Computational Hydraulics
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Lecture 44
Unsteady 1D Channel Flow

Welcome to this lecture of the course computational hydraulics. This is model number 4 surface water hydraulics and in this lecture class I will be covering unit 7 that is unsteady 1D channel flow.

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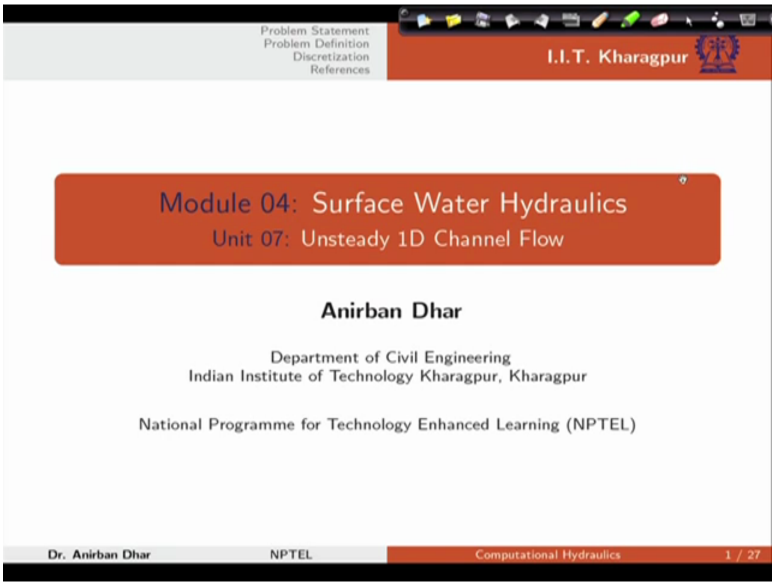

Learning objective, at the end of this unit students will be able to solve unsteady channel network problem using implicit approach.

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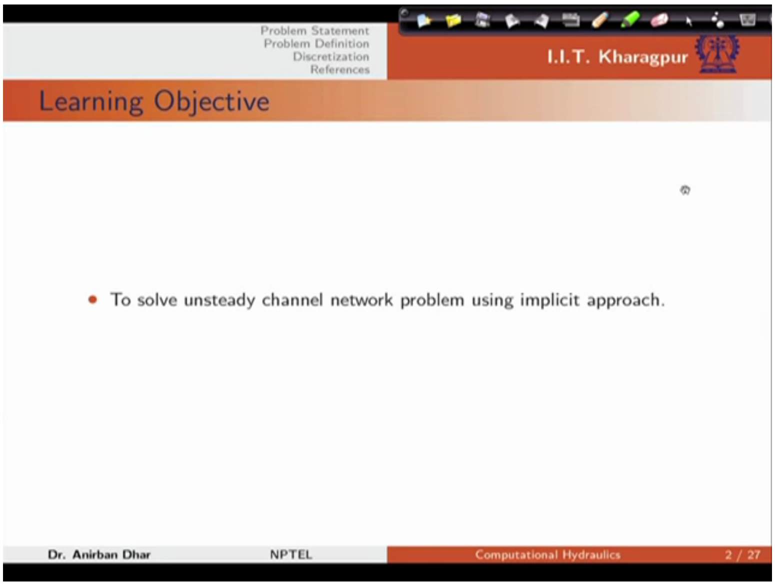

Problem definition to solution, this is our total structure. In this case we are considering that discharge is the function of x and t. At the same time y or flow depth that is also function of x and t. In our previous lecture class, lecture with steady channel network flow with or without reverse flow situation we have discussed the same problem. In that one we have conceptualized our problem as Q as function of x and y as function of x. So this is essentially steady problem and this is unsteady problem.

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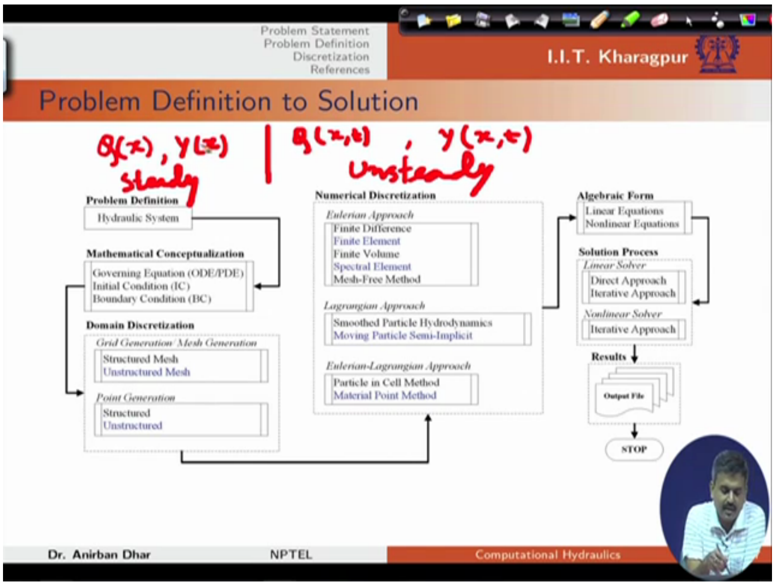

In steady problem the quantities Q and y these are not varying with t but in unsteady problem Q and y these two quantities are varying with time. So this is the first assumption that our discharge and flow depth these two values are varying with time. So what is our hydraulic system? Hydraulic system is natural maybe channel network. Let us say this is my channel network and let us consider a problem where I have some specified condition at this point and this point and end point. This is our junction as per definition.

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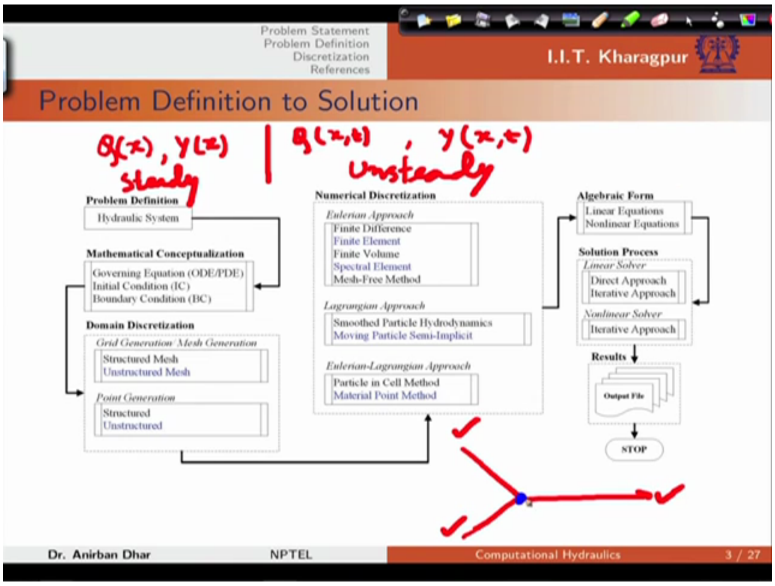

Now as per conceptualization in steady state flow situation or steady state channel network situation we have considered different Q values for different channel reaches. This is let us say channel reach 1, this is channel reach 2, this is channel reach 3 and for these three channel reaches our discharge and flow depth this two points it will vary in case of unsteady case also with time. But in steady state case we will have fixed values for discharge and y.

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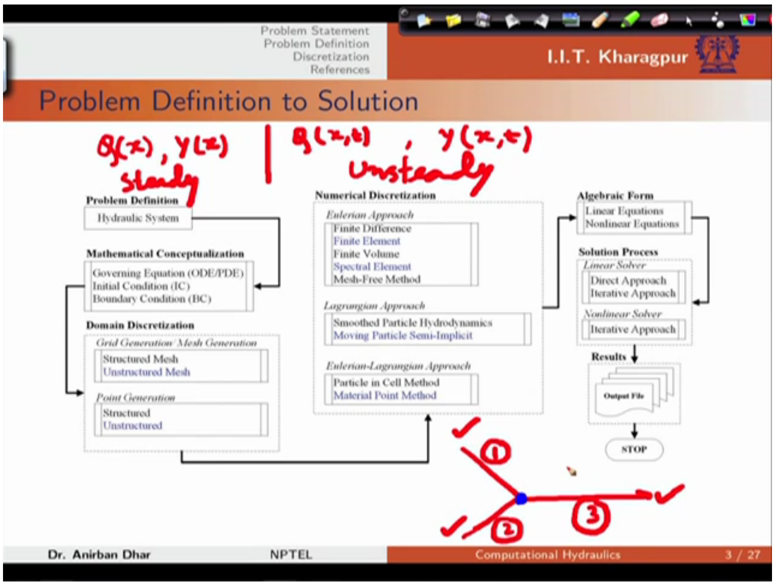

Now our steady state problem that was boundary value problem because we need boundary value only to solve that problem. But unsteady problem we need to specify the initial conditions. So during mathematical conceptualization so hydraulic system is channel network, this is channel network flow. Governing equation, we need governing equation for this one. We have two quantities that is Q and y. So we need two governing equations for this problem.

At the same time we also need initial condition because what is the initial step that is important for any unsteady problem. At the same time we also require boundary condition. Boundary condition may be fixed or time varying boundary conditions. Either it can be in the form of time varying discharge or varying depth or fix depth or time varying discharge condition. Now we can discuss our domain which is essentially 1D domain each channel network or channel reach in this case. We can consider uniform grid.

Also we can solve the same problem for non uniform grid system. And in this case I will be talking about special kind of finite difference technique. And finally the equations for this problem is nonlinear in nature. So we need to apply this nonlinear solver either that is iterative approach or Newton Raphson approach. I am utilizing for solution of this problem.

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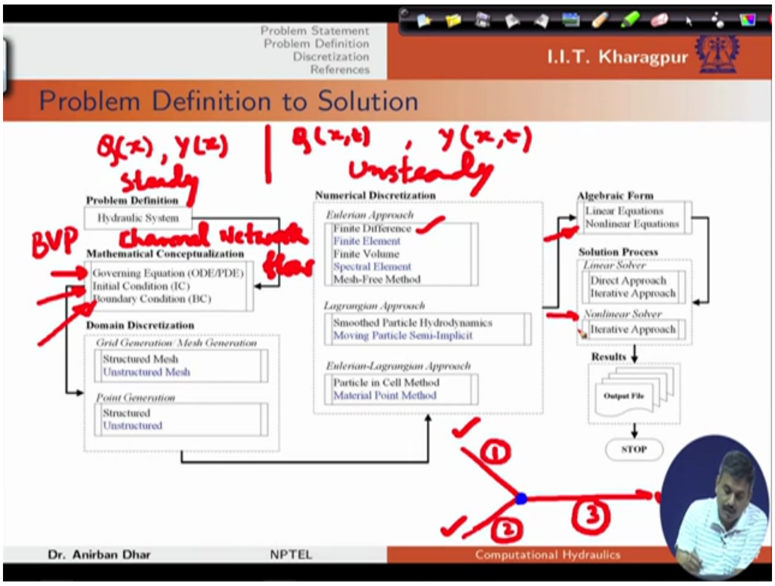

So let us see what is the problem we want to solve? Now let us consider this network. So we have channel 1, channel 2 and channel 3. All these channels are 5000 metres in length. And we have two specified discharge conditions at upstream. So let us say these red dots, red dot 1, red dot 2 these two are specified discharge conditions in the upstream. And we have specified depth condition in the downstream section. And internal blue node this is our internal junction condition.

