Computational Hydraulics
ProfessorAnirban Dhar
Department of Civil Engineering
Indian Institute of Technology Kharagpur
Lecture 23
Mesh - Free Method: Space - Time Moving Least Squares Method

Welcome to lecture number 23 of the course computational hydraulics. We are in module 2, numerical methods. This unitnumber 19, we will cover this mesh free method. Andspecific topicis space time moving least squares method. In our previous lecture class we have discussed only spatial discretization using or single variable discretization using moving least squares.

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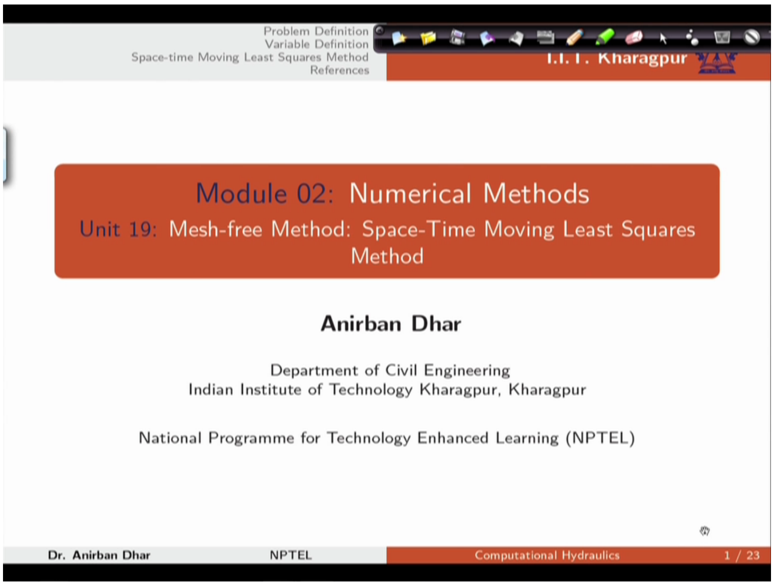

In this particular unit we will try to introduce the time concept within the discretization framework.Learning objectives, at the end of this particular unit the students will be able to discretize the spatial and temporal derivatives of single valuedmultidimensional function using mesh freeapproximations.And they will beable to derive the algebraic form using discretized partial differential equation or PDE, initial condition and boundary conditions.

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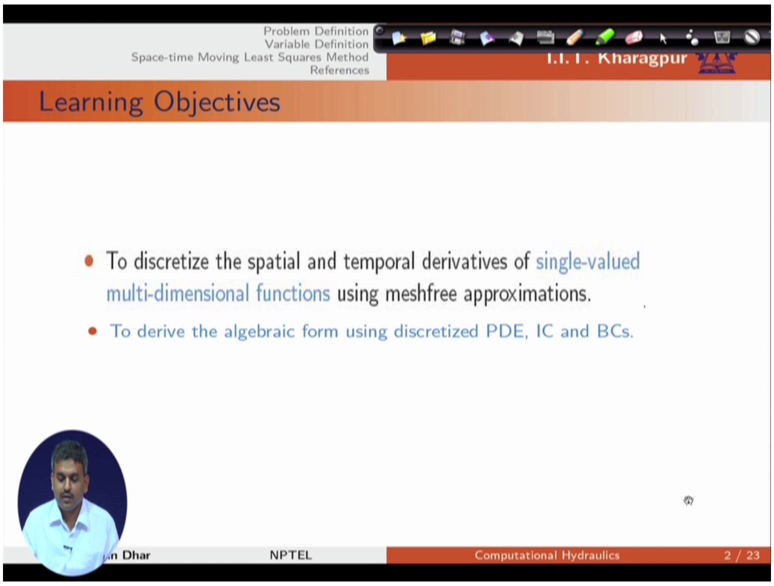

So let us consider our general form equationin this case. We already know that if we have a general variable phi which is a function of x, y, z and t in our physical system where x, y, z, these three are space dimension and t is the time dimension. So first term is temporal derivative, second term is advective term, right hand side first term is your diffusion or diffusing term and this is other forces and Sphi is our source sink term.

