

Course on Design of Steel Structures
Prof. Damodar Maity
Department of Civil Engineering
Indian Institute of Technology Kharagpur
Module 10, Lecture 50
Laterally Unsupported Beams

This lecture we will focus on laterally unsupported beam. So design strength of laterally unsupported beam will be calculated based on the code provisions, which is given in clause 8.2.2 of IS 800-2007. Now in case of laterally unsupported beam, the lateral torsional buckling will play an important role and because of lateral torsional buckling, the full plasticity of the section will not be developed that means the member will fail before at any of its full bending stress of the section, then it will fail due to laterally torsional buckling and this lateral torsional buckling happens.

In case of steel rolled section, unlike RCC section or the stocky section, their lateral torsional buckling does not come into picture, but in case of such type of section where we have made economic use in terms of the material, we face this type of lateral torsional buckling and if we do not provide support laterally then such type of buckling will come into picture.

So this buckling comes, because of the cross sectional shape then support conditions and the length effective length. So what is the effective length? What are the support conditions and what is the type shape depending on that the lateral buckling moment will be calculated and, because of that how it is going to fail that you mean how we are going to calculate the lateral buckling strength and then how we are finally going to find out the design bending strength of the section when the section is laterally unsupported will be discussed.

(Refer Slide Time: 2:21)

DESIGN STRENGTH

The design bending strength for laterally unsupported beams is

$$M_d = \beta_b Z_p f_{bd}$$

Where,

- Z_p = Plastic section modulus of the cross-section
- $\beta_b = 1.0$ for compact & plastic sections
- $= Z_e/Z_p$ for semi-compact sections
- f_{bd} = design bending compressive stress given by

$$f_{bd} = \frac{X_{LT} f_y}{\gamma_{m0}}$$

X_{LT} = bending stress reduction factor to account for lateral torsion buckling

So if we see the design bending strength, which we can write as M_d is equal to $\beta_b Z_p$ into.

(Refer Slide Time: 2:36)

Handwritten derivation of the design bending strength formula for laterally unsupported beams:

$$M_d = \beta_b Z_p f_{bd}$$

$$f_{bd} = \frac{X_{LT} f_y}{\gamma_{m0}}$$

$$X_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \lambda_{LT}^2}} \leq 1.0$$

$$\phi_{LT} = 0.5 \left[1 + \lambda_{LT} (\lambda_{LT} - 0.2) + \lambda_{LT}^2 \right]$$

Additional notes from the slide:

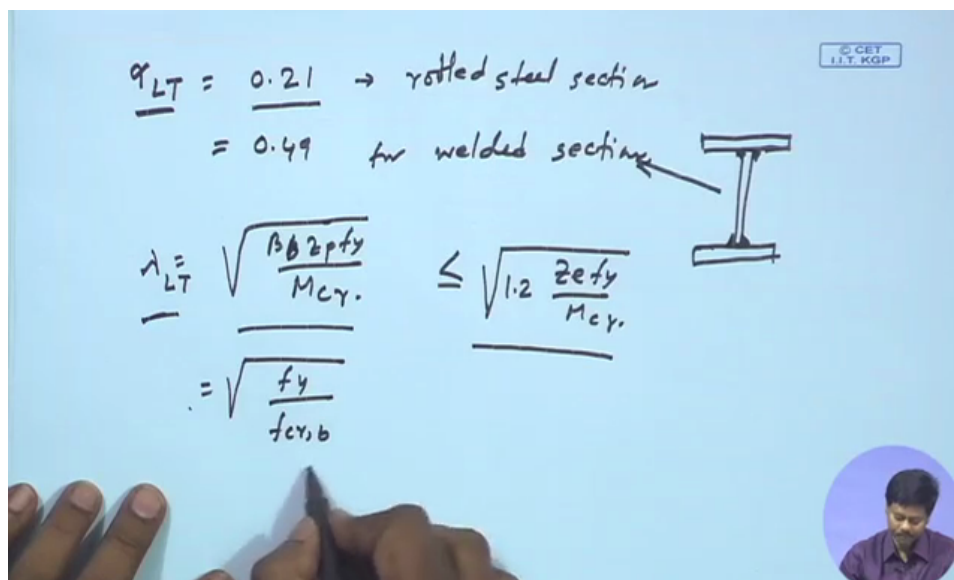
- $\beta_b = 1.0$ for compact & plastic sections
- $\beta_b = \frac{Z_e}{Z_p}$ for semi-compact sections

