Computational Hydraulics

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Lecture 40

Steady Channel Flow: Channel Network without Reverse Flow

Welcome to this lecture of the course computational hydraulics. We are in module 4, surface water hydraulics and this lecture is for unit number 5, steady channel flow channel network without reverse flow situation.

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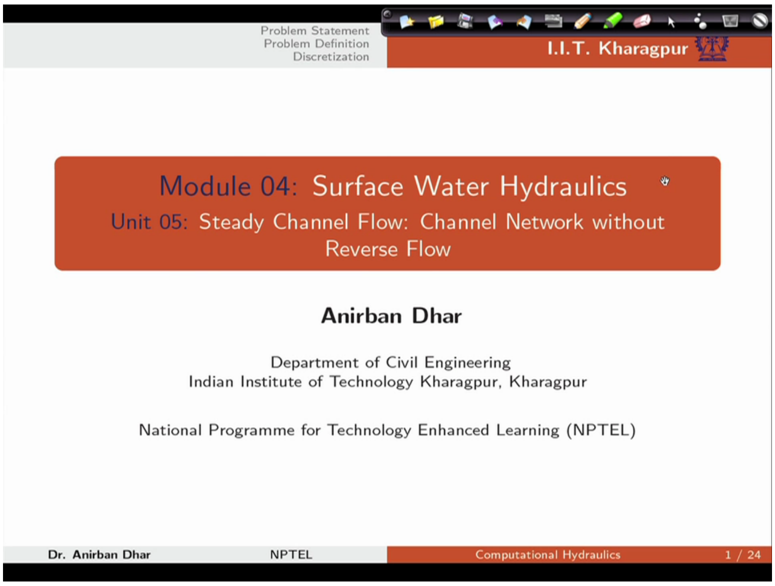

Learning objective of this particular unit. At the end of this unit students will be able to solve steady channel flow for channel network problem without reverse flow situation using implicit method.

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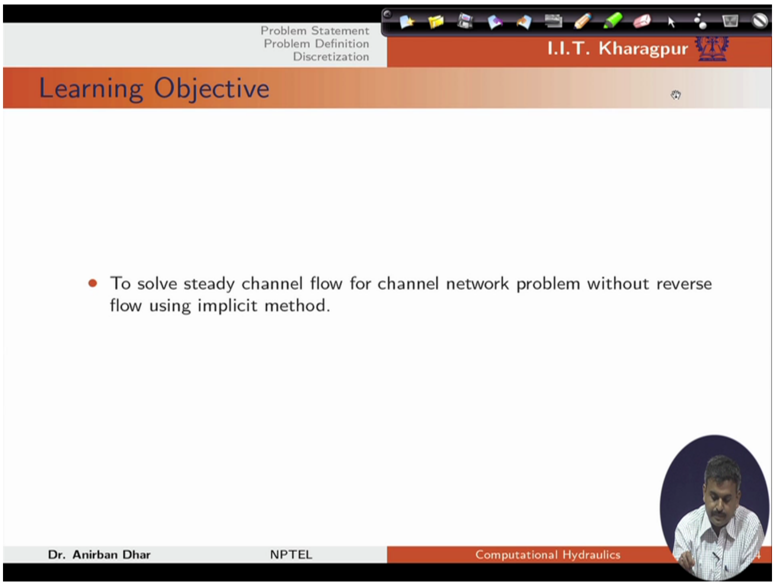

Now problem definition to solution. We have already defined our problem for steady channel flow in terms of continuity equation which is dQ by dx equals to zero and our momentum equation dE by dx is equal to minus SF. With this we have seen how to discretize the governing equation and how to solve this channel flow problem. In the first approach we have solved this problem considering that discharge is constant or discharge is not varying.

So in that case only flow depth was a variable. Now if we consider that both flow depth and discharge are varying for channel sections then we have two variables at each section. Now for any problem we need to find out these two variables and the resulting equation is nonlinear in nature. So we need to solve it using our iterative nonlinear solver or Newton Raphson technique.

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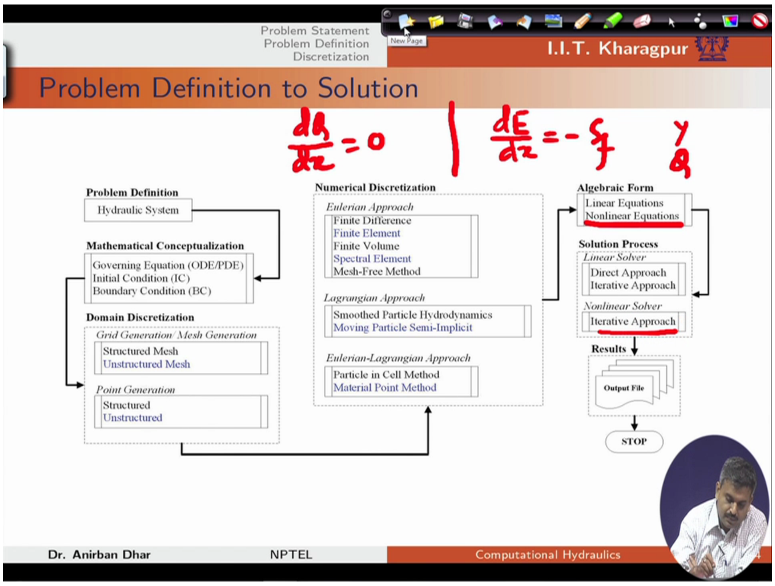

Now for this problem let us define this flow without a reverse flow situation. Let us consider this four channel reach situation. This is channel reach number 1, 2, 3 and 4. We have these four channel reaches. Now for these four channel reaches we have three junctions. One is with red dot that is external boundary condition and these blue dots are internal boundary condition or junction conditions.

So in this case we are considering this red dot as junction because the flow condition is such that one channel is joining at this point and we have another condition of exist or define condition exist for this particular node. So we will consider this red one as junction node but this is external junction and these blue ones are internal junctions.

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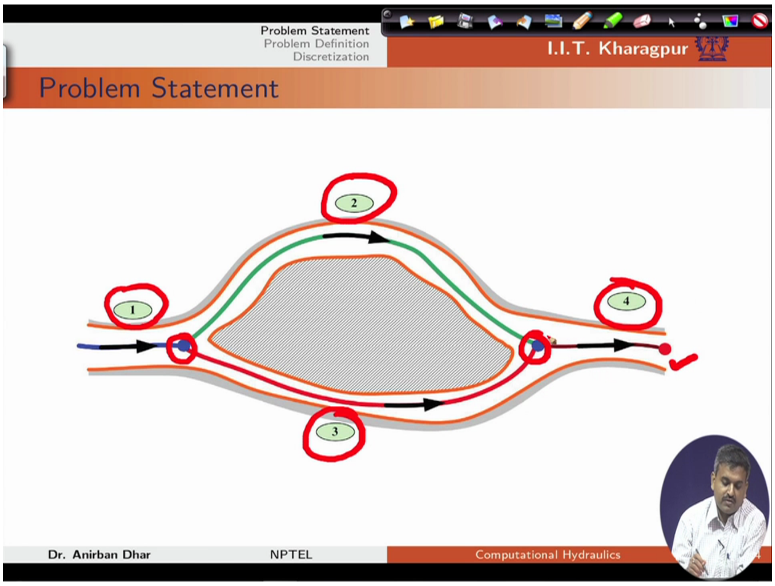

So we have 1, 2, 3, 4 these four channel reaches and for this channel reaches we are considering that for each channel reach we have starting from 1 to N1 plus 1 number of sections. Again 1 to N2 plus 1 number of sections. For third one we have N3 plus 1 number of section and for last one N4 1. So if we add all these sections so all total we have N1 plus N2 plus N3 plus N4 plus 4 total sections into 2. That means by considering y and Q as variables for each section we have these many variables. So this is variable number.

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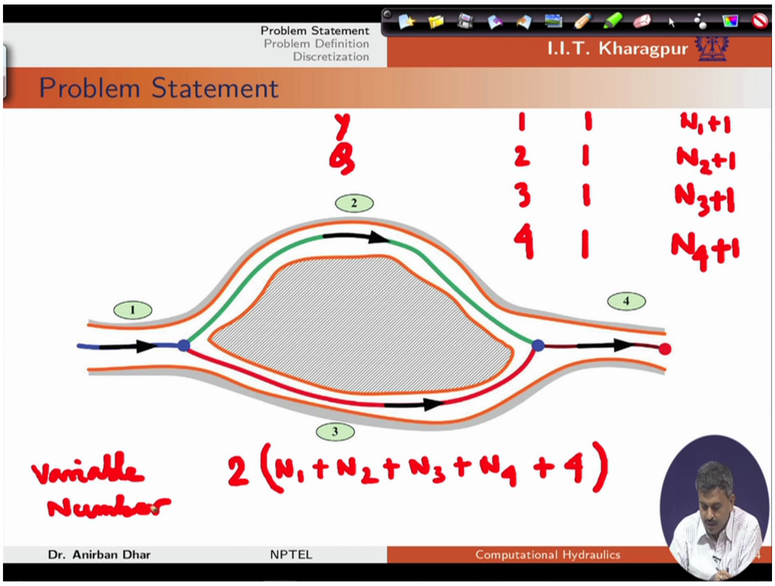

Now to solve this problem we have one continuity, one momentum equation for N1 plus N2 plus N3 plus N4 number of sections or in N1 plus N2 plus N3 plus N4 number of segments. For each segment we can write one continuity, one momentum. So we will have 2 into N1 plus N2 plus N3 plus N4 number of equations. So we need 8 equations more to complete the set. Then equations, so in this case if we consider these blue nodes these are actually our internal boundary conditions.

We will have one continuity and two energy conservation conditions. In this case also we will have one continuity and two energy conservation conditions because we have 3 channels or channel reaches meeting at this point and at this point.

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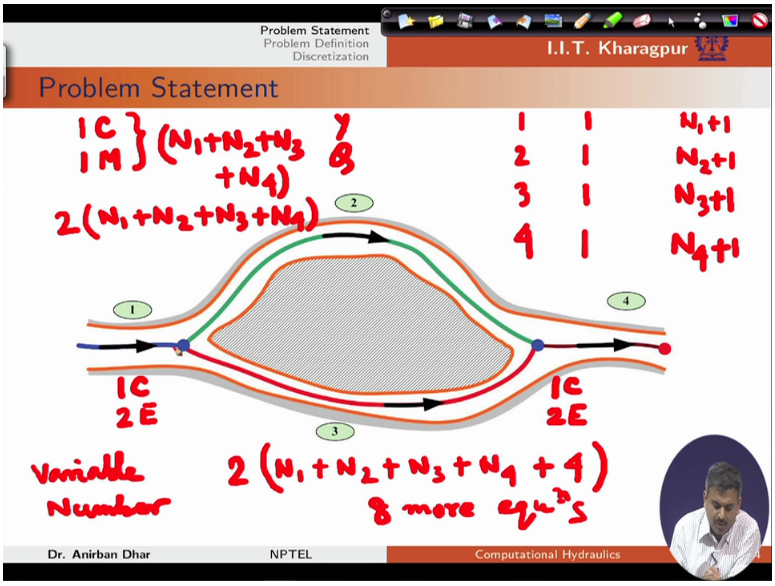

So we will get three equations each from these two internal conditions. So all total we have added six, so still we need two conditions. So how to define these two conditions? Either we can have discharge and depth at downstream specified. So if discharge in downstream is specified, depth in downstream is specified then we can get these final or last two conditions. And then we can solve these equations to get the final desired output that is the values of y and Q for different sections.

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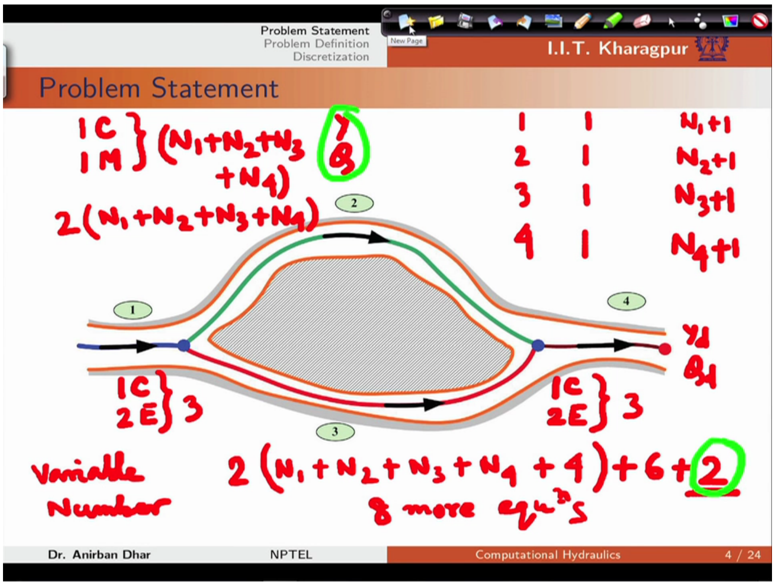

Now in this case we know what is the direction of flow. That is why we are not considering any reversal of flow situation. So flow is from left to right. So starting from channel section 1 at this point and ending at N1 plus 1 at this point. Again the channel 2 is starting at 1 and it is ending at N2 plus 1. Here the channel is starting at 1 so channel 3N plus 1 and channel 4 it is starting at 1 and it is ending at N4 plus 1.

