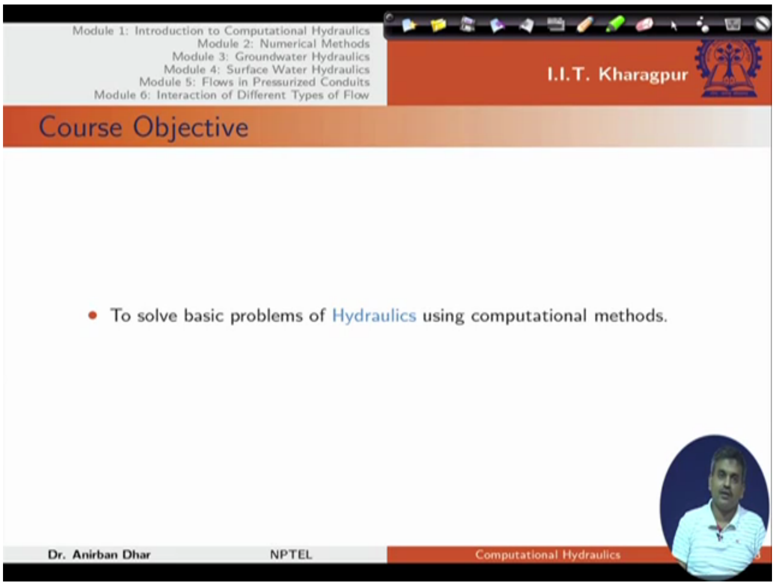
Computational Hydraulics
Professor Anirban Dhar
Department of Civil Engineering
Indian Institute of Technology Kharagpur
Lecture 51
Course Summary

Welcome to this final lecture of the course computational hydraulics. This is course summary. I will be covering all the modules in this particular lecture. So we have covered all total 6 modules starting from introduction to computational hydraulics, numerical methods, groundwater hydraulics, surface water hydraulics, flows in pressurized conduits and finally interaction of different types of flow. So what was the course objective? So at the end of this course students will be able to solve basic problems of hydraulics using computational methods.

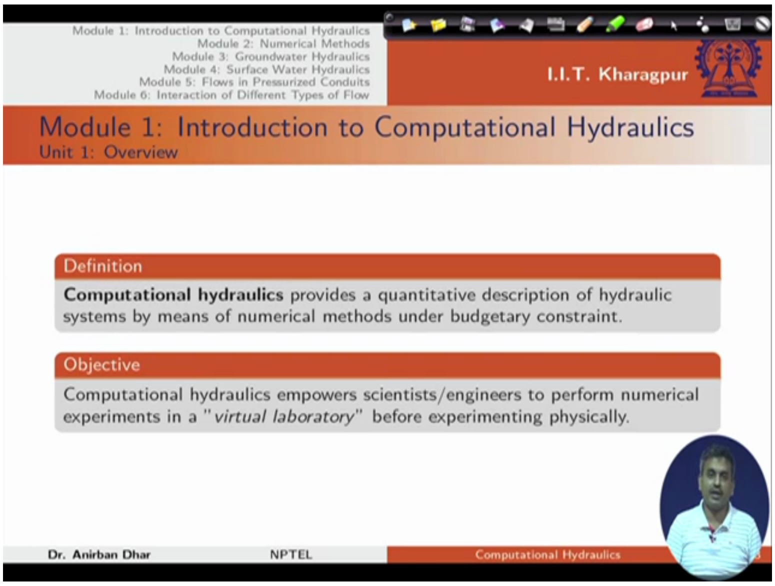
(Refer Slide Time: 01:17)



So at the same time they will understand the discretization used in different standard softwares and they can use their knowledge to identify the best suited software or standard code for their practical use. So in module 1 which was introduction to computational hydraulics we started with this overview unit and we defined our computational hydraulics. So what is computational hydraulics? Computational hydraulics provides a quantitative description of hydraulic system by means of numerical method and under budgetary constraint.

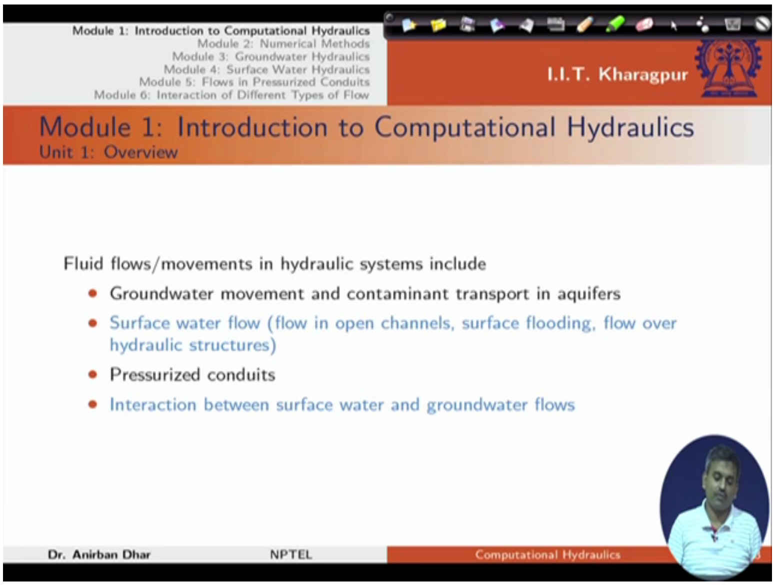
So obviously this budgetary constraint means that we have computational limitations or limitation of computational resources. So objectives of this computational hydraulics empowers scientists engineers to perform numerical experiments in a virtual laboratory before experimenting physically.

(Refer Slide Time: 02:40)



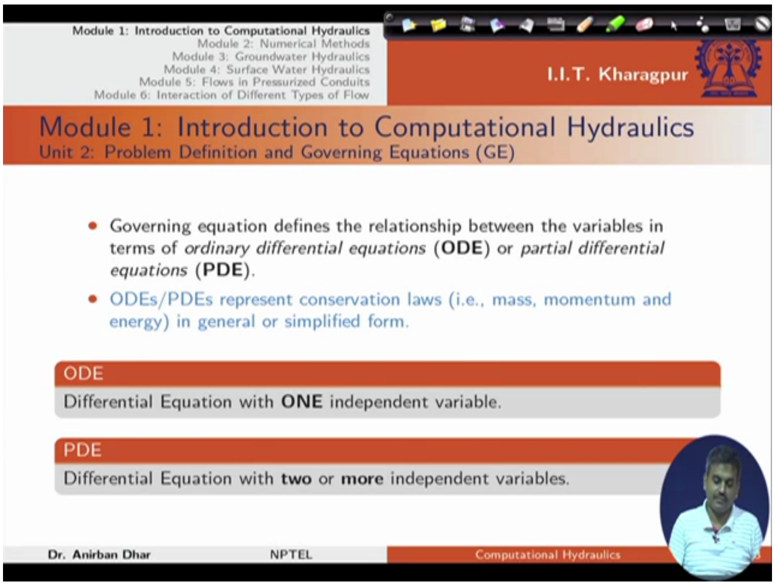
This unit 1 also we have summarized our different module names. This groundwater movements, surface water flow, flows in open channels, surface flooding, flow over hydraulic structures, pressurized conduits, interaction between surface water and groundwater flows.

(Refer Slide Time: 03:05)



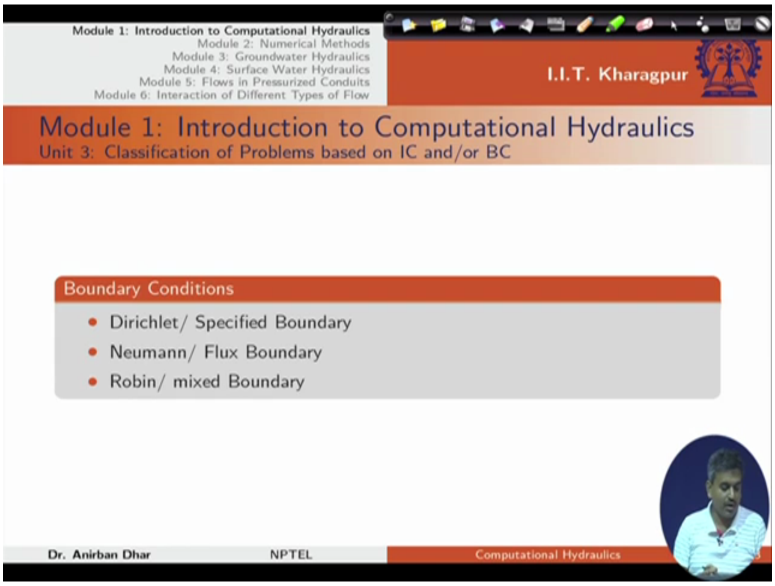
In unit 2 we have discussed about problem definition and governing equation. What was problem definition? Problem definition is in terms of mathematical conceptualization, in terms of ordinary or partial differential equations. And ordinary partial differential equations this represents conservation laws, mass, momentum and energy in general or simplified form. And ODE equation with one independent variable and PDE is equation with two or more independent variables.

(Refer Slide Time: 03:53)



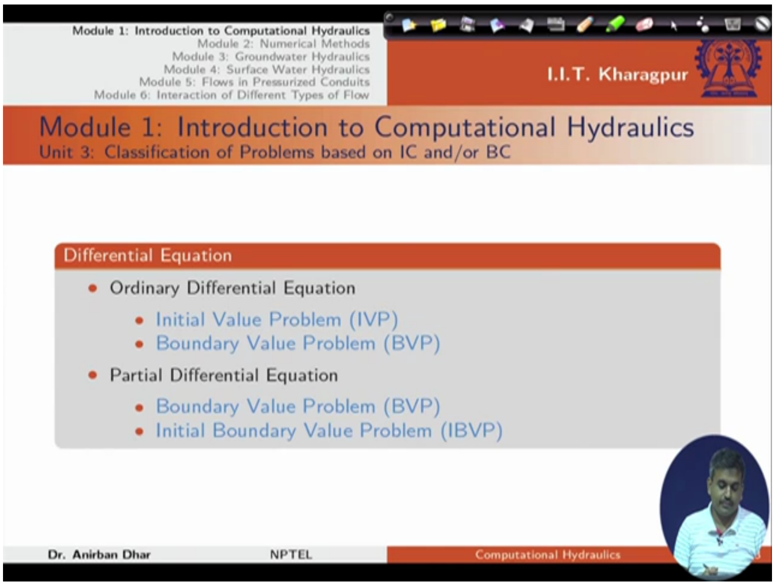
In unit 3 we have discussed the classification of problems based on initial and boundary conditions. So initial and boundary conditions that we have discussed. So for that one we started with this boundary conditions. Dirichlet or specified boundary, Neumann or flux boundary, Robin or mixed boundary condition. We have restricted ourselves to the Dirichlet and Neumann boundary condition in this particular course.

(Refer Slide Time: 04:33)



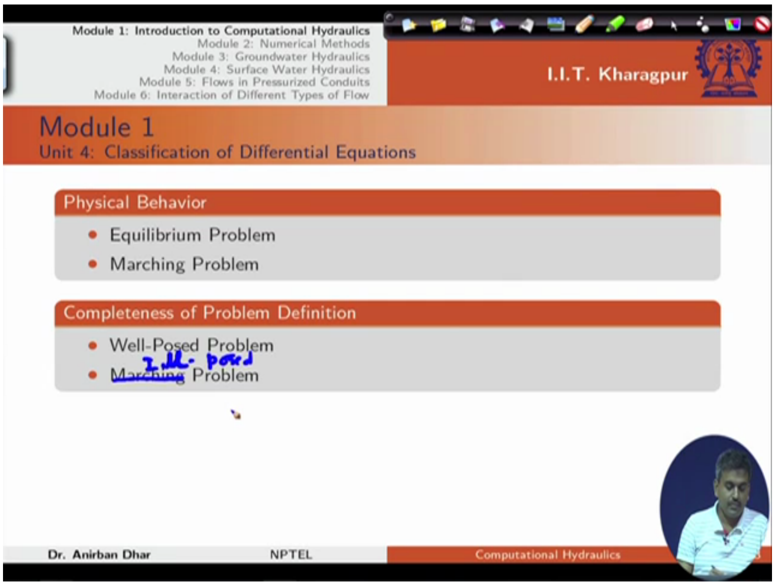
Classification, we have classified initial value problem, our boundary value problem, partial differential equation with boundary value problem, initial boundary value problem.

