

NPTEL
NPTEL ONLINE CERTIFICATION COURSE

Course
On
Reinforced Concrete Road Bridges

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Lecture 07: Limit State Method of Design
as per IS456:2000

Hello everybody so we have just now we have discussed on what is state method used in RC 21 and also IS456 also now, now we shall go for working stress method we shall go for on that and then that is as per IS456 because you are very much familiar with IS456 and then we shall go to IRC 112 that is for the bridge we used that we shall go for that concrete bridges that we shall go for it.

And then we shall compare that the difference method that whatever we are going to use it or whatever methods available so this is our lecture number 7 on reinforced concrete Road bridges and then we are coming back to that limit State method and which will be as per IS 456:2000 and we are again considering first limit state of collapse that is flexure because our objective here in this particular course but mainly it will go on by your clicks or and I have told it number of times.

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Assumptions

- **Plane sections normal to the axis remain plane after bending**
- **The maximum strain in concrete at the outermost compression fibre is taken as 0.0035**
- **The tensile strength of the concrete is ignored**

So let us click with the clicks are only and again we are having certain assumptions and those assumptions again you see that on the first assumption plane sections normal to the axis remain plane after bending the maximum strain in concrete at the outermost compression fiber is taken as 0.0035 so please note to this particular aspect that in working stress method we are totally silent only thing we have told that strain profile will be linear straight line that much only we have told.

But we have never given you any number that how much will be the strain but here we are giving certain limiting value of strain and that is 0.35 the tensile strength of the concrete is ignored as usual because there is no such difference with the working stress method elemental method here we are ignoring.

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Assumptions (contd..)

- The stress block may be assumed to be **rectangle, trapezoid, parabola** or any other shape which results in prediction of strength in substantial agreement with the results of test

Note : Triangular as in the Working Stress Method



That to the stress block may be assumed to be rectangle trapezoid parabola or any other shape which results in prediction of strength please note the next one the substantial agreement with the results of tests so that means the code agrees that it will not exactly it will not match so that is why the code means you that a very wise lead it has told the substantial agreement so this why that is the one issue the understand that record on that at left on the substantial agreement.


So that means it should come very close no doubt so but otherwise you can use any other one and I shall tell you that one also that's why I have taken this particular one that working stress method limited as per IS-456 and then we shall go for IRC 112 that we shall go for it so that we can I can show you the one way that you evolution another way the journey that how we are going from elastic to the beyond elastic that Parker 1 and real it is very interesting to note those things.

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Assumptions (contd..)

- **The stresses in the reinforcement are derived from representative stress-strain curve for the type of steel used.**

For design purposes, the partial safety factor γ_m , equal to 1.15 shall be applied.



Only thing I would like to say that note triangular as in the working stress method so this is the one we have taken that I have told you rectangle trapezoid parabola like that but here we are taking say that triangular in the working segment of this is the one difference that we are having the stresses in the reinforcement are derived from the representative stress curve for the type of steel used so which type of steel you have use it is dependent on that for design purposes the partial safety factor because you see that one last time in working strength at all.

Suddenly I have told you for fuel but I will know that partial we are not going to use it for work Lister's method but here we are going to use the term partial so the thing is that in the fifth lecture just I have given you idea though it may look like to you that why that particular that one lecture was given on the design principles where that problem because I feel that the coal one the there is no such scope to understand the that you say that how the code is made what is the basic principle the reliability index other things so that's why I have given you that just only happen over one though it may look like it is little bit very fast.

But anyway just to give you idea but the difference book which I have given from there you can get more information if you are interested and that is very interesting also but the basic idea they are there calibration of that partial safety factor that one is important because this partial safety factor it is coming for different materials for different cases were say loading you will get different actually factor you will get it.

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Assumptions (contd..)

