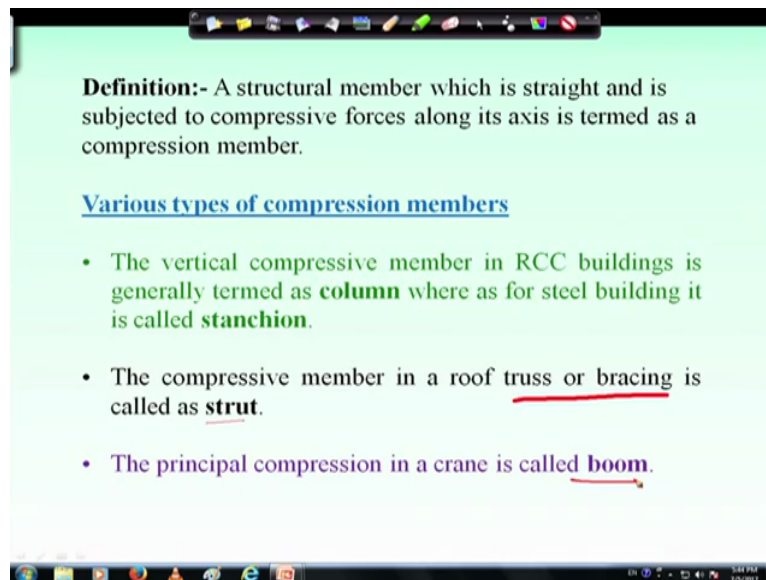


Course on Design of Steel Structures
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Lecture 29
Module 6
Compression Members

Hello today I am going to start a new module which will deal with compression member, a structure member when subjected to axial force only then such type of structure member is called compression member.

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Definition:- A structural member which is straight and is subjected to compressive forces along its axis is termed as a compression member.

Various types of compression members

- The vertical compressive member in RCC buildings is generally termed as **column** where as for steel building it is called **stanchion**.
- The compressive member in a roof truss or bracing is called as **strut**.
- The principal compression in a crane is called **boom**.

Now different type of structures (column) compression members are termed as different way like in case of RCC building such type of compression member is called column, column is or this in case of RCC building this compression member or column is basically vertical member which carries the load from beam or from the floor and transferring from upper floor to lower floor.

Similarly in case of steel building this type of compression member is called stanchion again the compressive member in a roof truss or bracing is called strut. Similarly the principal compression in a crane is called boom so what we could see that the same compression member are termed different in a in different name for different cases.

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MODES OF FAILURE OF COLUMN

1. Squashing
2. Local buckling
2. Overall flexural buckling
4. Torsional buckling
5. Flexural-torsional buckling

1. Squashing:
If the length of the compression member is relatively small then the column will be able to attain its full strength or 'squash load'. Squash load = yield strength $\times$ cross sectional area.

2. Local buckling:
The individual elements of a compressive member such as web, flange etc. may buckle locally.

Now when we will go for calculation of strength of the compression member or when we will go to design a compression member then we have to see that what are the type of failure come into picture for a compression member, depending on the type of failure we have to find out what are the strength can be carried by that type of member that particular member.

So in case of compression member there are different type of failures occurs out of them one is called squashing this squashing is basically happens when the length of the compression member is quite less compared to its transverse direction cross section area dimension like say for example if a member is like this I am taking compression load then in such case of member the crushing will come into picture mainly and full strength will attain at its yield strength and therefore the the failure load can be calculated in strength into the cross sectional area failure load will be calculated simply the yield strength into its cross sectional area.

So in such cases we could see that the member fails due to crushing of the material that is what one type of failure we could observe.

Another failure is the local buckling local buckling happens due to its cross sectional configuration in case of steel building we know that local buckling means the member we use different type of rolled sections steel rolled sections. Say for example we oftenly used the channel section, so in such case what will happen that due to compression this web may buckle individually, this flange may buckle individually or some other part of the member may buckle which is called local buckling. So it has to be taken care while calculating the compressive strength of the member.

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3. Overall flexural buckling:
Failure due to flexural buckling occurs due to excessive deflection of the member in its plane of weaker principle axis.

4. Torsional buckling:
Torsional buckling failure occurs due to torsional moment. The member gets twisted about the shear center in the longitudinal axis.

