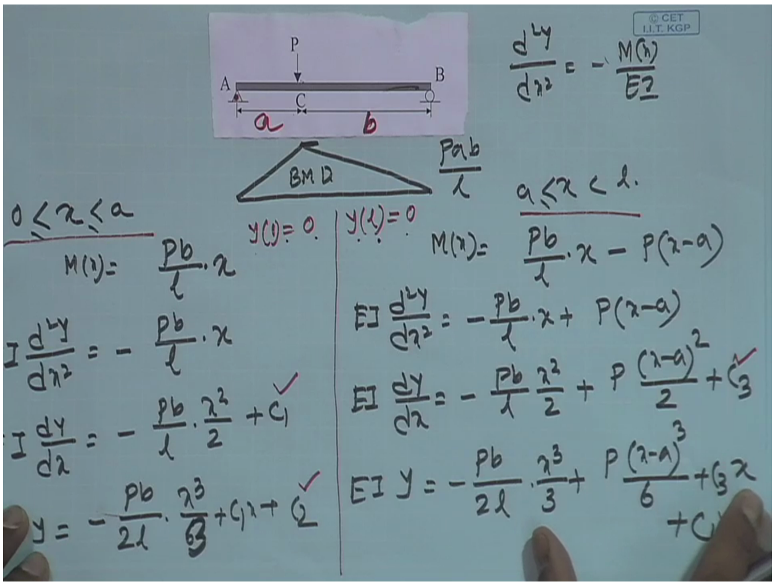
And for this part boundary condition is y at zero is equal to zero. Remember if it is y at L is equal to zero, this expression will not satisfy because this expression is only valid between A and C.

(Refer Slide Time: 27:14)



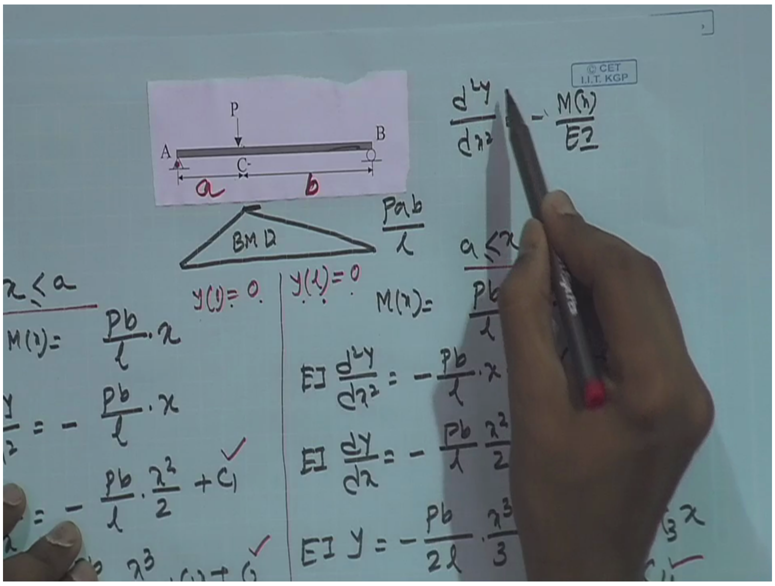
Similarly y at x is equal to zero is equal to zero will not be satisfied in this equation because this equation is only valid between C and B.

(Refer Slide Time: 27:22)



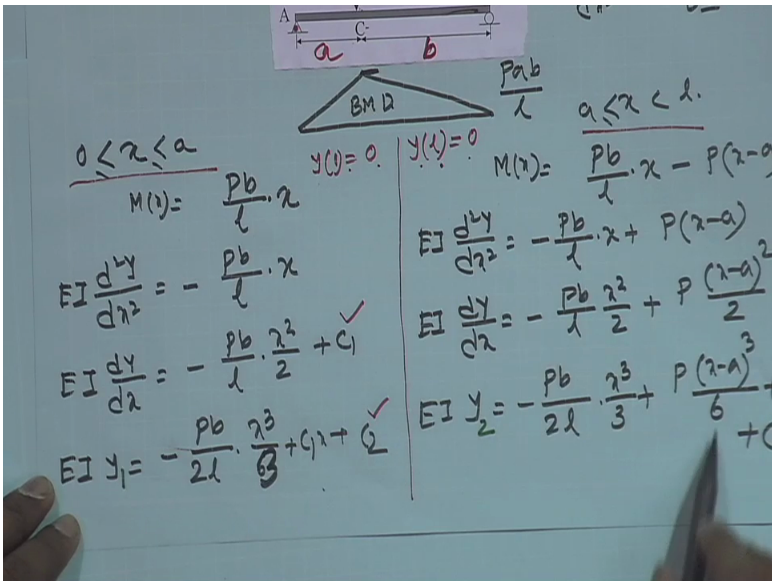
So these boundary conditions belong to this part and this condition belongs to this part. Now other than this two other conditions need to be satisfied these are the continuity at C. You see when I write this expression that d2y dx2 is equal to zero it is assumed that y is differentiable up to second order at least. If not then this equation itself is not valid.

