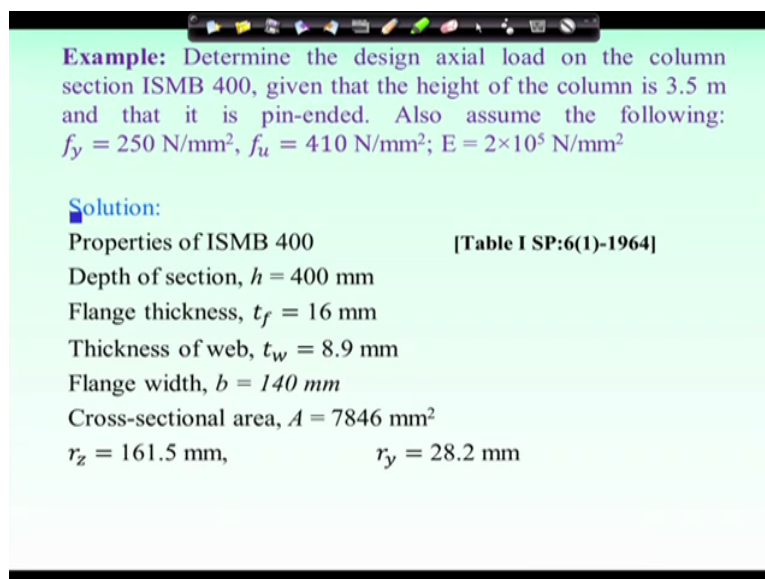


Course on Design of Steel Structures
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Lecture 30
Module 6
Design Strength of Compressive Member

Hello today I am going to discuss about the calculation of the compressive strength of a compression member. So in last lecture we have seen the design formula strength calculation formula of the compression member as per the IS code and we have seen this strength depends on three main factor one is the yield stress of the material that means the material properties, then the length of the member from which we can find out the slenderness ratio and and because of buckling the reduction of the strength occurs and then another is imperfection factor which depends on the buckling class of the member.

So three factors are introduced in the design strength calculation of the compressive member, these three factors is the slenderness ratio, then buckling class and the material properties that is yield strength. So based on that the formula has been derived which is basically similar to the British code. And today I am going to solve an workout example through which we will understand that the formulas that are used in the IS code, how to make use of those while calculating the compressive strength of a compression member.

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Example: Determine the design axial load on the column section ISMB 400, given that the height of the column is 3.5 m and that it is pin-ended. Also assume the following: $f_y = 250 \text{ N/mm}^2$, $f_u = 410 \text{ N/mm}^2$; $E = 2 \times 10^5 \text{ N/mm}^2$

Solution:

Properties of ISMB 400 [Table I SP:6(1)-1964]

Depth of section, $h = 400 \text{ mm}$

Flange thickness, $t_f = 16 \text{ mm}$

Thickness of web, $t_w = 8.9 \text{ mm}$

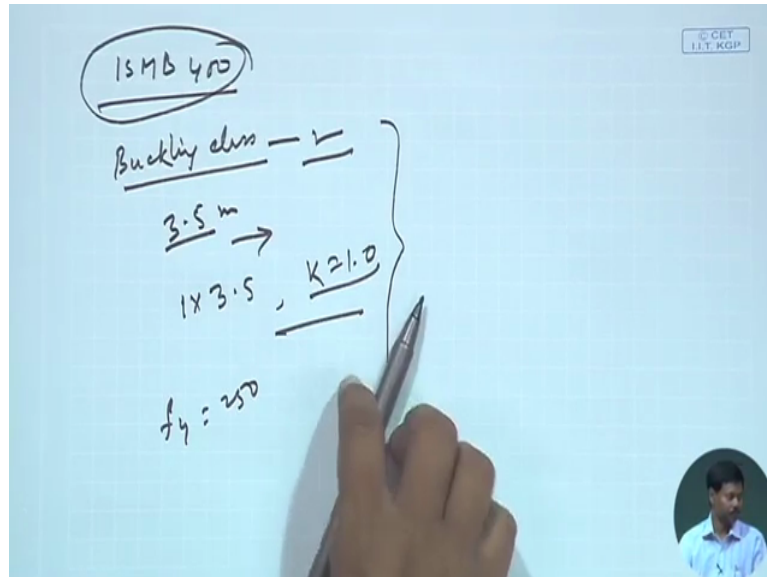
Flange width, $b = 140 \text{ mm}$

Cross-sectional area, $A = 7846 \text{ mm}^2$

$r_z = 161.5 \text{ mm}$, $r_y = 28.2 \text{ mm}$

So here the example is to determine the design axial load on the column section ISMB 400 given that the height of the column is 3.5 meter and that it is pin-ended also assume the following f_y is 250 newton per millimetre square, f_u is 410 newton per millimetre square and E is (200×10^3) newton per millimetre square.