5. Flexural-torsional buckling
The flexural-torsional buckling occurs when the member bends and twists simultaneously. Such type of failure happens with unsymmetrical cross sections

Then another type of failure will be that overall flexural buckling. When the member length along its axis is quite high compared to its cross sectional dimension then such type of buckling occurs which is called flexural buckling. Say for example we have a long column means compared to its lateral dimension, so in such cases it may buckle in this way. So before going to fail due to crushing it may fail due to buckling.

So such type of buckling phenomena has to be taken care while considering the strength of the compression member. Again if we see if we see the cross section say for example this if the cross section is something like this then we can see it may buckle about this axis, it may buckle about this axis. So we have means in which direction, it will first buckle it will buckle about the weaker direction weaker section.

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3. Overall flexural buckling:
Failure due to flexural buckling occurs due to excessive deflection of the member in its plane of weaker principle axis.

4. Torsional buckling: DELT
Torsional buckling failure occurs due to torsional moment. The member gets twisted about the shear center in the longitudinal axis.

5. Flexural-torsional buckling DELT
The flexural-torsional buckling occurs when the member bends and twists simultaneously. Such type of failure happens with unsymmetrical cross sections

The slide includes a diagram of a channel section labeled 'DELT' with arrows indicating deflection and twist. A small inset video shows a person speaking.

So in this case it will buckle about this then another failure may come due to torsional buckling torsional buckling failure occurs due to torsional moment, the member get twisted about the shear center in the longitudinal axis. So torsional buckling may occur may be in case of angle section or channel section depending on the type of load means acting in a particular place means load will be compressive but where it is acting whether it is acting in the member axis or it is C_g depending on that the torsion will come into picture torsional moment and because of that torsional buckling may occur.

Another scope of buckling is called flexural-torsional buckling. This is nothing but the buckling which occurs when the member bends and twists simultaneously, that means member will bend again it will twist means it may twist like this may be like this it may twist such type of failure happens generally in case of unsymmetrical cross section. So unsymmetrical cross section means say for example channel section it is symmetrical of this direction but it is unsymmetrical of this direction. So in which direction it is unsymmetrical depending on that we have to consider whether it is unsymmetrical cross section or symmetrical cross section and accordingly the torsional buckling will come into picture.

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Classifications of Compression Member depending on the length

- **Short compression member:** The failure stress will be equal to the yield stress and buckling will not occur.
- **Long compression member:** Stress will occur due to buckling of the member.
- **Intermediate compression member:** Failure will occur due to the combined effect of crushing and buckling.

Now while calculating the compressive strength of a compression member we have to find out means what type of effect is coming on a particular compressive member. It depends this effect depends in length of the compression member like one is called short compression member, short compression member means what I told earlier that suppose a member is length is quite short compared to its width and thickness means if its width and thickness is substantial with respect to moment that means the L by R ratio is quite low in that case the failure stress will be equal to the yield stress and there will be no buckling, buckling will not happen in this case.

So here it will fail due to yielding of the material so this is happens when short compression members are means when the members have short compression.

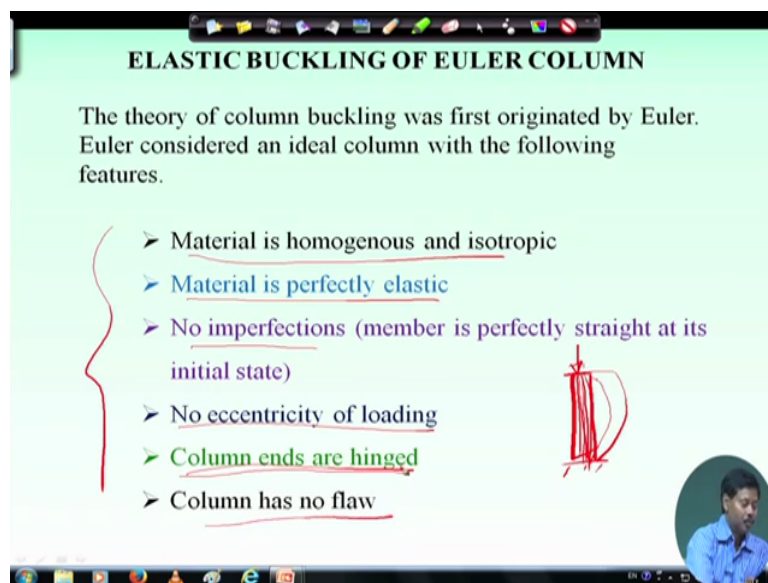
And another type of member which is called long compression member. In this case stress will occur due to buckling that means in this case the means may be its height is like this or length is like this but its cross section is quite less, right. So in this case buckling may happen before yielding of the stress that is why we need to consider whether it is long compression member or not and accordingly we have to find out what type of means stress is developing due to buckling or due to due to yielding accordingly the compressive strength of the member will be consider.

