Computational Hydraulics
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Lecture 47
Steady Flow in Pipe Network

Welcome to the lecture of the course computational hydraulics and this is a new module, module number 5. And in this particular module I will be discussing flows in pressurized conduits and this is unit number 1, steady flow in pipe network.

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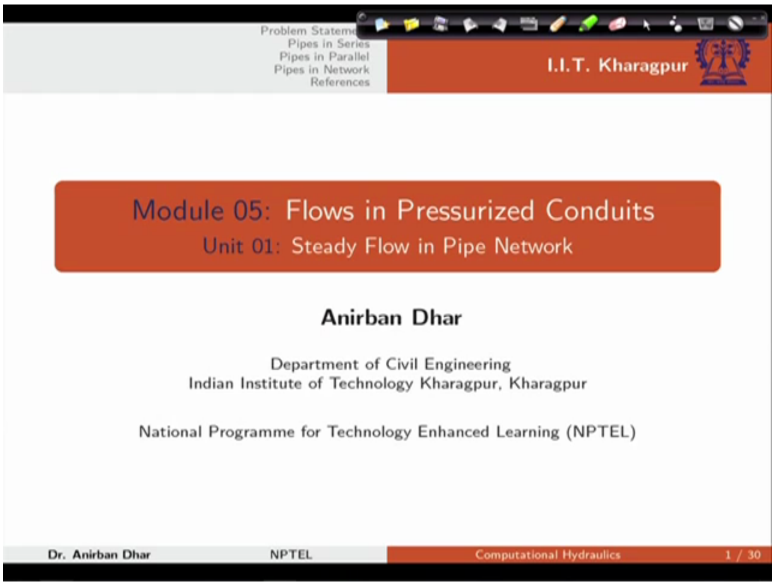

In our previous module we have discussed the surface water hydraulics and in surface water hydraulics mostly we have covered the free surface flow. That means in most of the cases our flow depth, in some cases velocity that was prominent, elevation head these are important. But in this case specifically pressure head component is the most important part. Learning objective for this particular unit. At the end of this unit students will be able to simulate steady flow in pipe networks using Hardy Cross method.

Hardy cross method which is the most popular method for steady state pipe network flow. In that one we consider a steady discharge condition and we start with some arbitrary values but we need to assign those arbitrary values by satisfying the continuity condition at different junctions.

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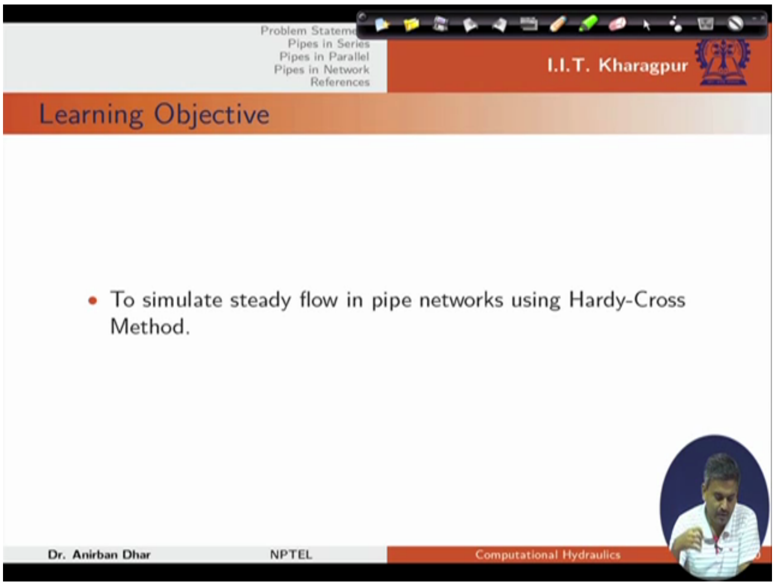

So let us discuss the basic of this particular method. If we see our problem definition to solution, this is the whole structure of our course. We have discharge condition Q in this particular case and it is varying with pipe. So if i is the pipe number then we are interested in defining or finding out the discharge value at different pipes. So obviously in this case whether the flow is from positive or negative direction that is important.

In our surface water hydraulics we have defined a specific kind of flow direction scheme based on node numbering. Let us say that we have a numbering scheme in our surface water hydraulics for Lth channel reach. If this is the first section L1 and this is our end section NL plus 1, in this case we have considered initially in that flow is from left to right.

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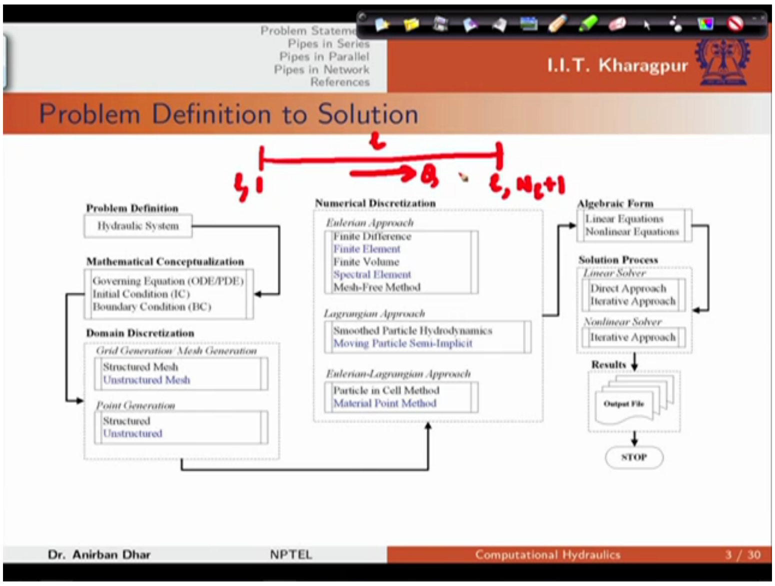

And in certain cases where we have reverse flow situation like our loop channel network, in that case we have considered different nodes and for those nodes we have tried to find out the discharge conditions and at the same time we have also considered internal points. So this was our structure. With this structure we have tried to find out reverse flow situation. In reverse flow situation for a particular configuration, let us say for configuration 1 if you check we have considered the flow is from this direction.

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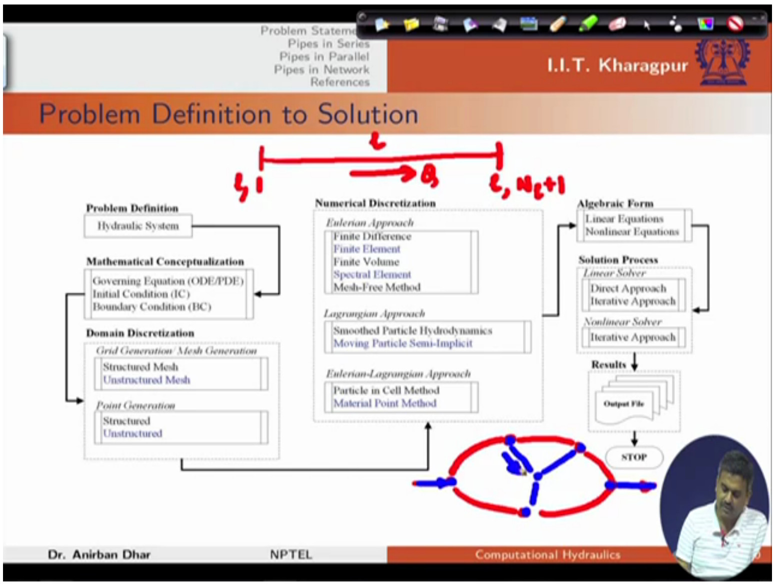

So this was our node numbering. This was node 1, 2, then 3, 4, 5 and 6. With this we have calculated everything. So in this case whether the flow is from 3 to 5 or 5 to 3 according to that convention we have defined whether this discharge value is positive or negative.

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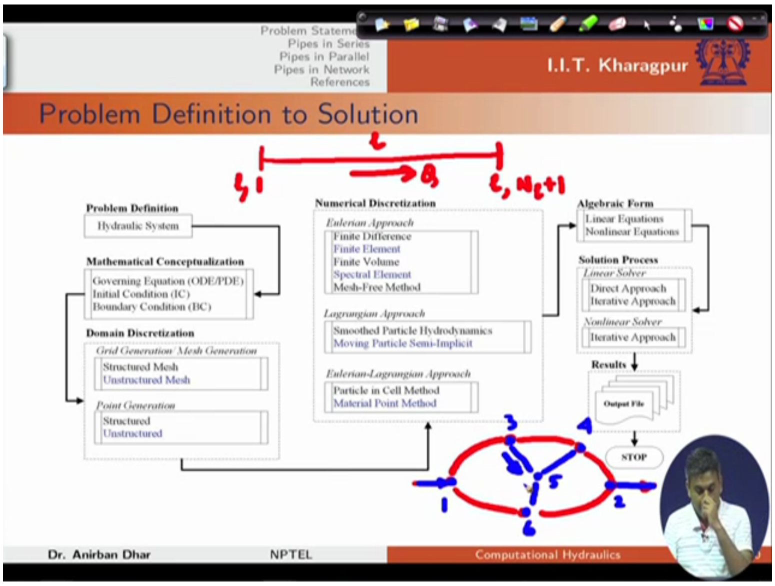

Now in pipe network case or specifically steady state pipe flow condition according to Hardy Cross method we will consider clockwise direction as positive direction for a particular loop. What is loop? Loop, if we have certain number of junctions, let us say we have three junctions in case of a pipe network then these are three pipes connecting node number 1, node number 2, node number 3.

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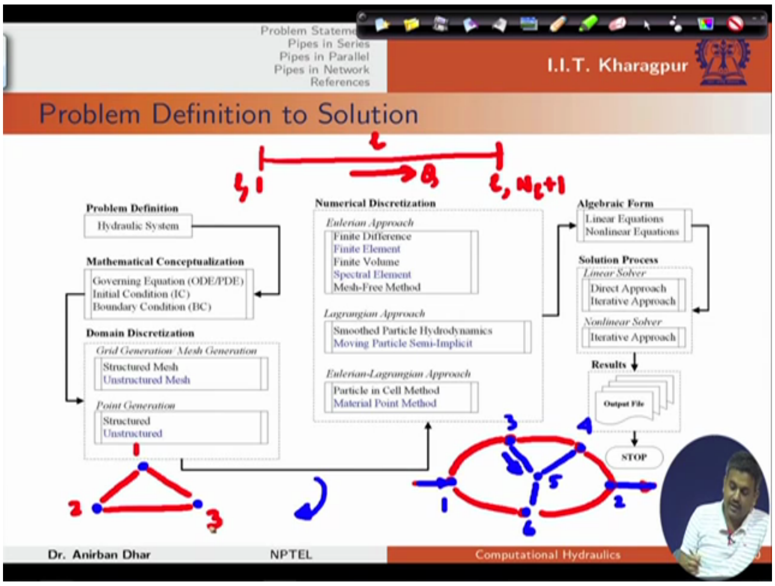

Then we will consider that the flow direction is positive for this loop or closed loop in this case and it is clockwise positive. So let us say that initially I am specifying some numbers as initial guess for my network. Let us say that at this node number 3 I have a demand of 100 units and flow which is coming from 2 to 3 it is 200. So the movement on this side should be 100 because inflow and outflow this mass conservation should be there in the network.

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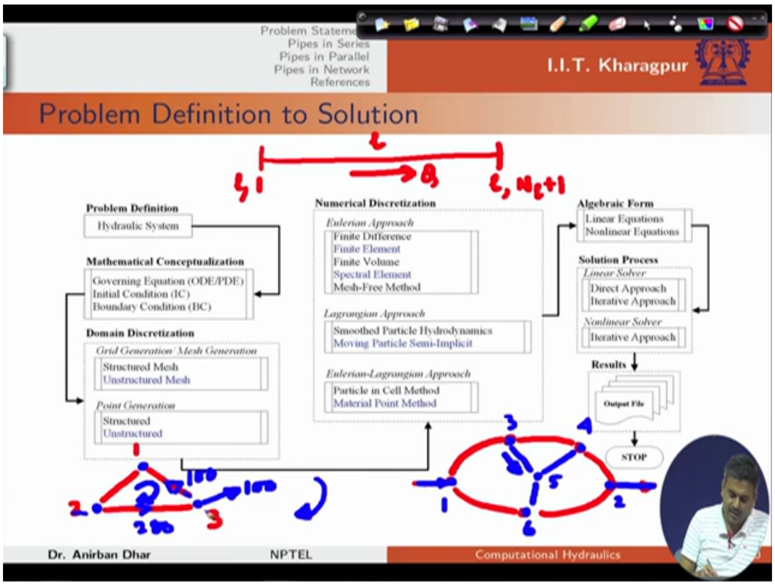

And if there is some other thing on the some other side let us say on this side we have 50 units of input. So all total 150 units we are getting from this one and on this side we have 200. So obviously this will be 50 on this side.

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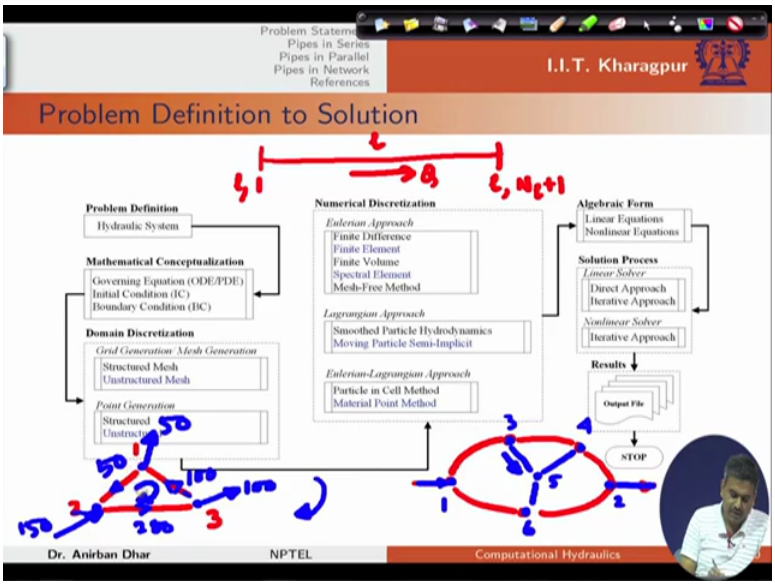

So in this case if we consider this discharge flow direction or initial discharge flow directions these are anticlockwise. So for this particular loop with the nodes 1, 2 and 3 we can consider this discharge values are negative during calculation process.

