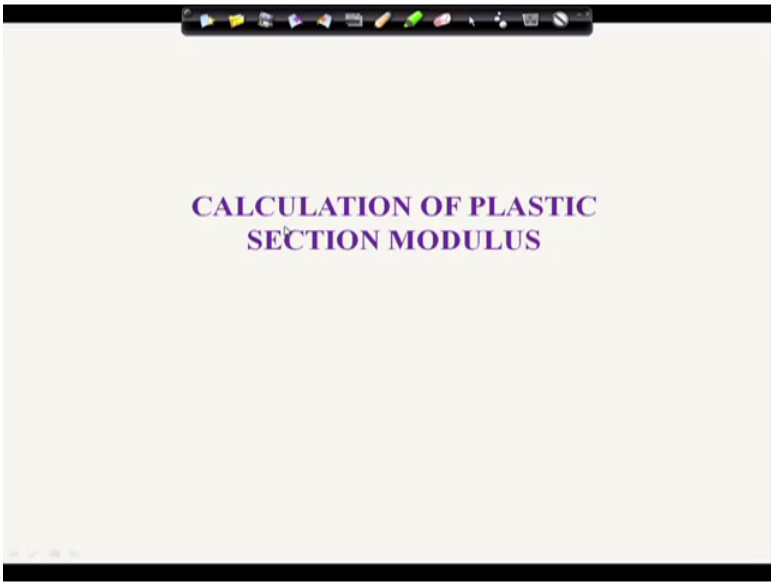
Course on Design of Steel Structures
Professor Damodar Maity
Department of Civil Engineering
Indian Institute of Technology Kharagpur
Lecture 53
Module 11
Calculation of Plastic Section Modulus

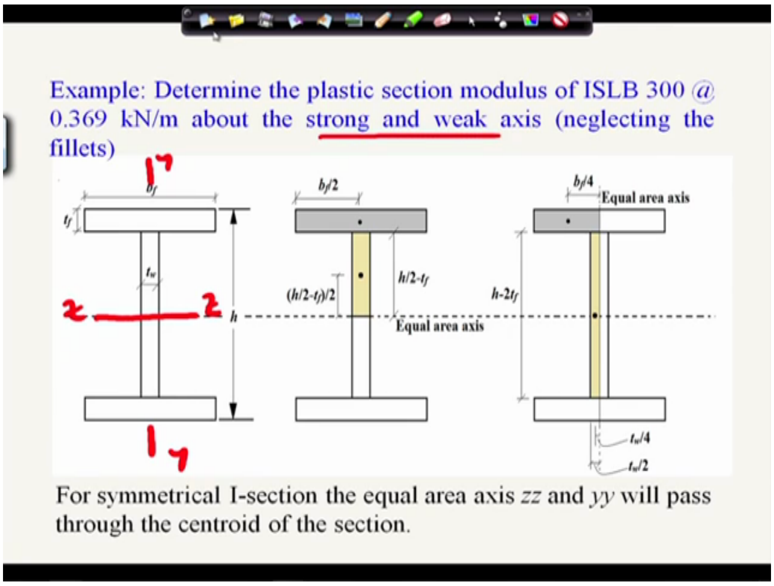
Hello so far we have discussed about the design bending strength calculation of a flexural member and we have seen when calculating the design bending strength we need to know the plastic section modulus of the section. And in case of built-up sections we do not have the plastic section modulus in the code so we need to calculate the plastic section modulus and to know the plastic section modulus of a section we need to know how to calculate that what is the theory behind that and that I will discuss today and that will be discussed through one workout example.

So first I will go through one workout example and I will see how to calculate the plastic section modulus of a section, then as time will permit I will go through another example where we will show that suppose if load is very high on the beam or the section is very long sorry the member is very long then bending moment is quite high. So in such cases sometimes the required section modulus not be able to take care means will not be means will not be sufficient on the available section that means whatever required section modulus is required the available section does not have that much section modulus.

So when the required section modulus is comparatively high then the available section then we have to go for built-up section that means we have to use certain plates on the top of the or top and bottom section to increase the section modulus and to take care that much load. So how to increase the section modulus and how to decide the thickness and width of the plate to withstand the required amount of load that will be demonstrated through that example so today we will go through these two examples.