(Refer Slide Time: 27:49)



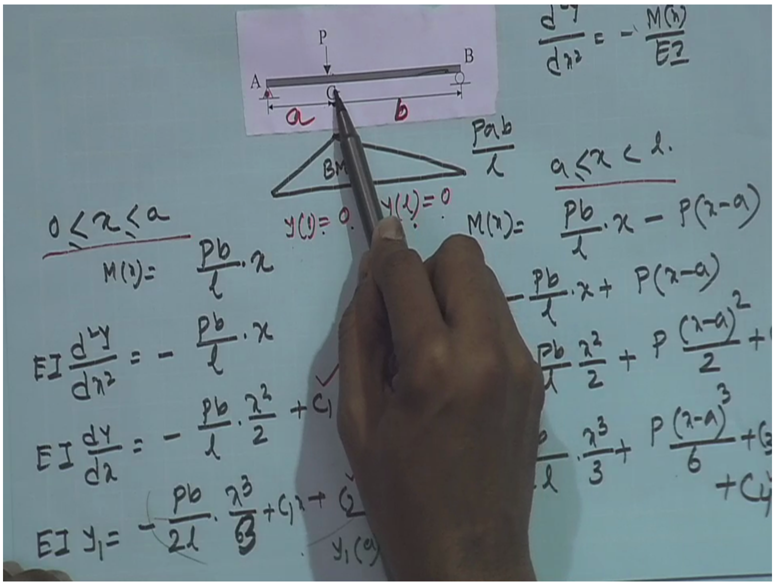
So if y has to be differentiable up to second order then y should be continuous at C and its derivative also should be continuous at C. So other two boundary conditions we have which is at this point that if we compute, suppose if we say this is y1 and this is y2. This expression is y1 and this expression is y2.

(Refer Slide Time: 28:16)



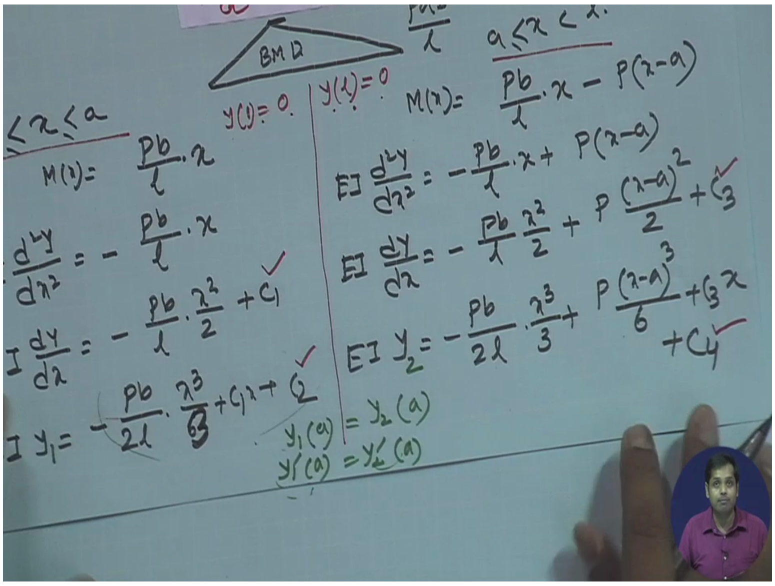
Then the boundary conditions are that y1 at A should be equal to y2 at A. So if we evaluate the deflection of the beam at point C which is at x is equal to A, from this expression and deflection at this at point C from this expression they should be same.