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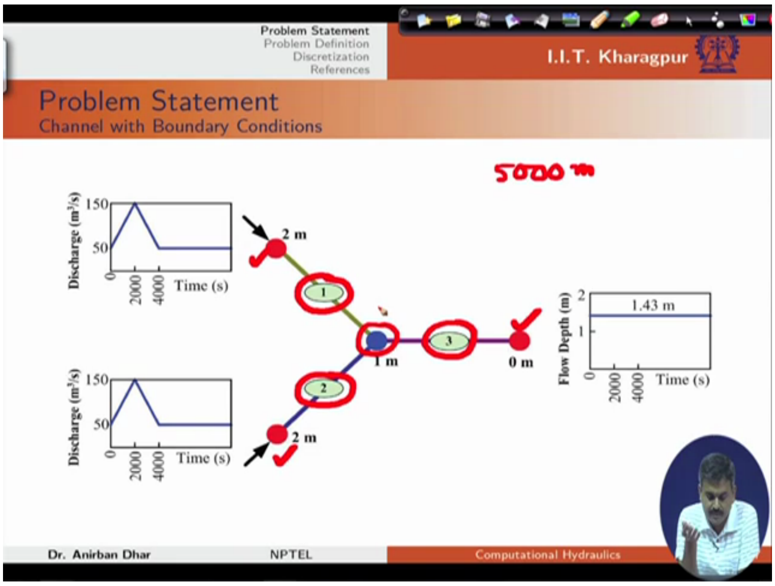

We have elevation of zero. This is plus 1. For this node we have plus 2, for this node we have plus 2 for elevation.

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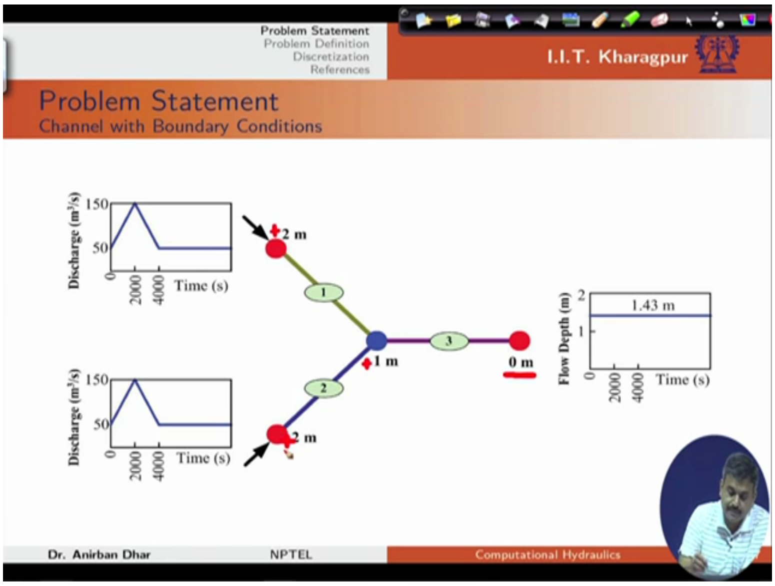

Now for this problem we can see that this discharge is in metre cube per second and it is varying from 50 and it is reaching maximum value at 2000 seconds. And again it is decreasing from 150 to 50 and after this 4000 seconds we have a constant 50 metre cube per second discharge for both the inflow sections or inflow junctions.

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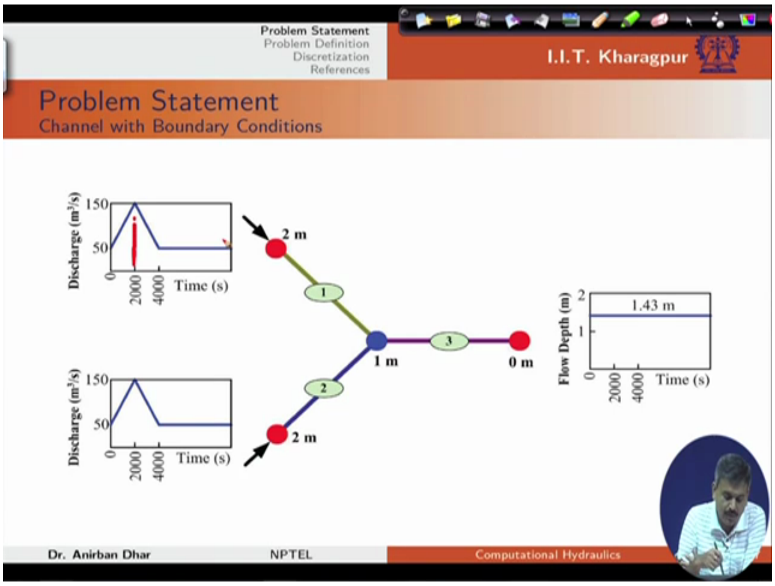

So we have two inflow junctions 1 and 2 and for both of these junctions we have specified discharge condition and this discharge condition is time varying.

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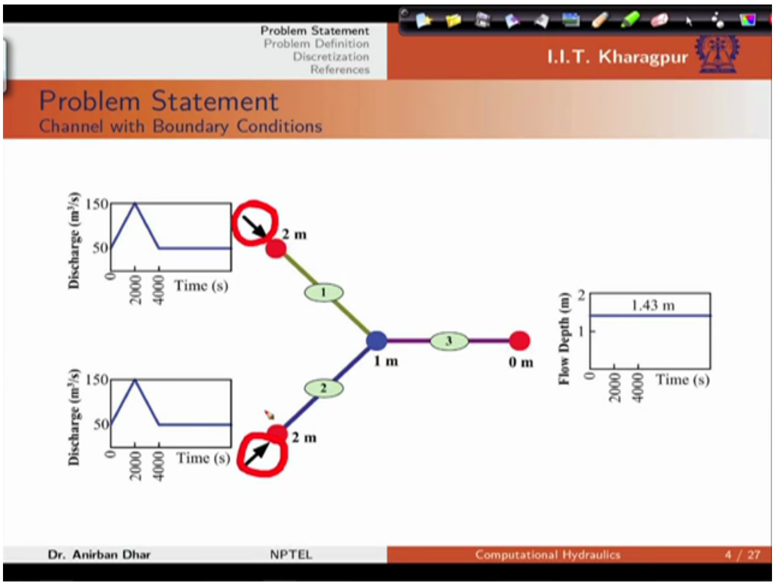

And downstream section that is at this point. We have specified flow depth and interestingly in this case we have constant flow depth overtime. So 1 point 43 metres this is constant flow depth at this point.

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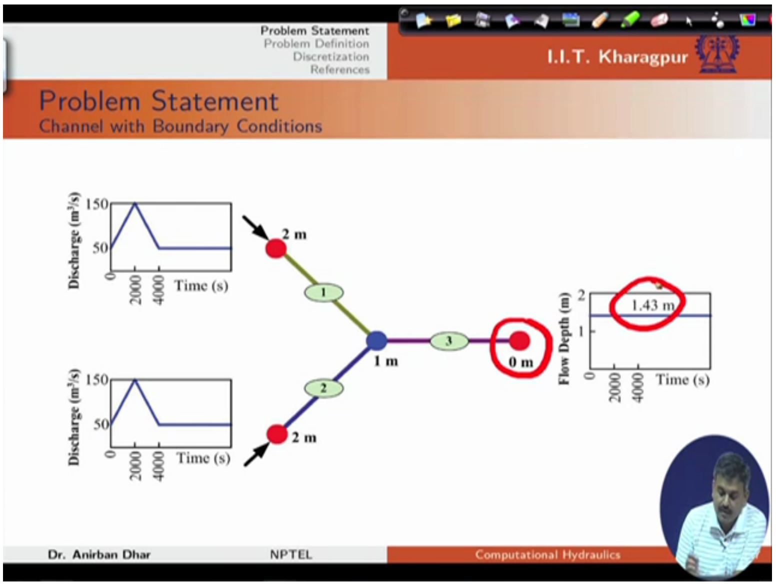

So it is not varying with time. Now we need to solve this problem. So for solution as I have already told we need the specification of initial conditions. So for this problem which is with these three channels let us say we have a positive flow direction.

This is our positive flow direction, this is our positive flow direction, this is 50 metre cube per second, this is 50 metre per second. On the downstream we have 100 metre per second and we have uniform flow condition in this channel network with flow depth of 1 point 43 metre for all the channel reaches.

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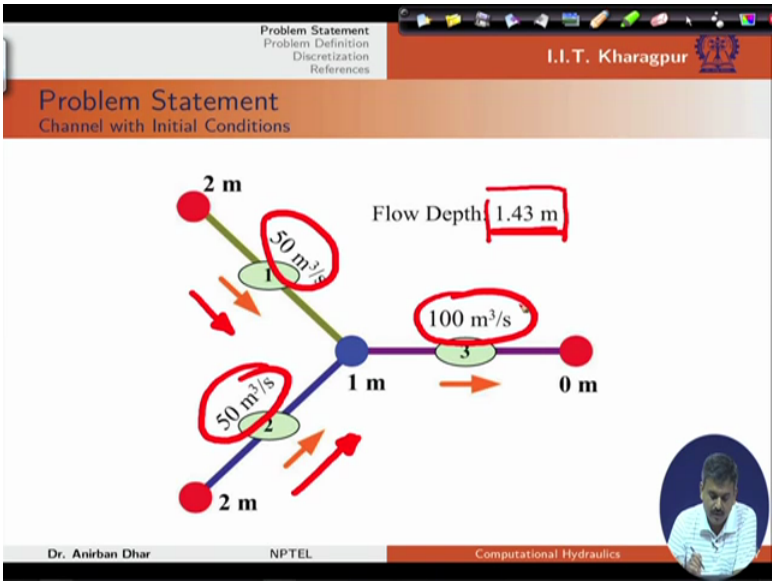

So we have a specified initial condition that is 50 metre cube discharge for these two upstream channels and for this downstream channel of this junction we have 100 metre cube per second.

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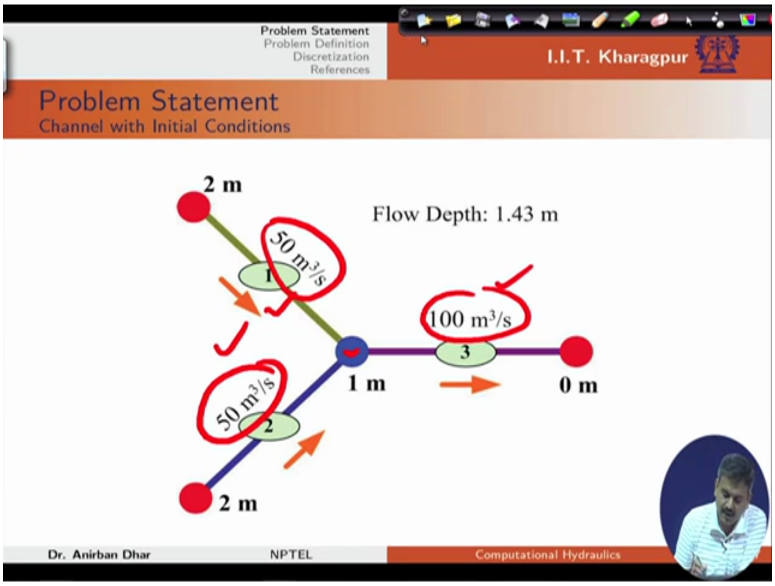

And this is our initial flow directions. So at this junction we can see that 50 metre cube, 50 metre cube, on this side it is going 100 metre cube. So unlike our steady state problem in this case we need to specify the initial condition by satisfying the continuity condition because initial condition means it should satisfy the physical problem. So in case of our steady state problem we have seen that from any arbitrary initial condition we can get the final result which is steady state flow condition.

So initial condition in case of steady state problem is nothing but initial guess for the steady state problem. But in case of (ini) or unsteady state problem initial condition is important because that should satisfy the physical constraints or physical equations. So discharge continuity is one equation that should be satisfied and we have a flow depth for all these channels as 1 point 43 metres.

That means at this point which is the blue point at the centre it is the internal junction point. For internal junction point our energy condition is also satisfied because from all sides this channel ends or the starting of the channel 3 these are at the same elevation.