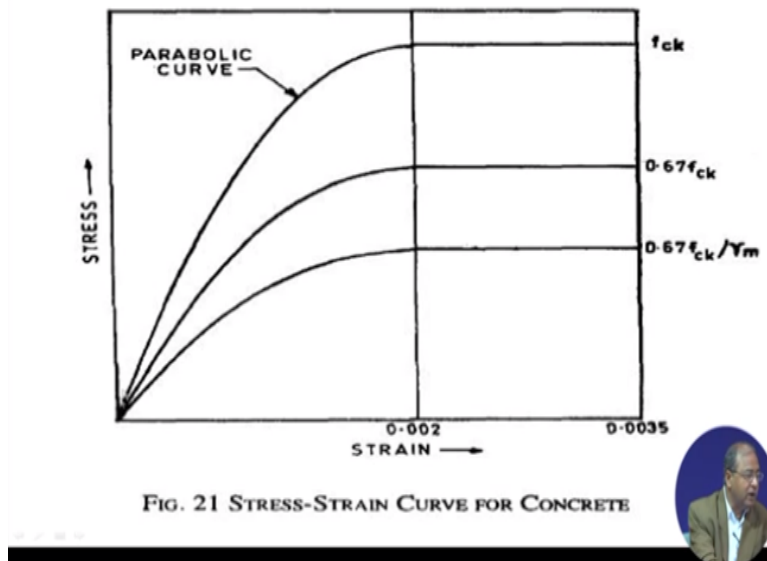
- The maximum strain in the tension reinforcement in the section at failure shall not be less than:

$$\frac{f_y}{1.15E_s} + 0.002$$



It is not like working stress method you are having only one value this is very important and here we shall get it say 1.15 that we shall take it here then maximum strain in the tension reinforcement in the section at failure shall not be less than if $F_y/1.15E_x+0.002$ what I would like to mention here for concrete irrespective of grade of concrete we always give .035 that is the limiting value of strain but in this case we are giving that value depending on the grade of steel so $F_y/1.15E_x+0.002$ this is the value that we shall consider that means for different grade of steel.

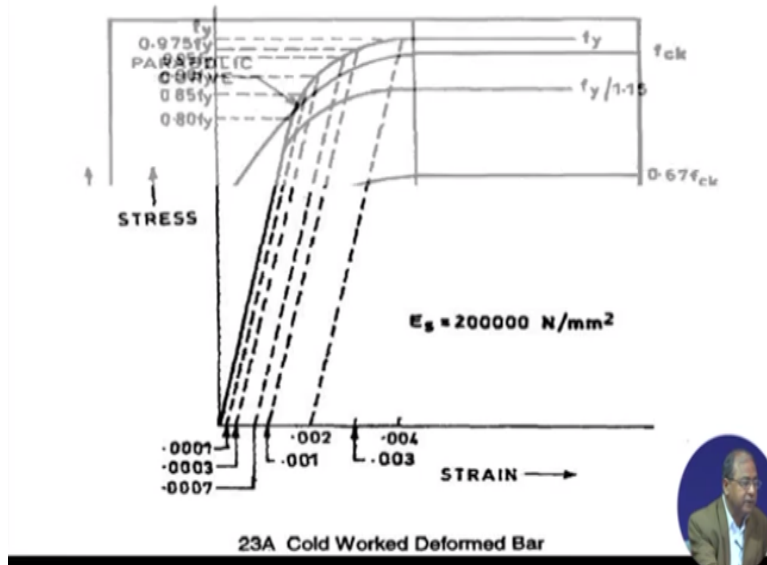
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We shall get different values so this is the one that figure which we have taken data here you are very much familiar with that so first this is the one f_{ck} next we are having from that designer point of viewpoint six seven f_{ck} which is nothing but $2/3$ of that then you are getting $.67 f_{ck}$ by γ_m we will give you 1.5 so we shall get which is nothing but $4/9 f_{ck}$ $2/3 f_{ck}$ into $2/3$ of that will come here.

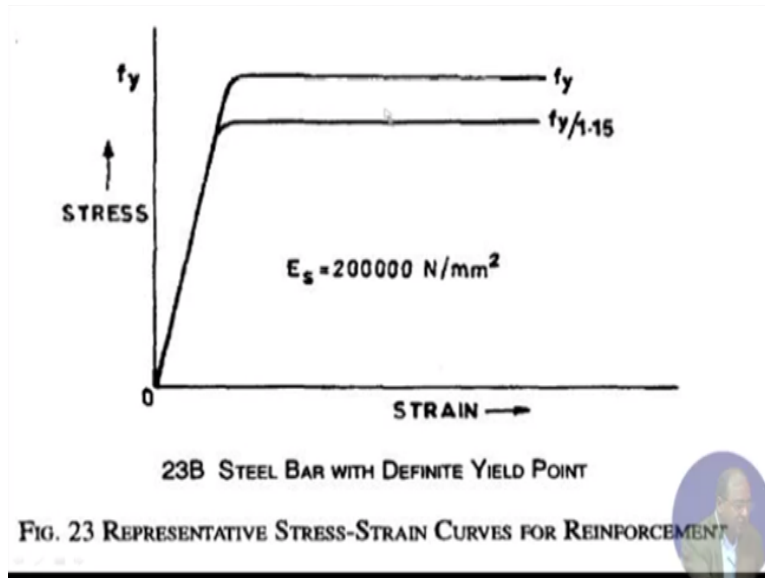
So $4/9$ 0.444 like that you will go so generally we write down one .445 or .45 f_{ck} that we write down that that means f_{ck} is the strength of the concrete and we are going for that point four five f_{ck} that is the value we take it here that for we can consider.

