Computational Hydraulics
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Lecture 42
Steady Channel Flow: Channel Network with Reverse Flow

Welcome to this course computational hydraulics. We are in module 4 surface water hydraulics. And in this particular lecture I will be talking about steady channel flow and specifically channel network with reverse flow situation. This is unit number 6 of this module number 4.

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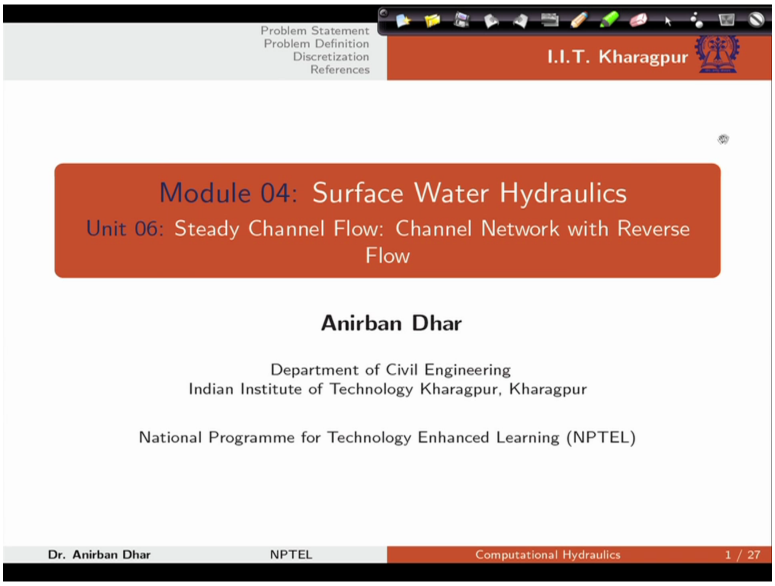

Learning objective of this particular lecture. At the end of this lecture students will be able to solve steady channel flow for channel network problem with reverse flow using implicit approach.

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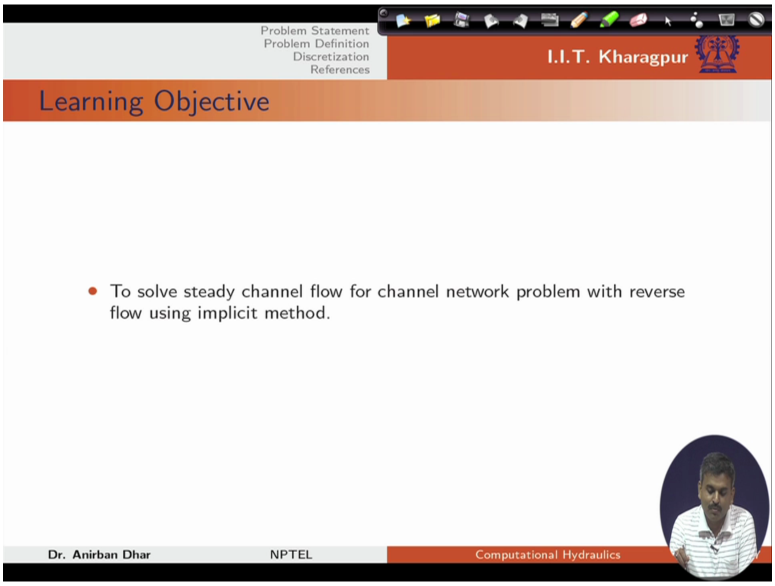

Problem definition to solution. In our previous lecture we have discussed one problem where we have considered two internal junctions, 1 this is 2. And one external junction. So this is external junction. This was the external junction.

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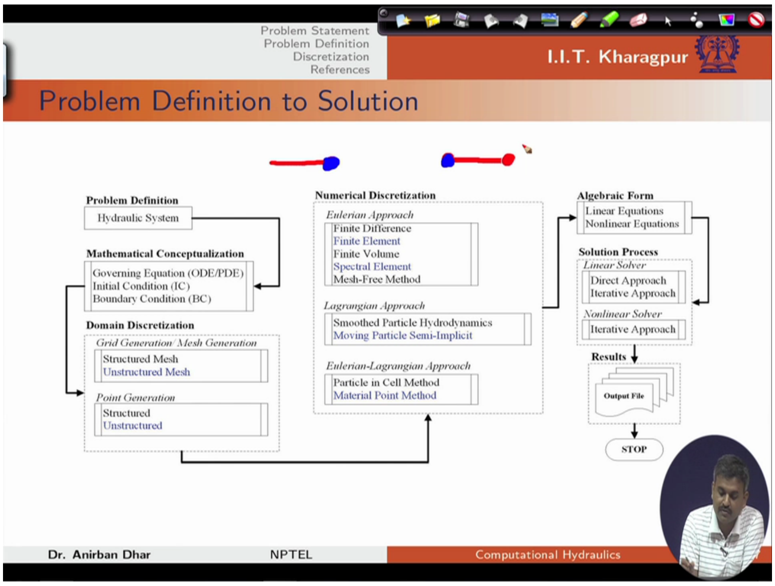

So now for this one we have considered four channel reaches 1, 2, 3, and 4. And in this case these are the channel junctions 1, 2 and 3.

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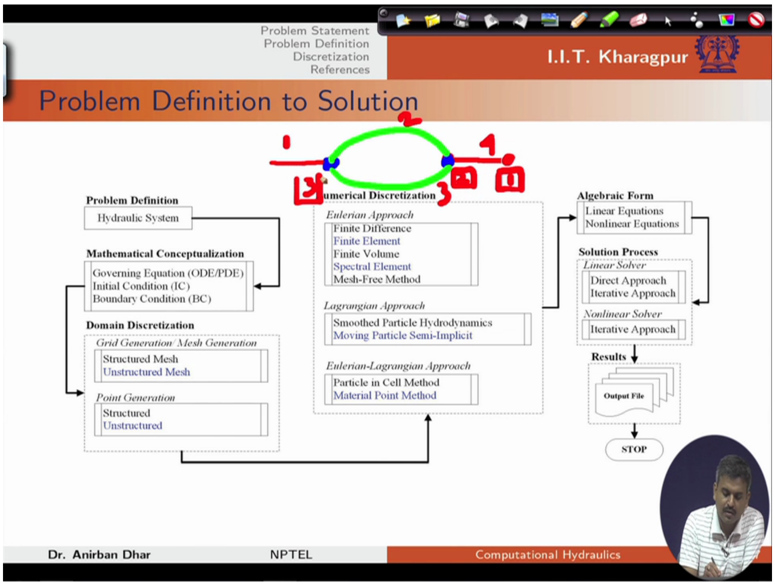

For these three channel junctions we have seen the flow is from left to right direction. So basically in this case we have a flow which is running from left to right direction and this is also the direction of sloping channel bed. That means we have sloping channel bed which is running from left to right.

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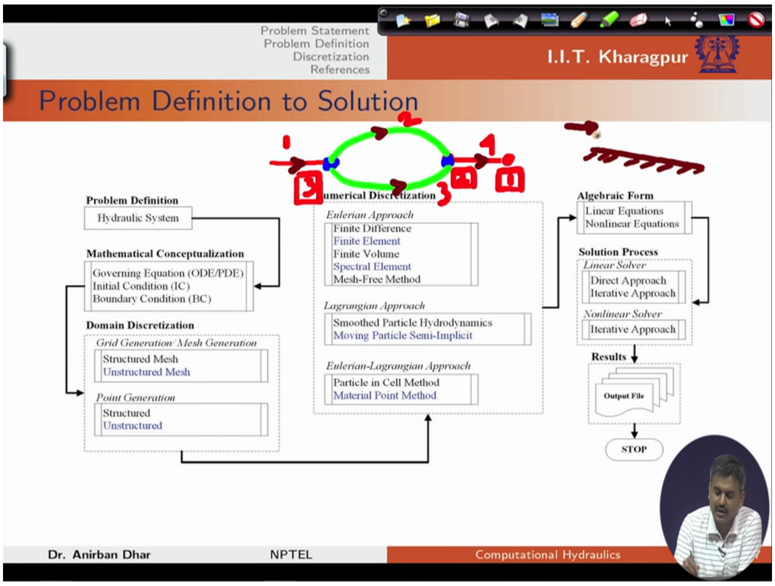

So if we consider this channel bed at junction 1 so this is the up gradient or upstream location and this part is the down gradient or downstream location. So for channel 1 this is 1, 1 and this is 1, N1 plus 1.

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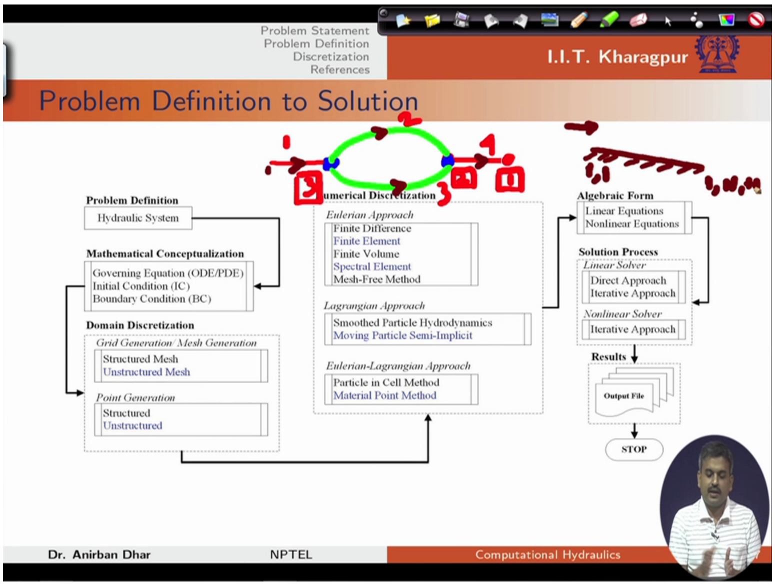

So we are considering that flow is from left to right and this is our positive flow. So according to our convention we have got all values positive because we know the flow direction. But let us consider a situation where we have reverse flow conceptualization. If I start the same problem with flow direction in the channel 2 as flow from right to left so this is reverse flow situation.

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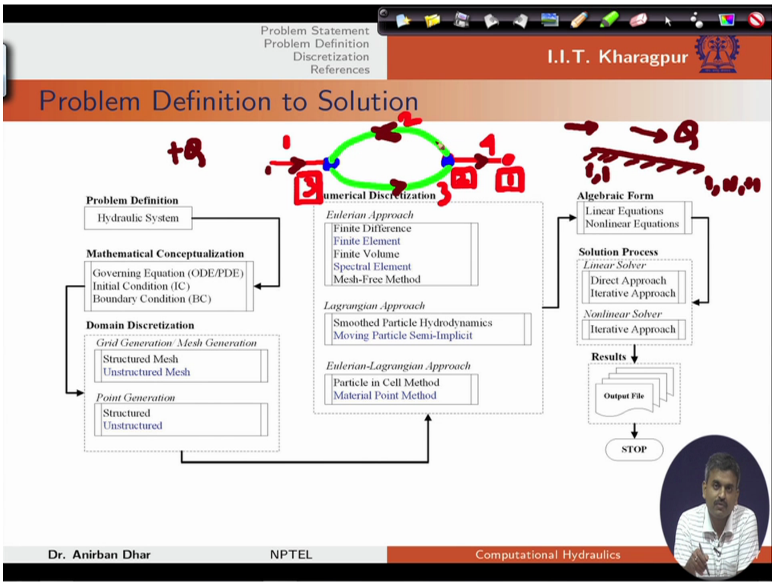

Now how to solve this kind of problems that we will discuss in this particular lecture class. So conceptualization wise governing equation are same but implementation wise we need to consider certain points so that we can include the plus and minus sign of these Q values.

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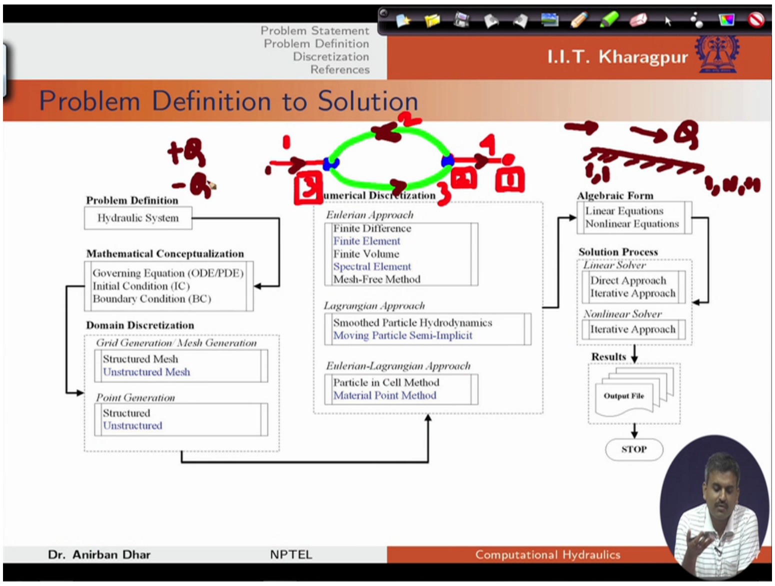

Because for y it is always positive. But we can have positive or negative Q values during our flow problem solution and we can have positive or negative Q values depending on the conceptualization if I consider that my flow in this small problem is from right to left obviously I should get a minus Q value at this location because the original flow direction is from left to right.

