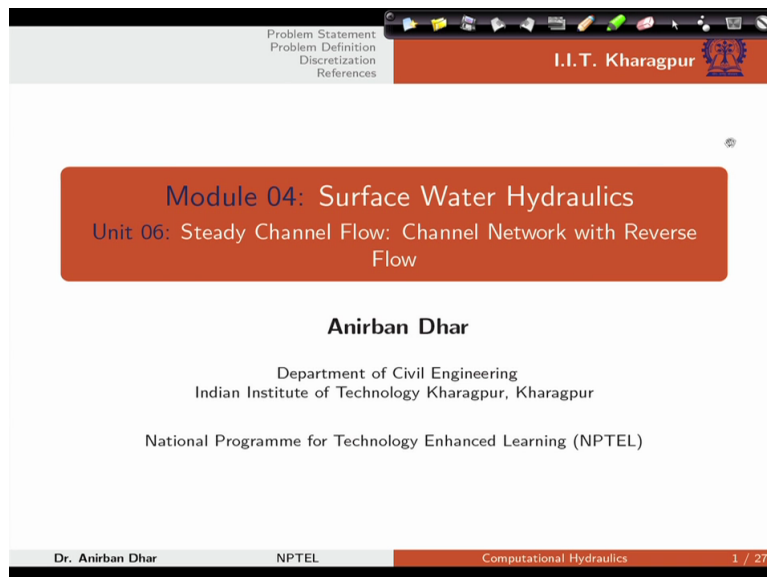


Computational Hydraulics
Professor Anirban Dhar
Department of Civil Engineering
Indian Institute of Technology Kharagpur
Lecture 42
Steady Channel Flow: Channel Network with Reverse Flow

Welcome to this course computational hydraulics. We are in module 4 surface water hydraulics. And in this particular lecture I will be talking about steady channel flow and specifically channel network with reverse flow situation. This is unit number 6 of this module number 4.

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The image shows a presentation slide with a white background and a red header bar. The header bar contains the text "I.I.T. Kharagpur" and the IIT Kharagpur logo. Below the header, there is a red box with white text that reads "Module 04: Surface Water Hydraulics" and "Unit 06: Steady Channel Flow: Channel Network with Reverse Flow". Below this box, the name "Anirban Dhar" is displayed, followed by "Department of Civil Engineering" and "Indian Institute of Technology Kharagpur, Kharagpur". Below that, it says "National Programme for Technology Enhanced Learning (NPTEL)". At the bottom of the slide, there is a footer with "Dr. Anirban Dhar", "NPTEL", "Computational Hydraulics", and "1 / 27".

Learning objective of this particular lecture. At the end of this lecture students will be able to solve steady channel flow for channel network problem with reverse flow using implicit approach.

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Problem Statement
Problem Definition
Discretization
References

I.I.T. Kharagpur

Learning Objective

- To solve steady channel flow for channel network problem with reverse flow using implicit method.

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Problem definition to solution. In our previous lecture we have discussed one problem where we have considered two internal junctions, 1 this is 2. And one external junction. So this is external junction. This was the external junction.

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Problem Statement
Problem Definition
Discretization
References

I.I.T. Kharagpur

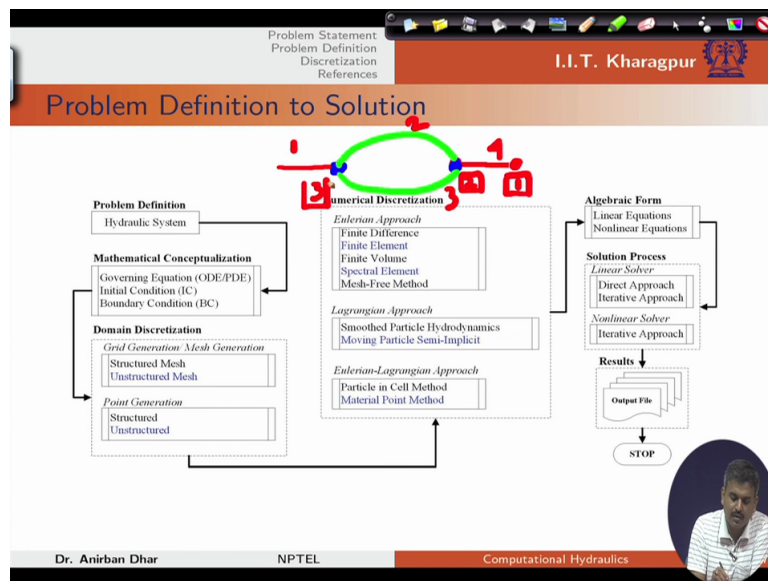
Problem Definition to Solution

```
graph TD
    PD[Problem Definition  
Hydraulic System] --> MC[Mathematical Conceptualization  
Governing Equation (ODE/PDE)  
Initial Condition (IC)  
Boundary Condition (BC)]
    PD --> DD[Domain Discretization  
Grid Generation / Mesh Generation  
Structured Mesh  
Unstructured Mesh  
Point Generation  
Structured  
Unstructured]
    MC --> ND[Numerical Discretization  
Eulerian Approach  
Finite Difference  
Finite Element  
Finite Volume  
Spectral Element  
Mesh-Free Method  
Lagrangian Approach  
Smoothed Particle Hydrodynamics  
Moving Particle Semi-Implicit  
Eulerian-Lagrangian Approach  
Particle in Cell Method  
Material Point Method]
    DD --> ND
    ND --> AF[Algebraic Form  
Linear Equations  
Nonlinear Equations]
    AF --> SP[Solution Process  
Linear Solver  
Direct Approach  
Iterative Approach  
Nonlinear Solver  
Iterative Approach]
    SP --> R[Results  
Output File]
    R --> STOP([STOP])
```

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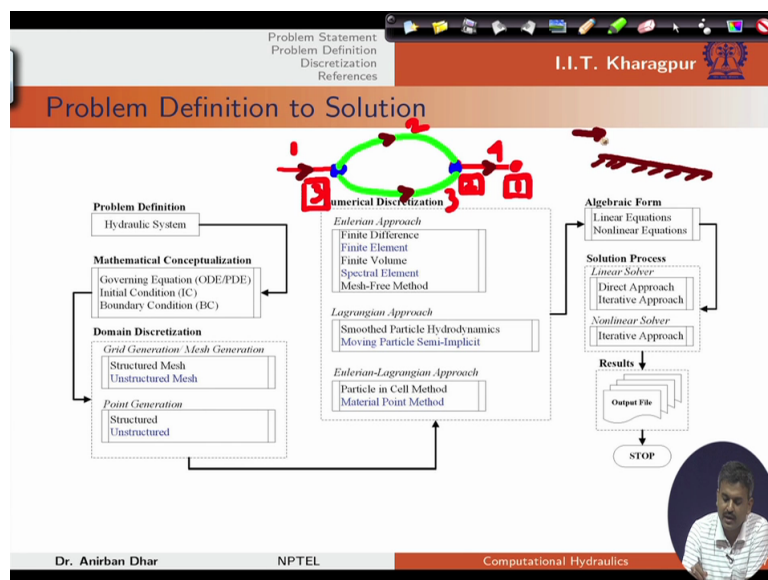
So now for this one we have considered four channel reaches 1, 2, 3, and 4. And in this case these are the channel junctions 1, 2 and 3.

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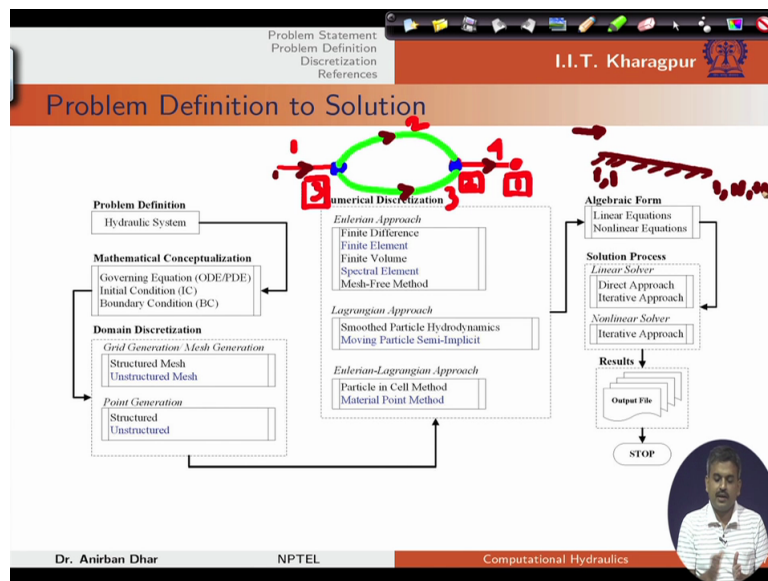
For these three channel junctions we have seen the flow is from left to right direction. So basically in this case we have a flow which is running from left to right direction and this is also the direction of sloping channel bed. That means we have sloping channel bed which is running from left to right.

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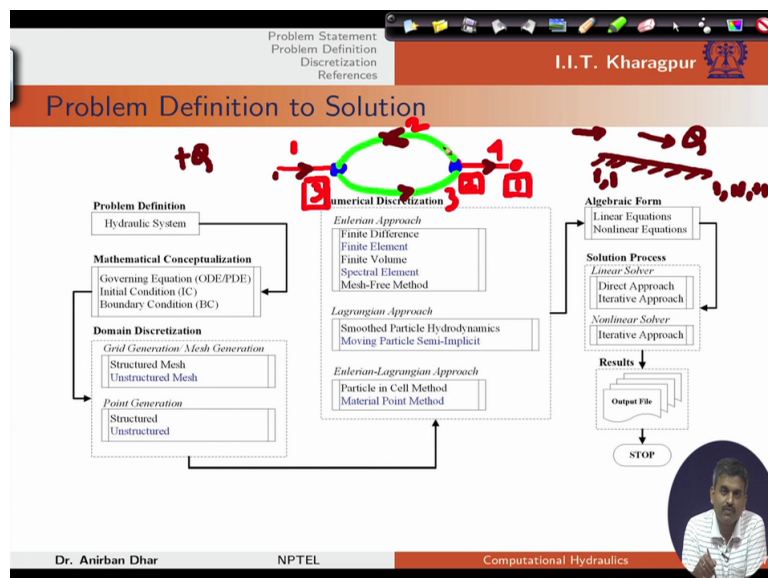
So if we consider this channel bed at junction 1 so this is the up gradient or upstream location and this part is the down gradient or downstream location. So for channel 1 this is 1, 1 and this is 1, N1 plus 1.