M_d is equal to $\beta_b Z_p$ into F_{bd} . So the design bending strength of laterally unsupported beam can be written as Z_p into F_{bd} into β_b , where β_b we can consider as 1.0 for compact and plastic section and we can consider as Z_e by Z_p for semi-compact section. so depending on the type of section, the β_b value will be calculated either 1 for compact or plastic section or the ratio of Z_e by Z_p for semi-compact section and this Z_p is basically the plastic section modulus of the cross section and F_{bd} is the design bending compressive stress.

So design bending compressive stress F_{bd} we can find out from this formula XLT into F_y by γ_{m0} . This is the bending stress reduction factor to account for lateral torsional buckling. So the design bending compressive stress F_{bd} can be found from this XLT into F_y by γ_{m0} , where XLT is the bending stress reduction factor. So what is a bending stress reduction factor that we need to calculate. This has been calculated as $1 / \sqrt{\phi_{LT}^2 + \lambda_{LT}^2}$ and in any case, it should be less than or equal to 1.

So the reduction factor XLT , we can calculate from this expression, where the ϕ_{LT} value can be calculated as this, that is $0.5 + \sqrt{1 + \alpha_{LT} \lambda_{LT}^2}$ right. So ϕ_{LT} value can be calculated from this expression and where we can see that there is a term which is α_{LT} and also λ_{LT} that non-dimensional slenderness ratio that we have to find out. So if we know the value of α_{LT} and λ_{LT} then we can find out ϕ_{LT} and if we know ϕ_{LT} then we can find out the value of reduction factor that is XLT . So bending compressive stress reduction factor XLT can be found from this. Now what α_{LT} ?

(Refer Slide Time: 5:43)



Handwritten notes on a blue background:

$$\alpha_{LT} = 0.21 \rightarrow \text{rolled steel section}$$

$$= 0.49 \text{ for welded section}$$

Below the text, there is a diagram of an I-section. To the left of the diagram, the following equations are written:

$$\lambda_{LT} = \sqrt{\frac{B \beta_2 p f_y}{M_{cr}}}$$

$$\leq \sqrt{1.2 \frac{Z_e f_y}{M_{cr}}}$$

$$= \sqrt{\frac{f_y}{f_{cr,b}}}$$

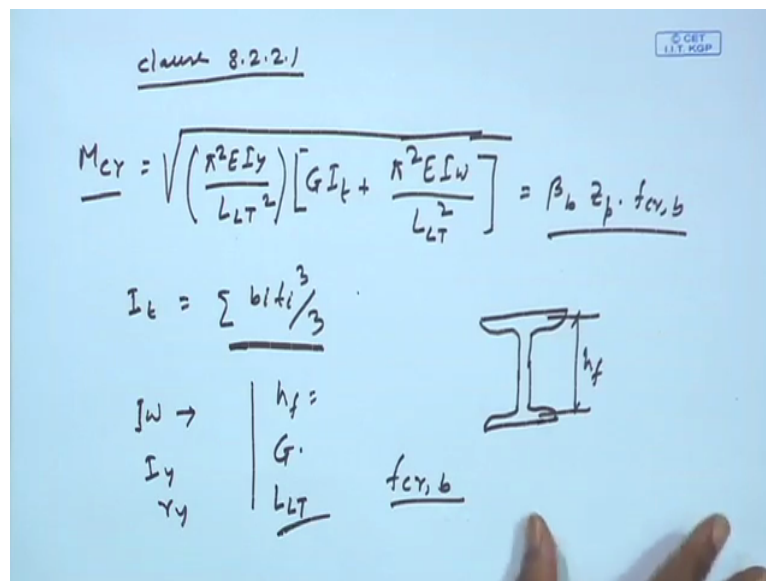
In the bottom right corner, there is a small circular inset photo of a man with dark hair, wearing a purple shirt.