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That means from this what we could see that ISMB 400 members are used. So from this we can find out what is the buckling class where ISMB 400 we know the thickness of the member that means thickness of the flange, width of the flange and the depth of the cross section those things we know, so from this we can find out the buckling class then another thing we can find out that is the slenderness ratio, slenderness ratio here the length is given 3.5 meter.

So we know the radius of gyration of the section ISMB 400, so from that we can find out slenderness ratio ofcourse the slenderness ratio to find out slenderness ratio we have to know the effective length. So effective length depends on the end connections, here ends are connected by pin-end joint, so effective length here will be will not be reduced that will be 1 into 3.5 because here K will be 1 as per the IS code, right.

So one is buckling class then slenderness ratio and grade of steel is f_u means $f_e 250$. So f_y value is 250 it was given. So based on these three we can find out the value of f_{cd} .

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Flange width, $b = 140 \text{ mm}$

Cross-sectional area, $A = 7846 \text{ mm}^2$

$r_z = 161.5 \text{ mm}$, $r_y = 28.2 \text{ mm}$



So if we see here that properties of ISMB 400 are obtained from SP:6 of 1964 where the h is 400 means overall depth. Then thickness of flange is 16 mm, thickness of web is 8.9 mm and width of flange b or b_f whatever we call is 140 mm. and also we can find out the cross sectional area that is 7846 millimetre square and radius of gyration about z - z axis is 161.5 mm and about y - y axis is 28.2 mm. So these are the properties of the cross sections are which are available in SP:6 in SP:6 we can find out.

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ISMB 400

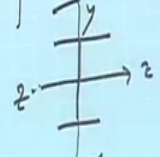
Buckling class - C

3.5 m \rightarrow 1×3.5 , $K = 1.0$

$f_y = 250$

$\left\{ \begin{array}{l} \frac{h}{b} = \frac{400}{140} = 2.86 > 1.2 \\ t_f = 16 < 40 \text{ mm} \end{array} \right\}$

$a \rightarrow z-z$ axis
 $b \rightarrow y-y$ axis

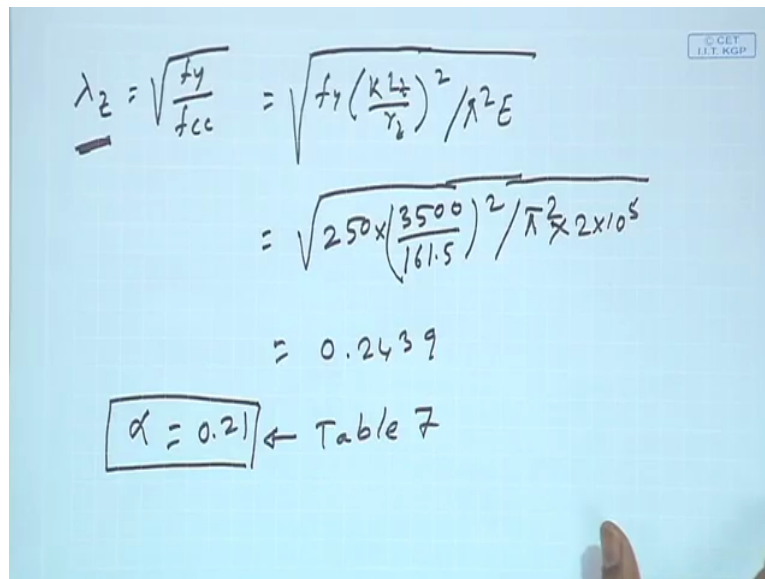


So now from this cross sectional properties we can find out h by b , here h by b is 400 by 140 which is becoming 2.86 and it is more than 1.2 and thickness of flange t_f is 16 mm which is (more) less than 40 mm. Therefore from these two we can find out the buckling class as per

table 10 from the table 10 if we see that we can use buckling class a about z-z axis and buckling class b about y-y axis, right.