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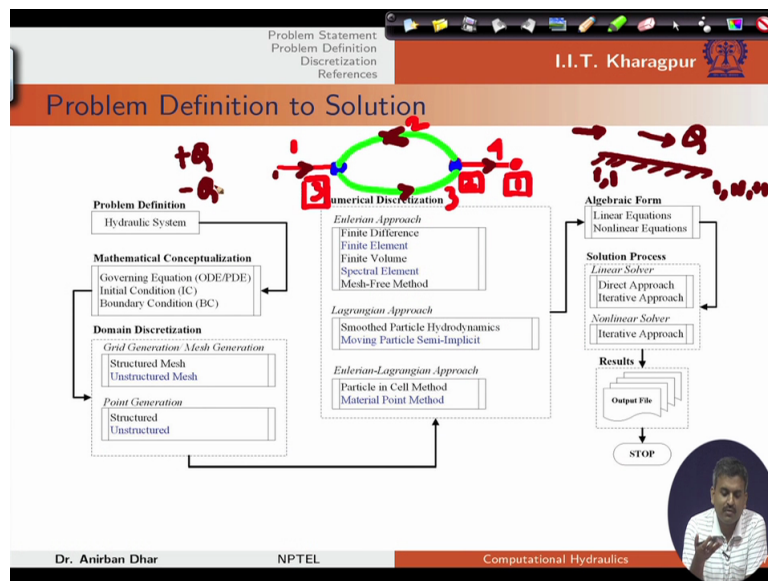
So we are considering that flow is from left to right and this is our positive flow. So according to our convention we have got all values positive because we know the flow direction. But let us consider a situation where we have reverse flow conceptualization. If I start the same problem with flow direction in the channel 2 as flow from right to left so this is reverse flow situation.

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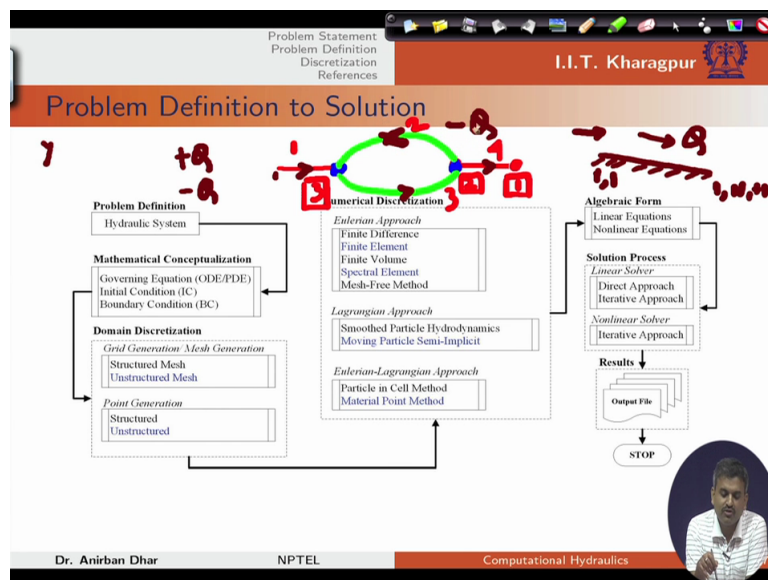
Now how to solve this kind of problems that we will discuss in this particular lecture class. So conceptualization wise governing equation are same but implementation wise we need to consider certain points so that we can include the plus and minus sign of these Q values.

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Because for y it is always positive. But we can have positive or negative Q values during our flow problem solution and we can have positive or negative Q values depending on the conceptualization if I consider that my flow in this small problem is from right to left obviously I should get a minus Q value at this location because the original flow direction is from left to right.

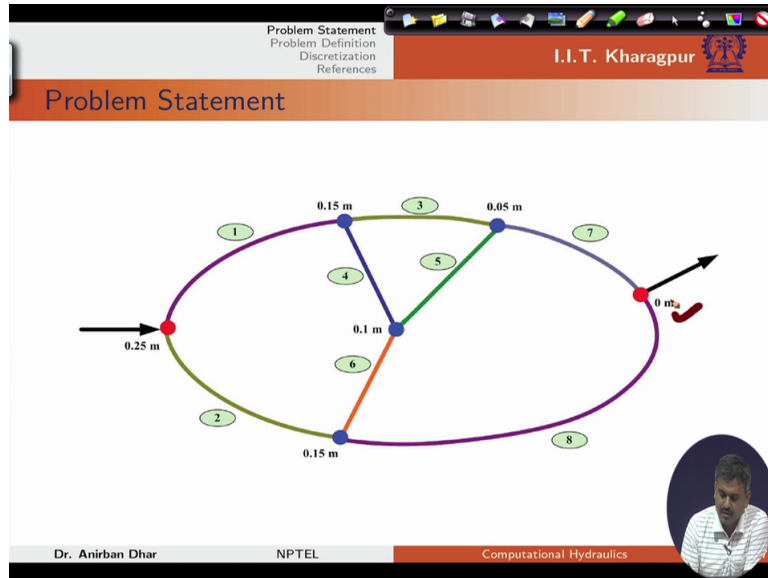
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So for this one again we will consider this finite difference discretization of the equations and we will get nonlinear equations and finally we will try to solve these nonlinear equations using nonlinear solution approach that is Newton Raphson. Now let us see what is the

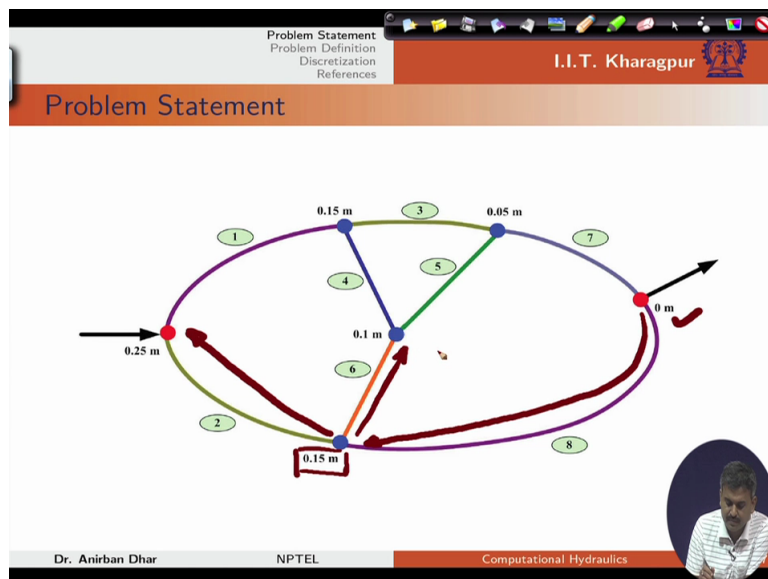
problem statement for our problem. Now let us consider this complicated network. As per this network directions this is the lowest point with zero metre bed elevation.

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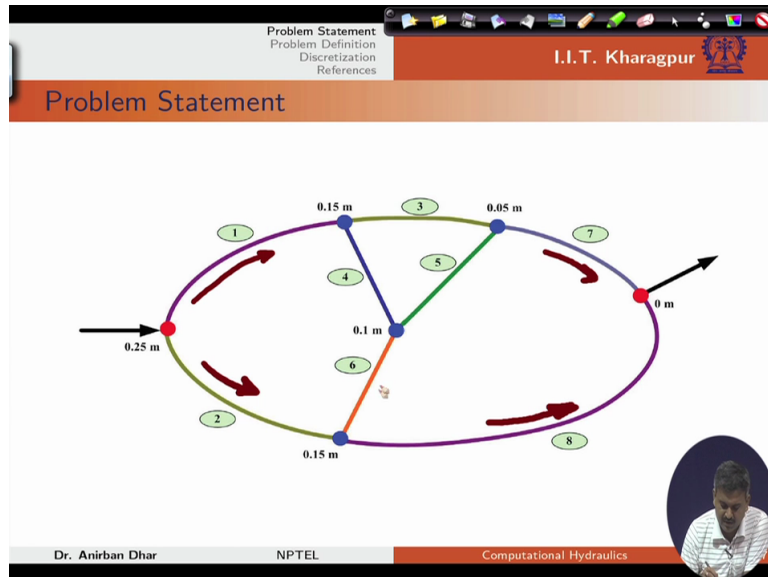
Now starting from the zero the values are increasing in this direction and the value of bed with respect to this downstream end point this point 15 metre. Again values are increasing and this is the highest point in this network. So obviously we have a slope which is towards this direction.

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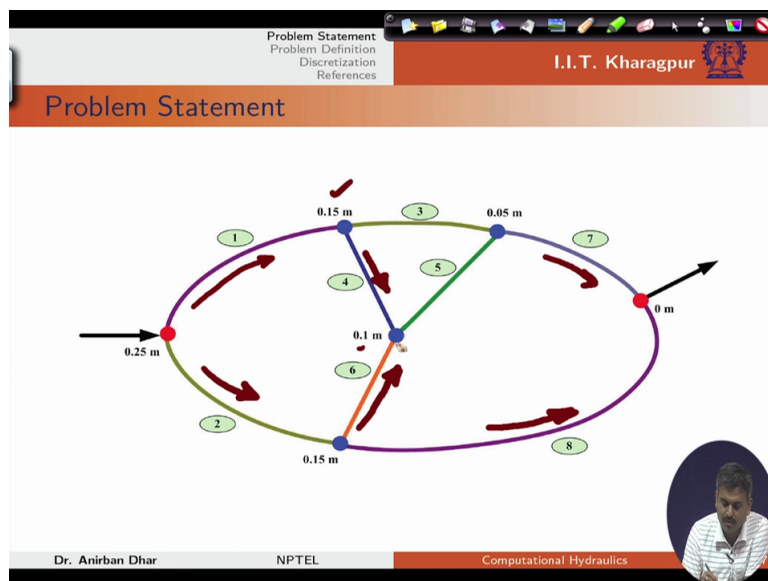
Again from this we have so obviously in this case your bed slope is in this direction. It is decreasing and again for this one it is decreasing in this direction, for 1 this is decreasing in this direction. So as per the natural flow direction these are the natural flow direction as per the slope consideration.