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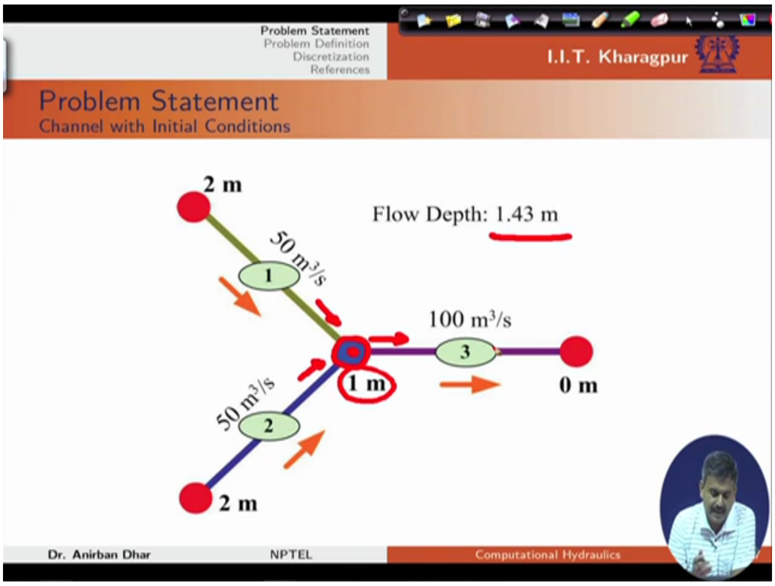

So we have energy continuity and discharge continuity. These two are satisfied at the beginning itself. Now we can list other things. So first assumption is our channel flow direction because we need to number our channel sections depending on the flow directions. So I will try to follow the same convention that I have used for our steady state flow situation. So in this case we will consider that we have 1, 2 and 3 these are junctions or boundary junctions and number 4 which is as internal junction point.

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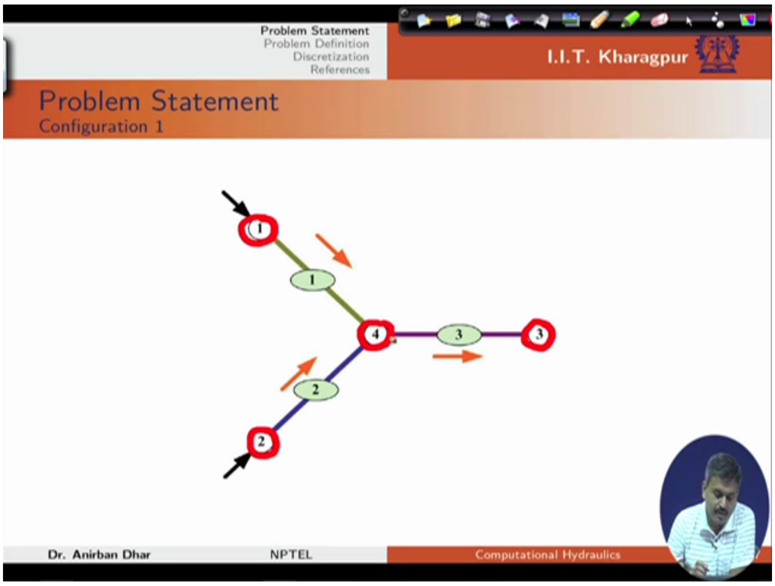

So we have three channel reaches 1, 2 and 3 and four junction nodes. Out of that three are boundary junctions. So let us see what are the background information for this one? So channel data we have channel number 1, this is 5000 metres length. So this is channel number 1 we have this is inflow, this is also inflow. So we have this as 1 junction node 2, 3 and this is 4. Channel 1, 2, 3. So now for this one for channel 1 length is 5000 metre, width is 50 metres.

So in this case we are considering rectangular channel section. So we have zero slope in m1 and m2 equals to zero. And this is B or width of the channel. But we do not know what is the depth? That is the function of x and t.

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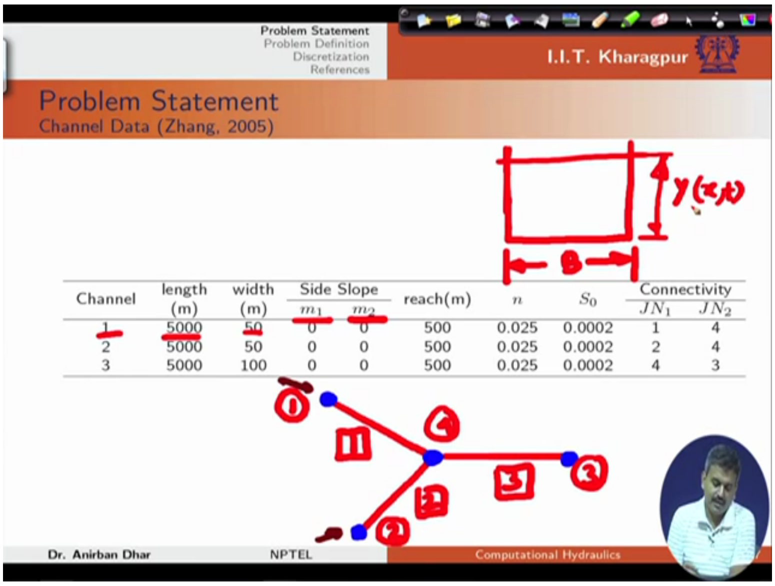

Channel	length (m)	width (m)	Side Slope m_1	Side Slope m_2	reach(m)	n	S_0	Connectivity JN_1	Connectivity JN_2
1	5000	50	0	0	500	0.025	0.0002	1	4
2	5000	50	0	0	500	0.025	0.0002	2	4
3	5000	100	0	0	500	0.025	0.0002	4	3

Now channel reach, that means del x1 for channel reach 1 is 500, del x2, del x3 all are 500 in this case. N value these are Mannings value and slope for all channels we have considered this slope.

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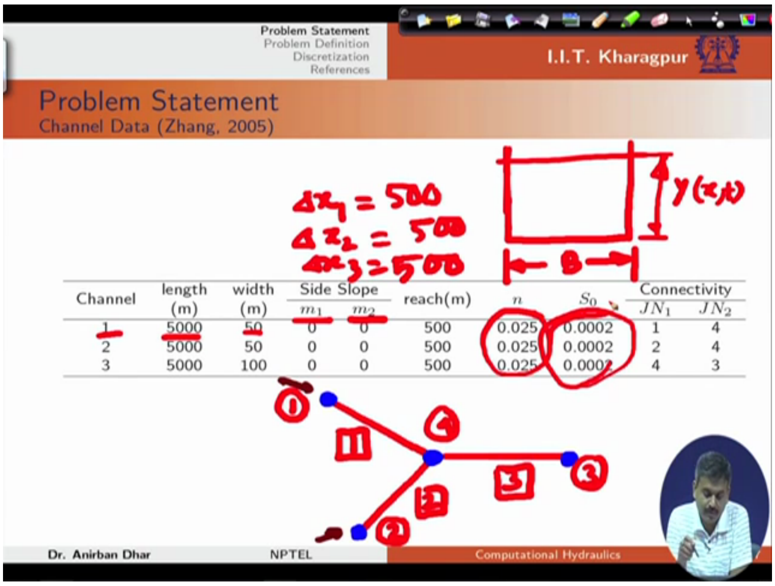

Now we need to specify the junction continuity or junction connectivity. For channel 1 it is connected to 1 and 4. As we have considered the flow direction from 1 to 4 that is why we are writing the starting node as 1 and ending node is 4. Channel 2 starting node is 2, ending node or node with the end section that is 4. Again for channel 3 we have 4 3.

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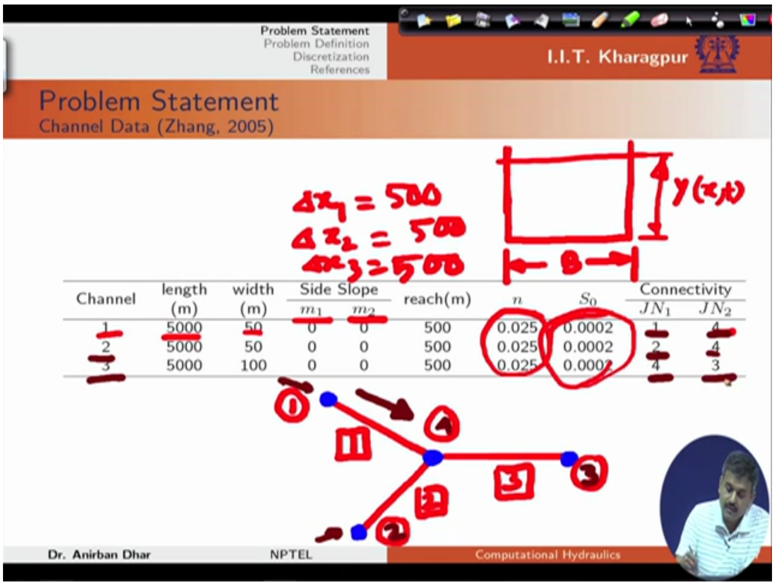

So these are the information that required for our problem. Now next thing is specification of boundary conditions. This is junction data. In our steady state case we have utilised same kind of matrix structure but the problem is in this case we have time varying boundary condition. So we cannot directly specify the values in single matrix. So for depth or flow depth we will write this is equivalent to 1. And discharge is equivalent to 2.

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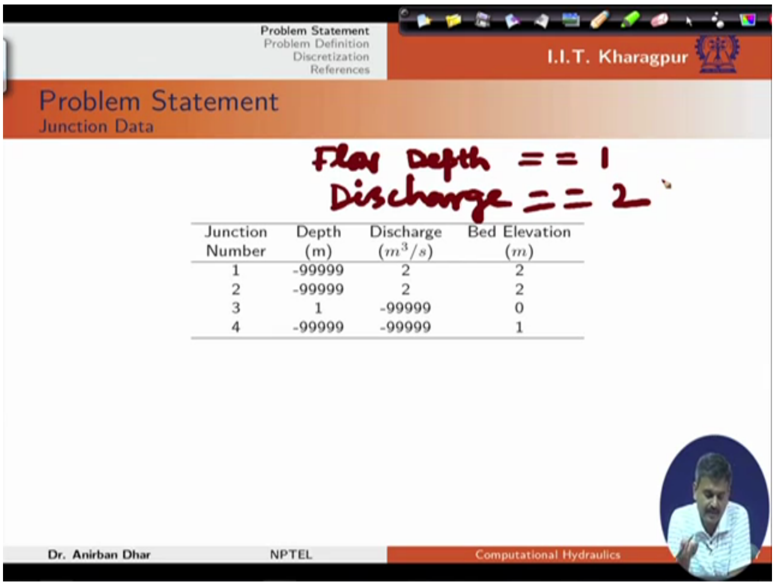

Now with this if I see this column 2, 3, column 2 is for depth. Column 3 is for discharge. Now for the particular junction this is junction number 1. Junction number 1 is inflow junction or specified discharge condition. So if there is specified discharge condition we will write it as 2. Otherwise we can directly write minus 5 9s. And for junction number 2 also that is again boundary junction we do not have any specified depth, so minus 5 9s. And in this case again we will write it as discharge equals to 2.

