Computational Hydraulics
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Lecture 49
Unsteady Flow in Pipes

Welcome to this lecture of the course computational hydraulics. And we are in module 5, flows in pressurized conduits. This is unit number 2, unsteady flow in pipes.

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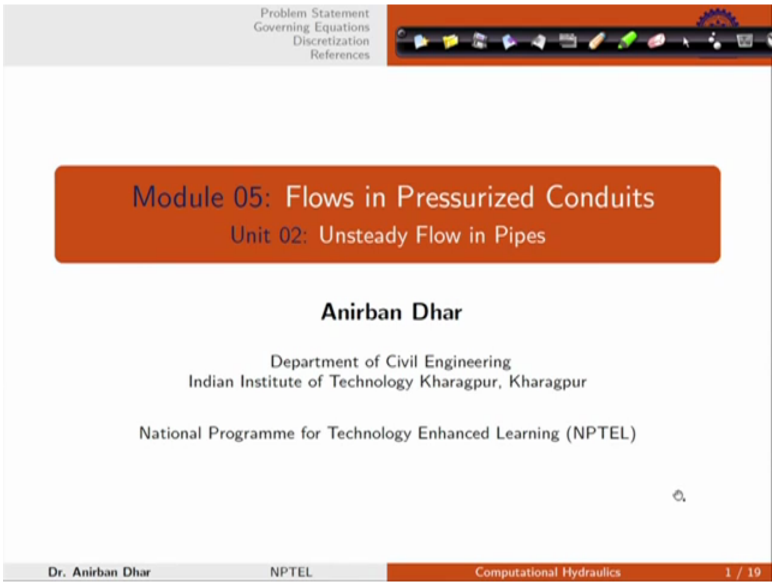

Learning objective, at the end of this unit students will be able to solve unsteady problem in pipes.

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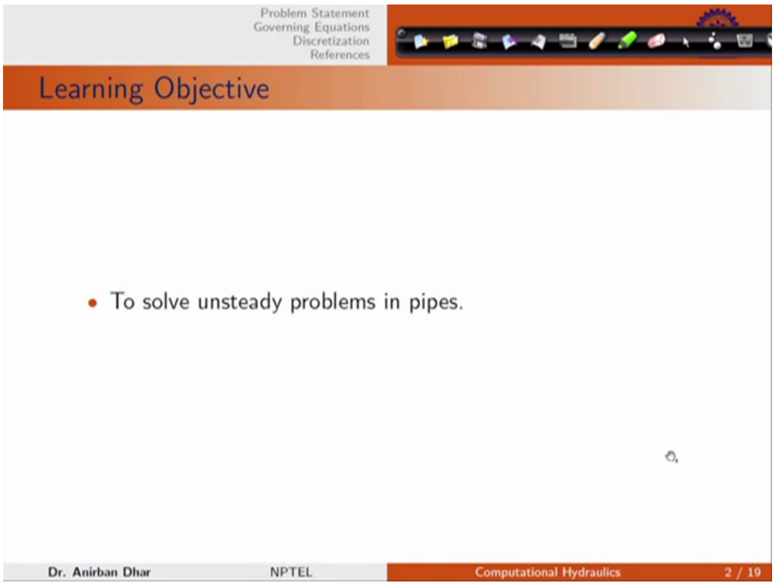

Problem definition to solution. In this case we will be considering one dimensional space as variable because along pipe length there will be variation but we will not consider 2D or radial direction in our calculations. So we will have continuity and momentum equation for our problem and we can discretize it using finite volume approach or Godunov scheme. And resulting thing we can solve using one predictor corrector approach.

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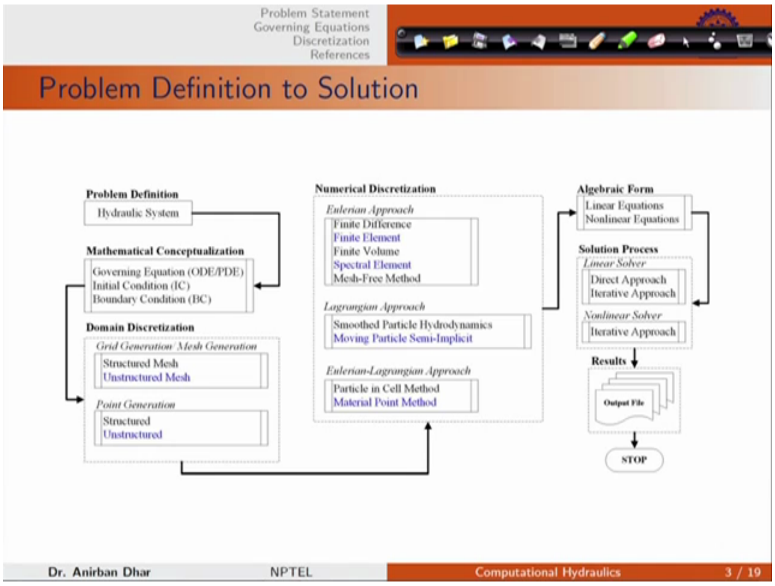

So problem statement, let us say that two reservoirs are connected. If I close wall in between suddenly obviously there will be changed in the pressure within the pipe system. And how to model that? That can be solved using our unsteady pipe flow equations. So variables H as a function of, H is piezometric head, H as a function of x and t and V is the velocity which is again function of x and t.

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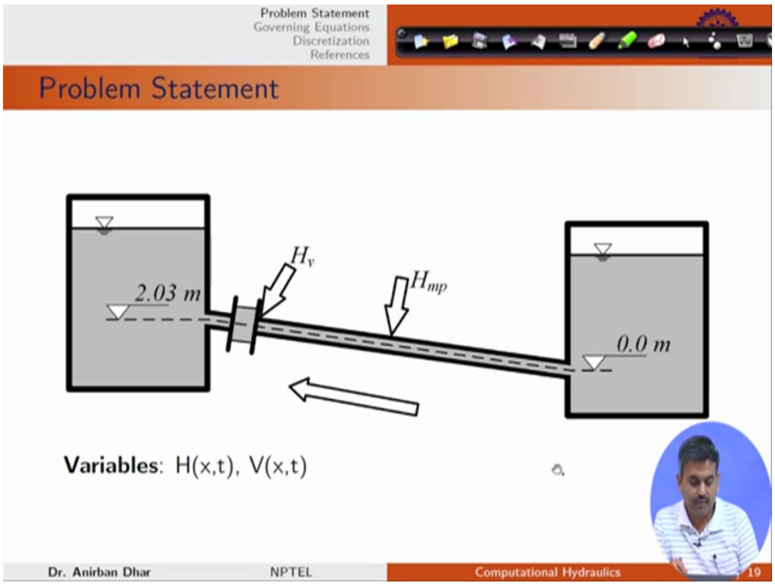

Pressurized conduits, one dimensional unsteady flow in pipes can be represented in terms of following differential equations. So this is, a is wave speed, H is piezometric head, V is cross sectional mean velocity and J is friction force at the pipe wall per unit mass, t is time, g is acceleration due to gravity. So in this case we need to discretize this equation. This is a non conservative form.