So these are actually local numbering for this problem. Now we need a global in numbering system for our problem because we need to solve these equations simultaneously.

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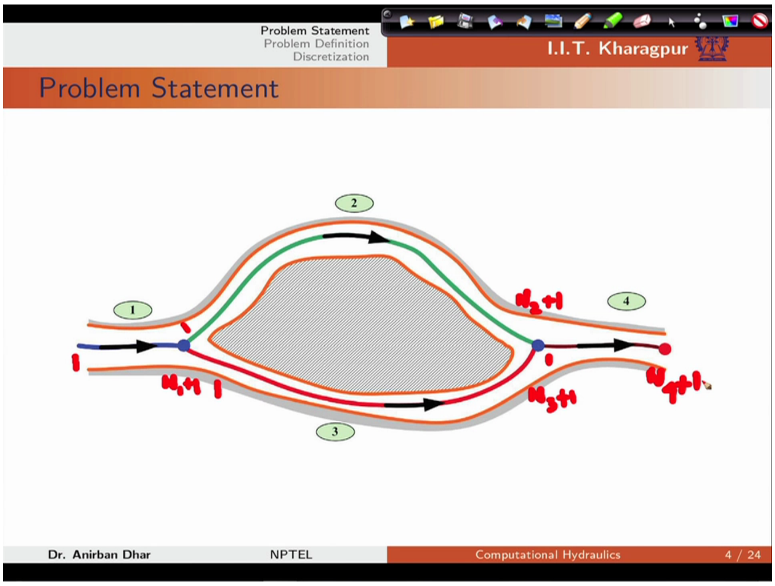

So let us see how we can define those numbering system or numbering scheme. So this is the simplified sketch of our problem. We have node number 1 which is our external boundary condition and this node number 2 and 3 these are actually our internal boundary condition or junction conditions. In this case 1 is meeting at 3, 2 is starting from this node 3 and 3 is again starting from node 3. And both 2 and 3 joining at 2 and 4 starting from this point node 2.

Again at node 1 we have the end point of channel reach 4. So at this point we need to specify this boundary conditions or there can be situation where we have one condition or flow depth condition here and inflow condition at upstream section. Then we need to consider this one also as our load.

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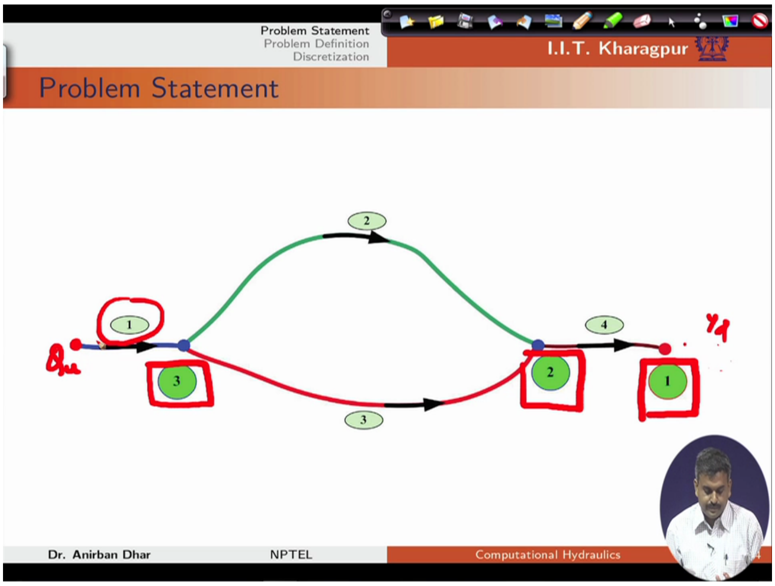

So in this case we are considering that flow depth and discharge both are specified at this downstream section. So this Qv and yd this is our flow depth, these two variables are defined at this red node.

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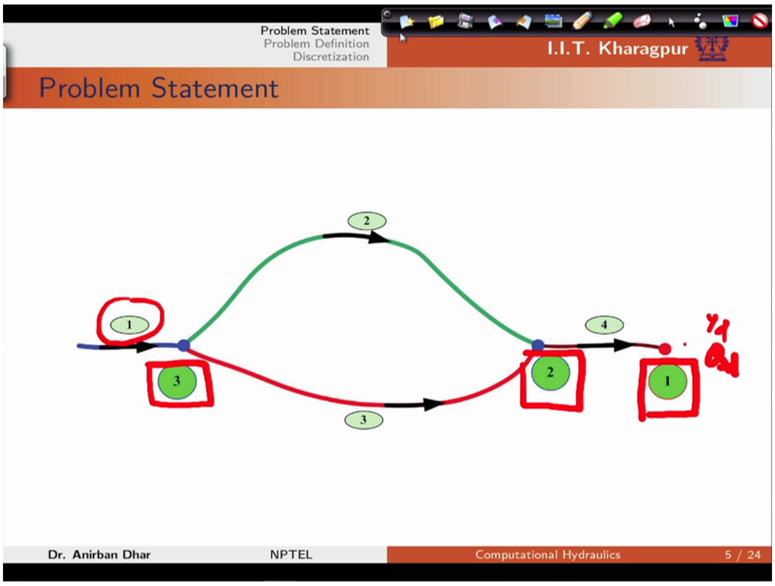

Now if we see our junction conditions so we have junction number 1, junction number 2, junction number 3. For junction 1 we have the end point of fourth channel reach. That is 4 comma N4 plus 1 and for junction 2 we have the end point of channel 2 or end section of channel 3 and starting section of channel 4. And junction 3 again for this one we have end section of channel 1 and starting section of channel 2 and channel 3. Now we can define our equations based on these internal and external junction or boundary conditions.

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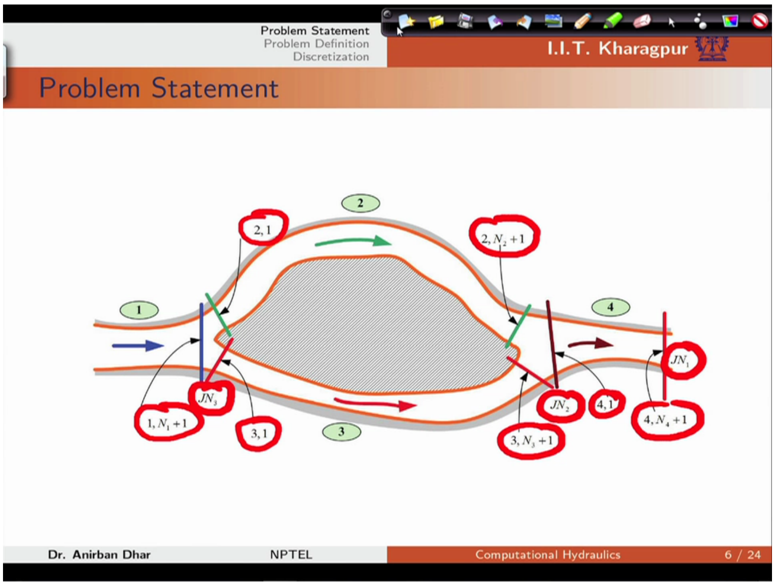

Now let us consider the case where we have trapezoidal channel cross section. In trapezoidal channel cross section we have seen in our gradually varied flow problem if 1 is to m1 and 1 is to m2, this is area this By. So this is the initial area here. And for this triangular sections if we add this m1 plus m2 into y square by 2, this will be the area for remaining portion.

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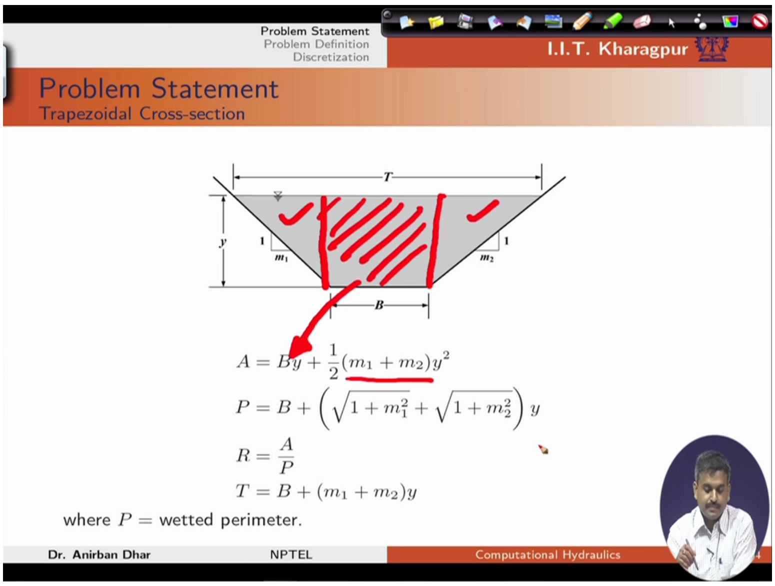

And weighted parameter, this is B and on these two sides we have square root of 1 plus m1 square plus square root of 1 plus m2 square into y. So weighted parameter will be up to this.

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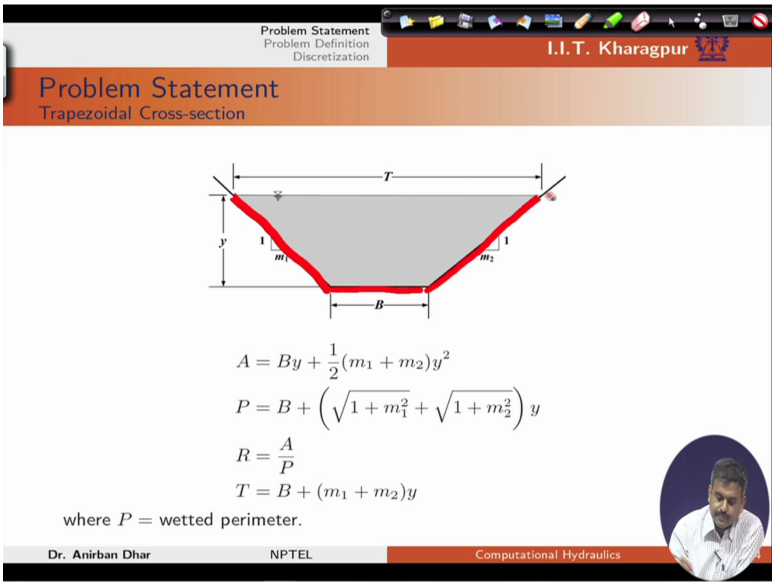

Now we have hydraulic radius which is R is equal to A by P. T is top width, B plus m1 plus m2 into y. We can define the values for our problem.

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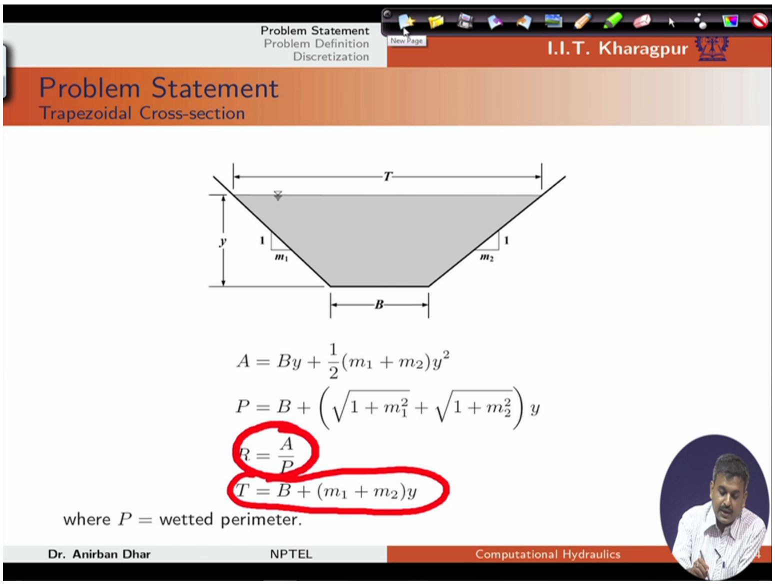

Till now we have defined the junction type, our external boundary condition type, number of channel reaches, number of sections. Now let us consider this problem. With this channel data we have this one. For channel reach 1 we have 100 metres length. So starting from 1 we have this junction. This is another junction and red one this is external boundary condition.