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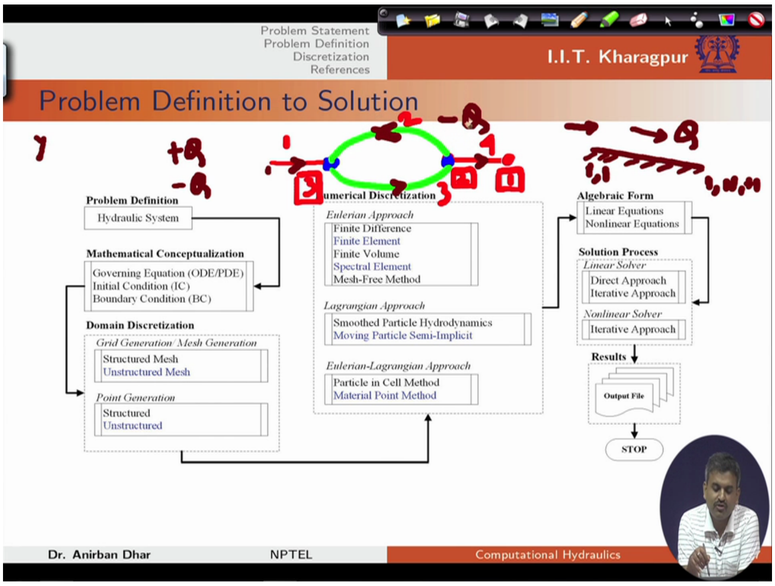

So for this one again we will consider this finite difference discretization of the equations and we will get nonlinear equations and finally we will try to solve these nonlinear equations using nonlinear solution approach that is Newton Raphson. Now let us see what is the problem statement for our problem. Now let us consider this complicated network. As per this network directions this is the lowest point with zero metre bed elevation.

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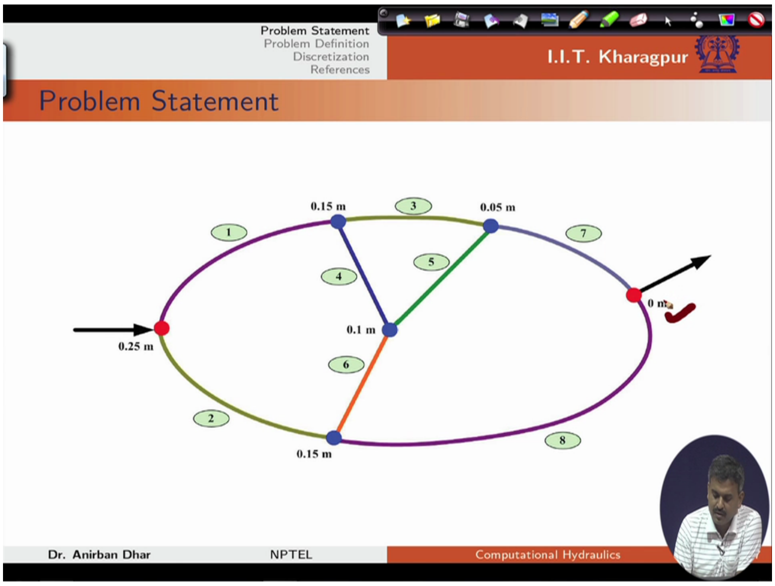

Now starting from the zero the values are increasing in this direction and the value of bed with respect to this downstream end point this point 15 metre. Again values are increasing and this is the highest point in this network. So obviously we have a slope which is towards this direction.

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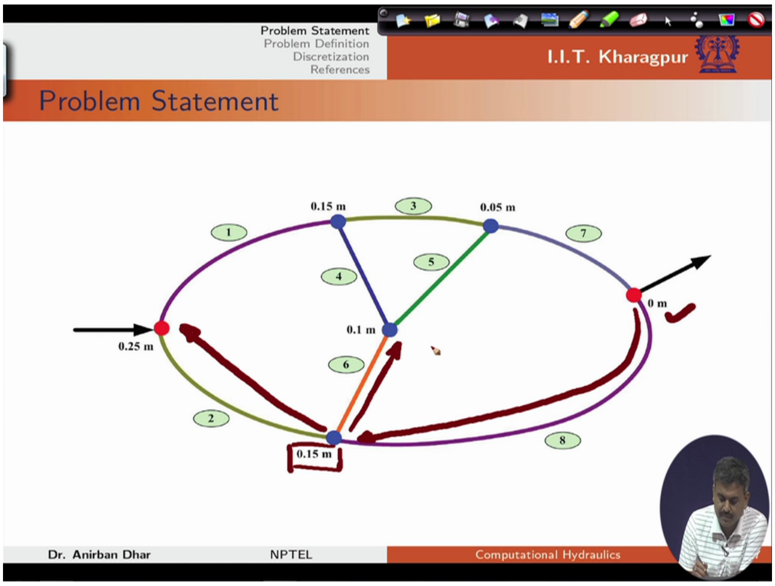

Again from this we have so obviously in this case your bed slope is in this direction. It is decreasing and again for this one it is decreasing in this direction, for 1 this is decreasing in this direction. So as per the natural flow direction these are the natural flow direction as per the slope consideration.

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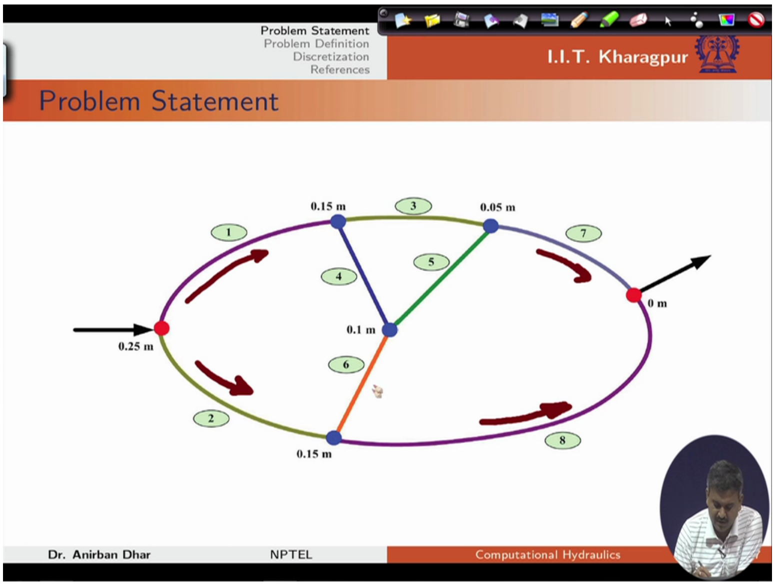

Let us say the slope discharge condition for this one. So this is the discharge condition for this one. Again this is higher elevation, this is lower so obviously flow will be from this point to this point.

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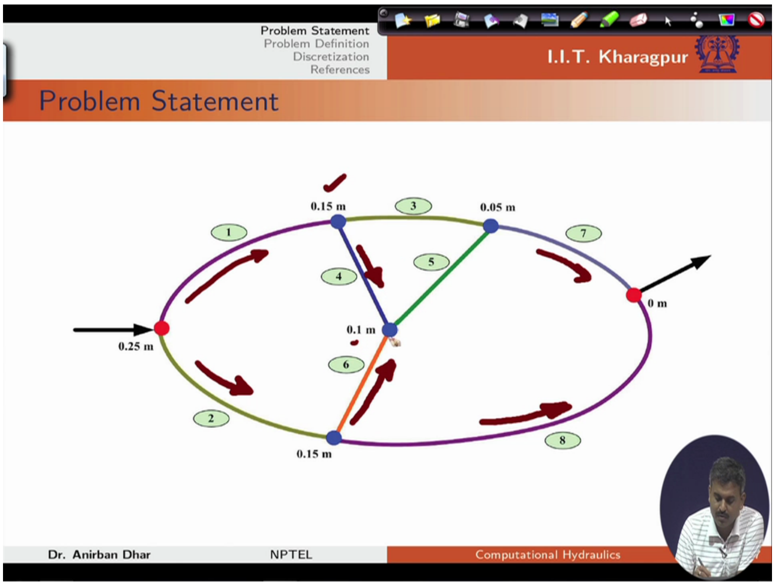

Again the flow will be here and this will be the flow direction for the third channel reach. So we have 8 channel reaches 1, 2, 3, 4, 5, 6, 7 and 8. For this we have 4 internal junctions. So this is 1, 2, 3 and 4 and red ones are external junctions or points where we need to specify the boundary condition.

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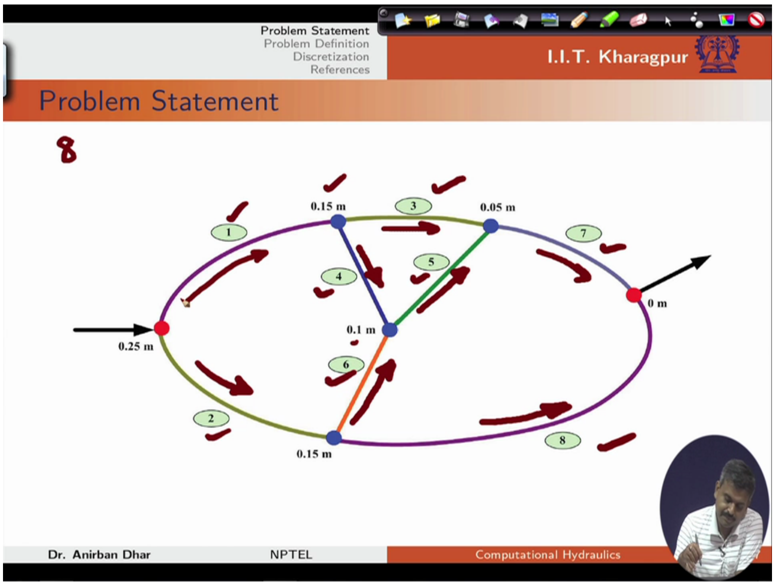

So this is our point number or junction number 1. So let us say this is junction number 2, this is junction number 3, this is junction number 4, this is 5, 6. So we have all total six junctions and for this junctions we can see that as per the bed elevation we have the natural direction of flow.

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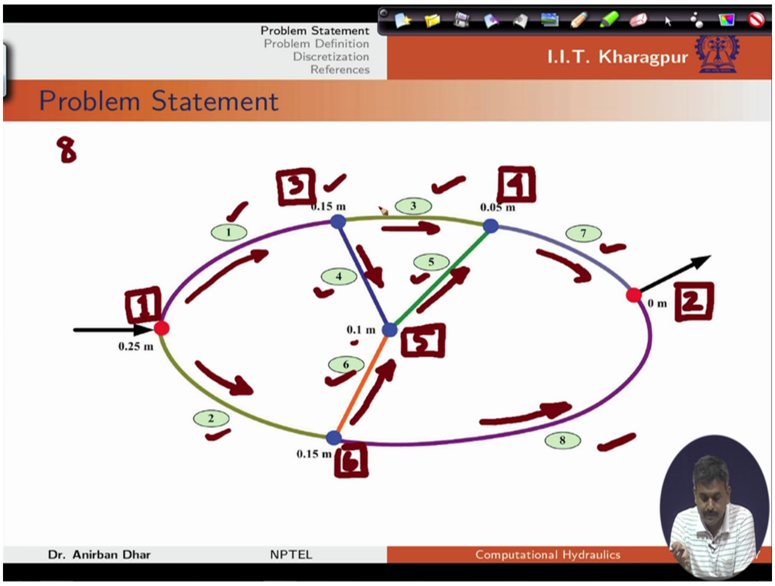

Let us say that we are considering channel reach 1. So if we start from node 1 and moves towards node 3 obviously I am considering the flow direction is according to the channel slope. So this will be the positive direction of flow.