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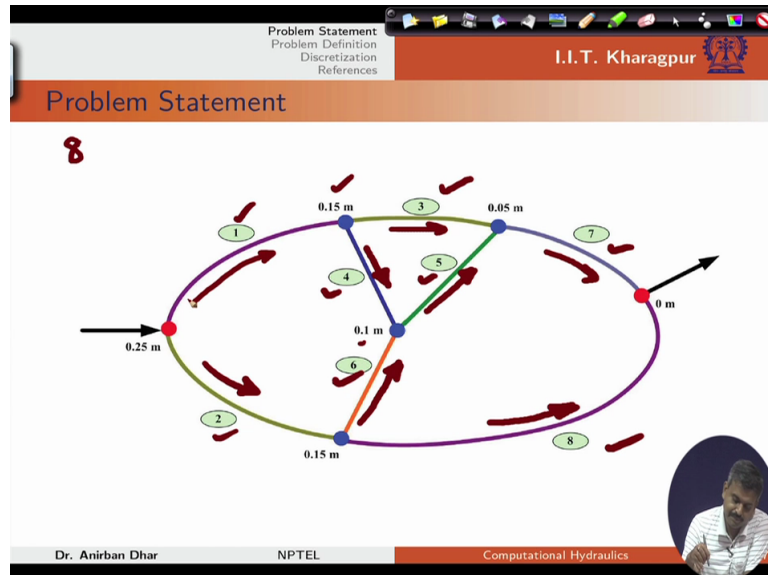
Let us say the slope discharge condition for this one. So this is the discharge condition for this one. Again this is higher elevation, this is lower so obviously flow will be from this point to this point.

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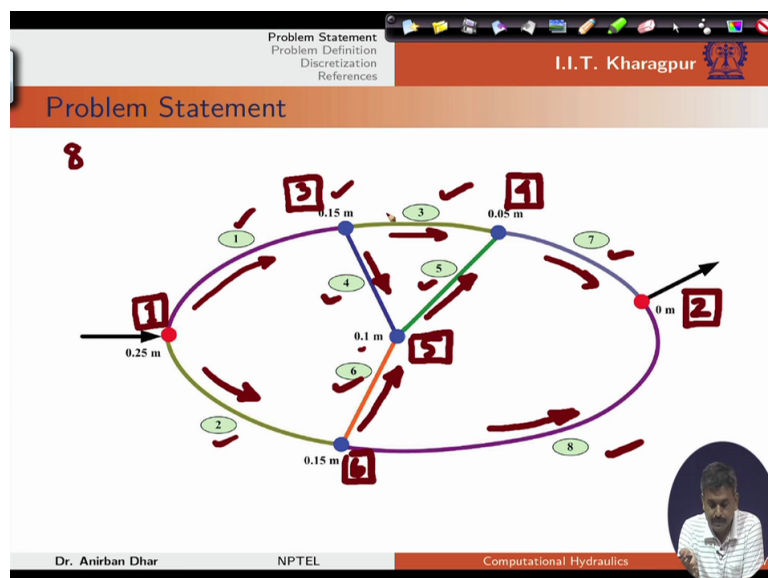
Again the flow will be here and this will be the flow direction for the third channel reach. So we have 8 channel reaches 1, 2, 3, 4, 5, 6, 7 and 8. For this we have 4 internal junctions. So this is 1, 2, 3 and 4 and red ones are external junctions or points where we need to specify the boundary condition.

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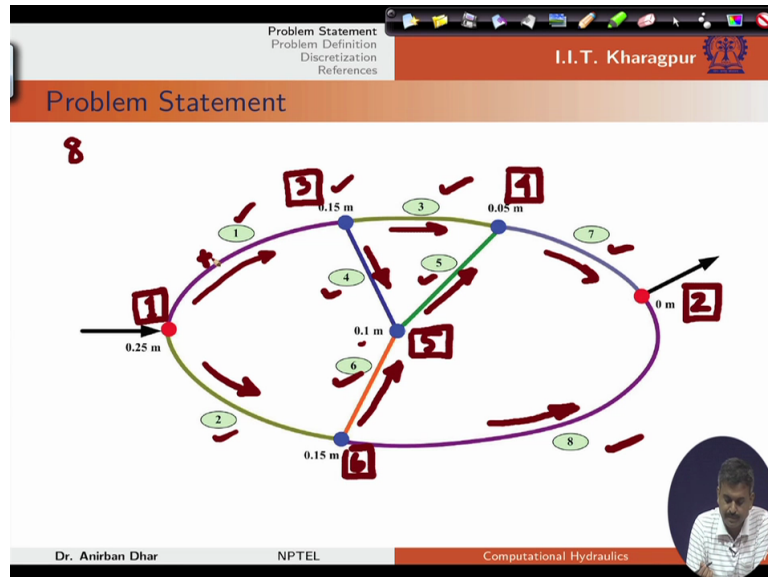
So this is our point number or junction number 1. So let us say this is junction number 2, this is junction number 3, this is junction number 4, this is 5, 6. So we have all total six junctions and for this junctions we can see that as per the bed elevation we have the natural direction of flow.

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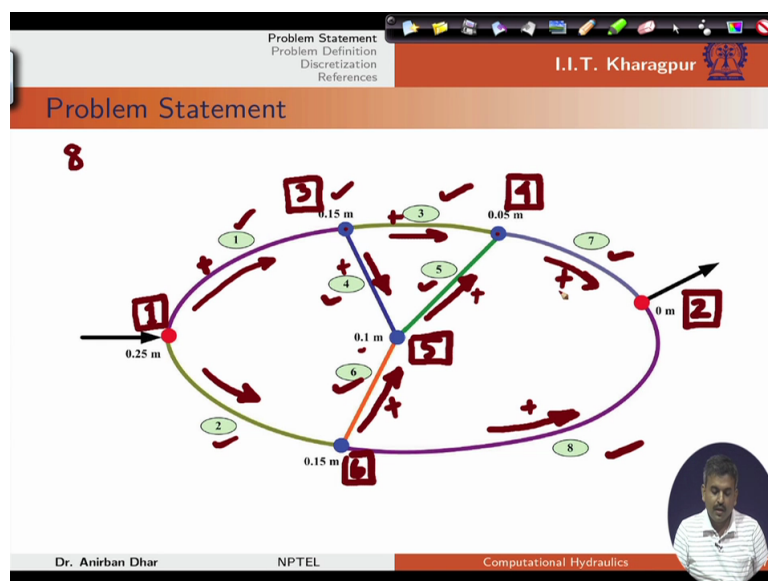
Let us say that we are considering channel reach 1. So if we start from node 1 and moves towards node 3 obviously I am considering the flow direction is according to the channel slope. So this will be the positive direction of flow.

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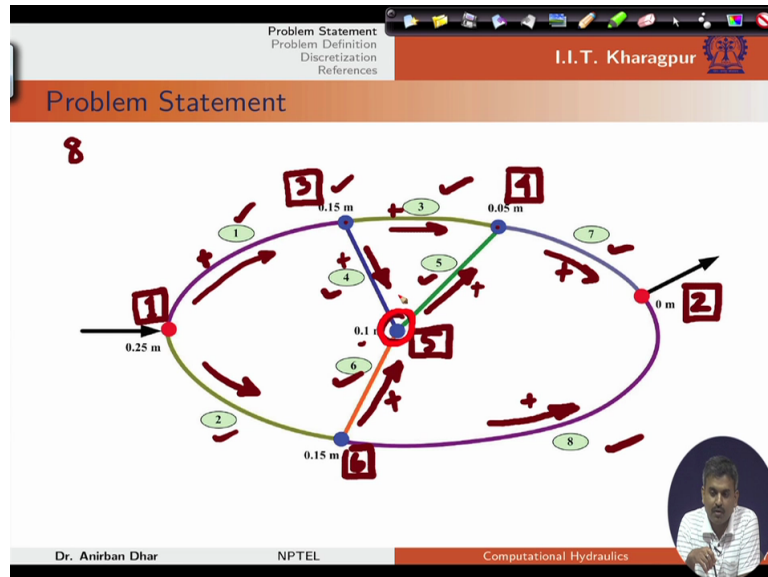
Again if I consider this channel 3, this is the positive direction of the flow because we are starting from higher elevation moving towards lower elevation. Again this side it is a positive direction of flow. For channel 5 this is positive direction, channel 6 this is positive, this is positive and this is positive for 8 and 7.

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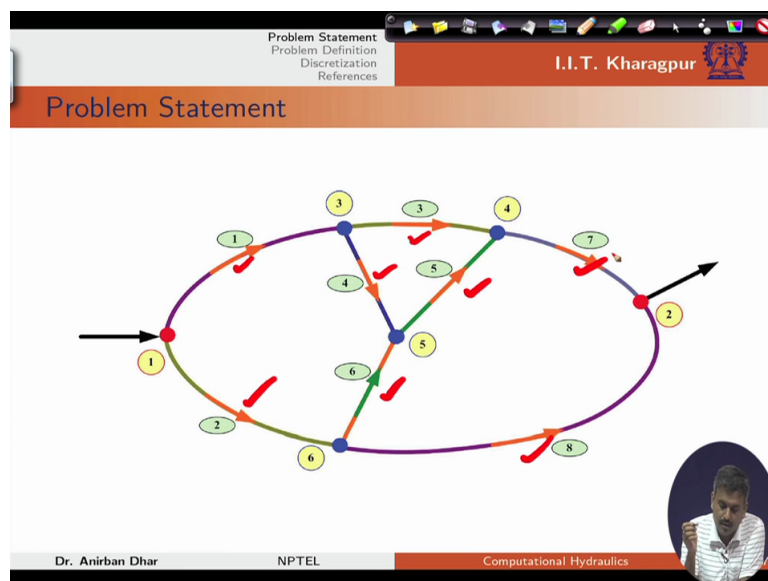
Now with this configuration we can start our problem. So we have a loop channel network because with this internal junction everything is connected there or every channel or internal channels that means 4, 5, 6 these are connected.