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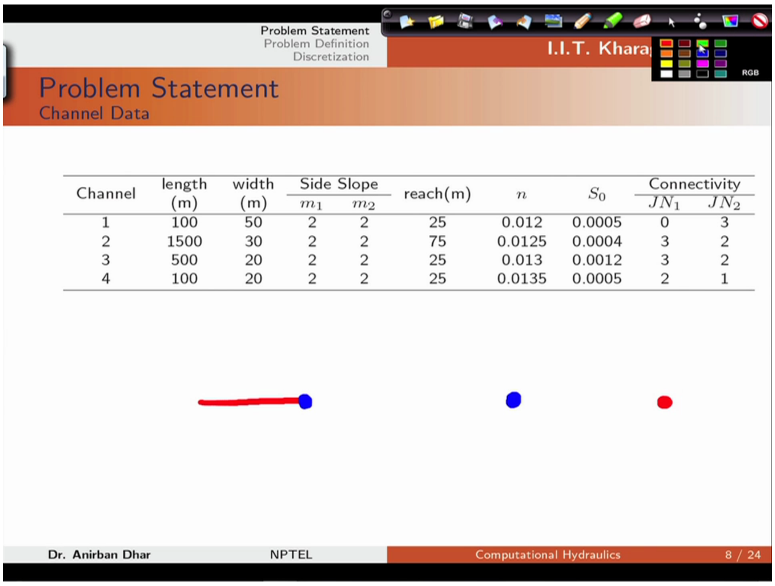

So for this problem we have this is channel reach 2, channel reach 3, channel reach 1 and this is our condition. Maybe last one I can use green so that we can easily identify this one. This is channel reach number 4 and junction condition wise this is 1, this is 2, this is 3.

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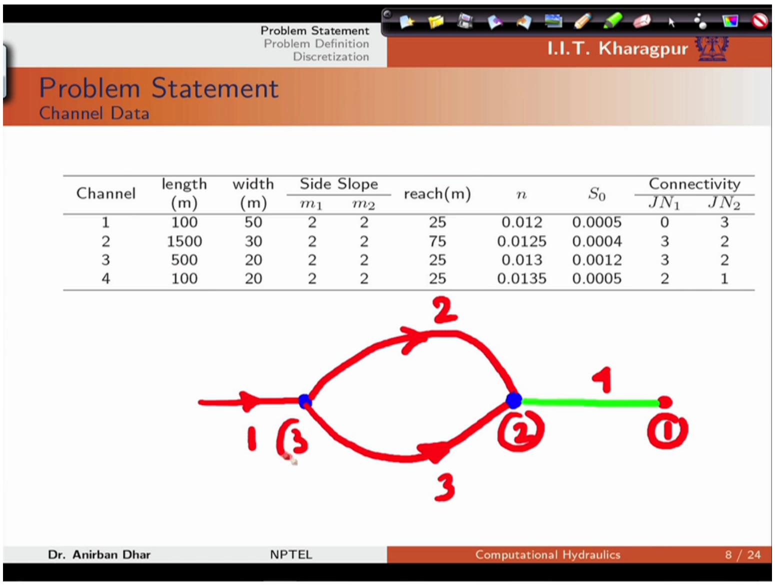

So length of channel 1 this is hundred metres, width is 50 metres that means for trapezoidal section we have width of 50 metres. And this is 1 is to 2. That means we have 2 and 1. Although from drawing it is not exact one.

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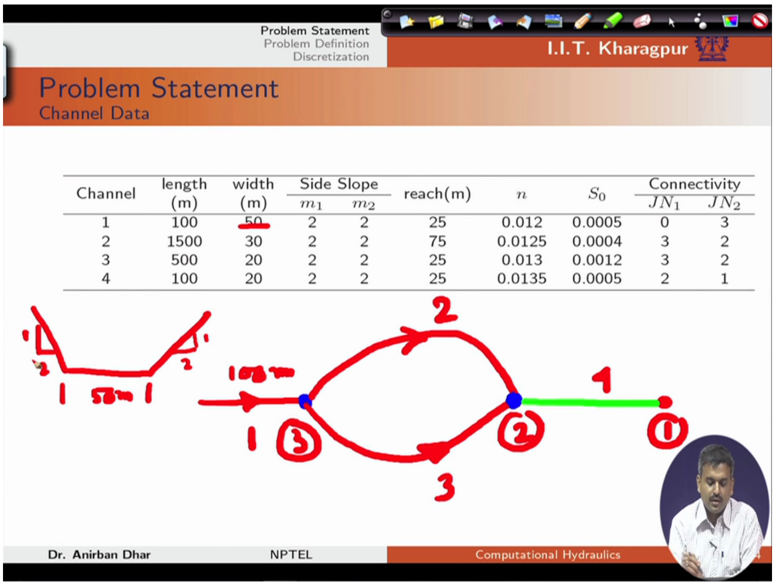

Channel reach length that means it is a segment length for each section. So for first one we have total 100 metres for channel 1. Channel one we have hundred metres. From this hundred metres we have 25 metres as our channel reach length or this is segment length. So we can just draw this is 3, 4 and one side we have this node.

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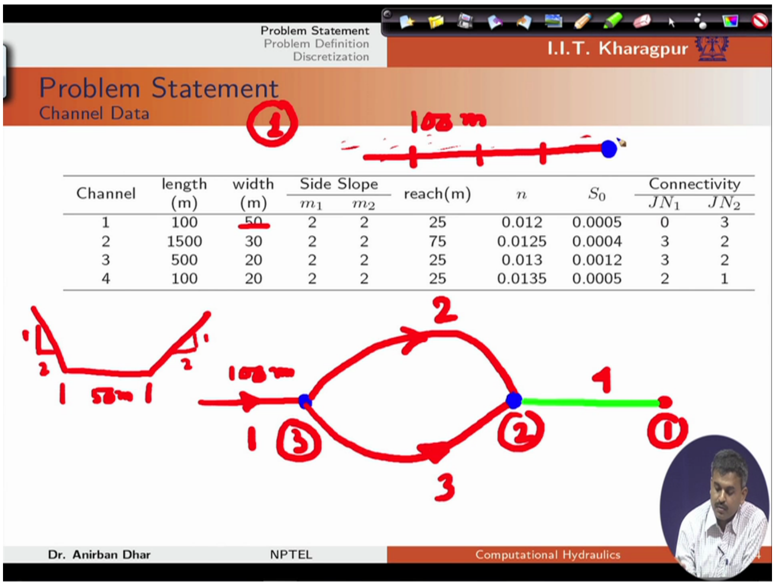

So this is for channel reach 1 starting from section 1, section 2, section 3, section 4 up to section 5. So in this case N1 equals to 4 and N1 plus 1 equals to 5.

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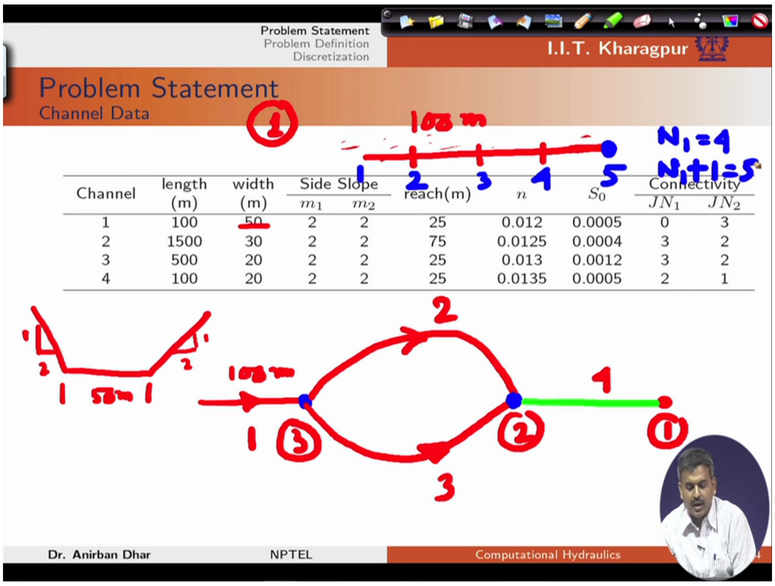

Similarly for other channel reaches or for next one channel reach 2 we have 1500 metres length, channel reach 3 we have 500 metres and channel reach 4 we have 100 metres length. So for channel reach 2 this segment length is 75 metres. That means we have N2 equals to 20 and N2 plus 1 equals to 21 sections for second channel.

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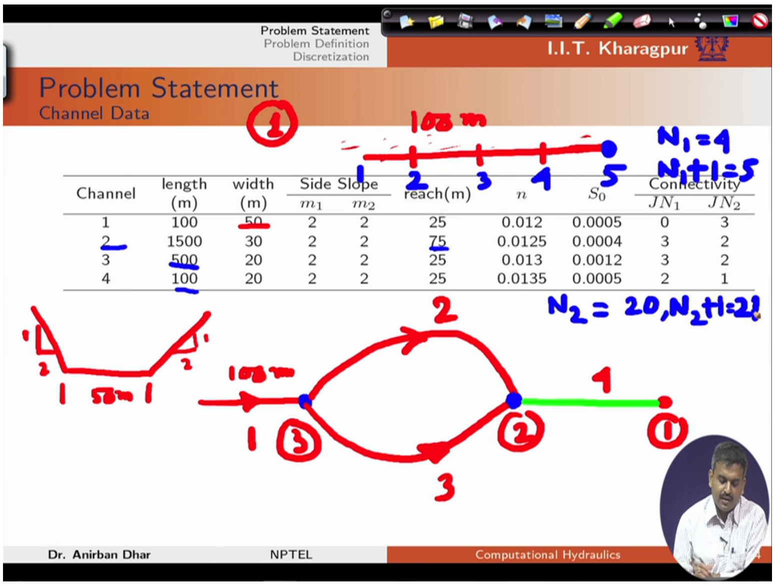

So in this case we have 5 sections. In this case 21. This is 500 that means four sections in 100 metres and again we have 21 sections here. And last one we have 5 sections. That means for this channel reach 1 this end section 5 of channel reach 1. That means 1 5 is joining at this point.

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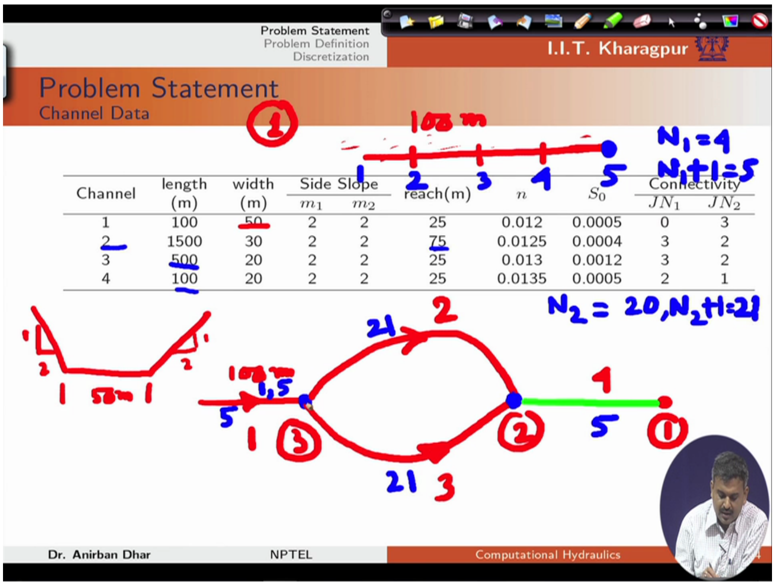

In this case 2 1 is there. In this case we have 3 1. At this junction we have 2 21. In this case we have 3 21 and starting point of channel reach 4 we have 4 1. So this is a basic information for our channel network. We have different n values for different sections and slopes are also different. And last one is our channel connectivity.

Channel connectivity this is for channel reach 1, this is zero and 3. That means it is starting from zero. There is no channel junction here. So I am writing it as zero. And next one channel junction is 3, so this is 3.

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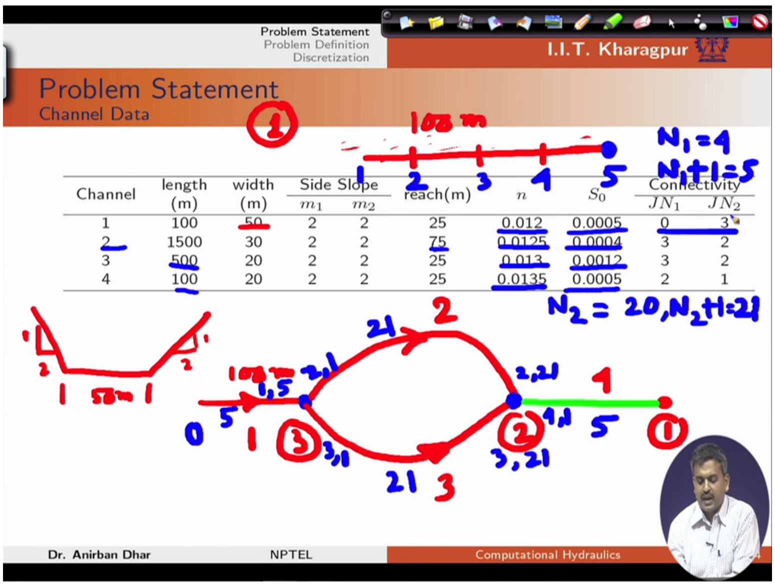

Channel reach 2 it is starting from 3 and ending at 2. Channel reach 3 it is starting at 3 ending at 2. Channel reach 4 it is starting at 2 and ending at 1. So with this information we can start the problem.