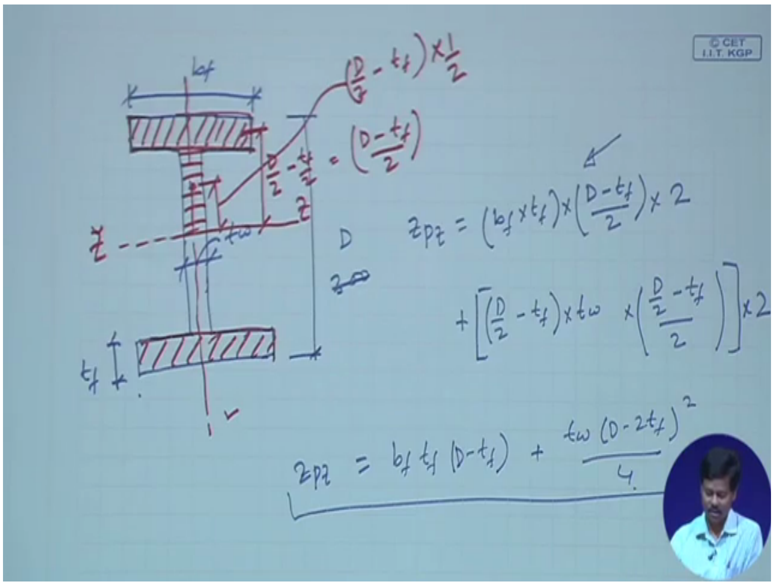
(Refer Slide Time: 2:44)





So first let us discuss about the calculation of plastic section modulus here you see we have given an example that is an I section ISLB 300 we need to find out the plastic section modulus about weaker and means weak and strong access weak and strong access, right that means about y-y and z-z, this section say we can say z-z and this section is y-y, right so we have to find out the plastic section modulus of this ISLB 300.

(Refer Slide Time: 3:37)



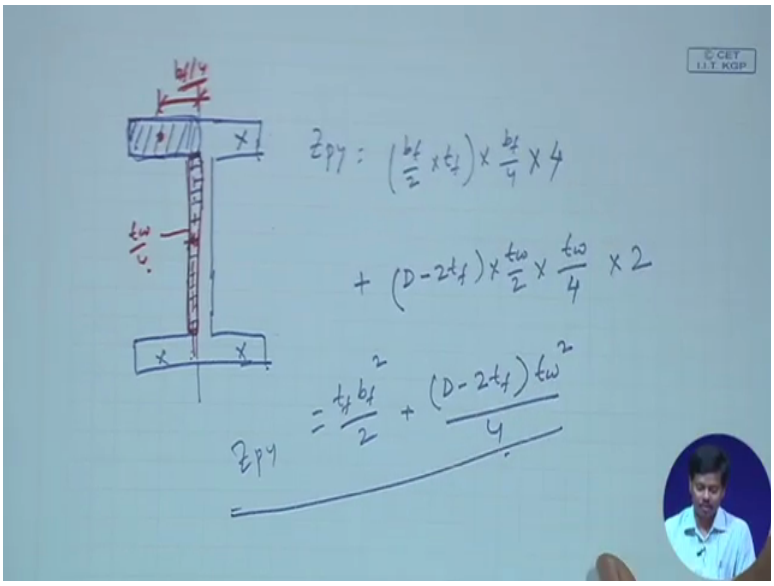
And so let us go through this example say this is an I section whose dimensions are given say for ISLB 300 we know that overall depth is 300 say let us make this is D, right. So overall depth is D, width of the flange is bf, thickness of flange is tf, thickness of web is tw so this so now this is an symmetrical I section, so it has equal area axis about z-z and y-y which will pass through the centroid of the section so we can say if this is the centroid of the section then for z-z axis about z-z axis we have to find out the plastic section modulus that means plastic section modulus means first we have to find out the section modulus of this, okay this flange okay that will be how much that will be the area of this flange into its (center) means its cg distance from the neutral axis.

So so this distance will be this is total D so this distance should be D minus 2f by 2, so that means D by 2 minus this is D by 2 minus tf by 2, right that means D minus tf by 2, okay right D minus tf by 2, okay D minus tf by 2. So this is one so I have to find out section modulus of this block into 2 because in this case also it will be then section modulus of this area, so this will be the area into its cg so cg if it is here then this distance should be D by 2 minus tf D by 2 minus tf by 2, right so this is how we will find out the section modulus.

So if I now try to find out the section modulus say Zf sorry Zpz so Zpz will be bf into tf is the area of the flange into distance between cg of the flange and cg of the section that is D minus tf by 2, right D minus tf by 2 sorry D by 2 minus tf by 2, right into 2 in two side, so this is the section modulus of the flange plus section modulus of web so area of the web will be D by 2 minus tf this is the height into thickness is tw into the cg will be D by 2 minus tf by 2, right.