(Refer Slide Time: 04:52)



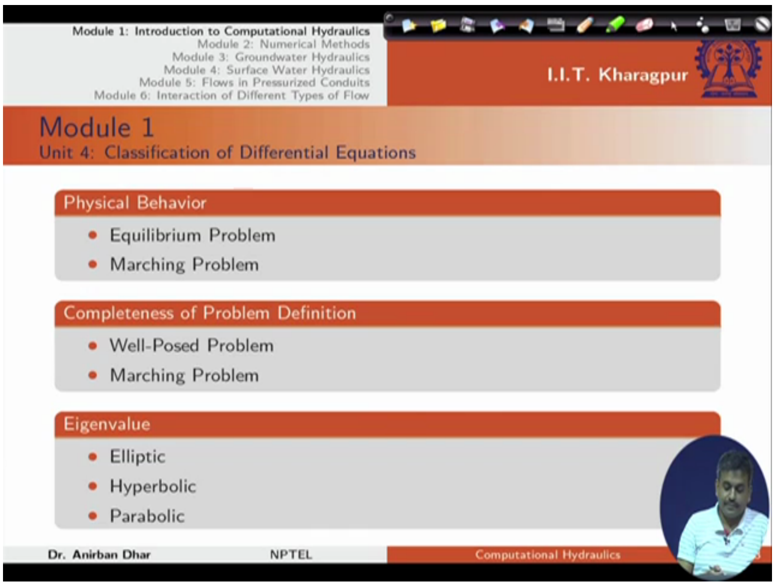
Then in unit 4 we have discussed about classification of differential equations based on physical behaviour, equilibrium problem, marching problem, completeness of problem definition, well posed and this was ill posed problem .

(Refer Slide Time: 05:30)



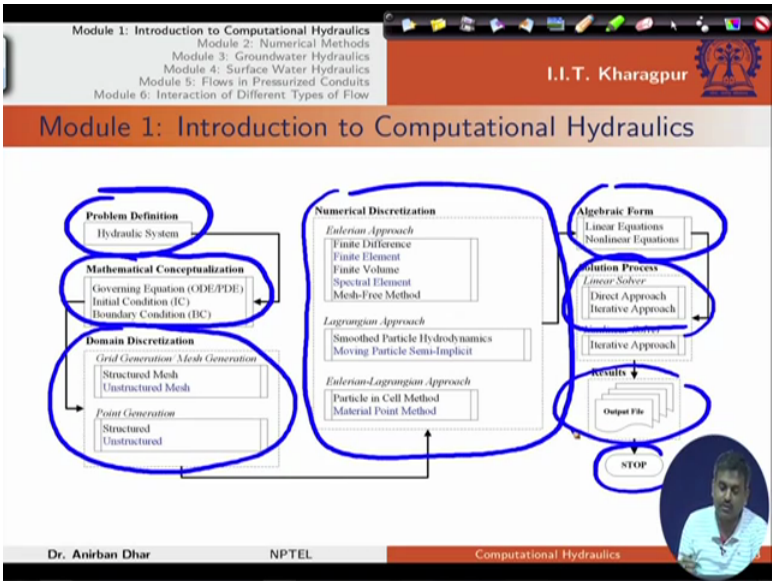
And next thing we have discussed about the classification of second order PDE whether it is elliptic, hyperbolic or parabolic. That we have discussed.

(Refer Slide Time: 05:49)



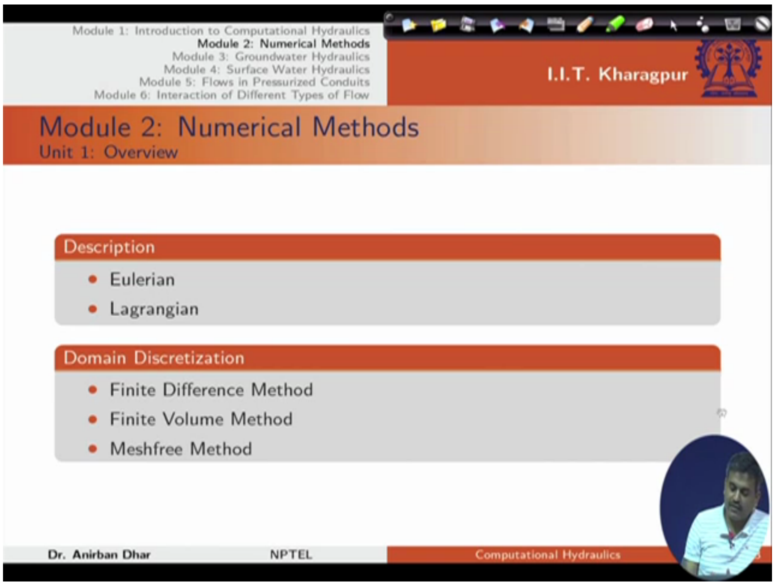
And at the end of module 1 we have summarised our total course in terms of this particular flowchart and we started with problem definition, mathematical conceptualization, domain discretization, numerical discretization, algebraic form, solution process whether it is linear or nonlinear, results and end of the structure.

(Refer Slide Time: 06:24)



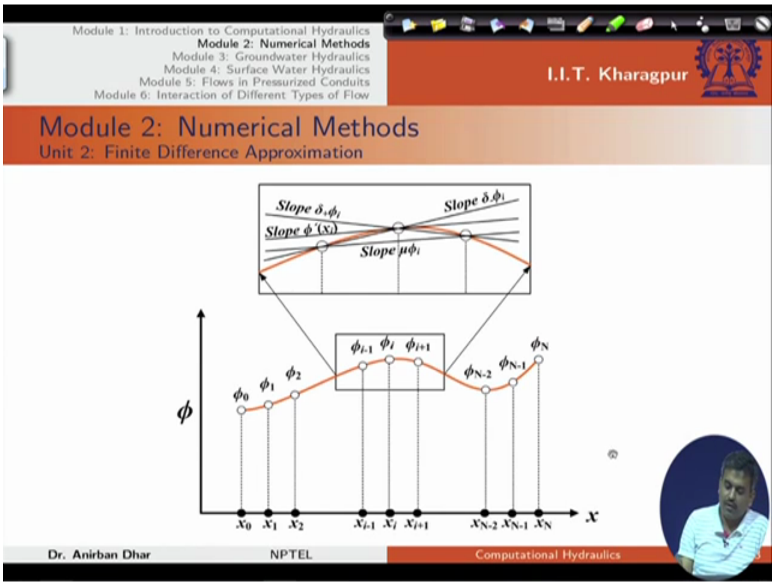
In module 2 we started with this (mod) unit 1 which was overview about the total numerical methods. Description, Eulerian or Lagrangian description. In Lagrangian description and Eulerian description or out of this we have covered only Eulerian description of the problem and we have solved only this type of problems. Domain discretization, I have covered finite difference, finite volume and mesh free methods.

(Refer Slide Time: 07:09)



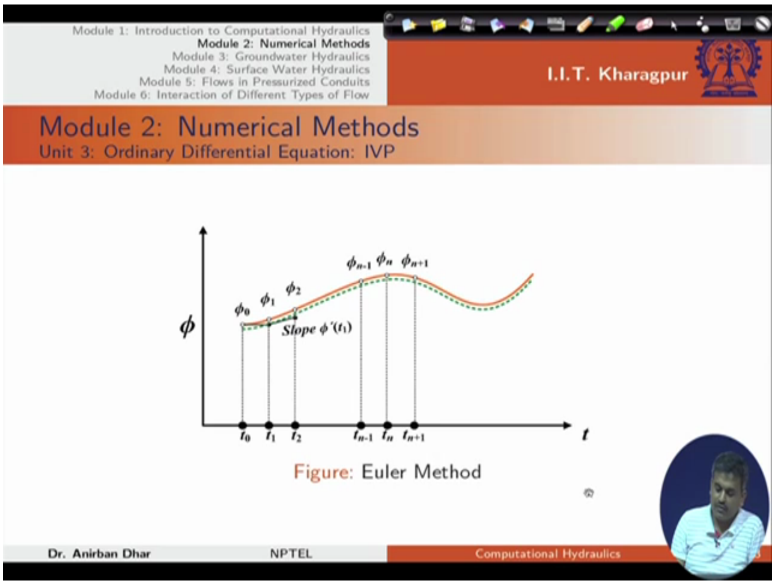
In finite difference approximation in unit 2 we have seen that for boundary node to get higher order (des) accuracy we need to incorporate more number of points. But for internal nodes we can get higher order accuracy with symmetric node distribution. That is i minus 1, i plus 1. And with this approximation we can solve our problems.

(Refer Slide Time: 07:53)



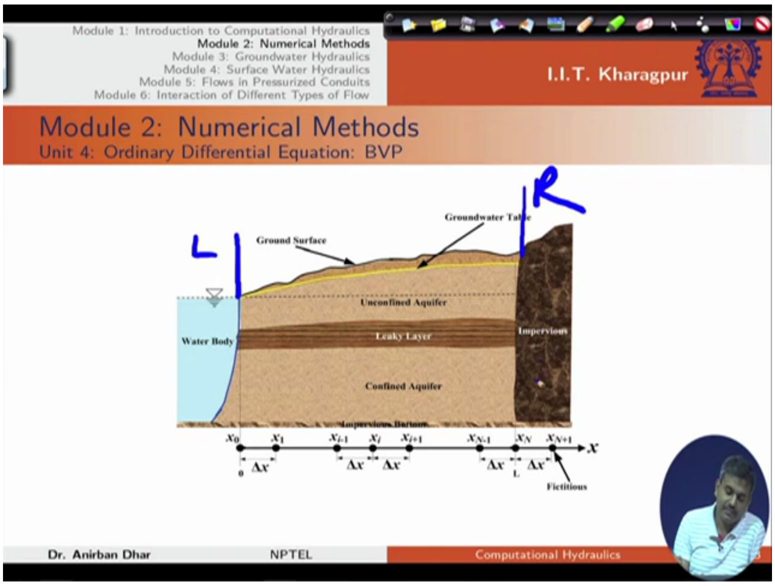
In unit 3 we have discussed this ordinary differential equation and first order ordinary differential equation specifically that is initial value problem. And initial value problem we have discussed modified Euler method. Also we have covered RK2, RK4 and RK3 also.