So z-z axis means if this is the I section this is called z-z, this is z-z and this is y-y, right. So about z-z axis it is class a, about y-y axis it is class b, as per the table 10 definition. The buckling curve classification has been defined in table 10 from which we can take the class a or class b.

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$$\lambda_z = \sqrt{\frac{f_y}{f_{cc}}} = \sqrt{\frac{f_y \left(\frac{KL}{r_z}\right)^2}{\pi^2 E}}$$

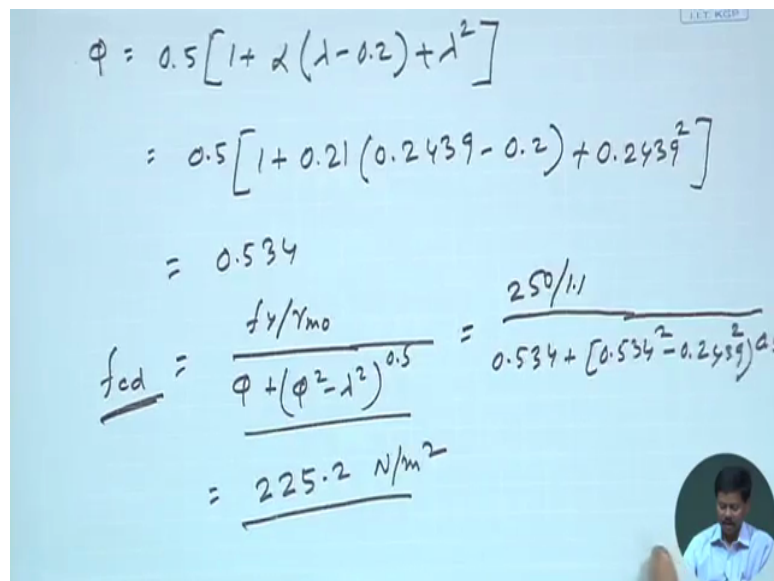
$$= \sqrt{\frac{250 \times \left(\frac{3500}{161.5}\right)^2}{\pi^2 \times 2 \times 10^5}}$$

$$= 0.2439$$

$\alpha = 0.21$ ← Table 7

Then effective length here effective length we have calculated 3.5, now the non-dimensional slenderness ratio also we can find out that is lambda z that will be fy by fcc, so this we can find out fy by fcc we know that will be fy into KLz by rz whole square by pi square E, right. So we can find out this value as 250 into KL is 3500 and rz was 161.5 by pi square into E means 2 into 10 to the power 5, right. so from this we can find out 0.2439 lambda z, lambda z we can find out, also the buckling class about z-z axis was a and for that alpha 0.21 that is found from table 7, from table 7 we can find alpha as 0.21 according to the buckling class a.

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The image shows handwritten calculations on a whiteboard. The first part calculates the value of phi using the formula $\phi = 0.5[1 + \alpha(\lambda - 0.2) + \lambda^2]$. Substituting $\alpha = 0.21$ and $\lambda = 0.2439$, the calculation proceeds to $\phi = 0.534$. The second part calculates the design compressive stress f_{cd} using the formula $f_{cd} = \frac{f_y/\gamma_{m0}}{\phi + (\phi^2 - \lambda^2)^{0.5}}$. Substituting $f_y/\gamma_{m0} = 250/1.1$, $\phi = 0.534$, and $\lambda = 0.2439$, the final result is $f_{cd} = 225.2 \text{ N/mm}^2$. A small inset video of a person is visible in the bottom right corner of the whiteboard image.

$$\begin{aligned}\phi &= 0.5[1 + \alpha(\lambda - 0.2) + \lambda^2] \\ &= 0.5[1 + 0.21(0.2439 - 0.2) + 0.2439^2] \\ &= 0.534 \\ f_{cd} &= \frac{f_y/\gamma_{m0}}{\phi + (\phi^2 - \lambda^2)^{0.5}} = \frac{250/1.1}{0.534 + [0.534^2 - 0.2439^2]^{0.5}} \\ &= 225.2 \text{ N/mm}^2\end{aligned}$$

So we can find out now phi as 0.5 into 1 plus alpha lambda minus 0.2 plus lambda square, here lambda means lambda z because about z-z we are doing. So 0.5 into 1 plus alpha means here 0.21 and lambda z we found already 0.2439 so putting all the values we can find out phi as 0.534.