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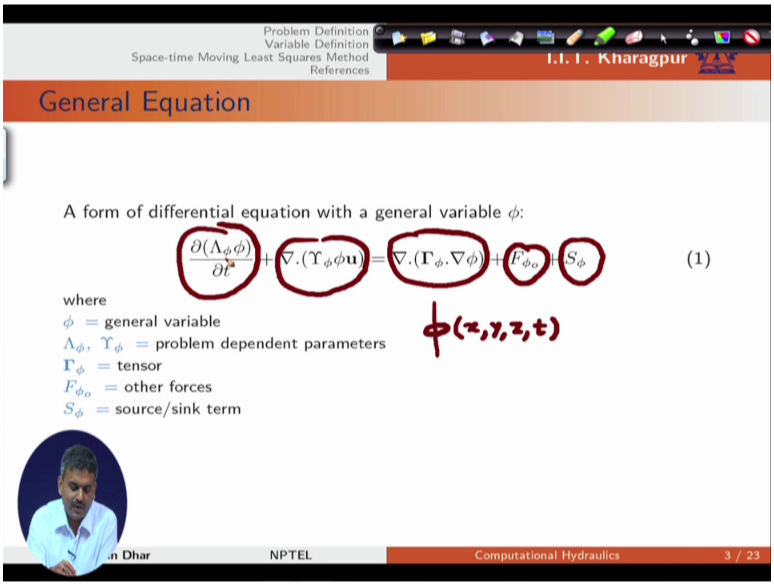

In this case capital Gamma phi and upsilon phi, these are problem dependent parameter and gamma phi these are tensors. We already know this from our previous lecture class. But in this casewe need to concentrate on thetemporal or spacetime discretization. In case of our finite difference approximation, we have discretizedthis temporal derivative using direct approximation.

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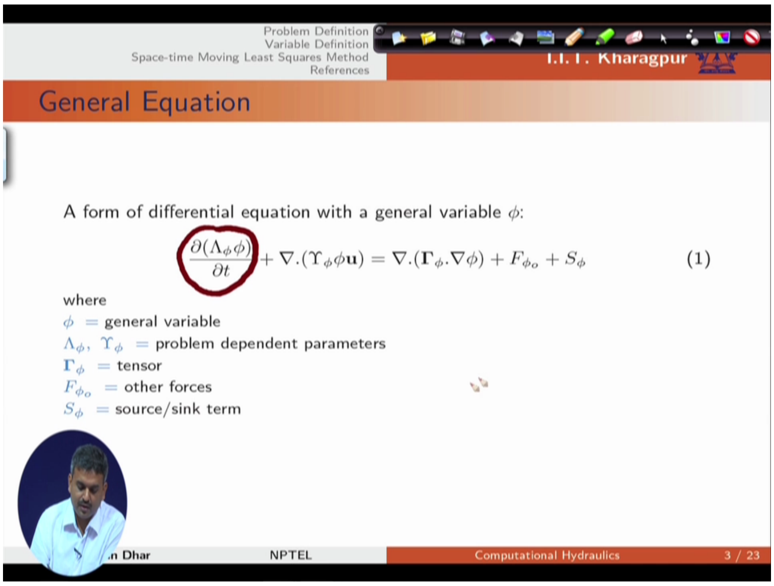

Let us say, if we have phiwhich is a constant, del phi, del t for three dimensional case, we can easily discretize by ijk and L plus 1 minus phi ijk L divided by delta t.L and our L plus 1, these are time index for this particular discretization where we are considering that phi is a function of x, y, z and t. But the problem is in this casewe are considering two derivatives.