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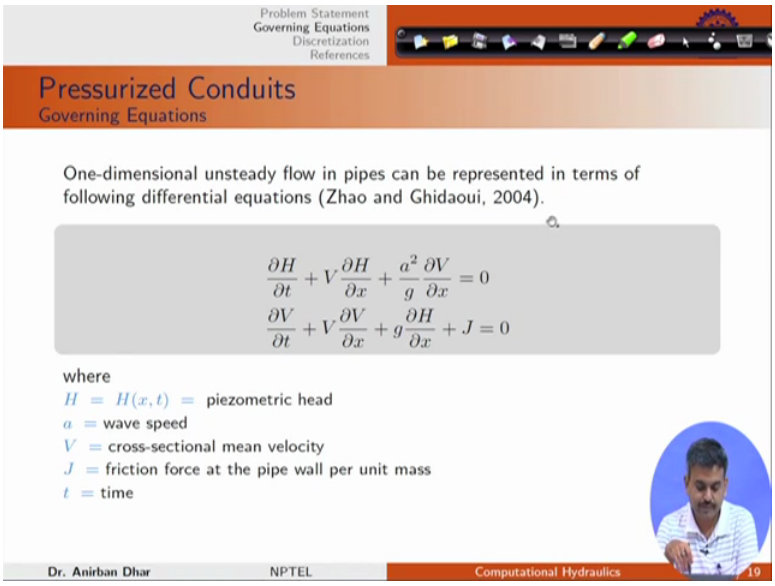

Now this equation can be written like this, which is our usual equation and we have non conservative from here. This means we have del H by del t, del V by del t, U as H and V. This A is V a g a square g V here.

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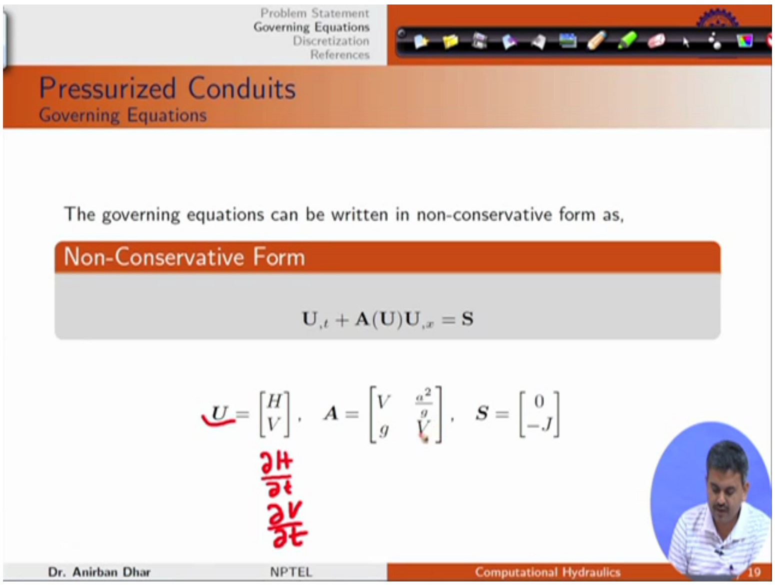

Now this non conservative form can be converted to conservative form with this approximation where A bar is the average velocity or approximated velocity. A bar and bar V and we can use this approximated conservative form for solution of unsteady flow in pipes. So A bar is V bar here.

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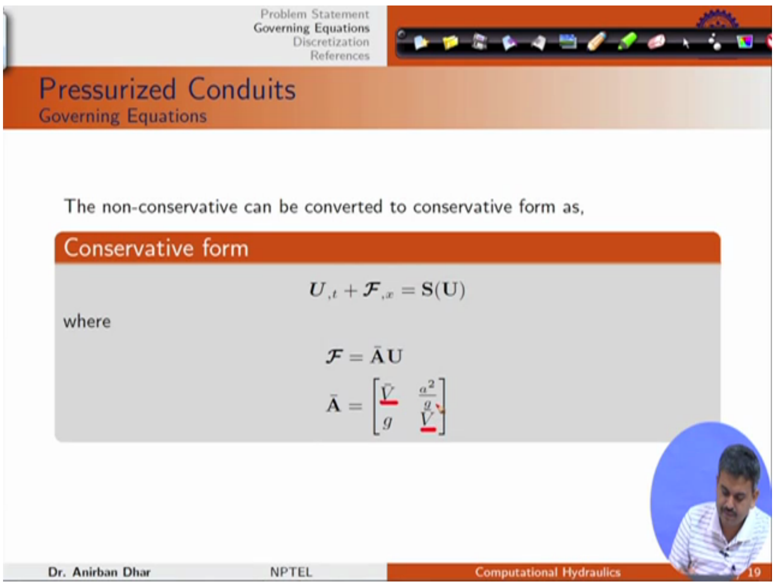

Now if V bar equals to zero this yields our classical water hammer equation. That means in this case A bar will be zero a square by g, g zero.

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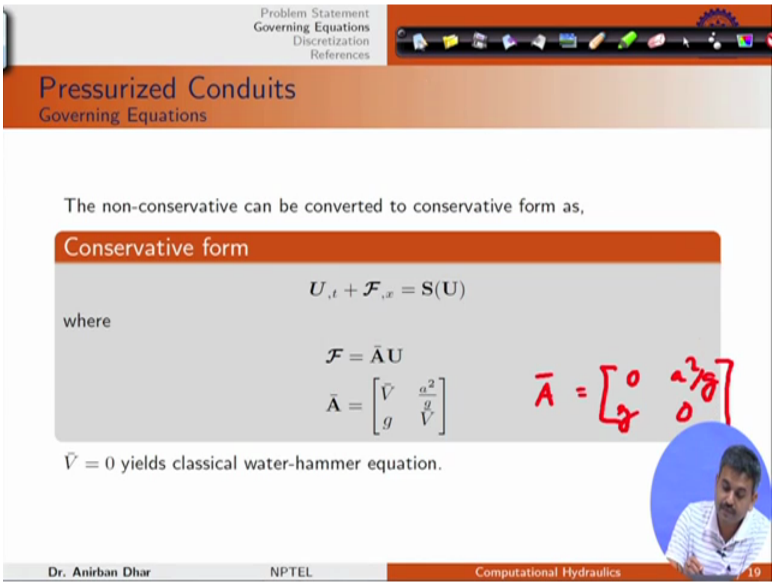

Now the governing equations can be discretized by finite volume method. Let us say this is our left boundary, this is my right boundary. Now with left and right boundary I have n number of cells within my domain. And this is a typical interior cell where we have P which is a central cell and east side I have E and west side I have W cell. And east and west faces are there in this case.