α_{LT} is the imperfection factor for lateral torsion. So α_{LT} is the imperfection factor for lateral torsional buckling of beam, we can consider this as 0.21 for steel rolled section, okay. So for all rolled section, the value of α_{LT} can be found as 0.21 and it will be 0.49 for welded section that means if we suppose, use plate to make a I section with the use of welded, say with welding we can make a I section from a plate, then for such type of section, we can use α_{LT} as 0.49 otherwise for the rolled section we can use α_{LT} as 0.21 and

lambda LT is, I have told that is non-dimensional slenderness ratio lambda LT. So this can be calculated from this that is beta b Zp Fy by Mcr, right. Beta b into Zp Fy by Mcr and it has to be less than or equal to in any case 1.2 Ze Fy by Mcr, right.

So we will calculate the non-dimensional slenderness ratio from this expression, which is beta b Zp Fy by Mcr and this lambda LT value should be less than or equal to this that we will go for 1.2 Ze Fy by Mcr. So this lambda LT also we can write as Fy by Fcr,b, where Fcr,b is the Mcr by beta b Zp, right. So lambda LT we can find from this, right.

(Refer Slide Time: 8:07)



Handwritten slide content showing the formula for M_{cr} and a diagram of an I-section.

Clause 8.2.2.1

$$M_{cr} = \sqrt{\left(\frac{\pi^2 E I_y}{L_{LT}^2}\right) \left[G I_t + \frac{\pi^2 E I_w}{L_{LT}^2} \right]} = \beta_b Z_p \cdot f_{cr,b}$$

$$I_t = \sum \frac{b_i t_i^3}{3}$$

Diagram of an I-section with parameters I_y , r_y , h_f , G , and L_{LT} indicated.

Now what is Mcr? Mcr is basically the elastic lateral buckling moment. This is given in clause 8.2.2.1 of IS 800-2007. We can find out the elastic lateral buckling moment Mcr and a formula has been given for calculating the value of Mcr that is as pi square E Iy by LLT square into GIt plus pi square E Iw by LLT square, right. So the elastic lateral buckling moment can be found from this expression. This can be writurn as beta b Zp into Fcr, b, right where here the different variables are given, which we can find out from clause 8.2.2.1 the parameters like It is a torsional constant. So It will be for open section, it will be bi ti cube by 3.

So how to calculate It that through one work out one example we will discuss details. Then Iw is the working constant Iw which is working constant Iy is the moment of inertia about weaker axis, similarly Iy is the radius of the erection (())(10:01) about weaker axis then hf is a center to center distance between flanges that means hf will be say, if I consider any I section then the hf value will be center to center distance of the flanges. This will be hf and G

will be shear modulus and LLT will be the effective length for lateral torsional buckling, which is given in clause 8.3, right. So effective length for lateral torsional buckling LLT can be found in clause 8.3 and $F_{cr,b}$ is the extreme fiber bending compressive stress $F_{cr,b}$ is a extreme fiber bending compressive stress corresponding to elastic lateral buckling moment.

(Refer Slide Time: 11:10)

The slide shows the following handwritten content:

$$F_{cr,b} = \frac{1.1 \pi^2 E}{(L_{LT}/r_y)^2} \sqrt{1 + \frac{1}{20} \left(\frac{L_{LT}/r_y}{h_f/t_f} \right)^2}$$

Below the formula, an arrow points to "Table 14". Next to it are the ratios $\frac{KL}{r_y}$ and $\frac{h_f}{t_f}$.