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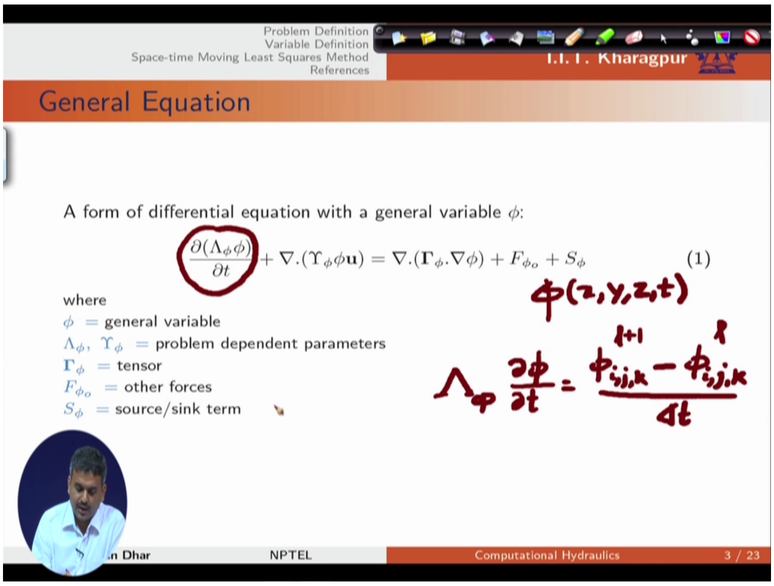

If you have one dimensional case then we have simply x and t in time dimension. Thenif this is our time level L and this is our time level L plus 1, then for any particular space level i, we can calculate this derivative where del phiby del t equals to our phi iL plus 1 and phi iL this is divided by del t.

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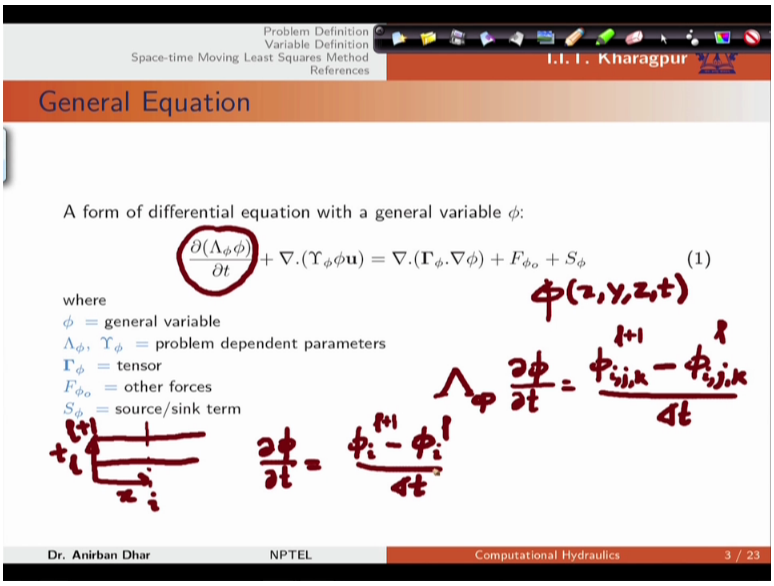

When there is increase in the number of dimensions we are considering two points. One at the future time level L plus 1, another one at present time level L. Now in this caseif we consider as you two dimensionalconsideration or discretization, the same thing we are repeating.We are considering thatin this caseif we have this time axis and these two are x and y, this is our time axis.

Then for discretizationif this is level L for any arbitrary point this one we can start our discretization and we will consider one in present, another one in future. So in finite difference case we are considering two points, one is in present, another one is in future.

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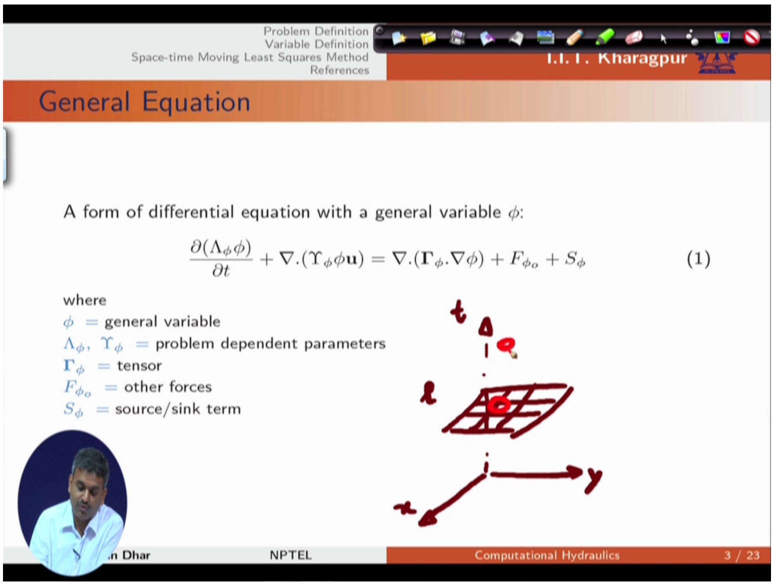