The discretized form of the equation our governing equation we can write like this by transferring this F term from left to right side. And this is our source sink term.

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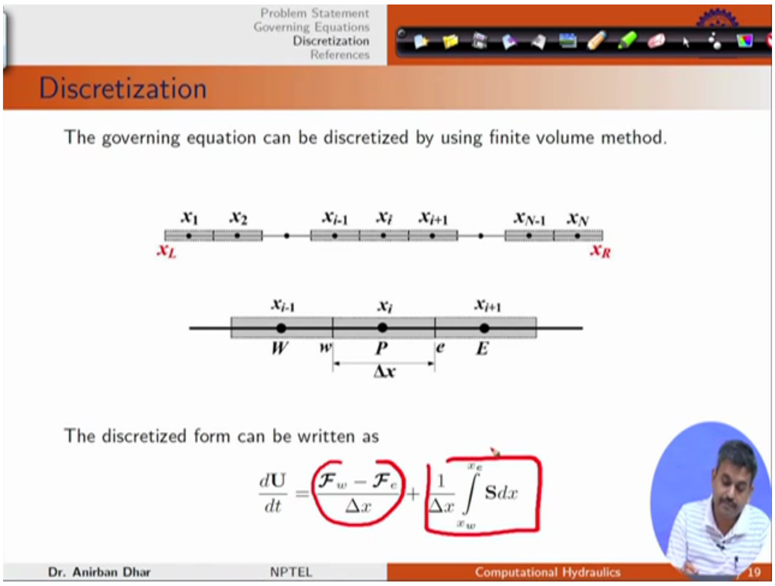

Now we have discussed this Riemann problem for one dimensional conservation laws where we are having this phi variable which was a general variable. Now you want to apply the same concept here. So Riemann problem this is our equation and U xt if we consider the value on the left side this is UL, if we consider the value of this side this is UL and if we consider the value on the right side this is UR.

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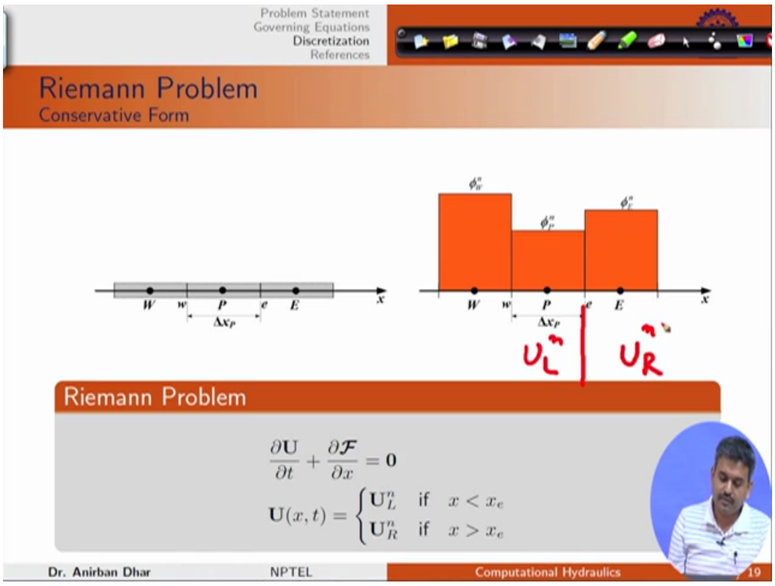

So this continuity is there. So this is called as Riemann problem. This is xe so obviously if x is less than on this side then we have UL, on this side we have UR.

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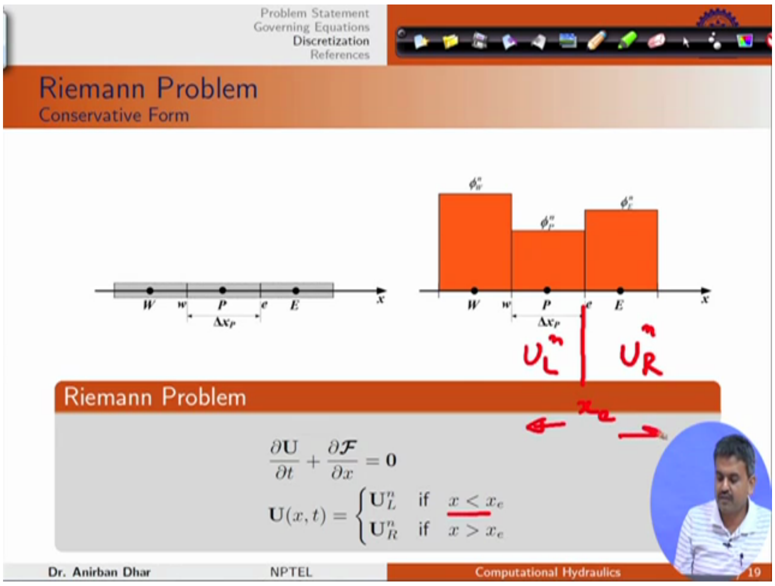

Now eigenvalues of the matrix A bar which is the approximated matrix there can be calculated like this. Lambda 1, lambda 2 these are V bar minus a, lambda 2 equals to V bar plus a. Applying Rankine Hugoniot conditions across lambda 1 and lambda 2 like this. Lambda 1 is having plus value here, lambda 2 is minus value. With this condition we can write our governing equations for a particular face using our Riemann problem.

So what is this? On left hand side on east side, this is my east face, whatever value is there on the left side, e minus L. This is L, this is R. V minus VL, He minus HL, V minus VL.

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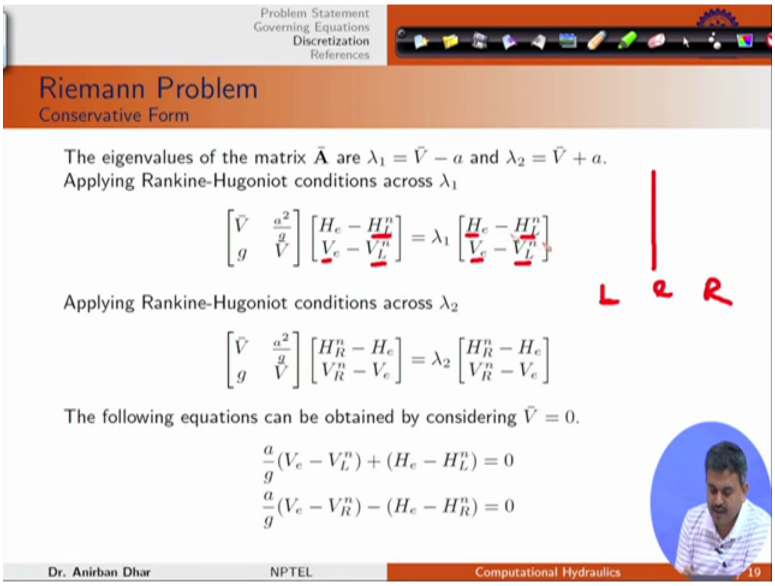

This is same but lambda is multiplied here. Now in this case we will consider that V equals to zero. During calculation of this one at this face we have a zero value.