Below "Table 14", an arrow points to "Table 13". Next to it are the terms $F_{cr,b}$ and F_y .

Below "Table 13", the term F_{bd} is written.

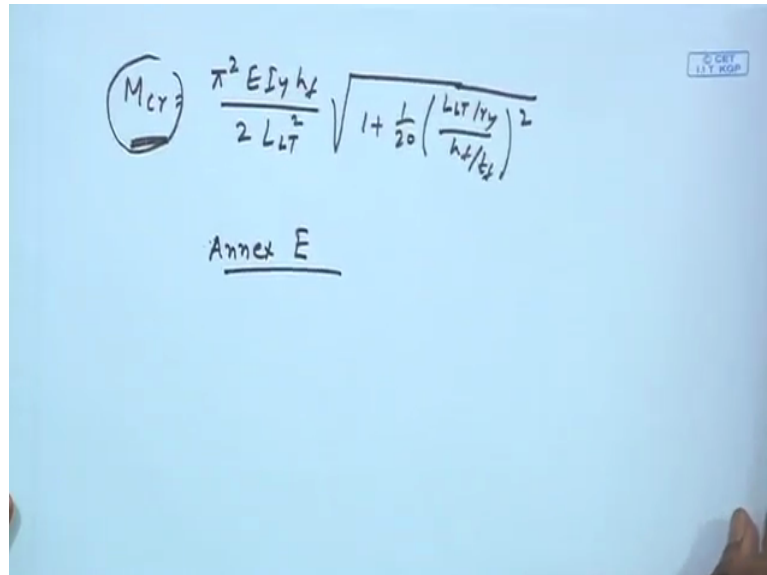
In the top right corner, there is a small logo that says "© CET I.I.T. KGP".

And this $F_{cr,b}$ can be expressed as like this $1.1 \pi^2 E$ by L_{LT} by r_y whole square into 1 plus 1 by 20 L_{LT} by r_y by h_f by t_f whole square, right. So $F_{cr,b}$ the extreme fiber bending compressive stress corresponding to elastic lateral buckling moment can be expressed from this, okay. This is also given in the code, where the details can be found and also we can find out this value the $F_{cr,b}$ value from table 14, instead of calculating all this we can find out the value from table 14 of IS 800-2007 with respect to KL by r_y and h_f by t_f . So for different value of h_f by t_f and KL by r_y , we can find out the value of $F_{cr,b}$ in table 14 that means in place of calculating through this expression directly we can find out the value of $F_{cr,b}$ from table 14.

And the F_{bd} value can also be found directly from table 13 corresponding to different $F_{cr,b}$ value and F_y value, for different $F_{cr,b}$ and F_y value we can find out the value of F_{bd} , F_{bd} the design compressive bending stress, right. So what we can do means without calculating through expression directly what we can do, first we can find out what is the KL by r_y ratio? What is the h_f by t_f and from that we can using we can use table 14 and using table 14, we can find the value of $F_{cr,b}$ and once the $F_{cr,b}$ value is obtained then corresponds to $F_{cr,b}$ value and F_y we can find out the value of F_{bd} . So directly I can find out the design

compressive bending stress and then I can find out the design bending strength of the member.

(Refer Slide Time: 13:54)



The image shows a handwritten formula for the elastic lateral buckling moment M_{cr} on a blue background. The formula is:

$$M_{cr} = \frac{\pi^2 E I_y h_f}{2 L_{LT}^2} \sqrt{1 + \frac{1}{20} \left(\frac{L_{LT}/r_y}{h_f/t_f} \right)^2}$$

Below the formula, the text "Annex E" is written and underlined. In the top right corner, there is a small logo that reads "© CRY 111 KOP".