But the concept of mesh free is such thatwe need to consider the concept of support domain. Now whenever we are considering this support domain we need to specify some radius for that one. So whenever we are specifying radius for any system, let us say this is one dimensional x and t system. So if you have different points at different levels, maybe if we move in this direction there will be increase in t, if we move rightward obviously there will be increase in x.

So we need to define the radius for this influence domain or support domain. Now wherever we are considering the central point, maybe this point is our centralpoint.

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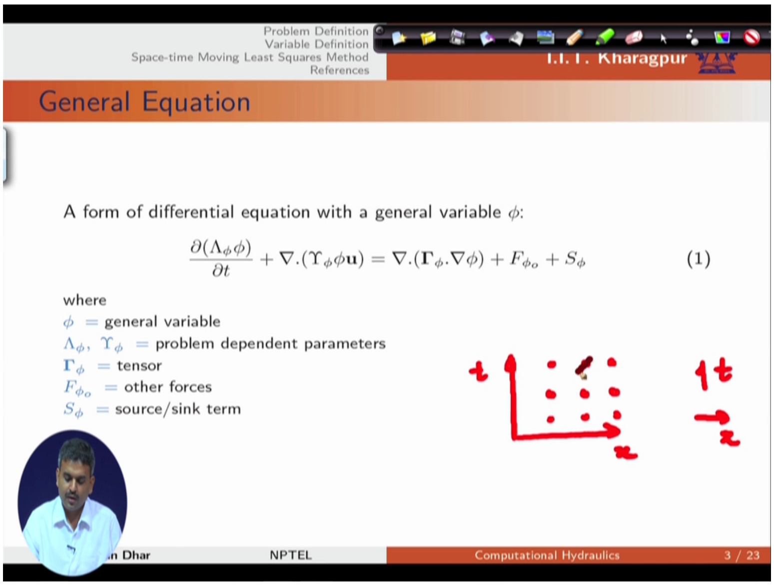

So if we consider this support domain we may need to consider multiplepoints at present and future time level in this case for calculation of derivatives. So the concept of mesh free and finite difference is similar. Only difference is that we are considering multiple number of points in this case.

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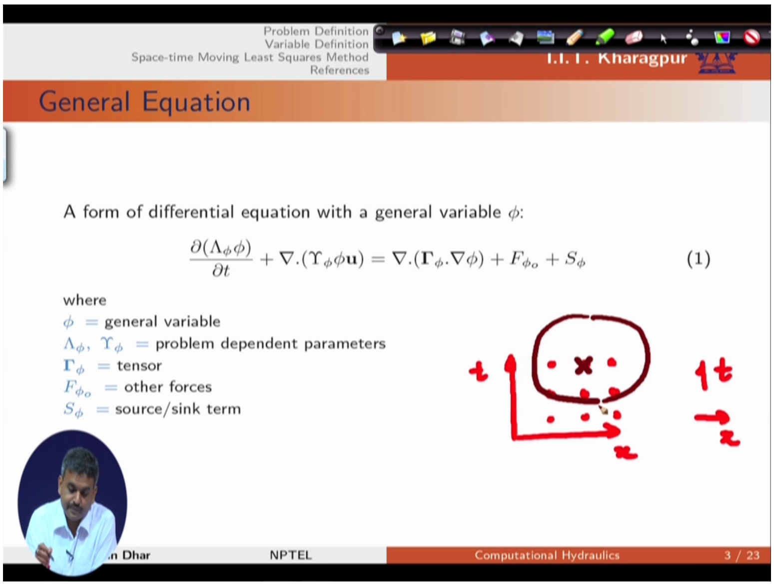

So nowwe can explain the reduced form that we have utilized in our finite difference approximations where governing equation is for IVBP or initial boundary value problem where this lambda phi isconstant. These gamma x, gamma y, these are also constant. And Sphi is somewhat specified quantity for any physical problem. Now with this information we can start our problem.