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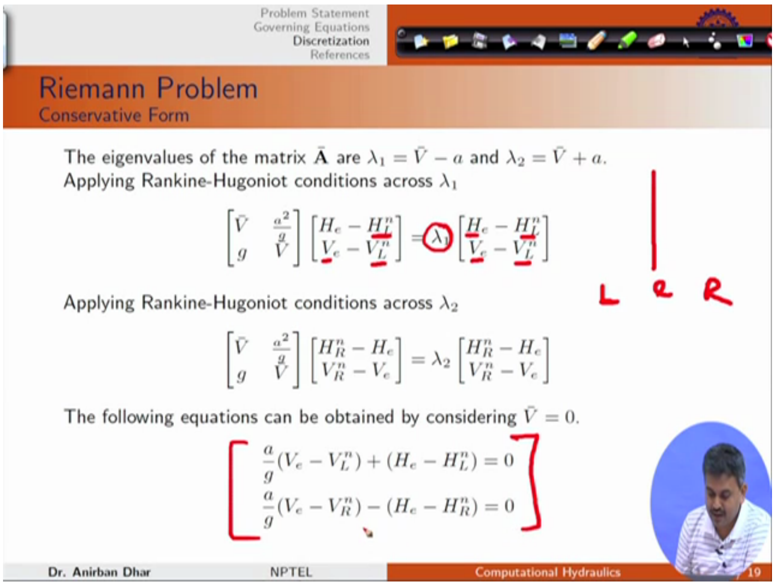

Let us consider that. And with this calculation we can get these two equations. Lambda 1 obviously this will be minus a, lambda 2 this will be plus a and these values will be zero here.

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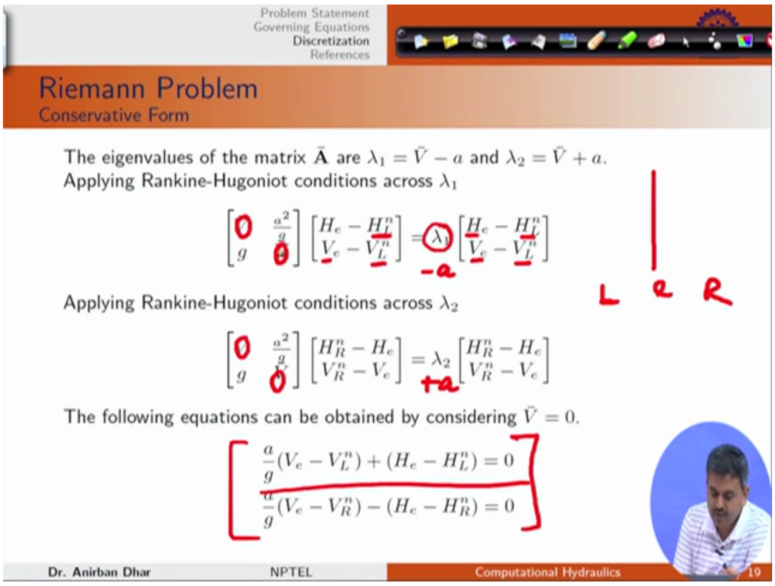

So if I apply this condition we can get these two equations. Now for all internal P cells the following solution can be written. So this value is directly coming from our Riemann problem. We have HL HR whatever value is there on the left side right side. Based on that we can calculate He and HV values. Obviously we can further simplify this with two matrixes where B equals to half 1 a by g, g by a, like this. That means on the left side whatever value is there, from the right side whatever value is there.

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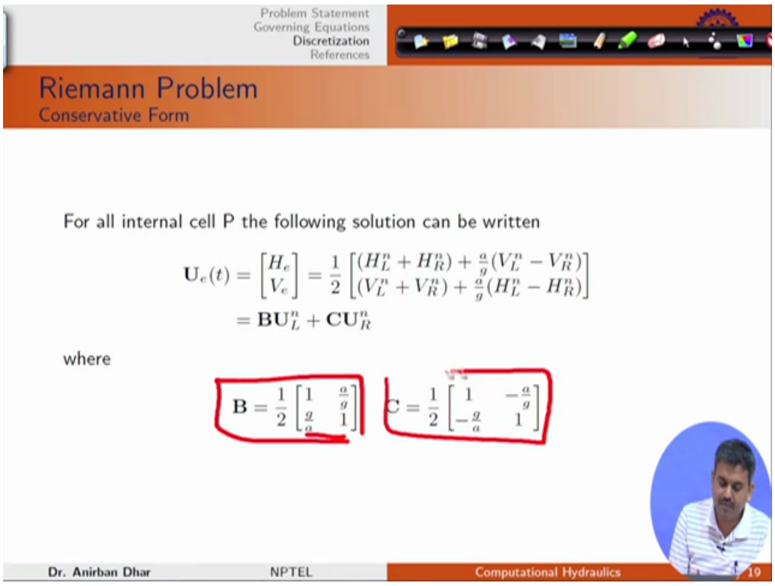

With this I can write the general form of the flux. Flux is Ae bar at east face that means A should be calculated at the face itself. The (ma) matrix which is essentially a Jacobian matrix it should be calculated at face only and this value is east face value. So I can just multiply from my previous slide. This is Ae bar into BU L n plus CU R n. So with this information I can calculate the flux there.

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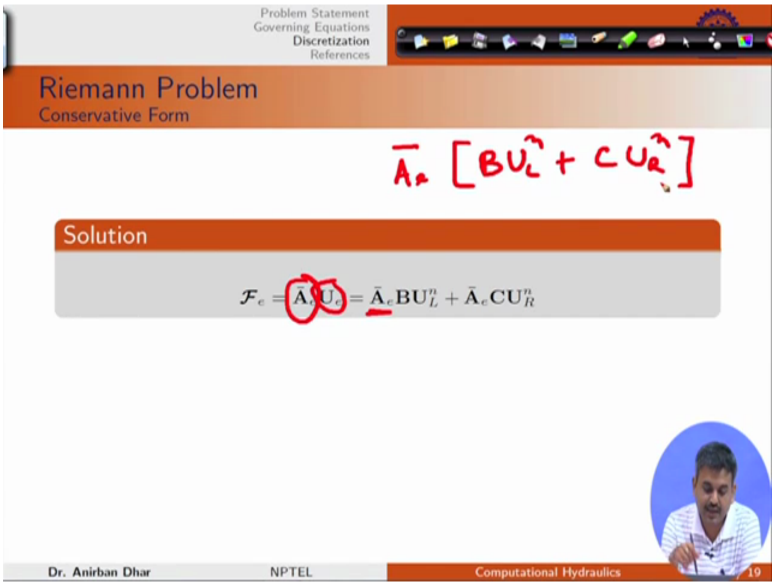

Ae can be computed by approach 1 by setting V equals to zero at any face or by setting V bar equals to half VP plus VE or approach 3 by setting V equals to VE from Riemann problem or Riemann solution.