Now also we can find out the M_{cr} value in a simplified manner from this formula that is $\pi^2 E I_y h_f$ by $2 L_{LT}^2$ square into $1 + 1$ by $20 L_{LT}$ by r_y , h_f by t_f whole square, right. This is the simplified equation which may be used in case of (14:26) member made of standard rolled section rolled I section and welded W symmetric I section. So to calculate the elastic lateral buckling moment M_{cr} , we can use this simplified formula, in case of (14:43) member made of standard rolled I sections and welded sections; however the M_{cr} for different beam section, considering loading support condition and non-symmetric section we can find out more accurately using the method given in annexure E of IS 800-2007. So in annexure E the details are given and more accurately we can find out the value of M_{cr} , see annexure E.

(Refer Slide Time: 15:19)

DESIGN STRENGTH

The design bending strength for laterally unsupported beams is

$$M_d = \beta_b Z_p f_{bd}$$

Where,

- Z_p = Plastic section modulus of the cross-section
- $\beta_b = 1.0$ for compact & plastic sections
= Z_e/Z_p for semi-compact sections
- f_{bd} = design bending compressive stress given by

$$f_{bd} = \frac{X_{LT} f_y}{\gamma_{m0}}$$

X_{LT} = bending stress reduction factor to account for lateral torsion buckling

So the things whatever we have discussed, I am just quickly going through this power point that is first we can find out design strength. So the design strength for lateral torsional buckling has been calculated as per the codal permission and I am very quickly going through this formula, which I have written in hand. So you can see that first we have to find out, the design bending strength for laterally unsupported beam from this formula that is M_d is equal to $\beta_b Z_p F_{bd}$.

Now this F_{bd} we can find out from this, the design bending compressive stress that is $X_{LT} F_y$ by γ_{m0} . Now again I am repeating that this design bending compressive stress is reduced by this factor X_{LT} . This X_{LT} is called bending reduction factor bending stress reduction factor to account for lateral torsional buckling. So if lateral torsional buckling is present then this factor has to be multiplied to find out the design bending stress, right. So the our main job is to find out the X_{LT} value.

(Refer Slide Time: 16:39)

$$X_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \lambda_{LT}^2}} \leq 1.0$$

Where, $\phi_{LT} = 0.5[1 + \alpha_{LT}(\lambda_{LT} - 0.2) + \lambda_{LT}^2]$

α_{LT} = imperfection factor for lateral torsional buckling of beams
 = **0.21** for rolled steel sections
 = **0.49** for welded steel sections

λ_{LT} = non-dimensional slenderness ratio given by,

$$\lambda_{LT} = \sqrt{\frac{\beta_b Z_p f_y}{M_{cr}}} \leq \sqrt{1.2 \frac{Z_e f_y}{M_{cr}}}$$

$$= \sqrt{\frac{f_y}{f_{cr,b}}}$$

Now this XLT value can be found from this expression, right where Phi LT can be found from this expression and alpha is can be consider as 0.21 or 0.49. Then lambda LT the non-dimensional slenderness ratio we can find out from this, right. So the non-dimensional slenderness ratio can be found from this expression, so if we find the lambda LT value and if we know the alpha LT then we can find out the value of phi LT, but to find lambda LT we need to know M_{cr} value..

(Refer Slide Time: 17:26)

Where,

M_{cr} = elastic lateral buckling moment (Cl. 8.2.2.1) is given by,

$$M_{cr} = \sqrt{\left\{ \left(\frac{\pi^2 E I_y}{(L_{LT})^2} \right) \left[G I_t + \frac{\pi^2 E I_w}{(L_{LT})^2} \right] \right\}} = \beta_b Z_p f_{cr,b}$$

I_t = torsional constant = $\sum b_i t_i^3 / 3$ for open section
 I_w = warping constant
 I_y = moment of inertia about weaker axis
 r_y = radius of gyration about weaker axis
 L_{LT} = effective length for lateral torsional buckling (Clause 8.3)
 h_f = centre-to-centre distance between flanges
 t_f = thickness of flange
 G = shear modulus