(Refer Slide Time: 08:23)



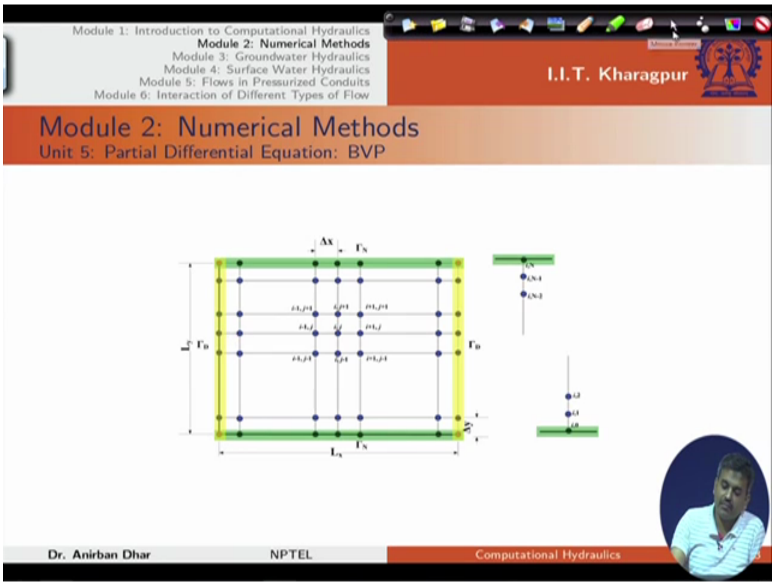
Unit 4 we have discussed this ordinary differential equation which is boundary value problem because it is a steady state problem and boundary values are defined at two boundaries. This is the left boundary this is the right boundary. Left boundary it is having specified boundary condition and on the right hand side it is a zero flux or zero Neumann condition because it is a impermeable boundary on the right hand side.

(Refer Slide Time: 09:05)



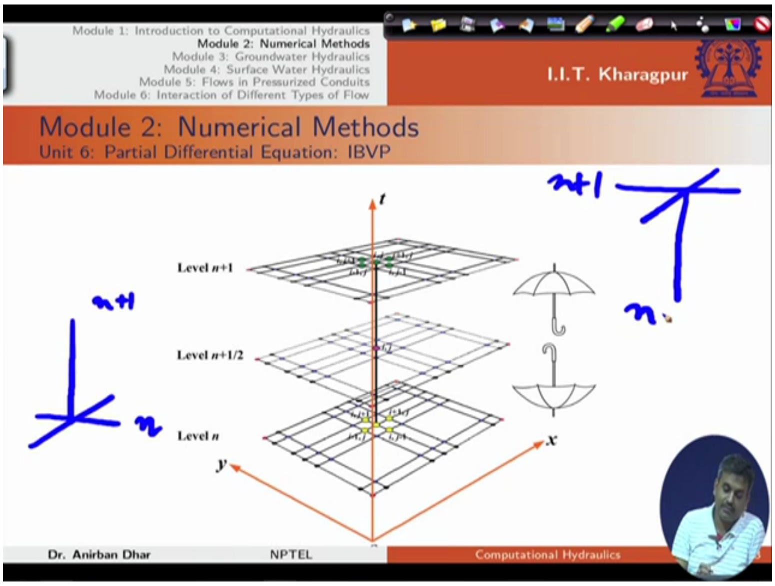
Next we have discretized our governing equation which was general governing equation in terms of finite difference grid and we have seen how to solve this boundary value problem or steady state governing equation using different discretization schemes.

(Refer Slide Time: 09:42)



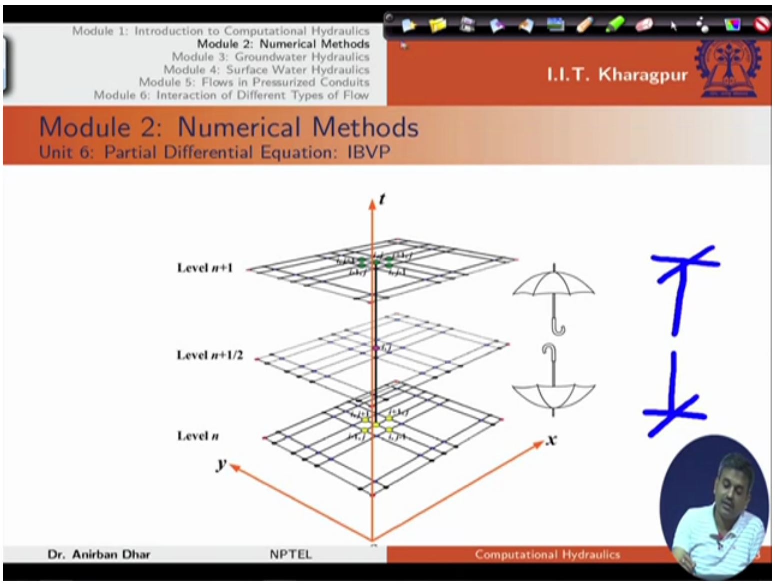
In unit 6 we discussed this initial boundary value problem. Obviously in case of our implicit scheme we have covered n plus 1 level and these are at nth level because spatial derivatives are discretized at nth level. For our implicit scheme this was for explicit. Implicit scheme this was n plus one that all special derivatives are discretized at n plus 1 level. And time derivative considers n plus 1 and nth level.

(Refer Slide Time: 10:28)



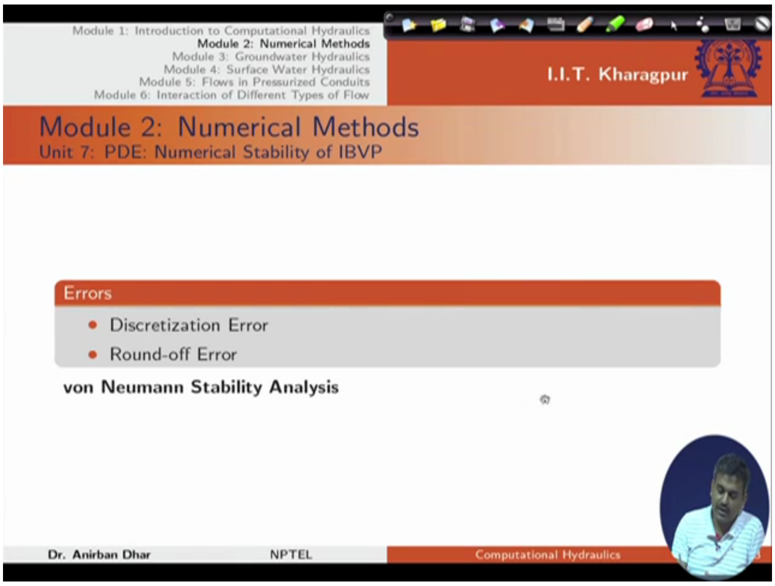
We have also discussed this Crank Nicolson scheme which considers n plus half level which is the intermediate level. And intermediate level we have used half step as explicit and half step as implicit to get the solution.

(Refer Slide Time: 10:54)



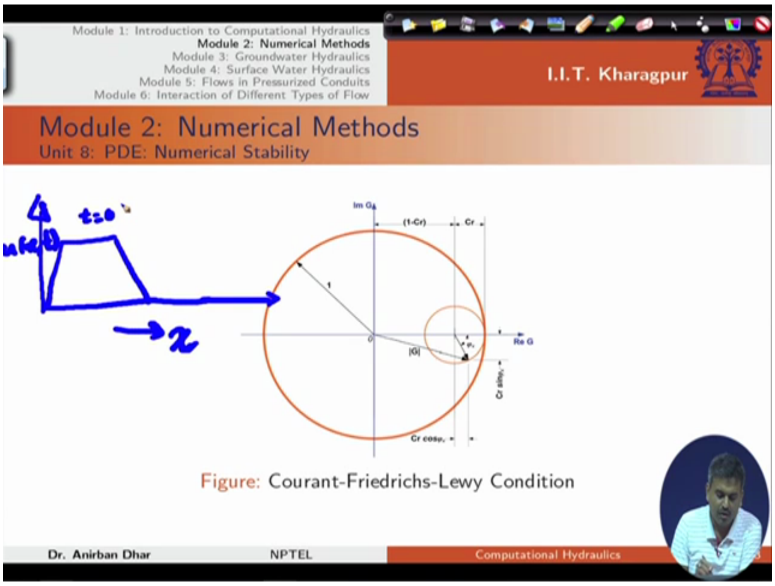
Unit 7 we have discussed this partial differential equation and numerical stability of initial boundary value problem using Von Neumann stability analysis and we have also considered this discretization error, round off error during this stability analysis.

(Refer Slide Time: 11:22)



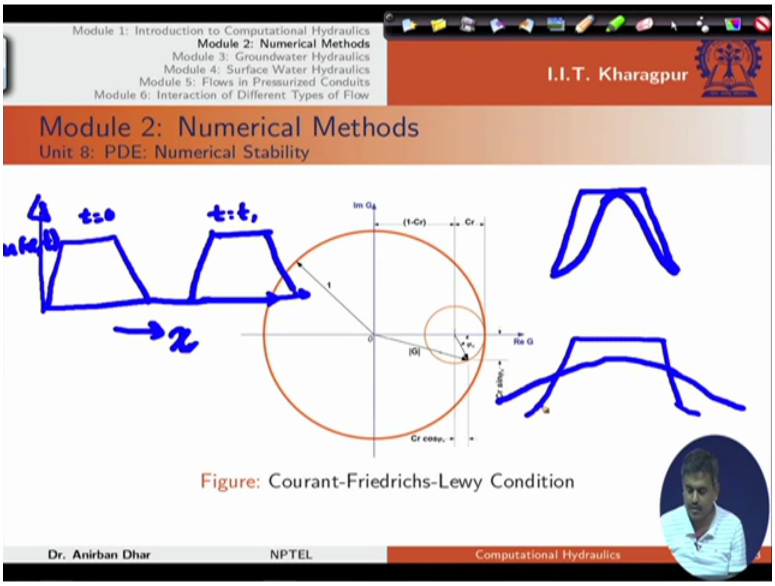
And numerical stability of PDE is we have seen what is this CFL condition or Courant Friedrich Lewy condition. So in this case if we have one pure advection problem without any loss of information, let us say that this (trap) trapezoidal wave this is travelling in the downward downstream direction. Let us say this is u xt and it is travelling in the (da) downstream direction with x and this is for different time. This is time is equal to zero.

(Refer Slide Time: 12:15)



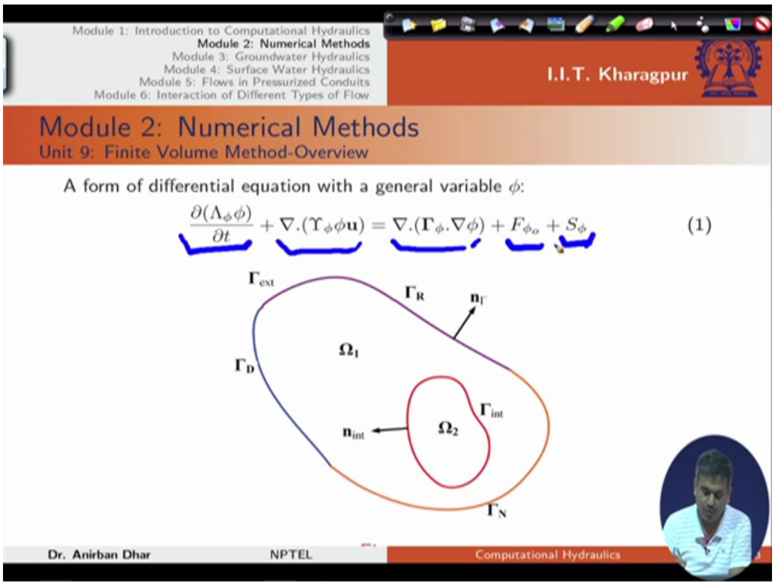
Obviously if we increase time and there is no change in the shape and it moves in the rightward direction which is at future time level then we can say that it is a pure (ad) advection kind of problem and there is no diffusion in this case is. If there is no change in the peak or no change in the shape then it is a pure advection thing. Otherwise there will be diffusion and then maybe dispersion case where there will be lowering of this one.