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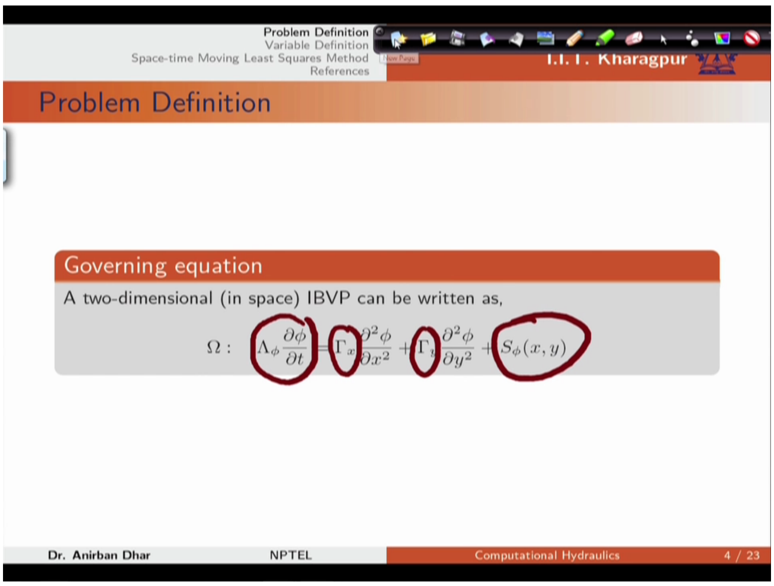

And if you see the initial condition, initial condition again phi is a function of x, y and at t equals to zero, we need to specify the value of phi which is phi0 and it's varying over x and y. And boundary condition, on the left side we have specified boundary or this is Dirichlet kind of boundary condition, D1, D2. And if we have on top in this case, this is Neumann and this bottom also we have Neumann kind of boundary condition.

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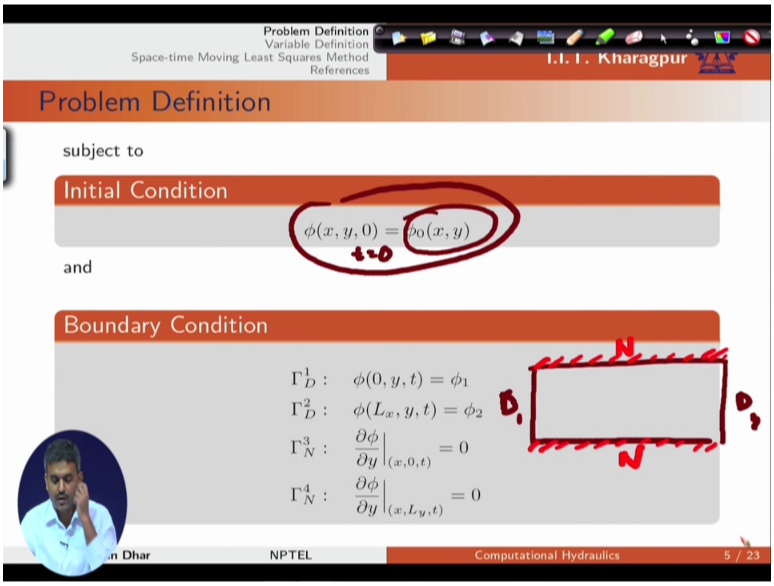

Now we need to discretize this derivatives or equations using our mesh free approximation approach. So again we can discretize our spatial domain into regularindividualpoints. Because these lines or these grid lines are not applicable for our mesh free approximations. We will consider only individualpoints for our calculation purpose. These points can be either in structured or placed in regular intervals or they are randomly distributed within thisdomain.