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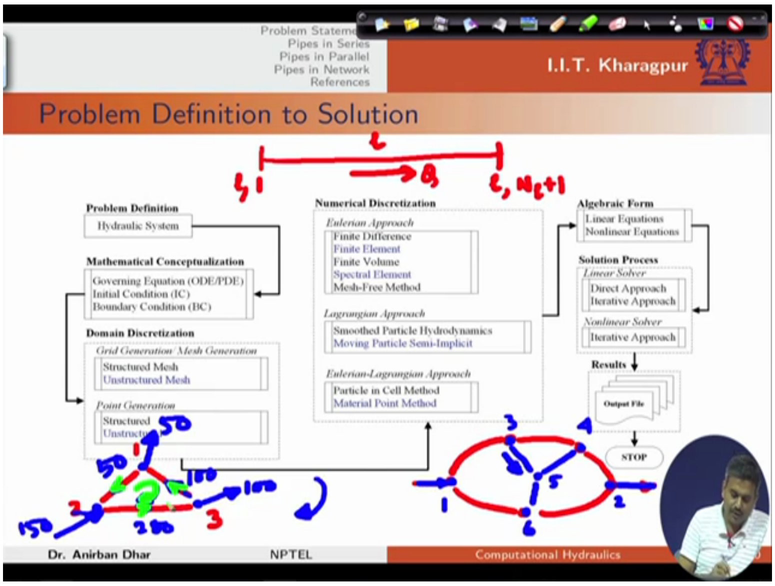

But if we define a different kind of structure let us say these are the nodes, the same problem with node number 1, node number 2, node number 3 and we have some inflow condition 150 units and demand at this node is 50, demand at this node is 100. So in this case let us say that initial distribution of our head that is 100 on this side, 50 on this side because 150 is entering into the system or I can consider this as this is very less, this is 125.

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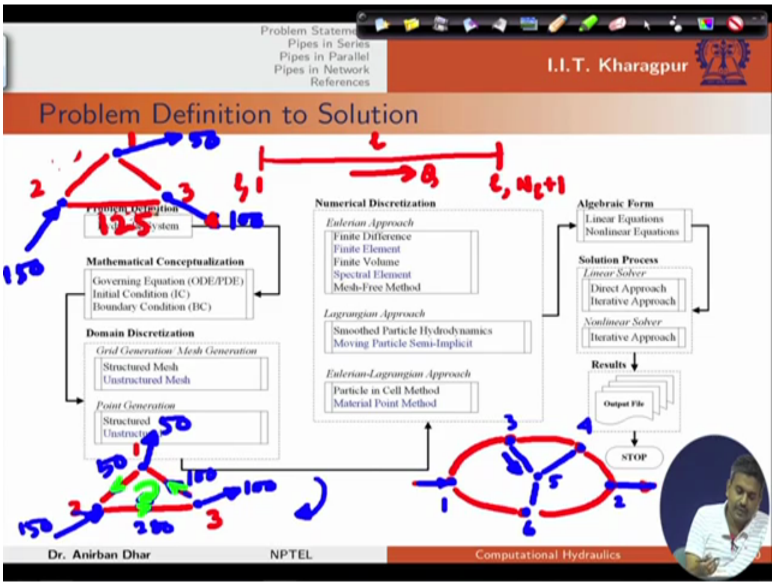

So remaining is this is 25, 125, demand is 100 on this case. So movement will be on the side of 25.

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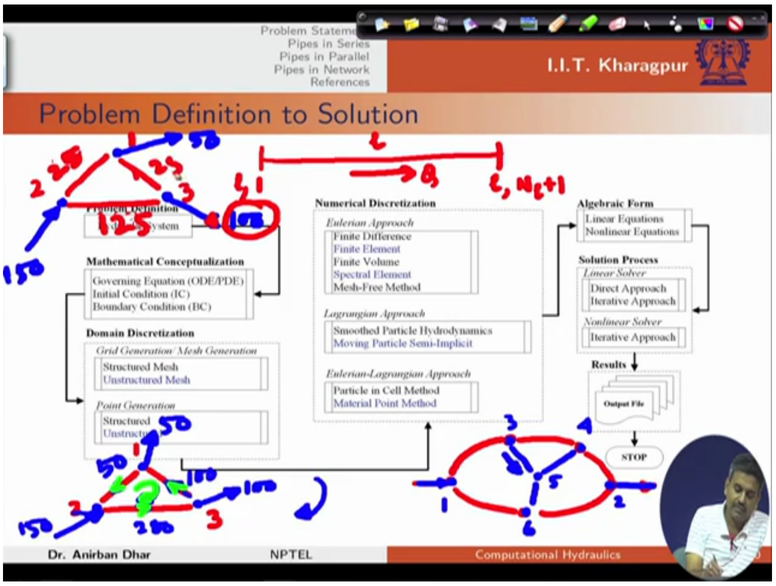

So if I consider this problem so flow direction in this case is in this direction it is clockwise, in this pipe it is anticlockwise, in this pipe it is anticlockwise. So the nodes or the pipes which are connecting the nodes this 2 and 1 specifically this particular pipe will consider that flow is positive and for these two pipes we will have negative discharge conditions.

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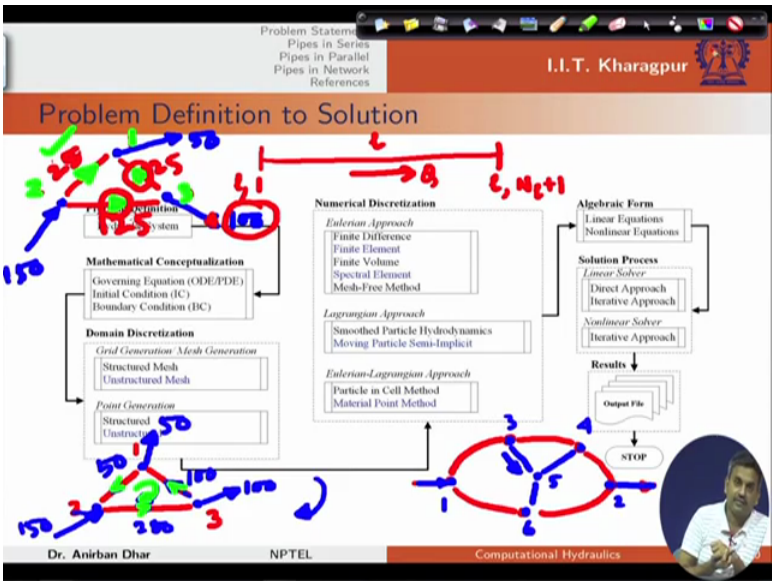

So if I compare the channel network flow condition that our specification of discharge direction depends on the direction of discretization. But in case of pipe flow we have fixed convention. The convention is that we will consider the flow in a clockwise direction for a particular loop as positive and flow which is in the anticlockwise direction will consider that as negative. Now in this case also we will see that we need to solve some nonlinear equations.

To solve nonlinear equations we will require nonlinear solver for our usual Newton Raphson method. But in Hardy Cross it is a modified Newton Raphson kind of method.

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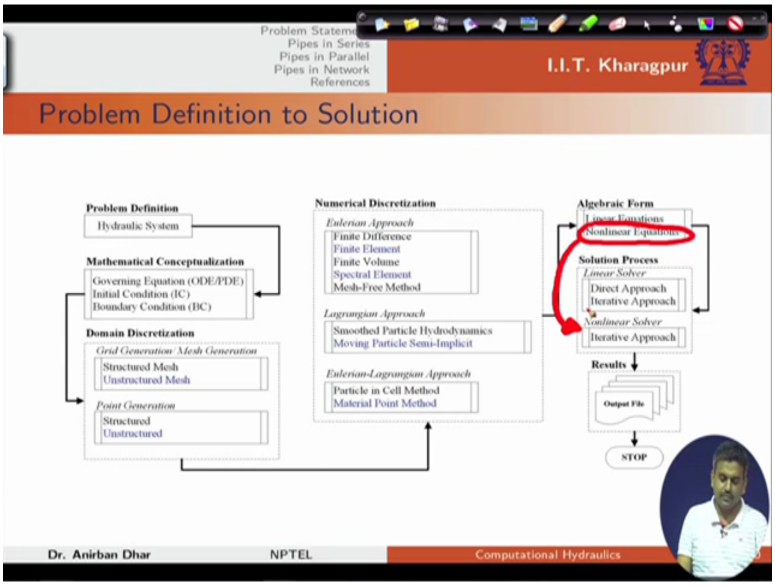

So let us consider this pipe network which is typical pipe network connecting all total seven nodes in this case. On this side we have tank, on this side we have one pump connected to this and difference in elevation is there for these two tanks.

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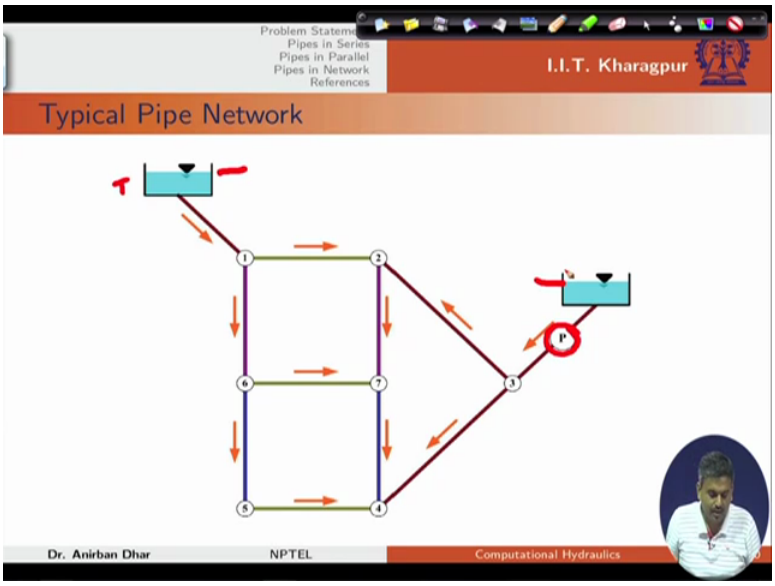

Now we have all total 1, 2, 3, 4, 5, 6, 7 these seven nodes. With these seven nodes we have 1, 2, 3, this is 4, 5, 6, 7, 8, 9 and 10 pipes. And with these 10 pipes we have one extra pump there. From this pump again we will get one discharge condition. So if we consider this problem we need to find out 10 discharge values starting from 1 to 10. Additional discharge is required for this pump, for P.

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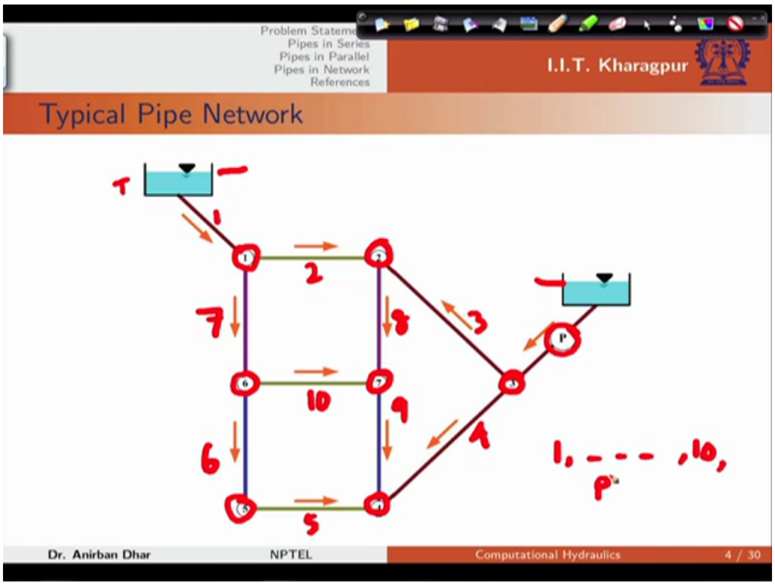

So we need to see how we can incorporate pump or difference in elevation in case of our pipe network and how we can solve that using our usual Hardy Cross method? So let us start with one typical problem. In this case we will consider these two tanks which are having this elevation 50 metres and 30 metres. Obviously del H or difference in elevation is 50 metres. This is 20 metres. 50 minus 30, this is 20 metres. This is the difference in elevation.

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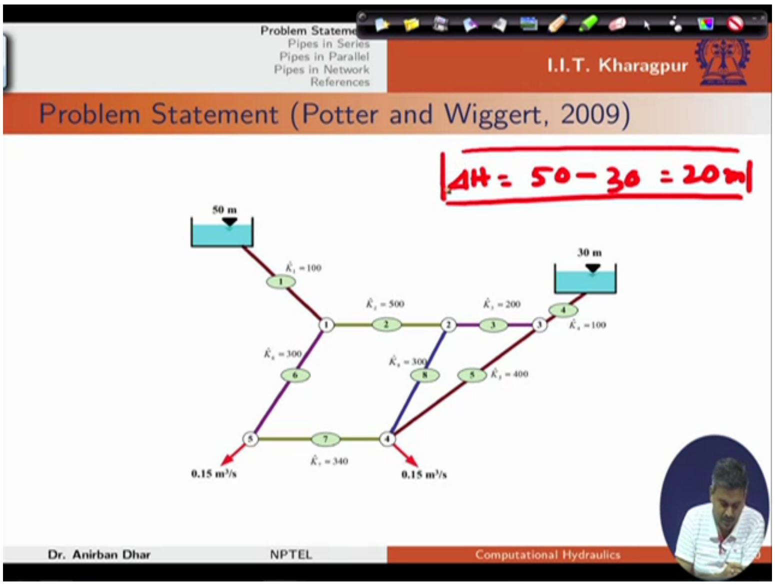

Now we have two demand nodes, node number 4, 5. The demand is point 15 metre cube per second in these case. And these values K1, K2, K3, K4, K5, K6, K7, K8 these are related to pipe. We can represent our head loss equation like this. K cap and Q to the power beta where this beta and K can be determined from different equations. Specifically this K value this depends on this pipe diameter. If we consider our Darcy Weisbach equation specifically K is dependent on area.

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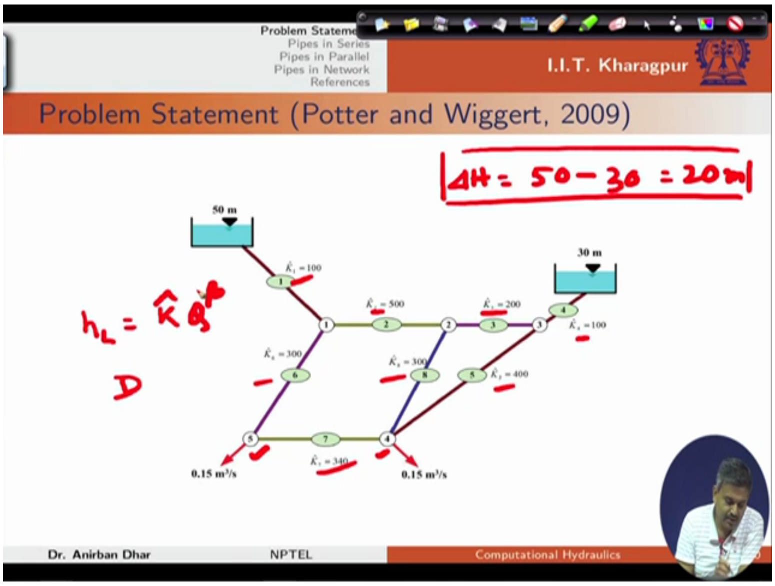

Another thing is there inside this K that is the loss or transition loss coefficients because at the junction or entrance there will be loss and we need to consider that loss in our formulation. So we can easily incorporate that using the simplified expression and K which is physical parameter, I should say it is a physical parameter because it has got different values depending on the nature of the pipe and the type of material.