So fcd the design compressive stress of the member now can be found as f_y by γ_{m0} by phi plus phi square minus lambda square whole to the power 0.5, so this is basically stress reduction factor. So if we put those value we can find out the compressive stress of the member about z-z axis. So phi is 0.534 plus 0.534 square minus lambda is 0.2439 square whole to the power 0.5, right.

So if we calculate this value we can get 225.2 newton per millimetre square, right. So fcd value about z-z axis we are getting as 225.2 newton per millimetre square millimetre square.

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y-y axis

$$\alpha = 0.34$$
$$\lambda_y = \sqrt{\frac{f_y}{f_{cc}}}$$
$$= \sqrt{250 \times \left(\frac{3500}{28.2}\right)^2 / (\pi^2 \times 2 \times 10^5)}$$
$$= 1.3968$$

Similarly I have to find out the design compressive stress about y-y axis about y-y axis because the member may fail if this this is the I section it may fail about z-z axis, it may fail about y-y axis. So the about weaker section it will fail so we have to find out which one is the weaker one.

So in this case y-y axis, the alpha value will be 0.34, alpha value will be 0.34. Now lambda y we can find out as f_y by f_{cc} , so if we put this value f_y by f_{cc} that we will get 250 into $K L_y$ is 3500 by r_y square, r_y is 28.2 by π square into E is 10 to the power 5, right. So from this I can find out lambda y as 1.3968. So the slenderness ratio about y-y axis we can find out not slenderness ratio lambda y sorry.

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$\alpha = 0.34$

$$\lambda_y = \sqrt{\frac{f_y}{f_{cc}}}$$
$$= \sqrt{250 \times \left(\frac{3500}{28.2}\right)^2 / (\pi^2 \times 2 \times 10^5)}$$
$$= 1.3968$$
$$\phi = 0.5 \left[1 + 0.34 (1.3968 - 0.2) + 1.3968^2 \right]$$
$$= 1.679$$

Then we can find out phi phi as 0.5 into 1 plus alpha alpha is 0.34 into lambda is 1.3968 which we have calculated minus 0.2 plus lambda y 1.3968 square, okay. So from this phi value can be found as 1.679, right. So phi value we could calculate from here, now we can find out the value of fcd.

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$$f_{cd} = \frac{f_y / \gamma_{m0}}{\phi + (\phi^2 - \lambda^2)^{0.5}}$$

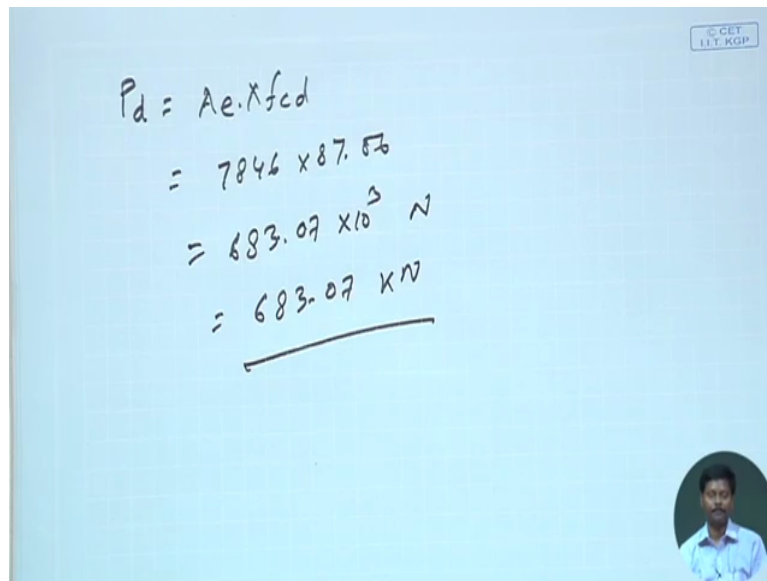
$$= \frac{250 / 1.1}{1.679 + [1.679^2 - 1.3968^2]^{0.5}}$$

$$= 87.06 \text{ N/mm}^2$$

$\frac{z-z}{y-y} \rightarrow \left. \begin{array}{l} 225.2 \\ 87.06 \end{array} \right\} 87.06$

So fcd value will be fy by gamma m0 by phi plus phi square minus lambda square to the power 0.5. So if we put this value 250 by 1.1 by 1.679 plus 1.679 square minus 1.3968 square whole to the power 0.5, from this we can find out 87.06 newton per millimetre square, right. So from this we can find out the design strength that is fcd we get about z-z axis about z-z axis we get 225.2 and about y-y axis we get 87.06. So the design compressive stress will be taken as 87.06 because it will fail about y-y axis first.