Another case is intermediate (compressive) compression member. In case of intermediate compression member failure occur due to the combined effect of crushing and buckling. Intermediate compression means in practise most of the members are considered as

intermediate compression member because in this case the member will undergo both the stress one is due to crushing, due to compression of the member it will shorten its length and it will crush and another is due to its length it will buckle something like this.

So buckling stress will come into picture as well as crushing stress will come into picture. So both the effect we have to consider and we have to find out the failure strength of the member and most of the cases the compression member acts as intermediate compression member where both the effects will be will have to be taken care.

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ELASTIC BUCKLING OF EULER COLUMN

The theory of column buckling was first originated by Euler. Euler considered an ideal column with the following features.

- Material is homogenous and isotropic
- Material is perfectly elastic
- No imperfections (member is perfectly straight at its initial state)
- No eccentricity of loading
- Column ends are hinged
- Column has no flaw

Now for finding out the compressive strength of a member as we see that one is the crushing value we have to find out that means yield strength we have to find out the yield strength of a particular material and accordingly the strength of the compression member can be calculated.

Another is it may buckle so due to buckling what will be the buckling force and what will be the stress that we have to find out and for that Euler has considered a ideal column and has find out a critical load for buckling that critical load which has been obtained due to buckling are derived on the basis of this few assumptions. That is one is the material is homogeneous and isotropic, that means the material along the throughout its length will be homogeneous and isotropic, there will be no change of material properties.

Then material is perfectly elastic that means upto elastic limit this buckling theory will be will be considered this will be true for upto elastic limit. Then no imperfection that means

member will be perfectly straight as its initial state and there will be no flaw of the geometry and material across the member.

Then there should not be any eccentricity of loading that means the (cons) in case of Euler theory the derivation has been made considering the concentric loading that means there will be no eccentric loading acting on the member. And another is column ends are hinged while developing this theory it has been consider that column ends are hinged and column has no flaw.

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ELASTIC BUCKLING OF EULER COLUMN

The governing differential equation is

$$\frac{d^2y}{dx^2} + \frac{P_{cr}}{EI} y = 0$$

The lowest value of the critical load

$$P_{cr} = \frac{\pi^2 EI}{l^2}$$

$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{Al^2}$

$$\sigma_{cr} = \frac{\pi^2 E r^2}{l^2} = \frac{\pi^2 E}{(l/r)^2} = \frac{\pi^2 E}{\lambda^2}$$

Thus, the critical stress is inversely proportional to slenderness ratio (λ)

With these assumptions Euler has with these assumptions Euler has suggested a buckling theory which is given here that is if a load compressive load is acting on this member along this member then and if there is a compressive force P and if at a if buckle happens like this then at a distance of x the displacement will be y and the governing differential equation will be $d^2y/dx^2 + P_{cr}/EI \cdot y = 0$, P_{cr} is nothing but the critical load critical load which will come means which will be derived, right.

That P_{cr} is the critical load which can be found from this governing differential equation and from the governing differential equation the lowest value can be found as like this the P_{cr} has $\pi^2 EI / l^2$, where l is the effective length effective length means I will come later because length between the end of the column will be the total length, this is total length.

