Computational Hydraulics
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Lecture 46
Unsteady 2D Surface Flow

Welcome to this lecture of computational hydraulics. We are in module 4 surface water hydraulics and this is unit number 8, unsteady 2D surface flow. And this is the last unit of our module 4.

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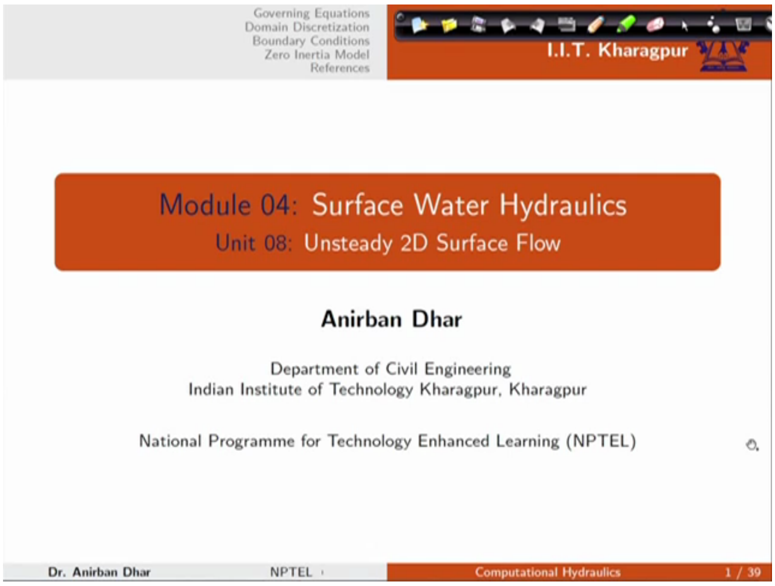

Learning objective of this particular unit. At the end of this unit students will be able to solve 2D unsteady shallow water flow using explicit approach.

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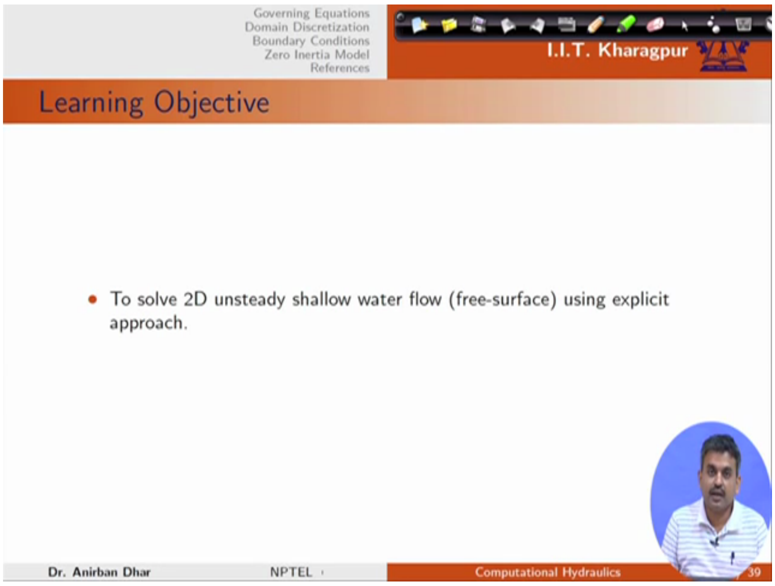

Problem statement, let us consider one river and irrigation command system. Let us say that this is my river system and this is one rectangular 2D area which is the command area and we have some hydraulic structure on the upstream side.

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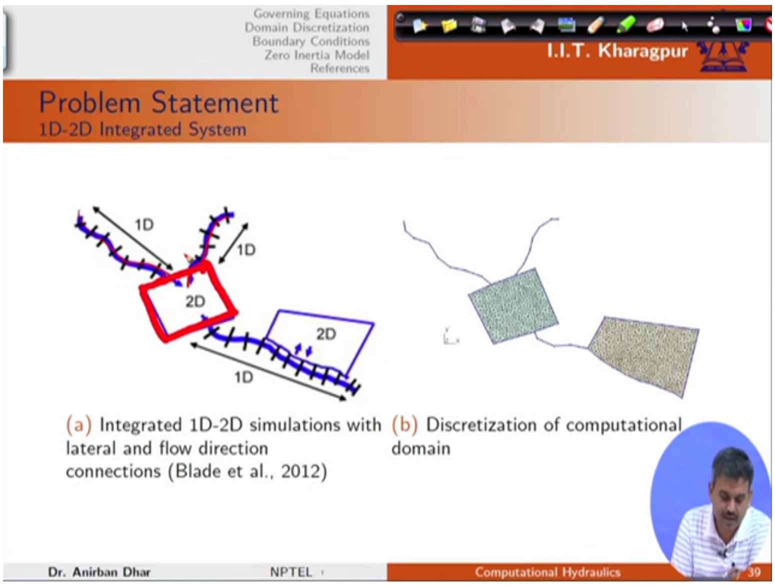

So the water is supplied from that hydraulic structure for irrigation purpose or the problems with severe flooding situation where due to upstream release from dams or barrages there will be inundation in the downstream areas. So we can conceptualize the problem as 1D and 2D case. Individually we can solve these systems. So in our previous lecture class I have already discussed 1D channel flow or unsteady channels flow case.

So now let us consider the case for 2D surface or free surface flow. We are talking about free surface flow because top portion of our water flows situation that is exposed to the atmosphere.

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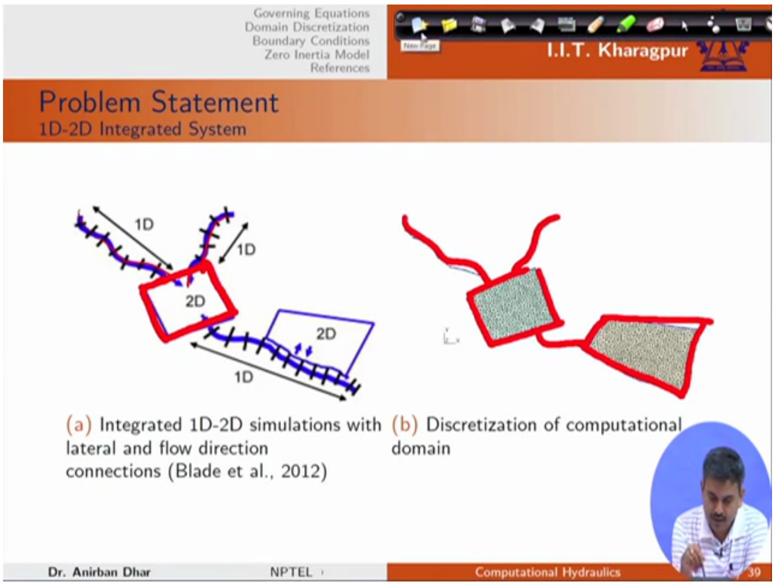

Now in this case let us consider our main course structure. In this one our hydraulic system is 2D or that is our mathematical conceptualization. So we need to write 2D governing equations. 2D problem is unsteady in nature in this case because time evaluation of surface flooding or surface water movement we can track using those governing equations. So we will have one governing equation, initial condition and boundary conditions.

So in this case we need to discretize our domain either with structured or unstructured mesh or with structured or unstructured point generation. So let us consider the case where we have utilized finite volume method. Now for using this finite volume method with a rectangular coordinate system or with uniform grading we need to consider structured mesh. So numerical discretization that is in terms of finite volume.

And algebraic form the resulting equations will be nonlinear in nature. So what we can do we can simply reduce the problem to pseudo linear or pseudo nonlinear problem and we can solve it using explicit approach. Obviously in explicit approach the technique is straight forward. We do not require any iterative method. Only iteration or time stepping is required for forward marching. And finally we can get the solution for the problem.

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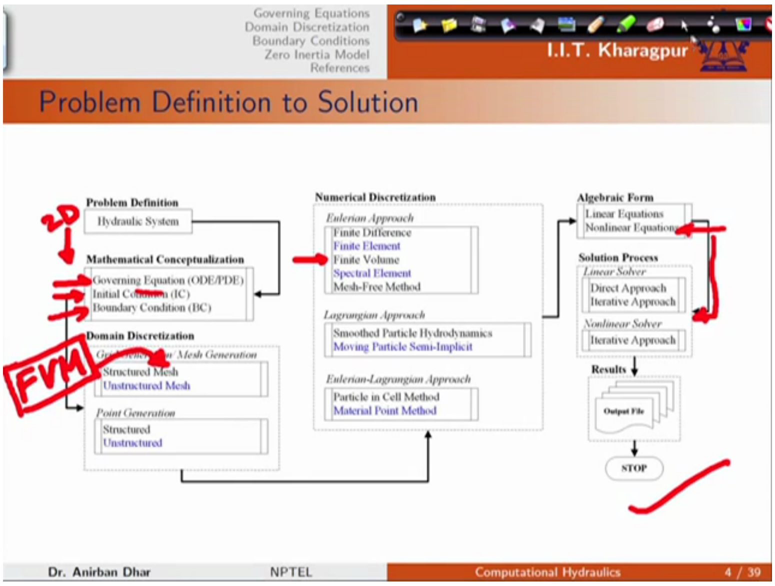

So in this case conservative form of the governing equation depth average mass and momentum conservation equations for surface water flow can be written like this where U is vector, E is vector, G and S all are column vectors. Individually if we see these components this is h, hu, hv, hu, this is hu square, gh square by 2, huv and this one G, hv, huv, hv square, gh square.

Now in this case on the right hand side we have minus qs. That means something is there which is going out of the system. We can consider infiltration as minus qs from the bottom of our ground surface.