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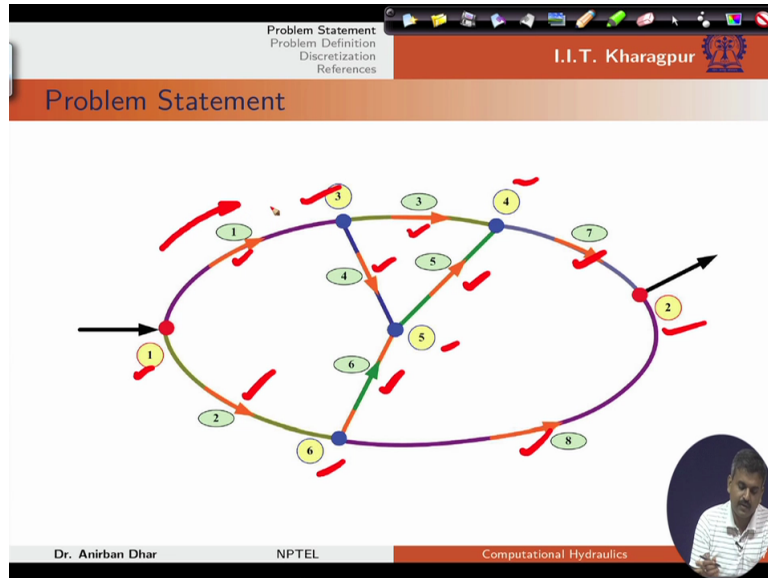
So flow is occurring through this junction and this junction is the main connector for the loop network. So in this case let us consider our flow directions. So as per our channel flow or channel bed slope these are the directions.

(Refer Slide Time: 12:09)



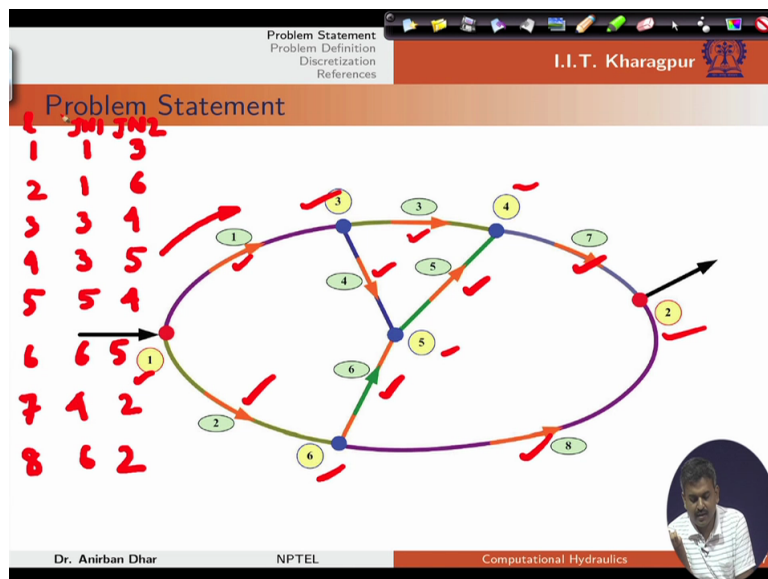
Now these are the junctions 1, 2, 3, 4, 5, 6. So if I say that my flow direction is from 1 to 3 that means I am considering that flow in this direction is positive.

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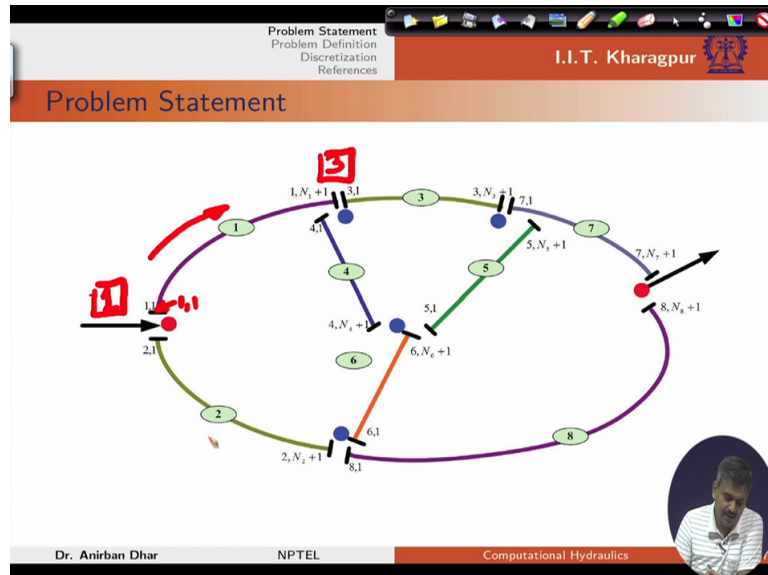
So obviously for channel 1 the connecting nodes are 1 and 3. For channel 2 this is 1 and 6. Channel 3 this is 3 and 4. Channel 4 it is 3 and 5. Channel 5 this is 5 and 4. Channel 6 we have 6 and 5. Channel 7 we have 4 and 2. Channel 8 we have 6 and 2. So this is the connectivity information or positive direction of flow. That means from 1 to 3 we are considering that the flow is positive. So this is junction 1 and this is jN2 and this is L or channel reach number.

(Refer Slide Time: 14:00)



Now in this case let us consider the individual channel reaches. If I consider 1 because I have considered that the flow is from this node number 1 to 3. So the first node should be 1, 1 that is the thing. Now node is $N_1 + 1$. Like that I can discretize all in channel reaches or individual channel reaches.

(Refer Slide Time: 14:53)



In this case what I am observing from this drawing this is 1 and 6. So connecting nodes are 1 and 6. So obviously starting one is 2 1, 2 $N_2 + 1$. In this case this is 3 1, 3 $N_3 + 1$. This is 4 1, 4 $N_4 + 1$. This is 5 1, 5 $N_5 + 1$. 6 1, 6 $N_6 + 1$. This is 7 $N_7 + 1$. 8 $N_8 + 1$. So these are starting and ending nodes and this is the discretization convention because we are starting with a positive flow.

(Refer Slide Time: 16:14)

Problem Statement
Problem Definition
Discretization
References

I.I.T. Kharagpur

Problem Statement

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Now in this case we will be considering rectangular channel. So obviously m_1 will be zero, m_2 will be zero in this case so we will get simplified expression out of this.

(Refer Slide Time: 16:31)

Problem Statement
Problem Definition
Discretization
References

I.I.T. Kharagpur

Problem Statement

Trapezoidal Cross-section

$$A = By + \frac{1}{2}(m_1 + m_2)y^2$$

$$P = B + \left(\sqrt{1 + m_1^2} + \sqrt{1 + m_2^2} \right) y$$

$$R = \frac{A}{P}$$

$$T = B + (m_1 + m_2)y$$

where P = wetted perimeter.

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So channel reach this is 1, total length is 200, width is in metres. So this is nothing but the bed width of the channel cross section. So this is B and these are m_1, m_2 . Obviously these are having zero values because we are considering rectangular channel. This is reach or segment length. If I consider that segment length is 50 and if I divide this 200 by 50 obviously I am getting 4 segments, 4 plus 1 so total 5 sections.

Second case also I am getting 5, third case I am getting 5, with 25 metres at segments length, fourth case also it is 5. In this case this is also 5, this is also 5 and last one this is also 5. So all total we have 1, 2, 3, 4, 5, 6, 7, 8, total 40 sections.

(Refer Slide Time: 18:21)

Channel	length (m)	width (m)	Side Slope		reach (m)	n	S ₀	Connectivity	
			m ₁	m ₂				JN ₁	JN ₂
1	200	30	0	0	50	0.013	0.0005	1	3
2	200	40	0	0	50	0.013	0.0005	1	6
3	200	20	0	0	50	0.012	0.0005	3	4
4	100	20	0	0	25	0.014	0.0005	3	5
5	100	20	0	0	25	0.013	0.0005	5	4
6	100	25	0	0	25	0.013	0.0005	6	5
7	100	30	0	0	25	0.014	0.0005	4	2
8	300	30	0	0	75	0.014	0.0005	6	2

$(4+1) \Rightarrow 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5$
 $= 40$

And all cases we have uniform bed slope and these are different n values for different channel reaches and this is connectivity information. This is 1 3, 1 6, 3 4, 3 5, 5 4, 6 5. I have changed the colour for these two channel reaches and that means channel which number 4 and 5.

(Refer Slide Time: 18:58)


Channel	length (m)	width (m)	Side Slope		reach (m)	n	S ₀	Connectivity	
			m ₁	m ₂				JN ₁	JN ₂
1	200	30	0	0	50	0.013	0.0005	1	3
2	200	40	0	0	50	0.013	0.0005	1	6
3	200	20	0	0	50	0.012	0.0005	3	4
4	100	20	0	0	25	0.014	0.0005	3	5
5	100	20	0	0	25	0.013	0.0005	5	4
6	100	25	0	0	25	0.013	0.0005	6	5
7	100	30	0	0	25	0.014	0.0005	4	2
8	300	30	0	0	75	0.014	0.0005	6	2

$(4+1) \Rightarrow 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5$
 $= 40$

I will try to show the effect of reverse flow by changing the flow direction in these two channel reaches. So all total you will have 40 plus 40, 80 unknowns y plus Q. But there will be another unknown that we will consider during discretization.

(Refer Slide Time: 19:36)

Problem Statement
Channel Data



Channel	length (m)	width (m)	Side Slope		reach (m)	n	S ₀	Connectivity	
			m ₁	m ₂				JN ₁	JN ₂
1	200	30	0	0	50	0.013	0.0005	1	3
2	200	40	0	0	50	0.013	0.0005	1	6
3	200	20	0	0	50	0.012	0.0005	3	4
4	100	20	0	0	25	0.014	0.0005	3	5
5	100	20	0	0	25	0.013	0.0005	5	4
6	100	25	0	0	25	0.013	0.0005	6	5
7	100	30	0	0	25	0.014	0.0005	4	2
8	300	30	0	0	75	0.014	0.0005	6	2

$(4+1) \Rightarrow 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5$
 $40 + 40 = 80 \Rightarrow 40$

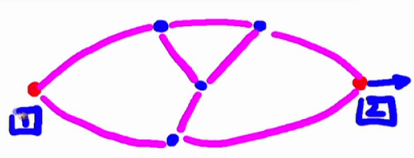
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Now we need to supply junction data in this case. What is this junction data? First one or first column this is junction number starting from 1 to 6. Depth is minus 99999 5 9s. So what is this? That means there is no information available for depth but discharge is 250. So as per our network this point 1 and 2 if I consider other junction nodes these are the junction nodes. In this case we have 1, 3, 7, 4, 5, 6, 2, 8 and externally from this node 2 this is 2, this is 1.

(Refer Slide Time: 21:03)

Problem Statement
Junction Data

Junction Number	Depth (m)	Discharge (m ³ /s)	Bed Elevation (m)
1	-99999	250	0.25
2	5	-250	0
3	-99999	-99999	0.15
4	-99999	-99999	0.05
5	-99999	-99999	0.10
6	-99999	-99999	0.15



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So as per this table we can see that the depth is specified at downstream end at junction number 2, this is 5 metres and we have specified discharge which is minus 250. That means water 250 metre cube per second discharge rate from this junction. And third column this is

bed elevation. Now in this case we have all total 40 sections from all channel reaches. For 40 sections we will get 40 into 2 that is 80 unknown values. That means y and Q .