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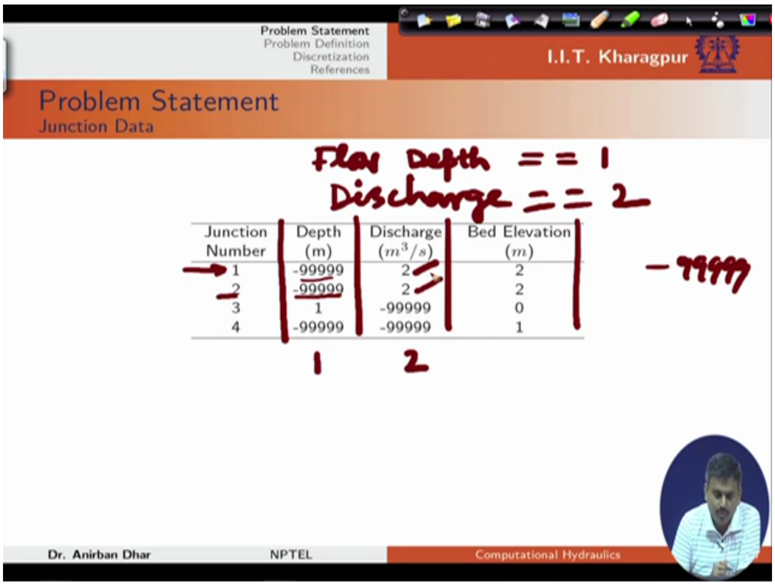

Now junction number 3 again we have specified depth value. That is why here we will write it as 1 and we do not have specified discharge condition for our problem that is why we have minus 5 9s in case of discharge. And for junction node which is internal junction node number 4 we have 5 9s because we do not have flow depth or discharge specified for this junction.

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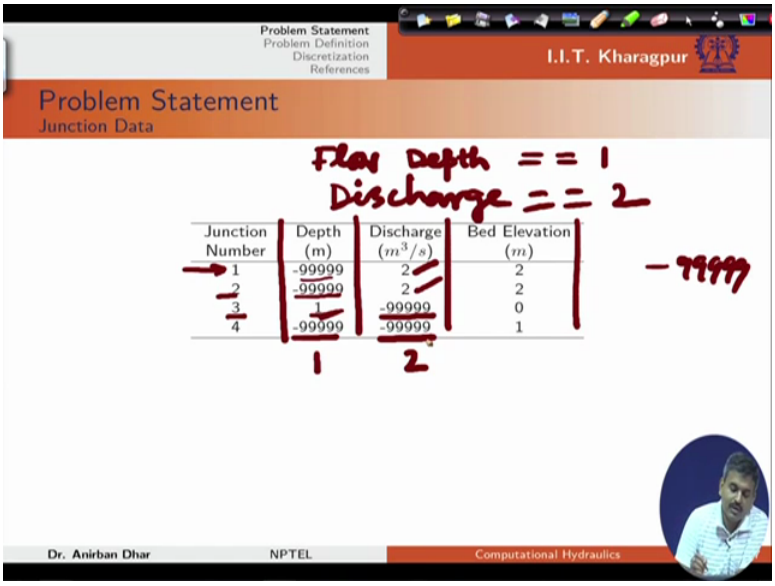

Now for our problem we have four junctions. Out of this so let us say that N1 plus 1 is the number of sections in channel reach 1. Channel reach 2 and channel reach 3 we have N2 plus 1, this is N3 plus 1. So in this case for a particular time step we have N1 plus N2 plus N3 plus 3 into 2 unknowns. That means we have depth and discharge. If we add depth and discharge values we have these many discharge and these many depth values are unknown.

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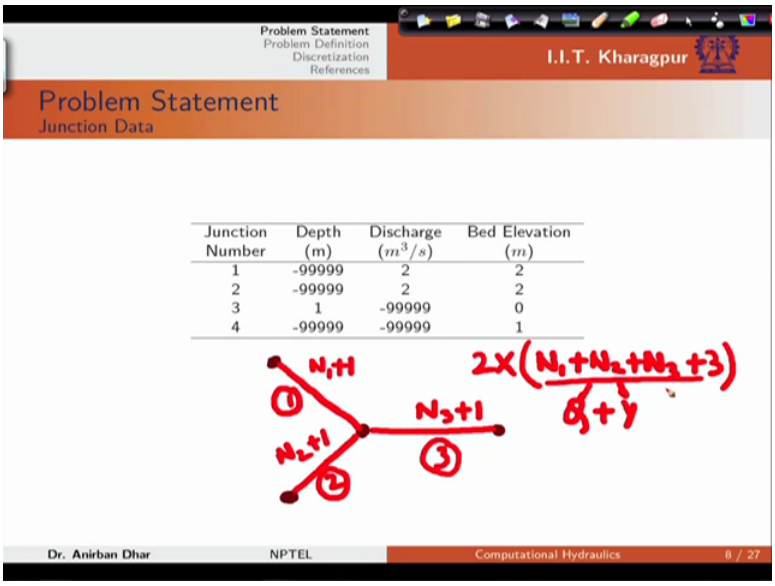

So all total we need these many equations to solve this problem. So we have 2N1 plus 2N2 plus 2N3 number of equations coming directly from N1 number of segments from channel 1, N2 number of segments in channel 2 and N3 number of segments in channel 3. So still we need 6 conditions. Out of that for this internal junction we will get junction continuity.

What is that junction continuity for this problem? For this problem we have this junction and at this junction this is our channel reach 1, this is channel reach 2 and this is channel reach 3. So we will have Q1 N1 plus 1 plus Q2 N2 plus 1 minus Q3 and 1, this should be zero.

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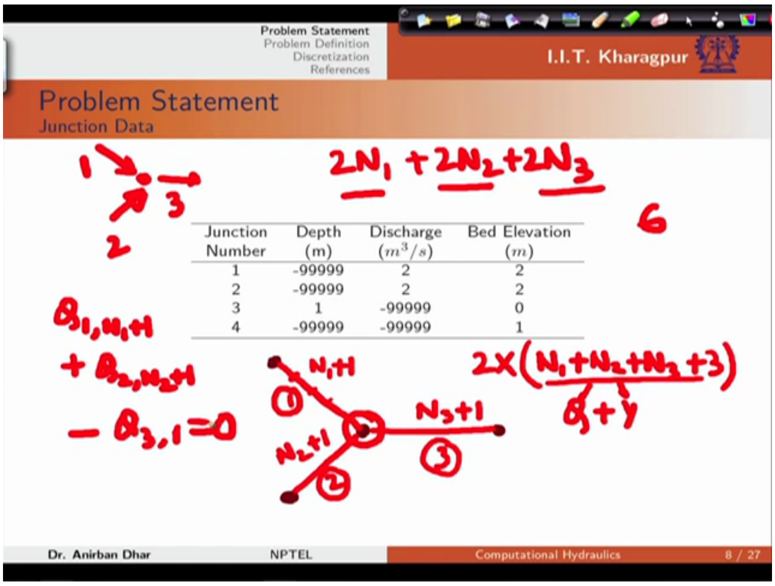

So inflow is coming from our channel reach 1, 2 and this flow is coming out from this section 3 1 of the channel reach 3. So we have this discharge condition. Next is energy condition because we have considered that junction or end sections are at the same elevation at the junction. So we can consider that y 1 N1 plus 1 equals to y 3 1. And third condition y 2 N2 plus 1 equals to y 3 1.

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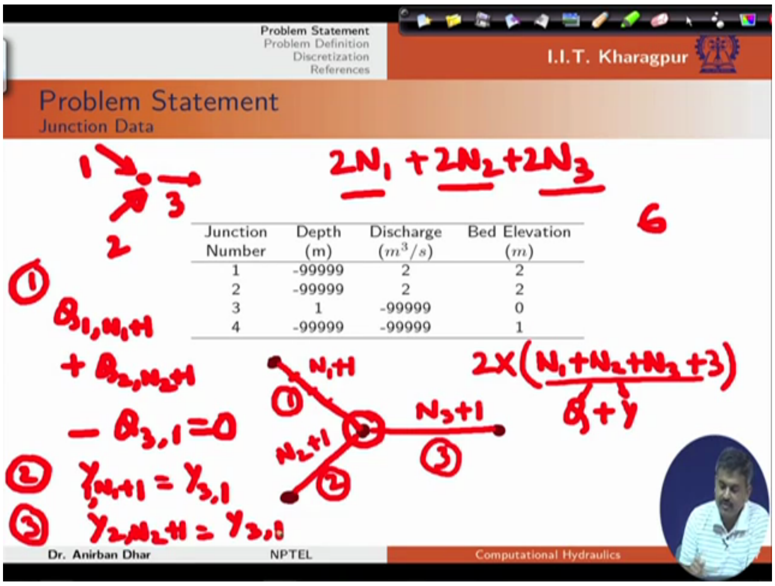

So we have three conditions out of this 6. Now still we need three conditions to solve this problem. So from our boundary conditions we have specified discharge at node 1, note 2 and specified flow depth at node 3. So from these we are getting extra three conditions. So now we can solve this problem. So 2N1, 2N2, 2N3 these number of equations will be coming from individual segments of different channel reaches and three equations for junction node and three boundary conditions. Two discharge and one flow depth condition for this problem.

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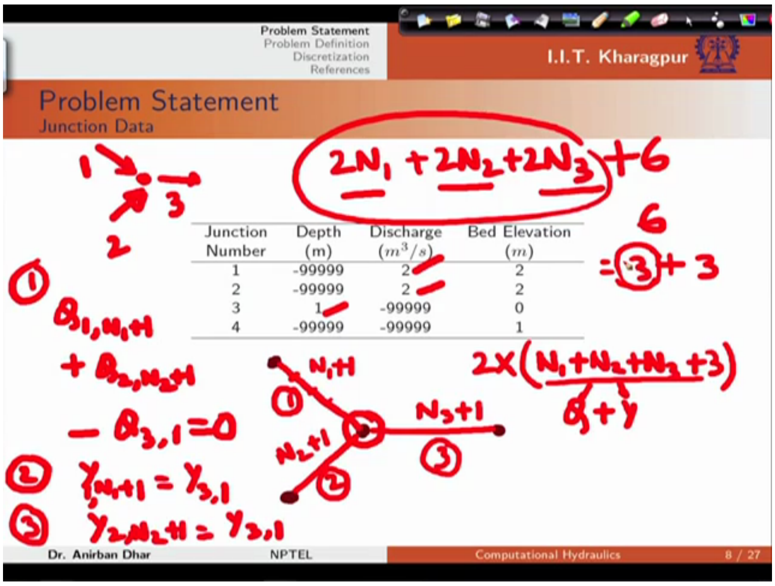

Now what is required out of this problem? We need to plot the discharge and depth hydrographs at x is equal to 4000 from internal junction node in channel 3 of the network. That means if we have this is junction node number 4, 4 to this is 3 at x is equal to 4000. That means this length is 4000 at this length of channel 3 we need to find out what is the variation of discharge x is equal to 4000, discharge with time. And this is x is equal to 4000 again flow depth with time.

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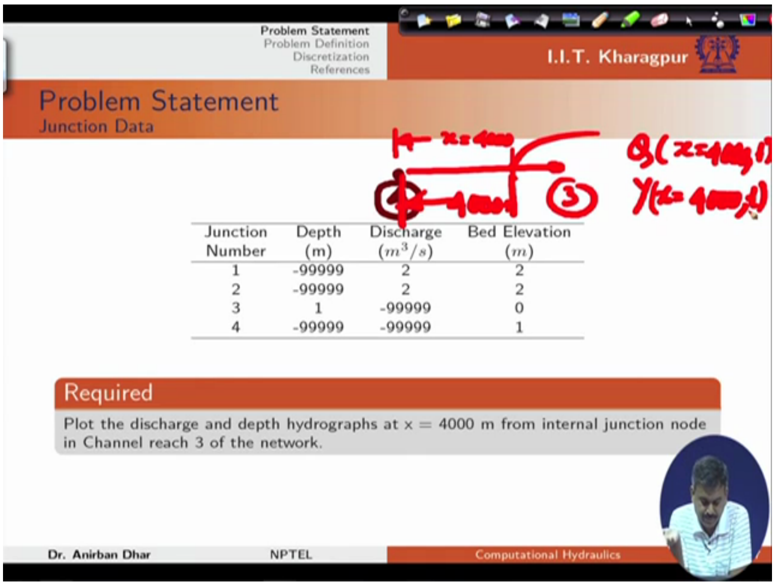

Now we need to plot these two values. To start with for this one we need to define the problem. So problem is essentially this is governing equation for 1D channel flow that is St.Venant Equations required. One for continuity and one for momentum. This is initial boundary value problem and this is our initial condition.