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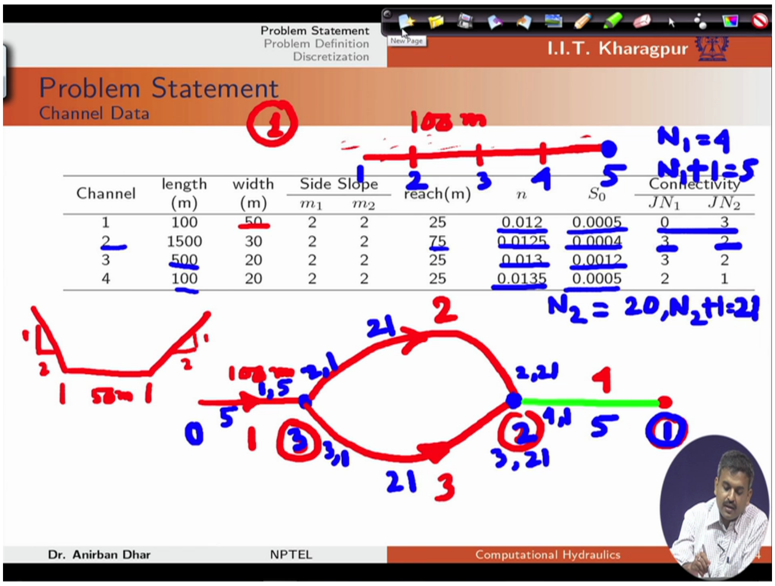

So next information required for our problem is the downstream boundary condition because more or less we have information about our internal boundary condition because we cannot directly specify the discharge or depth for our internal junction but we need that information for our external boundary condition. For external boundary let us say this yd or y depth is 3 metres and at this external junction we have a different situation. This is internal node 4 and if we have this external node at this level connect by line, this is channel reach number 4.

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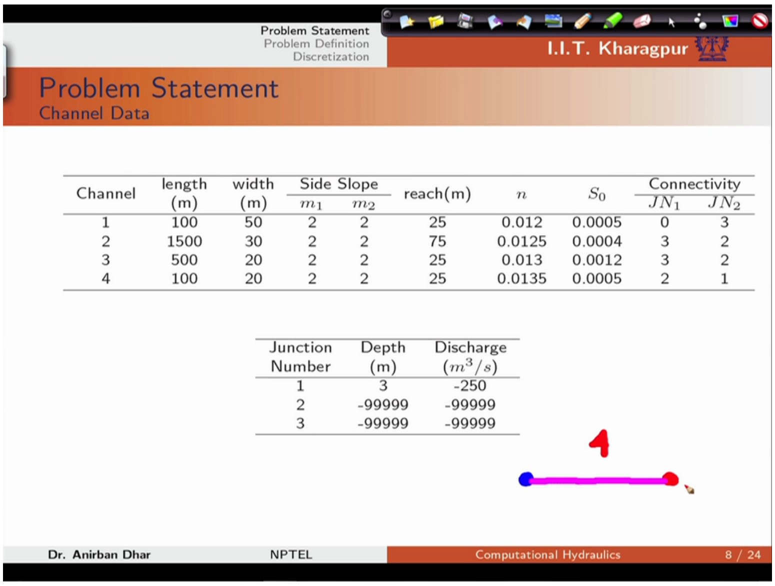

And we have specified depth yd that is okay because we can directly specify equals to 3. But the discharge is minus. Why this minus sign is there? Because we are extracting water from the system or water is coming out from the system. So this 250 metre cube per second this amount is negative. This is not added to the junction. This is a negative quantity for this junction.

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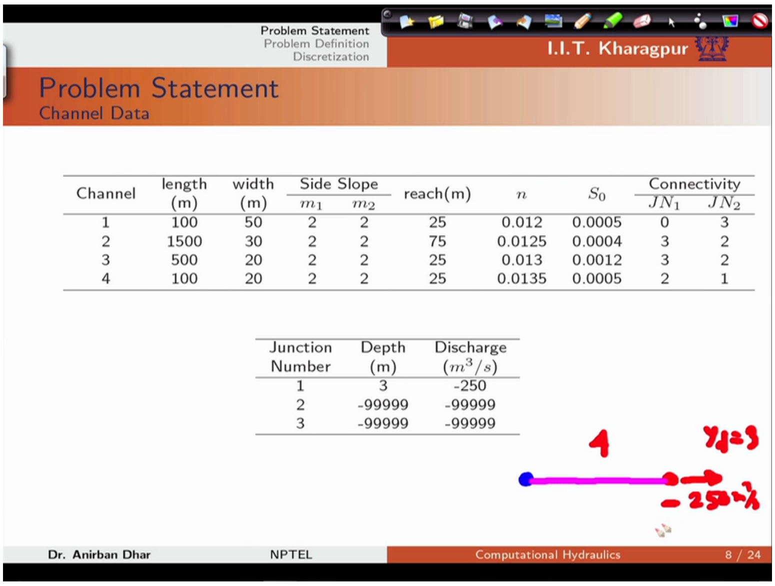

And what is these 5 9s, minus 99999? So to write a program we need to specify certain information or we need to input certain information for our understanding. What is that understanding? If minus 99999 which is different quantity if this is there within this matrix then we will consider that we do not have any specified boundary condition.

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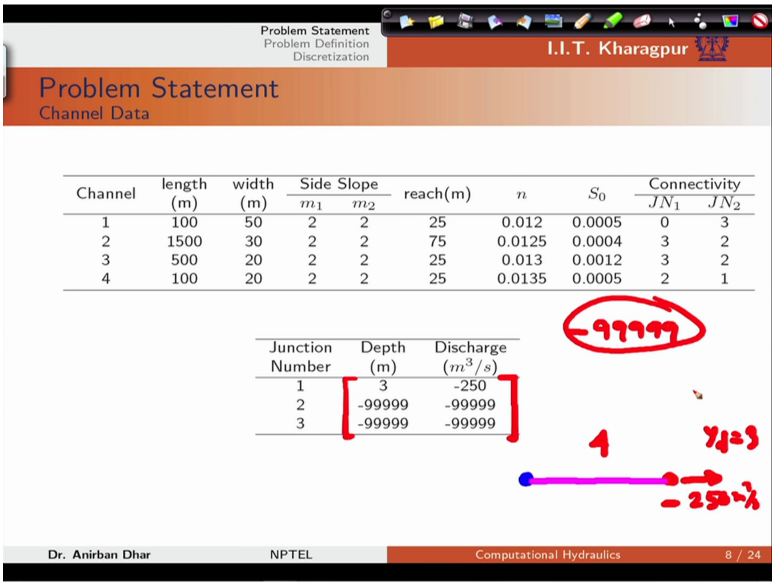

That means for this depth we are considering column number 1, discharge we are considering column number 2 and row numbers are actually for different junctions. This is junction number 1, 2, 3.

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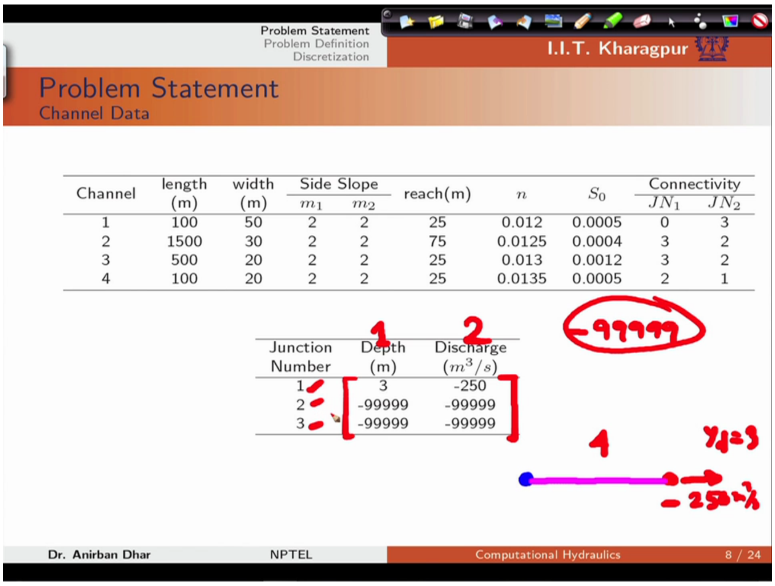

For junction 1 we have specified depth and discharge conditions. Let us say that we have a different problem where we have discharge condition or Qu condition at this inflow junction. This is internal node, internal load and this is again external load. So for this problem let us say that this is channel number 1, channel number 2, channel number 3, channel number 4. We have this node number 1, 2, 3. Now we need to consider this channel junction 4 also.

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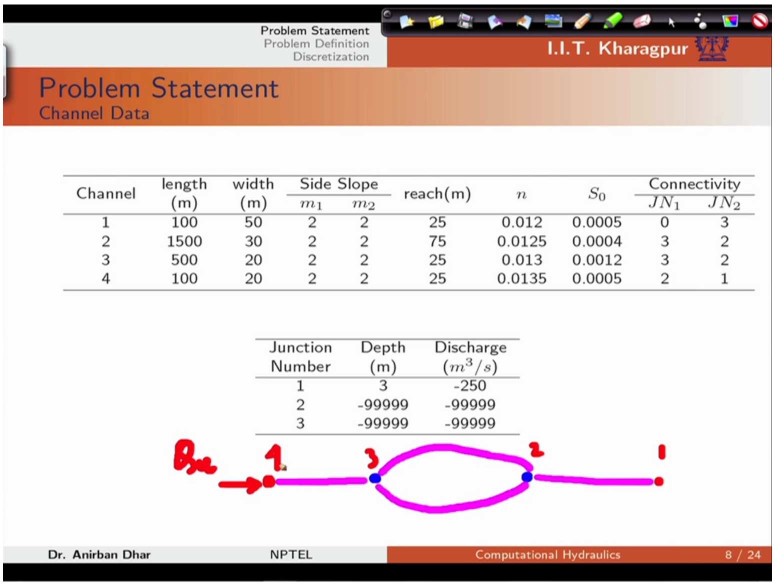

So if we consider channel junction 4 and specify the depth at this point we will have this condition. This is for depth, this is for our discharge. For depth and discharge we have 1, 2, 3, 4 junction points in that case.

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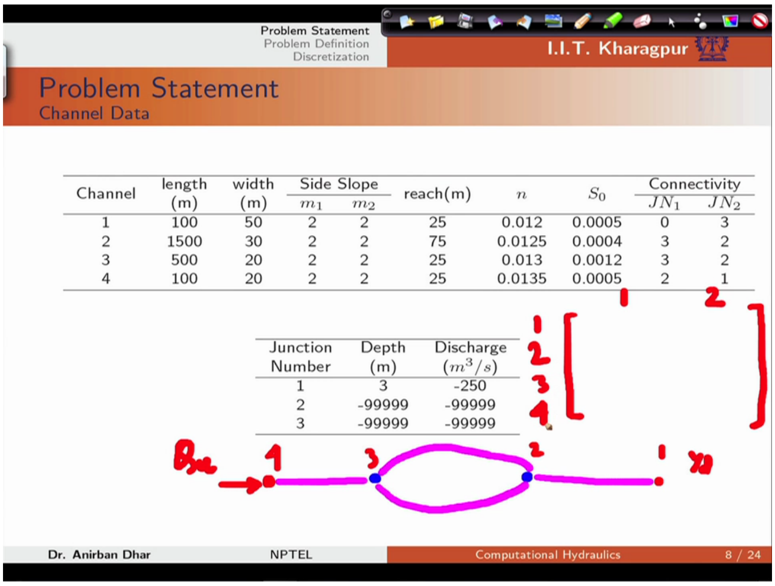

So for junction 2 and 3 this will be same, minus 4 9s 9, minus 4 9s 999. Or I can just write this for 3 also it will be minus 999, minus 99999. Discharge condition is specified for Qu. So Qu this quantity is positive so we will add this with positive thing.

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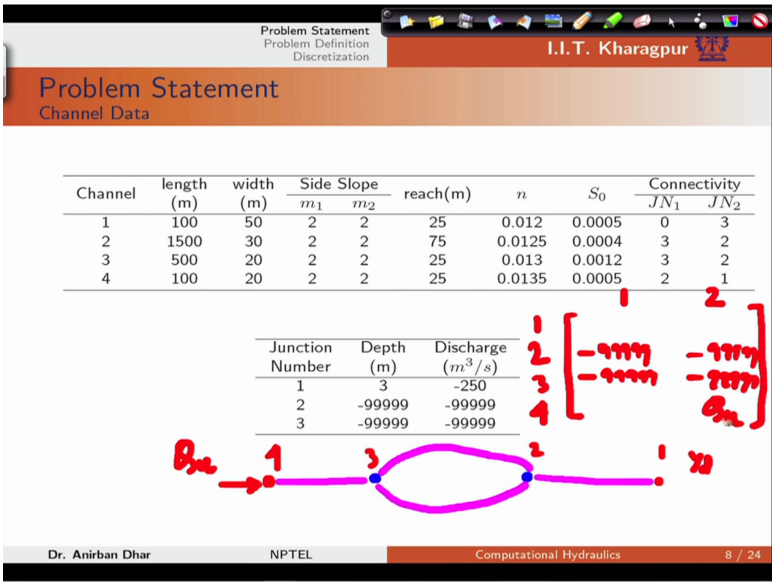

And in this case we do not have specified discharge condition at downstream end. So this is no specified depth value at upstream. So in this case we have this matrix.