So we need to solve this problem. For this problem we have 1, 2, 3, 4, 5, 6, 7, 8, eight pipes and these two are connected to (ro) reservoirs or tanks and for these tanks we have this elevation difference of or head difference of 20 metres.

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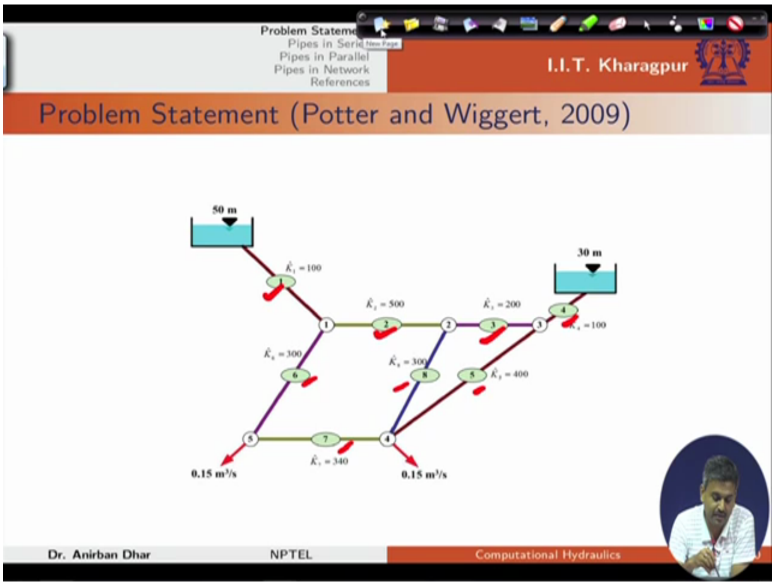

In this case we need to define our loops. So what is the first loop? First loop we will define as this pseudo loop. This is our pseudo loop connecting these two tanks. So in this pseudo loop or loop number 1 we will have 1, 2, 3 and 4.

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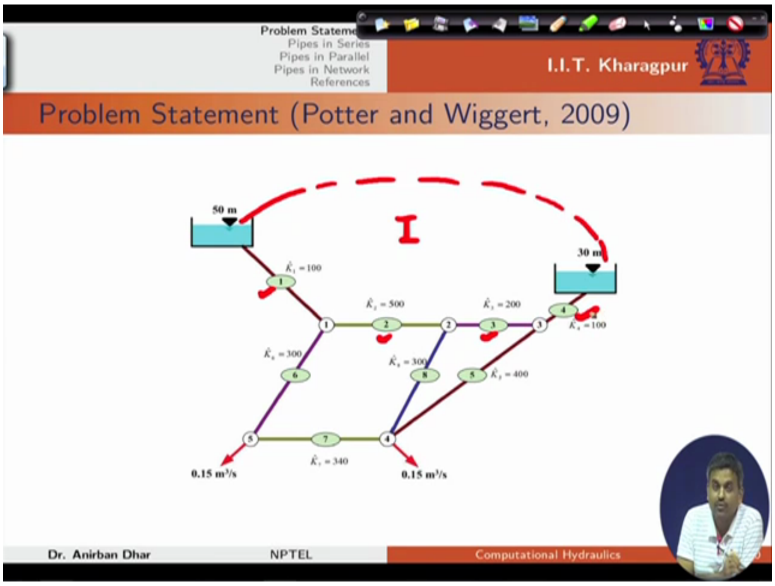

In case of this internal loop which is loop number 2 and loop number 3 we will have only internal pipes and we need to consider them during calculation. But in this i or loop number 1 which is a pseudo loop because we do not have direct connection between these two tanks. So in this case if we consider our initial discharge on this direction from tank towards this network obviously this discharge is negative.

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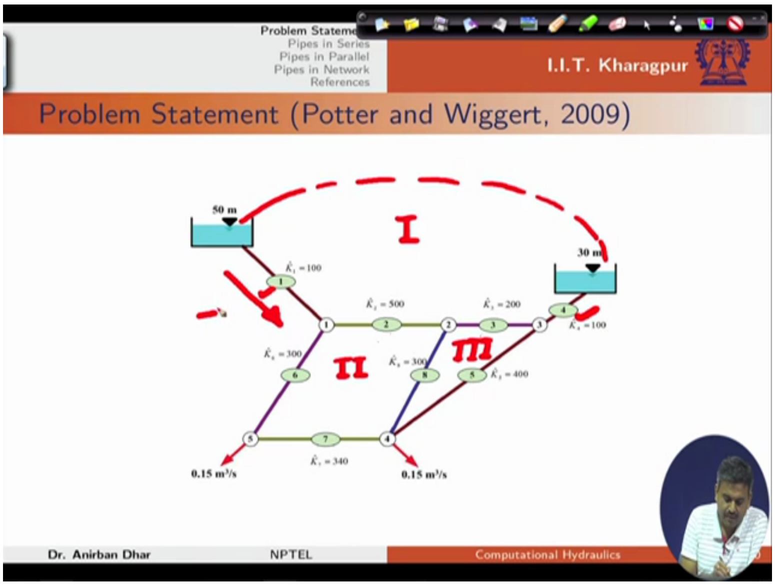

This discharge is negative because for this loop 1 we are considering a negative direction or counter clockwise direction. And for loop 2 one thing is interesting because in loop 2 if this pipe is having the flow direction like this, this is negative or negative discharge condition for loop 1 but it is positive for loop 2 because for loop 2 this direction is clockwise direction. But in case of loop 1 it is counter clockwise direction.

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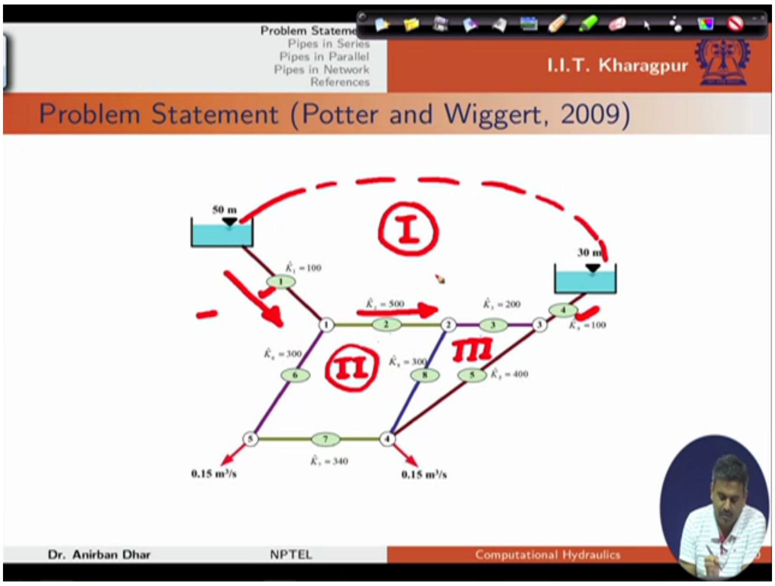

So depending on the loop we can define our sign for the discharge. So during calculation we need to consider this specific sign convention depending on the (na) nature or type of the loop. So let us start with the basic thing. Our general head loss equation which is hL equals to f L by D, V square by 2g. This is a well known equation where L is the length of the pipe, f is the friction factor, V is the average velocity, D is the diameter of the pipe.

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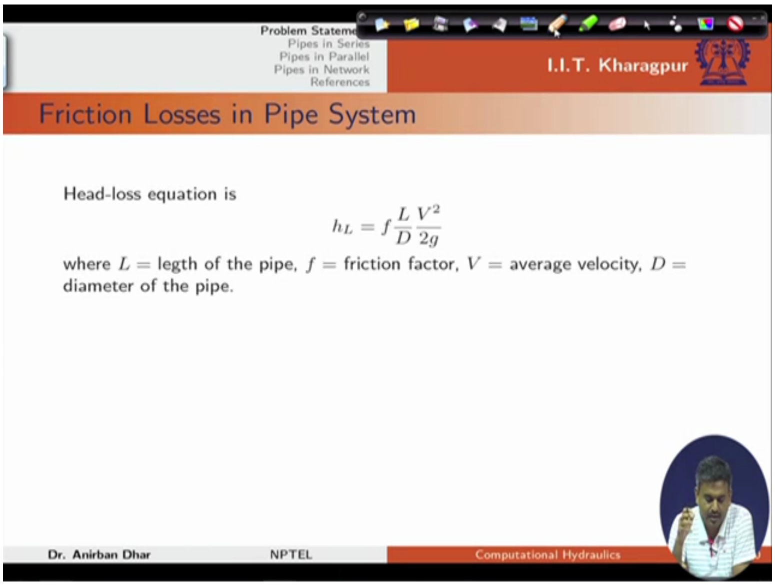

Now we can see that hL which is again head loss is directly related with this L length of the pipe, diameter of the pipe, this friction factor, so these are related to the properties of the pipe directly.

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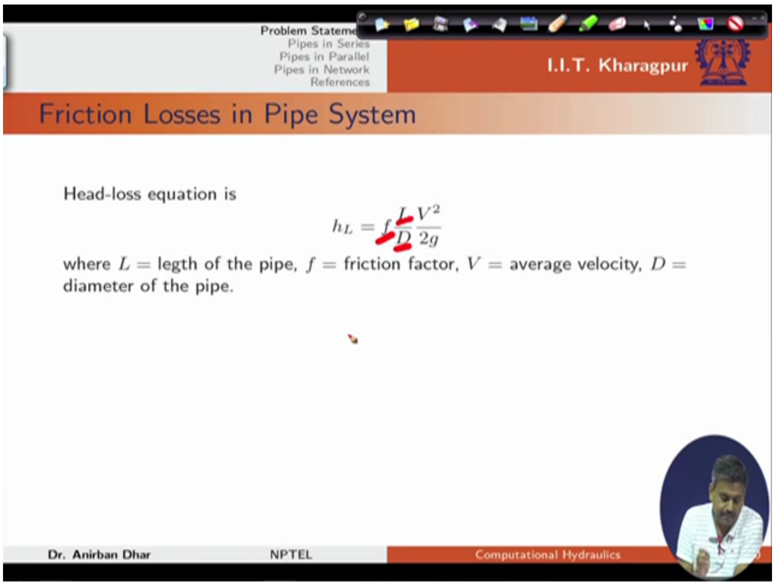

Now if we write this head loss equation in simplified form as we have used this hL which is K into Q to the power beta. But remember that in our problem (sta) statement we have utilised this K hat. K hat is nothing but the coefficient which considers the loss components in junction entrance or exit points. So that is the total loss thing. And this hL considers the loss in pipe only.

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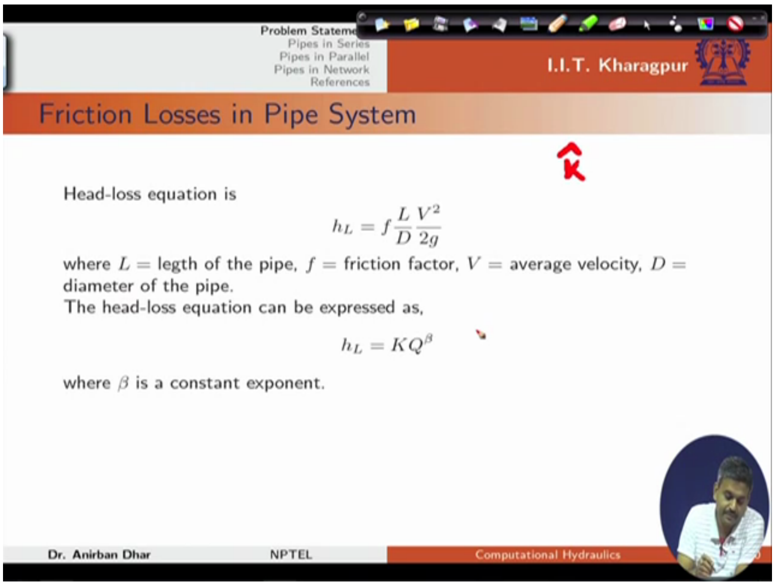

So if we equate these two so this is K Q to the power beta equals to f L by D. This is V square by 2g. Now in place of V I can write Q by A. So this is f L by D. This is Q square 2g A square. Now in A square this is pie by 4 D square. A square is in this case this is pie square by 16 D to the power 4.

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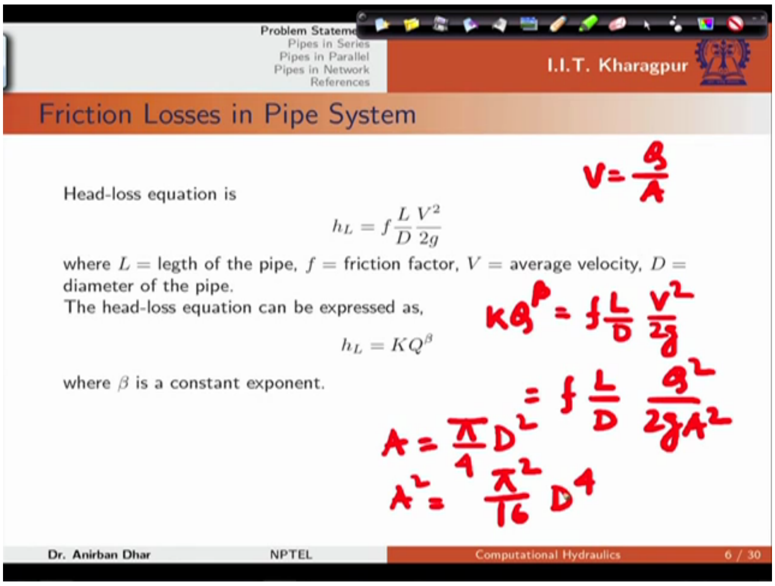

Now if we incorporate this thing here what you will get? This is fL then Q square and if we apply this here this is pie square and 8 because 2 is there. So 8, this is pie square g and D to the power 5. So pie square this is g, this is D to the power 5.

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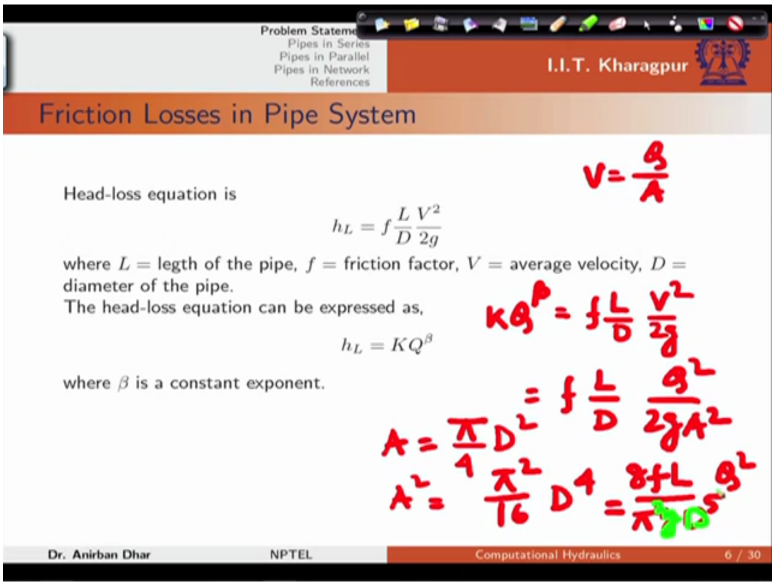

Now in this case we can say that this is our coefficient which is K and this Q square is there which means that beta equals to 2.