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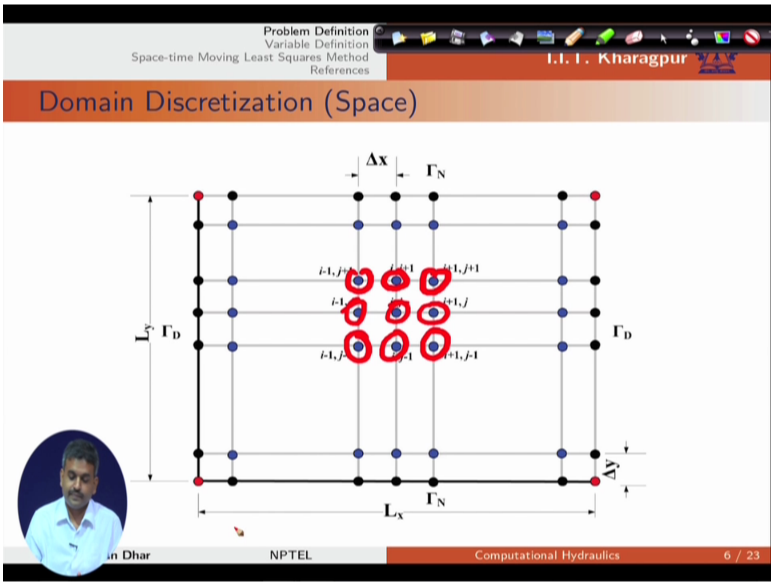

So we will have number of points within the domain and certain number of points will be there at the boundary. So forour governing equationwe need to select the interior points. And for boundary nodes we need to apply our boundary conditions. So this is all about space discretization. Now we need to consider different levelsas we have considered in our finite difference for implicit, explicit and Crank Nicolson scheme. So we will have different time slices.

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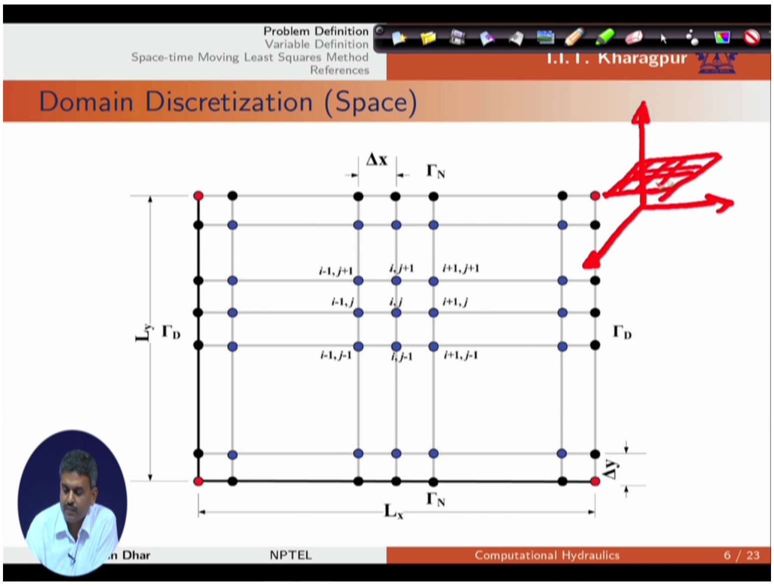

And for thatwe need to considerwe will have different time slices here. And there will be another sliceon top of this one. Let us say this is another one.

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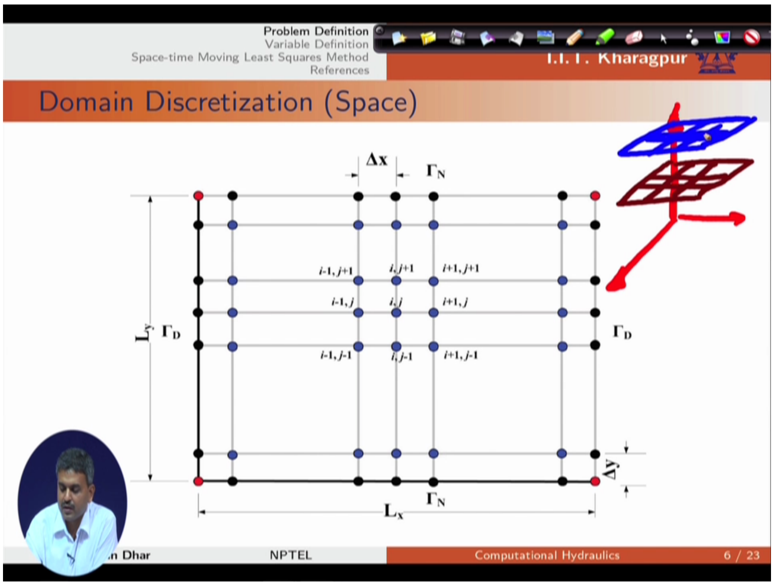