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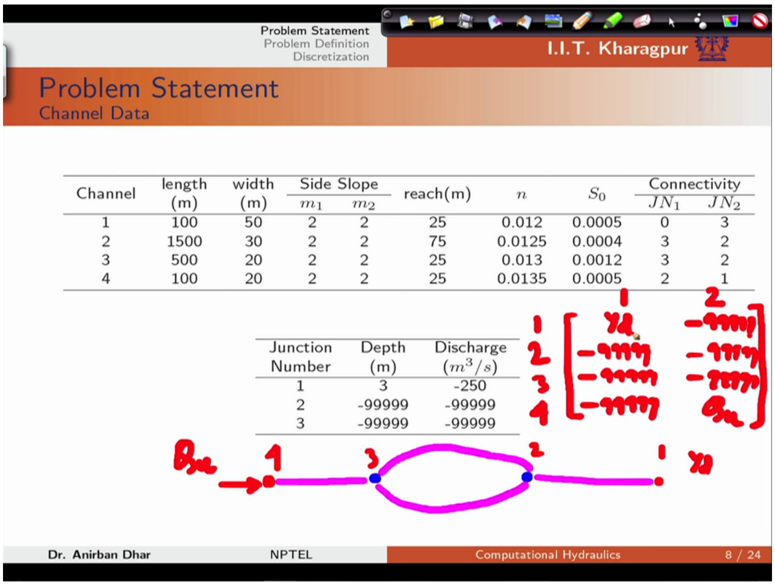

But in the present case under consideration we do not have this last junction point. So we have this matrix which is there to consider the boundary conditions for our junctions.

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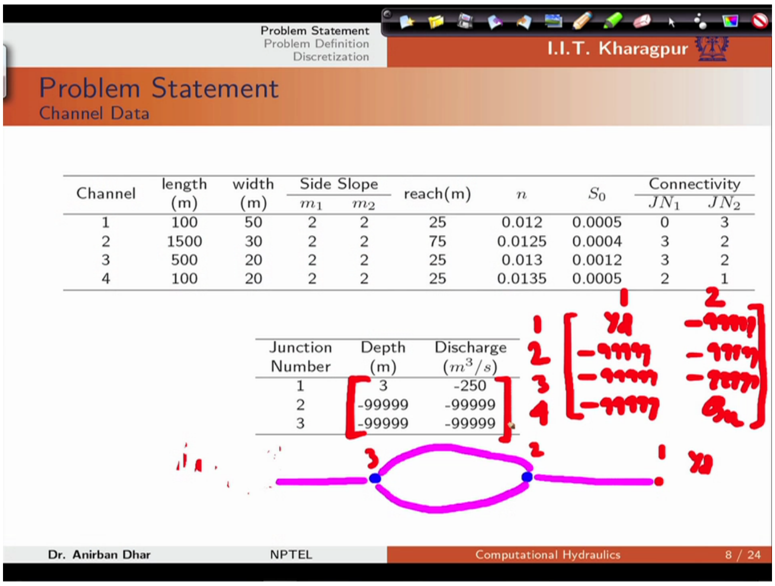

Now in this case what is required? Required is to estimate the flow depth and discharge across the channels. Now we already know what is the continuity and momentum equation under consideration for this channel network problem. We have this continuity equation, momentum equation.

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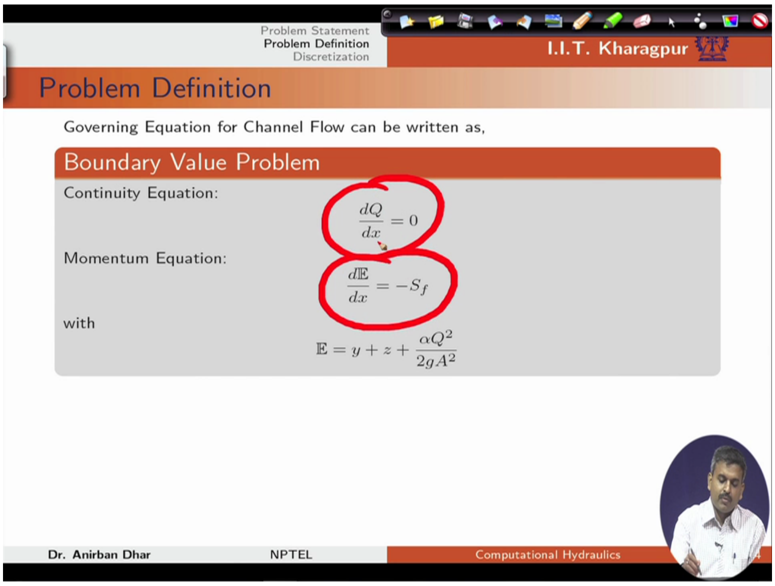

Now we need to discretize it. So this is our flow depth y and Q is the discharge for our case.

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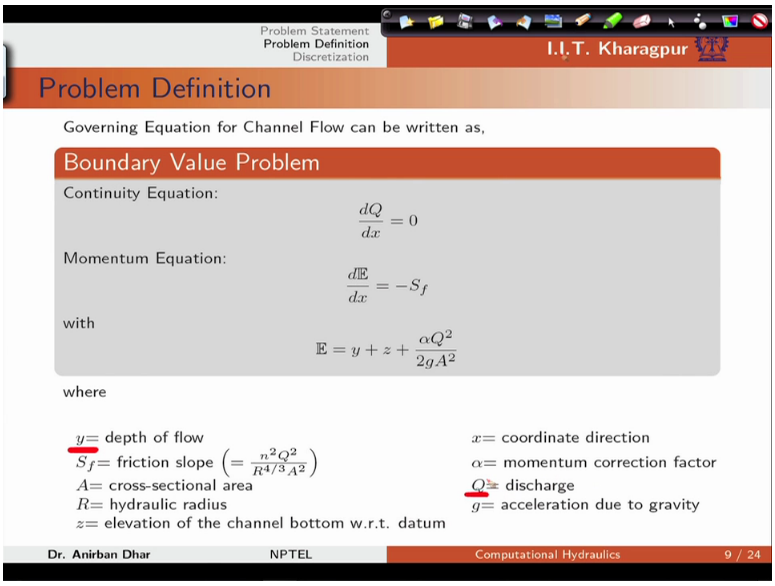

Now this is the general channel flow convention. That for any Lth channel and ith segment we have i and i plus 1, these are the section numbers. And we have elevation head. This is flow depth and this is our kinetic energy head or the case. We have z y in this case also. And this is our average value of friction slope.

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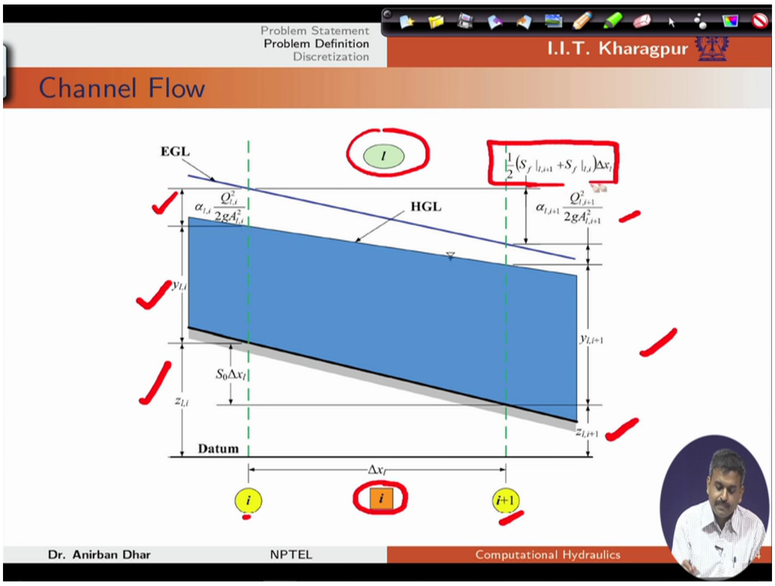

Now in this case without specifying z also we can solve this problem because the difference between these two sections will be S not into del x L. L is for specific channel reach.

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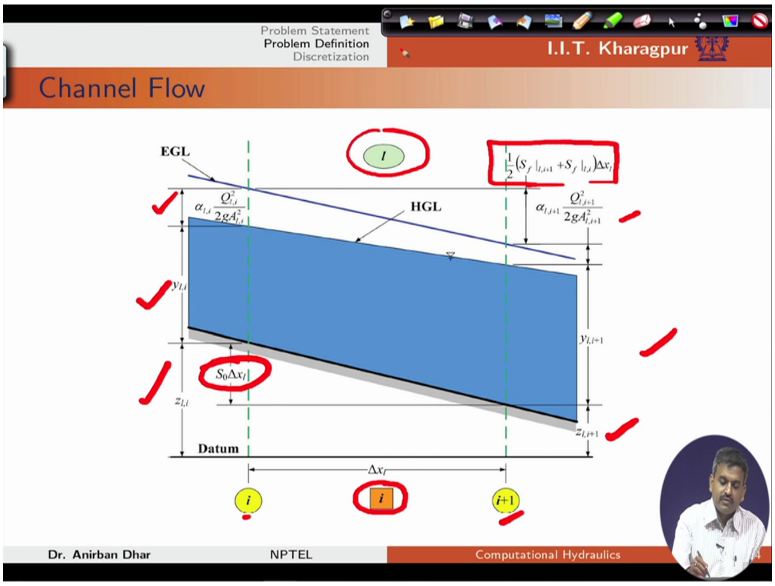

Now let us consider the channel flow conventions. If we have channel reach L and it is starting from 1. And NL plus 1 is the last section. Now in this case if we consider QL is positive so obviously in this case the flow will be from section 1 towards NL plus 1.

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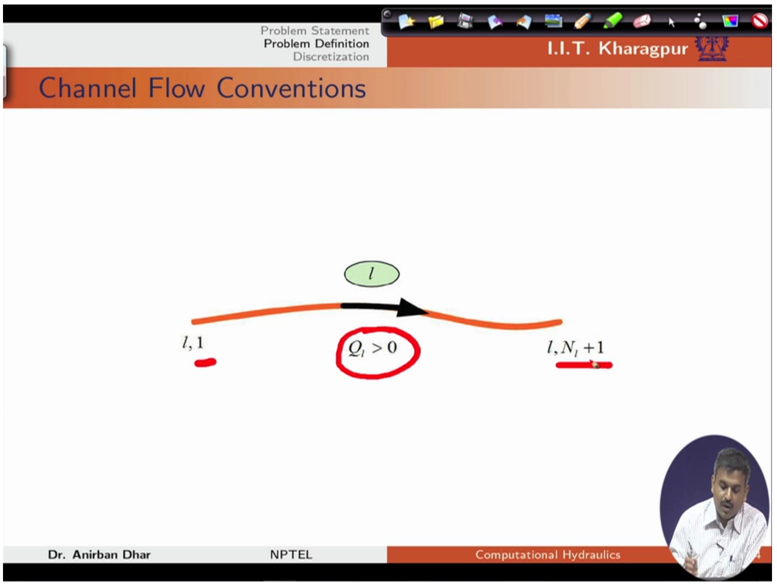

Now if we have a negative Q then we will have reverse flow. And for positive flow that means for flow from section 1 to NL plus 1 we have two junctions JNj and JNj plus 1.

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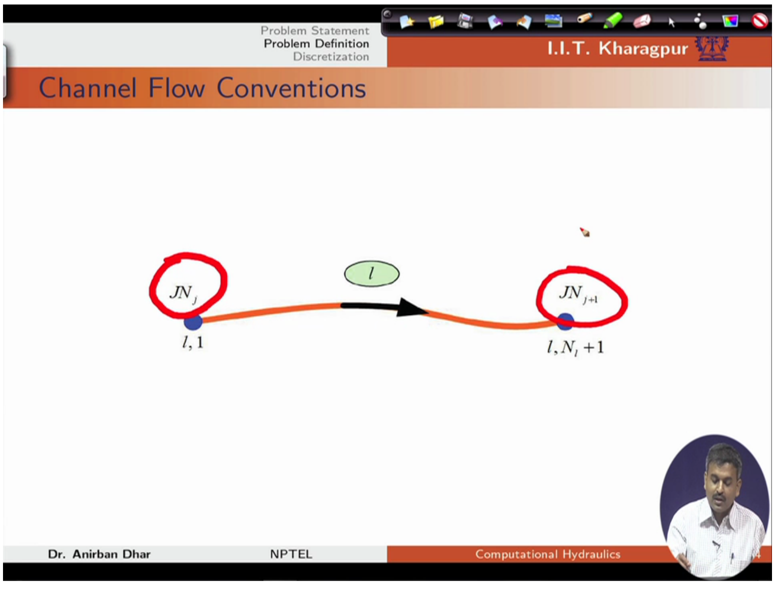

For these two junctions we will have two different discharge specification conditions. And what are these conditions? Because if we consider positive flow that means we are extracting water from this junction and we are adding some water to this junction.