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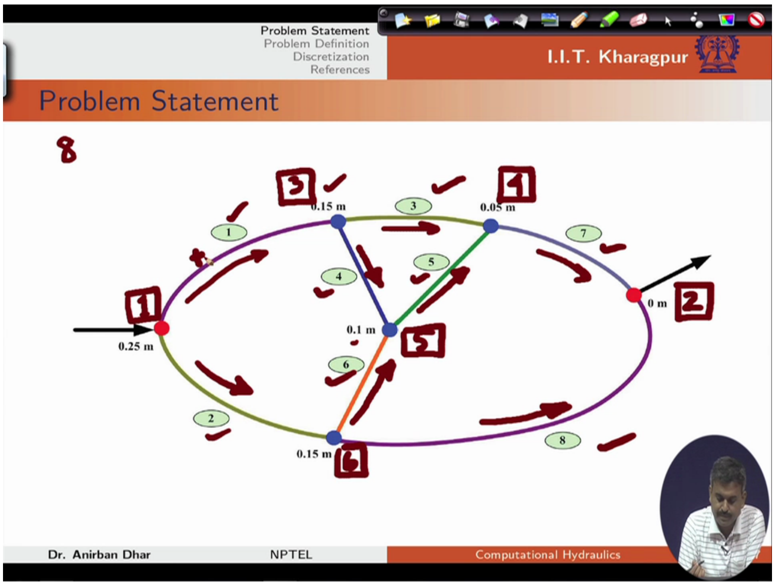

Again if I consider this channel 3, this is the positive direction of the flow because we are starting from higher elevation moving towards lower elevation. Again this side it is a positive direction of flow. For channel 5 this is positive direction, channel 6 this is positive, this is positive and this is positive for 8 and 7.

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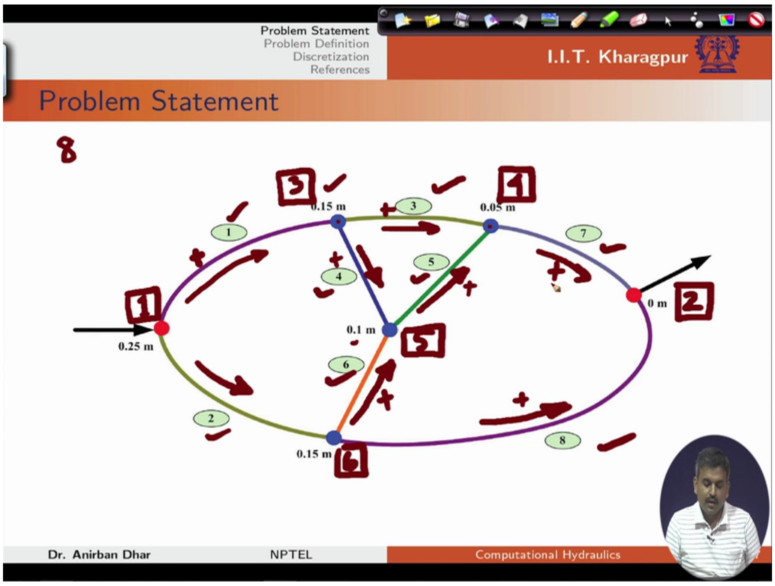

Now with this configuration we can start our problem. So we have a loop channel network because with this internal junction everything is connected there or every channel or internal channels that means 4, 5, 6 these are connected.

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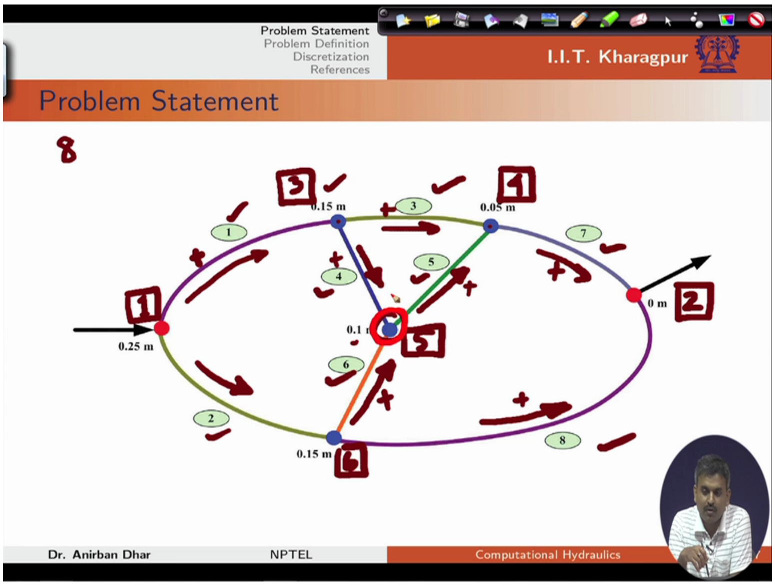

So flow is occurring through this junction and this junction is the main connector for the loop network. So in this case let us consider our flow directions. So as per our channel flow or channel bed slope these are the directions.

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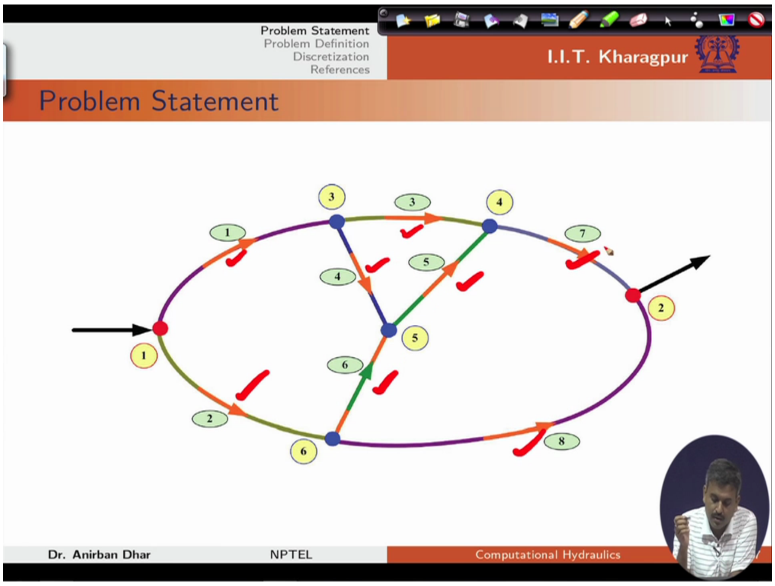

Now these are the junctions 1, 2, 3, 4, 5, 6. So if I say that my flow direction is from 1 to 3 that means I am considering that flow in this direction is positive.

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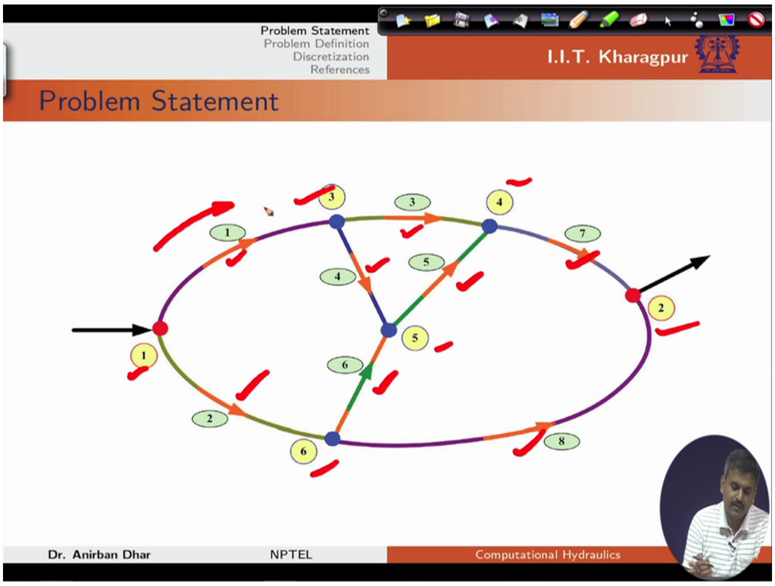

So obviously for channel 1 the connecting nodes are 1 and 3. For channel 2 this is 1 and 6. Channel 3 this is 3 and 4. Channel 4 it is 3 and 5. Channel 5 this is 5 and 4. Channel 6 we have 6 and 5. Channel 7 we have 4 and 2. Channel 8 we have 6 and 2. So this is the connectivity information or positive direction of flow. That means from 1 to 3 we are considering that the flow is positive. So this is junction 1 and this is jN2 and this is L or channel reach number.

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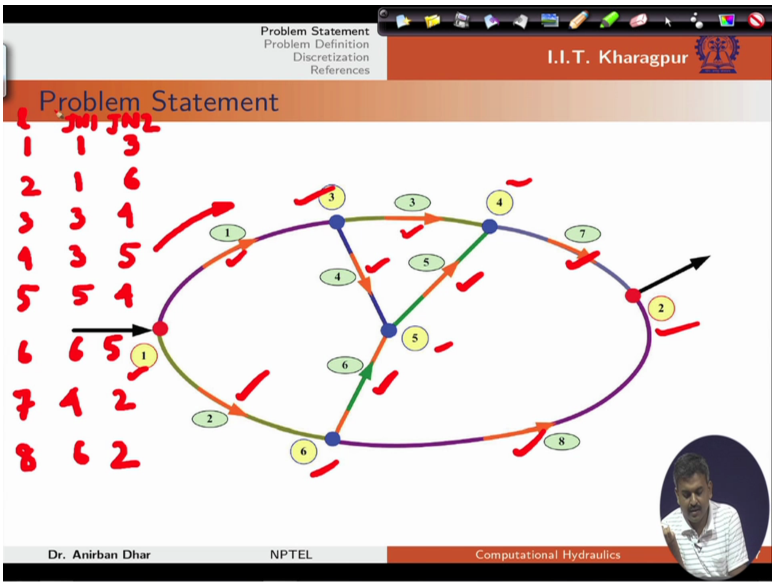

Now in this case let us consider the individual channel reaches. If I consider 1 because I have considered that the flow is from this node number 1 to 3. So the first node should be 1, 1 that is the thing. Now nnode is N1 plus 1. Like that I can discretize all in channel reaches or individual channel reaches.

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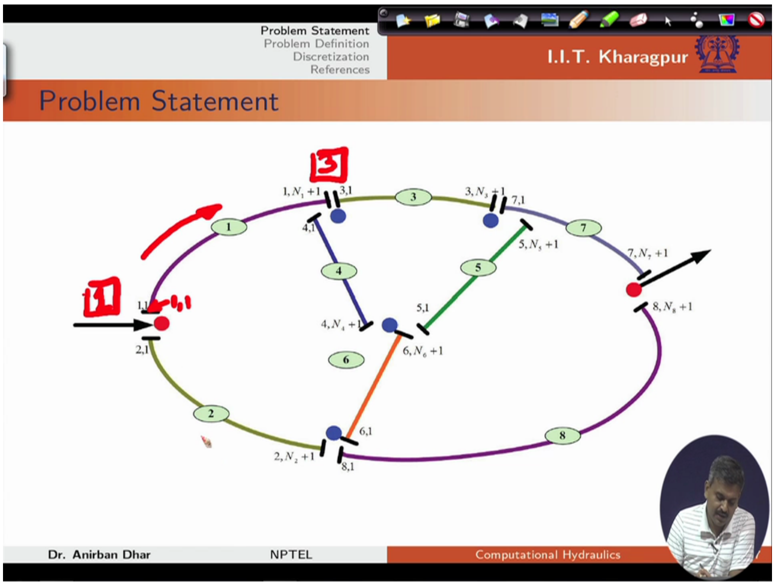

In this case what I am observing from this drawing this is 1 and 6. So connecting nodes are 1 and 6. So obviously starting one is 2 1, 2 N2 plus 2. In this case this is 3 1, 3 N3 plus 1. This is 4 1, 4 N4 plus 1. This is 5 1, 5 N5 plus 1. 6 1, 6 N6 plus 1. This is 7 N7 plus 1. 8 N8 plus 1. So these are starting and this 8 1 and 8 N8 plus 1. So these are starting and ending nodes and this is the discretization convention because we are starting with a positive flow.

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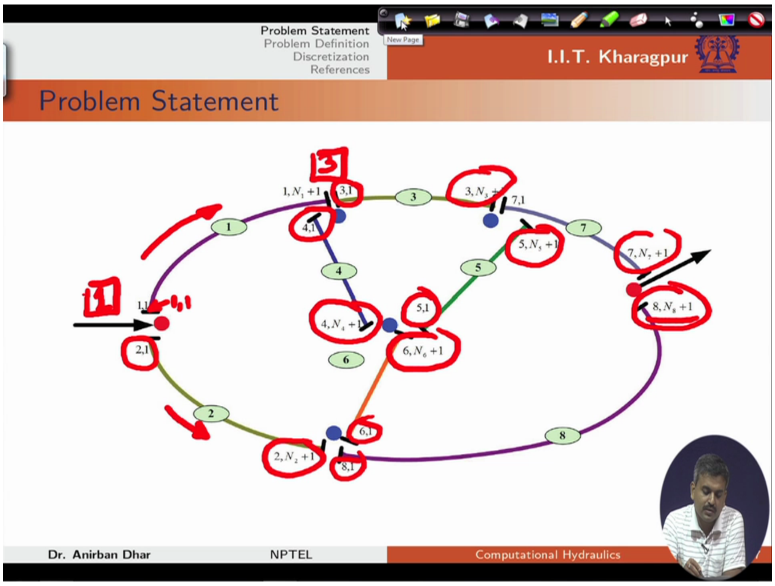

Now in this case we will be considering rectangular channel. So obviously m1 will be zero, m2 will be zero in this case so we will get simplified expression out of this.