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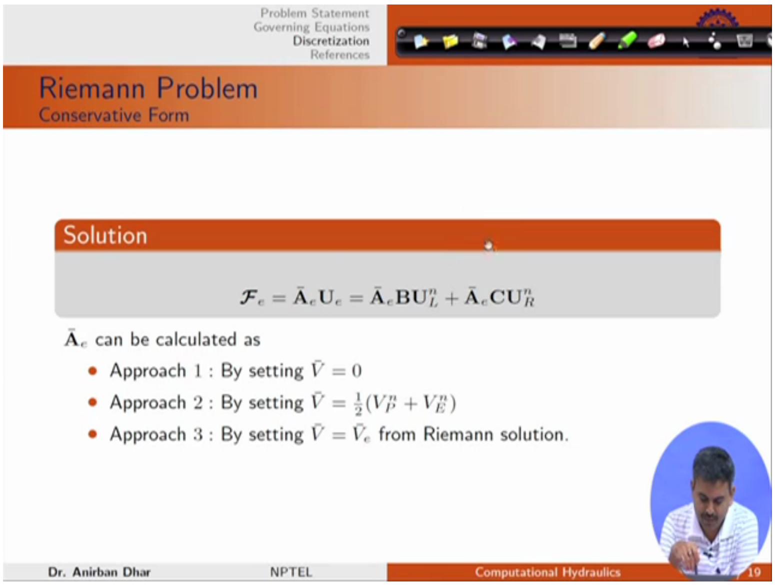

For interior cell in the first order Godunov approximation we already know from our lecture that on the left side we will have UP and on the right side he will have this UE. So with this information if I calculate the numerical flux I can simply change this subscript here. Instead of left I am writing it as UP and right as E.

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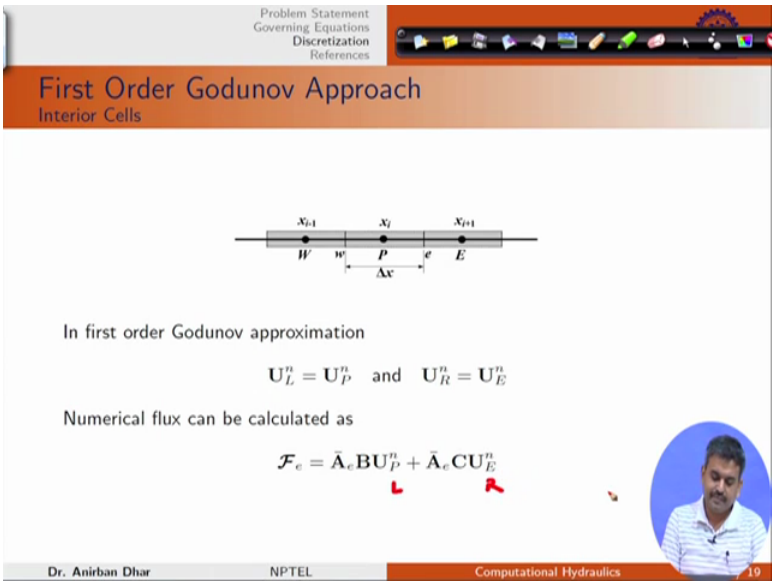

So with this I can calculate the flux at E face. So for flux calculation first we need to utilise the Riemann problem solution and we need to calculate the Jacobian matrix at the face based on different approaches. And further what is required is specification of UL and UR. So we can approximate UL and UR based on different schemes. In this case for Godunov approximation we have UL equals to UP, UR equals to UE.

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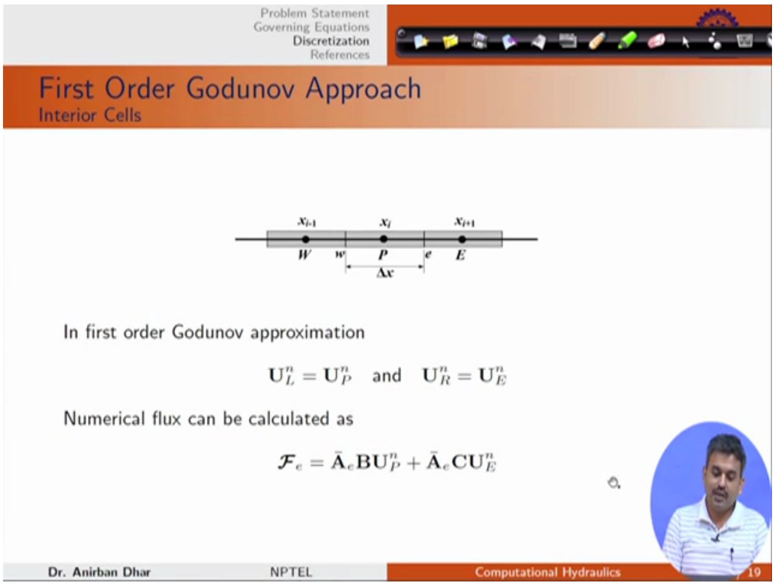

If you consider left boundary so left boundary we do not have a specific equation. So we need to find out that. So Riemann invariant associated with the negative characteristic line so we consider negative characteristic line for left boundary value. So in that case this is LB, VLB and next face we have H res. What is this H res? In this case let us say that at the starting left boundary I have specified reservoir head available H rest.

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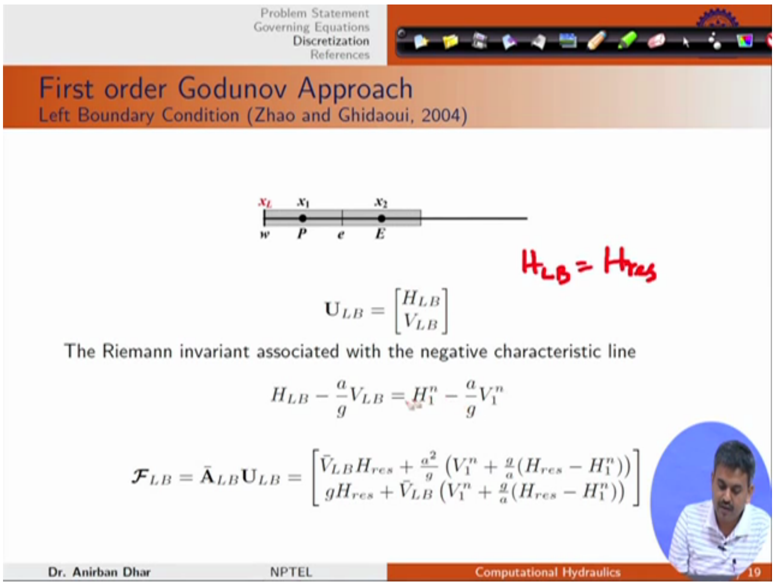

So from this equation I can get H res minus a by g VLB equals to H1n minus a by g into V1n. So from there I can get this VLB value. So if I utilise this VLB value which is H res minus H1n and plus a by g V1n divided by I have this a by g. Obviously in this case I will get the resulting thing as no coefficient for V1 and I will get only coefficient of this one as g by a.