So this will be the area into distance cg distance of the web of one side, so in two sides it will be multiplied by 2, right. So after simplifying the Zpz value finally we will get as bf into tf into D minus tf plus tw into D minus 2tf whole square by 4, right so Zpz we can find out as this.

(Refer Slide Time: 8:47)

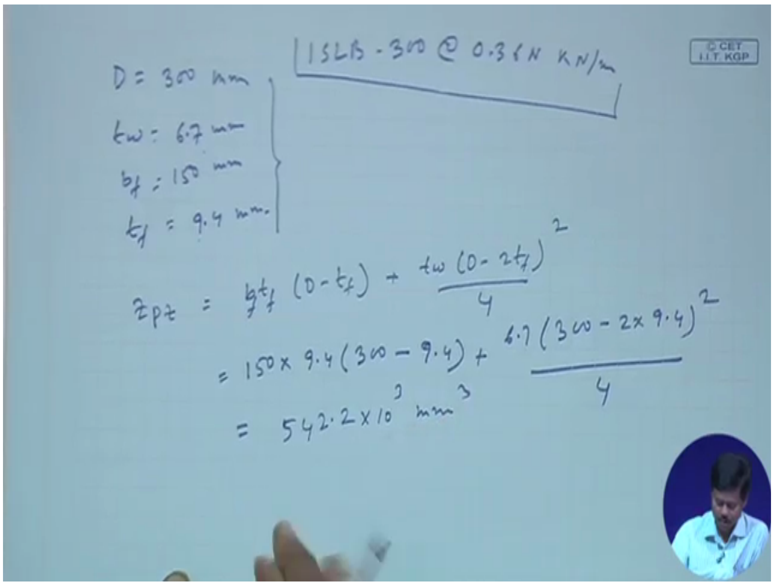


Similarly we can find out the value of Zpy, so for Zpy again let us draw the diagram and let us try to find out the area and its cg distance, say this is the I section and now about y-y we have to find out right so about y-y if we find out we will find out first this one okay and this one, right. So for this we can find out so the area of this will be bf by 2 into tf of this flange bf by 2 into tf and cg distance will be from here it will be it will be bf by 4, right because total is bf by 2 so half of that is will be bf by 4, okay.

And for this portion the cg will be at the middle of this, right. So for this the cg distance we can find out, okay right so that will be tw by 2 by 2 that means tw by 4 therefore we can find out Zpy also. So Zpy will be the plastic section modulus of this section into 4 because 4 this is 1, 2, 3, 4 so bf by 2 into tf this is the area into cg distance from the neutral axis that is bf by 4 into 4, right plus area of the web will be D minus 2tf this is the depth into tw by 2 into the distance tw by 4, right and into 2 side so into 2. So after simplification we can find out bf tf into bf square by 2 plus D minus 2tf into tw square by 4, so Zpy value we can find out.

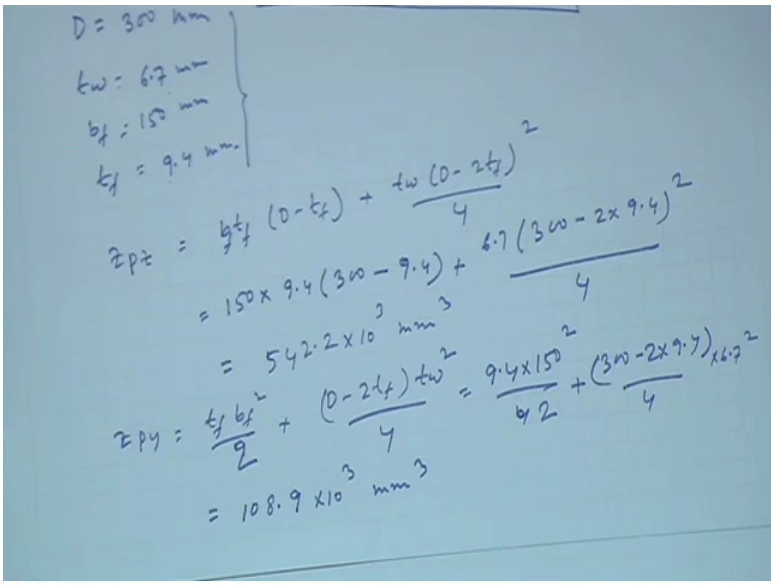
So that means here we have seen that for I section how to calculate the plastic section modulus about X axis or Z axis and about Y axis.