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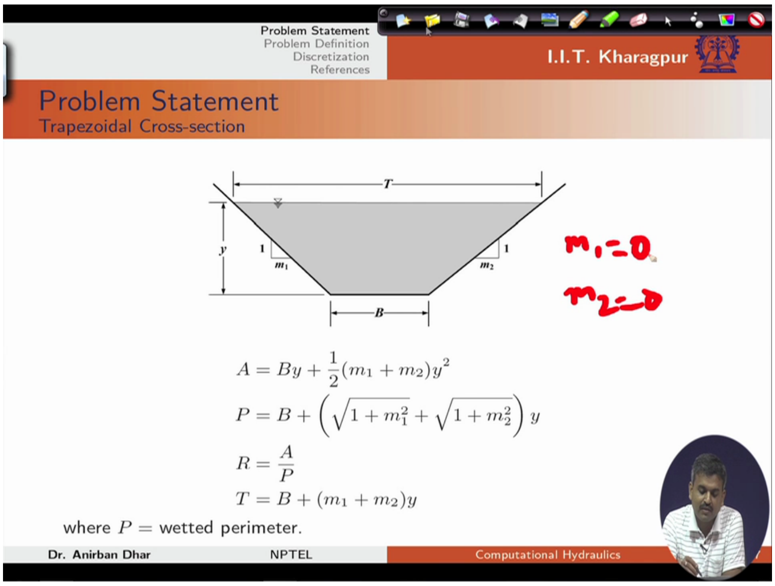

So channel reach this is 1, total length is 200, width is in metres. So this is nothing but the bed width of the channel cross section. So this is B and these are m1, m2. Obviously these are having zero values because we are considering rectangular channel. This is reach or segment length. If I consider that segment length is 50 and if I divide this 200 by 50 obviously I am getting 4 segments, 4 plus 1 so total 5 sections.

Second case also I am getting 5, third case I am getting 5, with 25 metres at segments length, fourth case also it is 5. In this case this is also 5, this is also 5 and last one this is also 5. So all total we have 1, 2, 3, 4, 5, 6, 7, 8, total 40 sections.

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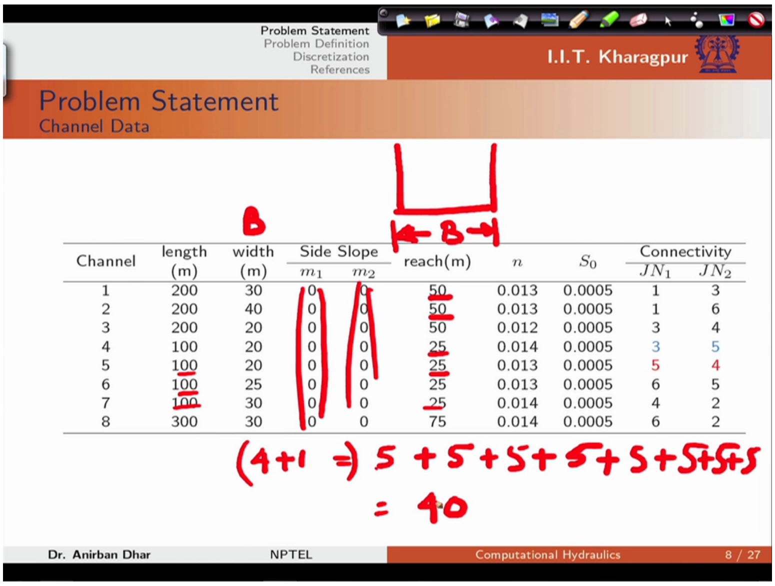

And all cases we have uniform bed slope and these are different n values for different channel reaches and this is connectivity information. This is 1 3, 1 6, 3 4, 3 5, 5 4, 6 5. I have changed the colour for these two channel reaches and that means channel which number 4 and 5.

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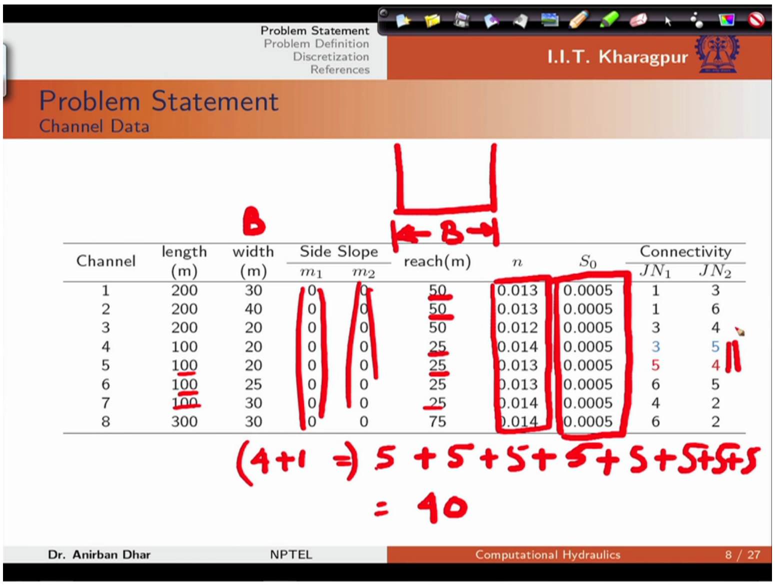

I will try to show the effect of reverse flow by changing the flow direction in these two channel reaches. So all total you will have 40 plus 40, 80 unknowns y plus Q. But there will be another unknown that we will consider during discretization.

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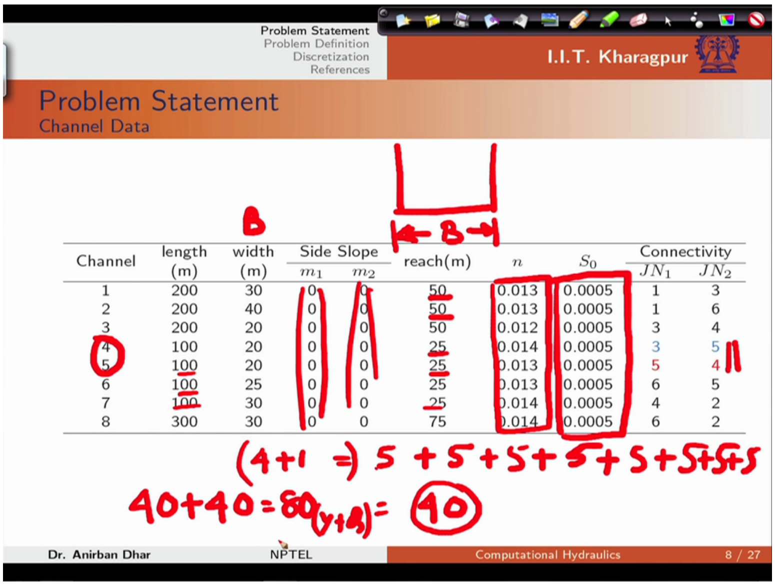

Now we need to supply junction data in this case. What is this junction data? First one or first column this is junction number starting from 1 to 6. Depth is minus 99999 5 9s. So what is this? That means there is no information available for depth but discharge is 250. So as per our network this point 1 and 2 if I consider other junction nodes these are the junction nodes. In this case we have 1, 3, 7, 4, 5, 6, 2, 8 and externally from this node 2 this is 2, this is 1.

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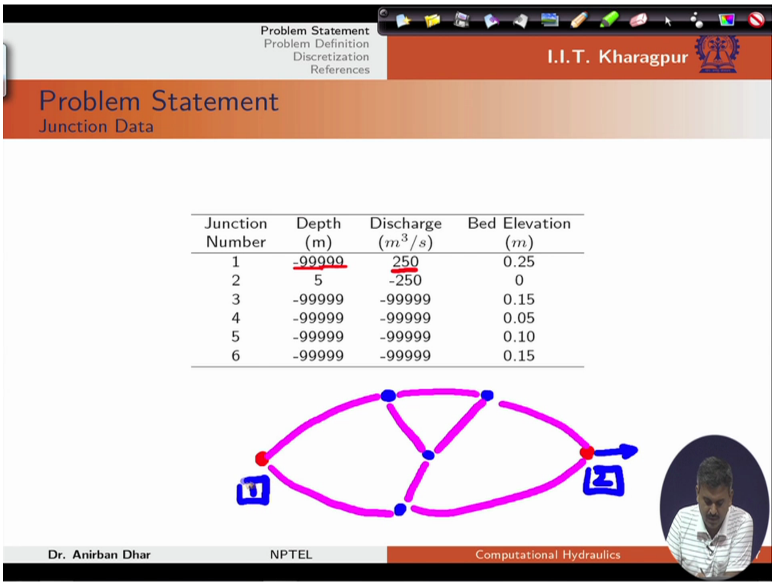

So as per this table we can see that the depth is specified at downstream end at junction number 2, this is 5 metres and we have specified discharge which is minus 250. That means water 250 metre cube per second discharge rate from this junction. And third column this is bed elevation. Now in this case we have all total 40 sections from all channel reaches. For 40 sections we will get 40 into 2 that is 80 unknown values. That means y and Q.

Another quantity is unknown here. That is inflow or Qu at the starting node 1 because in this case only y at downstream and Qd if I put negative sign because I am extracting water from this system from this particular point so this is minus Qd. So these two values are specified.

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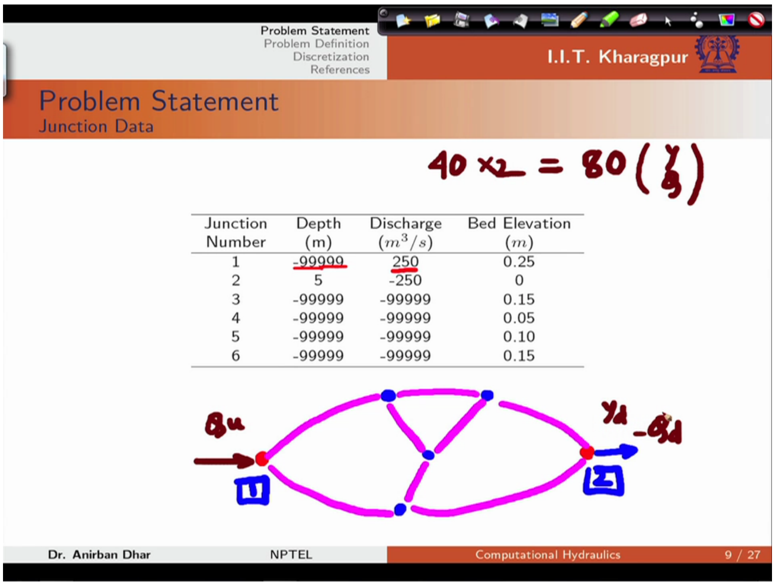

So this Qu is essentially unknown. Now if I add this Qu as unknown so all total we will have 81 unknowns in this case. Now for this channel 3 or node number 3, 4, 5 and 6. So for channel 1, channel 2, 3, 4, 5 and 6 we will have all total 4. 4 is the number of segments per channel reach, 4 into 2. Into 2 is for one continuity one momentum into 8. So this is the total number of equations that we will get from the individual segments. So 4 into 2 into 8. So total number is 64.

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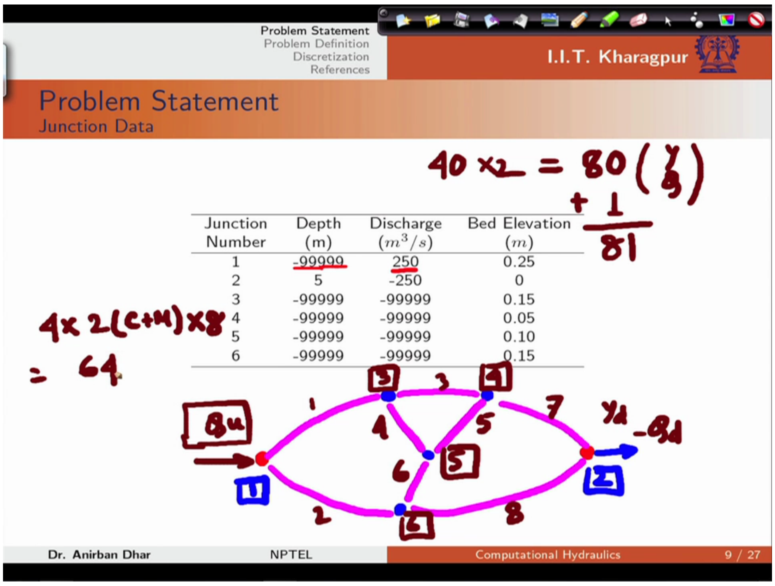

Now for this internal junctions 3, 4, 5, 6 that means we have four junctions or internal junctions. For four junctions we will get three conditions that is 1 continuity 2 energy conservation conditions and this we will get 12 conditions out of this.