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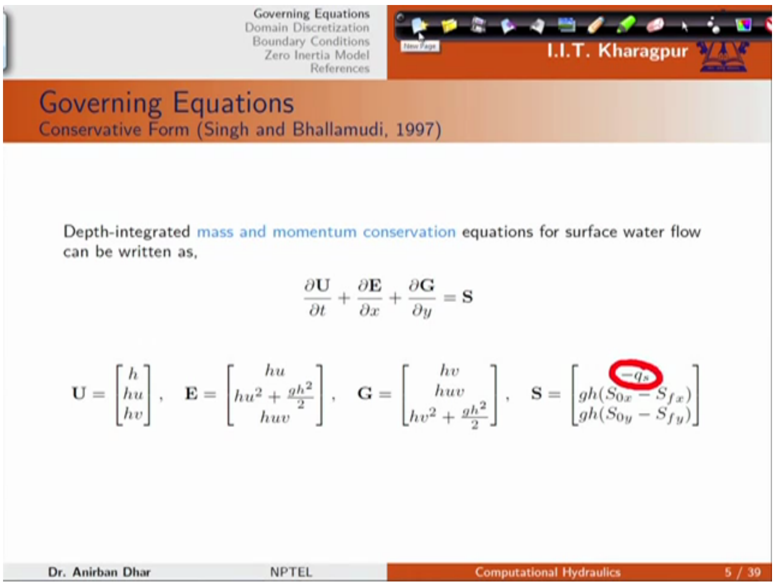

Now in this case first row that represents our continuity equation and second and third row these two considering momentum equation in x and y directions. We are not considering any variations in z direction. Obviously if we want to consider the variation in z direction we have to consider the full scale Navier Stokes equation. But this is depth integrated equation that is why we are not considering the variation within our system.

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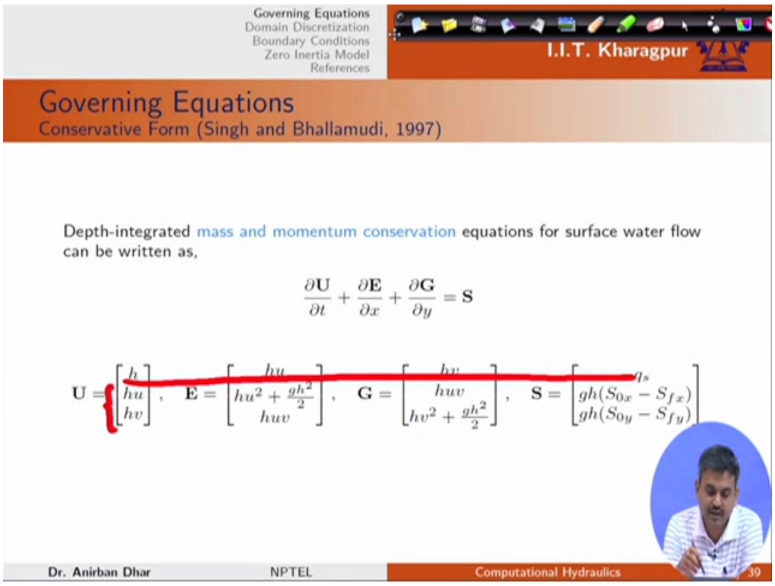

So non conservative form can be written from conservative equation like this. In this case obviously del F by del U this term is again vector. If we consider one dimensional case this is like Jacobian but in this case this Jacobian part is again vector.

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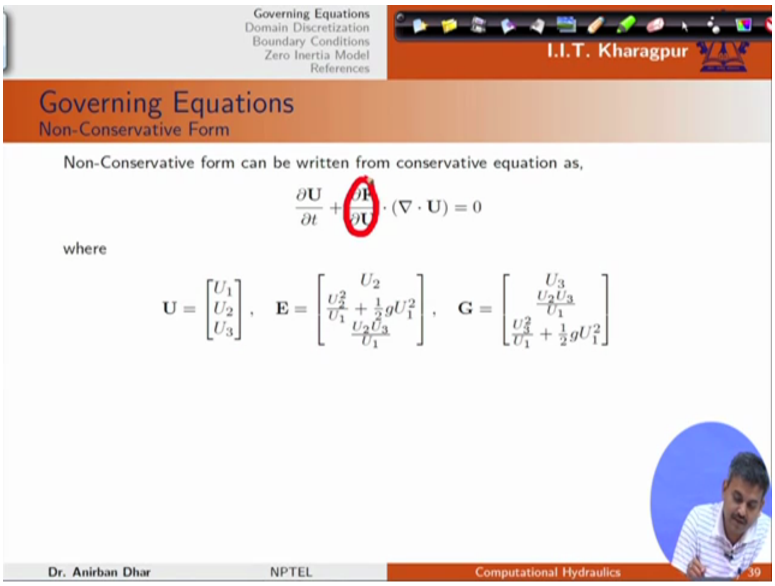

If I write this h as U1, U2 as uh, U3 as vh so in this case I can write the first vector as U1, U2, U3. Now E is nothing but U2. This is U2 square by U1. This was U square and this is U2 square means this is square divided by our h. So obviously this is u square h. So like that I have converted all the terms for this E and G in terms of U1, U2 and U3.

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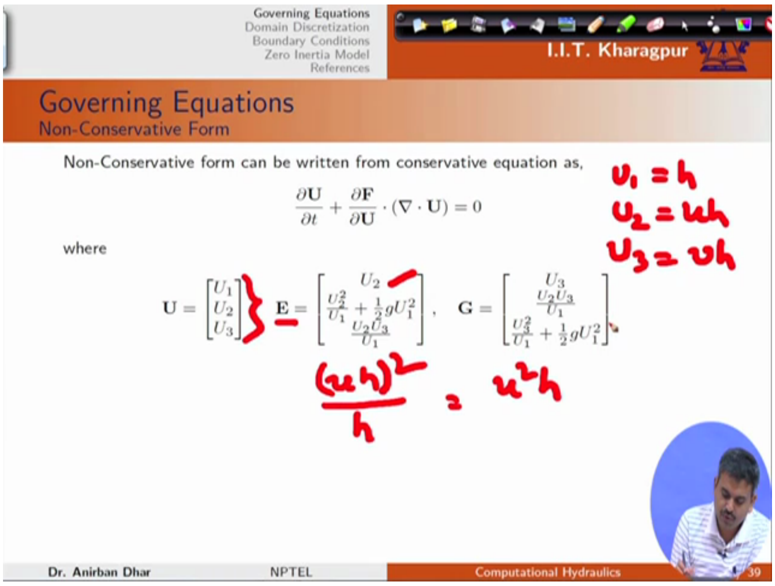

Obviously uvh this term will be U2 U3 divided by U1. So after converting this we can get the information about this del F by del U. So obviously Jacobian can be calculated like this. Here F is again vector. In this case this is Ei plus Gj.

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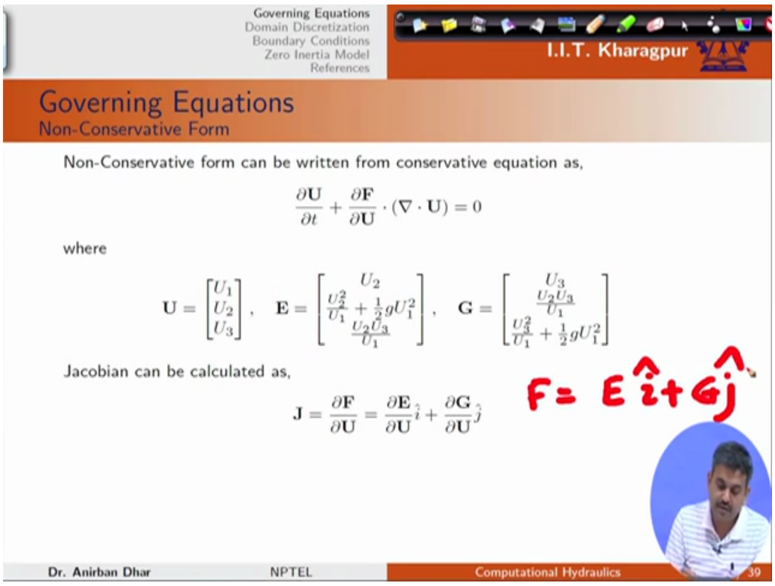

Now for this one we can individually calculate del E by del U and del G by del U components. So we can get this Jacobian matrix out of this. So how to calculate this one? This is essentially E1, this is E2, this is E3. This one is G1, this is G2, this is G3. Now for del E by del U, E and both are vectors.

In this case we will have del E by del U1, del E1 by del U2, del E1 by del U3, del E2 by del U1, del E by del U2, del E2 by del U3 and last one which is which is our del E3 del U1, del E3 del U2, del E3 del U3. Now in this case directly in this can be calculated like this.

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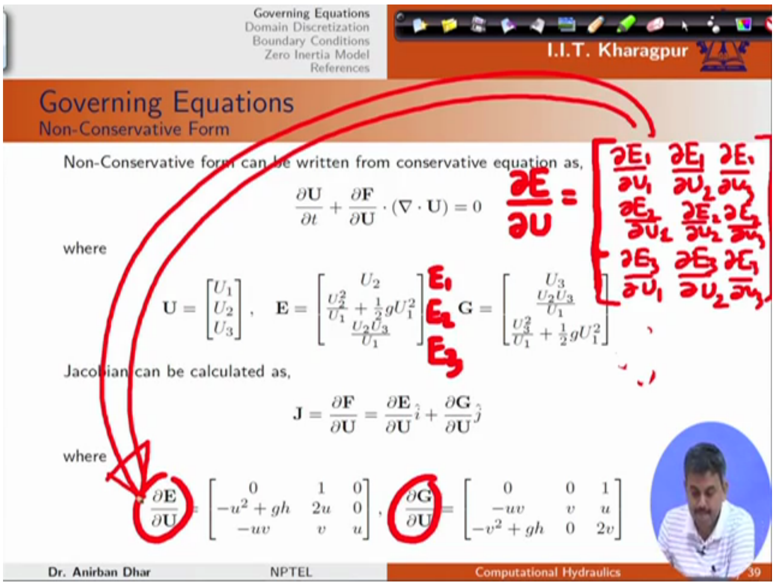

Now what is del E1 by del U1? There is no del U1 so obviously this is zero. Del E1 with respect to U2 obviously this is 1. Again we do not have any U3 so zero. Like that we can calculate individual components for this Jacobian matrix for individual E and G components.