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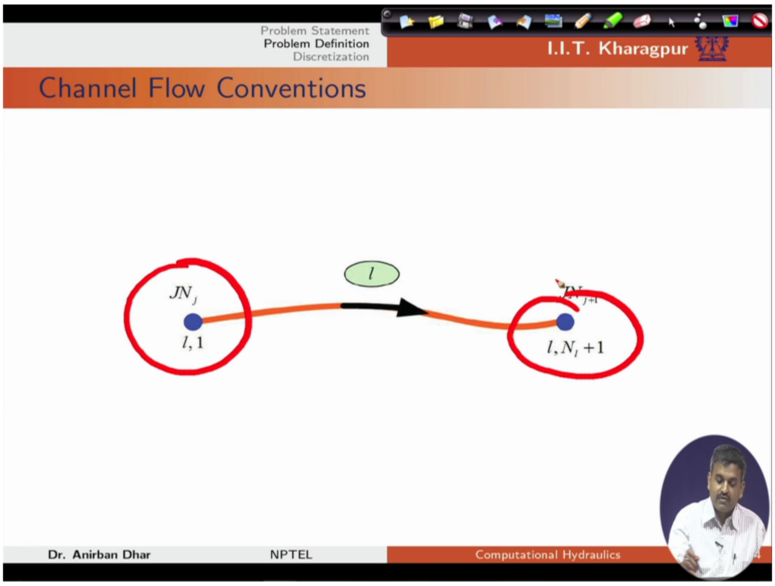

So we will have negative quantity here. This is negative quantity for our starting junction and this ending junction or ending segment we will have this positive Q.

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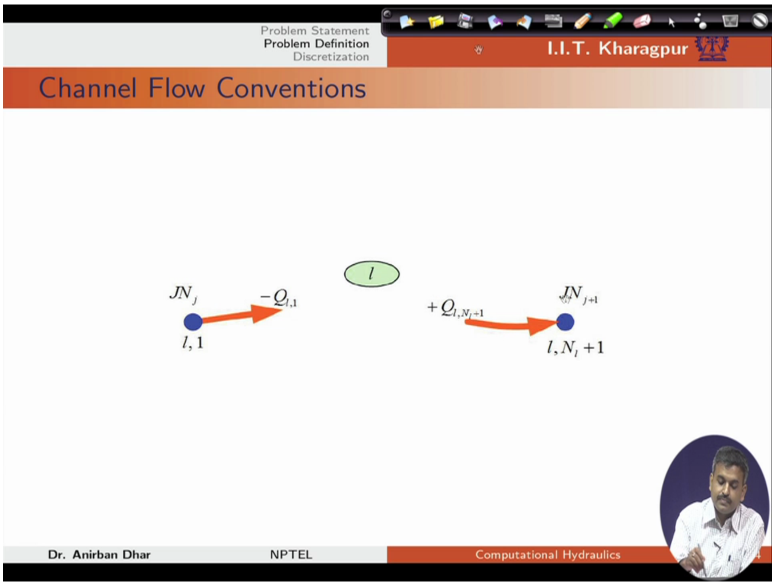

Now with this convention we can start discretizing governing equation for our problem. Algebraic form of continuity equation. If we have this CL i which is for Lth reach and ith segment we will have QL i plus 1 minus QL i. That means for any reach i we will have i and i plus 1 these two are sections. So this is the continuity equation or steady state continuity equation.

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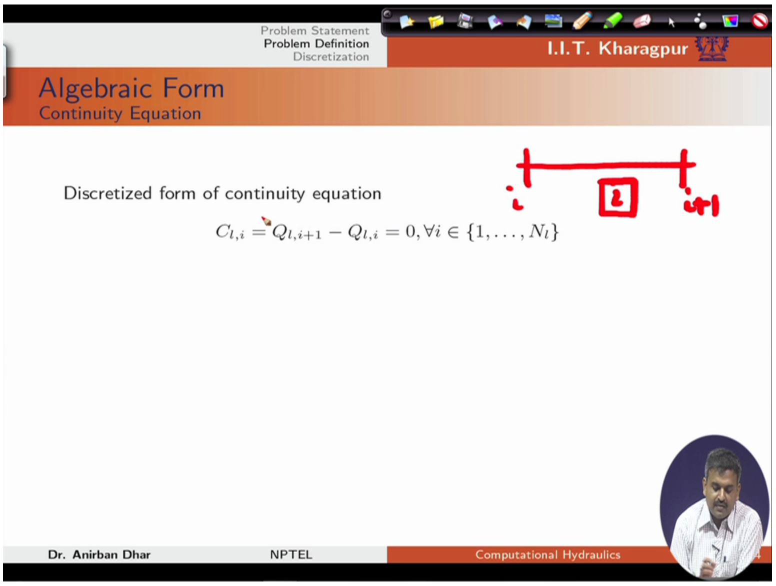

Now we have four unknowns for our problem. Starting from i i plus 1 so we will have yi, Qi, yi plus 1, this will be Qi plus 1. Now we need to differentiate the CL i with respect to these four variables.

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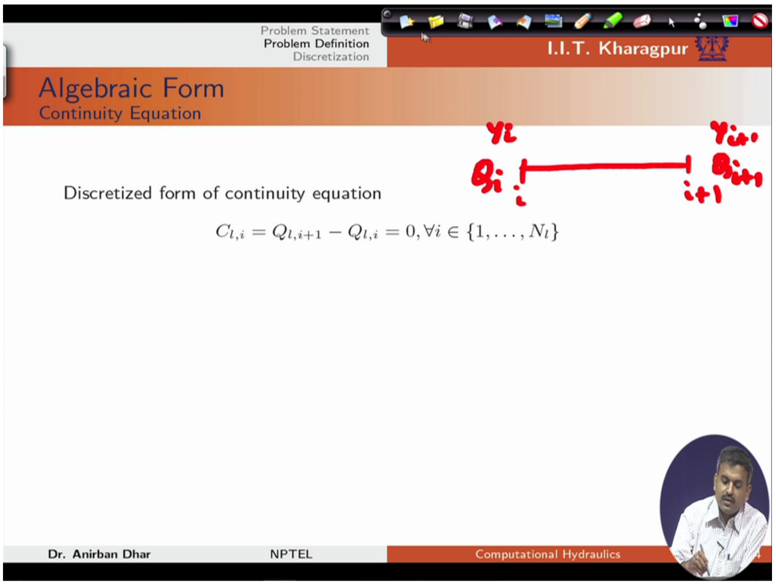

So what we will get? We will get four derivatives. So we do not have any y component here so this is zero. This is CL i del QL i this is minus 1. Next is again we do not have this yL i plus 1 so this is zero and we have L i plus 1 the coefficient is 1 so obviously this is plus 1.

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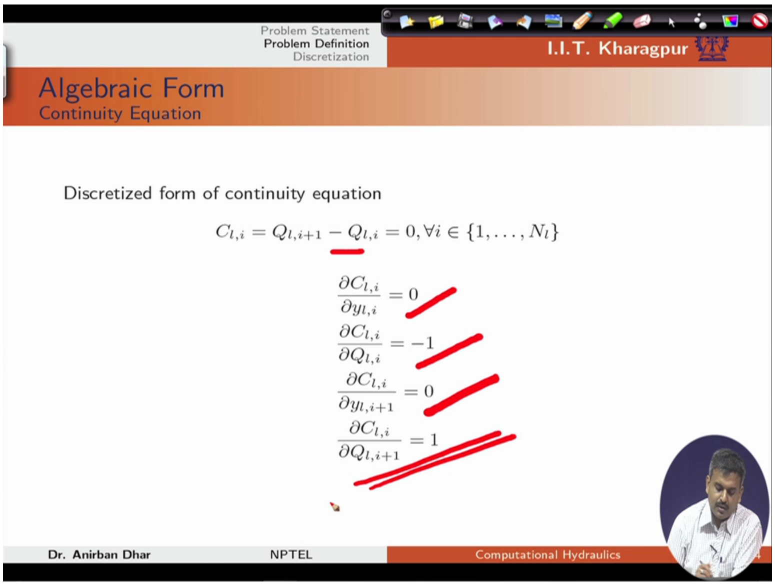

Now we can construct our Jacobian matrix to solve the nonlinear algebraic equation but essentially this continuity equation is not non linear in nature. And this is the discretized form of our momentum equation for ith segment for Lth channel reach.

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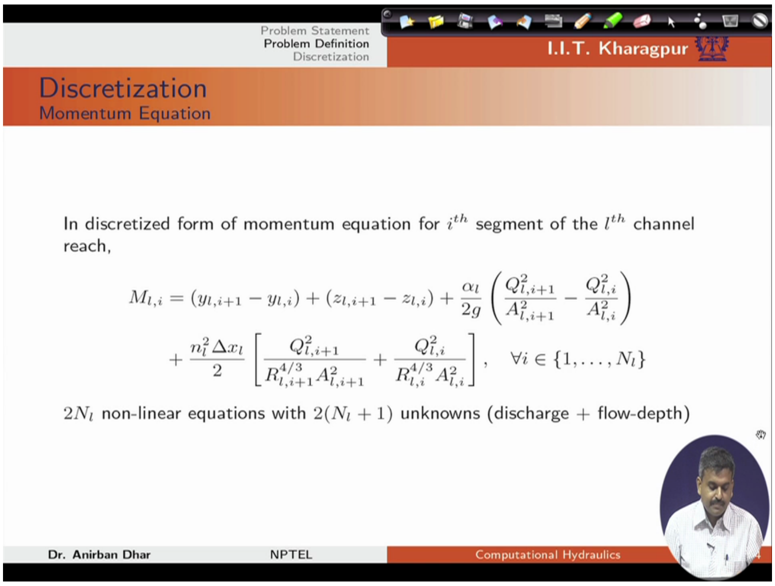

Now in this case this ML i we have difference between y z and this is alpha L. This is specific to a particular channel reach. This is Q square by A square minus Q square by A square. R is again hydraulic radius. These quantities are known quantities.

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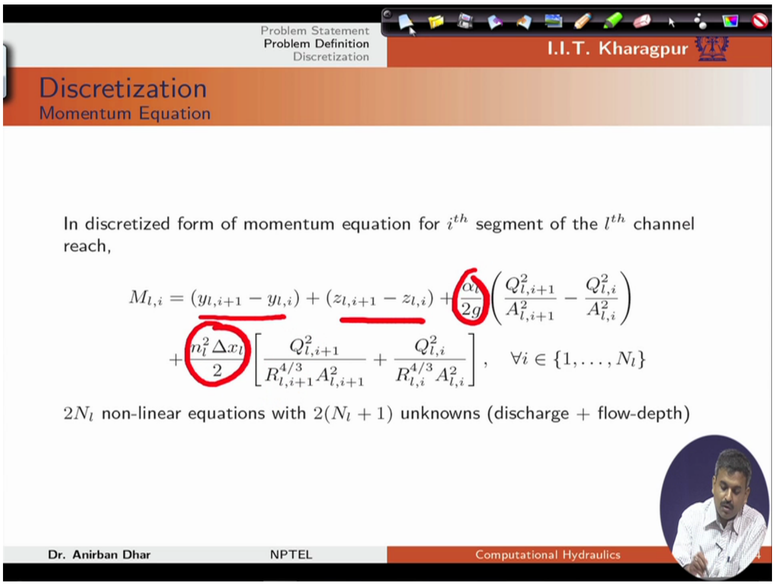

Now we have 2L or 2NL non linear equations with 2NL plus 2 unknowns. This is discharge or floor depth. This is for single channel reach for Lth channel reach. We will have multiple channels.

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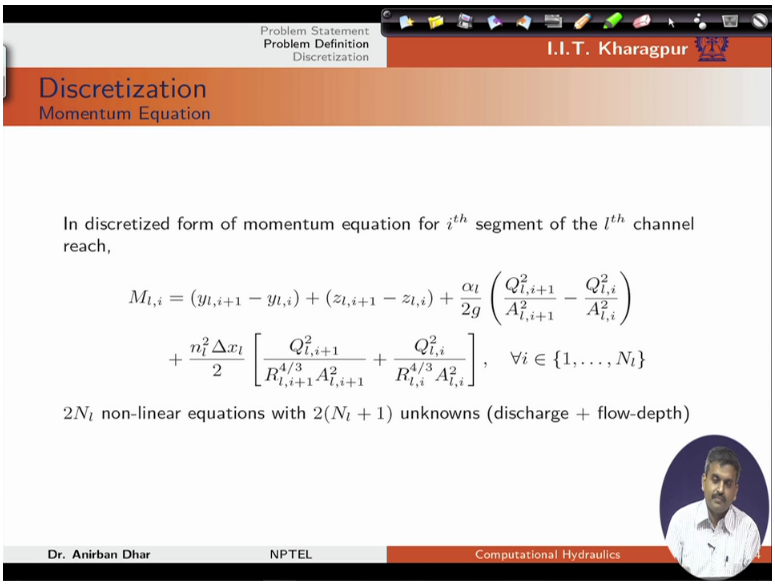

So this is our derivatives for the momentum equation function. So this is yL i, this is QL i, yL i plus 1, QL i plus 1. So derivative of ML i with respect to yL i, QL i, yL i, yLi plus 1, QL i plus 1. So we will get these terms.