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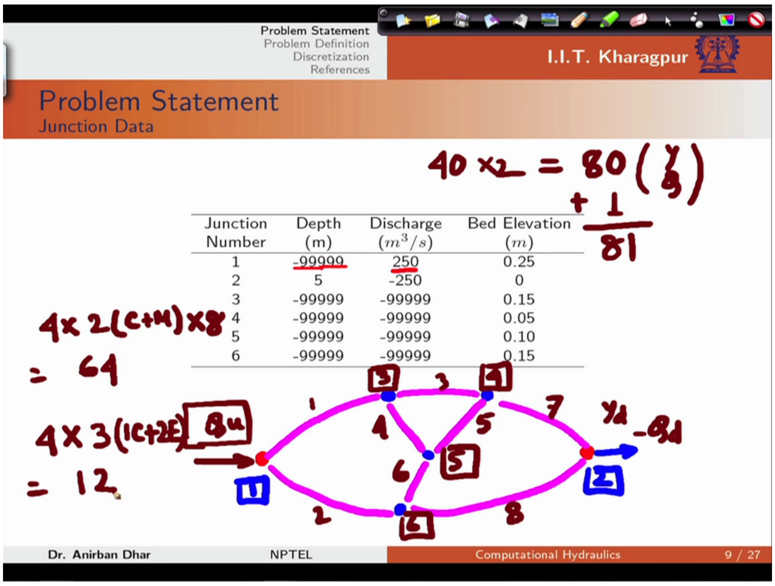

Now after that if we consider this external junction node 2 we have 1 depth and 1 discharge condition plus there will be energy continuity conditions. So all total we will get 2 conditions, 1 for specified depth. That means 2 for depth 1 for discharge continuity. So we are getting here 3 equations.

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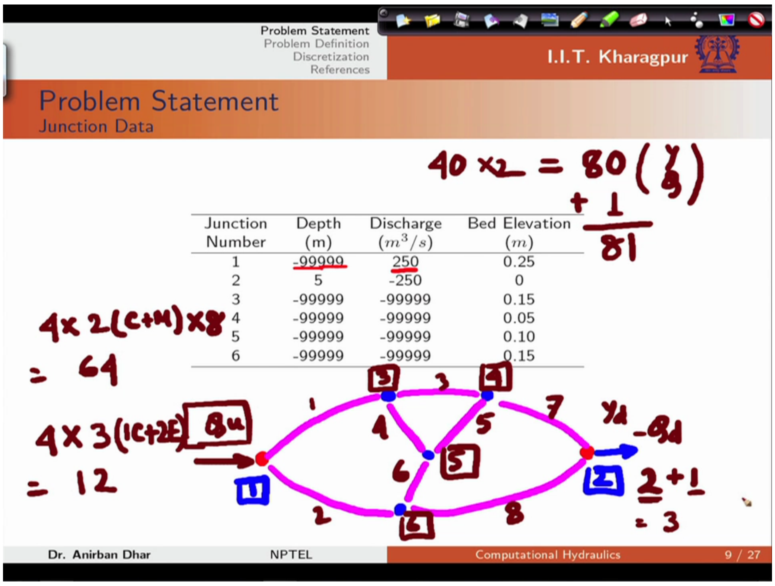

Now at this node number 1 so if I add now 64 plus 12 this is coming from segments. These are coming from internal junctions plus 3 here which is coming from external junction node plus 2, 1 for discharge continuity at node number 1 plus 1 energy continuity for this channel reach 1 and 2. So all total 2. So if we add this so we will get 17 plus 64 which is 81. So we have 81 equations and 81 unknowns.

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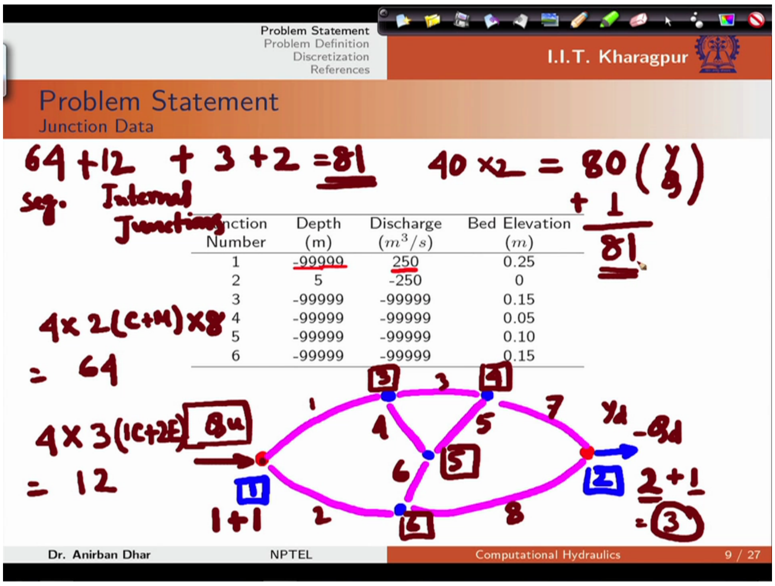

But what we can do, we can directly specify this Qu value during our calculations. So obviously we can directly specify this 250 metre cube per second value here and we can reduce one unknown variable because inflow from this particular node which is node number

1 or external junction node 1 should be equal to Qd which is coming out from this junction or external junction node 2.

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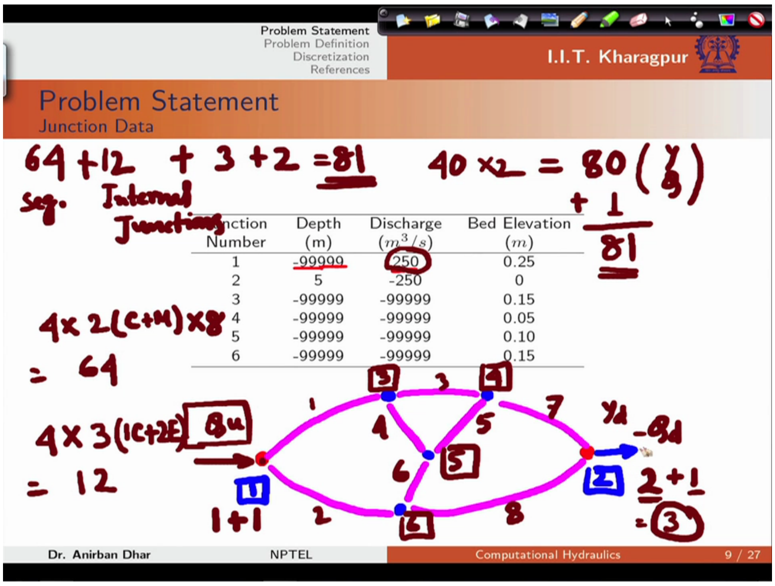

Now if we see this last column these are bed elevation for different junctions. Now what is required? We need to estimate the flow depth and discharge across the channels.

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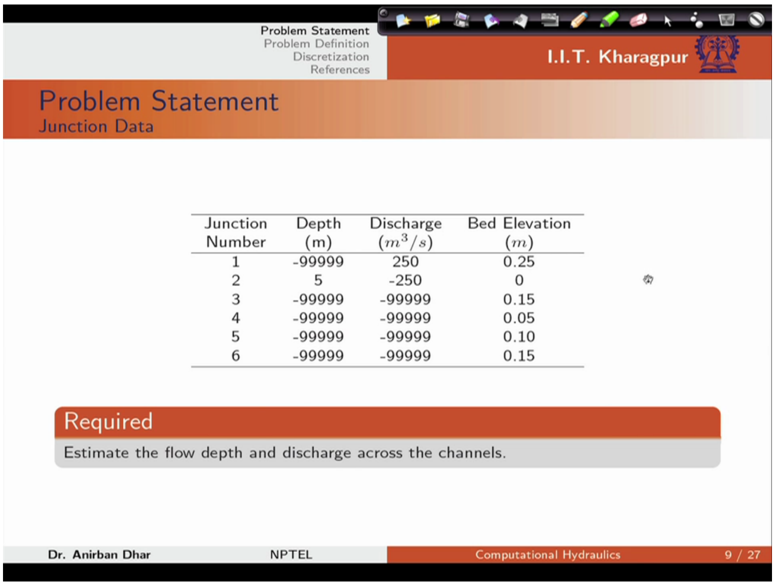

Problem definition that was the problem statement now we have governing equation for channel flow. This is continuity equation, this is momentum equation.

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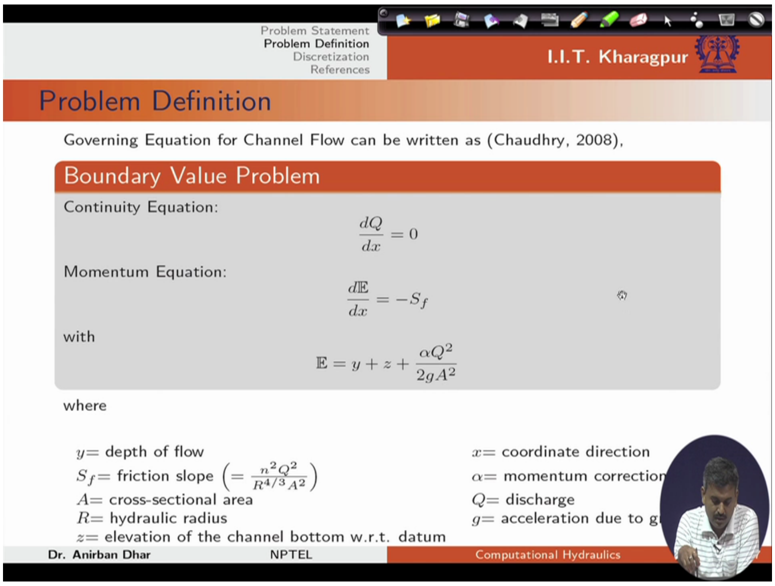

Now in discretized form for different channel reaches and different segments we can consider different sections and for different channel reaches if the flow is from section 1 to NL plus 1 we will consider that flow is positive.

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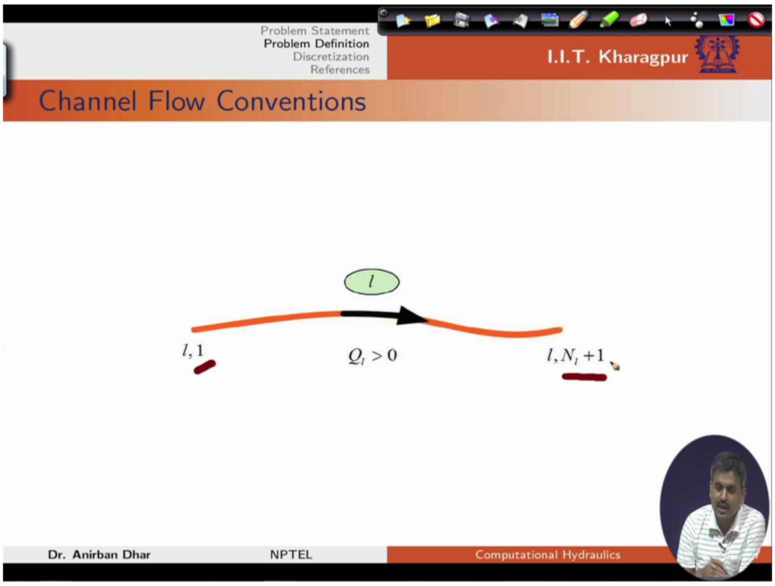

And if this QL is negative we will consider that this NL plus 1 to 1 is the flow direction. Now if we consider this as positive flow direction obviously we are extracting water from junction node J and we are adding water to this junction node J plus 1. So this is a sign convention for this one.

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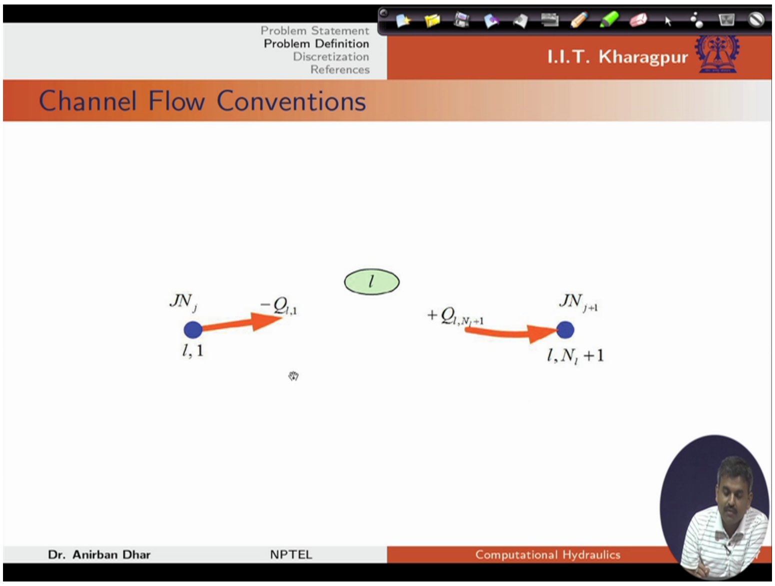

Now in discretized form for ith channel segment of the Lth channel reach we can write this continuity. This is CL i. This is valid for Lth channel reach and ith segment. Now we can get the coefficients of our Jacobian matrix from this one.