Another quantity is unknown here. That is inflow or Q_u at the starting node 1 because in this case only y at downstream and Q_d if I put negative sign because I am extracting water from this system from this particular point so this is minus Q_d . So these two values are specified.

(Refer Slide Time: 22:59)

The slide displays the following table:

Junction Number	Depth (m)	Discharge (m^3/s)	Bed Elevation (m)
1	-99999	250	0.25
2	5	-250	0
3	-99999	-99999	0.15
4	-99999	-99999	0.05
5	-99999	-99999	0.10
6	-99999	-99999	0.15

Handwritten equation: $40 \times 2 = 80 (y, Q)$

The diagram shows a channel network with six nodes. Node 1 is the inlet with inflow Q_u . Node 2 is the outlet with outflow Q_d . The channels are represented by pink lines connecting the nodes.

So this Q_u is essentially unknown. Now if I add this Q_u as unknown so all total we will have 81 unknowns in this case. Now for this channel 3 or node number 3, 4, 5 and 6. So for channel 1, channel 2, 3, 4, 5 and 6 we will have all total 4. 4 is the number of segments per channel reach, 4 into 2. Into 2 is for one continuity one momentum into 8. So this is the total number of equations that we will get from the individual segments. So 4 into 2 into 8. So total number is 64.

(Refer Slide Time: 24:36)

Problem Statement
Junction Data

Junction Number	Depth (m)	Discharge (m^3/s)	Bed Elevation (m)
1	-99999	250	0.25
2	5	-250	0
3	-99999	-99999	0.15
4	-99999	-99999	0.05
5	-99999	-99999	0.10
6	-99999	-99999	0.15

Handwritten notes: $40 \times 2 = 80 \left(\frac{7}{9}\right) + \frac{1}{81}$

Handwritten notes: $4 \times 2(C+M) \times 8 = 64$

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Now for this internal junctions 3, 4, 5, 6 that means we have four junctions or internal junctions. For four junctions we will get three conditions that is 1 continuity 2 energy conservation conditions and this we will get 12 conditions out of this.

(Refer Slide Time: 25:18)

Problem Statement
Junction Data

Junction Number	Depth (m)	Discharge (m^3/s)	Bed Elevation (m)
1	-99999	250	0.25
2	5	-250	0
3	-99999	-99999	0.15
4	-99999	-99999	0.05
5	-99999	-99999	0.10
6	-99999	-99999	0.15

Handwritten notes: $40 \times 2 = 80 \left(\frac{7}{9}\right) + \frac{1}{81}$

Handwritten notes: $4 \times 2(C+M) \times 8 = 64$

Handwritten notes: $4 \times 3(C+E) = 12$

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Now after that if we consider this external junction node 2 we have 1 depth and 1 discharge condition plus there will be energy continuity conditions. So all total we will get 2 conditions, 1 for specified depth. That means 2 for depth 1 for discharge continuity. So we are getting here 3 equations.

(Refer Slide Time: 26:06)

Problem Statement
Junction Data

Junction Number	Depth (m)	Discharge (m^3/s)	Bed Elevation (m)
1	-99999	250	0.25
2	5	-250	0
3	-99999	-99999	0.15
4	-99999	-99999	0.05
5	-99999	-99999	0.10
6	-99999	-99999	0.15

Handwritten notes and diagram:

- Equation: $40 \times 2 = 80 \left(\frac{1}{9}\right) + \frac{1}{81}$
- Equation: $4 \times 2 (C+M) \times 8 = 64$
- Equation: $4 \times 3 (I+2E) = 12$
- Diagram: A network of 6 nodes and 8 segments. Node 1 is the source with an inflow of 250. Node 2 is the sink with an outflow of 250. Nodes 3, 4, 5, and 6 are internal junctions. The diagram shows the flow paths and the number of equations associated with each node and segment.

Now at this node number 1 so if I add now 64 plus 12 this is coming from segments. These are coming from internal junctions plus 3 here which is coming from external junction node plus 2, 1 for discharge continuity at node number 1 plus 1 energy continuity for this channel reach 1 and 2. So all total 2. So if we add this so we will get 17 plus 64 which is 81. So we have 81 equations and 81 unknowns.

(Refer Slide Time: 27:22)

Problem Statement
Junction Data

Junction Number	Depth (m)	Discharge (m^3/s)	Bed Elevation (m)
1	-99999	250	0.25
2	5	-250	0
3	-99999	-99999	0.15
4	-99999	-99999	0.05
5	-99999	-99999	0.10
6	-99999	-99999	0.15

Handwritten notes and diagram:

- Equation: $64 + 12 + 3 + 2 = 81$
- Equation: $40 \times 2 = 80 \left(\frac{1}{9}\right) + \frac{1}{81}$
- Equation: $4 \times 2 (C+M) \times 8 = 64$
- Equation: $4 \times 3 (I+2E) = 12$
- Diagram: The same network of 6 nodes and 8 segments as in the previous slide, but with additional annotations. The inflow at node 1 is now explicitly labeled as 250, and the outflow at node 2 is labeled as 250. The diagram shows the flow paths and the number of equations associated with each node and segment.

But what we can do, we can directly specify this Q_u value during our calculations. So obviously we can directly specify this 250 metre cube per second value here and we can reduce one unknown variable because inflow from this particular node which is node number

1 or external junction node 1 should be equal to Qd which is coming out from this junction or external junction node 2.

(Refer Slide Time: 28:04)

Problem Statement
Junction Data

Handwritten calculations:
 $64 + 12 + 3 + 2 = 81$
 $40 \times 2 = 80 \left(\frac{7}{8}\right) + 1 = 81$
 $4 \times 2 (C+M) \times 8 = 64$
 $4 \times 3 (C+M) = 12$

Junction Number	Depth (m)	Discharge (m^3/s)	Bed Elevation (m)
1	-99999	250	0.25
2	5	-250	0
3	-99999	-99999	0.15
4	-99999	-99999	0.05
5	-99999	-99999	0.10
6	-99999	-99999	0.15

Diagram: A network of 6 nodes (1-6) connected by 8 channels (1-8). Node 1 is the source, node 2 is the sink. Nodes 3, 4, 5, and 6 are intermediate nodes. Handwritten annotations include flow directions and values like 84, 1+1, 2+1, and 3.

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Now if we see this last column these are bed elevation for different junctions. Now what is required? We need to estimate the flow depth and discharge across the channels.

(Refer Slide Time: 28:27)

Problem Statement
Junction Data

Junction Number	Depth (m)	Discharge (m^3/s)	Bed Elevation (m)
1	-99999	250	0.25
2	5	-250	0
3	-99999	-99999	0.15
4	-99999	-99999	0.05
5	-99999	-99999	0.10
6	-99999	-99999	0.15

Required
 Estimate the flow depth and discharge across the channels.

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Problem definition that was the problem statement now we have governing equation for channel flow. This is continuity equation, this is momentum equation.

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Problem Definition

Governing Equation for Channel Flow can be written as (Chaudhry, 2008),

Boundary Value Problem

Continuity Equation: $\frac{dQ}{dx} = 0$

Momentum Equation: $\frac{dE}{dx} = -S_f$

with $E = y + z + \frac{\alpha Q^2}{2gA^2}$

where

- y = depth of flow
- S_f = friction slope ($= \frac{n^2 Q^2}{R^4/3 A^2}$)
- A = cross-sectional area
- R = hydraulic radius
- z = elevation of the channel bottom w.r.t. datum
- x = coordinate direction
- α = momentum correction
- Q = discharge
- g = acceleration due to g

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Now in discretized form for different channel reaches and different segments we can consider different sections and for different channel reaches if the flow is from section 1 to NL plus 1 we will consider that flow is positive.

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Channel Flow Conventions

$l, 1$ $Q_i > 0$ $l, N_l + 1$

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And if this Q_i is negative we will consider that this $N_l + 1$ to 1 is the flow direction. Now if we consider this as positive flow direction obviously we are extracting water from junction node J and we are adding water to this junction node $J + 1$. So this is a sign convention for this one.

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Now in discretized form for i th channel segment of the L th channel reach we can write this continuity. This is CL i . This is valid for L th channel reach and i th segment. Now we can get the coefficients of our Jacobian matrix from this one.

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Now if we discretize our momentum equation this is somewhat different compared to our discretization that we have discussed in our previous lecture class. In our previous lecture class we have considered as Q square by A square Q square by AL square. This is also Q square because in that case all values were positive. But in the present case we will consider

this quantity which is the quantity for SF or friction slope, we will change this Q square with Q and mod of Q. In that case we can get the direction of your friction slope.

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Discretization

Momentum Equation

In discretized form of momentum equation for i^{th} segment of the l^{th} channel reach,

$$M_{l,i} = (y_{l,i+1} - y_{l,i}) + (z_{l,i+1} - z_{l,i}) + \frac{\alpha_l}{2g} \left(\frac{Q_{l,i+1}^2}{A_{l,i+1}^2} - \frac{Q_{l,i}^2}{A_{l,i}^2} \right) + \frac{n_l^2 \Delta x_l}{2} \left[\frac{Q_{l,i+1}^2}{R_{l,i+1}^{4/3} A_{l,i+1}^2} + \frac{Q_{l,i}^2}{R_{l,i}^{4/3} A_{l,i}^2} \right], \quad \forall i \in \{1, \dots, N_l\}$$

Considering reverse flow situation,

$$M_{l,i} = (y_{l,i+1} - y_{l,i}) + (z_{l,i+1} - z_{l,i}) + \frac{\alpha_l}{2g} \left(\frac{Q_{l,i+1}^2}{A_{l,i+1}^2} - \frac{Q_{l,i}^2}{A_{l,i}^2} \right) + \frac{n_l^2 \Delta x_l}{2} \left[\frac{Q_{l,i+1} |Q_{l,i+1}|}{R_{l,i+1}^{4/3} A_{l,i+1}^2} + \frac{Q_{l,i} |Q_{l,i}|}{R_{l,i}^{4/3} A_{l,i}^2} \right], \quad \forall i \in \{1, \dots, N_l\}$$

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Now obviously in this case we have NL. Nonlinear equations with NL plus 1 into 2 unknowns, discharge and flow depth. If we write the elements of our momentum equation this is important because in our previous lecture class we have considered all Q square values in the coefficients of this Jacobian matrix also or entries in the Jacobian matrix. But in this case the terms which are related to Q square by A square will not change or will not change that 2Q abs Q.