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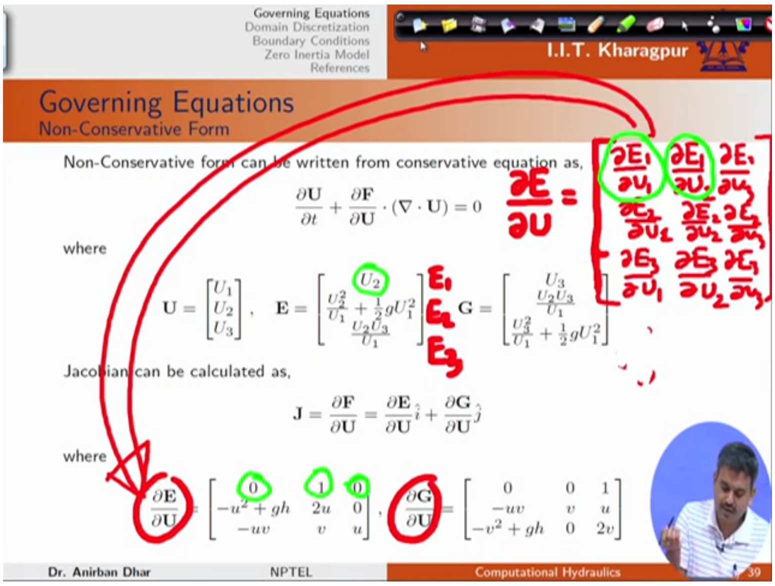

Now after this calculation we can discretize our domain because we need to use finite volume method. So obviously for finite volume method we need to divide our domain into number of cells. So let us say that on x direction I have total M number of cells, on y direction I have N number of cells. And like our classical problem we have this gamma N which is Neumann boundary, gamma D which is specified boundary.

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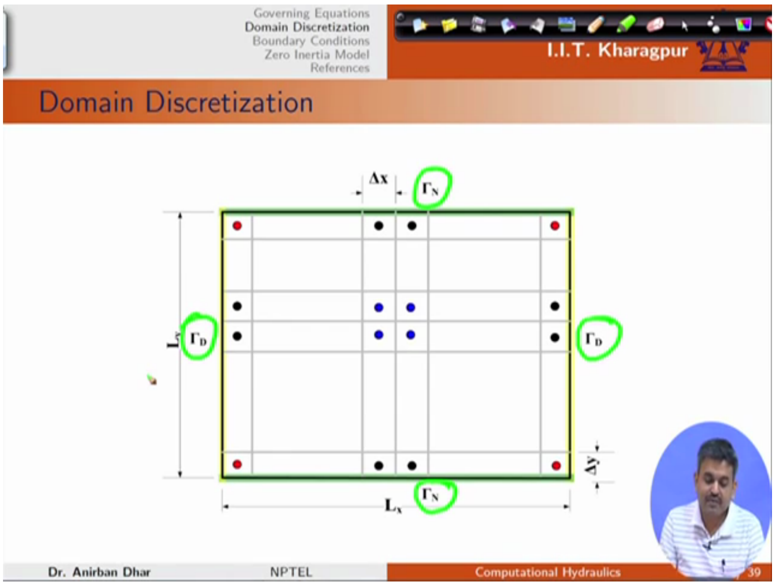

There can be situations where my domain is totally closed one. In that case I can consider all sides here this red. So all sides I have closed boundary or zero Neumann kind of condition. But obviously we need to see individual components in that case.

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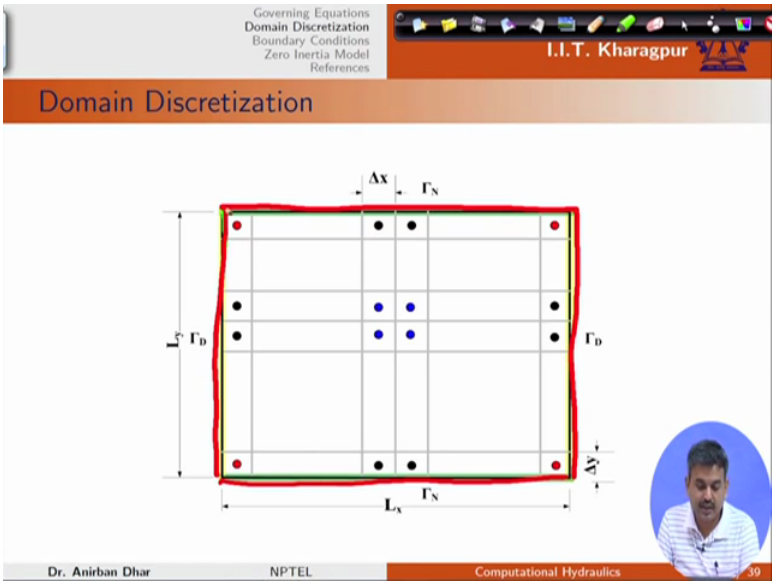

So starting with the discretization of the governing equation. So as per finite volume method the governing equation is integrated over the element volume in space. So what is that element volume? This omega P. Now in this omega P if I integrate starting from t to t plus delta t. That means nth time level to n plus 1 time level obviously we can write this with this integral sign. Now in this case I have changed this divergence of F and utilised it directly here.

So divergence of F this will give individual components. So this will be nothing but if I take del this is del by del x i plus del by del y j and if I take divergence of that so obviously I will get del E by del x plus del G by del y.

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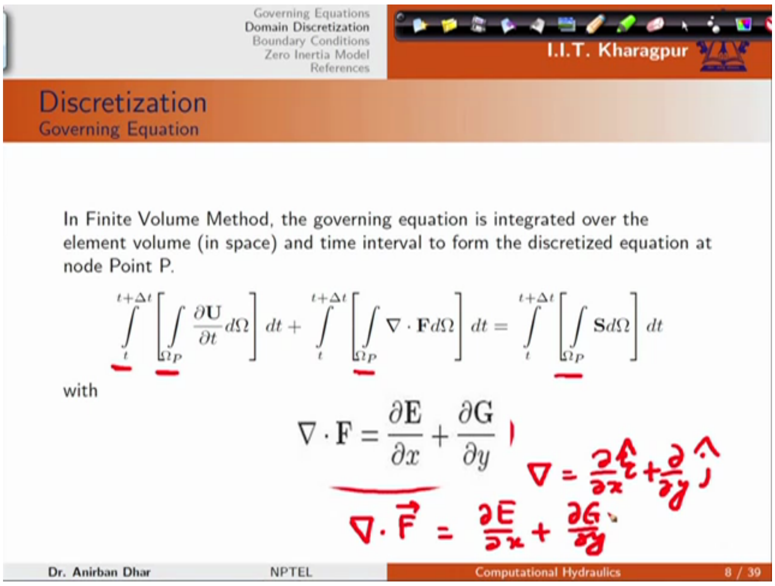

So after that I can start the discretization of my temporal term. So temporal term is nothing but in this case del U by del t directly and this is similar to the discretization that we have utilized for our groundwater equations. So for central cell which is the pth cell we can write this and UP L plus 1 minus UP L. And L represents our time level in this case. Again in this case I can write this for central cell P. This should be divergence of F, divergence of F can be written like this.

Again this F dot nF, nF means F represents a particular face. Divergence of F in that one and AF there. So for all faces for rectangular domain we have east, west, north, south. For all domains we can write these components. We will have four components out of this, east, west, north, south.

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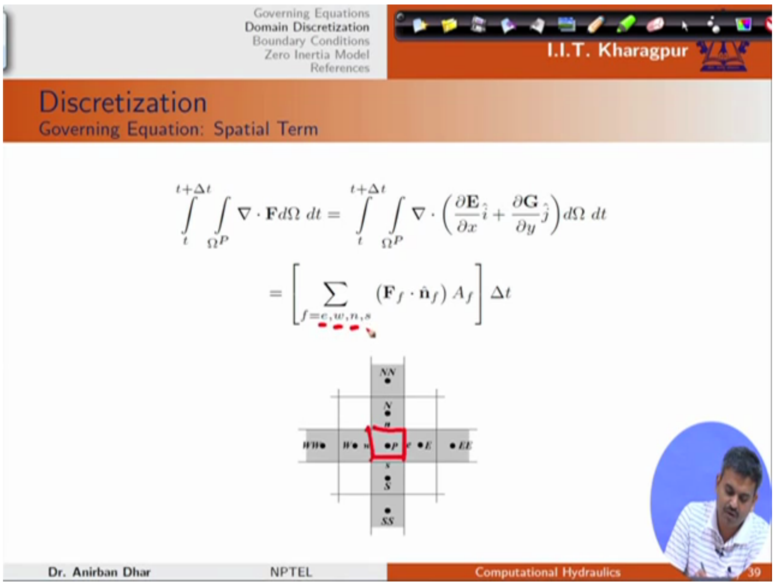

This is similar to a previous discretization. Now the difference is now we can calculate the flux values at the interface. For that calculation we need to consider flux at right face or right side, flux from left side and UR is the value on the right side, UL is the value from the left side. So with these values we can discretize our full flux terms because in groundwater equations we have discretized the derivatives directly. But in this case we need to find out the flux values at the interface. That is why this part is important.

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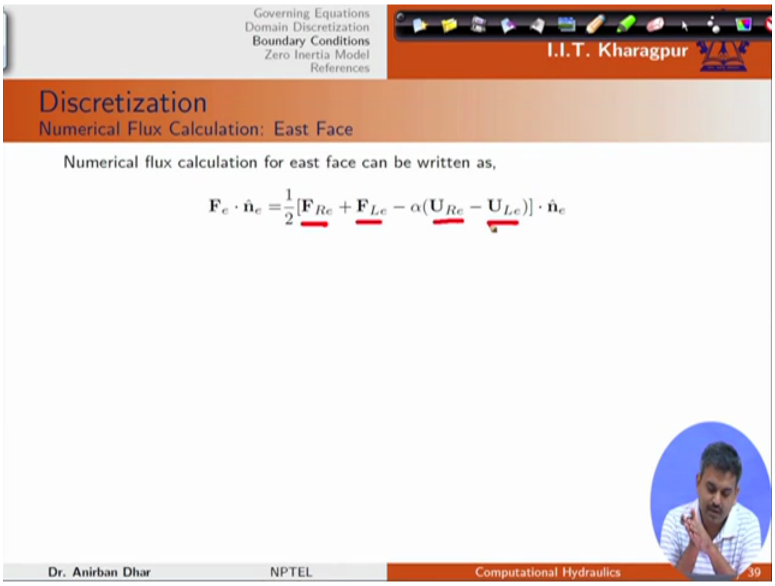

So in this case we can write this from the information from the right side, left side, this UL which is nothing but UP plus del UP. And right is it E minus del E. So if I see this notation this is nothing but this utilizes the minmod notation that we have discussed in our equations for conservation law. And UP and UE these values are at the cell centres on the on the east cell, west cell. And UEE this is for our east to east that means extreme east cell which is adjacent to east cell.