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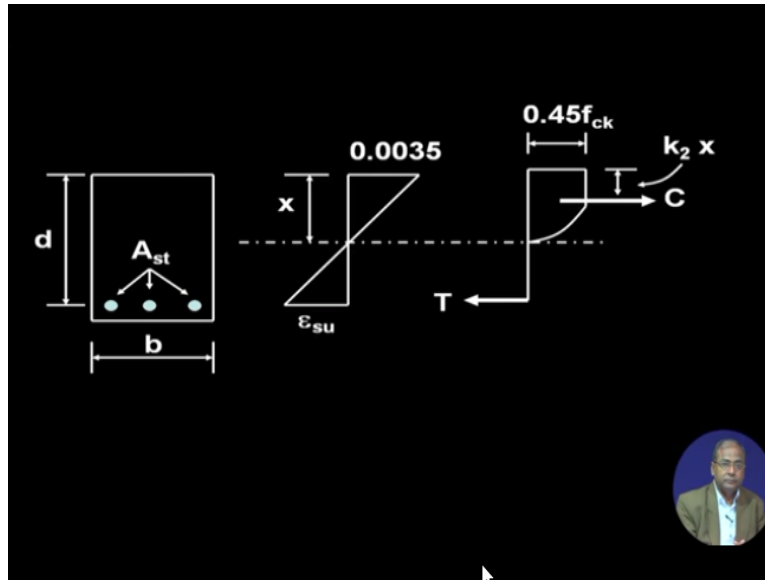
Now this is the four fold over the deform bar again he are getting this is the actual one and we are getting this one here $F_y/1.15$ and that value we get it here.

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This is for mild steel so this Parker one again you are getting $F_Y/1.15$ so this value we are getting up to the same after that we are getting little less coming to this one here.

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So this one whenever I have shown that figure I have shown you any value in working stress method here we are taking this factor on here a D and D X this is the neutral axis so this portion only we are getting that value because here we have not told or the value here but here I know how is the value this one for steel what is the value for the concrete there is always next from point 0 negative 3 5 this is 0.45 f-ck we have considered this one.

I am telling that what it wants okay to if they are actually of told this is the X so X by 3 that is very simple because this is a triangle but in this case it is not a triangle so that is why you are getting that value here that particular value you are getting here that you can find the that you are some value will be there k_2 that particular value we are to find out and this is the tensile stress tension of course in steel and as usual that you will say tensile stress in the concrete.

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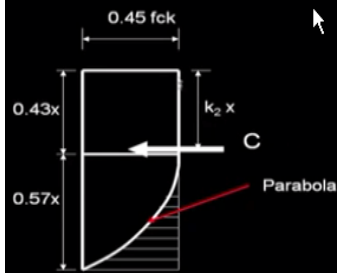
**k_2x is the distance of the
centre of compression in the
concrete from the top
compression fibre**



That is actually expert core portion ignored so only we are having compression concrete will take the compression and reason we take the tensile so $k_2 s$ is the distance of the center of compression in the concrete from the top compression fiber.

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Determination of constants k_1 and k_2 for compression stress block