So between these two we need to apply the theory ormaybe there will be inclusion of multiple time slicesin a particular calculation. So it all depends on our temporalinfluence domain definition. If we define the temporal influence with higher value, obviously we need to consider multiple time levels

Now let us defineour space time variable. Let us say this capital X, we are defining it with arbitrary spatial position x, y, d. This is applicable fortwo dimensional in space and one dimensional in time.

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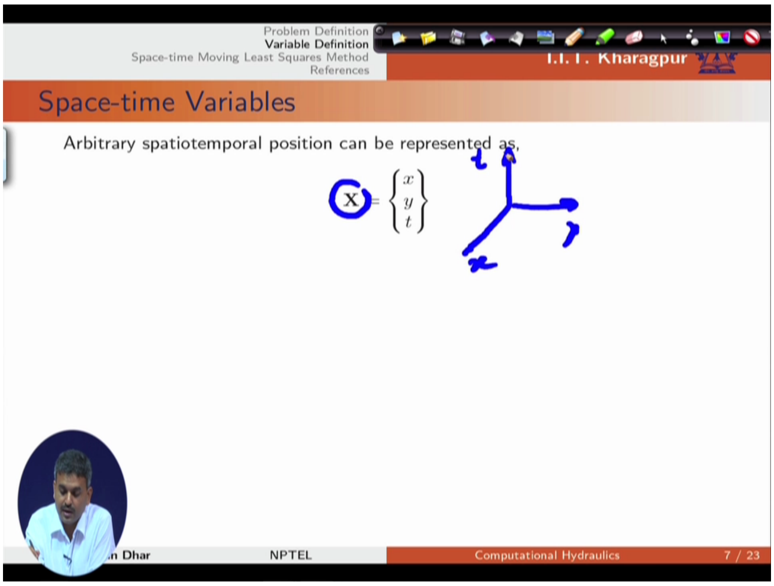

Let us define spatial temporal variable X tilde i. And in this case X tilde represents the difference between thispoint this X tilde specifically represents the difference of the spatial position with respect to any arbitrary variable x, y, t in this case. So xi minus x, yi minus y and ti minus t in this case.

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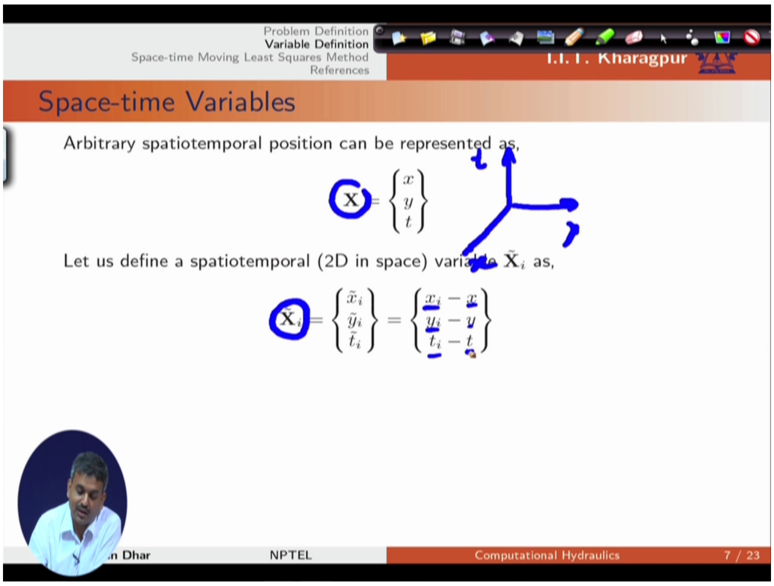

Soif we apply it for Taylor series expansion, so obviously for any general variable phithis can bewrittenwhere this is ourcolumn vector. Now in this casewe can havethis is our Jacobian and thisis our Asian matrix. Now this phi xi,if we write it as phi xplus xi minus x. So we can write this next to values that is xi minus x with this Xi tilde.