Only the terms which are related to friction slope will change the sign. In this case these two terms are related.

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Algebraic Form

Momentum Equation

$Q^2 = |Q|Q$

$$\frac{\partial M_{l,i}}{\partial y_{l,i}} = -1 + D_1 \frac{2Q_{l,i}^2}{A_{l,i}^3} \frac{dA}{dy} \Big|_{l,i} - D_2 \left[\frac{2Q_{l,i}|Q_{l,i}|}{A_{l,i}^3 R_{l,i}^{\frac{4}{3}}} \frac{dA}{dy} \Big|_{l,i} + \frac{4Q_{l,i}|Q_{l,i}|}{3A_{l,i}^2 R_{l,i}^{\frac{7}{3}}} \frac{dR}{dy} \Big|_{l,i} \right]$$

$$\frac{\partial M_{l,i}}{\partial Q_{l,i}} = -D_1 \frac{2Q_{l,i}}{A_{l,i}^3} + D_2 \frac{2|Q_{l,i}|}{A_{l,i}^2 R_{l,i}^{\frac{4}{3}}}$$


$$\frac{\partial M_{l,i}}{\partial y_{l,i+1}} = 1 - D_1 \frac{2Q_{l,i+1}^2}{A_{l,i+1}^3} \frac{dA}{dy} \Big|_{l,i+1} - D_2 \left[\frac{2Q_{l,i+1}|Q_{l,i+1}|}{A_{l,i+1}^3 R_{l,i+1}^{\frac{4}{3}}} \frac{dA}{dy} \Big|_{l,i+1} + \frac{4Q_{l,i+1}|Q_{l,i+1}|}{3A_{l,i+1}^2 R_{l,i+1}^{\frac{7}{3}}} \frac{dR}{dy} \Big|_{l,i+1} \right]$$

$$\frac{\partial M_{l,i}}{\partial Q_{l,i+1}} = D_1 \frac{2Q_{l,i+1}}{A_{l,i+1}^3} + D_2 \frac{2|Q_{l,i+1}|}{A_{l,i+1}^2 R_{l,i+1}^{\frac{4}{3}}}$$

with

$$D_1 = \frac{\alpha l}{2g} \quad \text{and} \quad D_2 = \frac{1}{2} n_l^2 \Delta x_l$$

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In the next case for the derivative calculation of ML i with respect to QL i we will consider this mod value here.

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Algebraic Form

Momentum Equation

$Q^2 = |Q|Q$

$$\frac{\partial M_{l,i}}{\partial y_{l,i}} = -1 + D_1 \frac{2Q_{l,i}^2}{A_{l,i}^3} \frac{dA}{dy} \Big|_{l,i} - D_2 \left[\frac{2Q_{l,i}|Q_{l,i}|}{A_{l,i}^3 R_{l,i}^{\frac{4}{3}}} \frac{dA}{dy} \Big|_{l,i} + \frac{4Q_{l,i}|Q_{l,i}|}{3A_{l,i}^2 R_{l,i}^{\frac{7}{3}}} \frac{dR}{dy} \Big|_{l,i} \right]$$

$$\frac{\partial M_{l,i}}{\partial Q_{l,i}} = -D_1 \frac{2Q_{l,i}}{A_{l,i}^3} + D_2 \frac{2|Q_{l,i}|}{A_{l,i}^2 R_{l,i}^{\frac{4}{3}}}$$


$$\frac{\partial M_{l,i}}{\partial y_{l,i+1}} = 1 - D_1 \frac{2Q_{l,i+1}^2}{A_{l,i+1}^3} \frac{dA}{dy} \Big|_{l,i+1} - D_2 \left[\frac{2Q_{l,i+1}|Q_{l,i+1}|}{A_{l,i+1}^3 R_{l,i+1}^{\frac{4}{3}}} \frac{dA}{dy} \Big|_{l,i+1} + \frac{4Q_{l,i+1}|Q_{l,i+1}|}{3A_{l,i+1}^2 R_{l,i+1}^{\frac{7}{3}}} \frac{dR}{dy} \Big|_{l,i+1} \right]$$

$$\frac{\partial M_{l,i}}{\partial Q_{l,i+1}} = D_1 \frac{2Q_{l,i+1}}{A_{l,i+1}^3} + D_2 \frac{2|Q_{l,i+1}|}{A_{l,i+1}^2 R_{l,i+1}^{\frac{4}{3}}}$$

with

$$D_1 = \frac{\alpha l}{2g} \quad \text{and} \quad D_2 = \frac{1}{2} n_l^2 \Delta x_l$$

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But for first term we are not changing any or we are not taking any absolute operator here. And yL i plus 1 also first term we are not changing that but for the second term we are changing it. So in this case we will get another QL i i plus 1. So obviously this term will change to mod value of Q.

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Algebraic Form Momentum Equation

$$\frac{\partial M_{l,i}}{\partial y_{l,i}} = -1 + D_1 \frac{2Q_{l,i}^2}{A_{l,i}^3} \frac{dA}{dy} \Big|_{l,i} - D_2 \left[\frac{2Q_{l,i}|Q_{l,i}|}{A_{l,i}^3 R_{l,i}^{\frac{4}{3}}} \frac{dA}{dy} \Big|_{l,i} + \frac{4Q_{l,i}|Q_{l,i}|}{3A_{l,i}^2 R_{l,i}^{\frac{5}{3}}} \frac{dR}{dy} \Big|_{l,i} \right]$$

$$\frac{\partial M_{l,i}}{\partial Q_{l,i}} = -D_1 \frac{2Q_{l,i}}{A_{l,i}^3} + D_2 \frac{2|Q_{l,i}|}{A_{l,i}^2 R_{l,i}^{\frac{4}{3}}}$$

$$\frac{\partial M_{l,i}}{\partial y_{l,i+1}} = 1 - D_1 \frac{2Q_{l,i+1}^2}{A_{l,i+1}^3} \frac{dA}{dy} \Big|_{l,i+1} - D_2 \left[\frac{2Q_{l,i+1}|Q_{l,i+1}|}{A_{l,i+1}^3 R_{l,i+1}^{\frac{4}{3}}} \frac{dA}{dy} \Big|_{l,i+1} + \frac{4Q_{l,i+1}|Q_{l,i+1}|}{3A_{l,i+1}^2 R_{l,i+1}^{\frac{5}{3}}} \frac{dR}{dy} \Big|_{l,i+1} \right]$$

$$\frac{\partial M_{l,i}}{\partial Q_{l,i+1}} = D_1 \frac{2Q_{l,i+1}}{A_{l,i+1}^3} + D_2 \frac{2|Q_{l,i+1}|}{A_{l,i+1}^2 R_{l,i+1}^{\frac{4}{3}}}$$

with

$$D_1 = \frac{\alpha l}{2g} \quad \text{and} \quad D_2 = \frac{1}{2} n_l^2 \Delta x_l$$

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Now so compared to our previous one what are the changes in this derivative calculation? So changed terms are 1, 2, 3, 4, 5 and 6. These 6 terms which are related to friction slope will change those terms.

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Algebraic Form Momentum Equation

$$\frac{\partial M_{l,i}}{\partial y_{l,i}} = -1 + D_1 \frac{2Q_{l,i}^2}{A_{l,i}^3} \frac{dA}{dy} \Big|_{l,i} - D_2 \left[\frac{2Q_{l,i}|Q_{l,i}|}{A_{l,i}^3 R_{l,i}^{\frac{4}{3}}} \frac{dA}{dy} \Big|_{l,i} + \frac{4Q_{l,i}|Q_{l,i}|}{3A_{l,i}^2 R_{l,i}^{\frac{5}{3}}} \frac{dR}{dy} \Big|_{l,i} \right]$$

$$\frac{\partial M_{l,i}}{\partial Q_{l,i}} = -D_1 \frac{2Q_{l,i}}{A_{l,i}^3} + D_2 \frac{2|Q_{l,i}|}{A_{l,i}^2 R_{l,i}^{\frac{4}{3}}}$$

$$\frac{\partial M_{l,i}}{\partial y_{l,i+1}} = 1 - D_1 \frac{2Q_{l,i+1}^2}{A_{l,i+1}^3} \frac{dA}{dy} \Big|_{l,i+1} - D_2 \left[\frac{2Q_{l,i+1}|Q_{l,i+1}|}{A_{l,i+1}^3 R_{l,i+1}^{\frac{4}{3}}} \frac{dA}{dy} \Big|_{l,i+1} + \frac{4Q_{l,i+1}|Q_{l,i+1}|}{3A_{l,i+1}^2 R_{l,i+1}^{\frac{5}{3}}} \frac{dR}{dy} \Big|_{l,i+1} \right]$$

$$\frac{\partial M_{l,i}}{\partial Q_{l,i+1}} = D_1 \frac{2Q_{l,i+1}}{A_{l,i+1}^3} + D_2 \frac{2|Q_{l,i+1}|}{A_{l,i+1}^2 R_{l,i+1}^{\frac{4}{3}}}$$

with

$$D_1 = \frac{\alpha l}{2g} \quad \text{and} \quad D_2 = \frac{1}{2} n_l^2 \Delta x_l$$

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For trapezoidal channel section this is dA by dy but we are considering m1 m2 (ze) zero. So obviously it will be dR by dy. So obviously we will get the simplified equation for rectangular case.

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Trapezoidal Section

For trapezoidal channel cross-section,

$$\frac{dA}{dy} = B + (m_1 + m_2)y$$

$$\frac{dR}{dy} = \frac{T}{P} - \frac{R}{P} \frac{dP}{dy}$$

with


$$T = B + (m_1 + m_2)y$$

$$P = B + \left(\sqrt{1 + m_1^2} + \sqrt{1 + m_2^2} \right) y$$

$$R = \frac{A}{P}$$

$$\frac{dP}{dy} = \left(\sqrt{1 + m_1^2} + \sqrt{1 + m_2^2} \right)$$

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Now this is the algebraic form. We have talked about our continuity momentum equation discretization. But we have not talked about the discretization of individual internal junctions and our external junction boundary conditions. So we have all total six junctions. In this case the outer one starting from 1 this is 2 we have 3, 4, 5 and 6. So obviously this is our condition in this case. So one by one let us consider the conditions for different junctions.