If at a particular junction if you have some extraction or injection into the system we can use this q. Momentum, this is the momentum equation and what is this H? H is nothing but this is y plus z. That means y is flow depth and z is our elevation of the channel bottom then H is y plus z.

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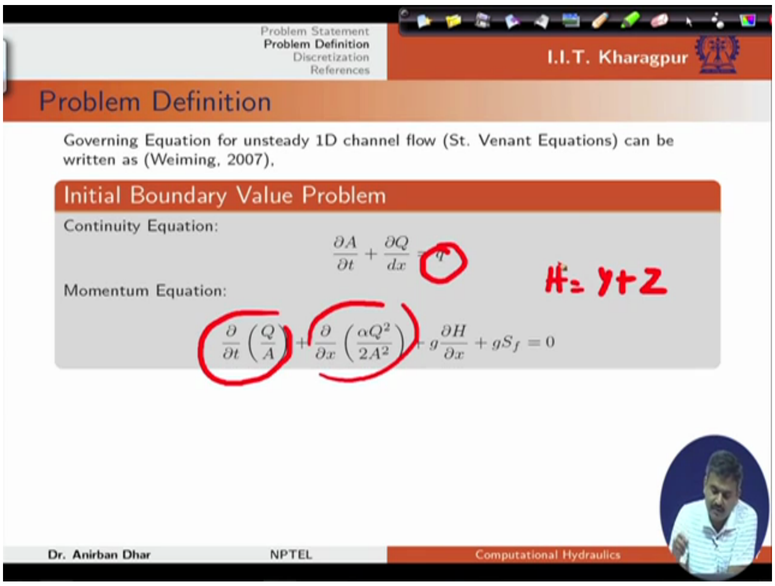

For any channel let us say this is a channel and we have this datum. For this datum at any section this is our z and this is our y. So the total thing is H or y plus z.

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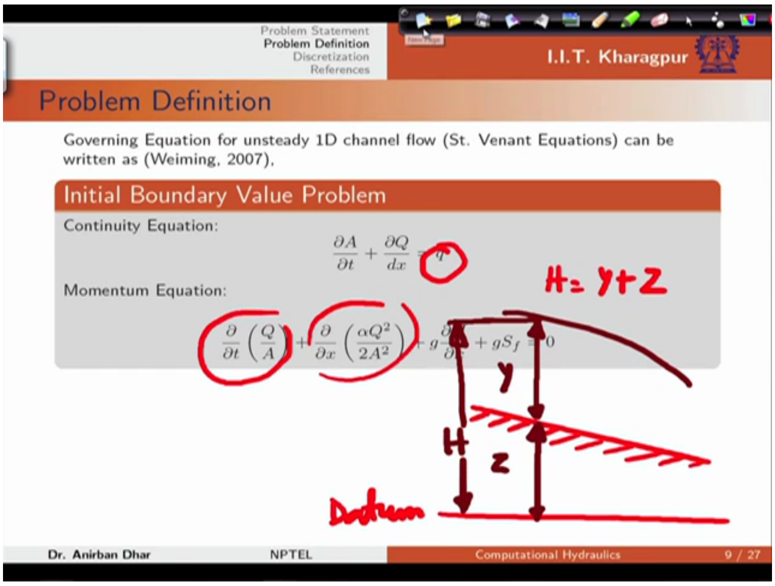

In this case SF is our energy slope or friction slope. So in this case we have y as flow depth, SF is friction slope, A is cross sectional area, q is lateral inflow to the system, z is elevation of the channel bottom with respect to datum, H is water surface elevation, alpha is momentum correction factor, Q is discharge, g is acceleration due to gravity and we are considering the flow in x direction only. X direction or one dimensional in space.

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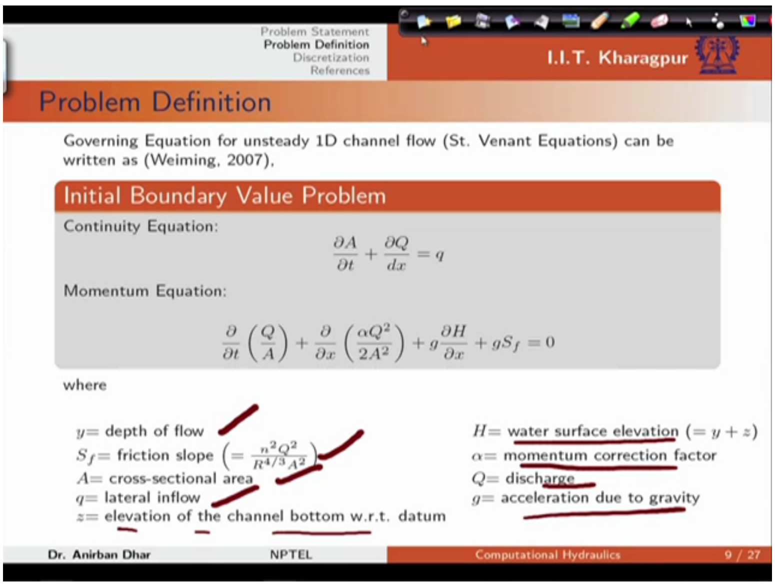

So this is the flow situation for our problem. In this case let us say we have two channel section i and i plus 1. Then for segment i for Lth channel reach we can write our discretized form of the governing equation. Now for this discretization I will utilise one special scheme.

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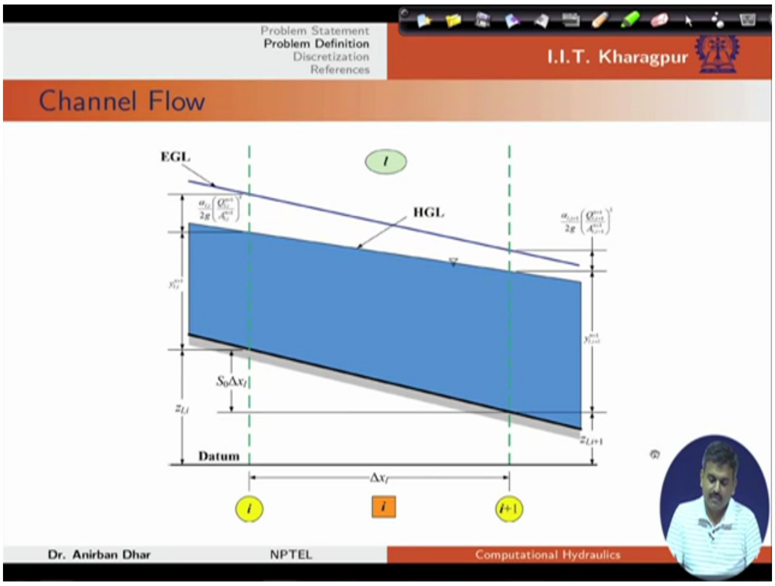

Before that this is our flow convention that we are utilising that is from L 1 to L NL plus 1 we have positive flow. If the flow direction or flow value is negative so obviously the direction of the flow will be opposite in this case.

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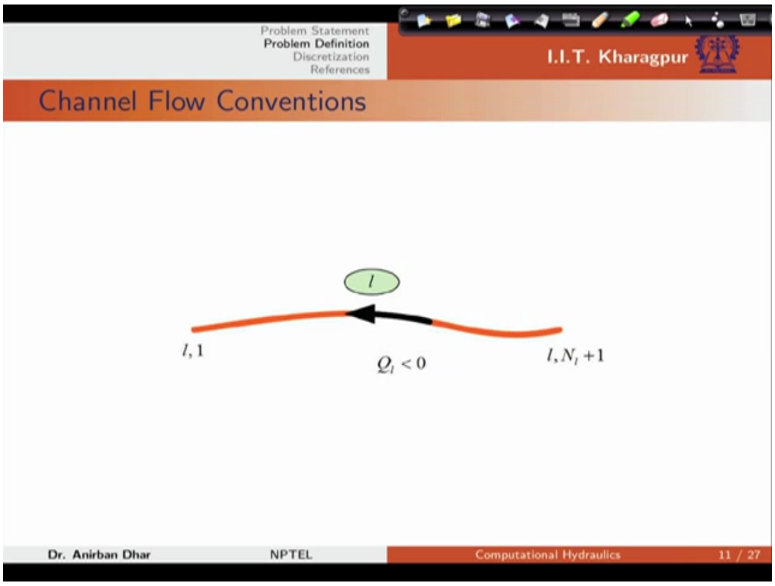

And from junction left to right this flow is occurring. So for one junction it is negative for another junction it is positive. For a general variable in this case I will just define this discretization. In this case we have i n is one section, another section is i plus 1 n. This is at future time level, i n plus 1, i plus n plus 1. So by considering these four points we can discretize our governing equation.

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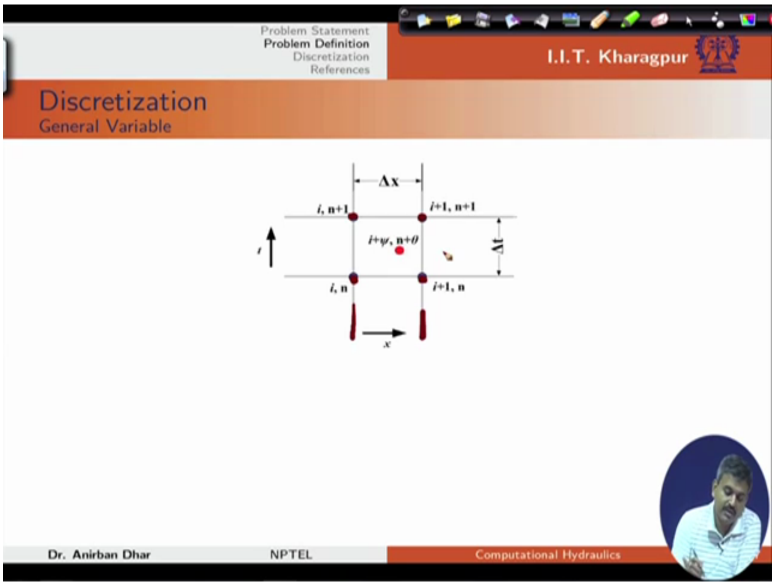

So for any general variable phi this Preissmann scheme can be written as for phi we have this psi i plus 1 n plus 1, this value and i n plus 1. So weighted addition between these two so some value in between we will get. Again weighted addition or weighted combination of these two we will get here and again this is for special combination again in time we can again check weighted combination here.

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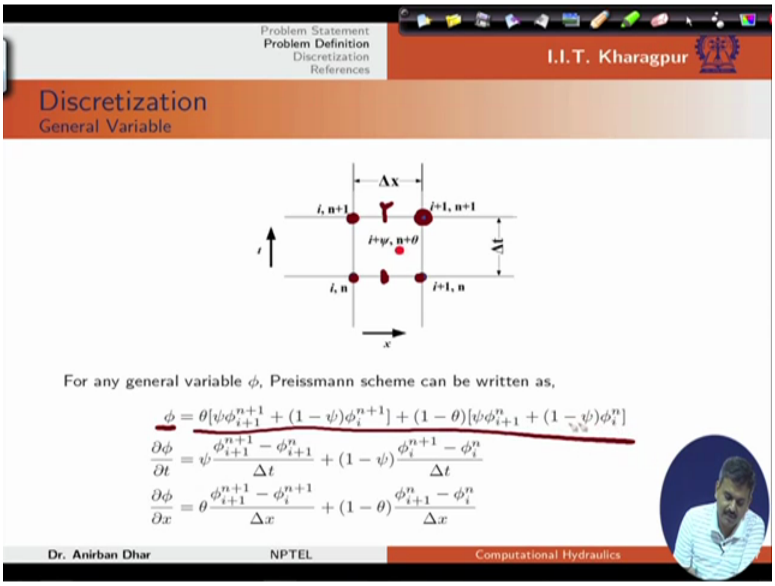

So with theta and psi we can define the problem. So i plus i n plus theta. So in this case again we can use our concept and what is that? This is again the combination. What is that combination? Combination is this psi into our phi i plus 1 n plus 1 plus 1 minus psi into phi i n plus 1. This quantity minus we have psi into phi i plus 1 n. And plus 1 minus psi into phi i n. So we can subtract this and divide it by del t. So this will give you this del t derivatives. Similarly for del phi by del x we will get the weighted combination here.