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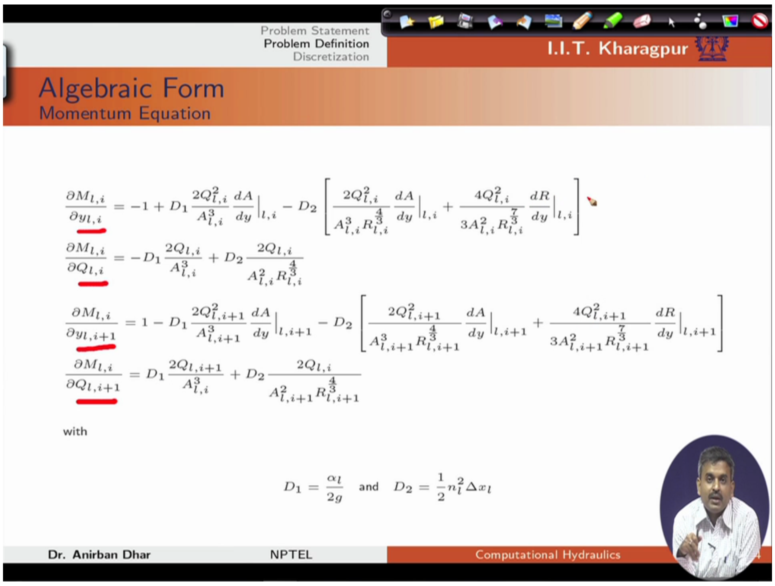

Now we need to implement it in our source code. So for our problem we have this D1 and D2. These two are channel dependent parameters. Now for trapezoidal channel we have dA by dy equals to this one. Again dR by dy is T by P, R by P, dP by dy. So these are the values. We can directly utilise these for calculation of dA by dy and dR by dy because we need these two values for calculation of the elements of the Jacobian matrix that is generated or that are generated from our momentum equation.

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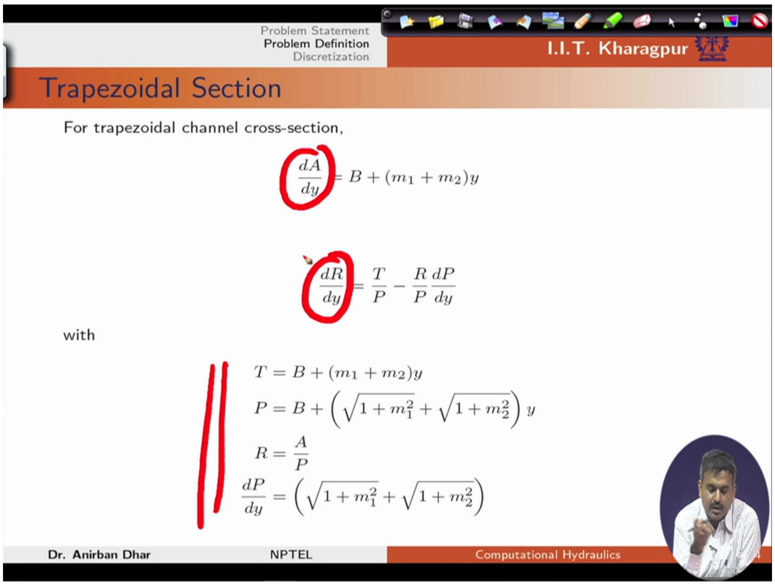

Now this is the boundary condition for our downstream end we have this channel reach number 4 and this is the n segment. This value equals to yd. We are not considering any difference in elevation there. So at this point downstream boundary DBy 4 N4 plus 1, this is one condition. So we can write this function as DB (pl) 4 N4 plus 1 minus yd equals to zero at junction 1.

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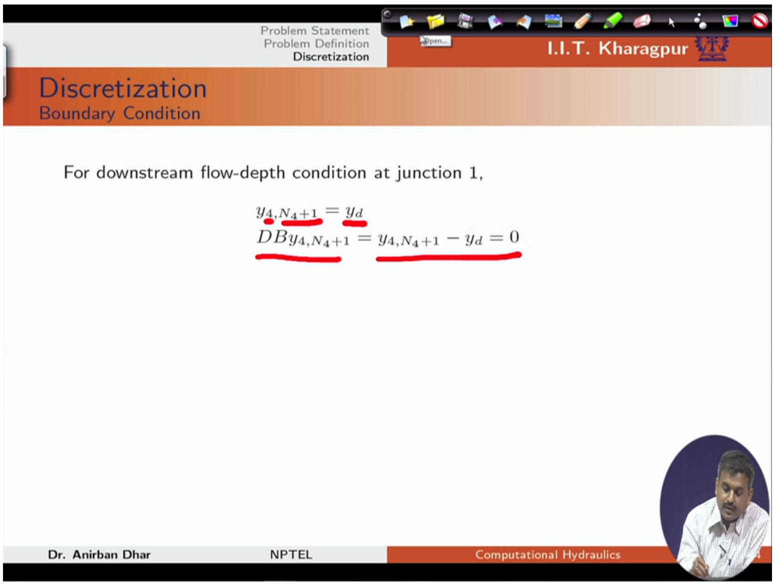

Now we can take derivative with respect to other section values like this. Now at this level we can define the downstream discharge condition. Downstream discharge let us say Qd I am adding here. So obviously Q4 N4 plus 1, this is Qd is minus 250. Obviously this quantity should be zero. This is downstream DBQ or downstream discharge condition DB4 N4 plus 1. And we need to utilise this information.

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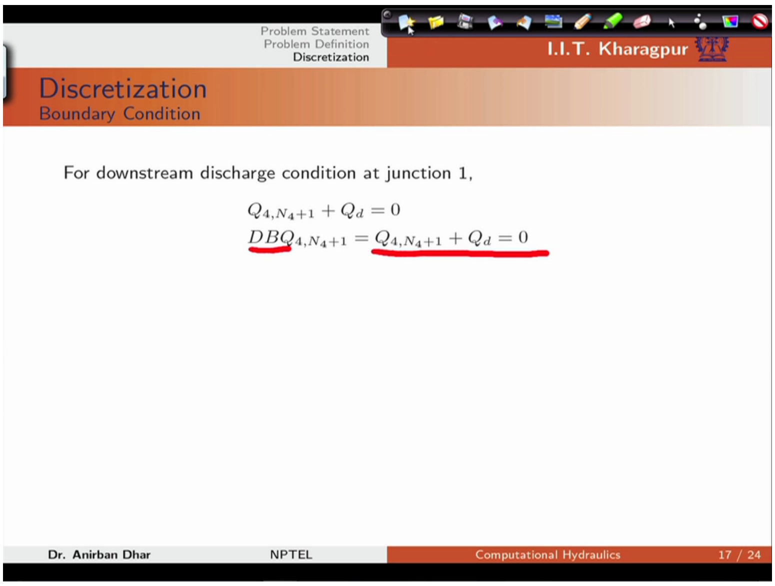

These are the elements of Jacobian matrix. Again we will have three conditions each for our junctions. For junction 2 that means with channels 4, 2, 3 this is 2, 3, 4 we have discharge conditions. That means this Q2 N2 plus 1 this is entering into the junction, this is entering and this is living from the junction. That is why negative.

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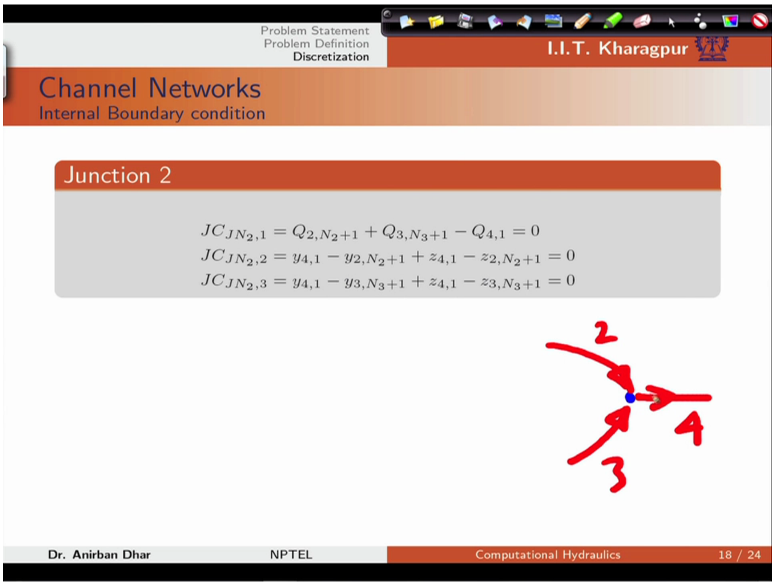

And we can equate this y4 at this level and y2 at this level. We can omit this z difference there. So this y4 and y3 again we can equate for our problem.

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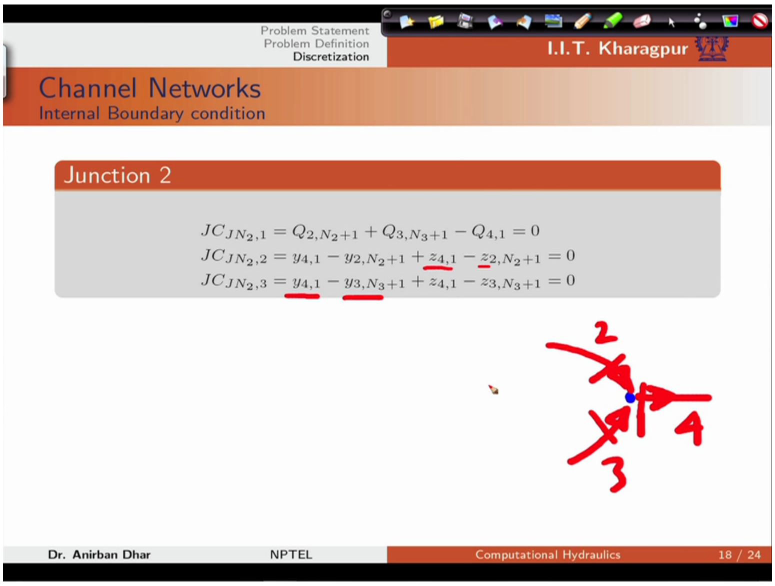

Similarly we can get elements of Jacobian matrix from there. Now from junction 3 again we will have channels 1, 2, and 3 connected to it. Channel 1 is entering there that is why positive. These two values are negative there.

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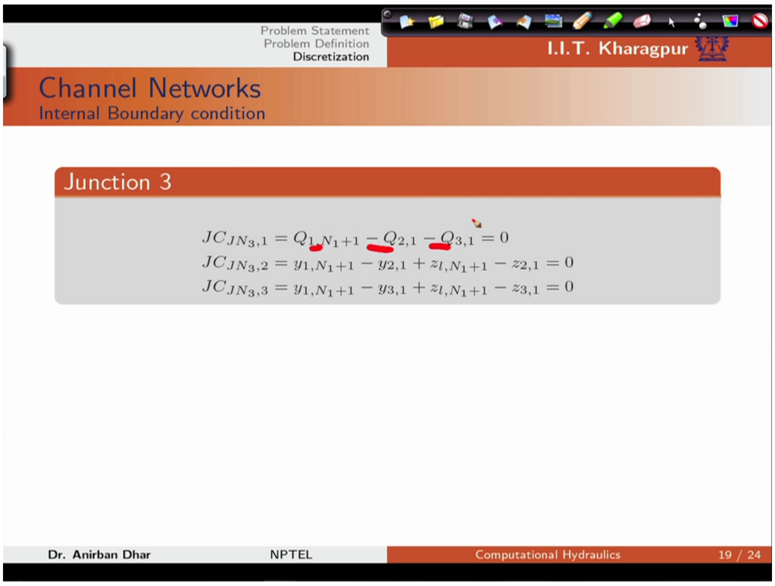

So we can define the condition or this (Jaco) elements of the Jacobian matrix. Now in general form we will have these equations. These are for interior conditions or interior segments starting from 1 to NL. And for all L in this case we will have 1, 2, 3, 4. This is all L. We need to write this.