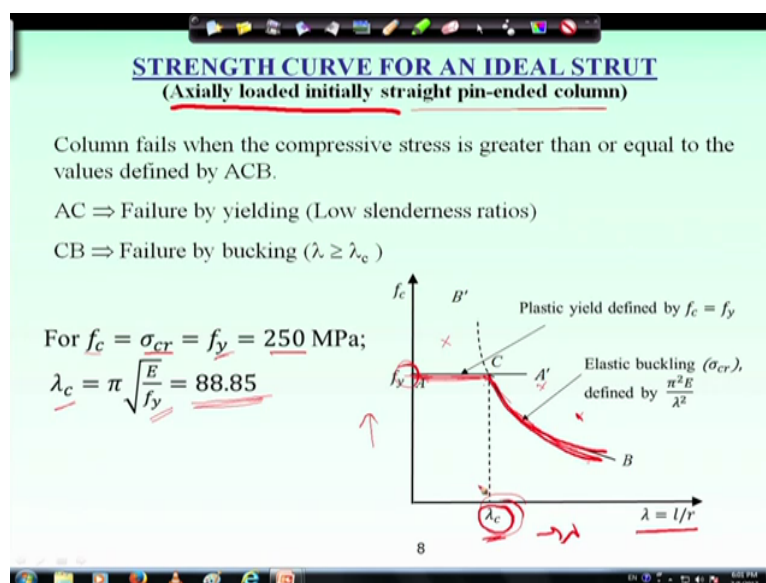
Now effective length will nothing but the the length where the two moment contraflexures are occurring distance between that two like in case of if it is fixed it will buckle like this, so

effective length will be this this is l , right. So P_{cr} will be $\pi^2 EI$ by l^2 , where l is the effective length and EI is the modulus of rigidity. Therefore the sigma shear the critical stress can be found as P_{cr} by A , where A is the cross sectional area of the column and if we use the value of P_{cr} here then it will be $\pi^2 EI$ by Al^2 .

So further if I write I as I by A as we can write I by A as r^2 , so sigma shear value will be $\pi^2 E r^2$ by l^2 and then again if we derived it will be $\pi^2 E$ by l by r whole square, which is $\pi^2 E$ by λ^2 , λ means λ is equal to l by r , λ is nothing but the slenderness ratio, r is the radius of gyration and radius of gyration means minimum radius of gyration. In two direction radius of gyration will occur and about minimum radius of gyration it will fail first that is why the minimum radius of gyration will be consider.

So sigma shear the critical stress can be found as like this $\pi^2 E$ by λ^2 , where λ is equal to l by r , so we can see that critical stress is inversely proportional to the slenderness ratio or inversely proportional to the its length, right. So critical stress will be increasing if the value of λ is less or reversely I can say the critical stress will be less if the slenderness ratio will be more. So the critical stress using Euler buckling theory can be found from this, which will be used for deriving the compressive strength of the column.

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Now for an ideal strut the strength curve of a column can be derived if the strut is axially loaded and initially straight with pin-ended then this can be derived in this way where x axis will be the slenderness ratio that is l by r and y axis will be the compressive strength of the of

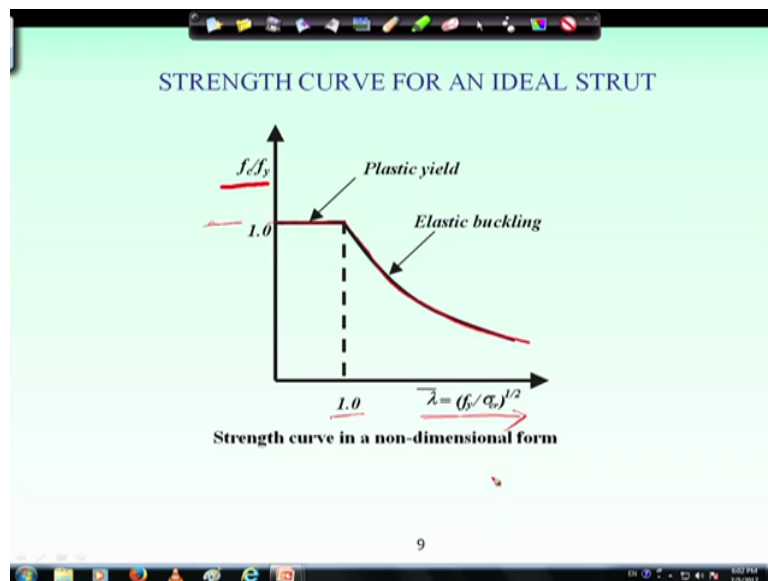
the material. So here we see the path is varying from A to C and then C to B, right. So column fails when the compressive strength is greater than or equal to the values defined by ACB that means this is the path defined and if column stress is coming somewhere here or here that means it is failed.