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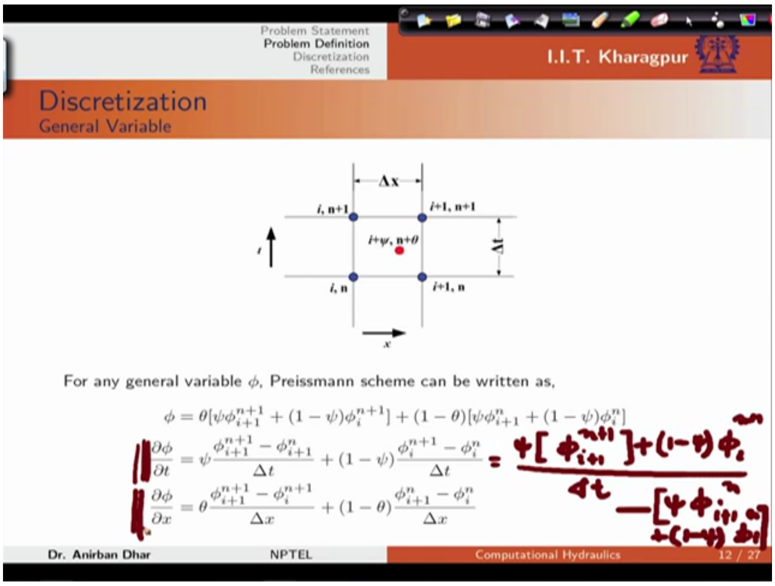

Now we need to discretize our continuity equation first. So continuity equation was del Q or del A by del t plus del Q by del x and minus q equals to zero. So del A by del t we can discretize like this. This is del A by del t. So again we can take that weighted combination with respect to psi and get this.

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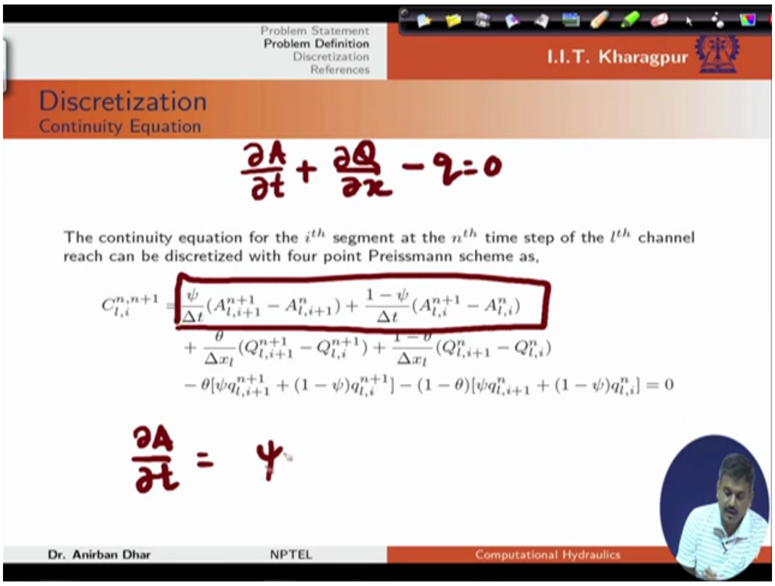

Similarly for del Q this is weighted combination for del A by del t. For del A by del x we have this theta weighted combination here, 1 minus theta and the last one this is again our inflow parameter.

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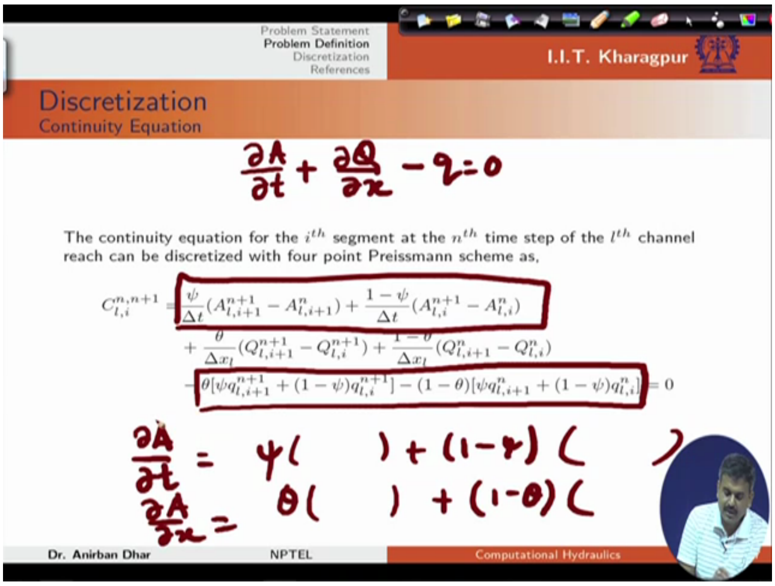

We can again take a weighted value for this one. So this is the discretization for the continuity equation. Although this part is linear in nature and the problem is totally dependent on theta and psi values in this case. Let us say we have psi equals to 1. Then we will have one combination here if theta is equal to 1 again that is the changing the special derivative.

If theta equals to 1 obviously we are considering implicit case. If theta equals to zero obviously we are considering explicit case because the explicit or implicit consideration depends on the time level of the spatial derivative.

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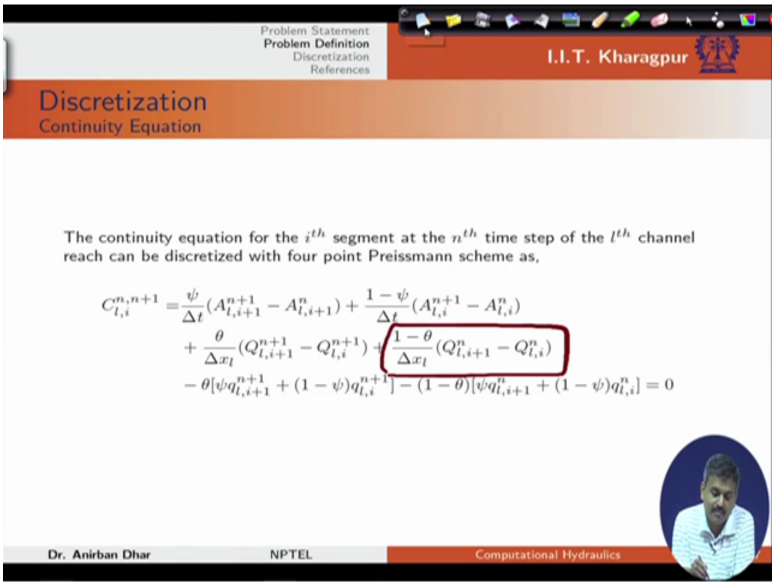

So to utilise this equation in our general Newton Raphson format we need to take derivative with respect to four variables. So what are these four variables? So for any segment we have variables yL. For Lth segment i n plus 1, then QL i n plus 1, then yL i plus 1 n plus 1 and QL i plus 1 n plus 1. These are three variables 1, 2, 3 and 4.

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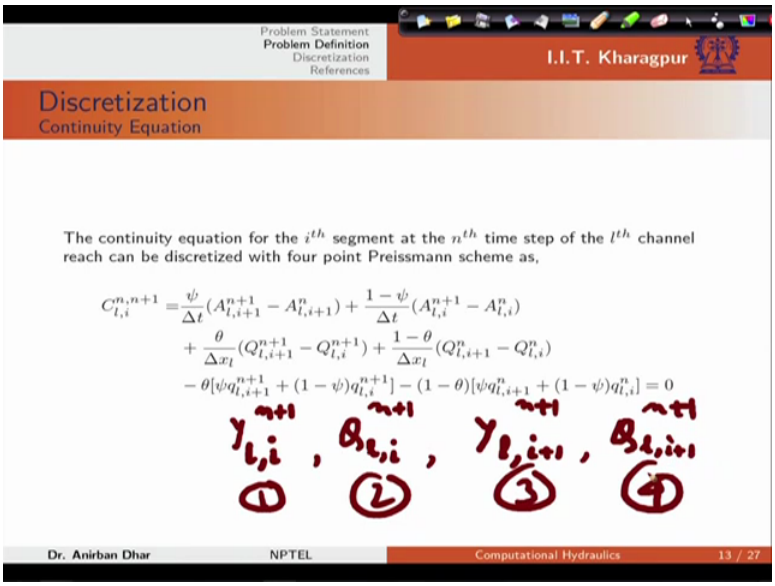

Now we need to take derivatives of this CL i n n plus 1 with respect to these four variables. So if you take derivative of our continuity equation we will get these four terms. Obviously in this case dA by dy this is the derivative of area with respect to y. And area is a function of y only, it is not a function of discharge in this case.

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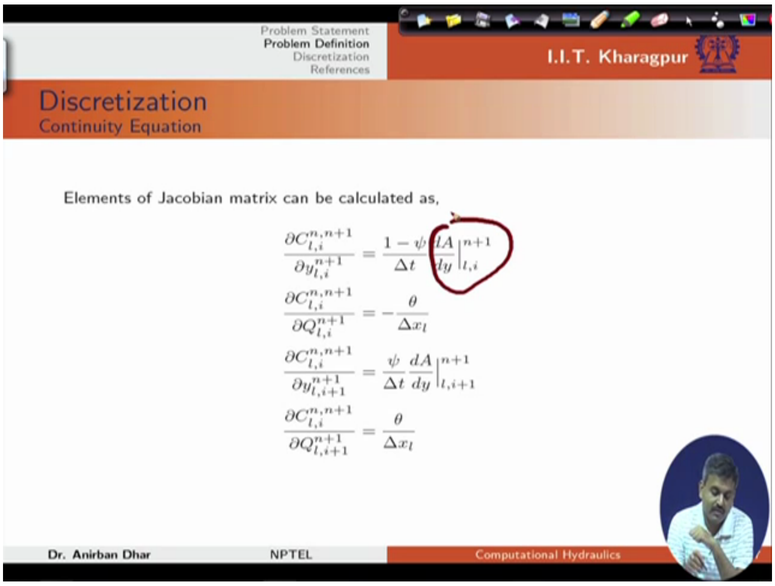

So momentum equation discretization, so momentum equation was dQ by A. So for dQ d by dt of A we can directly write this. Writing Q by A terms. Then we have this del by del x, this is alpha Q square, this is 2 and A square. Now the last one this is g del H by del x and plus g into SF this is equals to zero. So for all cases like temporal derivative we are taking weighted combination with respect to psi.

For spatial derivative we are taking weighted combination with respect to theta and for others like this one also we are taking weighted combination with respect to theta because it is a spatial derivative. And SF is the combination of theta and psi. And in this case I have not written the superscript n plus 1 for z because z is not wearing with time. Z is fixed bed elevation.