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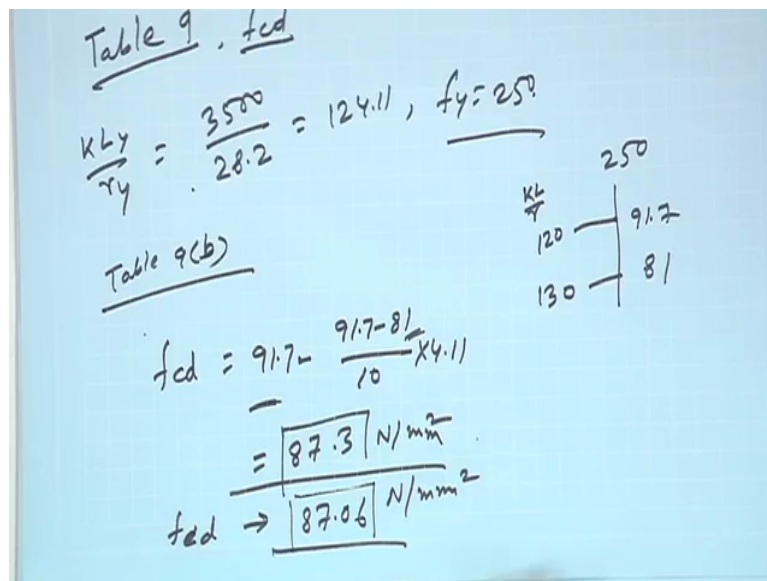
Handwritten calculation for design compressive strength P_d :

$$\begin{aligned} P_d &= A_e \cdot \sigma_{cd} \\ &= 7846 \times 87.06 \\ &= 683.07 \times 10^3 \text{ N} \\ &= \underline{683.07 \text{ kN}} \end{aligned}$$

A small circular inset photo of a man is visible in the bottom right corner of the slide.

Therefore P_d value the design strength P_d can be calculated as the area A_e into σ_{cd} that is 7846 into 87.06 is equal to 683.07 into 10 cube newton or 683.07 kilonewton. So this is how we can find out the design compressive strength of the member, right.

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Handwritten calculation for design compressive strength f_{cd} using Table 9:

Table 9, f_{cd}

$$\frac{KL}{r_y} = \frac{3500}{28.2} = 124.1, \quad f_y = 250$$

Table 9(b)

$\frac{KL}{r}$	f_{cd} (N/mm ²)
120	91.7
130	81

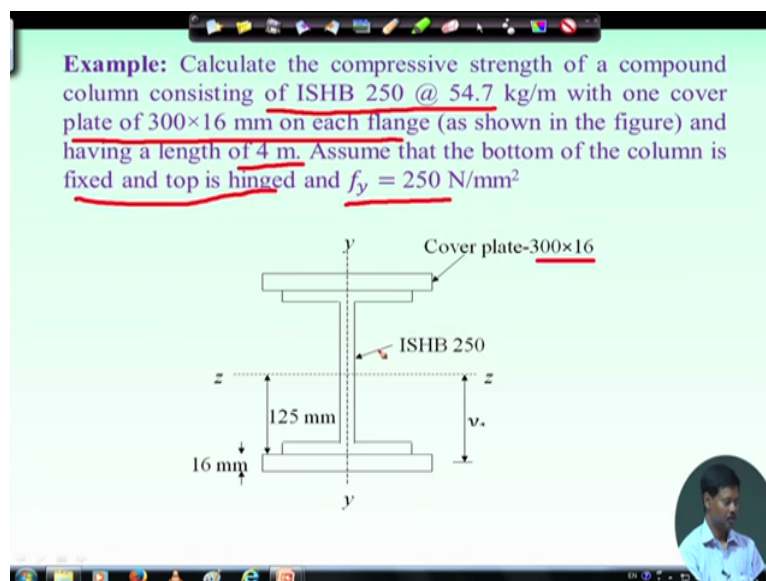
$$f_{cd} = 91.7 - \frac{91.7 - 81}{10} \times 4.11$$
$$= 87.3 \text{ N/mm}^2$$
$$f_{cd} \rightarrow \underline{87.06 \text{ N/mm}^2}$$

So using the formula we can find out the design compressive strength of the member of the given member and we could see that about y-y axis it is going to fail first that means about y-y axis is the weaker section and the same can be made by the use of the table 9, means in place of calculation of all these by f_{cd} we simply can find out using table 9 from where f_{cd} we can get.