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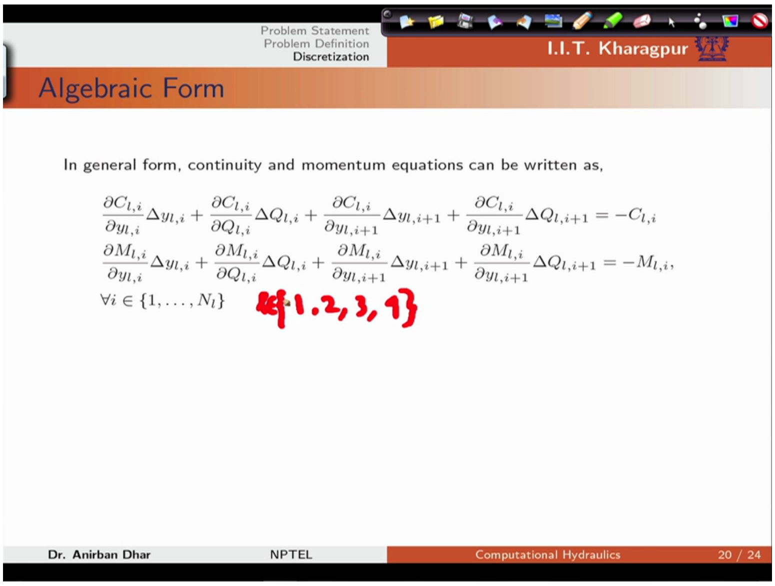

Now then we need to add this downstream boundary condition there. Then we need to add this junction conditions. So obviously these are increments and junction number 3 this condition is there.

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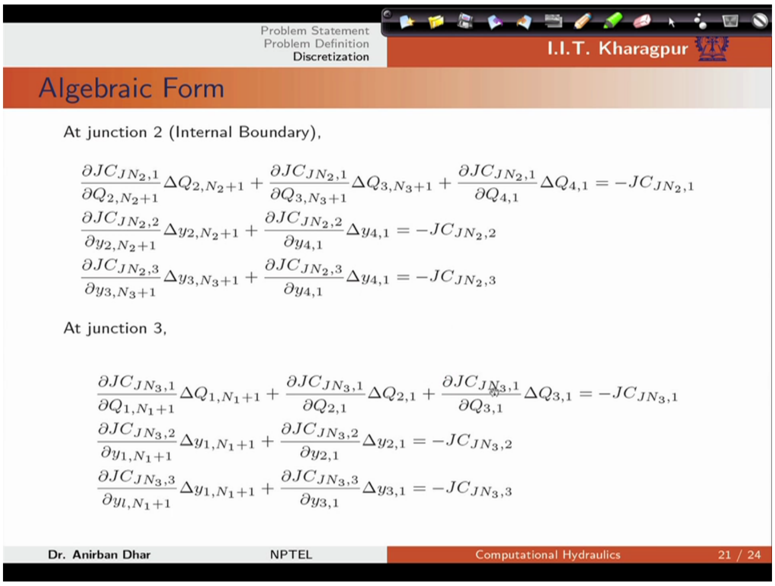

So now we need to implement everything in computer program. Now how to input this information in our system? So I can just directly transfer the initial values. This is channel reach number or channel reach number 1, 2, 3, 4. These are lengths, these are width values, these are slope values, this is segment length, these are n values, this is S not, this is jN1 or junction 1, jN2 junction 2 for connectivity.

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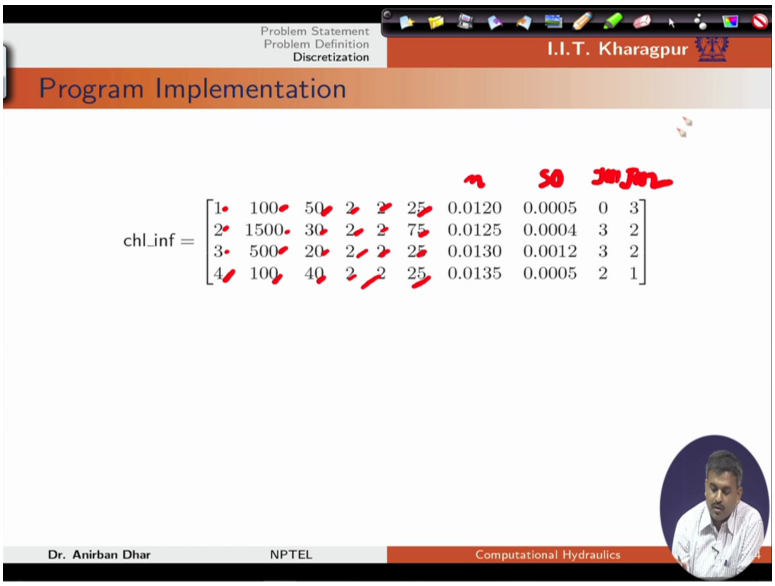

Another information matrix will be required for specification of boundary condition because at the downstream end or channel junction 1 we have this specified boundary. For others we do not have any information available. Last one is junction connectivity. In this case let us draw our original channel. This is our external node. These two are our internal nodes, okay.

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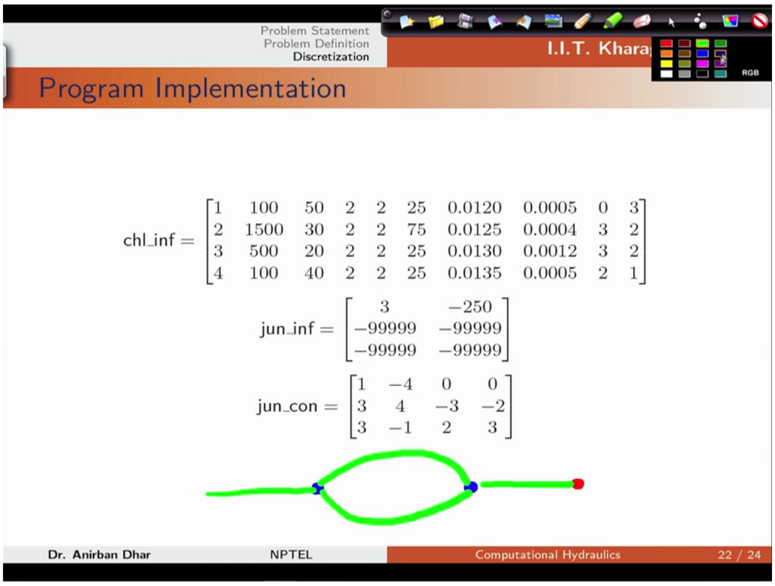

So in this case this is junction number, junction number 1, 2, 3. On this side first column provides information regarding number of segments connected to this junction. For this junction number 1 only one section is connected. What is that section? So this is section number N4 plus 1 of the fourth reach. So we will write this particular information in terms of plus and minus number.

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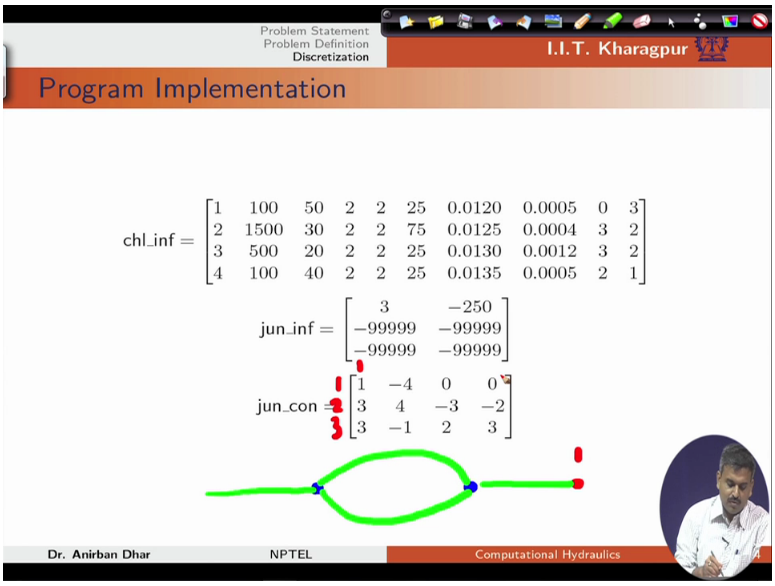

So in this case the channel is starting at plus 1, ending at minus 1. In this case channel is starting at plus 2, ending at minus 2. Channel is starting at plus 3, ending at minus 3. In this case channel is starting at plus 4, ending at minus 4. So minus 4 means that last section is connected. Plus 4 means first section is connected.

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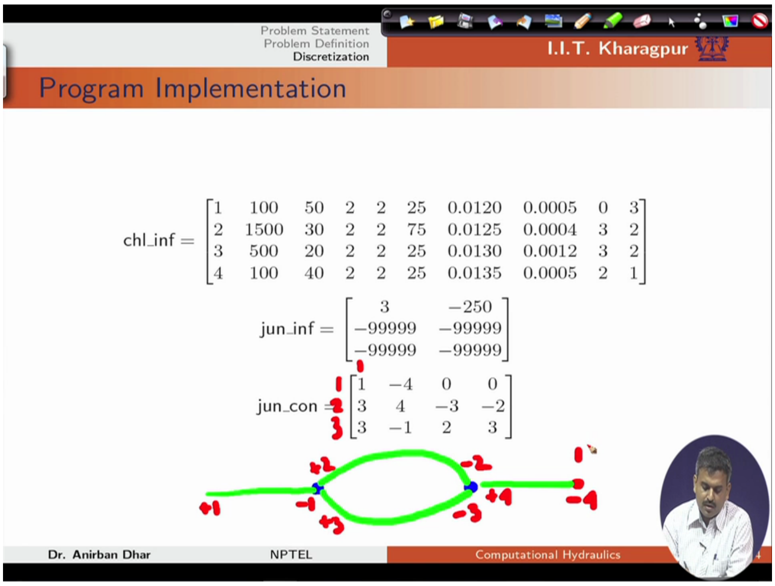

So channel reach for this particular junction we have last section connected. So in this case we are writing minus 4. Other two entries are zero. On this side this should be the maximum number of nodes connected to a particular junction. We have three junctions, out of that maximum number of nodes or sections connected is three. So this size is 3. So 1 plus 3, this is 4.

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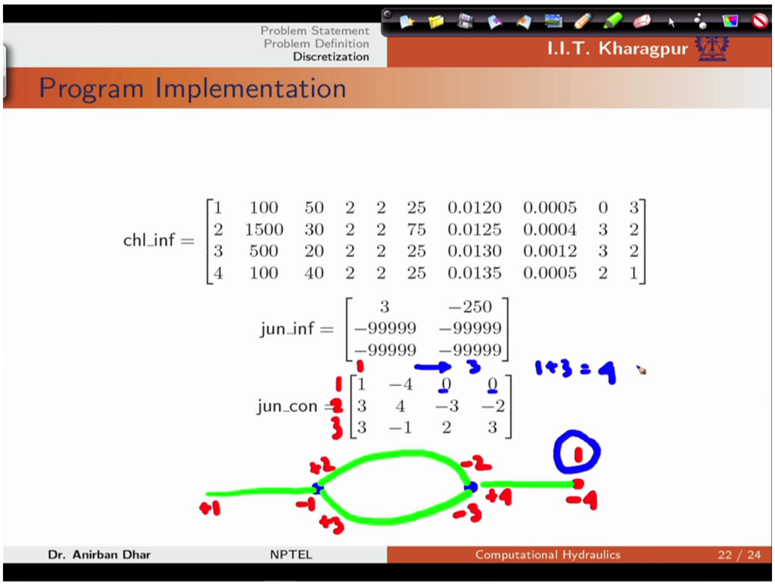

So in this case second junction. This is junction number 2. For junction number 2 we have three channel reaches there. So this is plus 4. Plus 4 means this is the starting point of channel 4, minus 3 this is the ending point of channel 3, minus 2 this is ending point of channel 2. Again third junction this is ending point of channel 1. This is starting point of channel 2 and starting point of channel 3.

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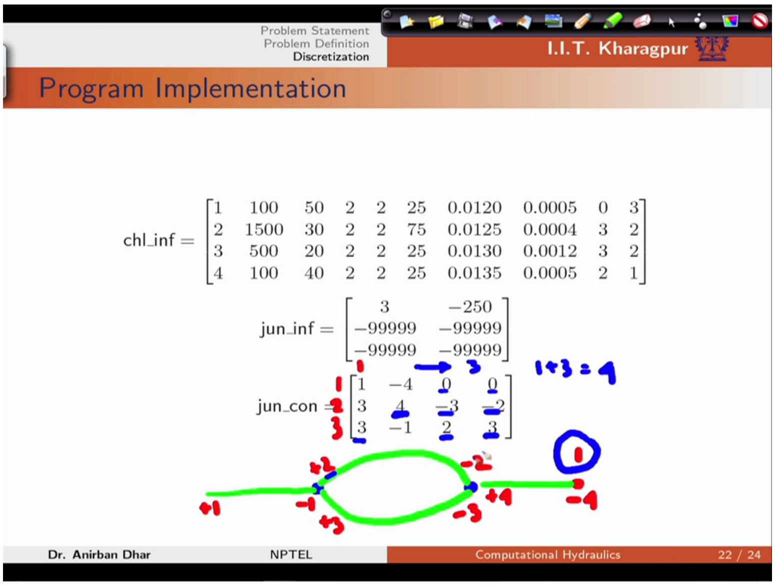

So with this we can start the programming thing.