$C = \text{area of rectangle} - \text{area of outer parabola}$

$$= 0.45 f_{ck} (x) b - 0.45 f_{ck} (0.57/3) (x) b$$

$$= 0.45 f_{ck} (x) b - 0.086 f_{ck} (x) b$$

$$= 0.364 f_{ck} (x) b$$

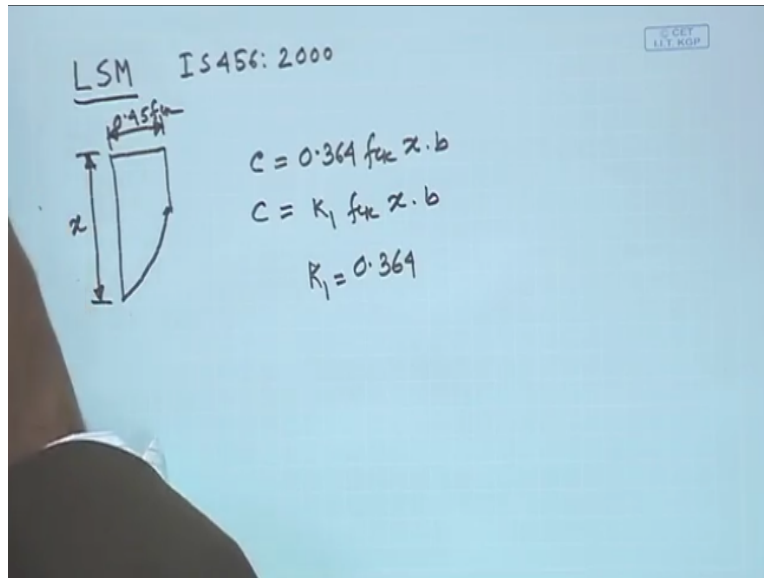
Therefore,

$$k_1 = 0.364$$



So we can find out that k_1 and k_2 this one area of rectangle that one- area of outer parabola so you can make this by culinary simple calculation then I can get it $0.364 f_{ck} X$ into B so q_1 means 0.364 .

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So this particular one please note this parameter 1.36 for again this is a very, very important one for limited method and we are talking here IS 456: 2000 in this case we are taking second portion and C which is coming as $0.364 f_{ck} X \cdot B$ so this is your X this is $0.45 f_{ck}$ and B is the width of the beam so this particular one we shall note down this particular one and which we are talking this one $K_1 F_{ck} X \cdot B$ further we can make many more things we can do it that I am not going to all detail all those things .

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Determination of constants k_1 and k_2 for compression stress block


$C = 0.364 f_{ck} (x) b$

Taking moment about NA

$$[0.364 f_{ck} (x) b] [x - k_2 x] = 0.45 f_{ck} (x) b (x/2) - 0.45 f_{ck} (0.57x)^2 b / 12$$

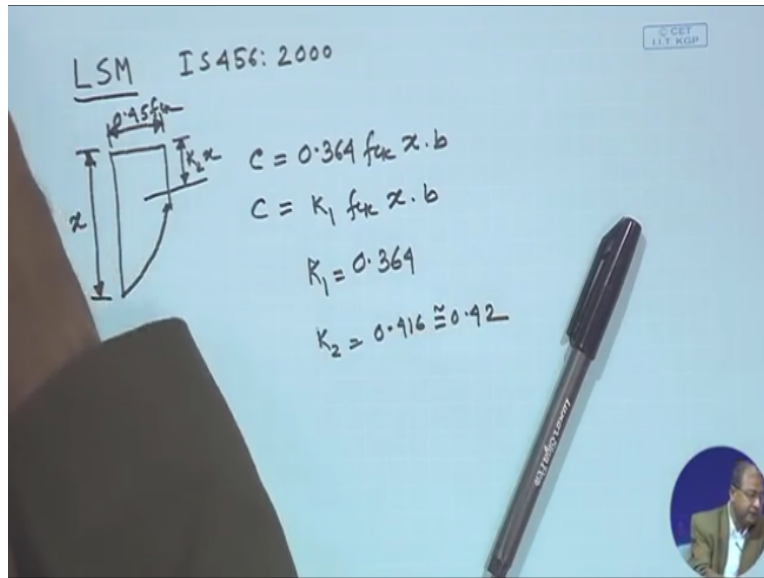
$1 - k_2 = 0.584$

$k_2 = 0.416 = 0.42$



But I am keeping this one for the time being that particular value where we are telling that $K_1=0.364$ because why I am looking down this what color on again I shall show you this 4-1 the current coming to this one here so another value of K_1 and K_2 so this K_2 we have to find out what is that $K_2 x$, $K_2 X$ we know taking moment our neutral axis you can find out all those things 0.36 for that area of this one $X * B X - K_2 X 0.45 f_{ck} X$ this is a rectangular part and then we are finding these four parabolic path this particular portion.

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We shall deduct so $1 - K_2 = 0.584$ so let us go down that $K_2 = 0.416 = 0.42$ so it is very, very important here and K_2 why I am try what I am trying to say the specter on here so that means see this one that all equation can be written in terms of K_1 and K_2 that is our objective that the current we have taken a parabolic shape certain say parabolic rectangular shapes you can say more precise way.