(Refer Slide Time: 13:19)



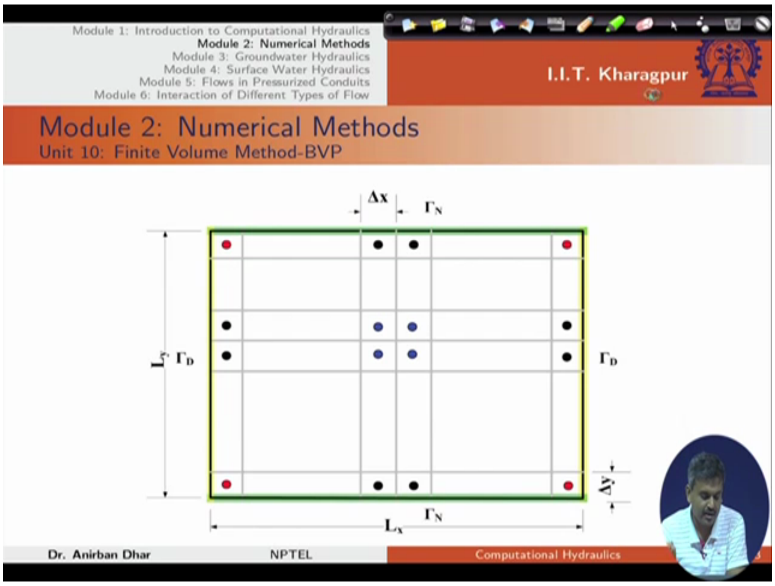
Finite volume method we have considered our general governing equation in that case and this was the advective term on the right hand side. We have diffusion term. These are general force terms, this is source sink term.

(Refer Slide Time: 13:48)



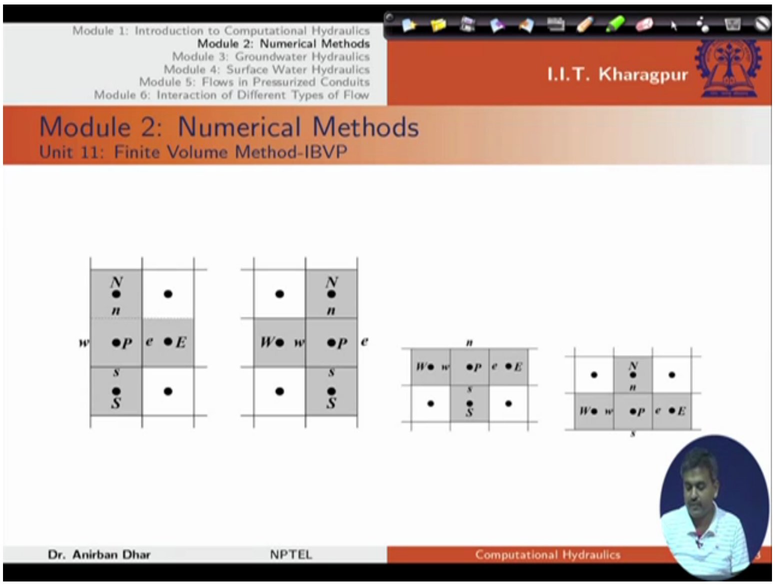
And we have considered one general system and we have tried to discretize or to see what is the basis of this finite volume method. In finite volume method again we have tried to discretize our boundary value problem unit 10.

(Refer Slide Time: 14:14)



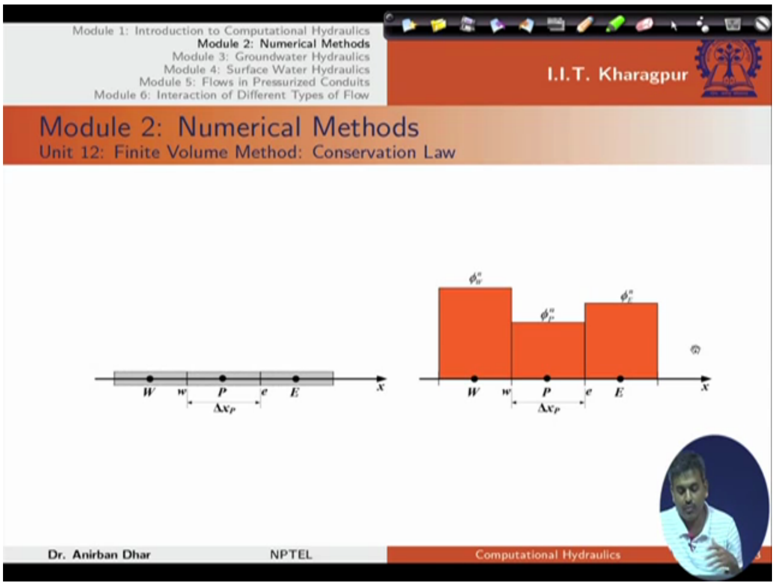
Then in unit 11 we have discussed the discretization of the same equation or equation that we have utilized in our finite difference case for initial boundary value problem and we have discretize it using finite volume method. But in this case near boundary the discretization scheme is somewhat different because we need to consider our no flow or specified flow conditions differently.

(Refer Slide Time: 14:51)



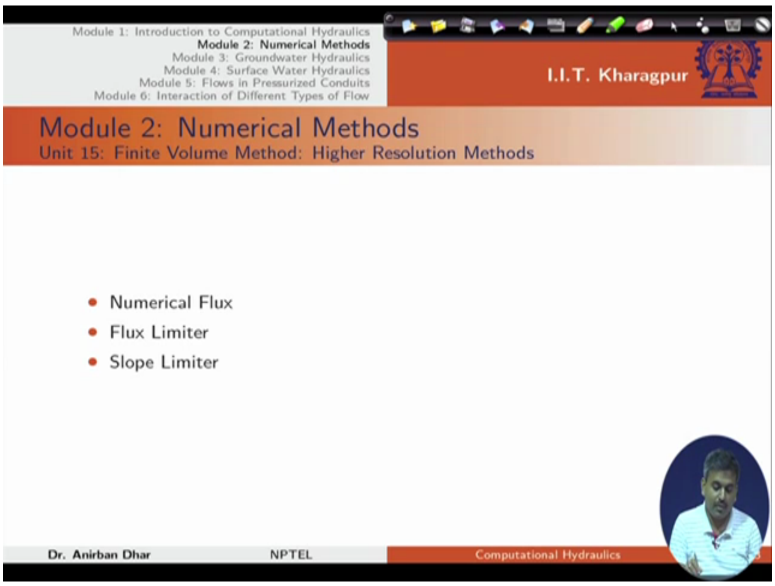
Finite volume method, this conservation laws we have discussed. If there is change in the values in the (down) up gradient and down gradient directions of the cell P then what will be the procedure for numerical flux calculation?

(Refer Slide Time: 15:18)



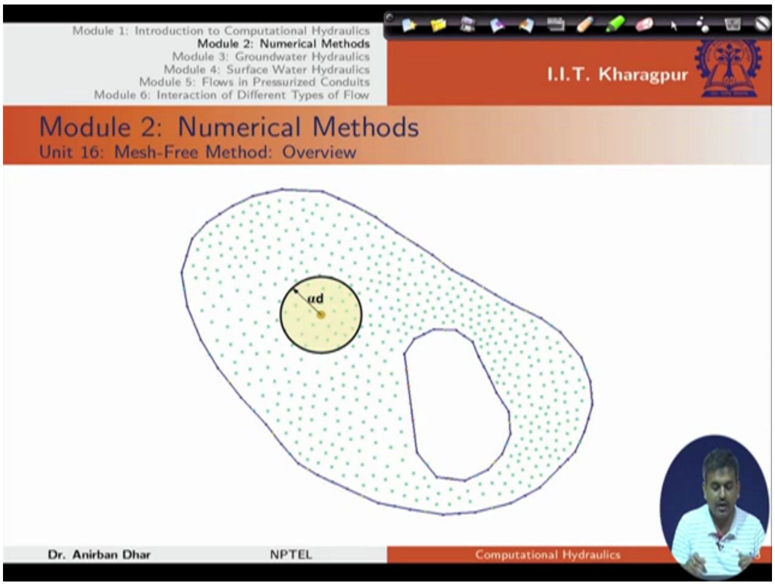
That we have discussed in these methods or in these units. Upwind method, then Godunov approach, then higher resolution methods. In higher resolution methods we have flux limiter and slope limiter scheme. But in this case we have discussed only linear problems. But during application we have considered our nonlinear problems.

(Refer Slide Time: 15:53)



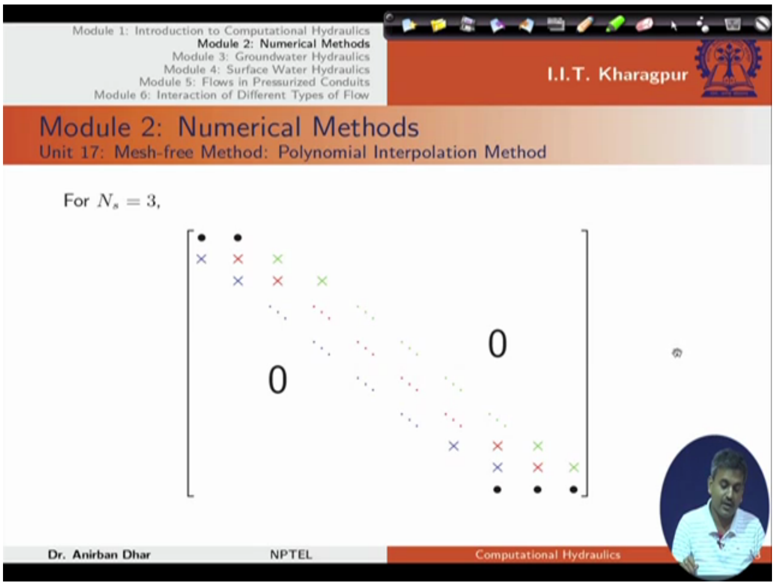
This is unit number 16 of module 2. We have discussed this mesh free method which is point based method and we do not require this grid or elements or cell for our problem. We can start the problem with number of points which are randomly or which are scattered within the domain under consideration.

(Refer Slide Time: 16:26)



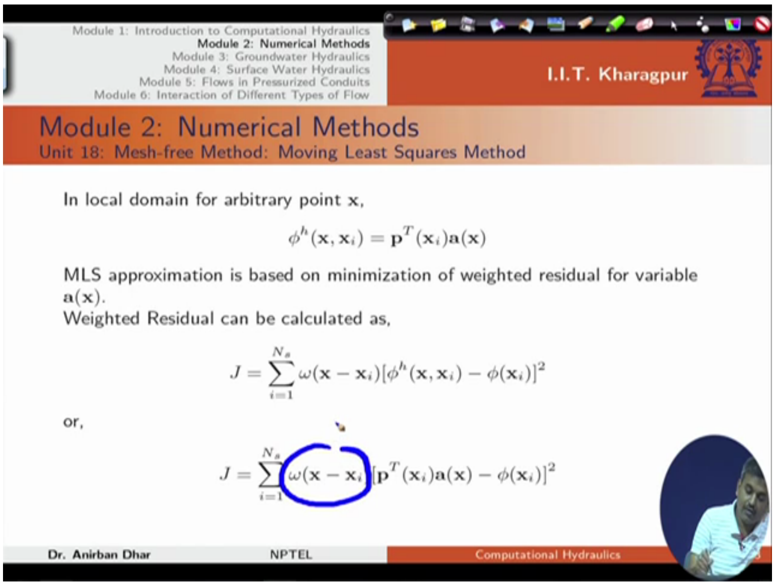
In mesh free method we have seen that polynomial interpolation method is equivalent to our finite difference method if we use NS equals to 3. If we increase the number of points in a particular domain obviously we need to consider more number of points and we will not get that Banded structure like finite difference case.