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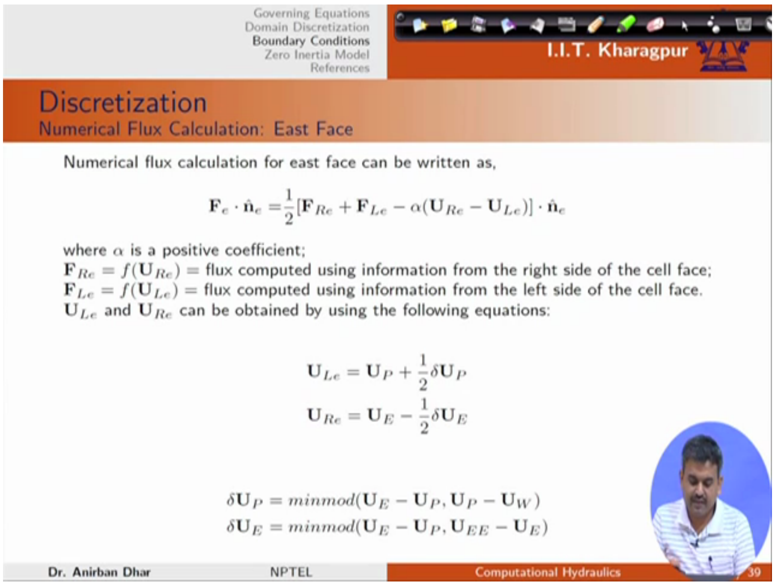

Now for this one we need to see one thing. What is that? That is our minmod calculation. As per our minmod calculations we have this a value. If modulus of a is less than our modulus of b and ab both greater than zero. And if it is b modulus a and ab greater than zero then it is b, otherwise it is zero in this case.

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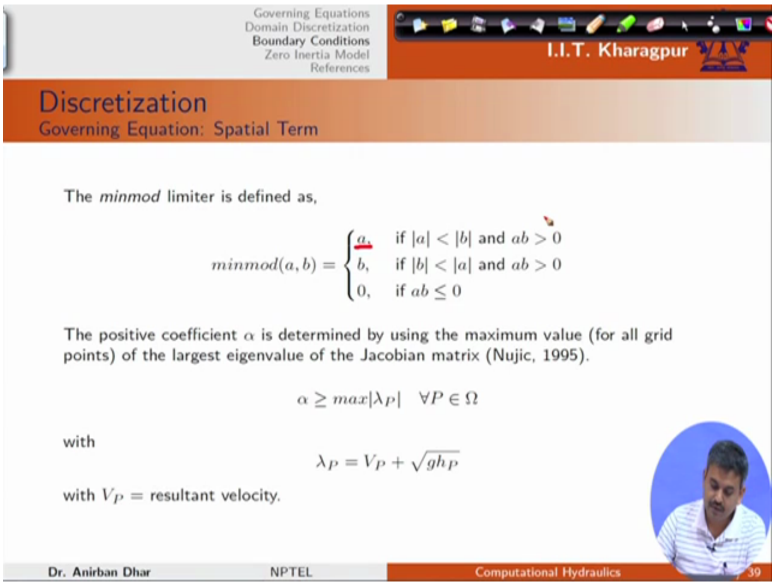

So if I utilise this concept for discretization I can obviously we have UP and UP UW, these are a and b values. If modulus of a is less than modulus of b and ab greater than zero then it is a. If modulus of b is less than modulus of a, ab greater than zero this is b. Otherwise this is zero.

So if I apply it directly in this case what I will get? This left side UP plus half of this one this is delta P. In first case if modulus of a is less than modulus of b obviously this is UP plus half UE minus UP because we have the first condition.

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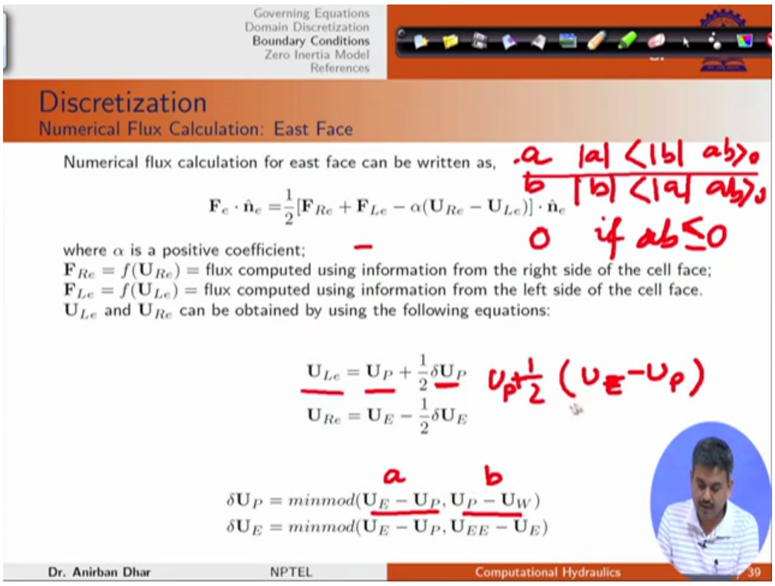

So for first condition this is UP plus UE divided by 2. Now if I have second condition that means if I have b there then what will be the situation? If I have b there then this is UP plus half UP minus UW. So that means this is 3 by 2 UP minus half UW. So what is this? Physically it means that if this is my interface east, this is pth cell and this is cell w whatever information is coming is from left side. This is my left side, this is my right side.

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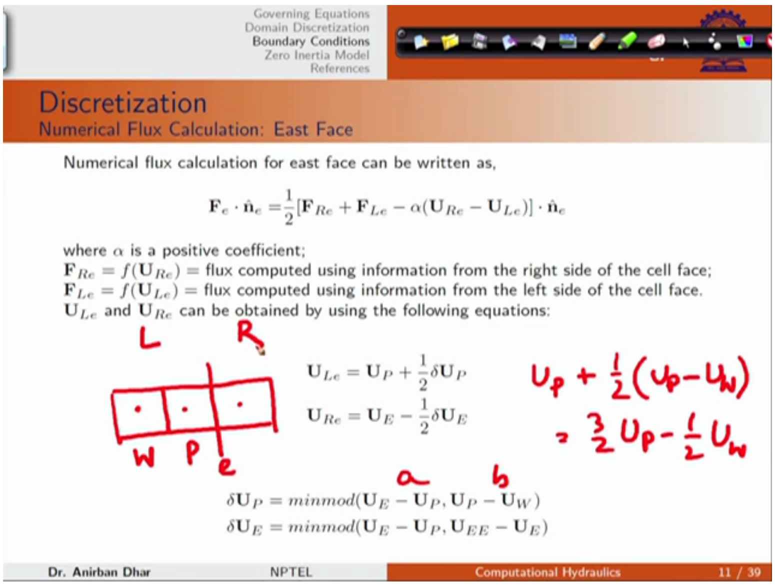

Initially when we have a value we have seen that this quantity is P and E these two values with average at the interface. But if it is b then we will have this value which is nothing but this value at interface. So this is nothing but interpolation or linear interpolation of UW, this is UW, this is UP and this is U from left side. Although it is UE but we are calculating ULE.

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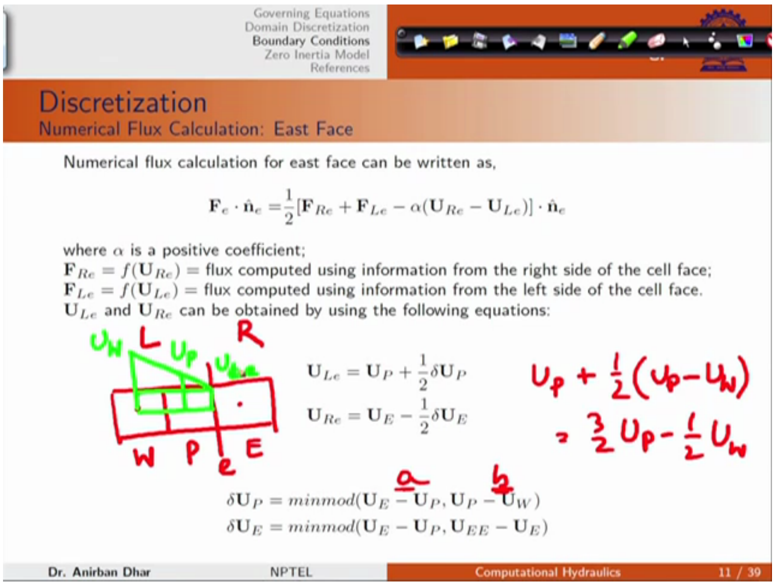

Similarly if I see the thing for right side, the right side again if the first quantity this is a, this is b and first one is my E minus half UE minus UP. So obviously in this case it will be UE plus UP divided by 2. From right side again if it is second quantity, second quantity is UE minus half UEE minus UE. So this is again 3 by 2 UE and minus half UEE.