That we can see for example here say $K_L y$ by r_y we can find out 3500 by 28.2, so we can find out 124.11 and we know f_y is equal to 250 and for plus b we can use table 9b for plus b, for plus c table 9c, accordingly we can make. So if we use table 9b we can find out f_{cd} value as because if we go through the table 9b we can see for f_y 250 the values are given, say for 120 KL by r for 120 this values are given as 91.7 and for 130 KL by r these values are given as 81.

So f_{cd} for 124.11 we can find out as 91.7 minus 91.7 minus 81 by 10 into 4.11, this is for 120 and this is for 130. So for 124.11 we can find out the value as like this, so these values are coming 87.3 newton per millimetre square and f_{cd} value by using numerical equations we could get 87.06 which are very closed to the earlier one. So either way we can do that either we can use the table and find out the value of f_{cd} or we can use the formula and we can find out the value of f_{cd} , both the way we can do.

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Another example we will go through quickly similar example but this is a built-up section, this example we will go through. Here we need to calculate the compressive strength of a compound column consisting of ISHB 250 with one cover plate on each flange of 300 by 16 as shown in the figure and having a length of 4 meter, assume that the bottom of the column is fixed and top is hinged, so boundary conditions are given and f_y is 250.

So here the cover plate of 300 by 16 are used in both the flange, right and the columns cross sections is ISHB 250 at 54.7.

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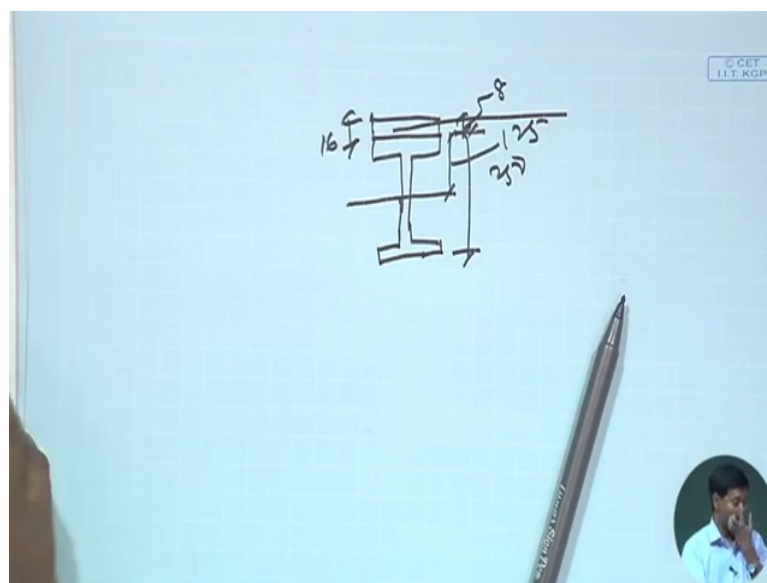
Solution:
Properties of ISHB 250 @ 54.7 kg/m : [Table I SP:6(1)-1964]
C/S area, $A = 6971 \text{ mm}^2$
 $I_{zz} = 7983.9 \times 10^4 \text{ mm}^4$
 $I_{yy} = 2011.7 \times 10^4 \text{ mm}^4$
 $t_f = 9.7 \text{ mm}$
a) Determination the radii of gyration for the compound section:
 $I_z \text{ for plates} = 2[I_a + A_p y_1^2]$
$$= 2 \left[\frac{300 \times 16^3}{12} + 300 \times 16 \times (125 + 8)^2 \right]$$
$$= 17001.92 \times 10^4 \text{ mm}^4$$

Total $I_z = 7983.9 \times 10^4 + 17001.92 \times 10^4$
$$= 24985.82 \times 10^4 \text{ mm}^4$$

So if we look back to the SP:6 we can find out the relevant properties of ISHB 250 at 54.7 kg per meter. Here from SP:6 we can find out the relevant properties like cross section area A, then I_{zz} and I_{yy} and thickness of flange, right. So from this I can find out first I_z for plate and total I_z of the section. Similarly I can find out I_y of the plate and total I_y of the section. So when calculating the I_z for plate we can see that we can consider 2 plates are there, so 2 into I_a about its own axis plus area of plate into Y_1 square, right.