(Refer Slide Time: 16:59)



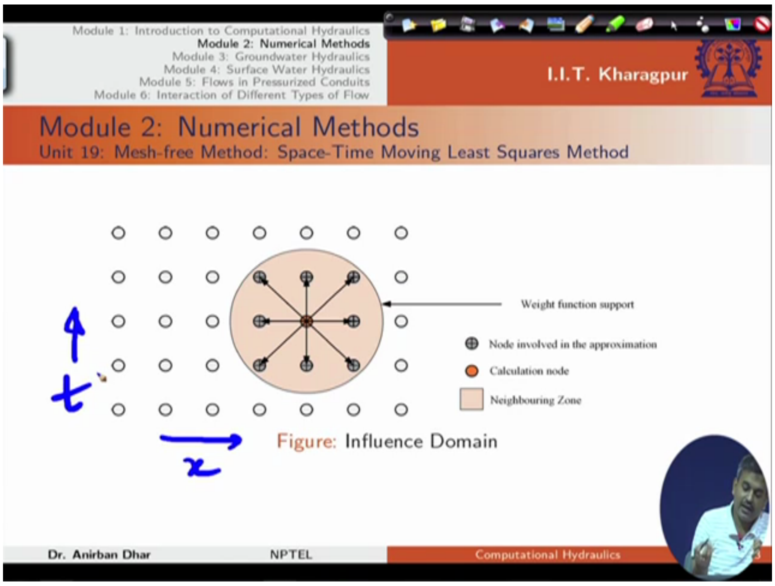
In moving least squares we need to solve one minimization problem but after solving that minimization problem we can get the values of this problem in terms of this weighted or weighting function. So waiting function is important in case of moving least squares.

(Refer Slide Time: 17:25)



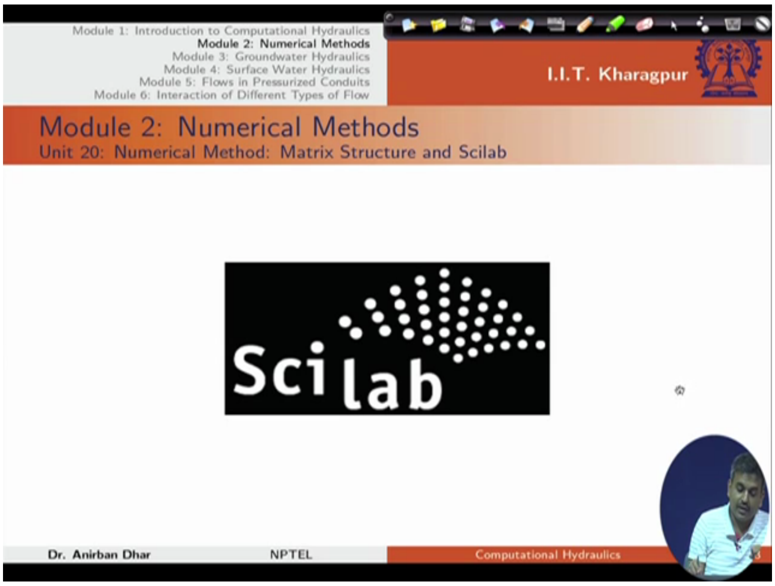
Then we have discussed about the space time moving least squares. In space time moving least squares we use this x domain, this was the t domain. So for any space time domain problem we have considered both points in spatial domain and temporal domain.

(Refer Slide Time: 18:02)



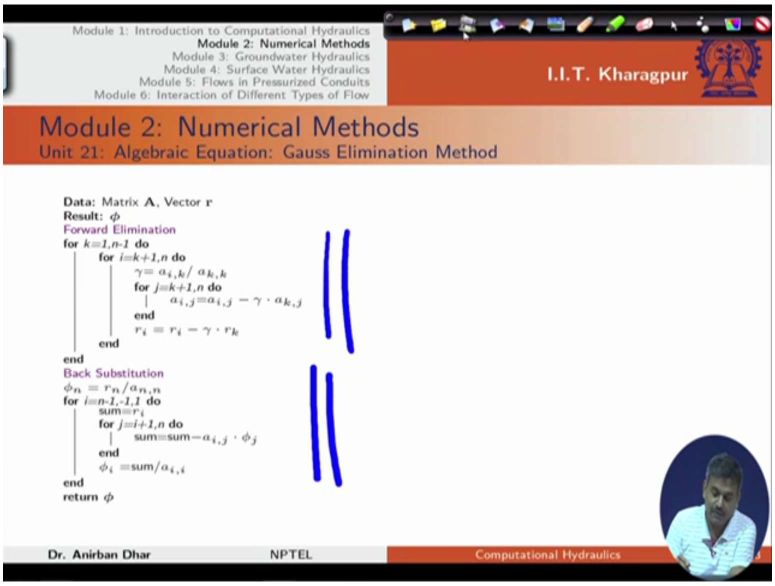
And from unit 20 onwards we used the scilab for writing small codes which is required for this computational hydraulics course. We have discussed about matrix structure and this scilab in unit 20.

(Refer Slide Time: 18:32)



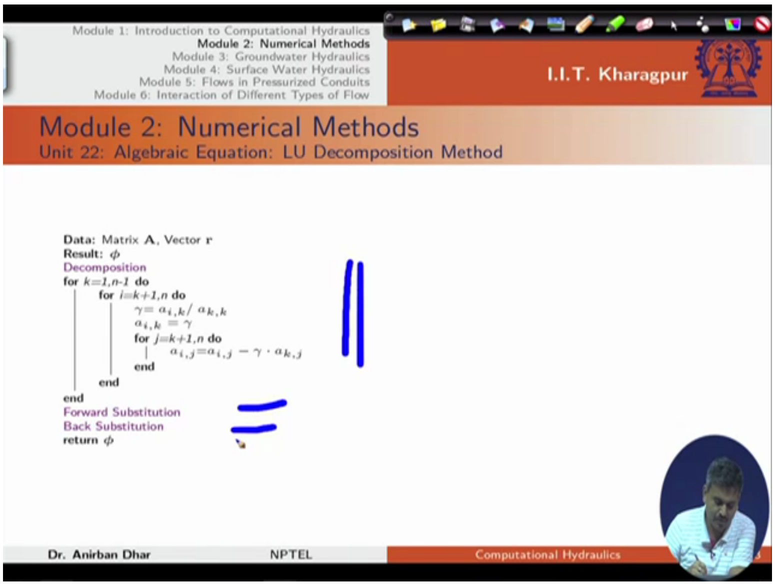
Then in unit number 21 we have discussed about this algebraic equation, Gauss elimination. Gauss elimination we have two steps. One is forward elimination next one is backward substitution or back substitution.

(Refer Slide Time: 18:52)



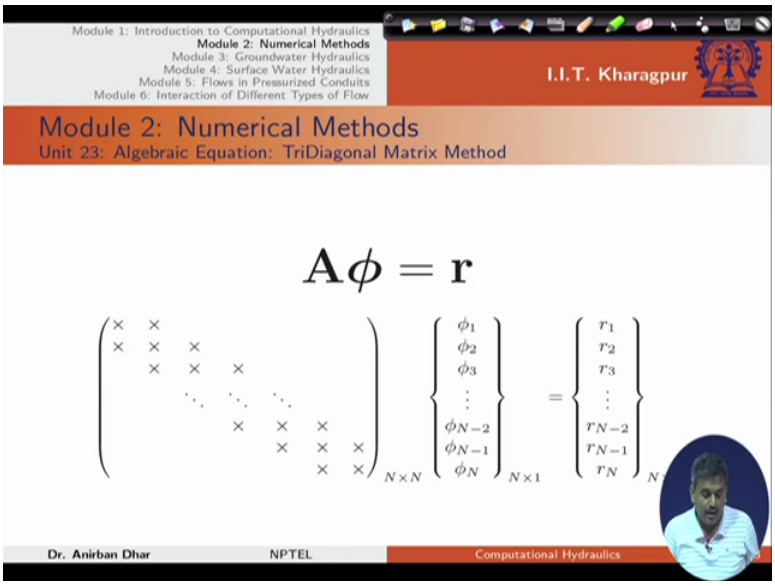
Unit number 22 we discussed this LU decomposition. The first step is decomposition then forward substitution then backward substitution.

(Refer Slide Time: 19:06)



Then we have discussed the solution of tri diagonal matrix which is by considering the Thomas algorithm or (tridia) with the tri diagonal matrix structure.

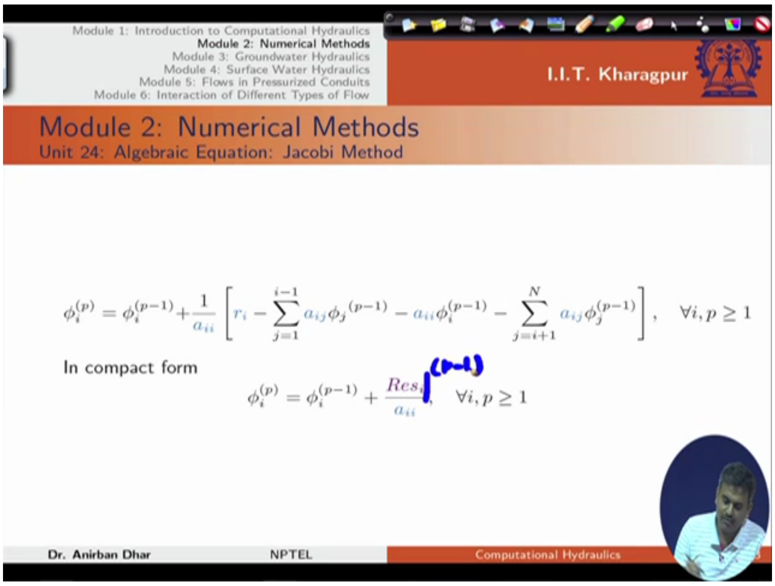
(Refer Slide Time: 19:34)



And unit number 24 I have discussed this Jacobis method. Jacobis method iteration, this is one iterative technique and in iterative technique this is our previous time level value and this is updated value. So updated value depends on only previous iteration values or previous iteration level values. But there is loss of information because we have already updated some values.

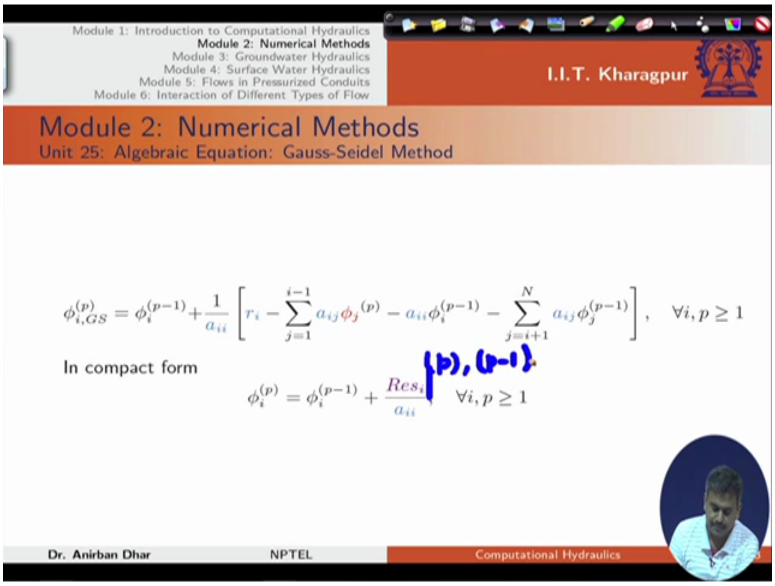
So there is loss of information is in this method. That is why this method is much slower compared to usual Gauss Seidel method. So in compact form we can write like this. Obviously this residual depends on only P minus 1 level values.