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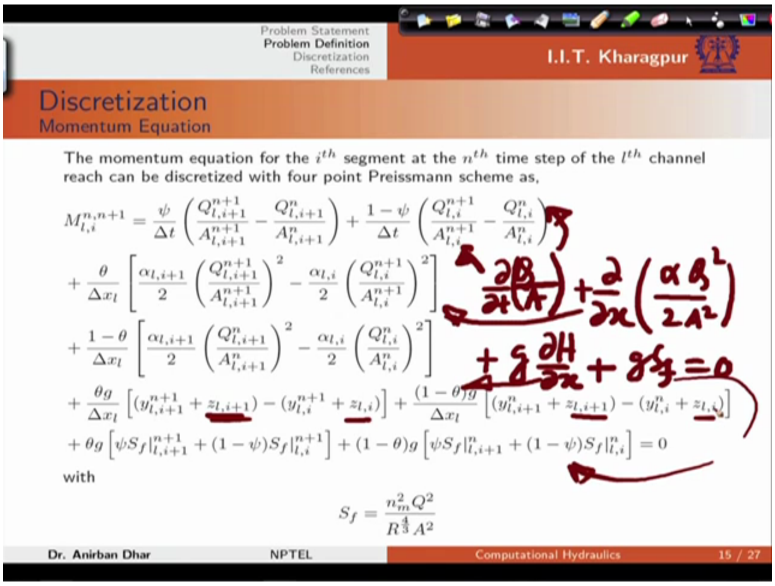

That means we are considering rigid bed channel here. Now SF can be calculated from nm square Q square R to the power 4 by 3 A square. And if we consider the sign of discharge then we can modify this one and modification will be there only in the case of SF. This is nm square Q Q mod R to the power 4 3rd and A square.

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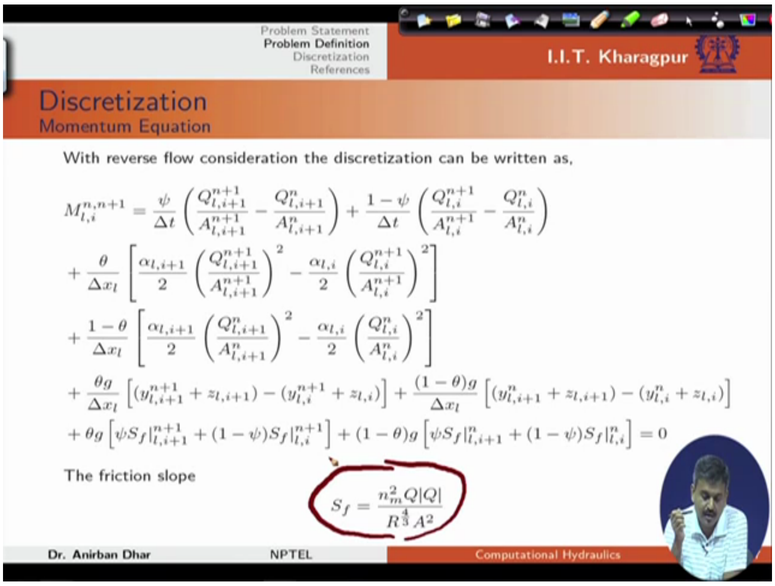

Now again we need to take derivative of ML i n n plus 1 with respect to the four variables that we have utilised for our continuity equation also. So first one is yL i n plus 1, next one is QL i n plus 1.

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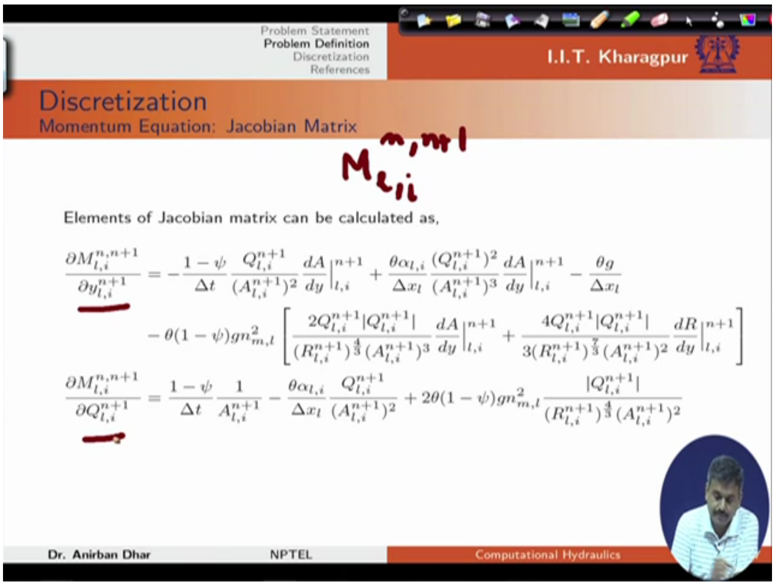

Now in this case one should know that in this case we are utilising these mod values only for the derivative terms which are related to SF. So in this case we have Q square term but we have replaced it with Q into mod Q. In this case also I have utilised this mod Q.

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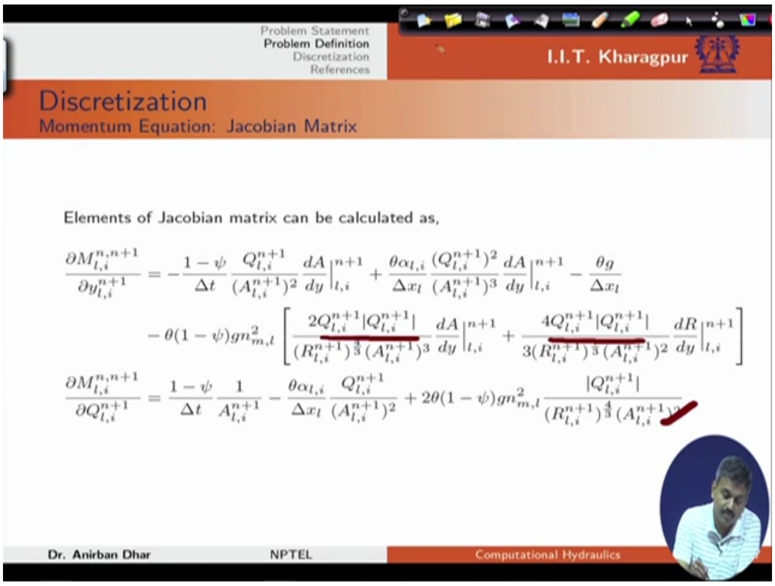

Another variable this is yL i plus 1 n plus 1, QL i plus 1 n plus 1. Again we can see that these mod values are utilised or used for this SF calculation or derivative of SF terms. And in this case we have dR by dy, dA by dy. These values are to be calculated from section dependent values. In this case we have a rectangular section. So dA by dy essentially that is width of the base or width of the channel for rectangular case.

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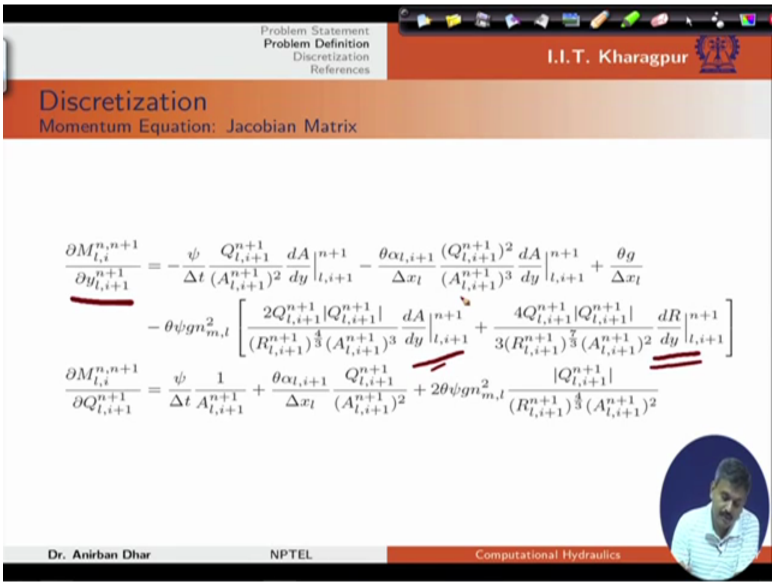

Now for each section we have two nonlinear equations with two NL plus 1 unknown that is discharge and flow depth. For trapezoidal section this is a general section because if we utilise different values of m1 m2, we can directly get the solution for different cases.

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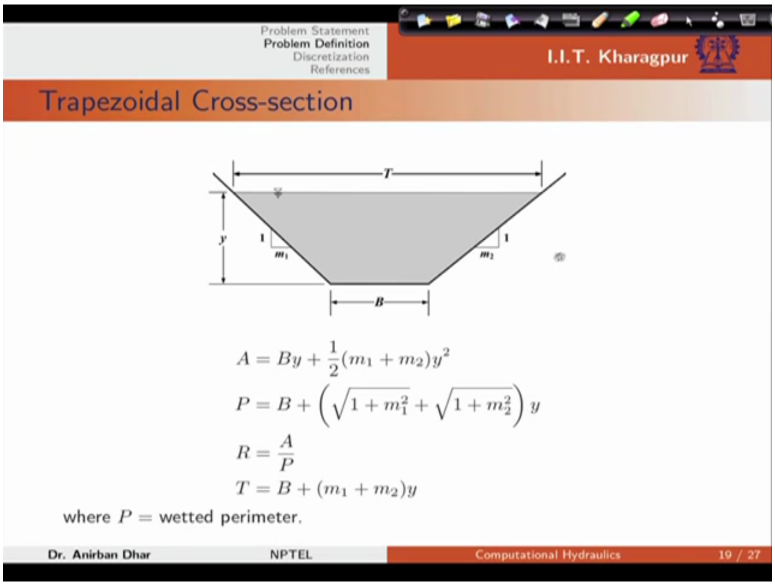

This is for trapezoidal section dA by dy and this is dR by dy we can directly get that from this expression. And by changing different values like dA by dy at L i n plus 1 means I should calculate this term with yL i n plus 1. So I can directly use the value here and I can get the derivative term.

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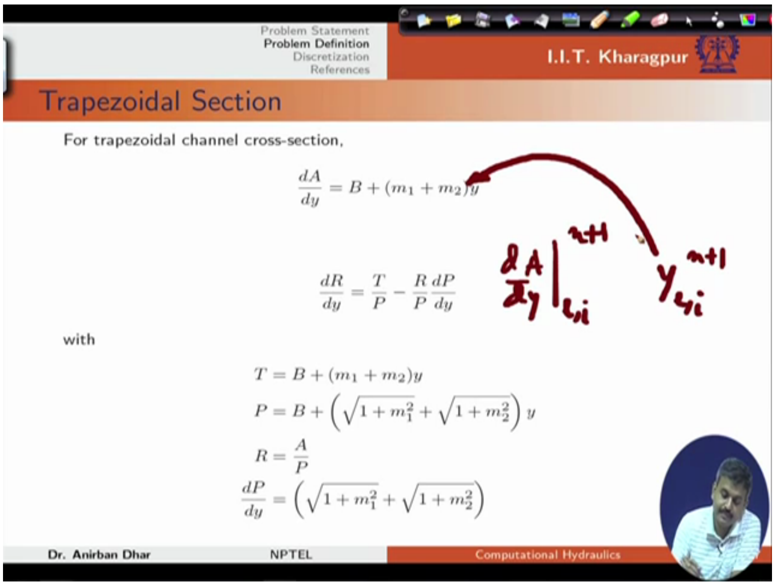

Similarly for others we can calculate the values. Now this is the algebraic form. If we have the equations from segments we can write it in general form in the format of Newton Raphson. So these are increment values and this is minus residual, minus residual for momentum, minus residual for continuity and these are the coefficients. Coefficients are essentially elements of Jacobian matrix.