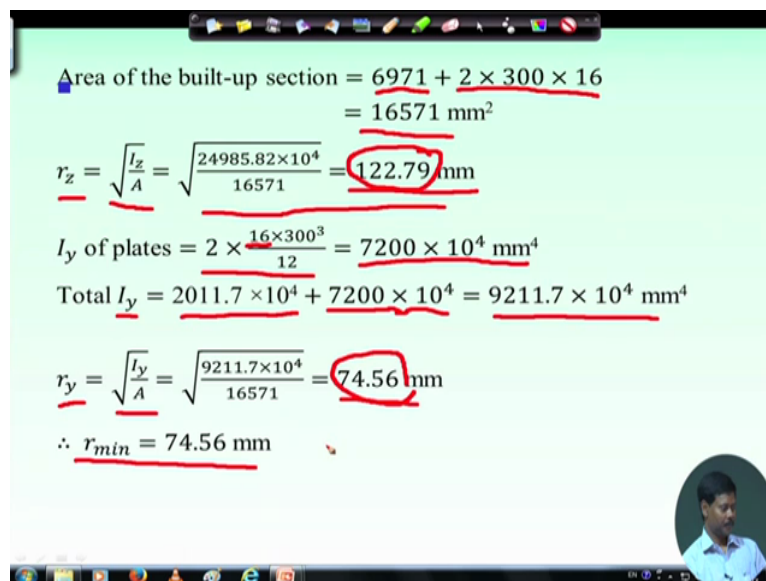
So 2 into I_a means about its own axis it will be 300×16^3 by 12 so 300 into d cube means 16^3 cube by 12 plus A is 300 into $16 \times r$ square r square is 125 plus 8 because 16 is the width.

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So if we see the section this total depth of the ISHB (225) 250 is 250, so from this it will be 125 and if I provide a plate of 16 mm, so the Cg of this will be 125 plus 8, right. So I_z for plate can be calculated from this formula which will be 17001.92 into 10 to the power 4 millimetre 4 . So total I_z will be the I_z of the member itself I section plus I_z of the plate. So summing of these two we can get total I_z as this, right. So to find out the radius of gyration about z-z section we need to know what is the I_z about z-z section.

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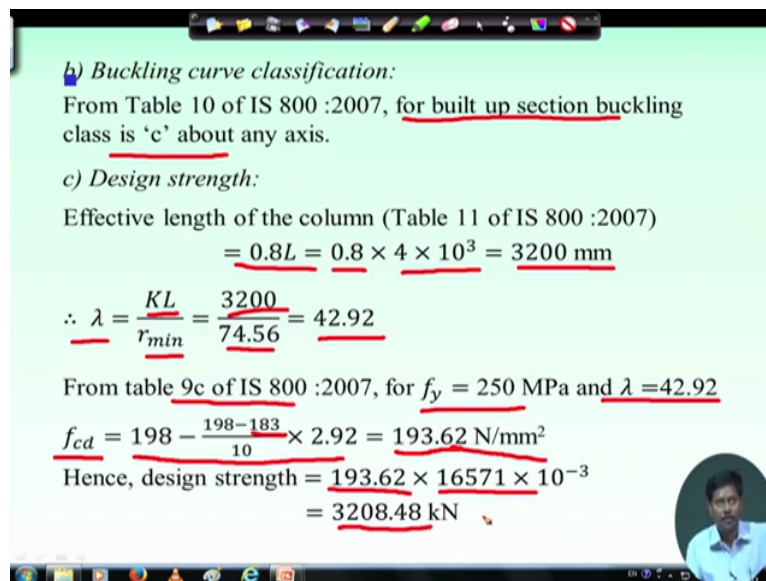
$$\begin{aligned} \text{Area of the built-up section} &= 6971 + 2 \times 300 \times 16 \\ &= 16571 \text{ mm}^2 \\ r_z &= \sqrt{\frac{I_z}{A}} = \sqrt{\frac{24985.82 \times 10^4}{16571}} = 122.79 \text{ mm} \\ I_y \text{ of plates} &= 2 \times \frac{16 \times 300^3}{12} = 7200 \times 10^4 \text{ mm}^4 \\ \text{Total } I_y &= 2011.7 \times 10^4 + 7200 \times 10^4 = 9211.7 \times 10^4 \text{ mm}^4 \\ r_y &= \sqrt{\frac{I_y}{A}} = \sqrt{\frac{9211.7 \times 10^4}{16571}} = 74.56 \text{ mm} \\ \therefore r_{min} &= 74.56 \text{ mm} \end{aligned}$$