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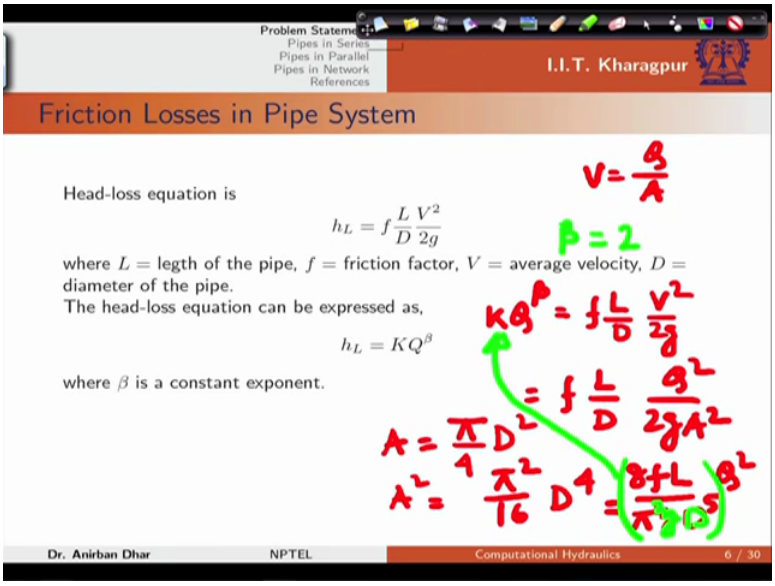

So with this convention we can say that this 8 fL by pie square g D to the power 5. Now we can directly utilise this during our pipe network calculations where beta equals to 2.

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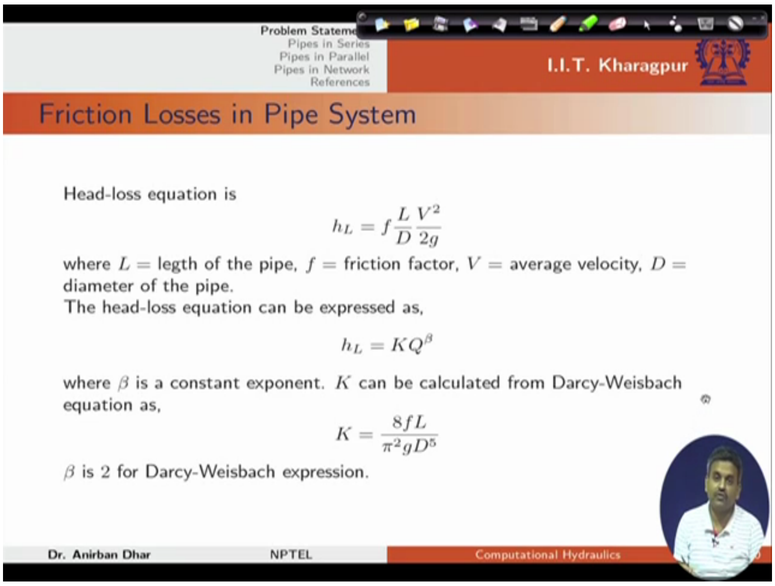

Now pipes in series. In pipes in series one thing is important that is whatever discharge is entering into the system that is coming out on the other end and obviously in case of pipe flow we are considering pressurized flow conditions. So what is this pressurized flow? Let us say we have a circular cross section for our open channel. Now if this is our free surface elevation obviously water will be there up to this.

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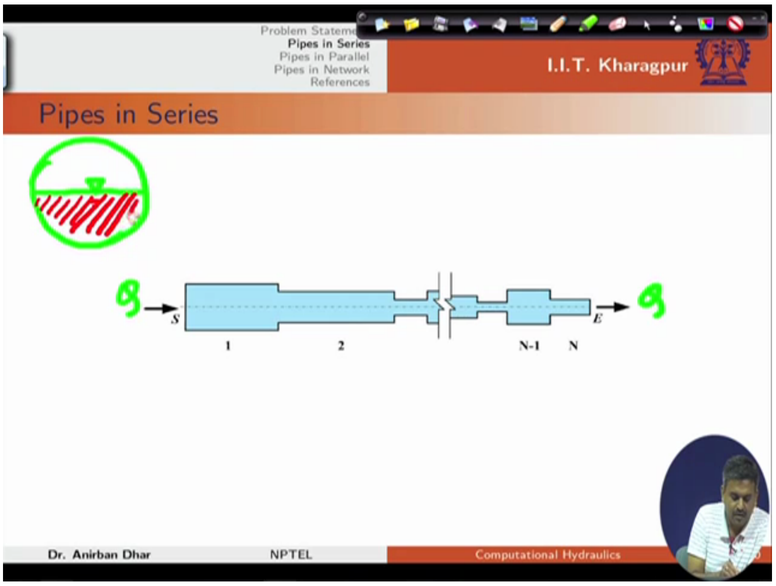

So same cross section is having free surface flow condition. This is open channel flow but it is running full with pressure then we can consider the same channel as pipe flow.

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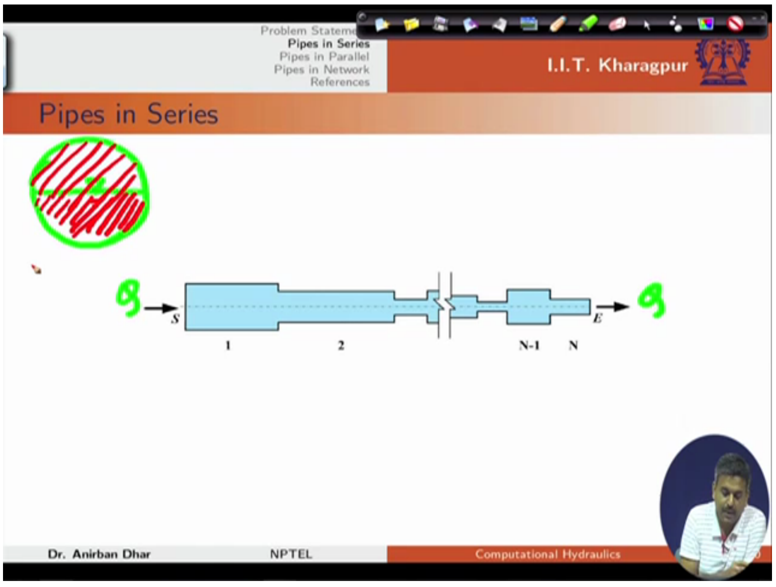

Now in pipe flow case we have starting and end points. This is end point, this is starting point. The head loss between the two end points, this HE and HS for the pipe connected in series. We are talking about pipes connected in series only. In this case this is head loss in the pipe 1, this is head loss in our pipe N.

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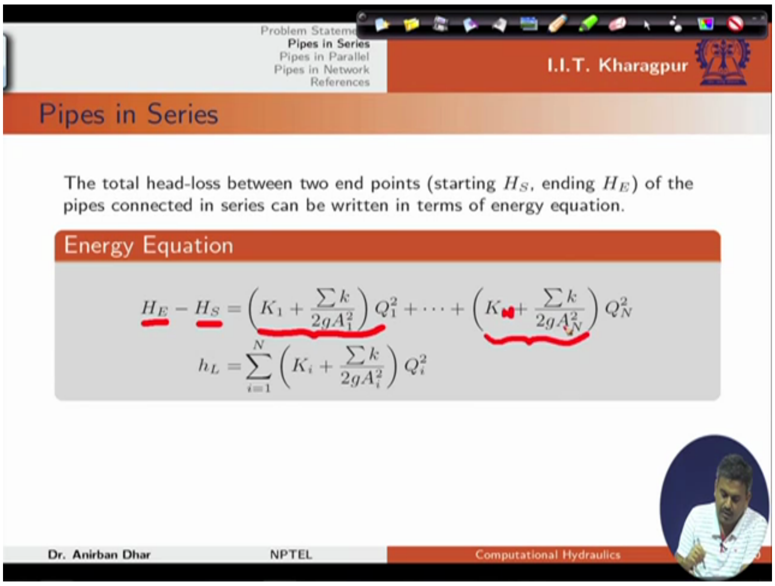

So like that we can generalize this and for pipes in series head loss between two end points can be calculated by adding the individual head losses. In this case the continuity equation is Q1 is equal to Q2 equals to QN, this is equals to Q.

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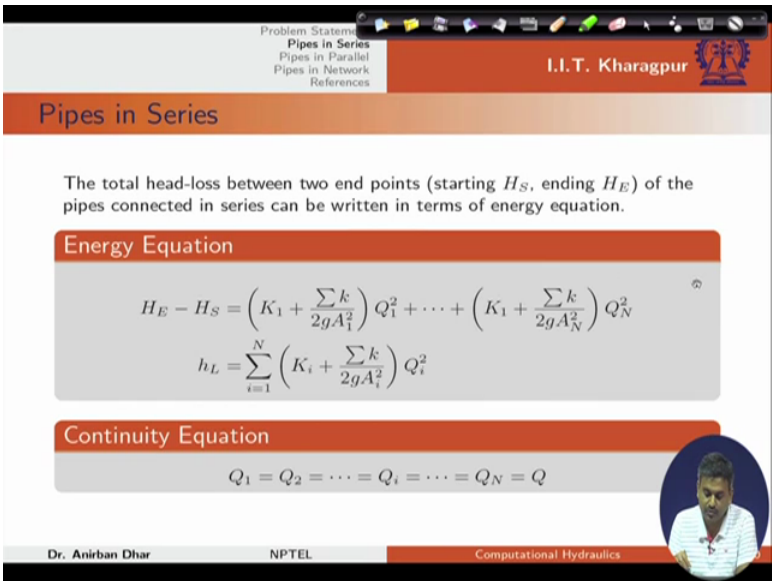

Now pipes in parallel. So obviously in pipes in parallel we have different discharge values for different pipes but the head loss that should be same in this case.

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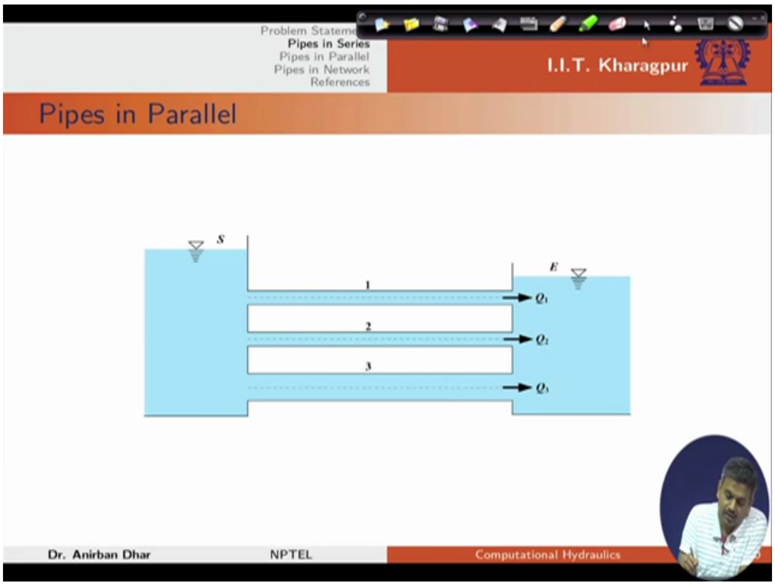

So head los between end and the starting points or starting sections this is same for all the pipes. And for discharge we can directly utilise the continuity equation. Now in this case remember that this quantity is nothing but K hat. This summation of small k this is nothing but the losses that we need to consider for junction, entrance on exit or other kinds of pipe fittings. So in this case this K1 plus this quantity we can directly write it as K hat.

And this is nothing but K hat into Q square because we are considering this Darcy Weisbach equation in our case. And discharge is summation of total discharge values from individual pipes.

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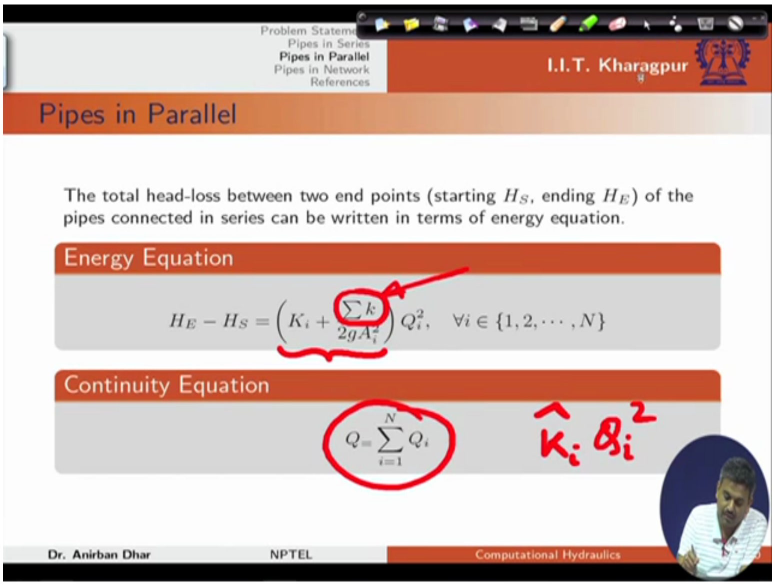

Pipes in network. Now pipe in series or pipes in our parallel we have different discharge and different energy conditions. But in pipes in network we need to satisfy our mass conservation at junction nodes.

So for any junction node j the pipe network the conservation of mass should be satisfied. In this case the J j in, this quantity is the set of all pipes connecting this junction J and these pipes are contributing or adding water to this junction. And j out this is set of pipes with outflow from the junction. And what is this small qj. Qj is the demand from that node.

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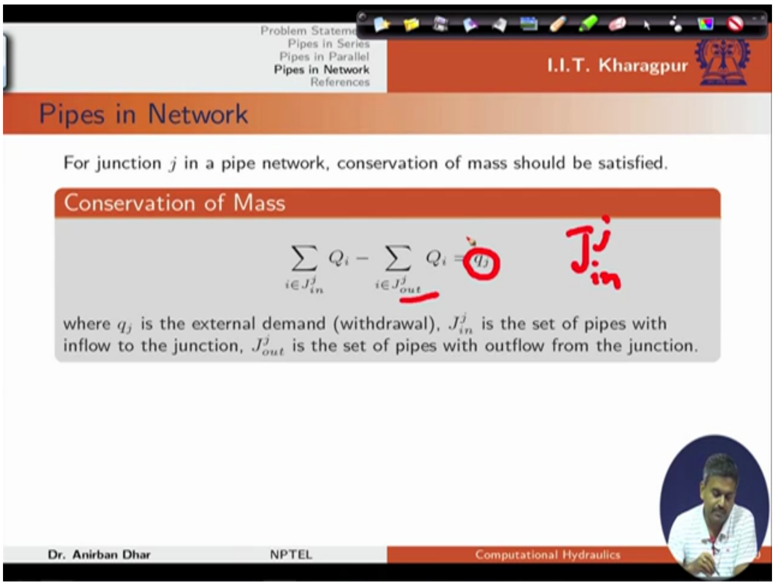

Now conservation of energy. Conservation of energy in case of pipe network we need to consider again two points that maybe some point within the network. Let us say this is our network. We are talking about these two points.