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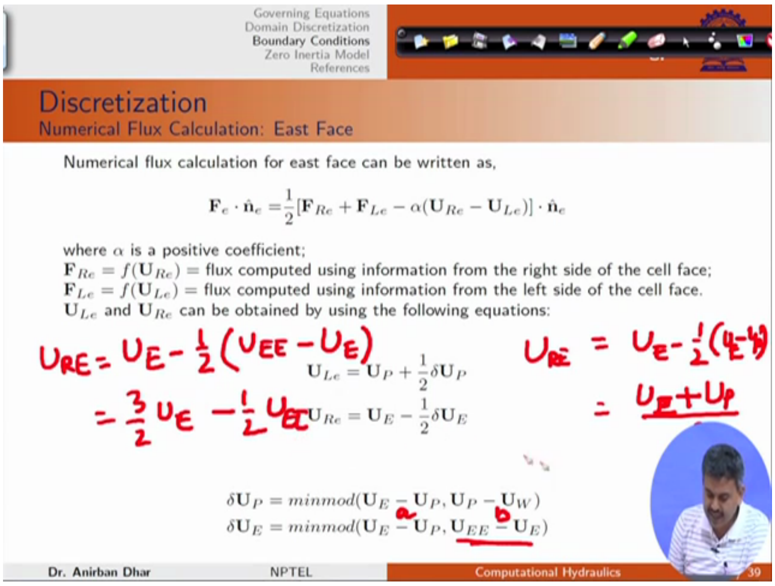

So again the information that is coming from my right side this is P, this is W, this is E and this is east cell. So for this cell if the information is there so I can take again linear interpolated value using this EE and E here. So from right side we are getting linearly interpolated value at the face or right side value. Like that we can have four combinations, two for the case on the left side, two for right side.

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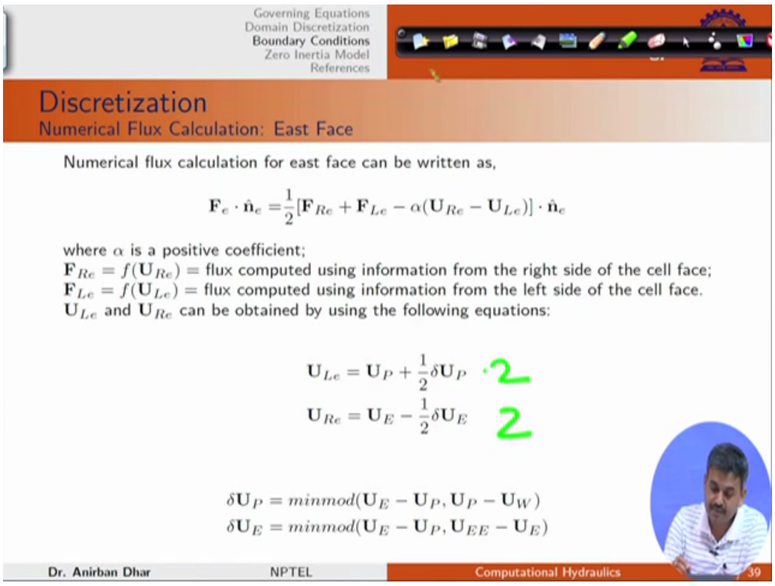

Now similarly when we will be calculating the value of west face so this is our P cell, this is west, this is ww and this is east one. So we are considering this w face. In case of w again we need to consider left and right one and in that case we have a and b. The first one if this is half UP minus UW then this is half of UP plus UW. But if it is UE minus UP or this is del P in this case. So del P is coming here on the right hand side it is just opposite. For left hand calculation it will be UW.

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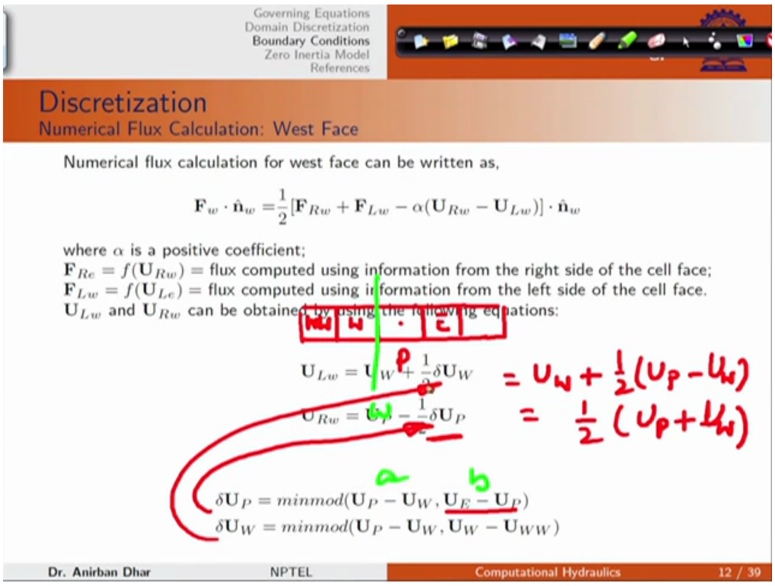

So either it should be average of P and W or the information should flow from other side. It considers WW so obviously from this side it will be WW.

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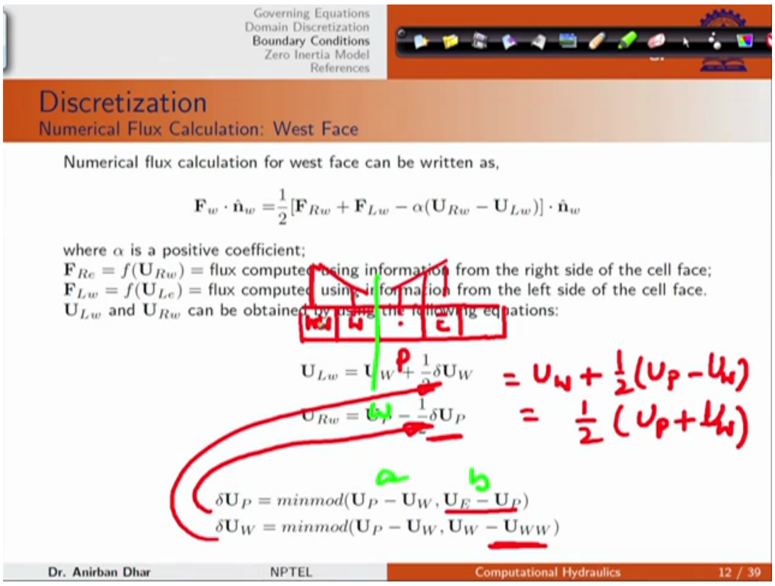

For north face this is again similar. We have top and bottom cell. This is P, this is south, this is north and we are talking about this particular north face. So for north face we need to consider NN that means north cell and UP UN, UP, US and UNN. So this is the structure there and we can calculate this UBn and UTn.

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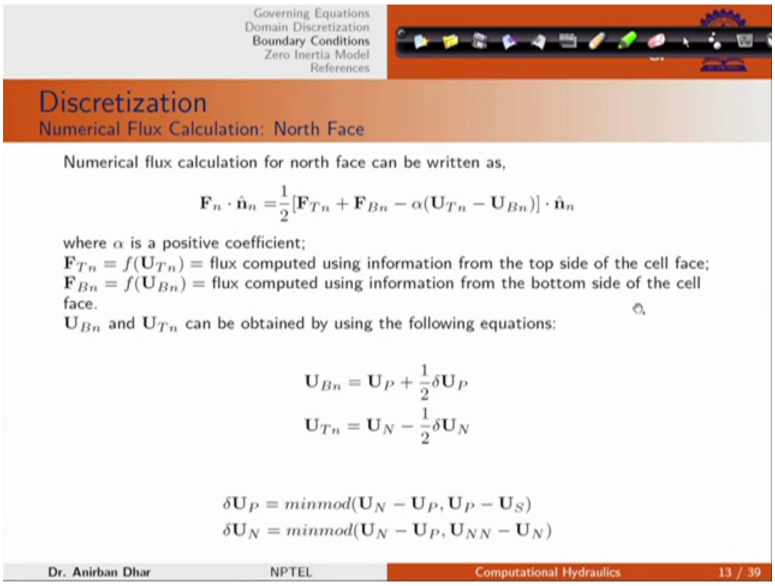

Similarly if we have component on this side so that means this is south face, we have P, this is north, this is south and then we can have SS for south-south cell there. So with this we can again calculate the flux values at the interface.

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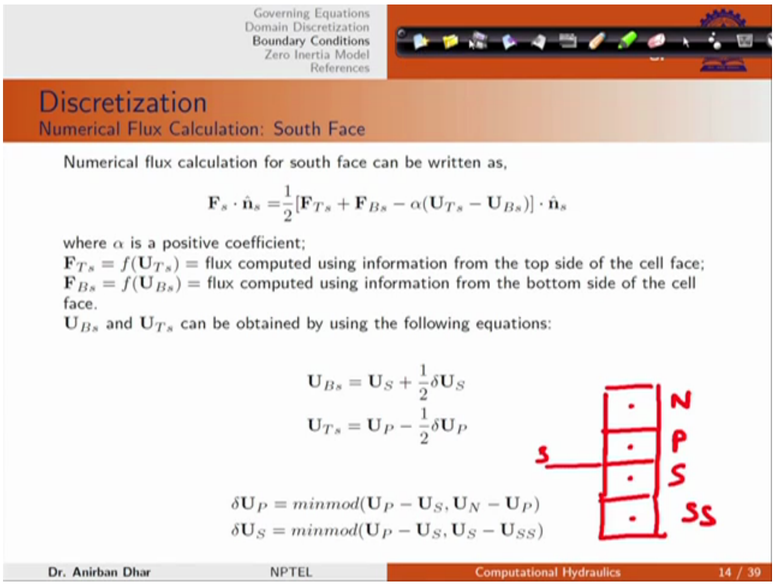

So for all faces we can calculate this. Now this is our general minmod limiter definition. Alpha value which is the coefficient can be calculated as per the suggestion of Nujic, 1995. So alpha should be greater than equal to maximum value of lambda P. Lambda P can be calculated from Jacobian matrix and it is the largest eigenvalues. Obviously this can be approximated with lambda P equals to VP plus root over hP. VP is the resultant velocity and hP this is our depth of flow in case of surface flooding or surface water flow.