(Refer Slide Time: 12:18)



So for this particular case we have this data like D is equal to 300 for ISLB 300 at 0.369 kilonewton per meter for this section D is equal to 300 millimetre, tw is equal to 6.7 millimetre and bf is equal to 150, tf is equal to 9.4 millimetre so from this data we can find out the value of plastic section modulus about Z axis that is Zpz. So Zpz will be bf into tf into D minus tf plus tw into D minus 2tf square by 4. So putting the value bf as 150 and tf as 9.4, D as 300, tw as 6.7 and after calculating we could find Zpz as 542.2 into 10 cube millimetre cube.

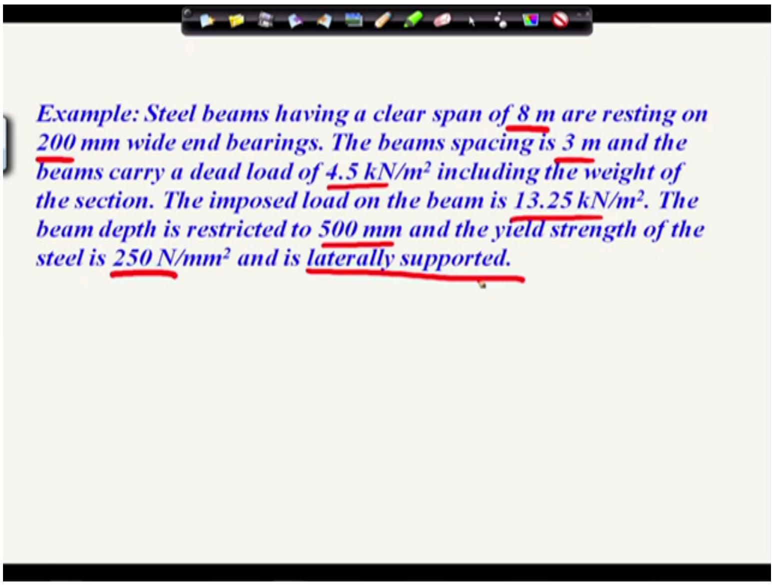
(Refer Slide Time: 13:50)



Similarly the Zpy value we can find out Zpy value will be tf into bf square by 4 plus D minus 2tf into tw square by 4, so thickness of flange was 9.4 into width of flange was 150 by 4 sorry by 2 this is by 2 plus 300 minus 2 into 9.4 by 4 into 6.7, right. So after calculation of this we can get the value of Zpy as 108.9 into 10 cube millimetre cube. So this is how one can find out the section modulus means plastic section modulus of a given section and this knowledge is useful means we will be useful for calculating the design bending strength of the member, right.

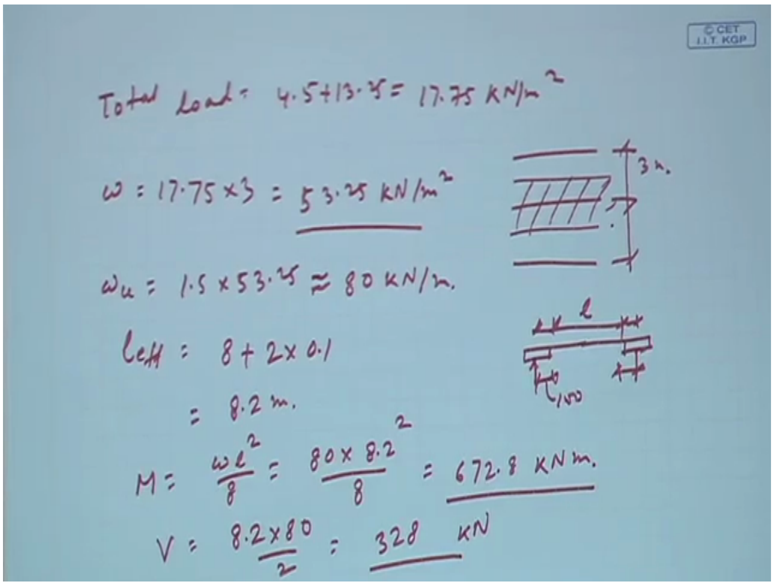
Now I will go through another example where the available section is less means is not available section is less than the required section. So there we need to provide certain plate on the section and we have to increase the section modulus by addition of that plate, right. So through this example we will see how to decide the thickness and width of the plate that means dimension of the plate for which the required load on the beam can be withstand withstood, okay.

(Refer Slide Time: 16:07)



Here we see the example is given like this that is a steel beam having a clear span of 8 mm and resting on 200 millimetre wide bearings, so bearing length is 200 mm and clear span is 8 mm, the beam spacing is 3 meter and the beam carry a dead load of 4.5 kilonewton per meter square including the weight of the section. The imposed load on the beam is 13.25 kilonewton per meter square and the beam depth is restricted to 500 mm that means the available beam depth is less than 500 mm so we have to restricted with this dimension and the yield strength of the steel is 250 newton per millimetre square and it is laterally supported, right. So these are the given things through which we have to find out a section size, right.