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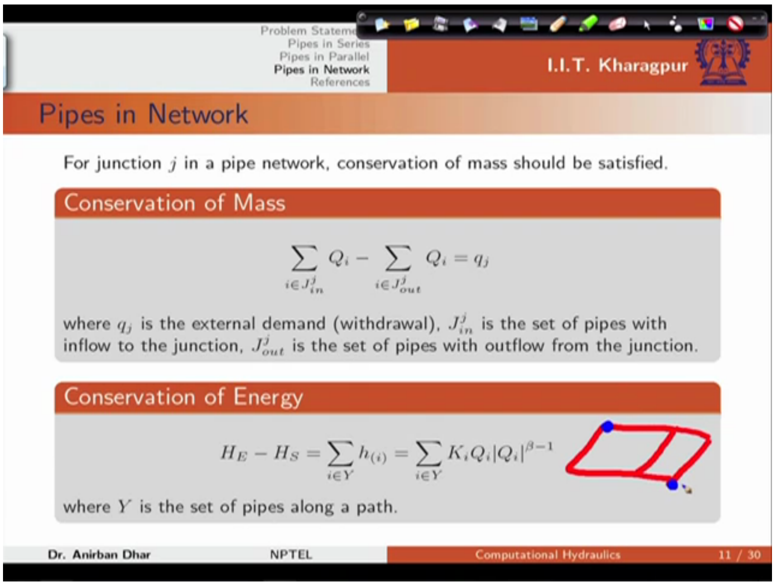

So we should follow some path and along this path there should be difference in head and that difference in head is nothing but the total friction loss from the pipes. Now in this case one thing should be considered that if we consider a particular loop. Let us say we have four nodes so I will connect these nodes with pipes. So these are let us say pipes.

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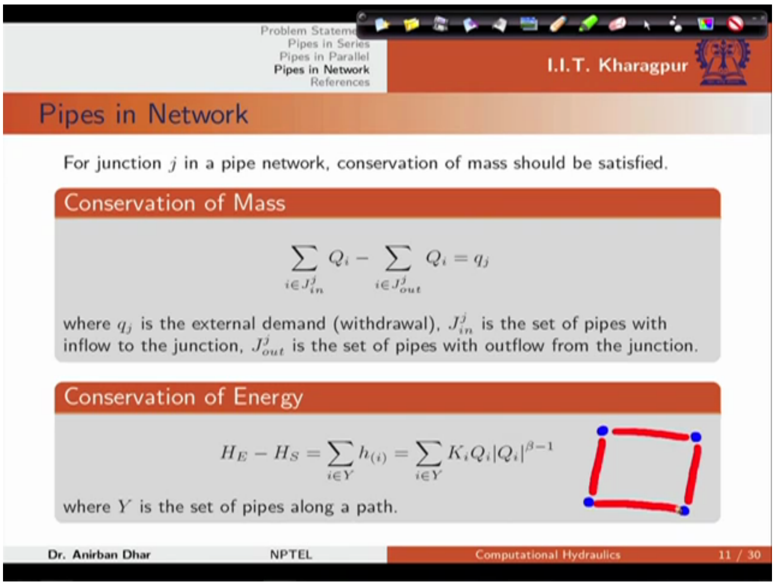

Now if I connect this nodes with pipe I have initial or starting point HE or HS is this one and again if I follow this loop, let us say this is clockwise direction or anticlockwise direction. This HE or end point both are same. So obviously if HE and HS these are same quantity so this should be equal to zero.

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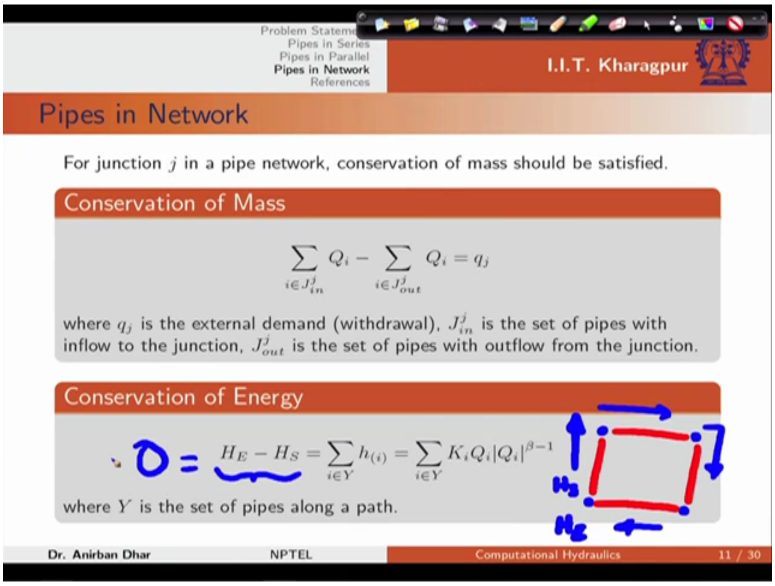

So for a particular loop in the network we should have a total head loss equals to zero. Now pipes in network for interior loop. Interior loop is the loop which consider only pipes without any reservoir or pumps. So in a closed loop the total head loss FL, let us say this is considered in terms of this FL.

So this quantity should be zero. I have used this mod sign. Ideally speaking it should be Q to the power beta. If we can get the actual direction of flow and we can assign that value directly. Otherwise if there is some error obviously there will be some problem and according to our convention in a particular loop will not get a zero value there.

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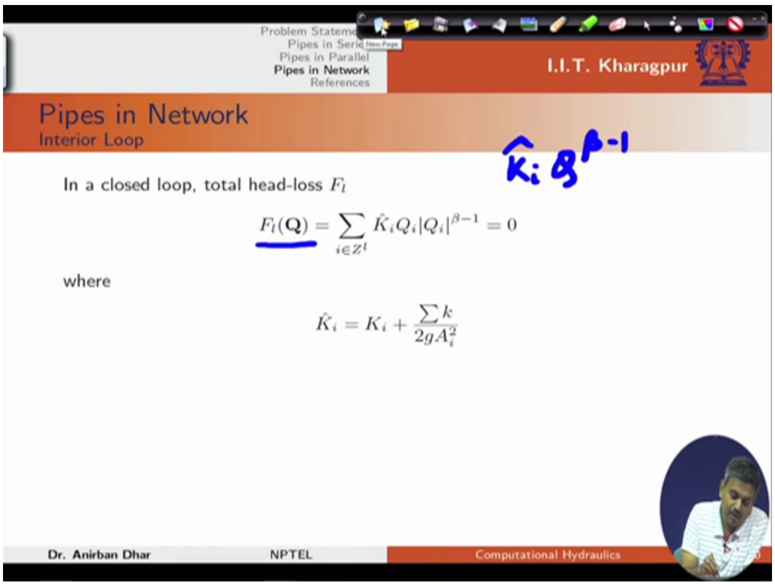

So as per Hardy Cross we are considering that in clockwise direction we have a positive discharge value. So in this case again this Ki hat, this is Ki plus this quantity. This quantity is again different losses.

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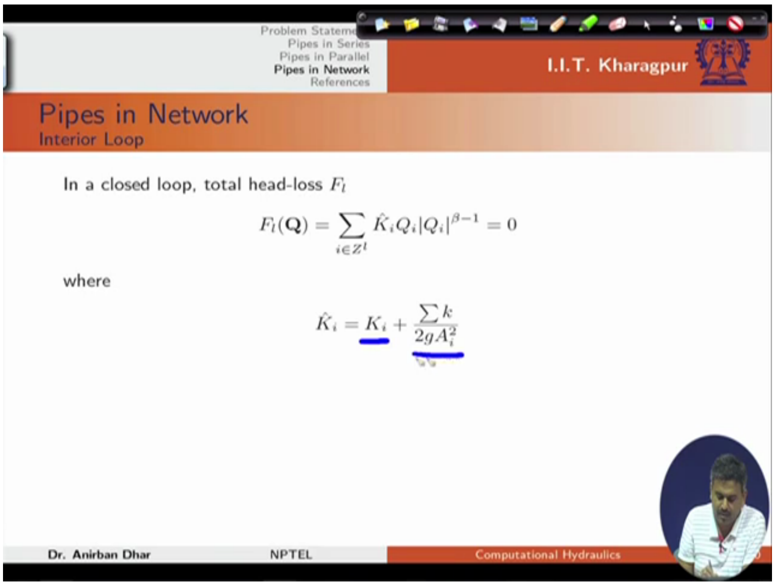

Now from Taylor series expansion, now this particular portion is important because we have utilised similar concepts during Newton Raphson calculation. So what is Newton Raphson? In Newton Raphson also we have used multivariate Taylor series. In this case we have single function but multiple variables.

So if we approximate it up to first order obviously this should be multiplied by difference in or discharge values between two consecutive iteration steps, P is the future iteration step and P minus 1 is the present iteration step. So this should be multiplied by this del because del should be there. We are considering this as multivariate function. So for multivariate function we need to use this del operator. So we need to calculate the derivative of this FL with respect to Qi for individual pipes.

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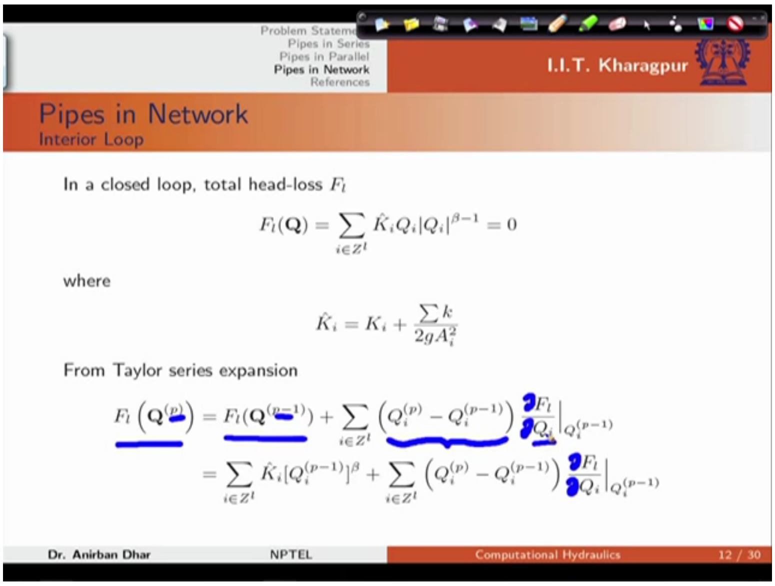

Now in this case one thing is important that in Hardy Cross method this part is approximated. If we compare this one with Newton Raphson, this is the head loss which is calculated based on previous iteration values.

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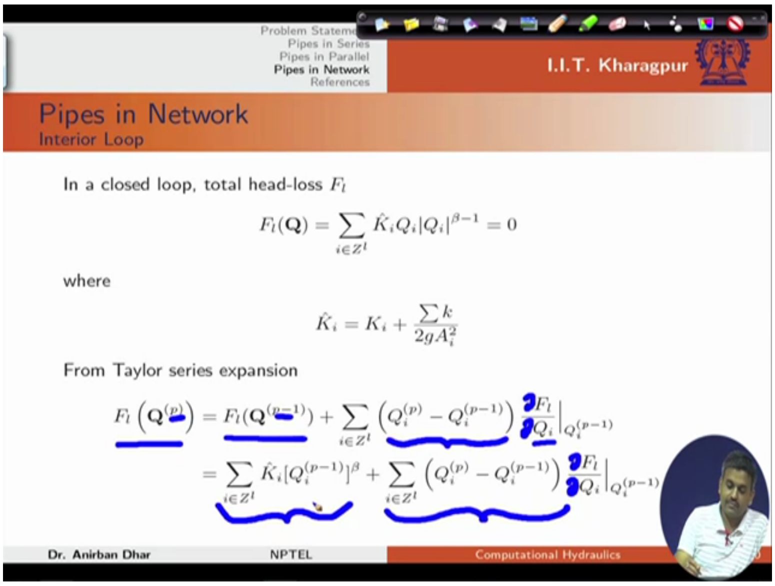

Now let us consider this approximation. In Hardy Cross method for all pipes for all i within this ZL. ZL is the set of pipes in a particular loop L. So for all pipes we have this del QL. So this increment is same for all. Now if I write that then actually that is equivalent to FL Q P minus 1 into del QL. So I can take out this del QL directly and I can add this quantity there.

On the right hand side as per our convention if iterate in the next step itself I am reaching to the desired value. So obviously right hand side equals to zero.

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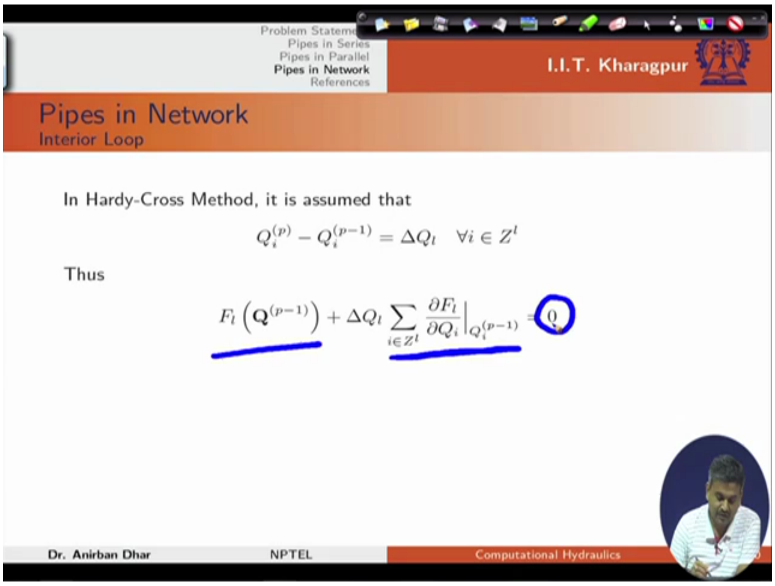

So with that if I write this del QL, del QL is nothing but FL Q P minus 1 and this is summation over all pipes connected in a particular loop. And this increment or change is applicable for Lth loop only which is again interior loop.