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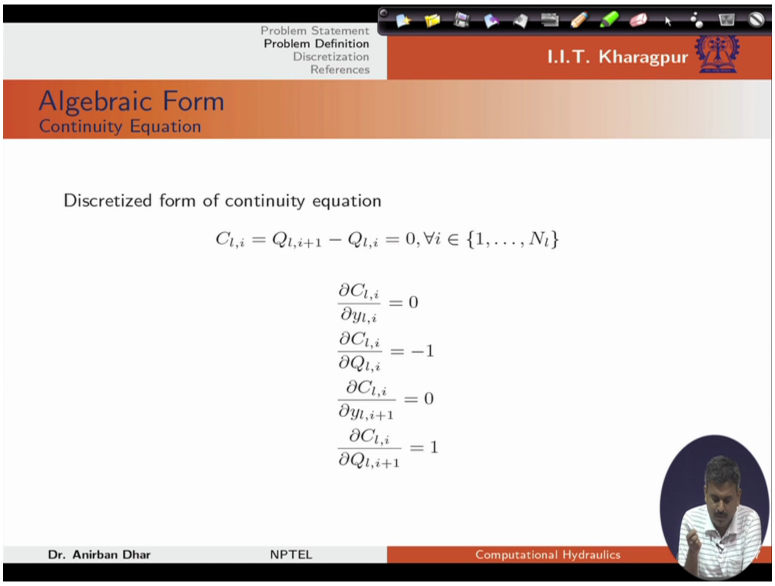

Now if we discretize our momentum equation this is somewhat different compared to our discretization that we have discussed in our previous lecture class. In our previous lecture class we have considered as Q square by A square Q square by AL square. This is also Q square because in that case all values were positive. But in the present case we will consider this quantity which is the quantity for SF or friction slope, we will change this Q square with Q and mod of Q. In that case we can get the direction of your friction slope.

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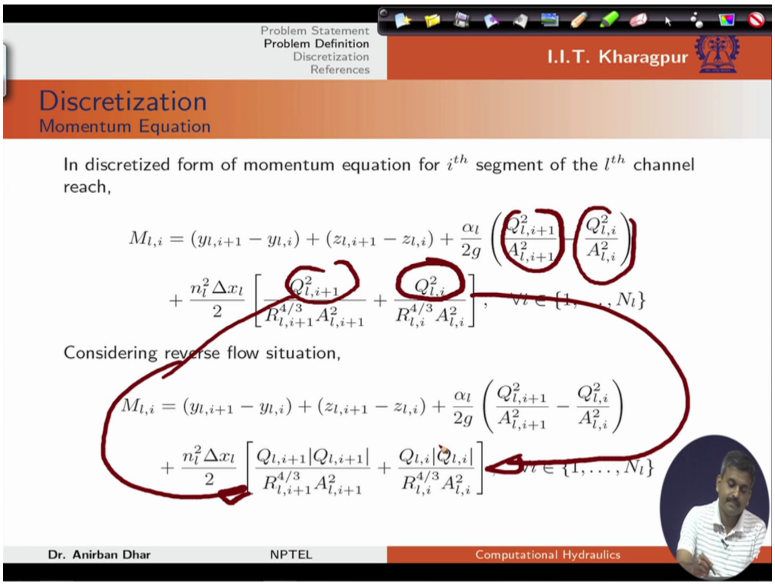

Now obviously in this case we have NL. Nonlinear equations with NL plus 1 into 2 unknowns, discharge and flow depth. If we write the elements of our momentum equation this is important because in our previous lecture class we have considered all Q square values in the coefficients of this Jacobian matrix also or entries in the Jacobian matrix. But in this case the terms which are related to Q square by A square will not change or will not change that 2Q abs Q.

Only the terms which are related to friction slope will change the sign. In this case these two terms are related.

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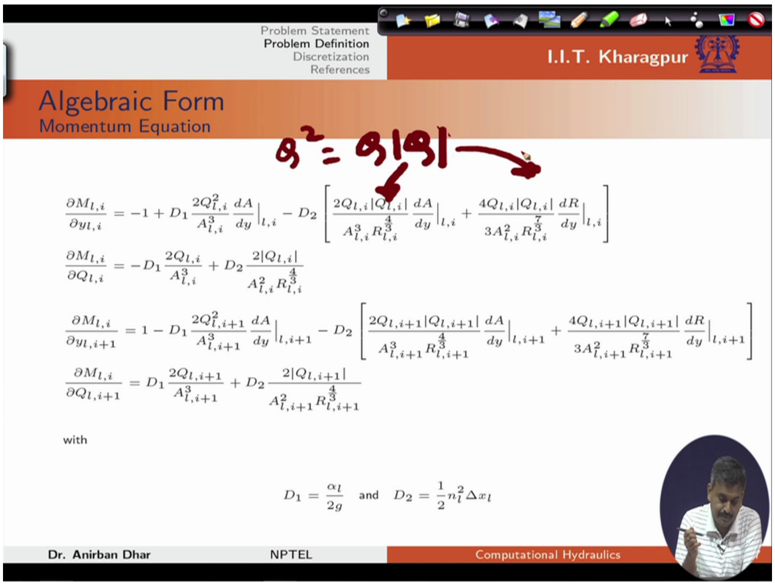

In the next case for the derivative calculation of ML i with respect to QL i we will consider this mod value here.

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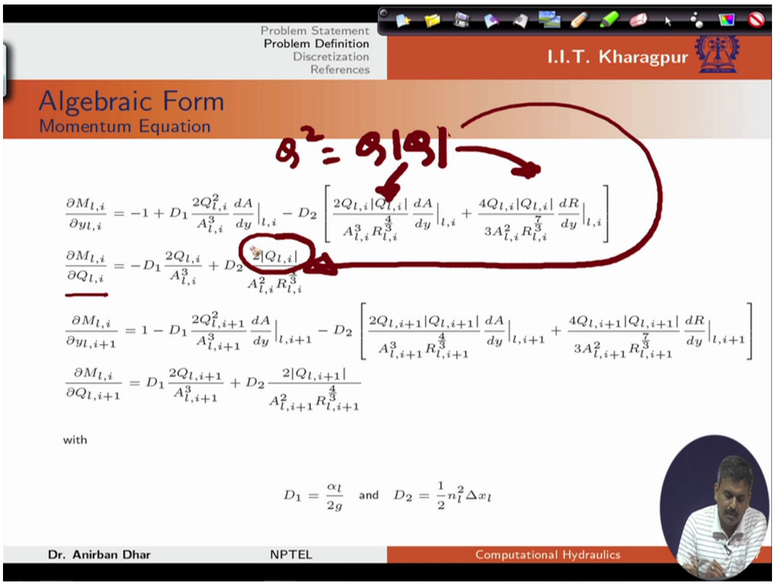

But for first term we are not changing any or we are not taking any absolute operator here. And yL i plus 1 also first term we are not changing that but for the second term we are changing it. So in this case we will get another QL i i plus 1. So obviously this term will change to mod value of Q.

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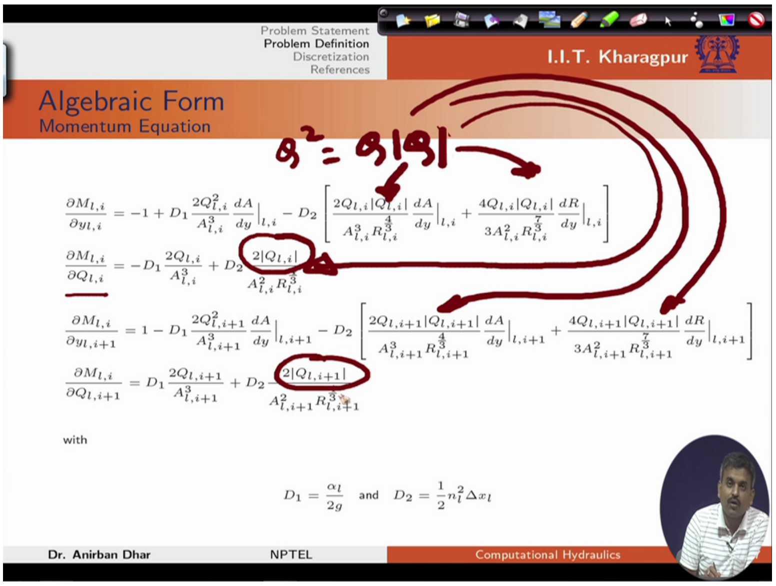

Now so compared to our previous one what are the changes in this derivative calculation? So changed terms are 1, 2, 3, 4, 5 and 6. These 6 terms which are related to friction slope will change those terms.

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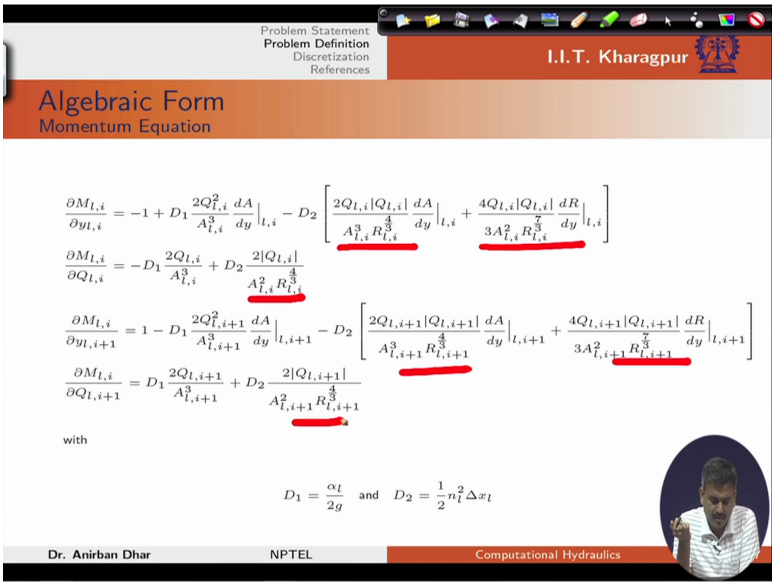

For trapezoidal channel section this is dA by dy but we are considering m1 m2 (ze) zero. So obviously it will be dR by dy. So obviously we will get the simplified equation for rectangular case.

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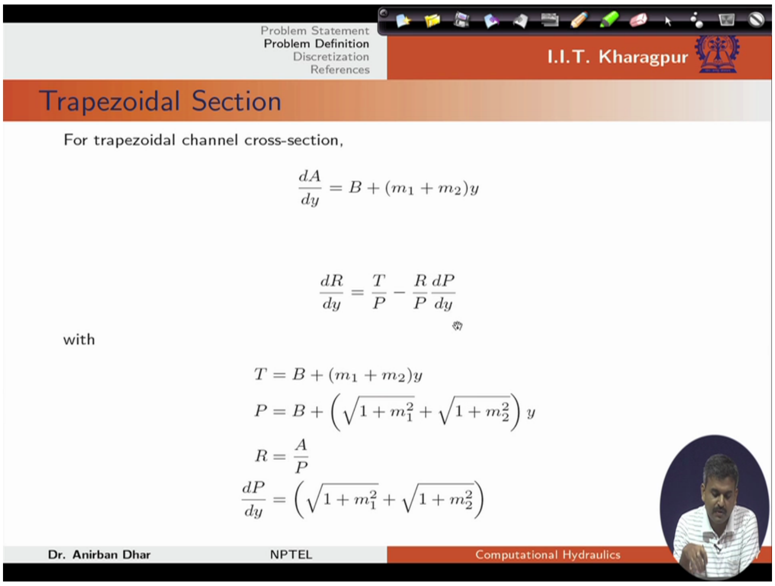

Now this is the algebraic form. We have talked about our continuity momentum equation discretization. But we have not talked about the discretization of individual internal junctions and our external junction boundary conditions. So we have all total six junctions. In this case the outer one starting from 1 this is 2 we have 3, 4, 5 and 6. So obviously this is our condition in this case. So one by one let us consider the conditions for different junctions.