(Refer Slide Time: 17:28)

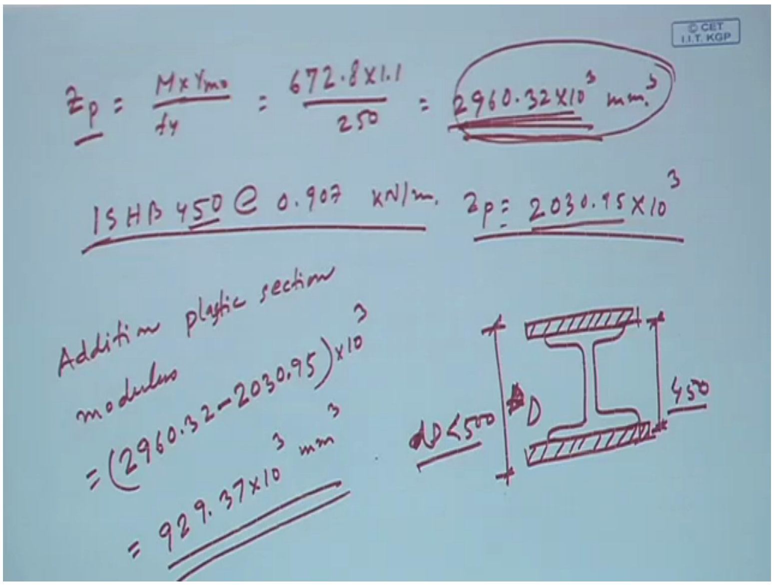


So with this we can write down that the total load is given as 4.5 plus 13.25 total load which is dead load plus imposed load as 4.5 plus 13.25 that means total load is 17.75 kilonewton per meter square and beams are spaced at 3 meter interval at 3 meter interval it is spaced, okay. So the load per meter run will be 3 into that, right so load per unit means meter load per meter will be that means W will become 17.75 into 3 this is the influence area so that is becoming (13) 53.25 kilonewton per meter square.

So total factored load again we can find out total factored load say Wu factored load so that is 1.5 into 53.35 because it is dead load and imposed load so load factor will be 1.5, so this is coming almost 80 kilonewton per meter. And effective span will be length of the span was given 8 meter plus bearing length was given into 2 sides there is bearing bearing length of 200 mm so from here it is 100 and from here it is 100, so total length will be the clear span length plus bearing length by 2 plus bearing length by 2, so 8 plus 2 into 0.1, okay so effective length is becoming 8.2 meter.

So the maximum moment will be at the mid span that will be as it is a simply supported beam so we can consider the maximum bending moment will be wl square by 8 wl square by 8, so maximum bending moment will be 80 into l is 8.2 square by 8, that is 672.8 kilonewton meter. Similarly the reactions are support that is reactions are support or shear force that will be wl by 2 so 8.2 into 80 by 2 that is 328 kilonewton, so maximum bending strength is 672.8 kilonewton and maximum shear force is 328 kilonewton.

(Refer Slide Time: 20:42)



Now we have to find out the required section modulus to find a approximate section size. So to do that we can find out Zp Zp value we know we can find out Zp as M by fy by gamma m0 that means if I put the value of M and fy then I can find the Zp required 2960.32 into 10 cube millimetre cube, right. So plastic section modulus required is 2960.32 into 10 cube millimetre cube.

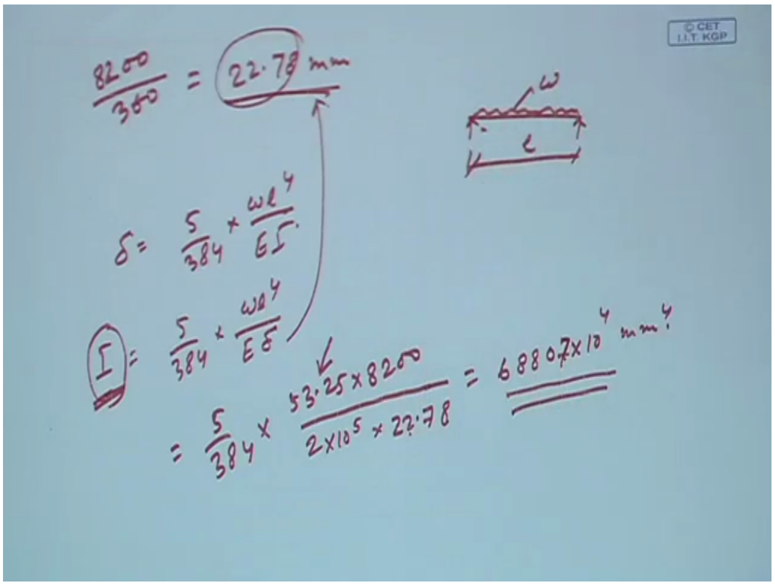
Now the depth restriction is given as 500 mm the maximum depth allowed is 500 mm therefore we can see the SP 6 and we can find out a section. Now when we look through the SP 6 we could see that largest plastic (modu) section modulus under 500 mm depth is ISHB 450 at 0.907, right largest section modulus we could achieve which is with the restriction of the depth as below 500 as ISHB 450, where the section modulus is 2030.95 section modulus means plastic section modulus, okay.