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Algebraic Form




In general form, continuity and momentum equations can be written as,

$$\frac{\partial C_{l,i}}{\partial y_{l,i}} \Delta y_{l,i} + \frac{\partial C_{l,i}}{\partial Q_{l,i}} \Delta Q_{l,i} + \frac{\partial C_{l,i}}{\partial y_{l,i+1}} \Delta y_{l,i+1} + \frac{\partial C_{l,i}}{\partial Q_{l,i+1}} \Delta Q_{l,i+1} = -C_{l,i}$$

$$\frac{\partial M_{l,i}}{\partial y_{l,i}} \Delta y_{l,i} + \frac{\partial M_{l,i}}{\partial Q_{l,i}} \Delta Q_{l,i} + \frac{\partial M_{l,i}}{\partial y_{l,i+1}} \Delta y_{l,i+1} + \frac{\partial M_{l,i}}{\partial Q_{l,i+1}} \Delta Q_{l,i+1} = -M_{l,i}$$

$$\forall i \in \{1, \dots, N_l\}$$

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So this is junction number 3, 4, 5 and 6. These are our channel reach numbers 1, 2, 3, 4, 5, 6, 8 and this is 7. So for channel reach number 1 what condition for discharge we are getting? For discharge or continuity we are getting that this Q_u this amount we are adding and this Q_1

this is being extracted from the system. So Q1 1 this is negative minus Q2 1 this should be equal to zero.

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Algebraic Form

In general form, continuity and momentum equations can be written as,

$$\frac{\partial C_{l,i}}{\partial y_{l,i}} \Delta y_{l,i} + \frac{\partial C_{l,i}}{\partial Q_{l,i}} \Delta Q_{l,i} + \frac{\partial C_{l,i}}{\partial y_{l,i+1}} \Delta y_{l,i+1} + \frac{\partial C_{l,i}}{\partial Q_{l,i+1}} \Delta Q_{l,i+1} = -C_{l,i}$$

$$\frac{\partial M_{l,i}}{\partial y_{l,i}} \Delta y_{l,i} + \frac{\partial M_{l,i}}{\partial Q_{l,i}} \Delta Q_{l,i} + \frac{\partial M_{l,i}}{\partial y_{l,i+1}} \Delta y_{l,i+1} + \frac{\partial M_{l,i}}{\partial Q_{l,i+1}} \Delta Q_{l,i+1} = -M_{l,i}$$

$\forall i \in \{1, \dots, N_l\}$

1: $Q_1 - Q_3 - Q_6 = 0$

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So Qu amount is added into the system and this amount is being extracted from the system as we have considered our flow directions from 1 to 3 and 1 to 6. Next condition because we are not considering the difference in elevation for this junction and our channel reaches. So for y we will have one condition. What is that? That is y1 1 this should be equal to y2 1. That means first section depth at channel 1 and channel 2 should be same provided that we have same elevation for these two channel sections.

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Algebraic Form

In general form, continuity and momentum equations can be written as,

$$\frac{\partial C_{l,i}}{\partial y_{l,i}} \Delta y_{l,i} + \frac{\partial C_{l,i}}{\partial Q_{l,i}} \Delta Q_{l,i} + \frac{\partial C_{l,i}}{\partial y_{l,i+1}} \Delta y_{l,i+1} + \frac{\partial C_{l,i}}{\partial Q_{l,i+1}} \Delta Q_{l,i+1} = -C_{l,i}$$

$$\frac{\partial M_{l,i}}{\partial y_{l,i}} \Delta y_{l,i} + \frac{\partial M_{l,i}}{\partial Q_{l,i}} \Delta Q_{l,i} + \frac{\partial M_{l,i}}{\partial y_{l,i+1}} \Delta y_{l,i+1} + \frac{\partial M_{l,i}}{\partial Q_{l,i+1}} \Delta Q_{l,i+1} = -M_{l,i}$$

$\forall i \in \{1, \dots, N_l\}$

1: $Q_1 - Q_3 - Q_6 = 0$

$y_{1,1} = y_{2,1}$

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If we consider the flow or the conditions for section or junction node 2 we will have Q7 this is N7 plus 1, this is added. Again Q8 N8 plus 1 this amount is added to the junction. This is for junction number 2 and extraction is Qd amount. So this is the continuity condition.

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Algebraic Form

In general form, continuity and momentum equations can be written as,

$$\frac{\partial C_{l,i}}{\partial y_{l,i}} \Delta y_{l,i} + \frac{\partial C_{l,i}}{\partial Q_{l,i}} \Delta Q_{l,i} + \frac{\partial C_{l,i}}{\partial y_{l,i+1}} \Delta y_{l,i+1} + \frac{\partial C_{l,i}}{\partial Q_{l,i+1}} \Delta Q_{l,i+1} = -C_{l,i}$$

$$\frac{\partial M_{l,i}}{\partial y_{l,i}} \Delta y_{l,i} + \frac{\partial M_{l,i}}{\partial Q_{l,i}} \Delta Q_{l,i} + \frac{\partial M_{l,i}}{\partial y_{l,i+1}} \Delta y_{l,i+1} + \frac{\partial M_{l,i}}{\partial Q_{l,i+1}} \Delta Q_{l,i+1} = -M_{l,i}$$

$$\forall i \in \{1, \dots, N_l\}$$

2: $Q_{7, N_7+1} + Q_{8, N_8+1} - Q_d = 0$

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Again you will have another condition that is y7 N7 plus 1 this is equal to yd and y7 N7 plus 1 equals to y8 N8 plus 1. That means we have two conditions for y and one condition for discharge at the downstream external junction boundary.

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Algebraic Form

In general form, continuity and momentum equations can be written as,

$$\frac{\partial C_{l,i}}{\partial y_{l,i}} \Delta y_{l,i} + \frac{\partial C_{l,i}}{\partial Q_{l,i}} \Delta Q_{l,i} + \frac{\partial C_{l,i}}{\partial y_{l,i+1}} \Delta y_{l,i+1} + \frac{\partial C_{l,i}}{\partial Q_{l,i+1}} \Delta Q_{l,i+1} = -C_{l,i}$$

$$\frac{\partial M_{l,i}}{\partial y_{l,i}} \Delta y_{l,i} + \frac{\partial M_{l,i}}{\partial Q_{l,i}} \Delta Q_{l,i} + \frac{\partial M_{l,i}}{\partial y_{l,i+1}} \Delta y_{l,i+1} + \frac{\partial M_{l,i}}{\partial Q_{l,i+1}} \Delta Q_{l,i+1} = -M_{l,i}$$

$$\forall i \in \{1, \dots, N_l\}$$

2: 1 $Q_{7, N_7+1} + Q_{8, N_8+1} - Q_d = 0$

2 $\begin{cases} Y_{7, N_7+1} = Y_d \\ Y_{7, N_7+1} = Y_{8, N_8+1} \end{cases}$

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Next is for junction number or junction node number 3. For 3 we have discharge from 1. From 1 this is $N_1 + 1$ which is being added there minus $Q_3 + 1$ this is being extracted then $Q_4 + 1$ this is also extracted from this node. So this continuity condition should be satisfied and we will have $y_1 N_1 + 1$ equal to $y_1 y_3 + 1$ and $y_1 N_1 + 1$ this is equals to $y_4 + 1$.

So we will have two depth related condition or energy condition, one our continuity condition. Obviously in this case I am considering our junction loss thing. in junction we do not have any loss condition.

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Algebraic Form

In general form, continuity and momentum equations can be written as,

$$\frac{\partial C_{l,i}}{\partial y_{l,i}} \Delta y_{l,i} + \frac{\partial C_{l,i}}{\partial Q_{l,i}} \Delta Q_{l,i} + \frac{\partial C_{l,i}}{\partial y_{l,i+1}} \Delta y_{l,i+1} + \frac{\partial C_{l,i}}{\partial Q_{l,i+1}} \Delta Q_{l,i+1} = -C_{l,i}$$

$$\frac{\partial M_{l,i}}{\partial y_{l,i}} \Delta y_{l,i} + \frac{\partial M_{l,i}}{\partial Q_{l,i}} \Delta Q_{l,i} + \frac{\partial M_{l,i}}{\partial y_{l,i+1}} \Delta y_{l,i+1} + \frac{\partial M_{l,i}}{\partial Q_{l,i+1}} \Delta Q_{l,i+1} = -M_{l,i}$$

$\forall i \in \{1, \dots, N_j\}$

3: 1 { $Q_{1,N_1+1} - Q_{3,1} - Q_{4,1} = 0$

2 { $y_{1,N_1+1} = y_{3,1}$
 $y_{1,N_1+1} = y_{4,1}$

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If I consider the channel 4 so obviously for 4 or junction node 4 we have three. That means $Q_3 N_3$ plus 1 this is being added. Also the amount which is added is $Q_5 N_5$ plus 1 minus Q_7 1. So this amount is extracted. So obviously in this case we will have $Q_3 N_3$ plus 1 or this is for depth condition. So obviously this will be y equal to $y_5 N_5$ plus 1 and $y_3 N_3$ plus 1 equal to y_7 1. So two conditions here and one for discharge.

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Algebraic Form

In general form, continuity and momentum equations can be written as,

$$\frac{\partial C_{l,i}}{\partial y_{l,i}} \Delta y_{l,i} + \frac{\partial C_{l,i}}{\partial Q_{l,i}} \Delta Q_{l,i} + \frac{\partial C_{l,i}}{\partial y_{l,i+1}} \Delta y_{l,i+1} + \frac{\partial C_{l,i}}{\partial Q_{l,i+1}} \Delta Q_{l,i+1} = -C_{l,i}$$

$$\frac{\partial M_{l,i}}{\partial y_{l,i}} \Delta y_{l,i} + \frac{\partial M_{l,i}}{\partial Q_{l,i}} \Delta Q_{l,i} + \frac{\partial M_{l,i}}{\partial y_{l,i+1}} \Delta y_{l,i+1} + \frac{\partial M_{l,i}}{\partial Q_{l,i+1}} \Delta Q_{l,i+1} = -M_{l,i}$$

$\forall i \in \{1, \dots, N_j\}$

4: 1 { $Q_{3,N_3+1} + Q_{5,N_5+1} - Q_{7,1} = 0$

2 { $y_{3,N_3+1} = y_{5,N_5+1}$
 $y_{3,N_3+1} = y_{7,1}$

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Similarly for junction node 5 this amount is added $Q_4 N_4$ plus 1. Also $Q_6 N_6$ plus 1 minus Q_5 1. This is equal to zero. So $y_4 N_4$ plus 1 this should be equal to $y_6 N_6$ plus 1 and $y_4 N_4$ plus 1 equal to y_5 1. So 2, 1 again 3.