(Refer Slide Time: 20:39)



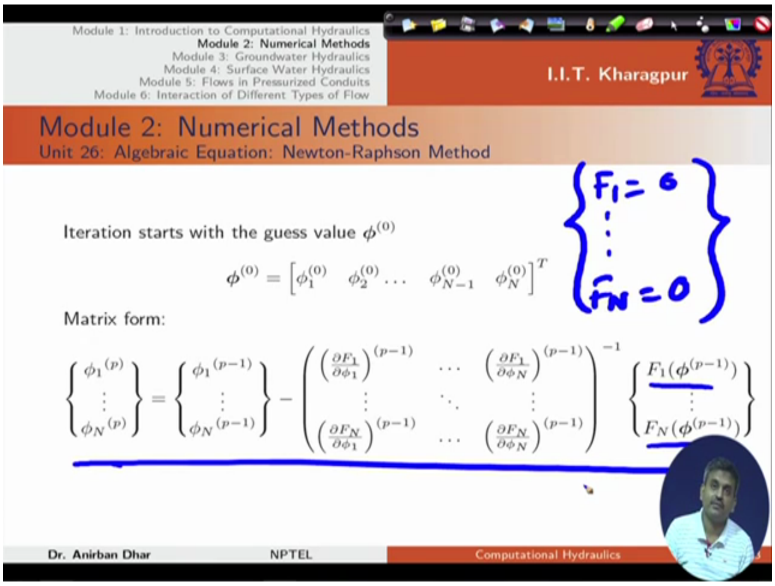
But if we consider Gauss Seidel method whatever value or whatever updated value is available or updated values are available for the problem we can directly utilize that value during calculation. So in this case the (converse) convergence is much faster. So obviously in this is residual this depends on P and P minus 1 both.

(Refer Slide Time: 21:14)



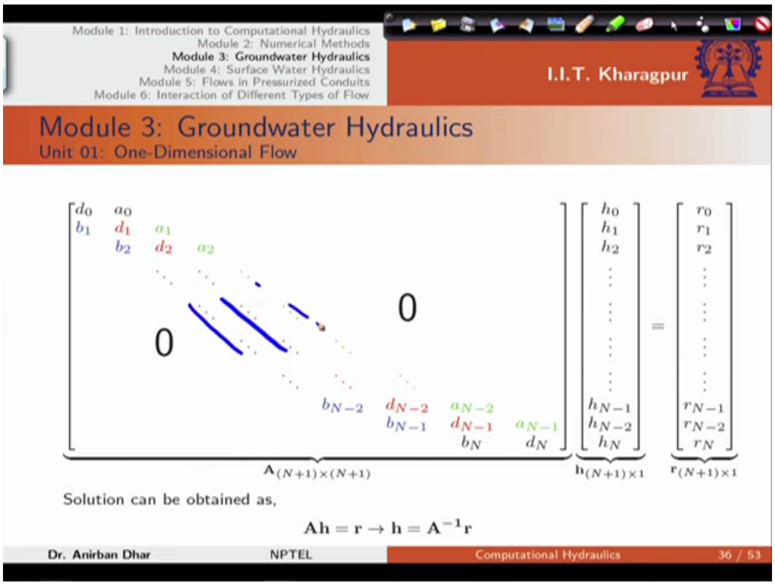
So in unit number 26 we have discussed the solution of algebraic equation using Newton Raphson method and in this Newton Raphson method we started with a guess value and we can invert the Jacobian matrix and finally with iterative approach we can get the solution starting from the guess value. These are individual functions. Individually F1 to FN this should be zero as per our consideration. But obviously there will be error and we need to update that value using this expression.

(Refer Slide Time: 22:14)



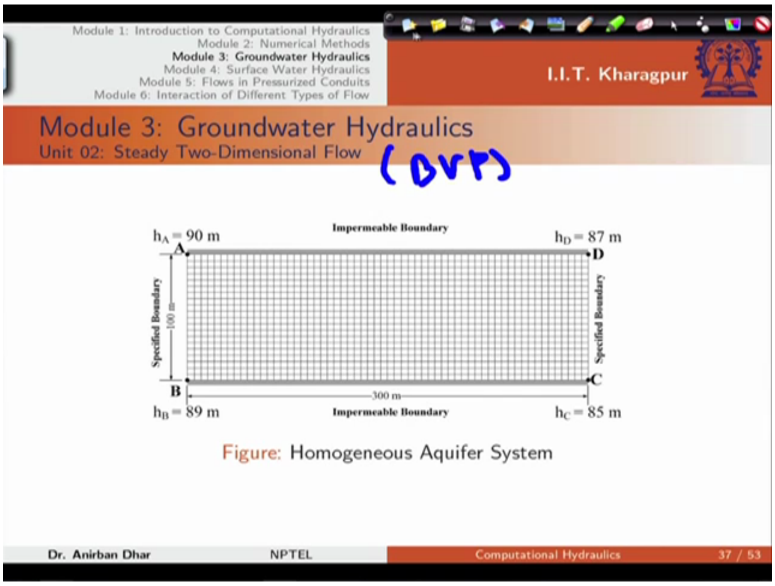
Now in module 3 onwards we have started using scilab as a tool for writing small source code for different kind of problems. So in unit 1 we have utilized scilab for writing this code for one dimensional flow. In one dimensional flow we have utilized this Gauss elimination, our tri diagonal matrix structure for solution of the problem.

(Refer Slide Time: 22:58)



In unit number 2 we have discussed the steady two dimensional flow of groundwater and we have solved boundary value problems or BVP.

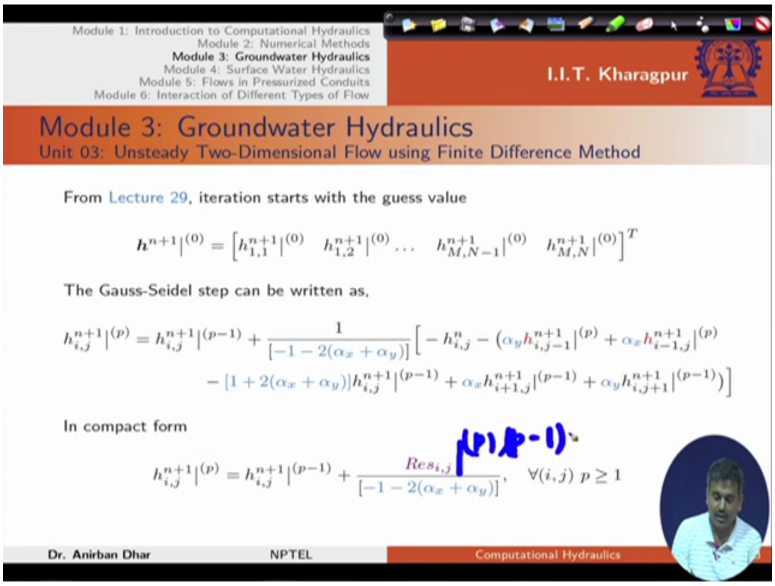
(Refer Slide Time: 23:22)



In unit number 3 of module 3 we have utilized unsteady flow equations for confined aquifer situation and we have solved those problems or solve that particular problem using this Gauss Seidel method and we have used iterative schemes for this method. Interestingly for the solution of this two index or two dimensional problem we need to convert the problem into single index notation for solution using our standard direct approaches.

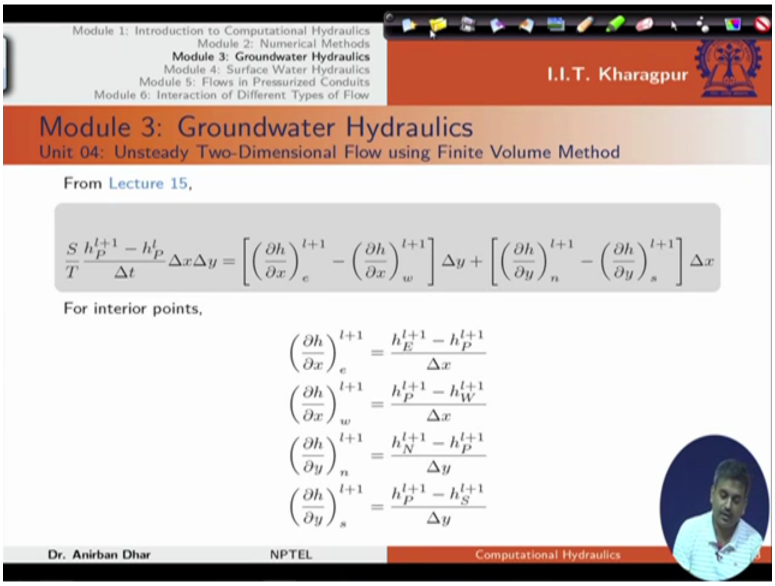
But in Gauss Seidel approach we can avoid that and we can get the solution without storing the coefficient matrix. In this case the coefficient matrix is constant matrix so that is why we do not need to store that matrix. We can avoid that by utilizing our Gauss Seidel approach. In compact form this is residual ij. Obviously we are utilising information from P and P minus 1 level.

(Refer Slide Time: 25:00)



Unit number 4 I have discussed this unsteady two dimensional flow using finite volume method and we can see that we have utilized the information that we have got from our finite difference method. We have utilized directly that one here which is forward difference or backward difference. And we have calculated this unsteady finite volume thing.

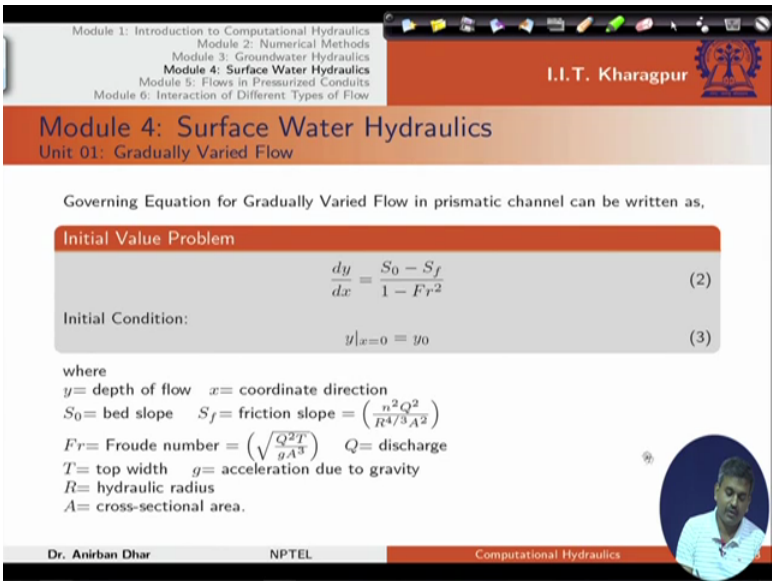
(Refer Slide Time: 25:52)



In unit number 4 this unsteady unconfined flow we have discussed and module 4 onwards we have started this surface water hydraulics. We started with this gradually varied flow which is initial value problem. First order ordinary differential equation with initial condition.