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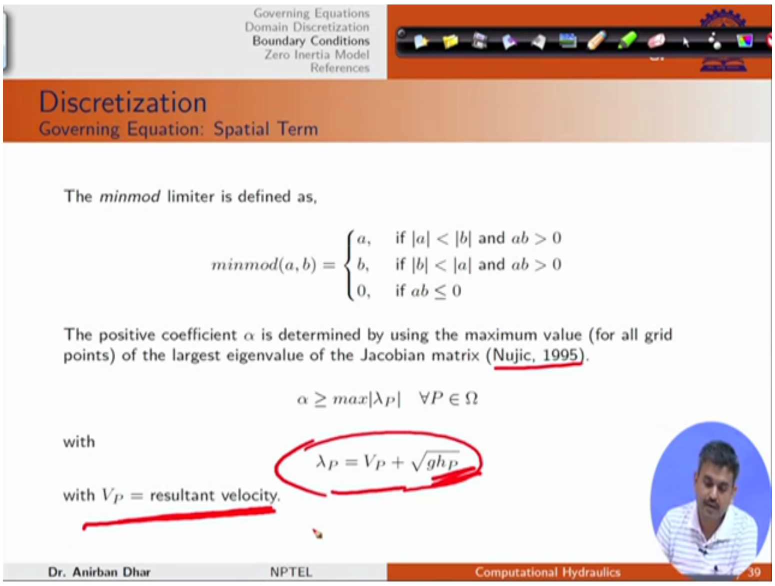

So we can utilize predictor corrector approach for explicit case. So obviously in this situation if we need to calculate the flux value at east face, west, north face so that is nothing but on east side it will be E, west side it will be W, north side it will be G, south side it will be S. So individually these components will be there for calculation and S value is evaluated at Lth or present time step. And this U star which is calculated (predic) or predictor step or intermediate step in this case.

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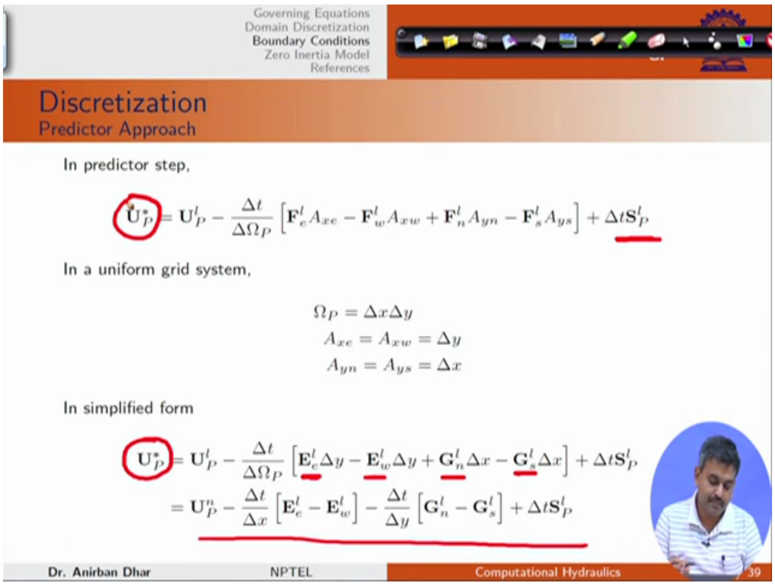

And in corrector step U star-star value is calculated. And U star-star value in this case again we have this one. So in simplified form again E, W, G, n, these values can be calculated there. At future time level or L plus 1 in our case which is the t plus delta t level we can simply add these values and take average of that.

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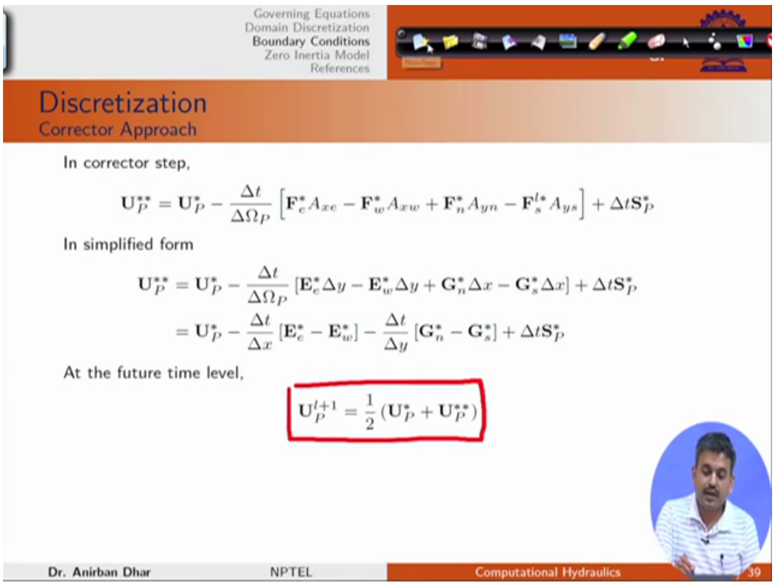

Actual variables now can be calculated after this step. So what is that actual variable? Actual variable is for pth cell UP 1 is the actual variable. But UP 2 is not the actual variable. We need to divide it by P1. Again vP we need to divide it by UP 1. So after this corrector step calculation we need to update this.

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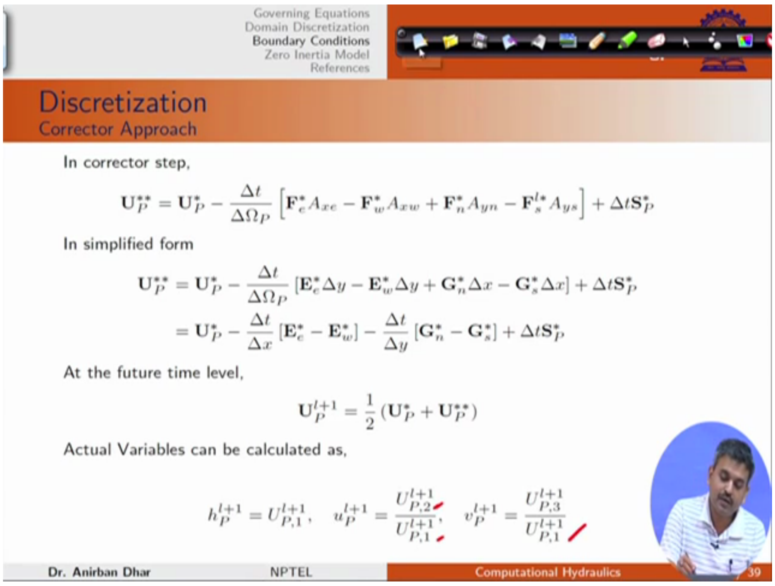

So in our predictor corrector step for both the cases we can use the same del E or del P these values because that will lead to numerical stability of the scheme in this case. Now we need to provide or specify no flow boundary condition. We know that if we have no flow boundary from all sides obviously uh e, uh w, vh n, this vh s these quantities will be zero individually. And if we want to implement this with our governing equation then let us see.

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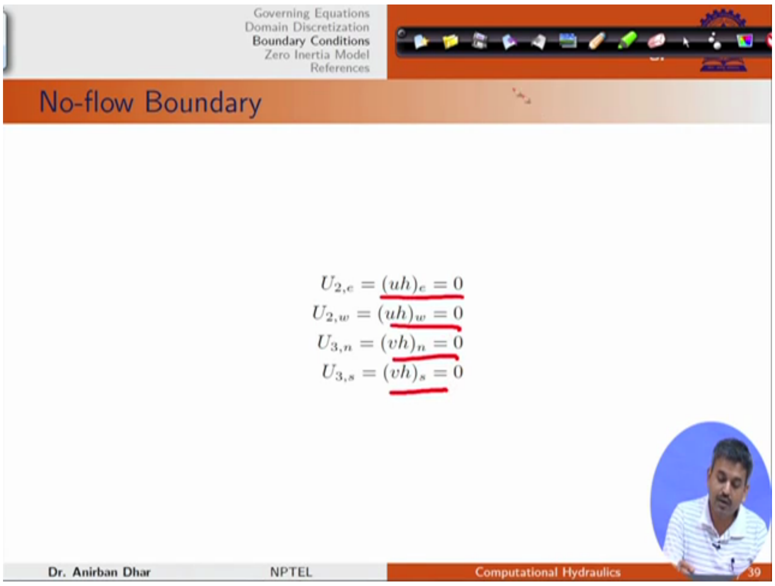

For east boundary obviously at east boundary we will have this flux component. We need to calculate at east boundary what will be the value for this case? Only this component will be there.

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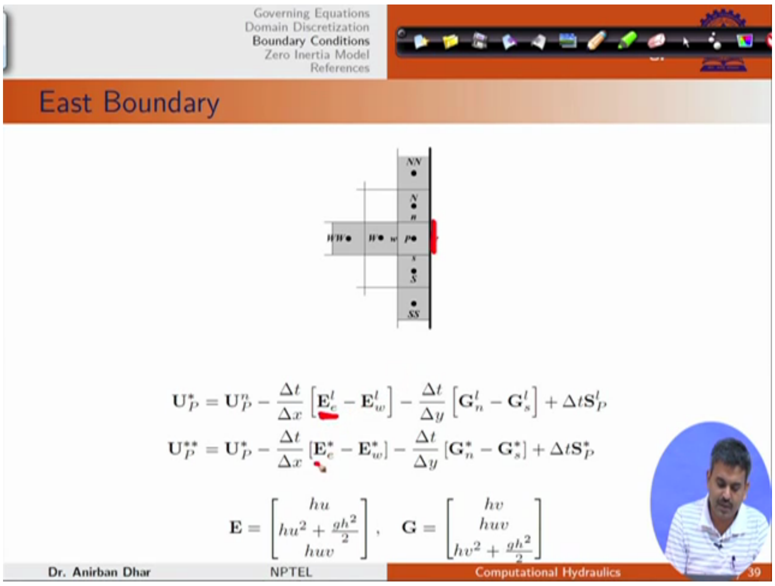

So at east boundary u is zero which is normal to this boundary. So this term is zero, this term is zero, this term is zero. Obviously E will be zero half gh square and zero. And G again this can be calculated for north and south boundaries. So there will be no change in this case.