So if the column stress is going to be greater than the stress defined by this path ACB then I can say that column is going to fail and this AC is basically failure by yielding and if we consider low slenderness ratio then failure may happen due to yielding and failure may happen due to buckling if for high slenderness ratio that is due to this and you see failure will happen due to buckling if λ is greater than λ_C , if λ this is λ if λ is greater than λ_C , then it will come due to means the failure may come due to buckling, right.

So this is basically plastic yield and this is defined by f_c is equal to f_y and this is defined by $\pi^2 E / \lambda^2$, that is elastic buckling stress. And if we consider that f_c is equal to σ_{shear} is equal to f_y that means the yield stress if we consider yield stress as σ_{shear} critical stress then from that I can find out λ_C as is equal to E / π^2 into root of E / f_y is equal to 88.85, it is a constant. This is constant for a particular this 88.85 will be for a particular value of E and f_y where E is equal to 2×10^5 to the f_y and f_y is equal to 250, for this value λ_C value could be found as like this however for other steel this λ_C value will be different.

So what we could see from this curve that if the λ value slenderness ratio value become more than 88.85 then it will fail by elastic buckling and if it is less than that it will fail by plastic yield due to plastic yield it will fail.

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Now the same can be written in a non-dimensional form as well, which is shown here where in y axis it is f_c by f_y and it will be 1 because where f_c and f_y will be 1 here means will be equal here and λC will be 1 here means λ bar will be 1 where λC λ bar will be 1 and in this direction in x direction λ bar which is square root of f_y by σ_{cr} have been plotted.

So this curve will be this curve will be elastic buckling and this will be plastic yield. So this is how the strength curve for an ideal strut can be developed by the Eulers theory.

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Factors affecting strength of a compressive member

- > Material property of the member
- > Length of the member
- > Cross sectional configuration
- > Support conditions
- > Imperfections
- > Residual stresses

The **imperfections** include the following:

- The material not being isotropic and homogenous
- Geometric variations of columns
- Eccentricity of load

Handwritten notes on the right side of the slide include: f_{cd} , f_y , and a diagram of a column with a square cross-section and a circular load applied at an eccentricity.

But this is this cannot be applied for a practical case because in case of practical it is there are certain parameters which will effect on the compressive strength of the member. Therefore we will consider means we will consider a separate means a different formula however this formula is based on this elastic means this Euler buckling theory as well as some other factor also has been included, so that we will come in next.

Now if we see the factors which are going to effect on the strength of compressive member we can see that first is the material property of the member. So the compressive strength if we see the σ_{fcd} if I see if I say f_{cd} compressive strength f_{cd} , the design compressive strength f_{cd} if we say it depends on f_y , that means yield strength of the member that means it depends on the material property of the member, this is one.

Another factor is the length of the member because we have seen the Euler critical load is inversely proportional to the radius of gyration and there by the length. So if length is more than definitely the compressive strength will be means compressive load carrying capacity will be less, so it effects on the member as well so this has to be taken consideration.

Then that means this is due to overall buckling and this is due to crushing, and this cross sectional configuration cross sectional configuration means in case of RCC member there is no problem because there will be no local buckling because generally in case of RCC member either rectangular section, square section, or circular section we use in general. But in case of steel member we use different type of built up section say for example built up section or rolled section, this is a built up section we are using channel facing each other channel face to face or we can make some I section also say I section also we can use.

So here what we can see that due to cross sectional configuration local buckling of the flange or web may happen so that has to be taken into consideration.

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Factors affecting strength of a compressive member

- > Material property of the member
- > Length of the member
- > Cross sectional configuration
- > Support conditions
- > Imperfections
- > Residual stresses

The **imperfections** include the following:

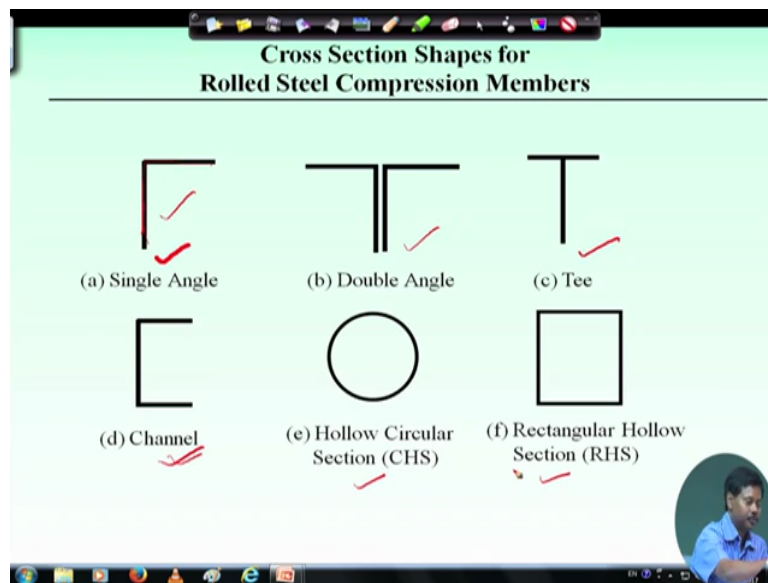
- The material not being isotropic and homogenous
- Geometric variations of columns
- Eccentricity of load

Another thing is the support condition, support condition why we should take because in case of means in case of hinge support the length will be the effective length will be simply l is equal to L , but if we see if this is a fix fix support then its buckling curve will be like this and we can we know that the l will be basically L by 2, right. So effective length is going to reduce. So as length of the member effects on the compressive strength in the member therefore support conditions also effects on that.

Next is the imperfection, now imperfection means that material may not be isotropic truly and homogeneous then geometric variation of the column may be there, that means cross sectional cross section throughout the length of the column may not be same exactly then eccentricity may not have exactly eccentricity. So these imperfections also effect on the strength on the member therefore that has to be also taken care in our design.

Another is the residual stress, if residual stresses are there in the member then the compressive strength is going to be different, so that aspects also has to be keep in mind.

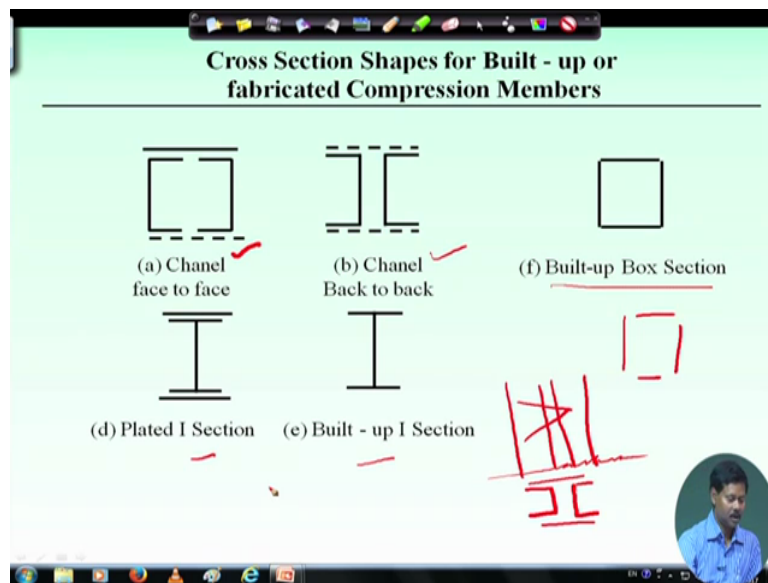
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Now coming to the cross section of the member if we see that in case of column or compression member different type of steel rolled sections which are available in the market can be used for compression member, like single angle section we can use single angle section however while using single angle section if the member means if the load come into means act on a on a leg of a member then the eccentricity will develop and therefore the torsional buckling will come into picture. So we have to take care the strength of the member accordingly.

Similarly for double angle also we can use in this way or Tee sections can be used for compressive member, most popularly used compressive member is channel section channel section we oftenly used for compressive member. Hollow circular section also we use, rectangular hollow sections also are used. So these are few steel rolled section which are commonly used for compressive member.