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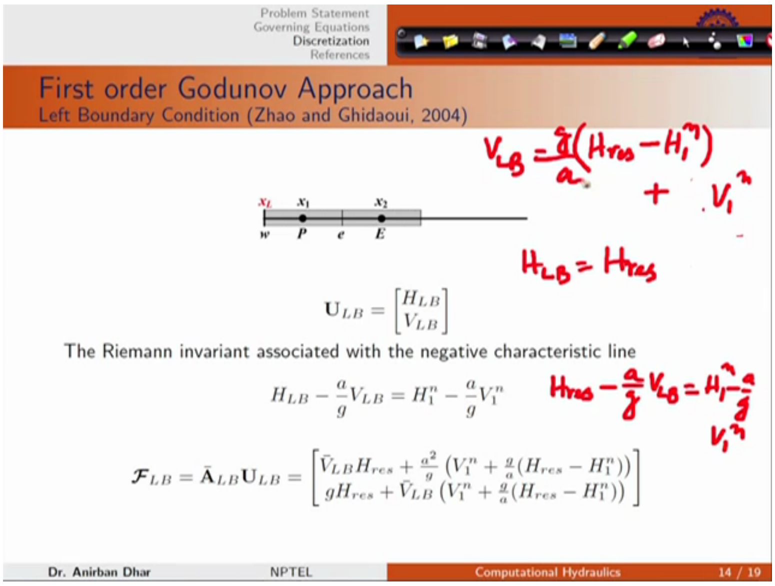

That is this quantity there.

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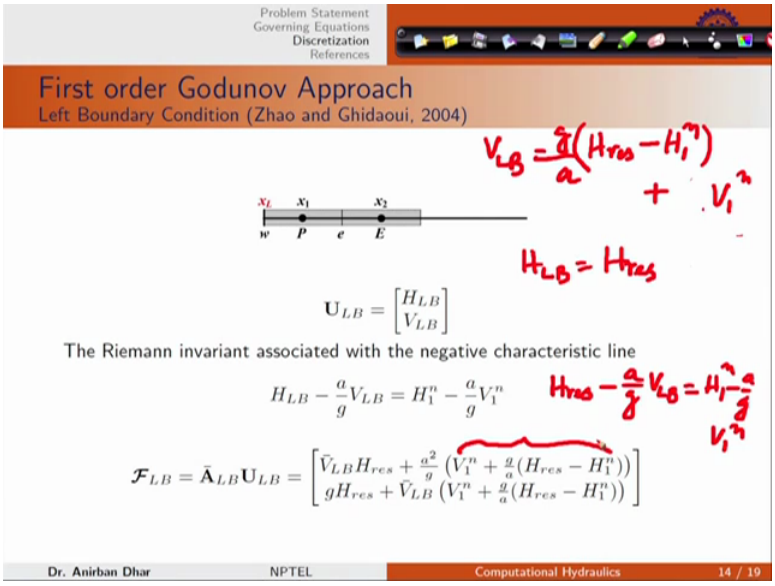

So in this case again I am considering that I have VLB present there. So in that case for left boundary I need to calculate this Jacobian matrix there. So this is VLB bar value into I have a square g, this is g, this is VLB bar.

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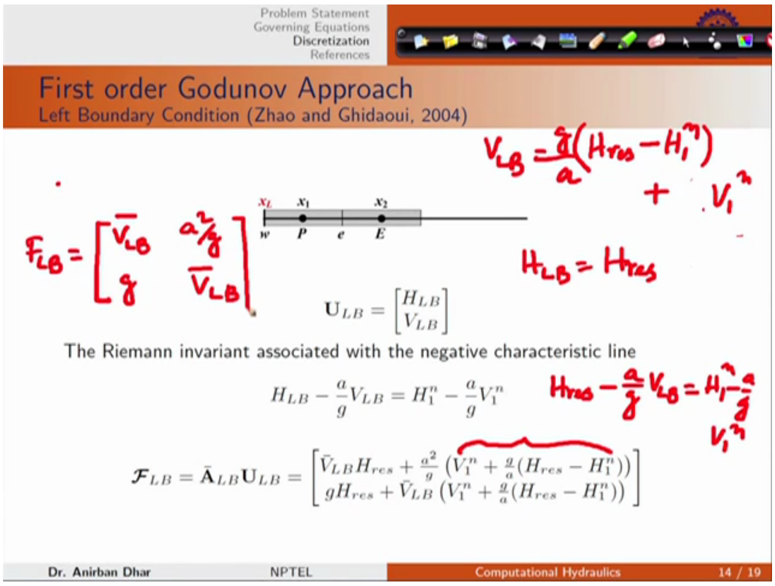

So from there if I multiply my H res which is known value at upstream section because a tank or a reservoir is connected. So I have known pressure level but velocity VLB this quantity I can directly calculate there based on this equation.

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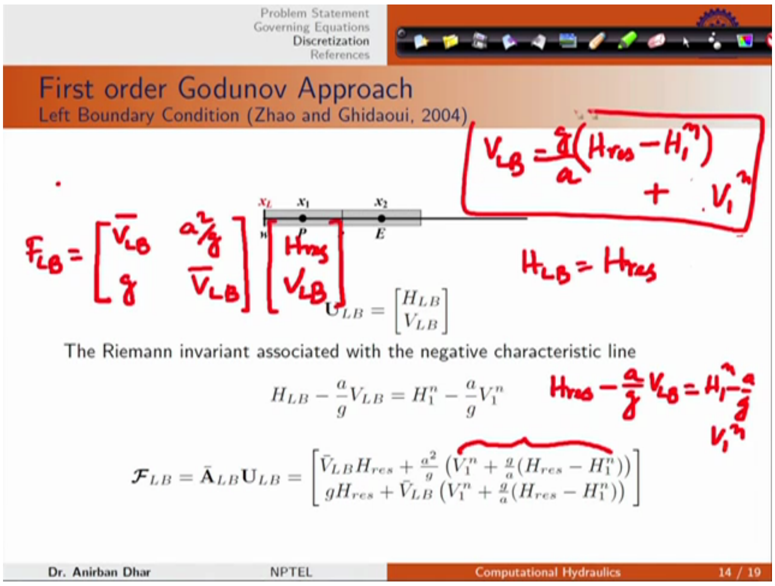

So remember that this VLB and VLB here there is difference. Obviously one approximation V bar can be this one.