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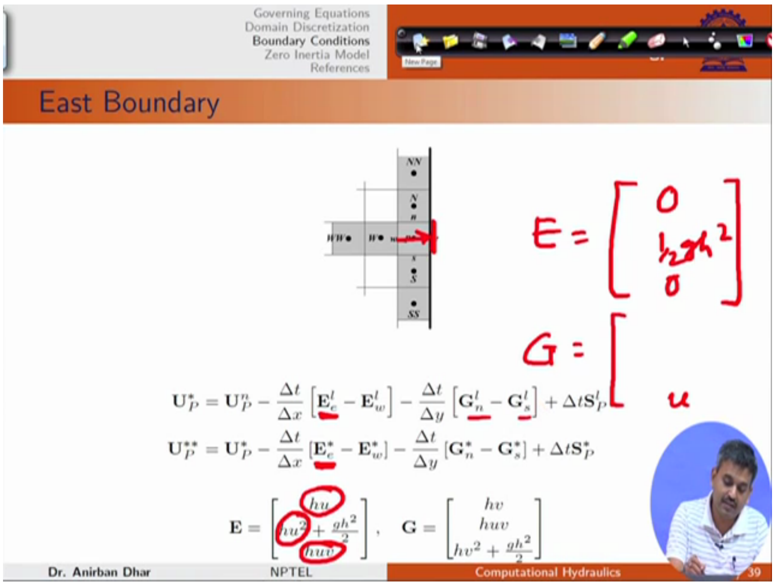

Similarly for west boundary again these quantities will be zero. So approximately to calculate the east face value we can specify half g P square which is the cell centre value there but this is approximate specification of boundary condition. We need proper characteristic or method of characteristics to specify the boundary condition for the explicit case.

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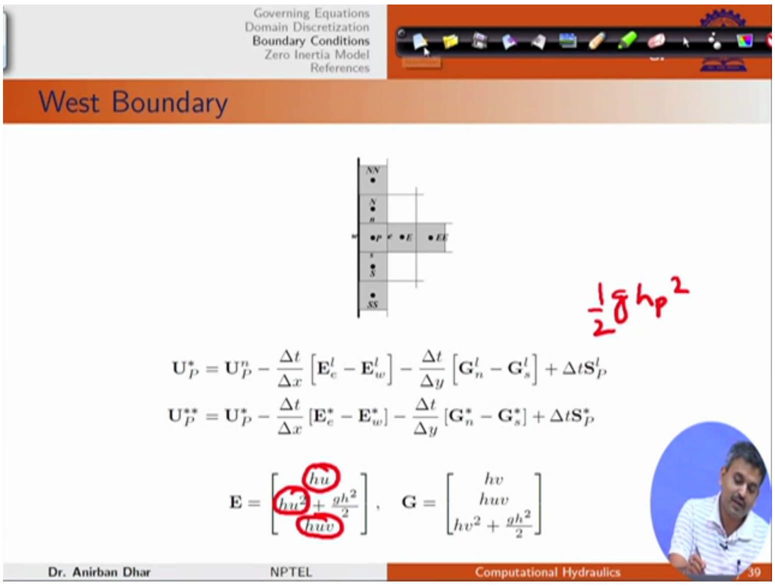

North boundary obviously north boundary case v is zero. So these quantities are zero. So G will be zero-zero half gh square for no flow case.

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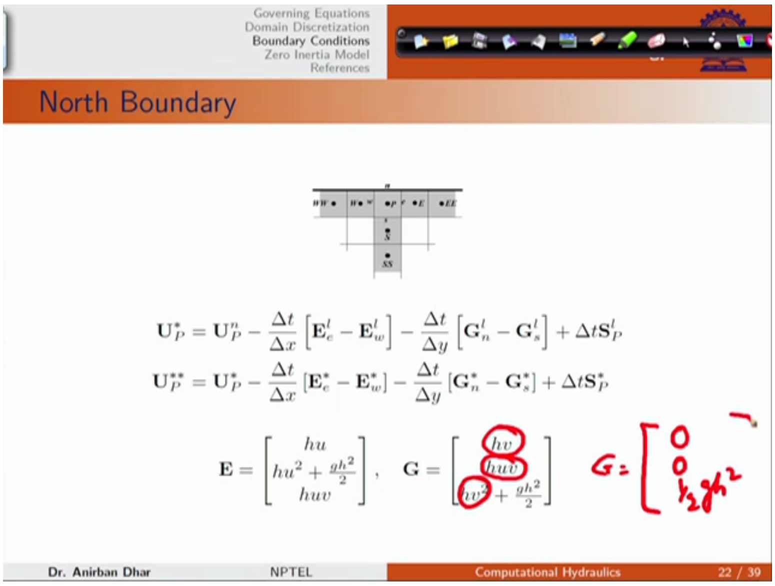

Now in case of our south boundary this is again north boundary. If I have a south boundary on this side we can have south boundary here. And for south boundary the thing is same. This GS this quantity will be zero for some components for no flow case.

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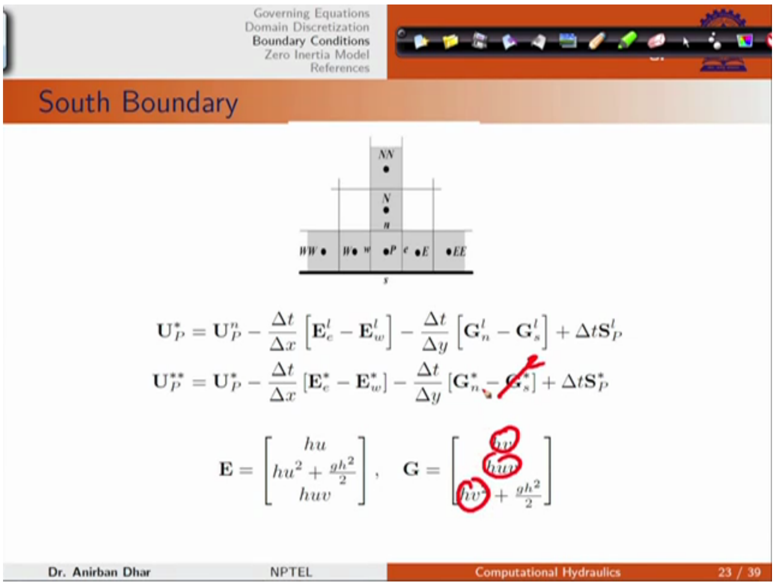

This next to east boundary we need to implement our governing equations there. Next to west boundary, next to north boundary, next to south boundary. Next to south boundary again we need to implement it. And north east corner, so obviously in this case for north corner this will be having zero components. East corner this will be having zero components.

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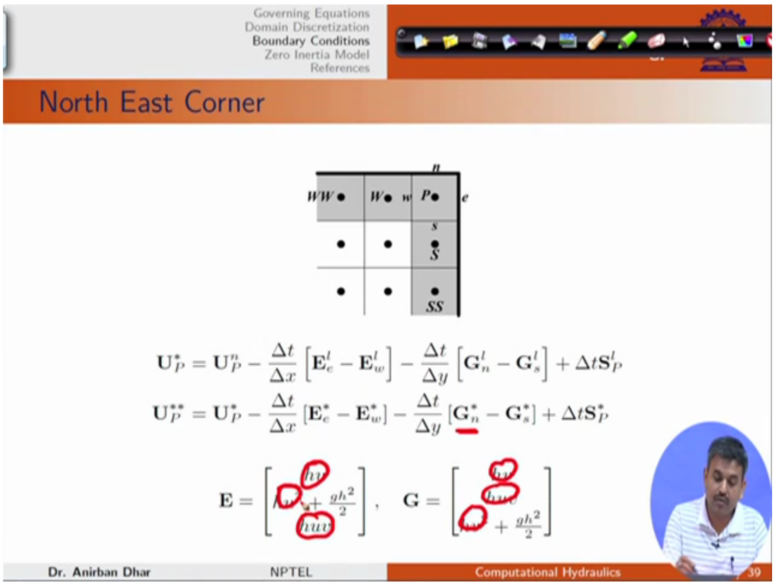

So north west then south east and south west corners we can have similar situations. So for our surface water flow specification of boundary condition and more or less the solution it is difficult due to nonlinearity present in the equation. So what we can do for basin flooding or simplified modelling case we can reduce our shallow water equations and we can drop some terms like acceleration terms and we can solve that equation for simple flooding situations.

So this is our usual shallow water equation. I have written it in terms of this qx qy. Qx qy is nothing but this uh and vh.

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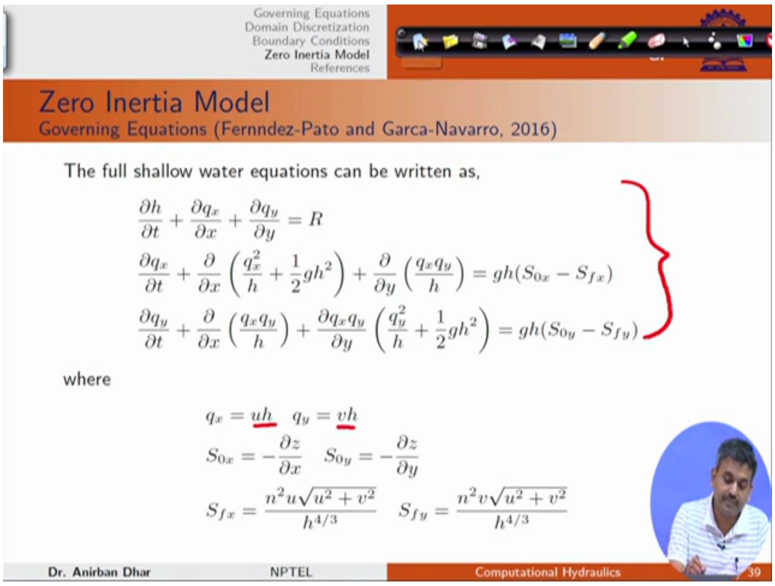

Now SF and Sy these quantities we can directly write here. Now what we can do we can drop some of the terms. Drop some of the terms and we can get the reduced form of the equation. So what is that? That is we can retain our continuity equation and we can drop other terms in our momentum equations by neglecting acceleration terms like this.