So plastic section modulus of ISHB 450 is becoming this and we cannot go beyond this size because the maximum depth is restricted to 500 mm therefore what we can do we can strengthen with additional plates to provide the required plastic modulus. That means we have a section with a size of means depth of 450 and it could go upto 500 and upto 500 we cannot achieve this required section modulus means if I consider some I section of 500 depth we are unable to achieve this therefore what we are doing we are taking a section lesser than 500 and we are adding some plate, right so that the total depth does not exceed 500 means less than 500 this is d so sorry capital D.

So what we have done the I sections we have considered 450 and we are considering certain plate so that the total depth of the section overall depth of the section does not go beyond 500, right. So additional plastic section modulus we have to means we need to know. So for this what we could see that for ISHB 450 the section modulus is 2030.95 and we need 2960.32 so additional plastic modulus we have to achieve through plate additional plastic section modulus so additional plastic section modulus will be the required section modulus minus the available section modulus for ISHB 450 into 10 cube.

So what we need is 929.37 into 10 cube millimetre cube section modulus from plate, so this is how. So now I can find out the plate dimension.

(Refer Slide Time: 25:51)

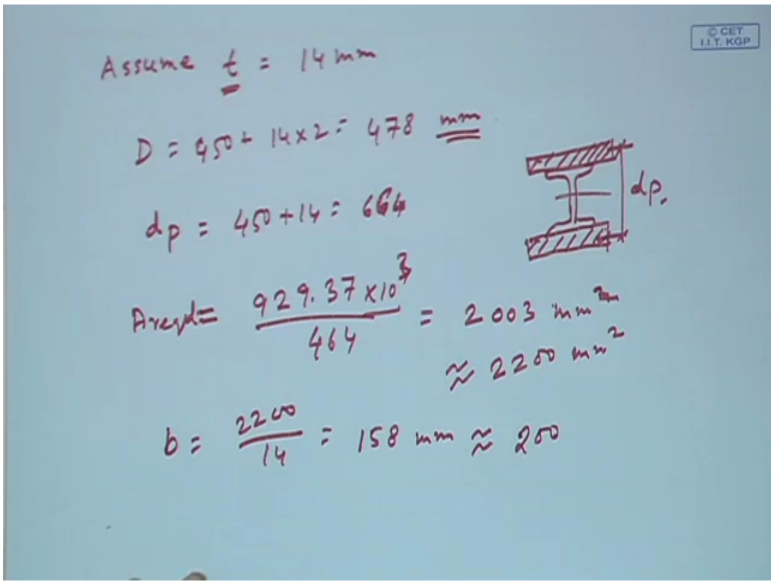


Another thing we have to consider that is what will be the maximum deflection, right. So maximum allowable deflection is the effective span 8200 by 360 the maximum deflection which can be allowed that is coming 22.78 millimetre, so this is the allowable deflection and we know what is the maximum deflection coming in the member maximum deflection on a member we can calculate.

Now from this if we equate the maximum allowable deflection with the maximum deflection then I can find out the value means value of I, okay. Now in this case so to find out the required I we can equate this, right now suppose for this case if this is w and if length is l, then we know delta is equal to 5 by 384 into wl to the power 4 by EI, right or I can find out I is equal to 5 by 384 into wl to the power 4 by E into delta, okay.

Now this delta will be the means this delta can be allowed upto this 22.78 so from this I can find out the required I of the section. So if I put this value that is w was calculated as 53.25 and length is 82, right and E is 2 into 10 to the power 5 into delta is the allowable displacement is 22.78. Now remember this w is the service load the unfactor load because deflection check has to be done through unfactor load. So from this I can find out I as 68807 into 10 to the power 4 millimetre to the power 4. So two conditions we got which we have to fulfil one is the section modulus means elastic (sec) sorry one is the moment of inertia of the section which is equal this to restrict the deflection within the limit and another is the plastic section modulus required to take care the moment carrying capacity, okay. So these two we need to check.