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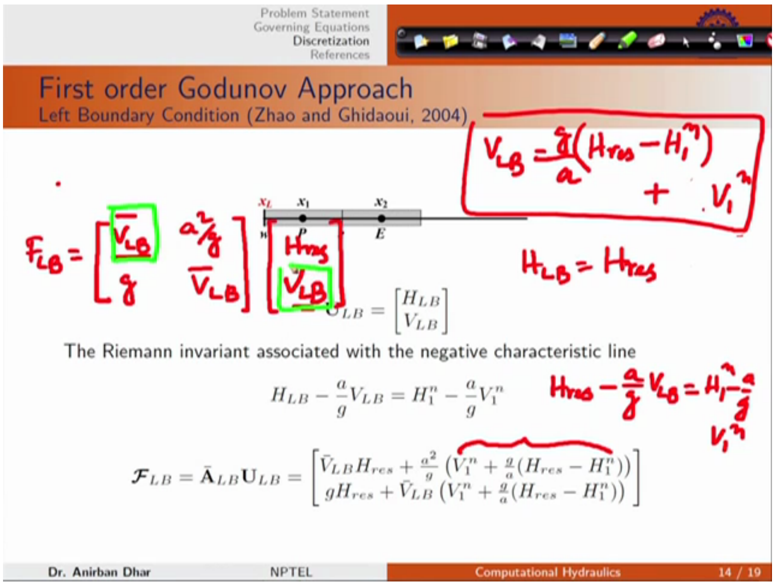

So I can get this flux values at interface for upstream boundary. Similarly if we have at no advective term or we can neglect advective term then VLB this bar this should be zero.

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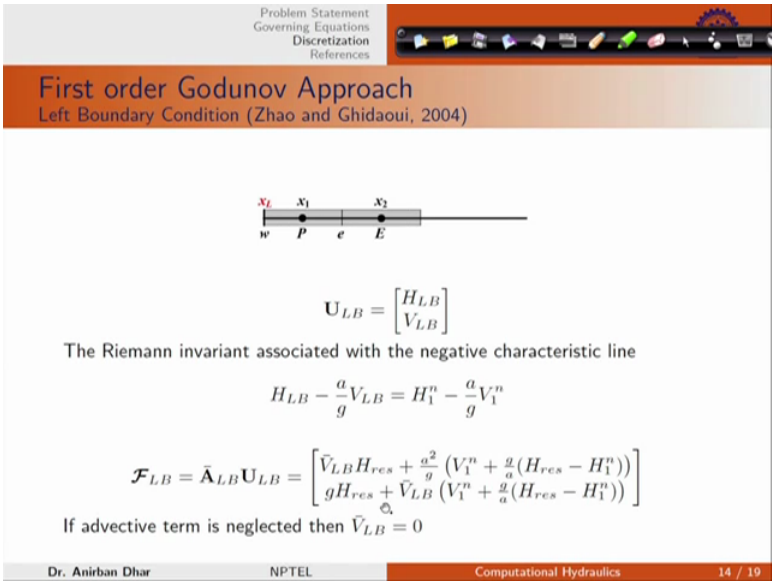

The right hand boundary again we have HRB and VRB. Right hand boundary this invariant associated with the positive characteristic line. So this can be calculated by Riemann invariant with a condition. This is not directly head flow condition. From one side we have head but other side we have if we have closure of wall so VLB will be zero.

So in that case where we will be having VRB as zero, we can directly get the value there. So in this case the matrix will be a square by g. This is constant term. We will have VRB, this is VRB, this quantity is g.

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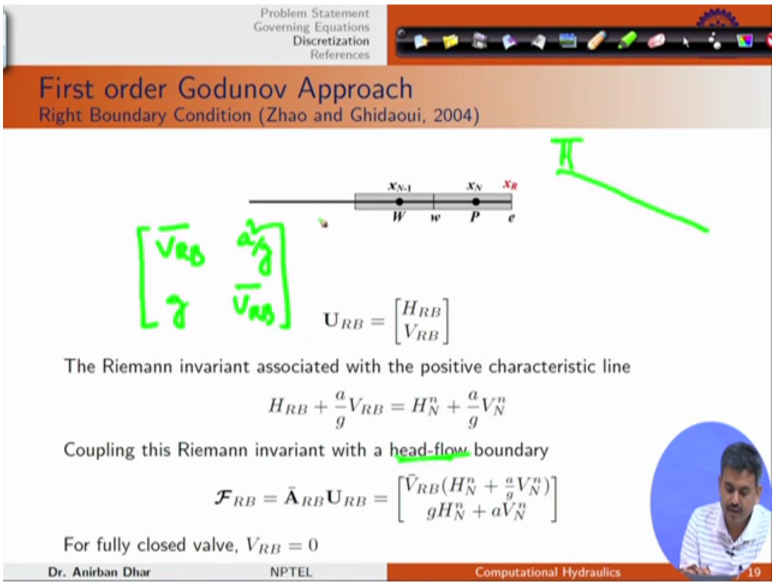

Now on the right hand boundary we know that VRB due to closure of wall we have zero value. If I put this zero value here obviously HRB on this side will be HNn plus a by g, this VNn.

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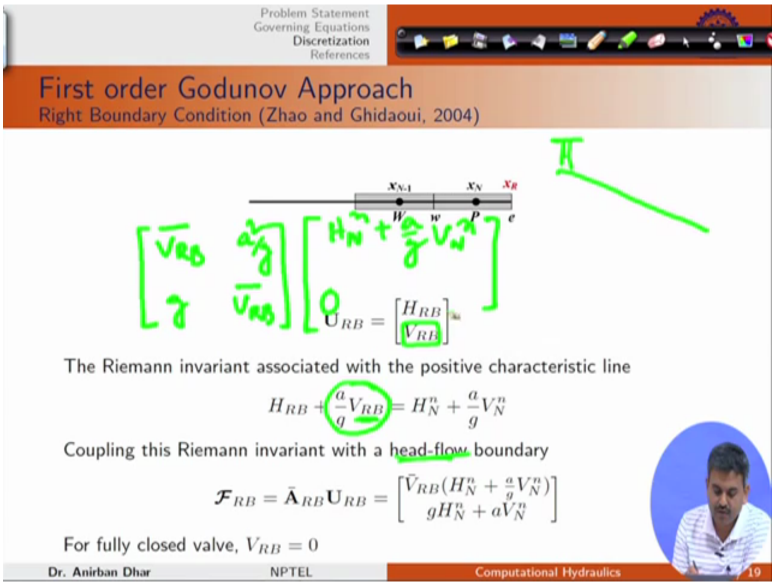

So after multiplying this on this side I will get these terms directly. This is multiplied by g and g will cancel here. So this is the value there on the right hand boundary.