So our next step is to find out the value of M_{cr}, which is given through this expression M_{cr}. This is available in the code in clause 8.2.2.1 the details are given, so from that the value of M_{cr} can be found. Now where here the different parameters are defined like It the torsional

constant are defined as summation $\sum b_i t_i^3$ and I_w is a working constant, I_y is the moment of inertia about weaker section, r_y is the radius of gyration about weaker section, L_{LT} is the effective length for lateral torsional buckling and h_f is a center center distance between flanges, t_f is the thickness and G is the shear modulus.

So from these parameters, I can find out the value of M_{cr} and (18:24) to mention here that this elastic critical buckling moment depend on the few factors like one is L_{LT} that is effective length of lateral torsional buckling then the shape of the section for which, the I_y value and other value will be dependent right. So depending on those factor finally we can find out the value of M_{cr} , okay.

(Refer Slide Time: 18:59)

$f_{cr,b}$ = extreme fiber bending compressive stress corresponding to elastic lateral buckling moment and is given by

$$f_{cr,b} = \frac{1.1\pi^2 E}{(L_{LT}/r_y)^2} \sqrt{1 + \frac{1}{20} \left(\frac{L_{LT}/r_y}{h_f/t_f} \right)^2}$$

For different values of L_{LT}/r_y & h_f/t_f corresponding values of $f_{cr,b}$ is given in **Table 14, IS 800:2007**

Values of f_{bd} can also be found from **Table 13(a) and 13(b), IS 800: 2007** corresponds to different values of $f_{cr,b}$ and f_y

Md

Also we can find out the $F_{cr,b}$ value, which is the extreme fiber bending compressive stress corresponding to that elastic lateral buckling moment. So $F_{cr,b}$ value also we can find out from this expression that is $1.1 \pi^2 E$ by L_{LT} by r_y whole square into root over $1 + 1$ by 20 into L_{LT} by r_y by h_f by t_f whole square. So $F_{cr,b}$ value can be found from this expression otherwise very quickly, we can find out the F_{bd} value from table, the code as made us, easy to find out the value of F_{bd} without calculating through that those expressions which are given. So what we can do? We can find out the value of L_{LT} by r_y and h_f by t_f .

So once we find the L_{LT} by r_y and h_f by t_f , we can find out the value of $F_{cr,b}$, which is given in table 14. So in table 14 correspondings to these value we can find out the extreme fiber bending compressive stress $F_{cr,b}$. So once we find $F_{cr,b}$ value then corresponding to that $F_{cr,b}$ and F_y we can find out the design compressive bending stress. So design bending

compressive stress F_{bd} can be found from this F_{bd} from table 13(a) or 13(b) depending on the type of member section, okay. So the F_{bd} value once we get we can find out the value of M_d . So this is how we can find out the section, sorry bending moment.

So in today's lecture what we can see that due to unsupported uhh unsupported length of the member laterally unsupported length of the member. There is a chance of lateral torsional buckling if the member the cross section is not stocky in nature then lateral torsional buckling may play an important role and because of lateral torsional buckling before developing the full bending stress, the member may fail and therefore, we have to find out what is the lateral torsional buckling moment. So what we have done we have calculate the lateral torsional buckling moment.

Then we have found the extreme bending stress due to this lateral torsional buckling moment F_{crb} and then we can find out the F_{bd} . So F_{bd} the design compressive bending stress is nothing but F_y by γ_{m0} into some reduction factor, which is called XLT. So because of the presence of lateral torsional buckling of the section, the reduction factor has to be calculated, which is call XLT and once reduction factor is calculated then we can find out the design bending stress compressive design bending compressive stress and then from that we can find out the design bending moment. The design bending strength of the section, which is the section, which is laterally unsupported can be found in this way. So in next lecture, we will go through one example then the detail of the things will be clear. Thank you.