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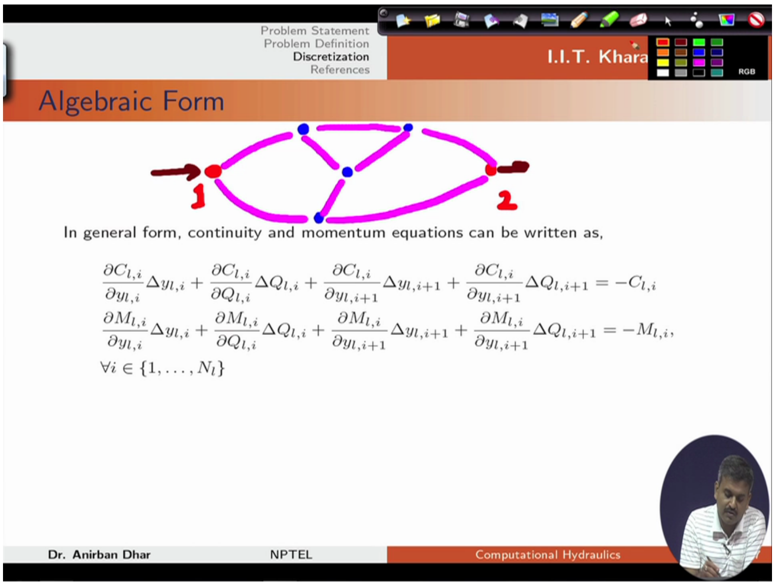

So this is junction number 3, 4, 5 and 6. These are our channel reach numbers 1, 2, 3, 4, 5, 6, 8 and this is 7. So for channel reach number 1 what condition for discharge we are getting? For discharge or continuity we are getting that this Qu this amount we are adding and this Q1 this is being extracted from the system. So Q1 1 this is negative minus Q2 1 this should be equal to zero.

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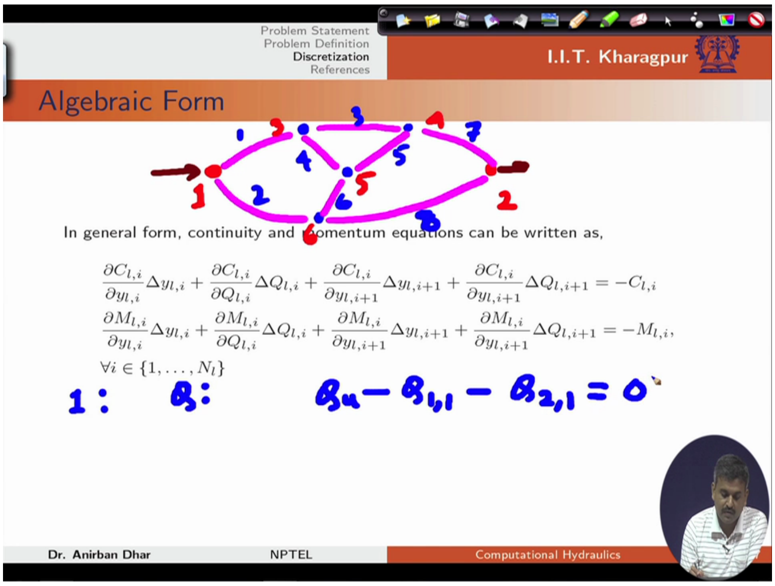

So Qu amount is added into the system and this amount is being extracted from the system as we have considered our flow directions from 1 to 3 and 1 to 6. Next condition because we are not considering the difference in elevation for this junction and our channel reaches. So for y we will have one condition. What is that? That is y1 1 this should be equal to y2 1. That means first section depth at channel 1 and channel 2 should be same provided that we have same elevation for these two channel sections.

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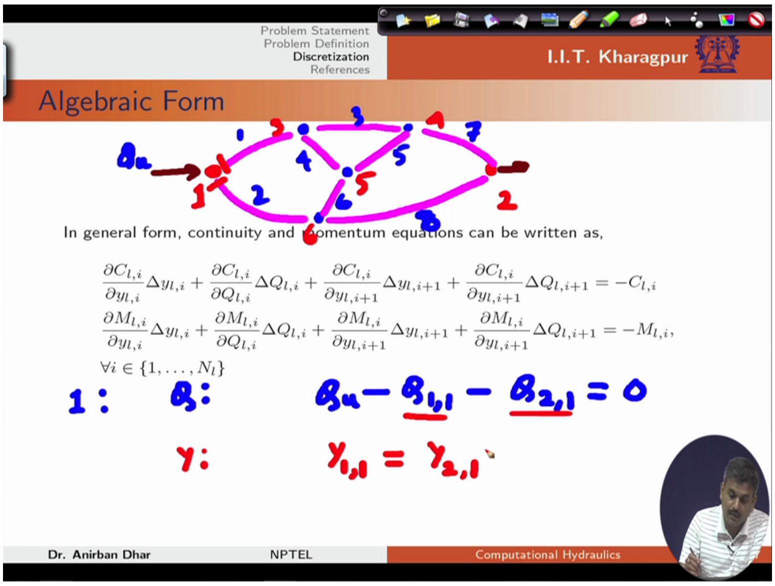

If we consider the flow or the conditions for section or junction node 2 we will have Q7 this is N7 plus 1, this is added. Again Q8 N8 plus 1 this amount is added to the junction. This is for junction number 2 and extraction is Qd amount. So this is the continuity condition.

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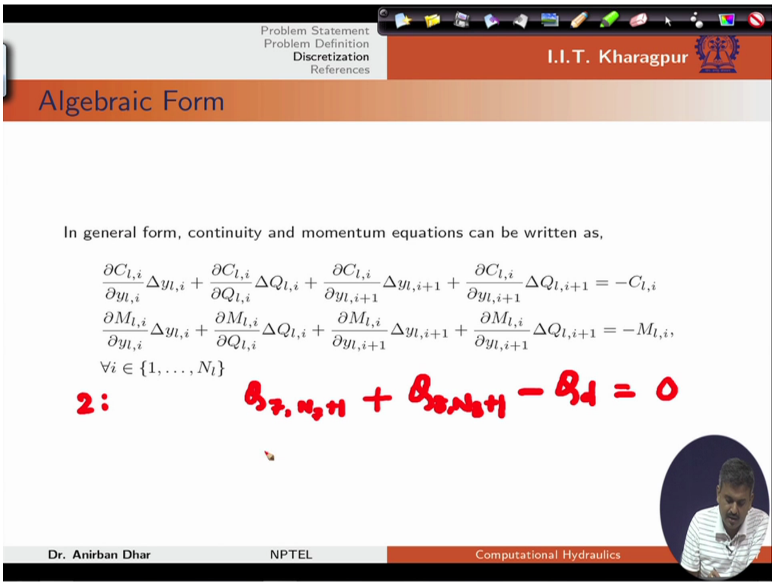

Again you will have another condition that is y7 N7 plus 1 this is equal to yd and y7 N7 plus 1 equals to y8 N8 plus 1. That means we have two conditions for y and one condition for discharge at the downstream external junction boundary.

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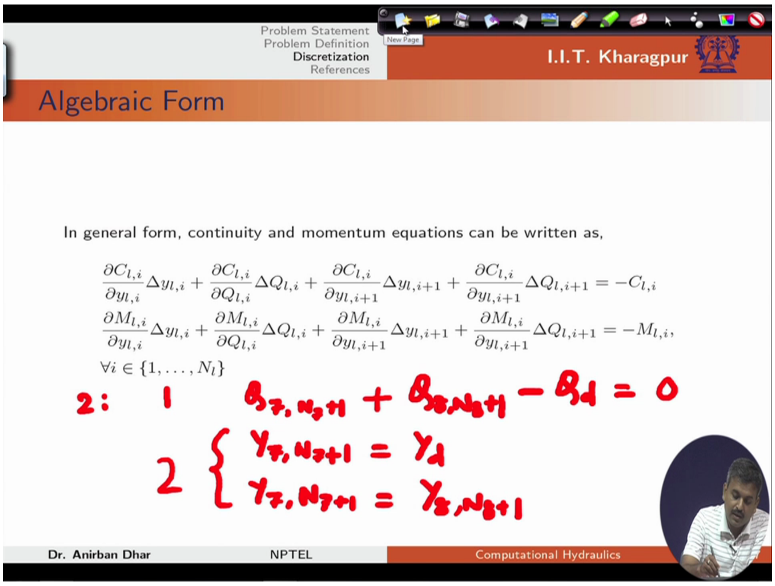

Next is for junction number or junction node number 3. For 3 we have discharge from 1. From 1 this is N1 plus 1 which is being added there minus Q3 1 this is being extracted then Q4 1 this is also extracted from this node. So this continuity condition should be satisfied and we will have y1 N1 plus 1 equal to y1 y3 1 and y1 N1 plus 1 this is equals to y4 1.

So we will have two depth related condition or energy condition, one our continuity condition. Obviously in this case I am considering our junction loss thing. in junction we do not have any loss condition.

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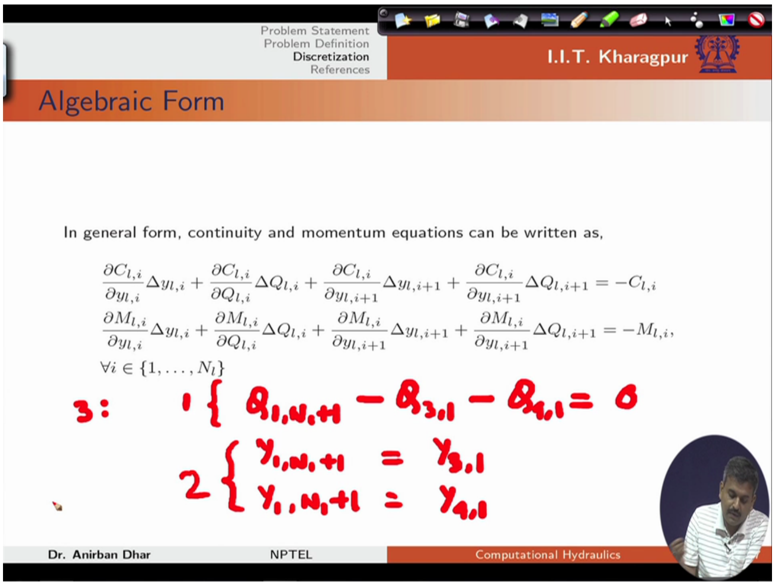

If I consider the channel 4 so obviously for 4 or junction node 4 we have three. That means Q3 N3 plus 1 this is being added. Also the amount which is added is Q5 N5 plus 1 minus Q7 1. So this amount is extracted. So obviously in this case we will have Q3 N3 plus 1 or this is for depth condition. So obviously this will be y equal to y5 N5 plus 1 and y3 N3 plus 1 equal to y7 1. So two conditions here and one for discharge.

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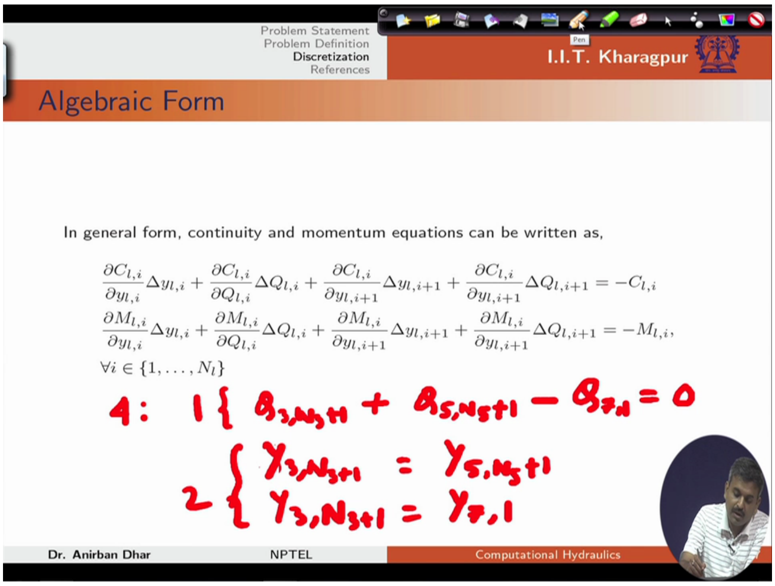

Similarly for junction node 5 this amount is added Q4 N4 plus 1. Also Q6 N6 plus 1 minus Q5 1. This is equal to zero. So y4 N4 plus 1 this should be equal to y6 N6 plus 1 and y4 N4 plus 1 equal to y5 1. So 2, 1 again 3.

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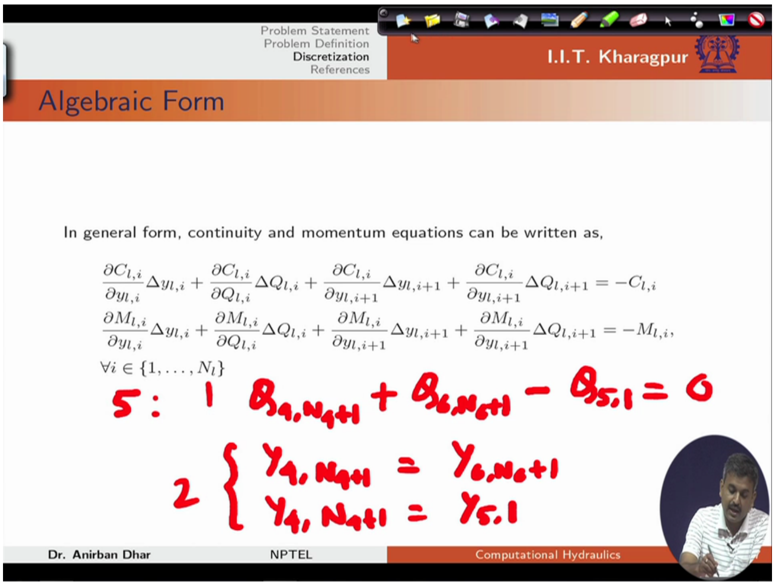

And the last junction that is 6. For 6 this is 6 this is connected to 1, 6 and 8. So in that case this is 2, 6 and 8.