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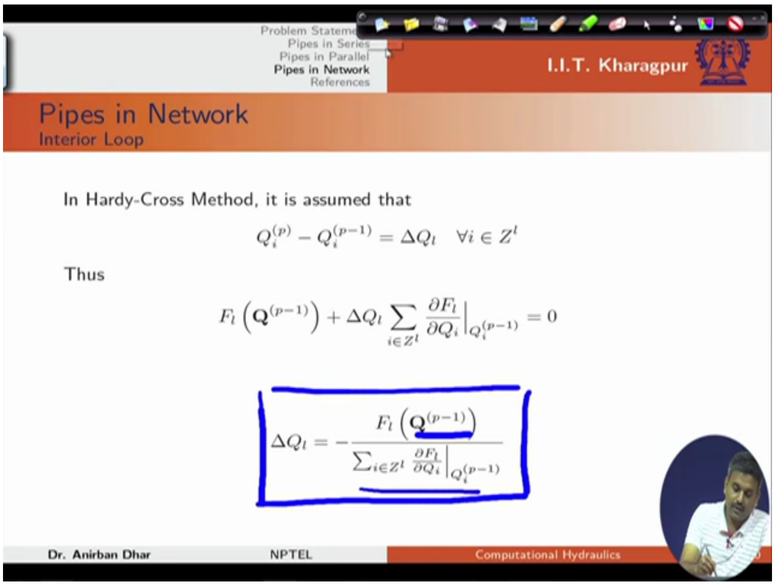

Now for this interior loop we need to calculate the derivative. Obviously if I take a derivative of this FL, FL is nothing but Ki Qi individually we need to take derivative of this term only.

Other terms it will be zero. So beta Ki hat and Qi beta to the power minus 1 or Q to the power beta minus 1. In this case this is beta into Ki and mod to the power beta minus 1. This expression is utilized during calculation because we need to consider the direction of flow during calculations.

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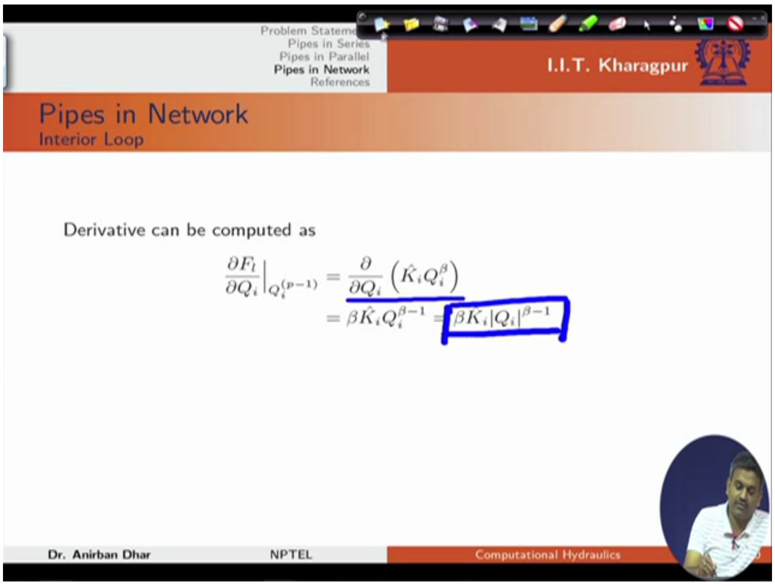

So now everything is clear because for head loss or F calculation with Q P minus 1 obviously this is the thing and for individual derivative calculation we are utilising this. So directly we are getting this quantity. So for a particular pipe we have to add this value and if the same pipe is shared by another loop then during the calculation of discharge of that particular pipe in that loop we need to subtract this quantity. So if this increment is within loop for a particular pipe we have to add it. If it is coming from another loop then we have to subtract it.

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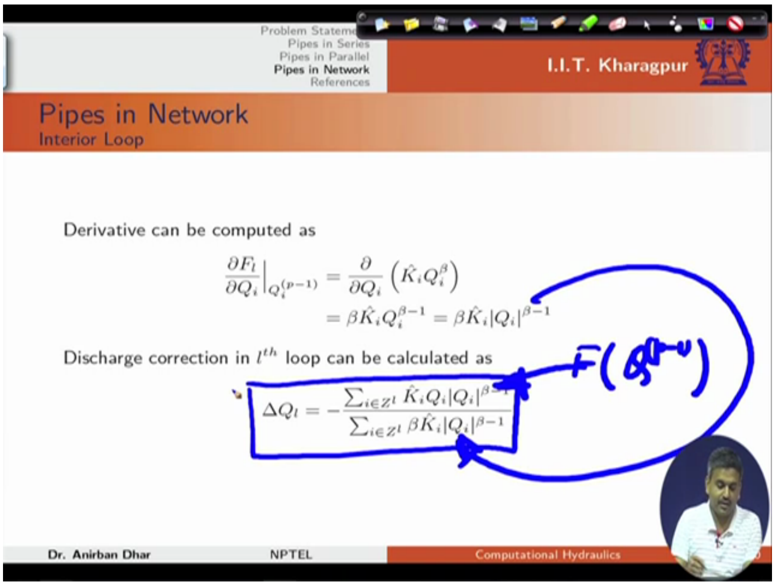

Now this pseudo loop. Let us consider this pseudo loop thing. In pseudo loop the total head loss considering head difference between two fixed grade nodes. So for fixed grade nodes we will have this delta H as constant. So we will add this with the head loss equation and this should be equal to zero.

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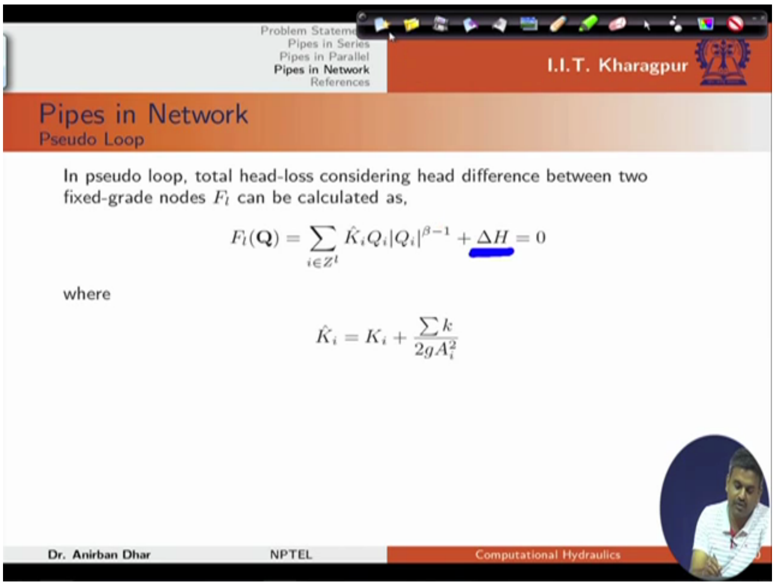

So again we can expand it but after expanding it we are basically calculating the previous iteration value which we need to add this del H here and this part is same.

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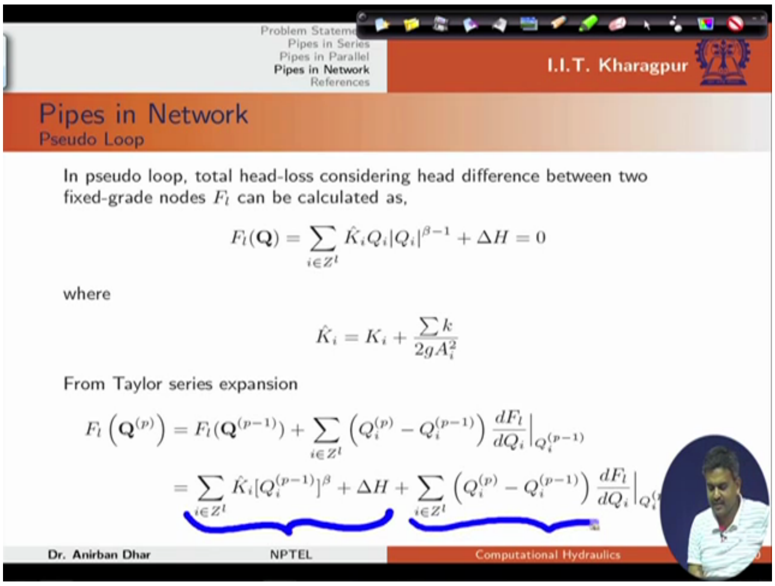

But again for Hardy Cross method we need to approximate this part with a single increment for a particular loop. So in Hardy Cross method in this case we are considering only one increment. This is similar to the previous one. So this increment calculation expression is also similar to the previous one.

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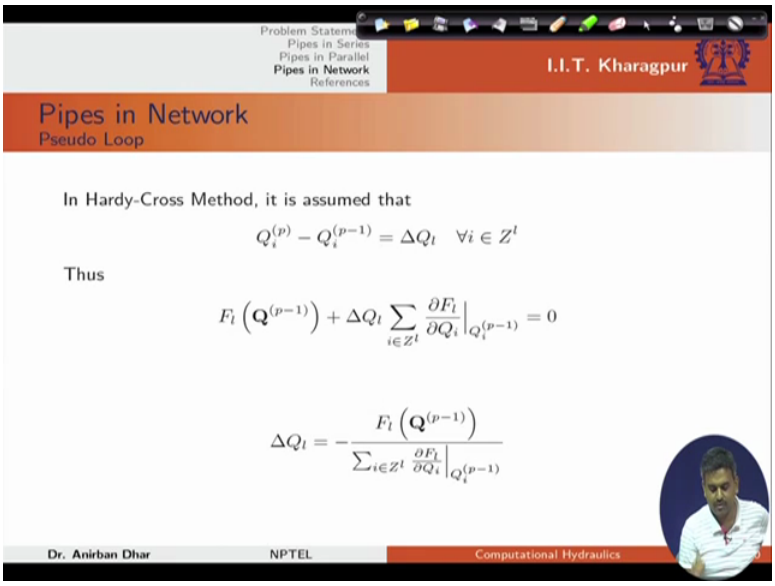

But only difference is there in terms of addition of the head difference term during del QL calculation. So again we can find out the derivative and derivative will be same because del H is a fixed quantity so there will be no effect of del H on this calculation or derivative calculation.

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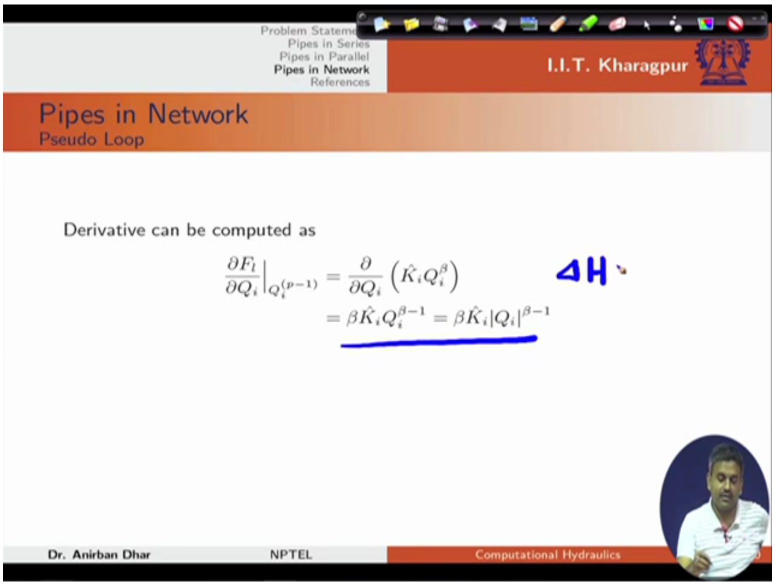

So discharge correction in Lth loop can be calculated as, this is our usual head loss for pipes and this is the difference. And divided by this is again coming from here. So again we need to add this quantity for the loop and we need to subtract it during the calculation of the neighbouring loop.

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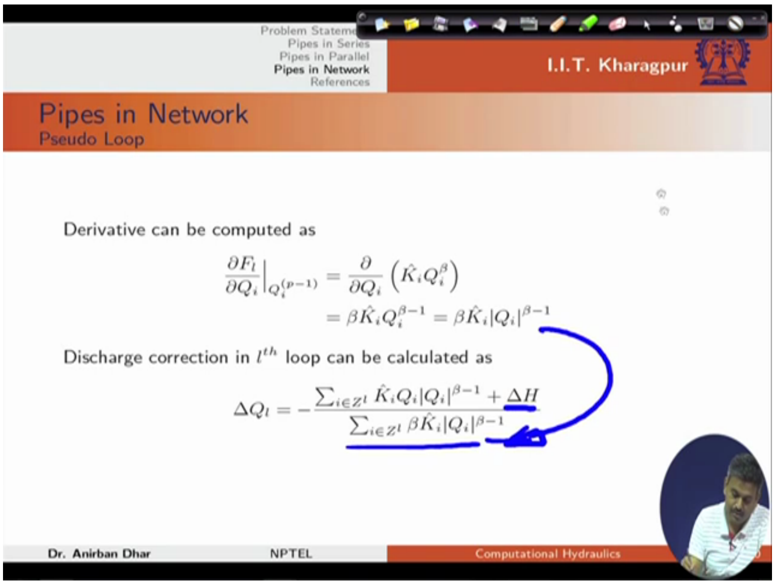

So in pseudo loop this total head loss considering head difference between two fixed grade notes and pump we can calculate like this. Now in this case we need to add this head for pump. This should be negative quantity and head versus discharge this condition is imposed during our head loss calculation.

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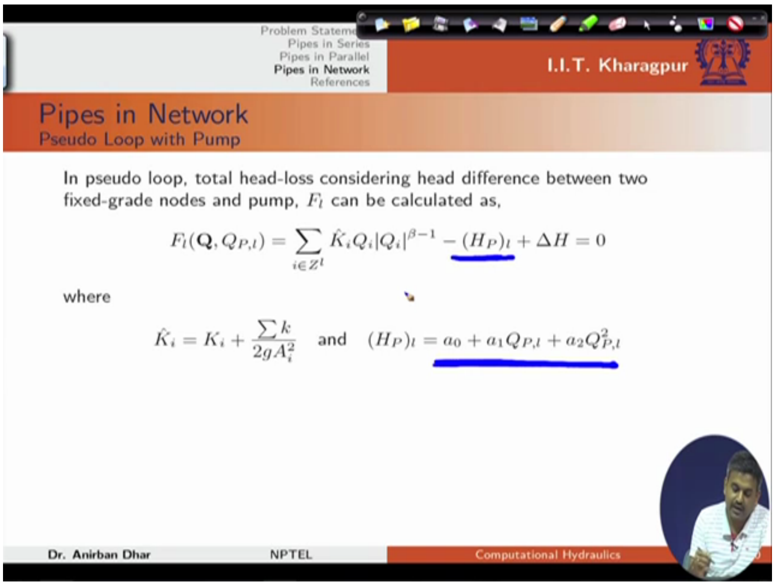

So again if we expand it using Taylor series we can see that in this case we have one extra variable. If we have n number of pipes in a network so for one pump we need to add 1. So N plus 1 number of variables will be there in this case. So I have included it in this function again here on the left hand side also.