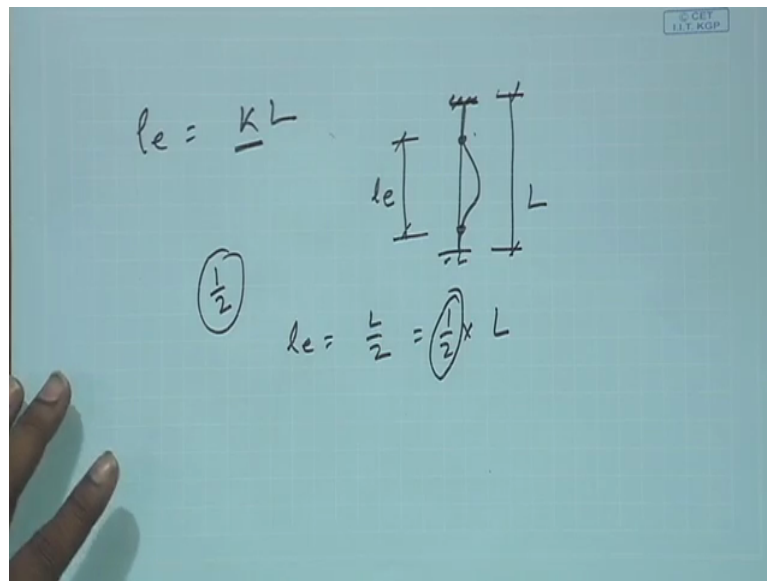
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Then some built-up sections are also used, like channel face to face this is one type of built-up section we use, then channel back to back this also use. So now suppose channel face to face or back to back if we use say for example this, now we cannot use simple like this because if we see in the elevation it will be something like this, right. So unless we tie then unless we provide some lacing then it will not act as a monolithic. Therefore we have to provide certain either some joint means in terms of batten plate or some lacing has to be provided, so that throughout the length it acts as a monolithic member. So this has to be taken care.

And another is built-up box section means with four plates one can make built-up box section. Then plated I section, then built-up I section like this some of the commonly used built-up compression members are there.

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Then we will come to the effective length factor effective length factor means here we see the l_e as we have written the effective length l_e is equal to K into L , where K is the effective length factor. Now this K depends on the (25:46) condition of the member, as I have told that in case of say suppose fixed fixed column it will be it will buckle like this, so point of contraflexure will be here, another point of contraflexure will be here, point of contraflexure means where the moment is becoming 0.

Now this distance, distance between two point of contraflexure becoming as effective length l_e , right where capital L is the total length of the member, right. So what will be the value of K , so here theoretically we got K value as half, that means l_e is equal to L by 2 that means 1 by 2 into L so this is half.

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Effective Length Factor for Centrally Loaded Columns with various End (Table 11 of IS 800 2007)

End Conditions	Theoretical K value	IS-800 provisions
Columns with both ends pinned	1.0	1.0
Columns with both ends fixed	0.5	0.65
Columns with one end fixed and other end pinned	0.7	0.8
Columns with one end fixed and other end free	2.0	2.0
Columns partially restrained at each end	1.0	1.2
Columns with one end unrestrained and other end rotation partially restrained	2.0	2.0

However in IS code this is consider as 0.65 because it will not be perfectly fixed and it will not be perfectly 0.5, theoretically though we are getting 0.5 we are going to consider as 0.65 with certain conservative factor of safety, right. Similarly when columns with both ends are pinned we are considering 1, K value as 1 that means l_e is equal to L . Again columns with one end fixed and other end pinned in this case theoretical value though it is coming 0.7, but in IS probably in codal provision it is 0.8.

Again column with one end fixed and other end free means like cantilever beam cantilever column theoretical value is 2 also we are considering 2, means in IS code also it is considering 2. Columns partially restrained at each end will be 1, however it is consider as 1.2. Similarly columns with one end unrestrained and other end rotation other end rotation partially restrained it is 2.

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Effective Length of Compression Member (Table 11 IS 800 :2007)

Table 11 Effective Length of Prismatic Compression Members
(Clause 7.2.2)

Boundary Conditions				Schematic Representation	Effective Length
At One End		At the Other End			
Translation	Rotation	Translation	Rotation		
(1)	(2)	(3)	(4)	(5)	(6)
Restrained	Restrained	Free	Free		0.5L
Free	Restrained	Free	Restrained		0.7L
Restrained	Free	Restrained	Free		1.0L
Restrained	Restrained	Free	Restrained		1.0L
Restrained	Restrained	Restrained	Free		0.6L
Restrained	Restrained	Restrained	Restrained		0.4L

So when we are going to consider the effective length of a member we have to go to table 11 of IS 800:2007 and as per table 11 of IS 2007 we have to find out the effective length of the member. This snapshot of the code of table 11 of IS 800:2007 has been shown here here the effective length factor has been given here.