(Refer Slide Time: 29:14)



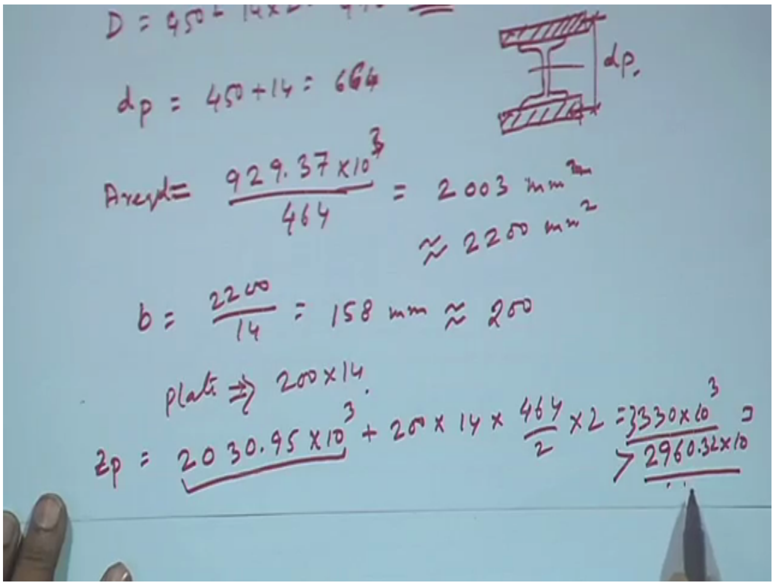
So now what we can do we can assume because we do not know the plate thickness and plate dimension, so we can assume anyone of this say for example thickness of the plate as say 14 mm, right this is again a completely trial and error method means we have to find out the thickness from the assumed value of width or we have find out the width from the assumed value of thickness.

So if we assume the thickness of the plate as 14 mm then total depth of the beam will be 450 plus 14 into 2, 478 mm so total depth of the beam is less than 500 mm so with this we can try, right and distance between the center to center distance of the plate the d say dp distance between center to center of the plates will become 450 plus 14, 464 464 this is means if we have a I section like this and if we have plate then distance between center to center of the plate which we are telling as dp, okay.

So if we have this then the required area of plate because from the plastic section modulus the required area of the plate we can find as section modulus the additional section modulus which has to be achieved through the plate 10 cube divided by 464, right because we need to find out the section modulus of the plate as the additional section modulus which we need to achieve, right.

So we know the section modulus of the plate will be the area into cg distance from here okay so from that if we equate we can find out the area, right. So area will be 2003 the area of the plate is 2003 millimetre square. Now we can provide certain area say let us increase say 2200 millimetre square, right now the width of the plate b width of the plate will be 2200 divided by 14, so that is becoming 158 so let us use as say 200, right.

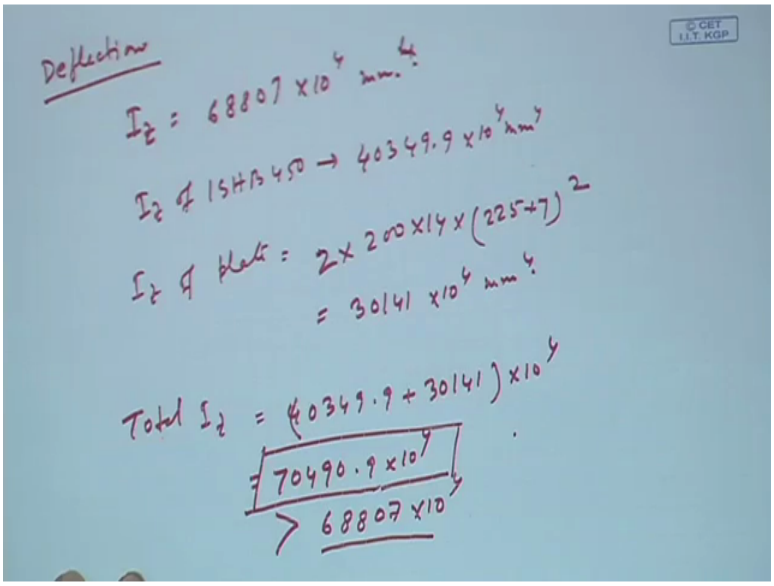
(Refer Slide Time: 32:51)



So plate size will be finally 200 by 14, so we can use plate size as 214 and then the plastic section modulus of the whole section we can find out that is 2030.95 into 10 cube plus this is the section modulus of the I section then 200 into 14 is the area into cg distance 464 by 2 into 2, okay. So the total plastic section modulus is coming as 3330 into 10 cube, which is greater than the required section modulus which is coming 2960.32 into 10 cube, right.