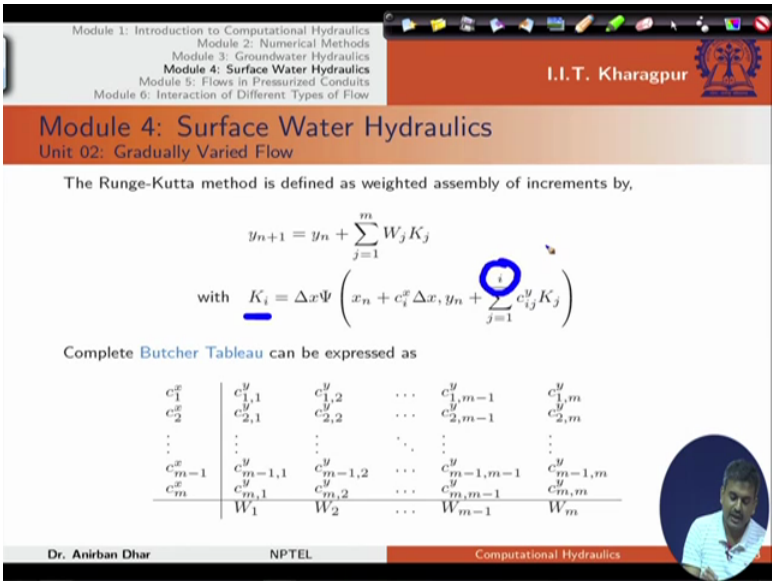
And these are the information like flow depth, x coordinate, bed slope, friction slope, Froude number, discharge, top width, acceleration due to gravity, hydraulic radius, cross sectional area. In this case we have discussed the formulation using usual information for the prismatic channels.

(Refer Slide Time: 26:53)



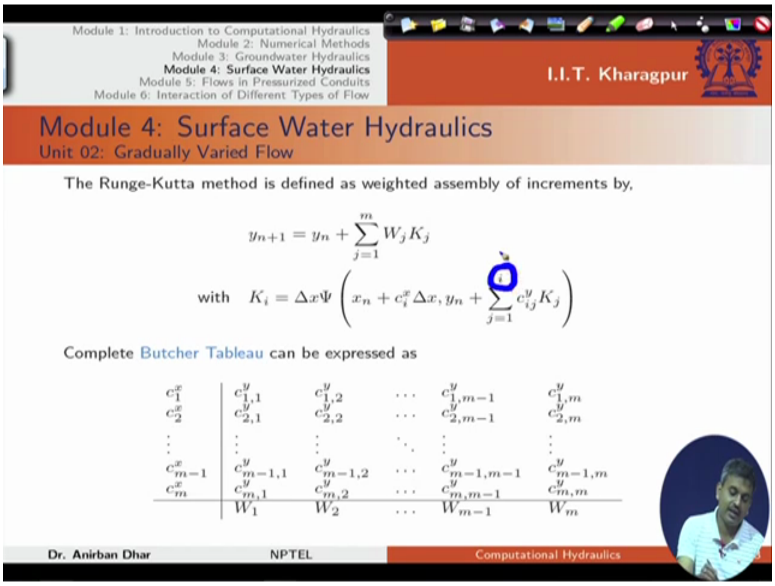
In unit number 2 we have discussed this gradually varied flow and we have utilized the Runge Kutta method as implicit and explicit both the cases we have utilized. So previous one we have utilized explicit one. This one we have utilized implicit one. For implicit one we need this Butcher Tableau. In this case this Ki is dependent on this K value which is up to i.

(Refer Slide Time: 27:41)



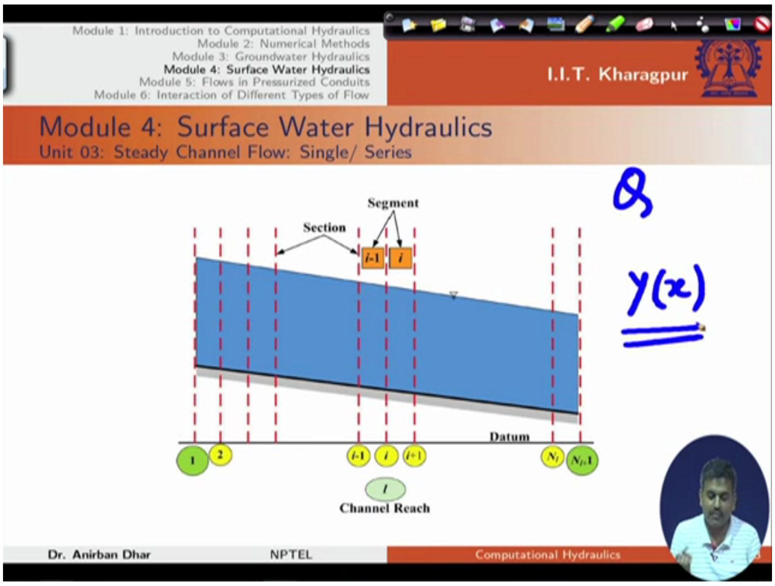
In explicit case this Ki should be up to the summation i minus 1 which is again known one. But in this case it is unknown because we are utilizing i as an index here or maximum value.

(Refer Slide Time: 27:59)



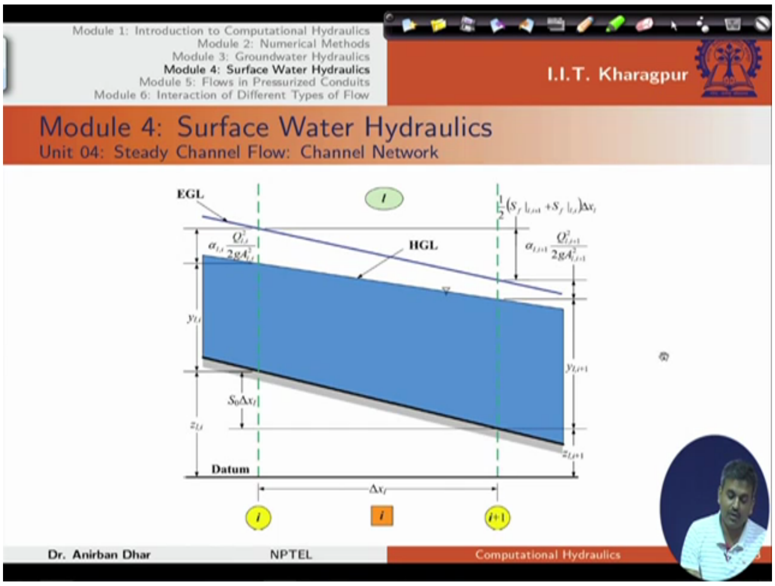
In unit number 3 we have started this solution of steady channel flow single and series and we have solved the problem considering the flow as function of x only. We have considered discharge as constant so with single variable we have solved this problem.

(Refer Slide Time: 28:39)



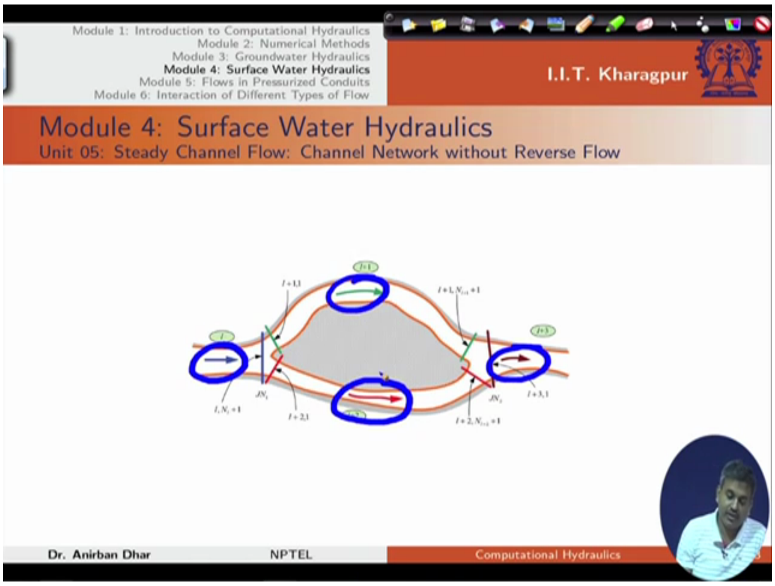
In steady channel flow we have utilized this Q and y as functions because in that case there will be changed in the Q values, different channels in the network.

(Refer Slide Time: 29:05)



Unit 5 considers the steady channel flow and channel network without reverse flow. If we know the flow direction like this we can directly solve our problem by specifying the flow direction here.

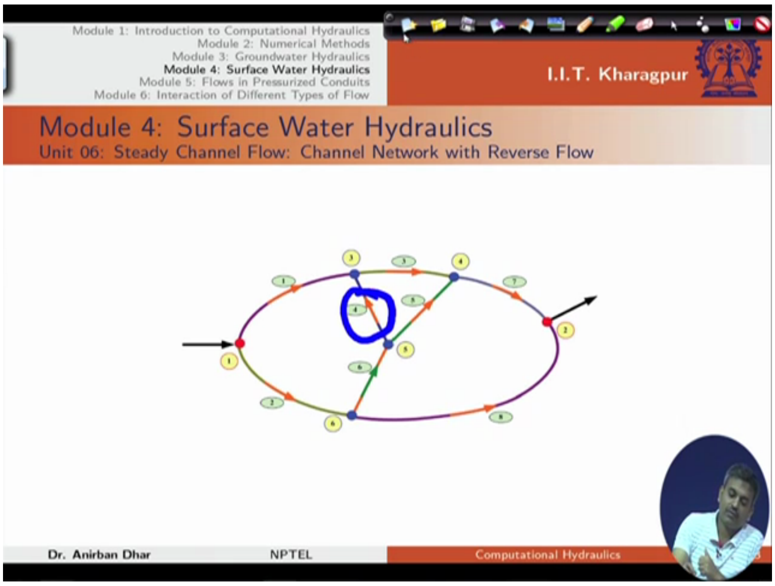
(Refer Slide Time: 29:26)



And we have also use scilab for writing codes for this one. In unit number 6 we have discussed the steady channel flow and channel network with reverse flow. In this one we have considered reverse flow situation. If we do not know the actual flow direction we can start with a assumed direction and depending on the final answer we can comment on the direction of flow.

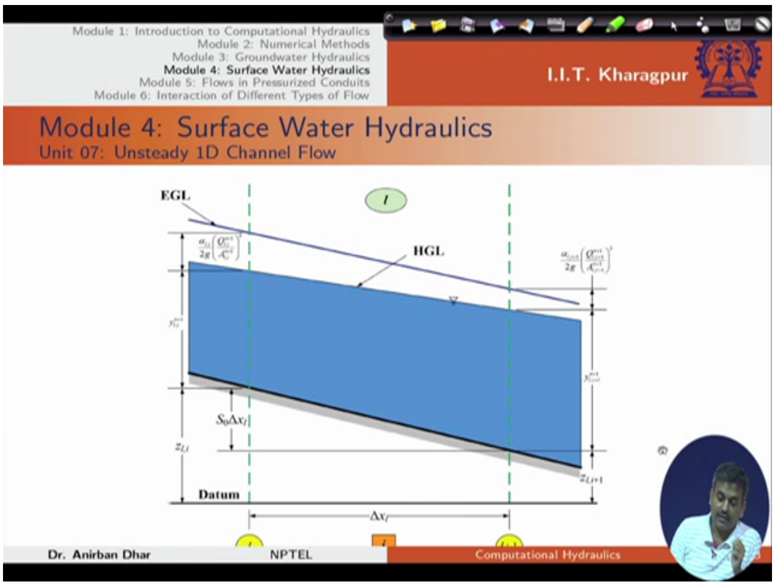
And in this case we have utilized three configurations to solve these problems. And you can see that we have got answers in terms of negative value. That means the flow direction is just opposite to the assumed flow direction. So channel network with reverse flow we have solved.