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Effective length of column in frames
ANNEX D (clause 7.2.2) – IS 800 :2007

IS 800, gives the following equations for the effective length factor k , based on Wood's curve:

- For non-sway frames (braced frames):

$$K = \left[\frac{1 + 0.145(\beta_1 + \beta_2) - 0.265 \beta_1 \beta_2}{2 - 0.364(\beta_1 + \beta_2) - 0.247 \beta_1 \beta_2} \right] \quad \text{(D-1 IS 800:2007)}$$
- For sway frames (moment – resisting frames):

$$K = \left[\frac{1 - 0.2(\beta_1 + \beta_2) - 0.12 \beta_1 \beta_2}{1 - 0.8(\beta_1 + \beta_2) + 0.6 \beta_1 \beta_2} \right]^{0.5} \quad \text{(D-1 IS 800:2007)}$$

Where β_1, β_2 are given, $\beta = \frac{\sum K}{\sum K_c + \sum K_b}$

K_c and K_b are the effective flexural stiffness of the columns or beams meeting at the joint at the ends of the columns and rigidly connected at the joints.

Now another thing we will discuss here that is the effective length of column in frame because in practical cases we have to calculate the effective length in a frame and it is not a separate member column member is not separate or compressive member is not separate, it is inside the frame. So what should be the effective length of the column in a frame that has to be that we have to know.

And that is given in annexure D of the IS 800:2007 annexure D in clause 7.2.2 it is mentioned, so in annexure D for non-sway frame the K value are given that K value is $1 + 0.145 \beta_1 + \beta_2 - 0.265 \beta_1 \beta_2$ by 2 minus $0.364 \beta_1 + \beta_2 - 0.247 \beta_1 \beta_2$. And in annexure D-1 of IS 800:2007 this formula is given which will be required to find out the value of K for non-sway frame that means that is braced frame when bracings are there.

If a frame is there and non-sway means bracings are there in such cases the K value can be calculated by this and this is basically proposed by Wood which is known as Woods curve from which the K value has been derived. And for sway frames that means the moment-resisting frame that means sway frame means just it is like this say it is a 2 story frame, so if we see like this. So this is a sway frame or moment-resisting frame.

So in this case K value will be like this that is $1 - 0.2 \beta_1 + \beta_2 - 0.12 \beta_1 \beta_2$ by $1 - 0.8 \beta_1 + \beta_2 + 0.6 \beta_1 \beta_2$ whole to the power 0.5. So this is also given in annexure D from which we can find out the value of K, here β_1 and β_2 can be found from this that is the stiffness of the member and stiffness of the column and stiffness of the flexural member which is written here that K_c and K_b are the effective flexural stiffness of column or beam, K_c is for column beam K_b is for beam, meeting at the joint at the ends of the columns and rigidly connected at the joints.

So this is how we have to find out the stiffness relative stiffness of the effective stiffness of the beam and column connecting at that point. Then we have to find out β_1 β_2 , right so once we find β_1 β_2 we can find the value of K. So this is how effective length of column in frame whether it is sway or non-sway based on that we can find out the effective length of column, right.

So this is what in today's lecture I would like to discuss I wanted to discuss so in short again I am repeating the (32:01) of today's lecture that in a compression member the compressive strength depends on different factors like length, (32:09) condition, imperfection, residual stress, then material properties. So all these consideration has to be done also the cross sectional configuration is an important part which come into picture for considering the strength of the member. So again the slenderness ratio which depends on the length effective length and the radius of gyration.

So as per the means slenderness ratio the compressive strength will vary, we have seen that it is inversely varying with slenderness ratio. So those are the some parameters which have to be taken care and then while finding out the slenderness ratio we need to know the effective length and effective length for different (32:59) condition that has been given in the code.

So as per codal provision we have to find out the effective length of a particular member and also in case of frame depending on the basis on sway or non-sway the K value the effective length factor can be found. So this is how we can decide the length of the means effective length of a column or a compression member which will in turn will be require for calculating the column strength or compression member strength. In next class we will find out means we will try to find out the compressive strength of a compression member, thank you.