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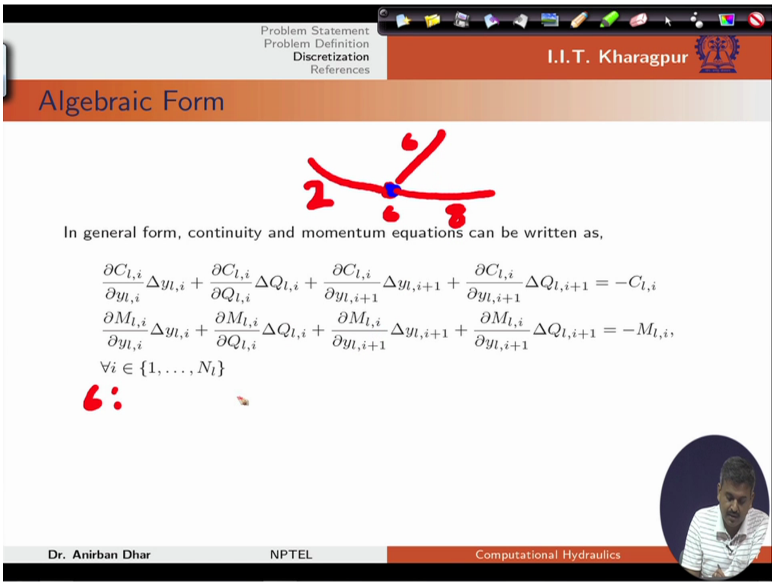

In this case we will get Q2 N2 plus 1 minus Q6 1 minus Q8 1 equals to zero. And this will be y2 N2 plus 1 equals to y6 1, this is y2 N2 plus 1 equals to y8 1. So these are the conditions. Obviously if you want to get the elements of Jacobian matrix for this one we can differentiate with respect to the variables in this case.

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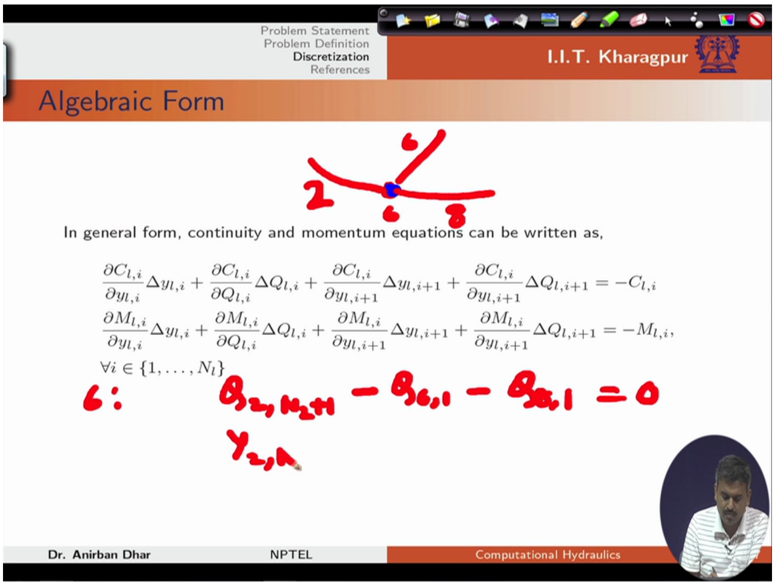

So obviously all these values are unknown. So we have this is 1, 2, 3, 4, 5 and 6, total 6 unknowns are there in these three equations. So we can write the elements of the Jacobian matrix from this calculation.

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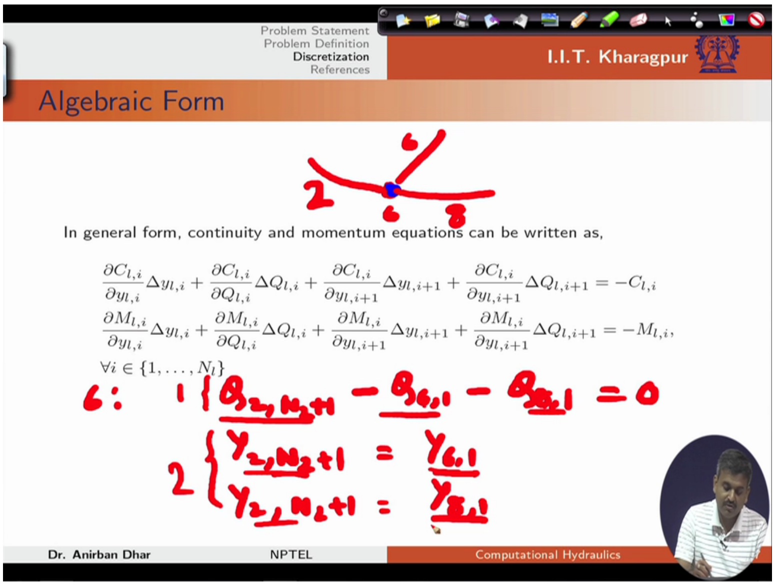

Now after getting these elements of the Jacobian matrix now we can construct the total Jacobian matrix and we can get the solution out of that. But for implementation purpose we need to input certain information in certain format. First is channel chl inf which is channel information.

First column is channel reach number, second column is total length of the channel, third one is the width of the channel, fourth and fifth one these are m1 and m2 values, this sixth one this is segment length, this is n, this is S not and the last two these are junction node number 1 and junction node number 2 for any channel reach. That means channel is connected from 1 to 3, 1 to 6.

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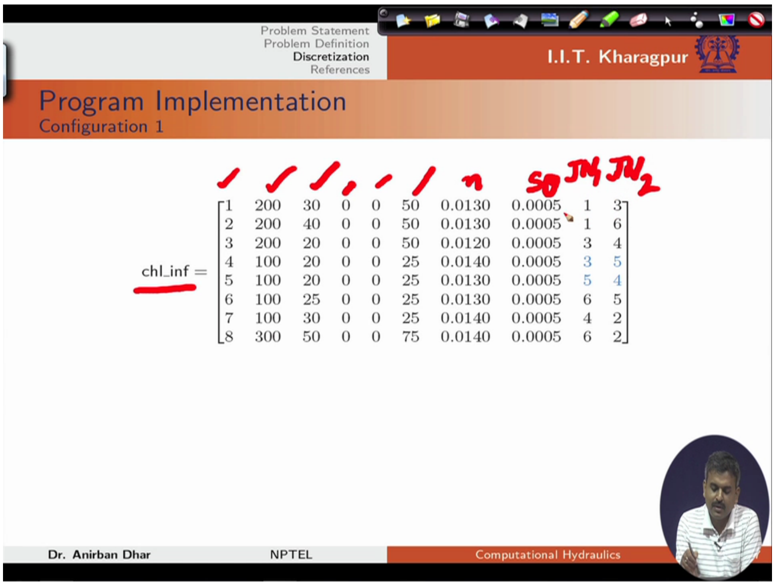

Next information we required is that junction information and in junction information we can include the boundary condition. And in the last column we are providing this elevation values for different junctions.

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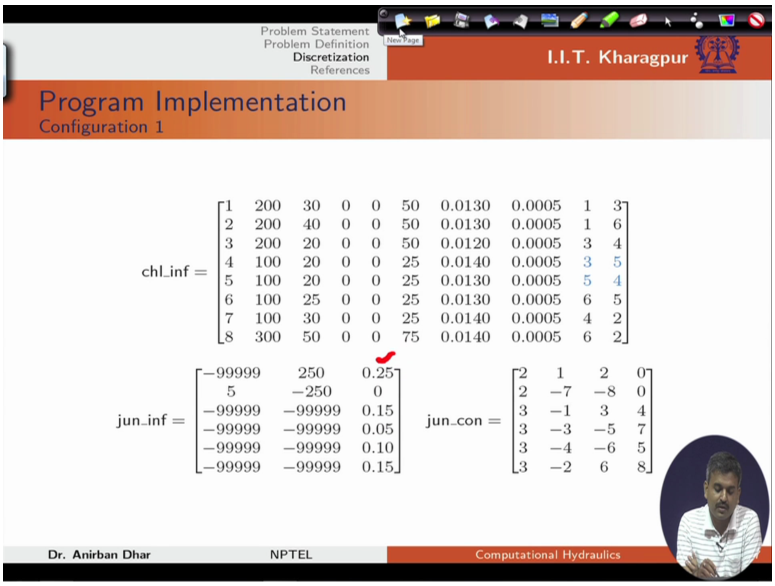

Now junction connectivity, this is another important parameter. For junction 1 this is starting from junction 1, 2, 3, 4, 5, 6. For 6 junctions we have this condition. So for junction 1 we have two channel reaches connected to it that is channel reach 1 and 2. So this is the starting sections for channel reach 1 and 2.

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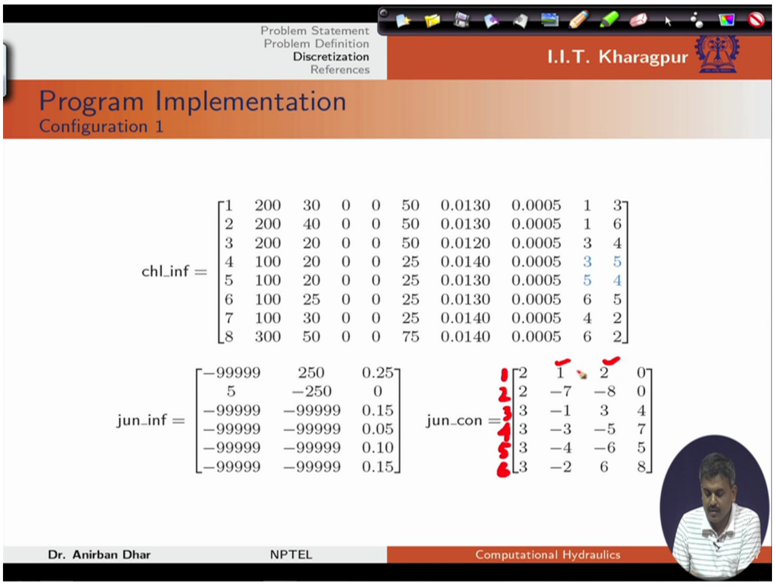

So we have plus 1 and plus 2. For channel reach or junction node 2 we have two connected channel reaches. In this case minus 7 and minus 8 means end sections are connected to this channel reach 2. Channel junction 3 in this one the end section of 1, starting section of 3 and 4 are connected. Channel 4 we have end section of 3, end section of 5 and starting section of 7 that is connected. Channel junction 5 we have again 3 connected channel reaches.

Minus 4 means this is the ending section or end section of channel reach 4, end section of channel reach 6 and starting section of channel reach 5 these are connected. And channel reach or junction node 6 we have 3 connected channel reaches. This is the end section of 2, starting section of 6 and starting section of 8.

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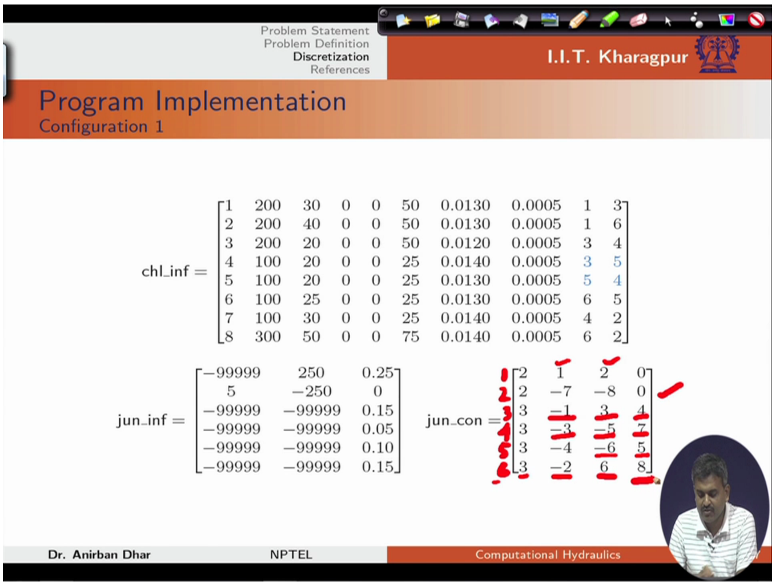

So these three matrixes these are required as input in our program. With this information we can calculate the values.