(Refer Slide Time: 30:44)



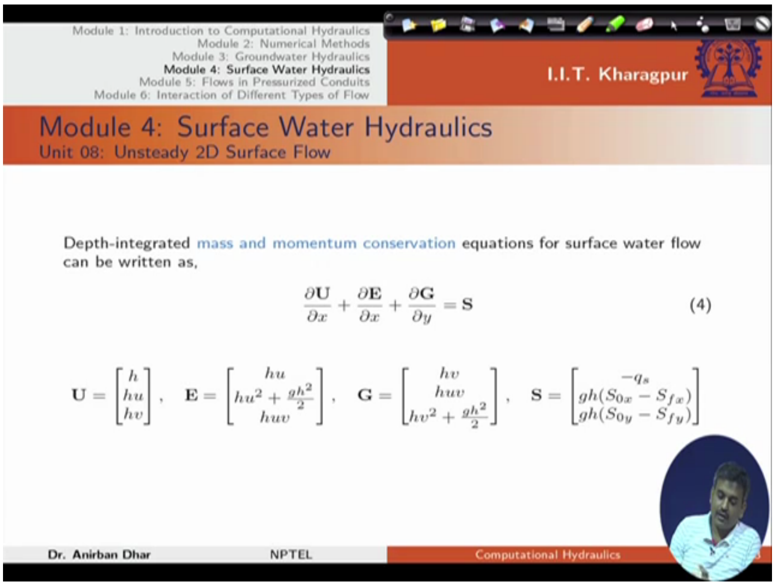
Then we have discussed this unsteady 1D channel flow problem. In unsteady 1D channel flow we have considered inflow and outflow conditions varying or time varying conditions at inflow section. And outflow section we have a specified flow depth and we have solved that problem and we have seen the variation of y which is flow depth and discharge at a particular section in a particular channel.

(Refer Slide Time: 31:33)



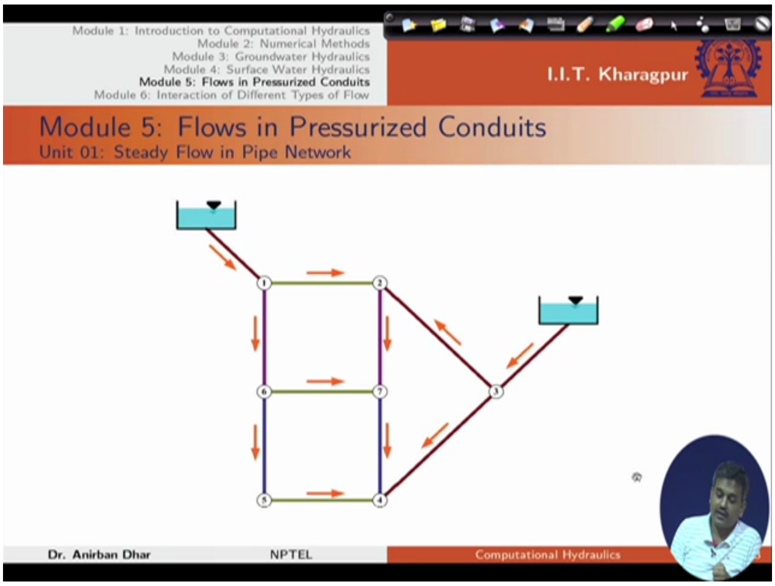
In (cha) unit number 8 we have discussed this 2D surface flow.

(Refer Slide Time: 31:43)



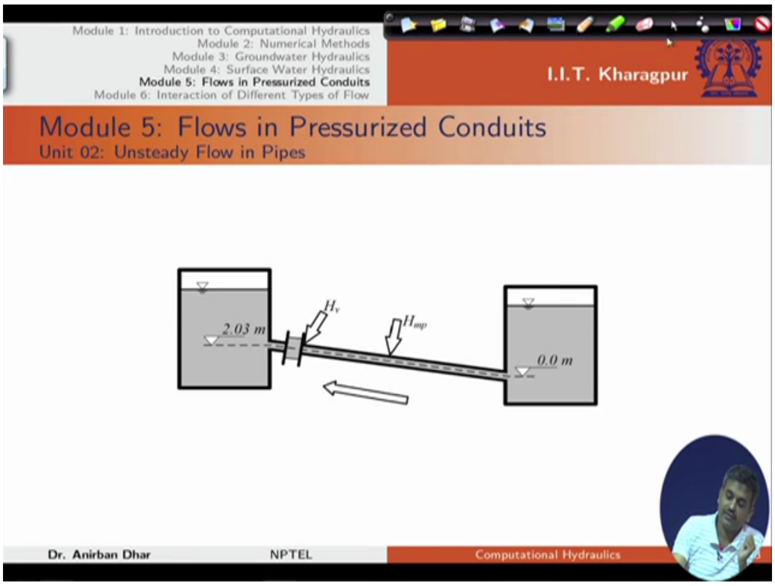
And unit number or module number 5 we have started this unit number 1, the steady flow in pipe networks. That means discharge is not varying with time. It is a steady state network.

(Refer Slide Time: 32:08)



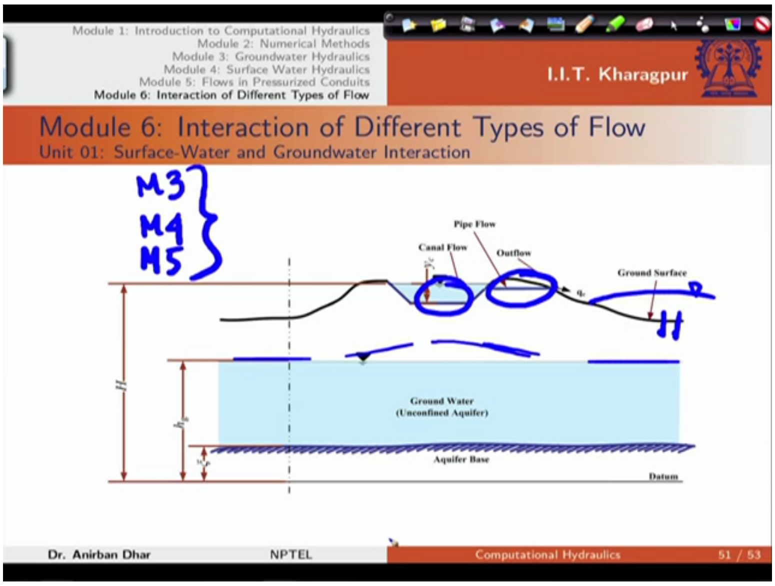
In unsteady state case we have considered piezometric head and in piezometric head depending on the flow situation in transient system this is one wall there during operating conditions we can get different head values. This is at wall and this is at midpoint. And this is one unsteady flow in pipe problem. This is one important problem for flows in pressurized conduits.

(Refer Slide Time: 32:54)



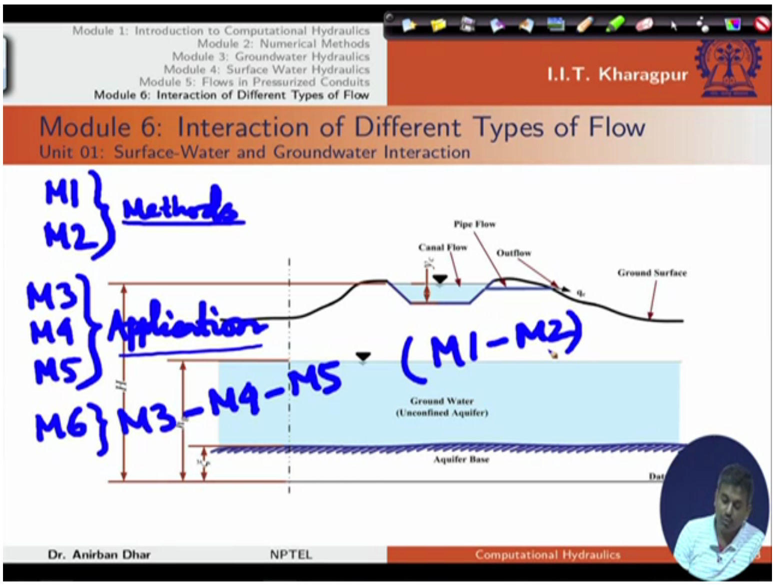
And in the final module which was our interaction of different types of flow we have considered channel flow, pipe flow, surface flow and from there recharge and finally groundwater flow with the effect from surface ring in terms of recharge there. And in that case in that particular interaction related case we can combine whatever we have learnt in the subject maybe in applied form is M3, M4, M5, we can combine these three modules and we can solve this interaction problem using our usual concepts.

(Refer Slide Time: 33:54)



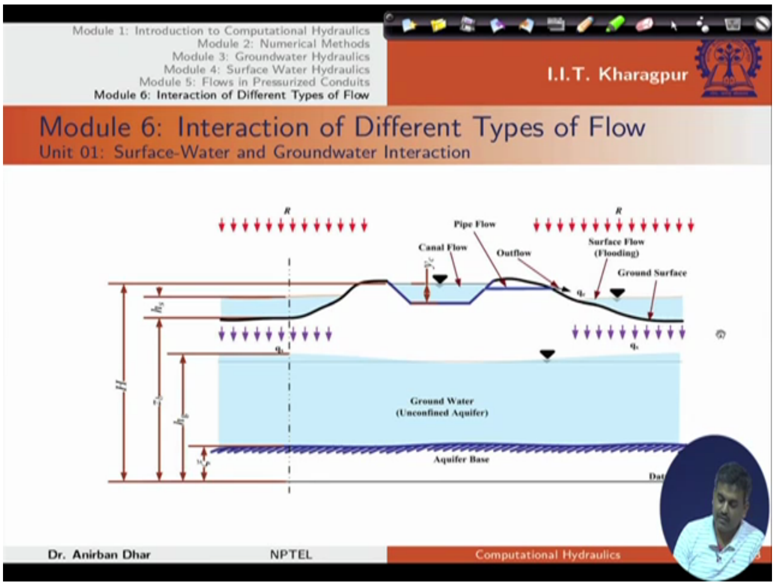
So starting from module 1, module 2 we have discussed the background methods. In module 3, module 4, module 5 we have discussed application of the methods that we have learnt in module 1 and 2. And finally module 6 covers the integration of module 3, module 4 and module 5. Obviously it considers the basic methods that we have studied in module 1 and module 2.

(Refer Slide Time: 35:11)



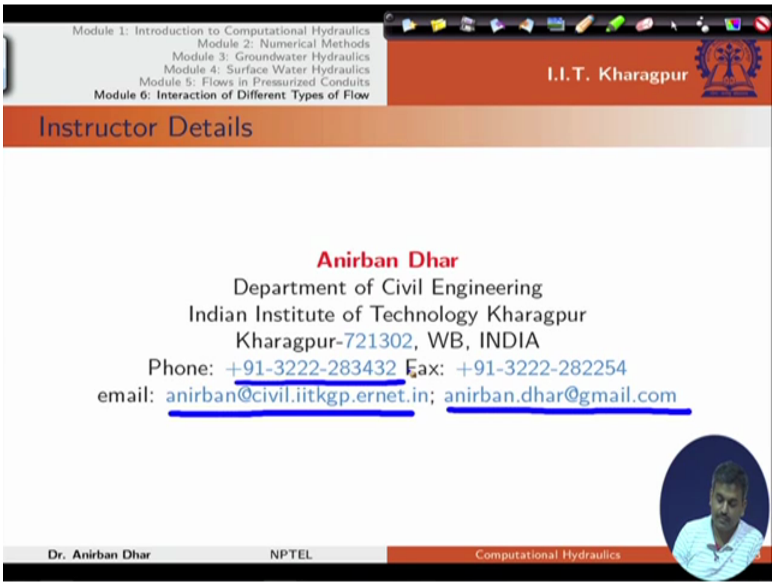
So this is all about the course computational hydraulics. If you have any query or further questions we can discuss it. You can contact me for that one. And in this one interaction flow we have considered all the situations there. This is groundwater flow.

(Refer Slide Time: 35:39)



And this is the instructed details. I am Anirban Dhar and you can contact me through this email or you can call me on this number for further discussion.

(Refer Slide Time: 36:03)



Thank you.