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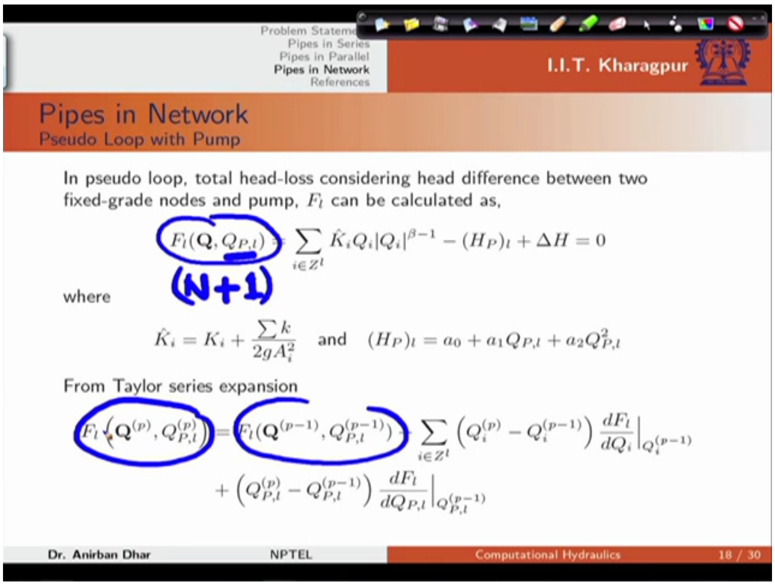

But this derivative calculation also we need to add this. So this is the quantity which is coming for individual pipes and this is the quantity which is coming for pumps.

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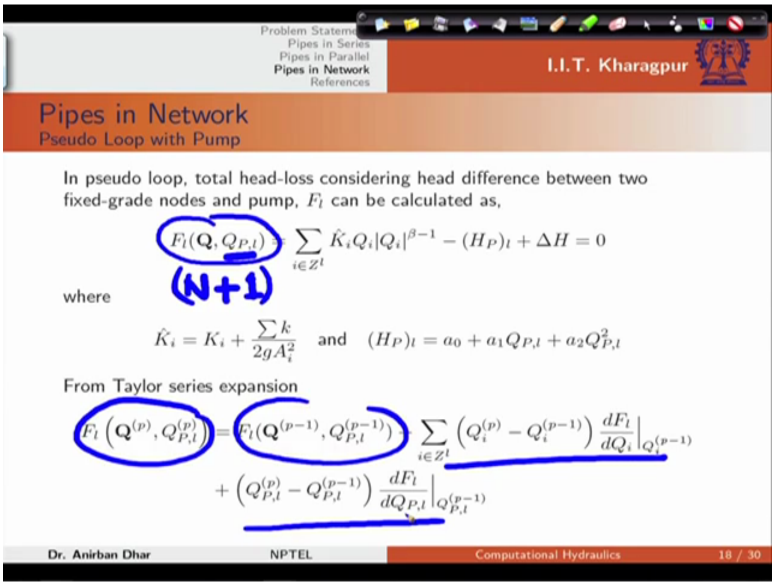

Now again in Hardy Cross method it is assumed that this difference in discharge between two consecutive iterations for pumps and all pipes same and we can approximate it as del QL. Now using that del QL we can again calculate our increment value. So del QL is nothing but this quantity. But in this case if I compare it with the interior loop I have one extra term. This is the extra term that is coming there.

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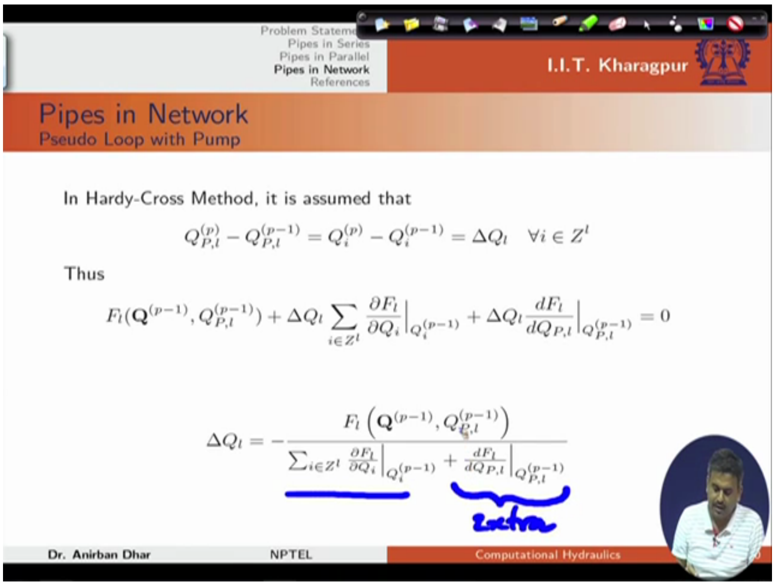

Again we need to add another extra term on the numerator. So in that case again we have partial derivatives here. So with this partial derivative one negative sign because whenever we are including the pump term that is negative. So negative a not, a1 QPL a2 QP square L and if I take derivative of that obviously these two terms will be there with the coefficients.

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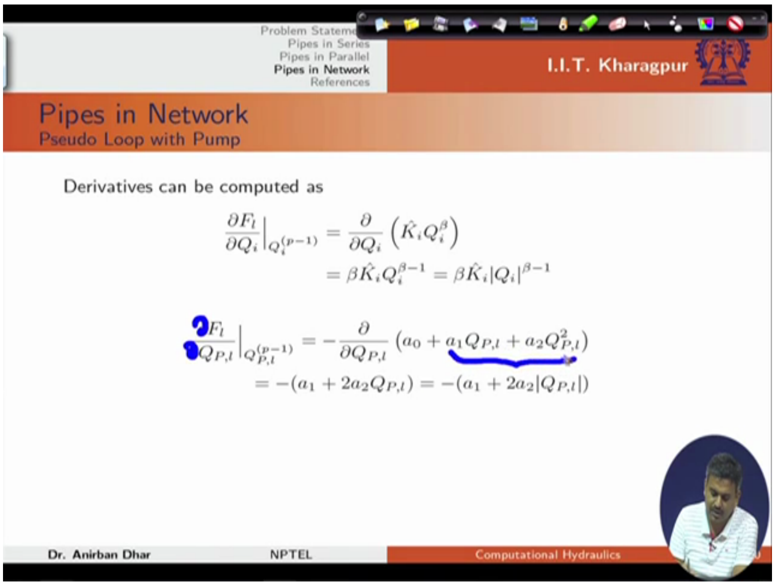

And again in this case we need to consider the sign of the discharge because this sign is important during calculation.

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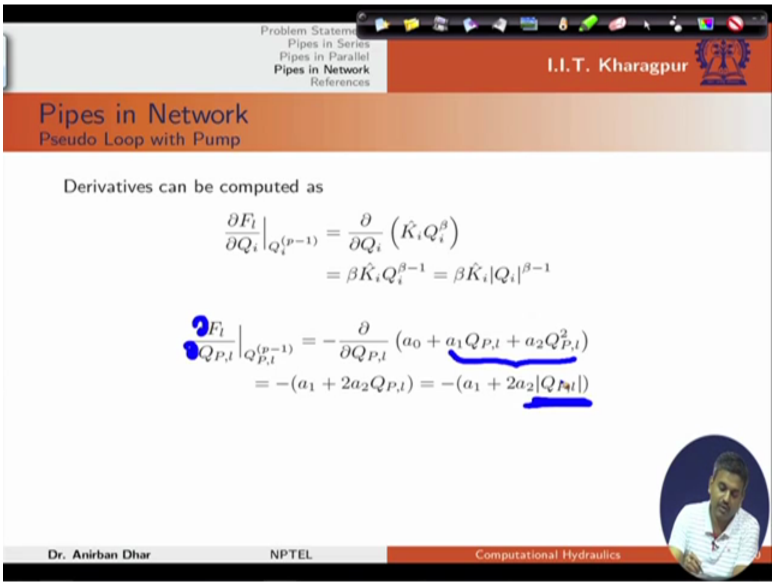

So discharge correction for the Lth loop can be calculated like this. So in this case the first component is for pipes connected in the pseudo loop. This component is for pump and this is for the usual head difference.

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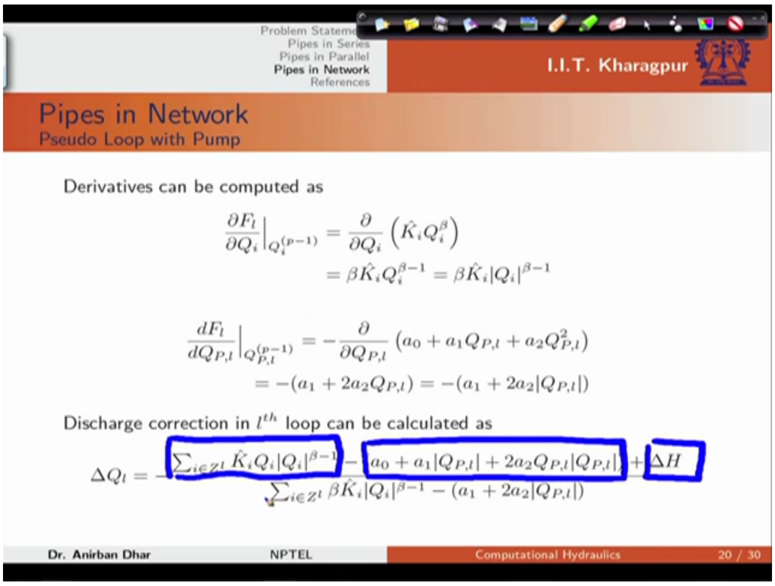

Again the denominator we have this component which is our first component is for pipes and the second component is there to consider the effect of pump.

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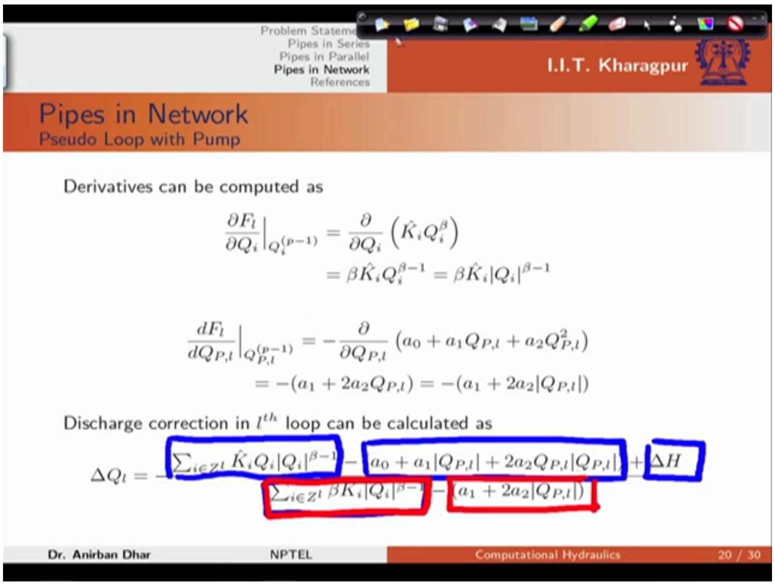

Now with this we can start developing our source code for Hardy Cross method. For Hardy Cross method we need to start with this first step. First we need to assume initial flow distribution in the network that satisfies the junction condition because we need to satisfy initial discharge.

Unlike our gradually varied flow there we have utilised our arbitrary conditions and from there we have calculated the discharge values. But in Hardy Cross method we need to specify discharge values directly with sign and we need to satisfy the junction conditions there.

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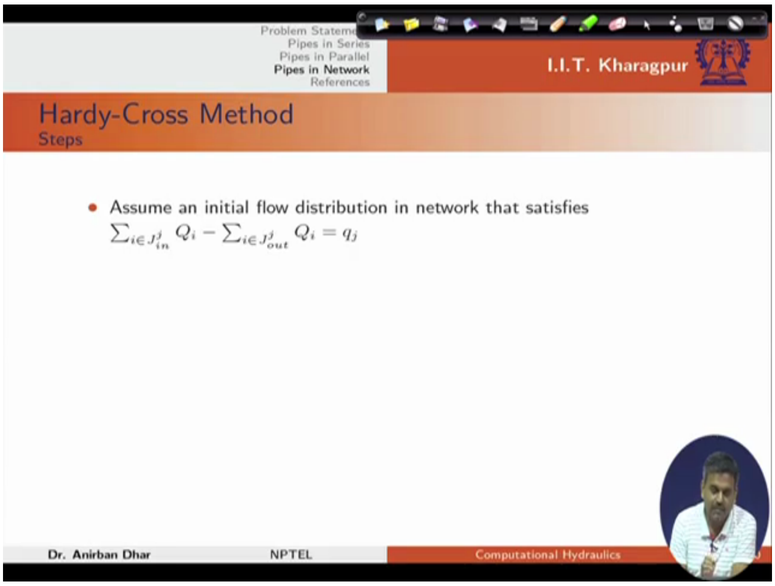

Closer the initial estimates fewer iterations are required and Q will decrease for higher K values. Obviously in this case this one hL equals to Ki to the power this Q square. We can easily see that if I have higher Q value then I will get lesser discharge in this case.

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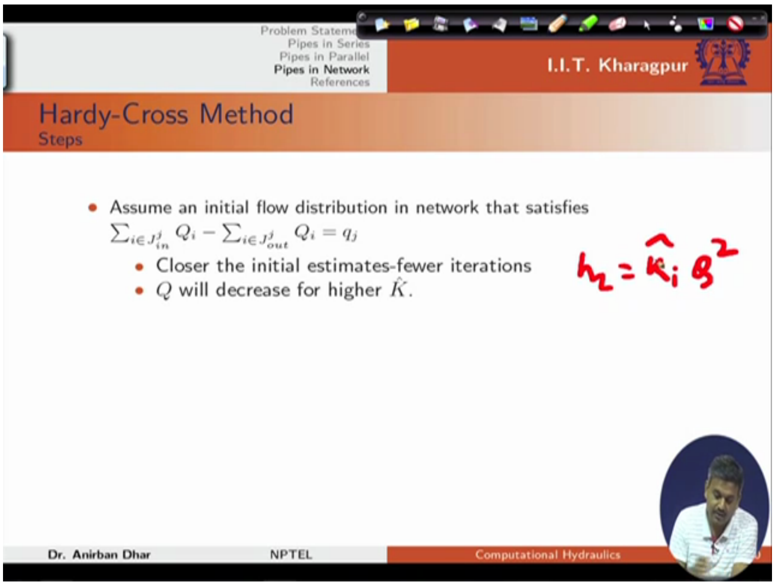

So for each loop we need to determine this QL or del QL quantity. I have written it for path and loop because path that is applicable for our pseudo loop.