So if we take a parabolic rectangular one and there but code 1 we can define with the term sum 1 and K_2 then C will be equal to $K_1 f_{ck} x / d$ similarly we can find out your say value of NIEM also so C equal to $K_1 f_{ck} x / d$ $BD t = 0.87 F_Y$ so let us write down that one here also $t = 0.87 F_Y$ ast so this is your C and this compression in concrete and tensile in steel and this is called again I think I have not told so far again I am killing this paper on here that a singly reinforced section.

So this is your single reinforced section that compression side even though steel is there that on in ignore that means or that it will contribute or not in your moment of resistance we do not know so but, but that force are actually ignore that one so just to complete this worker on I would like to say similarly we are having doubling and for section also reinforcement the compression will the compression side also steel will be available all will be provided sometimes we do it.


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$$C = k_1 f_{ck} (x/d) bd$$

$$T = 0.87 f_y A_{st}$$

$$0.87 f_y A_{st} = 0.364 f_{ck} b x_u$$

$$M_u = k_1 f_{ck} \left(\frac{x}{d}\right) \left(1 - k_2 \left(\frac{x}{d}\right)\right) b d^2$$

$$j = 0.5 + \sqrt{0.25 - \frac{M_u}{0.87 f_{ck} b d^2}}$$


So coming to this on here this is a situation we are getting it here and as usual the C and T that one should be equal that C and T that will be equal so C and T we know and we know this one equivalent $0.87 F_Y$ ast equal to 0.36 for $f_{ck} b x_u$ that parker one we can find out and then we can write down here M_u equal to $K_1 f_{ck} \left(\frac{x}{d}\right) \left(1 - k_2 \left(\frac{x}{d}\right)\right) b d^2$ I have taken all of them to the virtually who are your $k_1 f_{ck} \left(\frac{x}{d}\right) b d^2$ that economy can take it here.

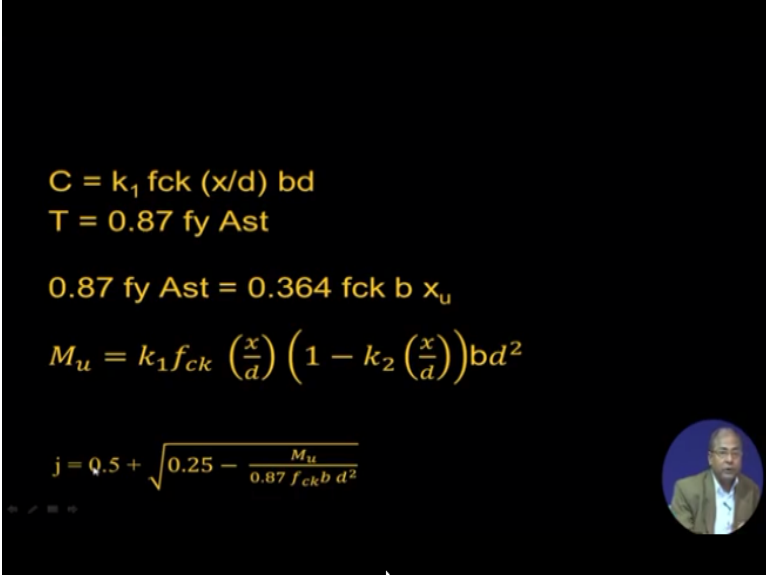
So this one just to complete this packer on i can write down here what I can do $.364 f_{ck} \left(\frac{x}{d}\right) b d^2$ that is whatever take a one point three six fold is nothing but $K_1 f_{ck} \left(\frac{x}{d}\right) b d^2$ and this portion is coming from the lever arm so this portion is coming from the lever arm so that means actually whatever I am doing M_u equal to C times Z, C this much C is this much into Z, Z means this Parker one here.

This is your Z so Z equal to D minus $k_2 X$ equal to 1 minus $K_2 \left(\frac{x}{d}\right)$ into D so 1 minus $K_2 \left(\frac{x}{d}\right)$ into D a colonial here on there I am getting one D and from that I am get another D so that is how it is coming any equal to this particular one $b d^2$ so this is the equation you are getting for in you so similarly just I would like to make it in one page that is why I am becoming a little miser.