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And the last junction that is 6. For 6 this is 6 this is connected to 1, 6 and 8. So in that case this is 2, 6 and 8.

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In this case we will get $Q_2 N_2 + 1 - Q_6 - 1 - Q_8 = 0$. And this will be $y_2 N_2 + 1 = y_6$, this is $y_2 N_2 + 1 = y_8$. So these are the conditions. Obviously if you want to get the elements of Jacobian matrix for this one we can differentiate with respect to the variables in this case.

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Algebraic Form

In general form, continuity and momentum equations can be written as,

$$\frac{\partial C_{l,i}}{\partial y_{l,i}} \Delta y_{l,i} + \frac{\partial C_{l,i}}{\partial Q_{l,i}} \Delta Q_{l,i} + \frac{\partial C_{l,i}}{\partial y_{l,i+1}} \Delta y_{l,i+1} + \frac{\partial C_{l,i}}{\partial Q_{l,i+1}} \Delta Q_{l,i+1} = -C_{l,i}$$

$$\frac{\partial M_{l,i}}{\partial y_{l,i}} \Delta y_{l,i} + \frac{\partial M_{l,i}}{\partial Q_{l,i}} \Delta Q_{l,i} + \frac{\partial M_{l,i}}{\partial y_{l,i+1}} \Delta y_{l,i+1} + \frac{\partial M_{l,i}}{\partial Q_{l,i+1}} \Delta Q_{l,i+1} = -M_{l,i}$$

$\forall i \in \{1, \dots, N_l\}$

6: $\theta_{2, N_2+1} - \theta_{6,1} - \theta_{2,1} = 0$
 $y_{2,1}$

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So obviously all these values are unknown. So we have this is 1, 2, 3, 4, 5 and 6, total 6 unknowns are there in these three equations. So we can write the elements of the Jacobian matrix from this calculation.

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Algebraic Form

In general form, continuity and momentum equations can be written as,

$$\frac{\partial C_{l,i}}{\partial y_{l,i}} \Delta y_{l,i} + \frac{\partial C_{l,i}}{\partial Q_{l,i}} \Delta Q_{l,i} + \frac{\partial C_{l,i}}{\partial y_{l,i+1}} \Delta y_{l,i+1} + \frac{\partial C_{l,i}}{\partial Q_{l,i+1}} \Delta Q_{l,i+1} = -C_{l,i}$$

$$\frac{\partial M_{l,i}}{\partial y_{l,i}} \Delta y_{l,i} + \frac{\partial M_{l,i}}{\partial Q_{l,i}} \Delta Q_{l,i} + \frac{\partial M_{l,i}}{\partial y_{l,i+1}} \Delta y_{l,i+1} + \frac{\partial M_{l,i}}{\partial Q_{l,i+1}} \Delta Q_{l,i+1} = -M_{l,i}$$

$\forall i \in \{1, \dots, N_l\}$

6: $1 \left\{ \theta_{2, N_2+1} - \theta_{6,1} - \theta_{2,1} = 0 \right.$
 $2 \left\{ \begin{aligned} y_{2, N_2+1} &= y_{6,1} \\ y_{2, N_2+1} &= y_{2,1} \end{aligned} \right.$

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Now after getting these elements of the Jacobian matrix now we can construct the total Jacobian matrix and we can get the solution out of that. But for implementation purpose we need to input certain information in certain format. First is channel chl inf which is channel information.

First column is channel reach number, second column is total length of the channel, third one is the width of the channel, fourth and fifth one these are m1 and m2 values, this sixth one this is segment length, this is n, this is S not and the last two these are junction node number 1 and junction node number 2 for any channel reach. That means channel is connected from 1 to 3, 1 to 6.

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Program Implementation
Configuration 1

$$chL.inf = \begin{bmatrix} 1 & 200 & 30 & 0 & 0 & 50 & 0.0130 & 0.0005 & 1 & 3 \\ 2 & 200 & 40 & 0 & 0 & 50 & 0.0130 & 0.0005 & 1 & 6 \\ 3 & 200 & 20 & 0 & 0 & 50 & 0.0120 & 0.0005 & 3 & 4 \\ 4 & 100 & 20 & 0 & 0 & 25 & 0.0140 & 0.0005 & 3 & 5 \\ 5 & 100 & 20 & 0 & 0 & 25 & 0.0130 & 0.0005 & 5 & 4 \\ 6 & 100 & 25 & 0 & 0 & 25 & 0.0130 & 0.0005 & 6 & 5 \\ 7 & 100 & 30 & 0 & 0 & 25 & 0.0140 & 0.0005 & 4 & 2 \\ 8 & 300 & 50 & 0 & 0 & 75 & 0.0140 & 0.0005 & 6 & 2 \end{bmatrix}$$

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Next information we required is that junction information and in junction information we can include the boundary condition. And in the last column we are providing this elevation values for different junctions.

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Program Implementation
Configuration 1

$$chL.inf = \begin{bmatrix} 1 & 200 & 30 & 0 & 0 & 50 & 0.0130 & 0.0005 & 1 & 3 \\ 2 & 200 & 40 & 0 & 0 & 50 & 0.0130 & 0.0005 & 1 & 6 \\ 3 & 200 & 20 & 0 & 0 & 50 & 0.0120 & 0.0005 & 3 & 4 \\ 4 & 100 & 20 & 0 & 0 & 25 & 0.0140 & 0.0005 & 3 & 5 \\ 5 & 100 & 20 & 0 & 0 & 25 & 0.0130 & 0.0005 & 5 & 4 \\ 6 & 100 & 25 & 0 & 0 & 25 & 0.0130 & 0.0005 & 6 & 5 \\ 7 & 100 & 30 & 0 & 0 & 25 & 0.0140 & 0.0005 & 4 & 2 \\ 8 & 300 & 50 & 0 & 0 & 75 & 0.0140 & 0.0005 & 6 & 2 \end{bmatrix}$$

$$jun.inf = \begin{bmatrix} -99999 & 250 & 0.25 \\ 5 & -250 & 0 \\ -99999 & -99999 & 0.15 \\ -99999 & -99999 & 0.05 \\ -99999 & -99999 & 0.10 \\ -99999 & -99999 & 0.15 \end{bmatrix}$$

$$jun.con = \begin{bmatrix} 2 & 1 & 2 & 0 \\ 2 & -7 & -8 & 0 \\ 3 & -1 & 3 & 4 \\ 3 & -3 & -5 & 7 \\ 3 & -4 & -6 & 5 \\ 3 & -2 & 6 & 8 \end{bmatrix}$$

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Now junction connectivity, this is another important parameter. For junction 1 this is starting from junction 1, 2, 3, 4, 5, 6. For 6 junctions we have this condition. So for junction 1 we have two channel reaches connected to it that is channel reach 1 and 2. So this is the starting sections for channel reach 1 and 2.

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The screenshot shows a software interface with a menu bar (Problem Statement, Problem Definition, Discretization, References) and a header for I.I.T. Kharagpur. The main content is titled "Program Implementation Configuration 1". It displays two matrices: ch_inf and jun_inf , and a junction connectivity matrix jun_con .

ch_inf matrix:

1	200	30	0	0	50	0.0130	0.0005	1	3
2	200	40	0	0	50	0.0130	0.0005	1	6
3	200	20	0	0	50	0.0120	0.0005	3	4
4	100	20	0	0	25	0.0140	0.0005	3	5
5	100	20	0	0	25	0.0130	0.0005	5	4
6	100	25	0	0	25	0.0130	0.0005	6	5
7	100	30	0	0	25	0.0140	0.0005	4	2
8	300	50	0	0	75	0.0140	0.0005	6	2

jun_inf matrix:

-99999	250	0.25
5	-250	0
-99999	-99999	0.15
-99999	-99999	0.05
-99999	-99999	0.10
-99999	-99999	0.15

jun_con matrix:

2	1	2	0
2	-7	-8	0
3	-1	3	4
3	-3	-5	7
3	-4	-6	5
3	-2	6	8

The interface also includes a small video feed of a person in the bottom right corner and a footer with "Dr. Anirban Dhar", "NPTEL", and "Computational Hydraulics".

So we have plus 1 and plus 2. For channel reach or junction node 2 we have two connected channel reaches. In this case minus 7 and minus 8 means end sections are connected to this channel reach 2. Channel junction 3 in this one the end section of 1, starting section of 3 and 4 are connected. Channel 4 we have end section of 3, end section of 5 and starting section of 7 that is connected. Channel junction 5 we have again 3 connected channel reaches.

Minus 4 means this is the ending section or end section of channel reach 4, end section of channel reach 6 and starting section of channel reach 5 these are connected. And channel reach or junction node 6 we have 3 connected channel reaches. This is the end section of 2, starting section of 6 and starting section of 8.

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Program Implementation

Configuration 1

$$\text{chLin.f} = \begin{bmatrix} 1 & 200 & 30 & 0 & 0 & 50 & 0.0130 & 0.0005 & 1 & 3 \\ 2 & 200 & 40 & 0 & 0 & 50 & 0.0130 & 0.0005 & 1 & 6 \\ 3 & 200 & 20 & 0 & 0 & 50 & 0.0120 & 0.0005 & 3 & 4 \\ 4 & 100 & 20 & 0 & 0 & 25 & 0.0140 & 0.0005 & 3 & 5 \\ 5 & 100 & 20 & 0 & 0 & 25 & 0.0130 & 0.0005 & 5 & 4 \\ 6 & 100 & 25 & 0 & 0 & 25 & 0.0130 & 0.0005 & 6 & 5 \\ 7 & 100 & 30 & 0 & 0 & 25 & 0.0140 & 0.0005 & 4 & 2 \\ 8 & 300 & 50 & 0 & 0 & 75 & 0.0140 & 0.0005 & 6 & 2 \end{bmatrix}$$
$$\text{jun_inf} = \begin{bmatrix} -99999 & 250 & 0.25 \\ 5 & -250 & 0 \\ -99999 & -99999 & 0.15 \\ -99999 & -99999 & 0.05 \\ -99999 & -99999 & 0.10 \\ -99999 & -99999 & 0.15 \end{bmatrix}$$
$$\text{jun_con} = \begin{bmatrix} 2 & 1 & 2 & 0 \\ 2 & -7 & -8 & 0 \\ 3 & -1 & 3 & 4 \\ 3 & 3 & -5 & 7 \\ 3 & -4 & -6 & 5 \\ 3 & -2 & 6 & 8 \end{bmatrix}$$

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So these three matrixes these are required as input in our program. With this information we can calculate the values.