(Refer Slide Time: 28:43)



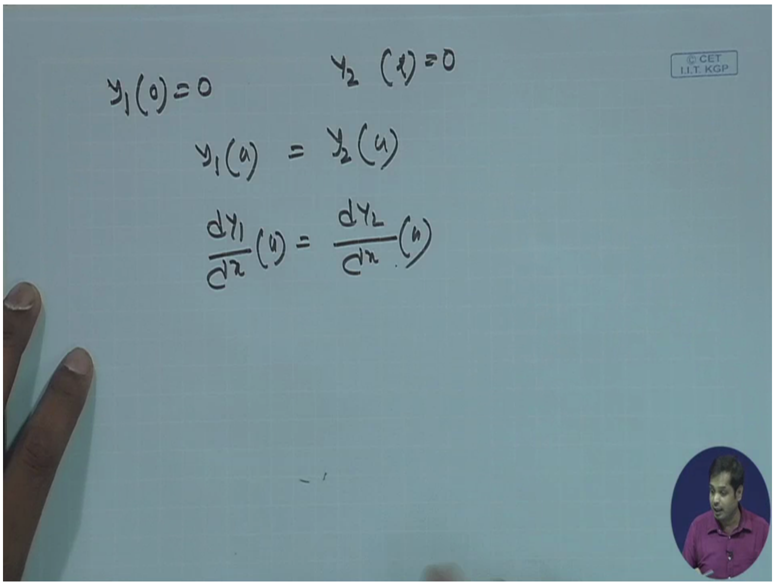
And the next boundary condition is y1 dash at A is equal to y2 dash at A. Means y1 dash is dy1 dx and y2 dash is dy2 dx. So these are another 2 boundary conditions.

(Refer Slide Time: 29:01)



So total (bol) four boundary conditions what we have is, one is y1 x is equal to zero is equal to zero and y2 L is equal to zero at B is equal to zero. And continuity equation, y1 A should be equal to y2 at A and dy1 dx at A should be equal to dy2 dx at A. So these are four boundary conditions.

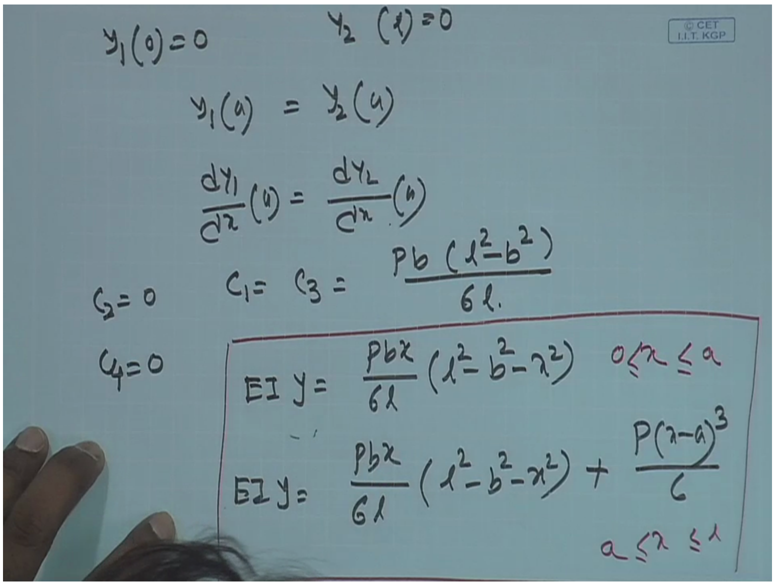
(Refer Slide Time: 29:28)



And if you substitute these four boundary condition what you get is, C2 is equal to zero, then C3 is equal to pb L square minus b square by 6L and again C4 is equal to zero. So final expression you will get is this. Final expression you will get is EI y is equal to pbx by 6L L square minus b square minus x square and this is for x less than equal to A, zero.

And EI y is equal to pbx by 6L L square minus b square minus x square plus p x minus A whole cube by 6. And this is for x less than equal to L equal to A. So these are the (exp) solutions, okay.

(Refer Slide Time: 31:00)



Again you can differentiate that y to get the point where y is maximum, okay. So like this you can apply this equation to many other problems with many other boundary conditions. You try yourself all the example that we have done here. We assume that EI is constant. But as I said EI may not be constant and if they are not then what happens to this equation and how this equation needs to be integrated? Please do it yourself.

And if you see any book there are many examples given, okay. So the equation of elastic curve is very important not only for this course you will see in future there are many occasion you will come across this equation and many practical problem this equations are being used, okay. So what we do next is we study two other methods. One is moment area method and conjugate beam method for determining deflection of statically determinate beam.

But we will see that the premise of these methods are these equations, okay. So there should be any confusion as far as derivation and application of these equations are concerned. Okay then, see you in the next class where we will discuss the moment area method for determination of statically determinate beams. Okay, thank you.