So let me write down that one M_u equal to get $0.87 F_Y$ ast into Z, Z again will be equal to 1 minus a $K_2 \left(\frac{x}{d}\right)$ so this is the one term which you are getting due to steel so this one you are getting so now what I would like to say that $k_1 f_{ck} \left(\frac{x}{d}\right) b d^2$ all these equations whatever you are writing that one you will find out later on that it will be independent of any method.

Whether it is working stress method or it will limit State method whether you be IS 456 IRC 112 you will get this type of that equation that format of equation in you get it every time that is what I would like to say and that one we shall come back later on if we get time because the main objective will find out that is not just only a formula you are using you try to understand the formula and from there you find out the little difference little modification.

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$$C = k_1 f_{ck} \left(\frac{x}{d}\right) bd$$

$$T = 0.87 f_y A_{st}$$

$$0.87 f_y A_{st} = 0.364 f_{ck} b x_u$$

$$M_{uL} = k_1 f_{ck} \left(\frac{x}{d}\right) \left(1 - k_2 \left(\frac{x}{d}\right)\right) bd^2$$

$$j = 0.5 + \sqrt{0.25 - \frac{M_{uL}}{0.87 f_{ck} b d^2}}$$

And how it actually affects or how which the particular parameter actually affects that also you can find out this one is very, very important this particular one you can say very, very important here that J equal to 0.5 plus root over point to 5 minus nu by 0.87 fck bd² so I think this particular one we can find out from the equation.

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$$M_u = 0.36 f_{ck} \left(\frac{x}{d}\right) \left(1 - 0.416 \frac{x}{d}\right) b d^2$$

$$\frac{M_u}{f_{ck} b d^2} = 0.36 \left(\frac{x}{d}\right) \left(1 - 0.416 \frac{x}{d}\right)$$

$$0.36 \times 0.416 \left(\frac{x}{d}\right)^2 - 0.36 \left(\frac{x}{d}\right) + \frac{M_u}{f_{ck} b d^2} = 0$$

$$\left(\frac{x}{d}\right)^2 - \frac{1}{0.416} \left(\frac{x}{d}\right) + \frac{1}{0.36 \times 0.416 f_{ck} b d^2} = 0$$

$$\left(\frac{x}{d}\right)^2 - 2.40 \frac{x}{d} + \frac{6.68 M_u}{f_{ck} b d^2} = 0$$

$$\frac{x}{d} = 1.2 - \sqrt{1.2^2 - \frac{6.68 M_u}{f_{ck} b d^2}}$$

In $M_u = 0.36$ we can write down here $f_{ck} x/d (1 - 0.416 x/d) b d^2$ these equation it will give you a $f_{ck} b d^2$ so I am rearranging this particular equation and from there you can find out here so I am taking these equation in one place plus f_{ck} I could omit this portion but I feel that one that whenever you are going through a problem so it is wise to go for that that is why I have taken this particle one.

I could refer this one to your reinforced-concrete one that way we can find out but any way so we can find out this one here taking this particular one here we get it here I am not going to show you are advised to go in detail you can get this particular one in PC verges book of limit State design of concrete and this one will give you just I am writing the final one $6.68 M_u / f_{ck} b d^2$, so this is the one you will get it this particular one you will get here.

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$$\begin{aligned}
 j &= 1 - 0.416 \frac{x}{d} \\
 &= 1 - 0.416 \times 1.2 + 0.416 \sqrt{(1.2)^2 - \frac{6.68 M_u}{f_{ck} b d^2}} \\
 &= 1 - 0.499 + \sqrt{0.249 - \frac{1.156 M_u}{f_{ck} b d^2}} \\
 &= 0.5 + \sqrt{0.25 - \frac{1.15 M_u}{f_{ck} b d^2}} \\
 j &= 0.5 + \sqrt{0.25 - \frac{M_u}{0.87 f_{ck} b d^2}} \\
 K &= \frac{M_u}{f_{ck} b d^2} \\
 j &= 0.5 + \sqrt{0.25 - \frac{1.15 K}{0.87}}
 \end{aligned}$$

Now harder I would like to do it the $j=1-0.416 x/d$ equal to $1-0.416$ into one point two plus 0.416 root over one point two whole square minus $6.668 M_u / f_{ck} b d^2$ which comes as $1- .499$ plus if we just inside if you take it $.249-1.156 M_u / b d^2$ which is nothing but you can say $.5$ plus 0.25 minus 1.15 I am writing down $M_u / f_{ck} b d^2$ this particular one we can write down here so J further that very interesting way I can write down 0.5 plus root over $0.25-M_u$ 1.15 divided by that one.