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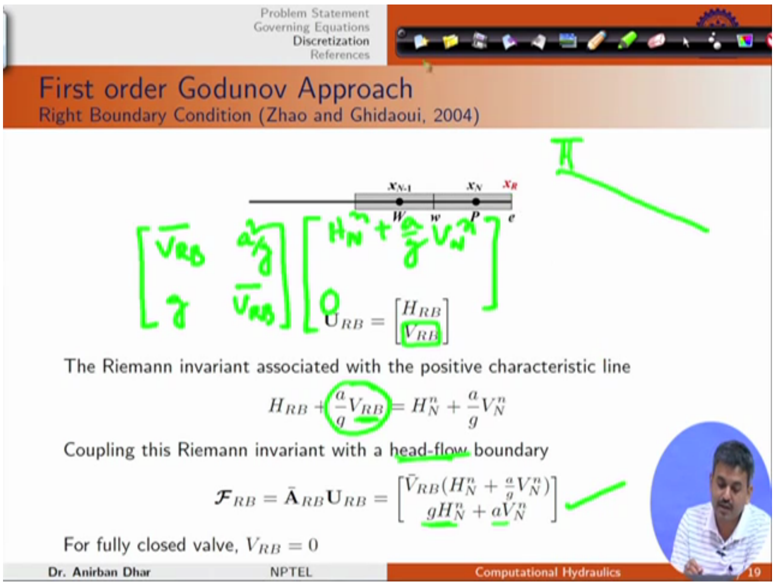

Similarly by specifying different conditions on the boundary side we can solve this problem. So next part is numerical discretization and final solution. After finding out these flux values we can update this future time level that is n plus 1 in terms of nth level values.

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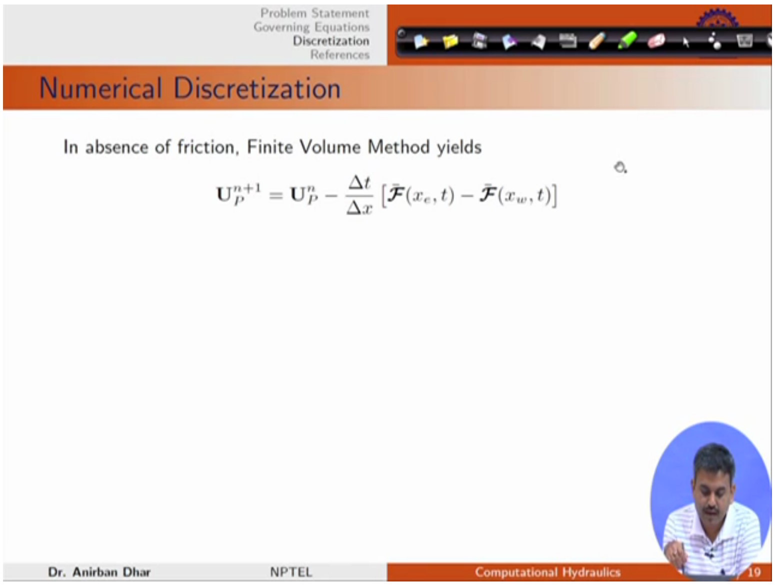

And the previous one was for in absence of friction we can directly calculate like this. But if we have friction source sink term on the right hand side so first step is without source sink term, second step is with source sink term based on the first step and final step we have this final calculation which is again with source sink term there.

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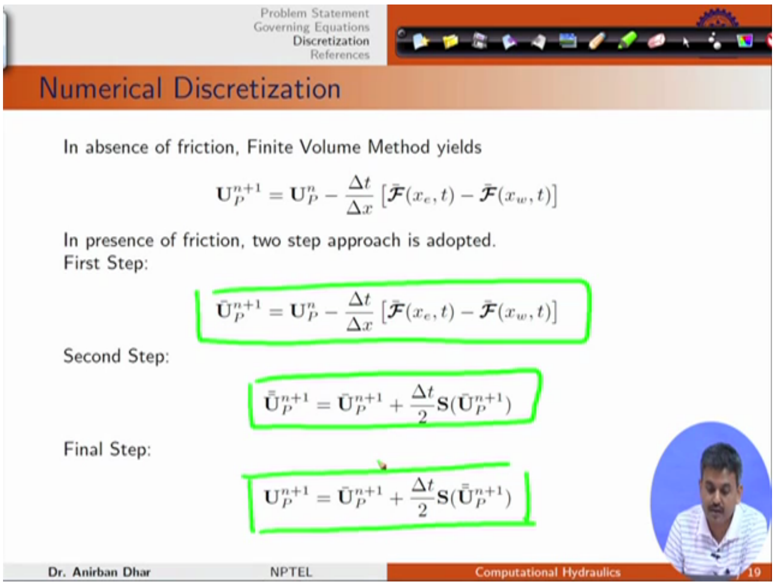

So we need to satisfy stability condition for this problem because this is explicit approach and first order approach. So CFL condition should be satisfied, a delta t by delta x, this should be less than equals to 1.

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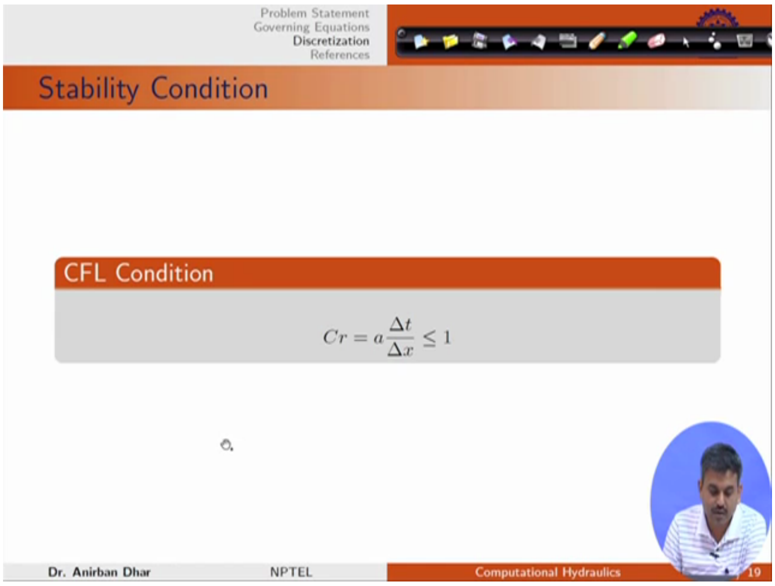

So with this approach you can solve unsteady state pipe flow problem with simple Riemann solution and Godunov approach. Thank you.