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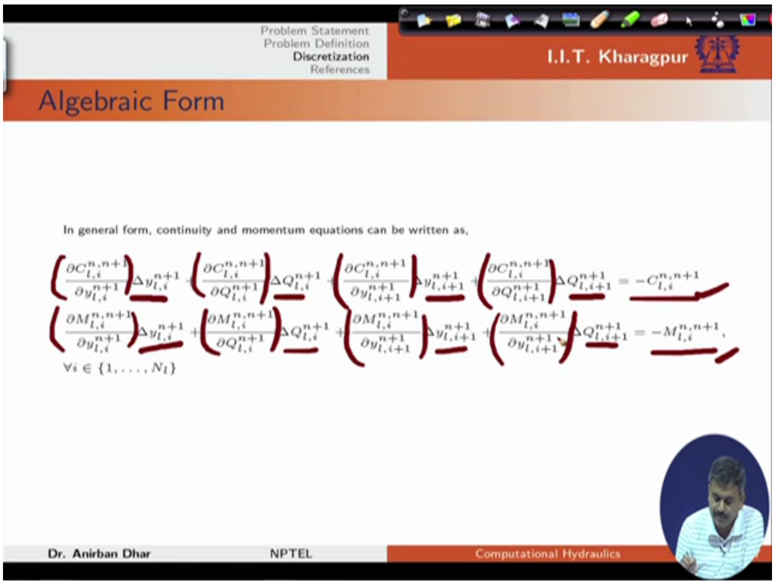

Now we can solve this problem with a guess value. And for the problem guess value for next time level should be the value which is specified for the initial time level. And for consecutive times steps we can consider guess value as (fut) previous time level value. Now after getting this we can directly add it.

So we can start with a QL i n plus 1, this is yL i n plus 1, Q this is L i plus 1 n plus 1, this is again this is yL i plus 1 n plus 1. Now in this case after getting these increment values we can directly add it with a previous time or previous level iteration values like this, plus del yL i.

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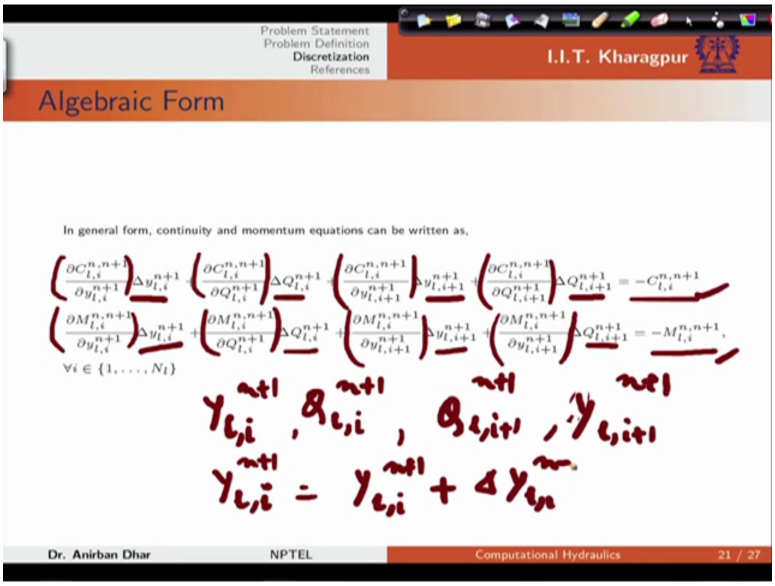

Similarly for other variables this thing is repeated or this expression is repeated. Now for this one we have only 2NL number of equations. Now I have already discussed that we have three internal conditions and three boundary conditions. So we need to incorporate those values within our calculation or expression. So configuration wise we have started with this configuration starting with three channels.

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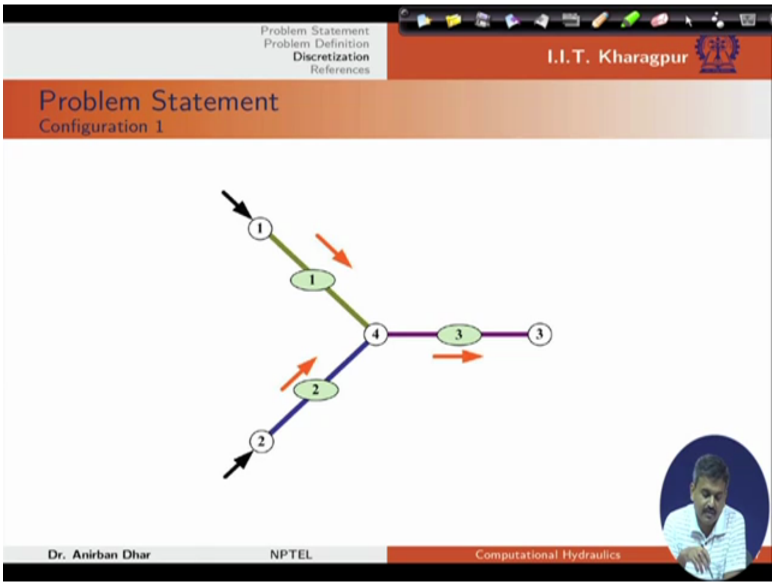

First channel linking this node numbers 1 and 4, second channel 2 4, third channel 4 3. So if we consider our usual discretization approach, so discretization is in the direction of the flow. So this is 1 1, this is 1 NL plus 1, this 2 N2 plus 1, first one 1 N1 plus 1, this is starting is 3 1, this is 3 N3 plus 1. So obviously we have these nodes 1, 2, 3.

These nodes are internal nodes. We should utilise these three nodes for specifying the junction condition at 4. And these three nodes should be utilised for specifying the boundary conditions at boundary junction nodes.

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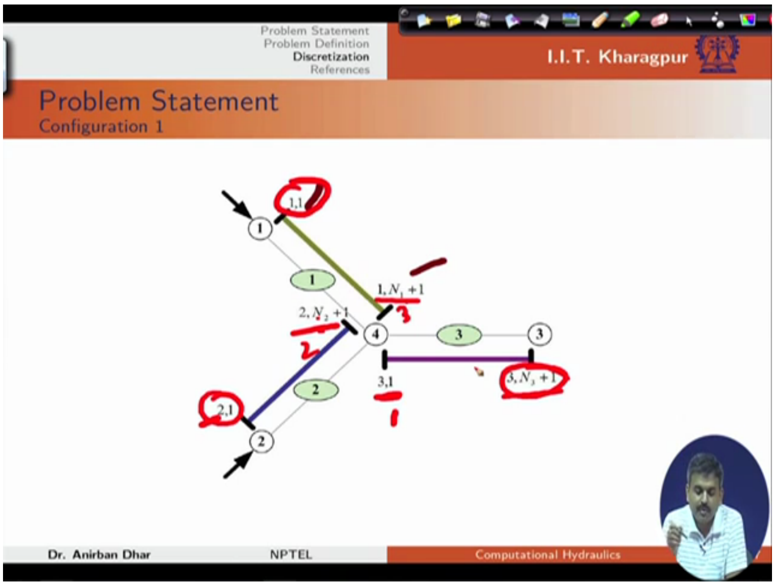

As per our previous program structure from the steady state flow condition we have chl inf which is channel information matrix. This is 1, 2, 3. This is the same matrix or same table that we have directly utilised.

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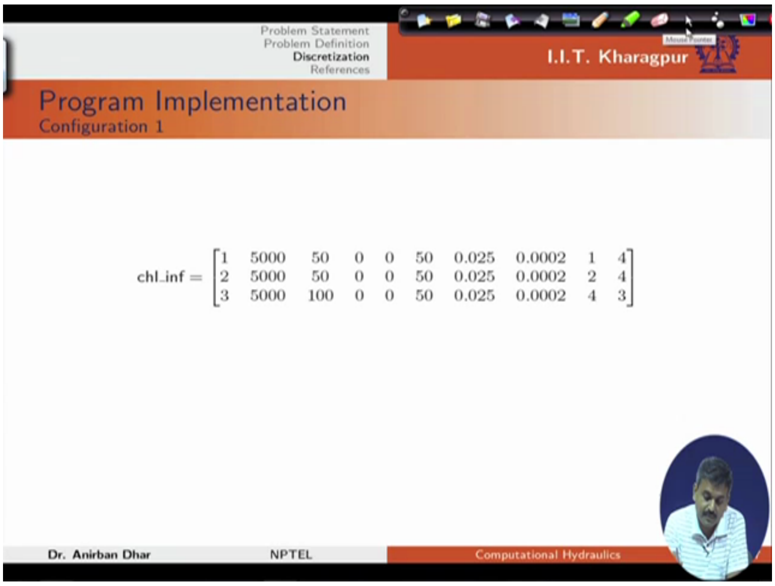

Next is junction information. Only change is there in this junction information. If I compare this junction information matrix with our steady state case we are not directly specifying the values in junction information. We are specifying the type of junction information available at this node with this junction information matrix. And the third column obviously that is our elevation of junction elevation information. And this is junction continuity.

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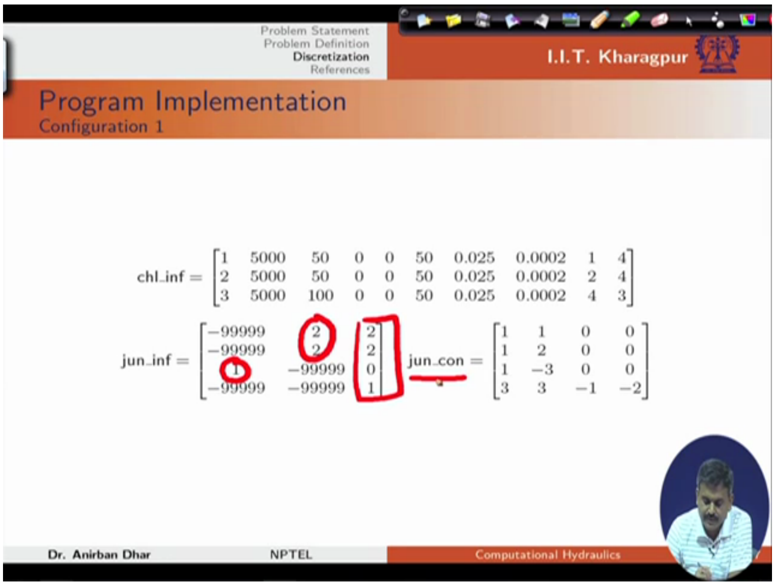

Junction continuity in this case it is simple. As per our usual convention we have 1, 2, 3 and this is 4 junction wise and this is channel number 1, 2, 3. So for (chan) channel number 1 or node number 1 this is junction continuity. So node number 1 we have only 1 starting from plus 1. For node number 2 we have only connected channel is 2 and it is starting from plus 2, channel number 3 which is connected to this node number 3 and the end section is connected that is why minus 3 is there.

And internal junction we have three channels connected with this particular junction. So this is plus 3, minus 1, minus 2 because end section of 1, so this is minus 1, end section of 2 that is minus 2 and starting section or first section of channel 3 connected with this one so this is plus 3.

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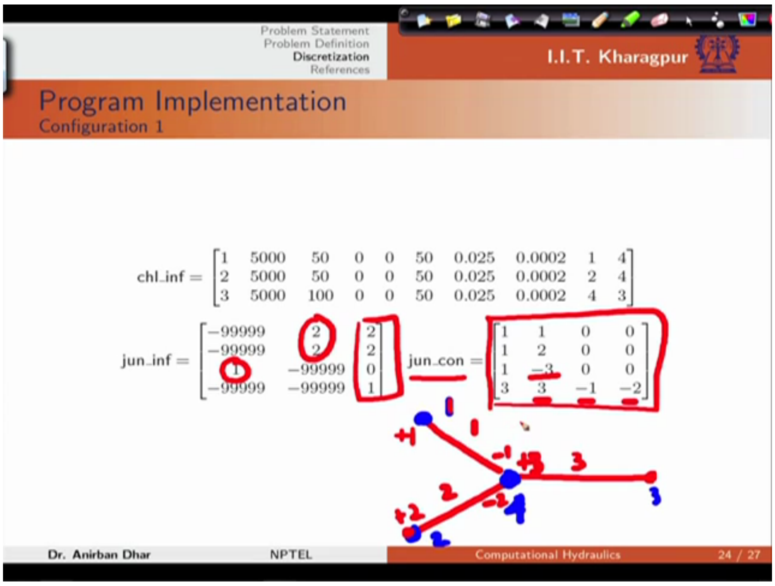

So this is our junction continuity or junction connectivity information.