So $0.87 f_{ck} b d^2$ so $M_u / f_{ck} b d^2$ that M_u which over the force over the moment actually applied well due to load and $f_{ck} b \& d$ on the basis of that I can find out and then I can decide the value of j and whenever we shall decide the j and then on the basis of that we can find out there many more things so this particular one j we can find out so j is equal to this one so that means here point five plus root over point two five very interesting that beautiful number easier to remember also.

And we did like to see that particular one whether it is valid for other cases also so only these portion we change that will change in lot of things will influence the whole idea $M_u / f_{ck} b d^2$ this parker 1 we can tell something this $M_u / f_{ck} b d^2$ so we can say this one as you say $k = M_u / f_{ck} b d^2$ so we can write down this one as j 1 plus just let us distinguish this $1.5+0.25-1.15/K$ so k by k by 0.87 or we can write down $1.15 k$ also.

We can write down this one so that all together I do not either $1.15 K$ or point okay by 0.87 that well so you can write down this work on equation so this key is a very, very important one because that it depends on that whether this one is going to be negative or that negative I fit

becomes negative so we shall not be able to solve it so that is why this $M_u / f_{ck} b d^2$ this is a very, very important aspect because I know M_u because it is coming from the load I know x EK because which grade of concrete you are going to use is also already or beside it b also we know only thing that one that you can change it that is the depth of the beam.

So this changing this depth d so that one should be always less than 0.25 if it is greater than 0.25 then it will be a complex number and which is then means it is not possible it is not feasible so this particular aspect we have to find out such a way so that i can get certain value of j so that means here J can be 0.5 because if we make such I that point over 0 so that means you can get same minimum there is a J value of that point 5 here.

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LIMITING VALUES OF x_u/d

Type of steel	f_y	Yield strain (ϵ_{su})	X_u/d
Mild steel	250	0.0031	0.53
High yield strength	415	0.0038	0.48
High yield strength	500	0.0042	0.46

So coming to this here just to give you idea as per IS 456 mild steel 250 0.0031, 0.53 high yield strength is 415 and yield strength is 0.0038 and 0.48 high yield strength 500 0.0042 and 0.46 we are getting this one F_y by 1.15 yes, yes that modulus of elasticity of steel that one we consider here all the same 2 into 10 to 5 Newton per square millimeter $X Y D$ it changes with respect to the respect to steel.

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Balanced Sections

Sections, in which the tension steel reaches yield strain simultaneously as the concrete reaches the failure strain in bending are called balanced sections



That X_u/d this changes and then x/d that your F_y/f_{ck} this value it gives you m or that that value it will give you that balance x later on whenever we solve the problem that time we shall come comeback to that work for one here and the maximum limiting values of compression this four column we can say so we can say steel for our $X_u/d = k_1$ same for all of them k_2 and if that occur one we can find out x/d is varies with respect to that similarly F will also vary with respect to that.

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PERCENTAGE OF LIMITING STEEL AREAS FOR BALANCED DESIGN

Steel	x/d	$\rho_t(f_y/f_{ck})$
Fe250	0.53	21.97
Fe415	0.48	19.82
Fe500	0.46	18.87



So as usual the same thing balance sections already i have told we have already told so that i do not want to come to that under reinforced sections and that also we have discussed that an over reinforce section that particular one we have considered already we have told these values so this is very, very important here so that means here whatever we are considering here why I have chosen that IS 456 because it will because since you are familiar with that may be that you are not familiar with working stress method those who are taking but that is also another important actually method.

That we should never forget because there is a very, very basic one after that we have come and considering all these aspect you know what I can say that what want from the linear triangular one to the parabolic rectangular one we have considered and because then finally we shall come to that IRC 112 and not only that you will find out that code that is actually you know code where you will find out that per core 1 or 0 code there will be get also similar kind of thing.

Whatever is when you will find out in IRC 112 so considering all aspects oh I thought I can tell you together CL but also that we shall tell you the you know when we shall solve the problem that time we shall discuss so with this you know I think whatever we can say that factor I would like to actually conclude this particular one with respect to Is 456 the next one we shall go for it and that is IRC 112 and what is the difference that we shall find out so with this we conclude this particular session thank you very much.