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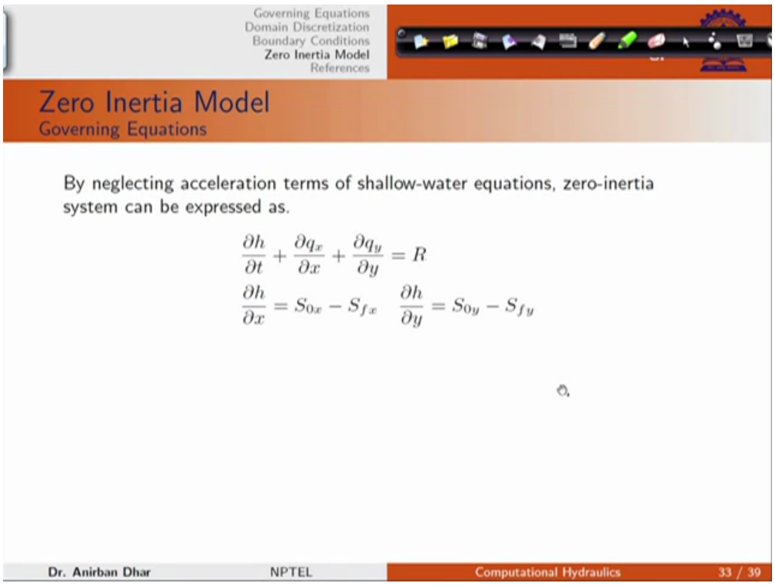

Now if I combine this Sy and Sx these are related to water surface slopes. And water surface slopes if I equate it with energy slope then I can write this simple equation there. And finally by combining discharge and continuity equations final form of inertia equation can be written like this and you can see that this has got similarity with our groundwater flow equations.

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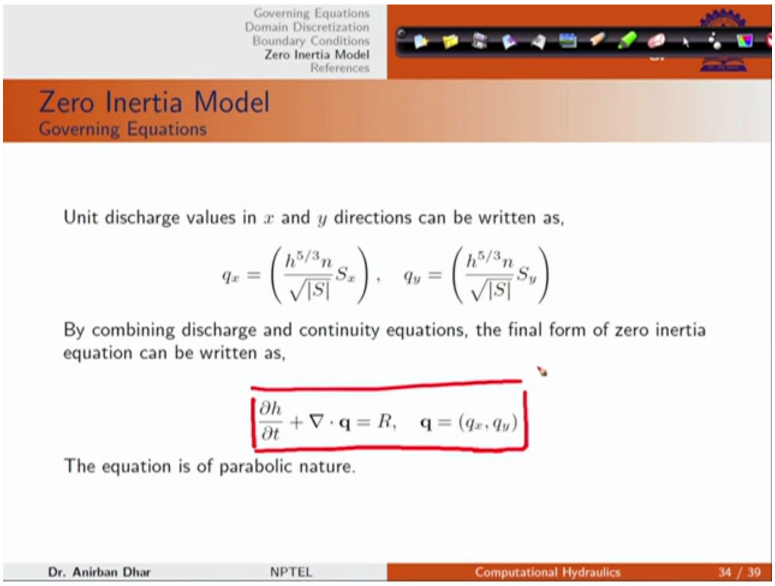

It is near to or it is more or less equivalent to that equation. If I simplify with this consideration which is alpha h which is function of h only and zero inertia equation can be further simplified like this. I can transfer this h term on the right hand side. Obviously this has got only one variable for zero inertia model.

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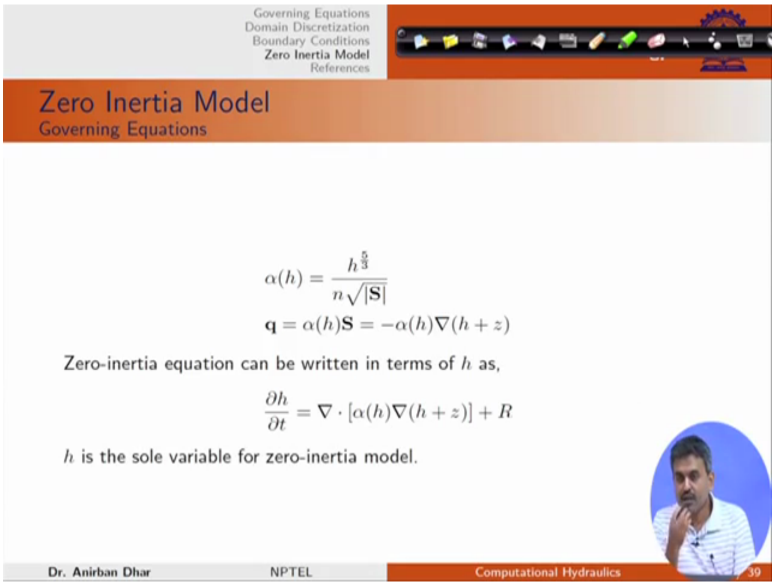

Usual surface water flow equations we need to solve huv but with a zero inertia case we can reduce our problem to one variable case and we can solve that problem. So again if I apply our finite volume method here I can directly write this integration or integrate a form of the equation.

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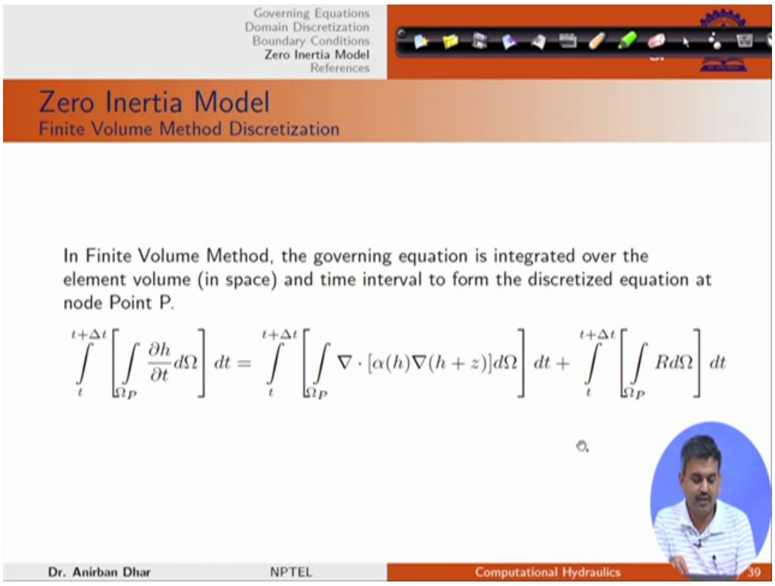

And we can discretize that for solution of different kinds of problems. But obviously in this case the movement or acceleration terms we are neglecting. Obviously in that particular case we have to consider slow movement of water. So slow flooding or irrigation application of water through canal system can be considered as one can example for this particular equation.

This is my irrigation system and for this irrigation system I can (sup) supply water from the canals. So the movement of water from this canals will be slower. So I can model this using this simplified version of our shallow water equation or 2D surface flooding equation as zero inertia model. So after flooding obviously the water will move from one cell to another cell and there will be flooding situation in the total area if water is applied through this canal outlets.

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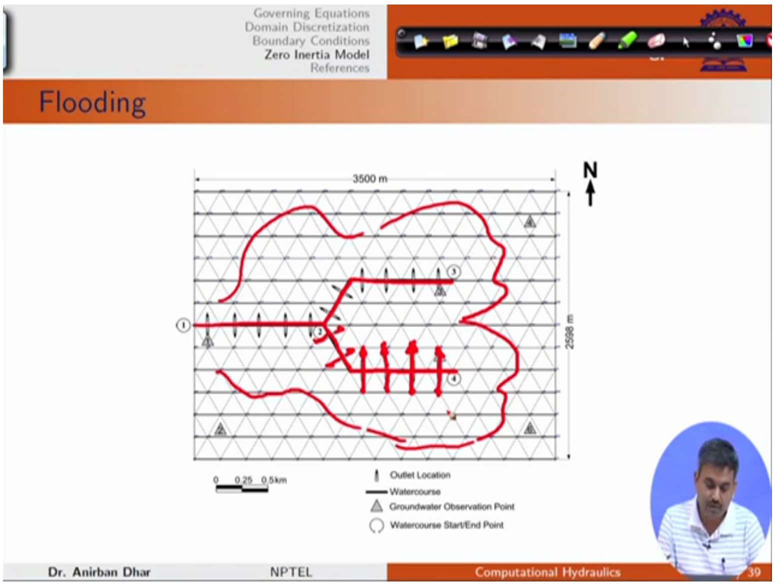

So this kind of problem can be solved with zero inertia model and we can directly apply our numerical approach for solution of that. So this is all about our 2D surface water hydraulics with unsteady term. This is the flooding situation in a typical canal system. So obviously there will be slow moment of water.

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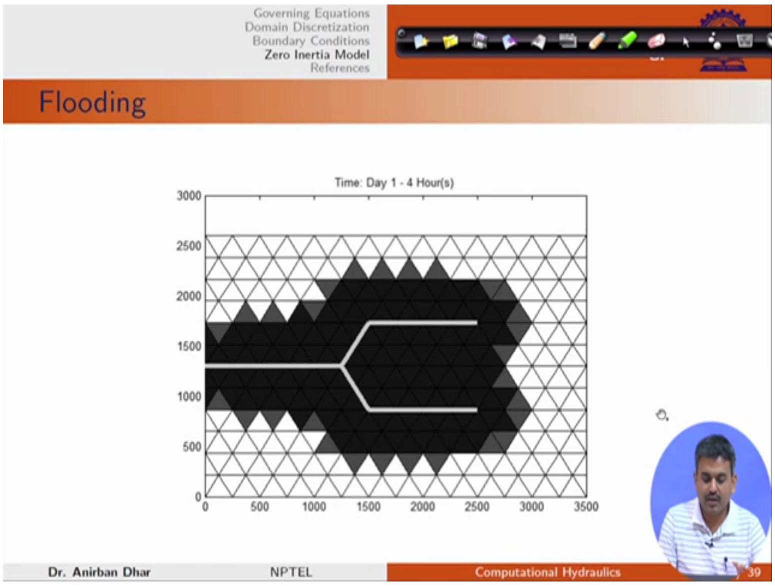

And this can be model using zero inertia. Thank you.