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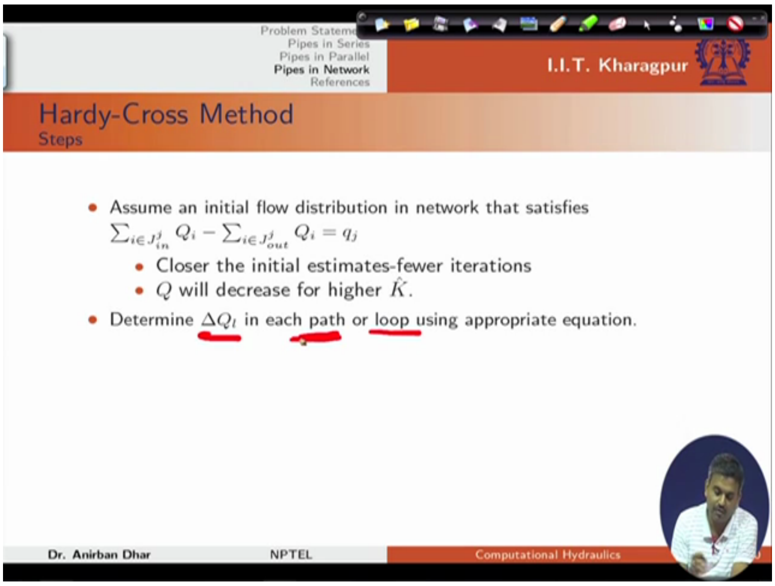

So adjust the flow for each pipe elements in all loop and paths using the relation. QLP this is the condition when we are considering updating this QL i. So QL i P is updated value. This is our previous citation value for Lth loop only.

So for Lth we need to add this QL quantity or del QL quantity but we need to subtract other del QL values because there will be only one loop because if this quantity is there obviously if we are considering the flow on the left side this is our calculation part then the contribution from other loop. So obviously there will be only one loop from other side. So in this case we need to subtract that quantity which will be the discharge from del QK.

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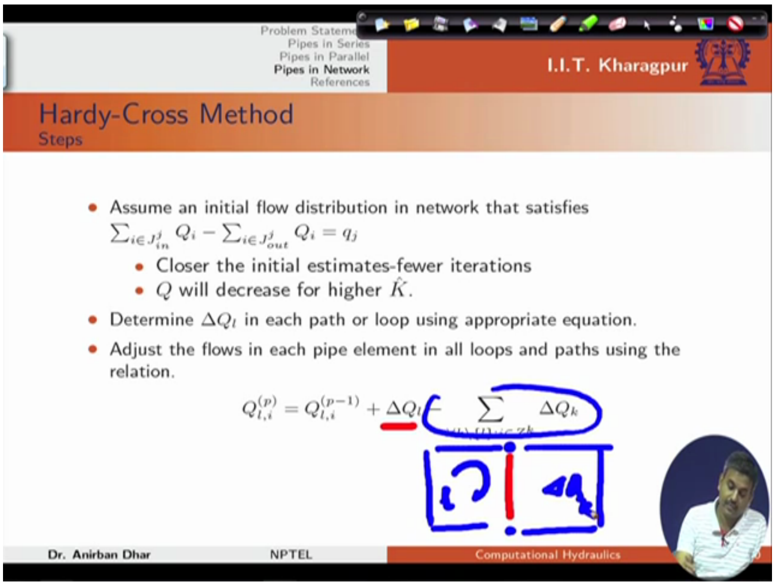

Now we need to repeat this thing until we get convergence up to desired accuracy. Now this is our configuration 1. We know that we have external demands at junctions 4 and 5. So by satisfying the conditions we can get this thing. So at this point obviously if we add the values we will get zero value here because no external demand. So obviously the inflow here and minus outflow that should be same.

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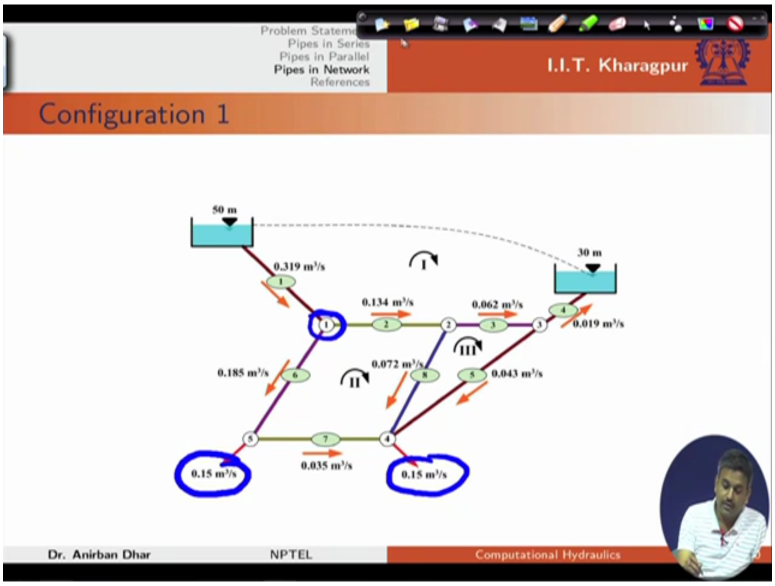

Like in this case point 185 and this is point 15 and on this side it is going like point 035. So if I add this two I will get point 185.

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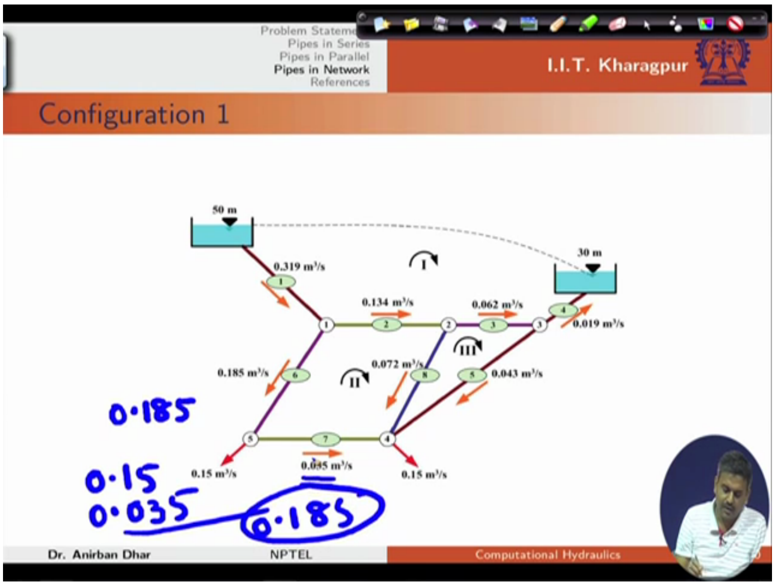

So like that we need to specify initial discharge values here. And these are arbitrary discharge values for this case. Now in this case we have three loops. So 1, 2 and 3. For these three loops we need to calculate our discharge values.

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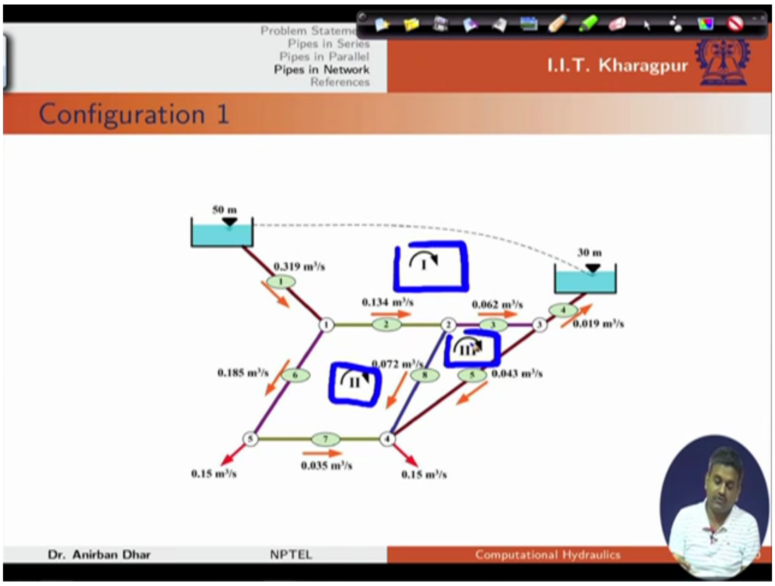

So obviously if I consider this pipe 1 this is having negative flow for loop 1. So pipe 1 if I consider and these are loop numbers 1, 2 and 3 obviously for pipe 1 it is negative for 1, pipe 2 it is negative for 1 but positive for 2, pipe 3 is again negative for 1 and positive for 3, pipe 4 is connected to only pipe or loop 1 so it is negative again. This is pipe number 5, pipe number 5 is connected to loop number 3 and it is positive.

Pipe number 6 it is counter clockwise direction so this is negative. 7 is again counter clockwise for 2, it is negative. 8 is clockwise for 2 so it is positive, counter clockwise for 1 so we have negative. 9th one or we have only 8 pipes in this case. So these are the conditions that we need to impose for individual loop specific calculations.

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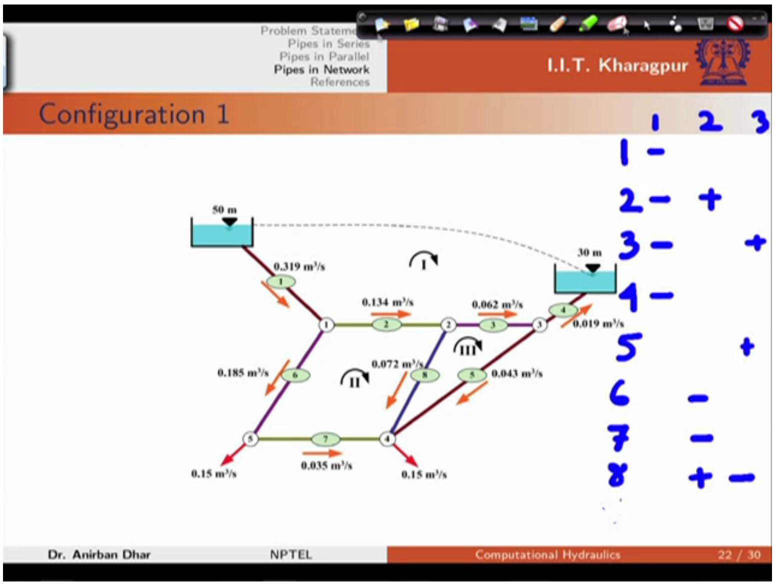

So we need to define it or we need to transfer these positive negative information to our problem or program structure. So we can transfer these concepts directly with loop number on this row, this is loop number row, this is 1, 2, 3. And first column contains the number of pipes connected to the loop and starting from second column up to the number of maximum pipes connected to a particular loop we have this information.

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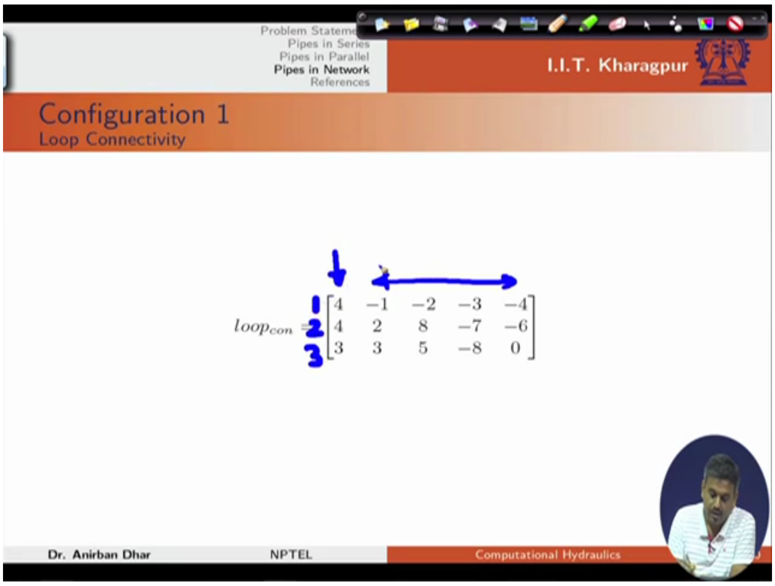

So for loop 1 we have four pipes connected to this particular loop and pipe 1 that is having negative discharge, pipe 2 is having negative discharge, pipe 3 is having negative discharge, pipe 4 is having negative discharge.

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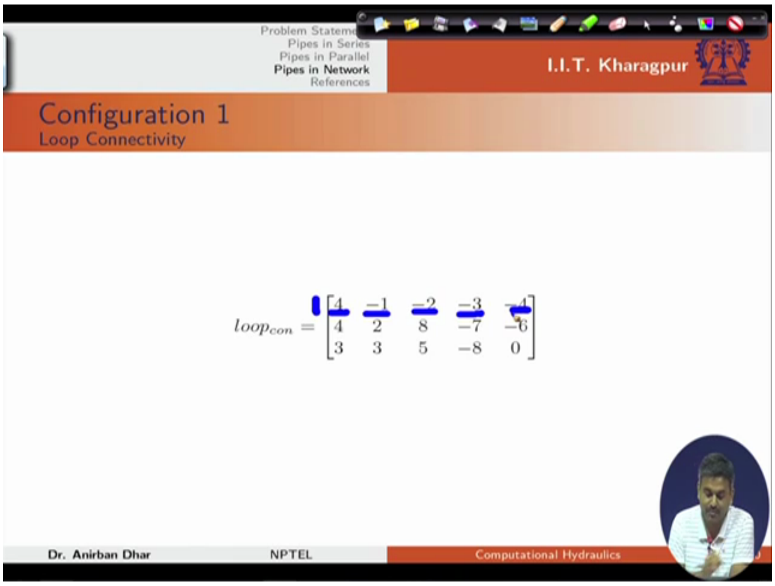

Loop 2, for loop 2 again we have 4 connected pipes. We have this 2 discharge in pipe 2 that is positive, in 8 it is positive, 7 and 6 it is negative.

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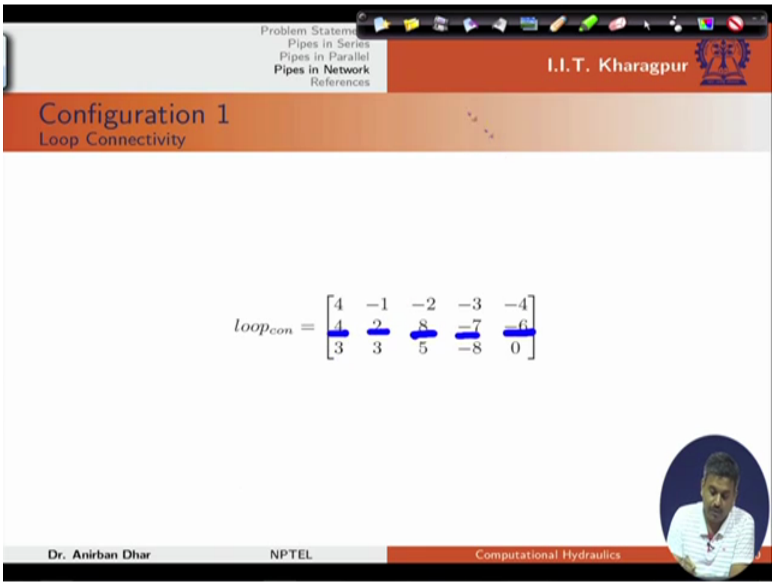

For loop 3 we have positive discharge in 3, our positive discharge in 5, negative discharge in 8 and as we have only three connected pipes that is why the fourth entry is zero.