So what we could see the required section modulus we calculated earlier as 2960.32 and we are providing section modulus 3330 which is greater than this through these two additional plates at top and bottom, right. So now from section modulus point of view this is okay.

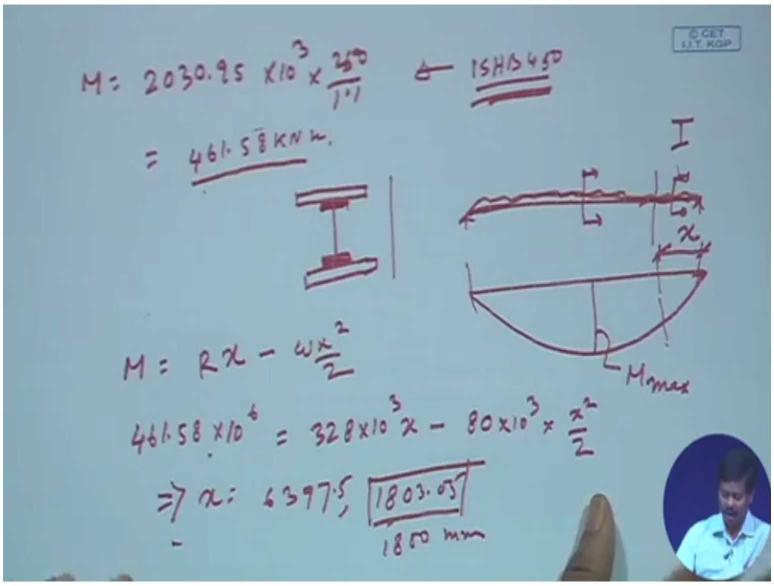
(Refer Slide Time: 34:18)



Now we have to check for deflection, right so with this we have to check whether the deflection is okay or not. So here we can see that maximum Iz required was 68807, right this is the requirement and Iz provided by rolled section that is ISHB 450 is 40349.9 millimetre to the power 4, right. So maximum Iz we need to provide this, so Iz provided by plate has to be this one that is Iz of plate will be Ar square say 2 into 200 into 14 into r square r means 225 plus 7, right that is 30141 into 10 to the power 4, right.

So total Iz of the built up section will be the Iz of ISHB 450 plus Iz of plate that means that is 40349.9 plus 30141 into 10 to the power 4, right which is becoming 70490.9 into 10 to the power 4, which is greater than the required Iz that is 68807 into 10 to the power 4. So we are providing the section such that its Iz is 70490 and our required Iz was 68807, so the section is okay from the deflection point of view.

(Refer Slide Time: 36:52)



Moment carrying capacity of the beam of ISHB 450 section is we can find out that is 2030.95 into 10 cube into 250 by 1.1, so this is the moment carrying capacity of ISHB 450 means the the rolled section ISHB 450 has certain moment carrying capacity which can be calculated as 461.58 kilonewton meter, right.

Now see for a beam the bending moment is varying and maximum bending moment is coming at the center, right and to accommodate that bending moment to take care the maximum bending moment we need to provide a section which consist of I section and some additional plates, right to withstand that much moment but the moment is not constant it is varying and becoming 0 at support so we do not need to provide the plate throughout the length of the section because unnecessary we will be providing, right because the I section which is ISHB 450 it can take 461.5 kilonewton meter itself.

So we can find out that at what distance this moment is developing upto that we can provide this ISHB section after that we can provide the plate. So to find out this distance we can equate M is equal to say reaction force at support R into x minus Wx square by 2. So if if I put those value say 461.58 into 10 to the power 6 is equal R into x, so 328 into 10 cube into x minus 80 into 10 cube into x square by 2, right. So I am making all newton millimetre.

So from this I can find out x as 6397 and 1803.05, right so I can find out that that x will be 1803.05, okay that means the cut-off point I can make upto say 1800 mm say upto this I can provide simply I section and after that I can provide the built up section, so in this way I can make economy of the use of the steel, right. So in plate girder in fact this concept will be used so that at a particular section we will provide the required amount of steel to withstand the bending moment and we will be go on increasing or decreasing the section size according to the requirement of the bending moment, okay. So today let us conclude with this thank you.