Similarly we can find out I_{yy} also, now the area of the built-up section also we can find out, area of the section was 6971 that is rolled section area ISHB 250 and area of the plate is 300 into 16 into 210. So total area of the built-up section will be 16571 millimetre square. So we can find out, the radius of gyration about z-z axis that is I_z by A square root of that, so after calculation we can find out r_z as 122.79 millimetre.

Similarly I_y of the plate also we can find out I_y of the plate will be 2 into bd^3 by 12 , here b is 16, d is 300. So this will become 7200 into 10 to the power 4 millimetre to the 4 and total I_{yy} also we can find out I_y of the rolled section plus I_y of the plate, so I_y of the built-up section will be 9211.7 into 10 to the power 4 millimetre to the 4 . So I_y this is how we can find out.

Now r_y can be found as square root of I_y by A , so that is becoming 74.56. Now the member will fail about the least radius of gyration or here r_z we found as 122.79 and r_y we found 74.56. So it will fail about y-y because minimum r_y is 74.56 which is about y-y axis.

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b) *Buckling curve classification:*
From Table 10 of IS 800 :2007, for built up section buckling class is 'c' about any axis.

c) *Design strength:*
Effective length of the column (Table 11 of IS 800 :2007)
 $= 0.8L = 0.8 \times 4 \times 10^3 = 3200 \text{ mm}$

$$\therefore \lambda = \frac{KL}{r_{min}} = \frac{3200}{74.56} = 42.92$$

From table 9c of IS 800 :2007, for $f_y = 250 \text{ MPa}$ and $\lambda = 42.92$

$$f_{cd} = 198 - \frac{198-183}{10} \times 2.92 = 193.62 \text{ N/mm}^2$$

Hence, design strength = $193.62 \times 16571 \times 10^{-3}$
 $= 3208.48 \text{ kN}$

So now we will consider r_{min} and we will find out the radius of gyration, ok. Now radius of gyration we can find out λ as KL by r_{min} and here KL we have to find out, now KL is as per the distant condition it will be $0.8L$, so 0.8 into 4 into 4000 so 3200 millimetre will be the effective length of the column. So effective length of the column is 3200 and r_{min} is this, so λ will be 42.92 , right.

So another thing is that buckling class we have to find out, from table 10 we know for builtup section buckling class will be c about any axis, therefore according to buckling class c we can find out the f_{cd} value from table 9c that is for f_y is equal to 250 , λ is equal to 42.92 , f_{cd} value can be found as this. Here if we see the table 9c of IS 800:2007 we can see that for λ is equal to 40 this value is coming 198 and λ is equal to 50 this value is coming 183 .

So by interpolating these values we can find out f_{cd} value as 193.62 , right. So f_{cd} value we can find out from table 9c directly as 193.62 . So the design strength we can find out the f_{cd} value into effective area that is becoming this 3208.48 kilonewton, thus the design strength of this built up column is becoming 3208.48 kilonewton. So from this what we could see that when built up section is used the buckling class will be c in any direction.

Therefore we do not need to calculate f_{cd} value for both the direction, because buckling class is same for both the direction. So we will consider r_{min} in which direction it is coming, the r_{min} will be the failure criteria. So according to means about the r_{min} or r_{min} in which (\cdot) (26:48) is coming about that axis failure will happen. So we will

straight calculate the r minimum value and then we will find out λ and then according to buckling class we will find out the f_{cd} value and once we get f_{cd} value we can find out the value of compressive force which can be carried by the that particular member, ok.

So this is how we can calculate the compressive strength and compressive force of a compression member using IS 800:2007, ok this is all about the calculation of compressive strength of the member, thank you.