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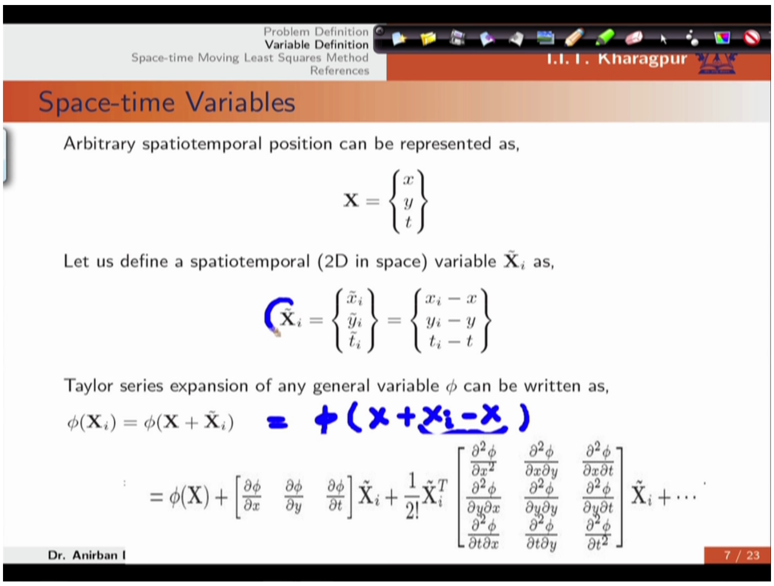

And Xi tilde is our increment from x plus this xi tilde. So let us expand it with respect to x. For any arbitrary phi xi, there will be x and del x by del x, now del phiby del x, del phi by del y, del phi by del t. So these three terms will be there. Andin Asian matrix nine terms will be there to considerthetemporal variation.

Now we can utilize this for our calculations. So in space time polynomial basis we can write in terms of 1, xi tilde, yi tilde, ti tilde. Obviously xi tilde, individually this is our xi minus x. Similarly for other terms and others are xi square, yi square, ti square, xiyi, yiti, tixi. So these terms will be there.

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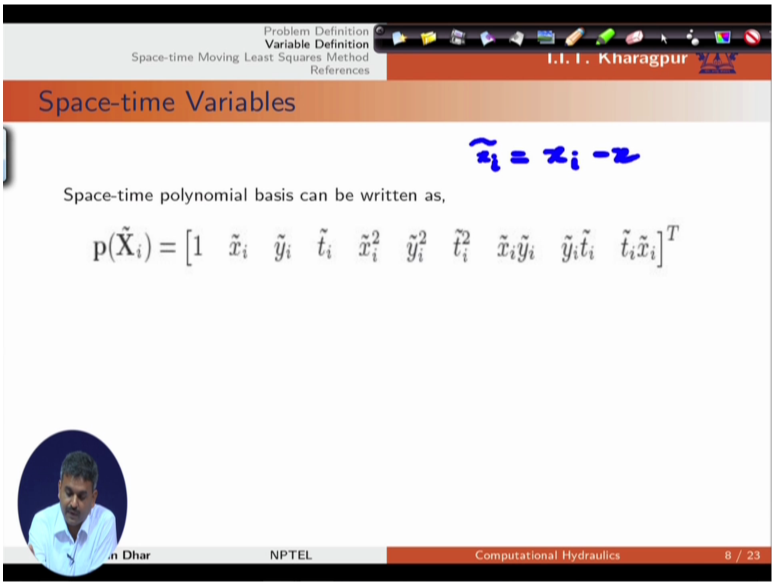

Now let us define another corresponding vector that is A vector which incorporatesall derivative terms including Jacobian, our normal function and Asian term. Soin Jacobian,obviously single derivative terms will be there, three.

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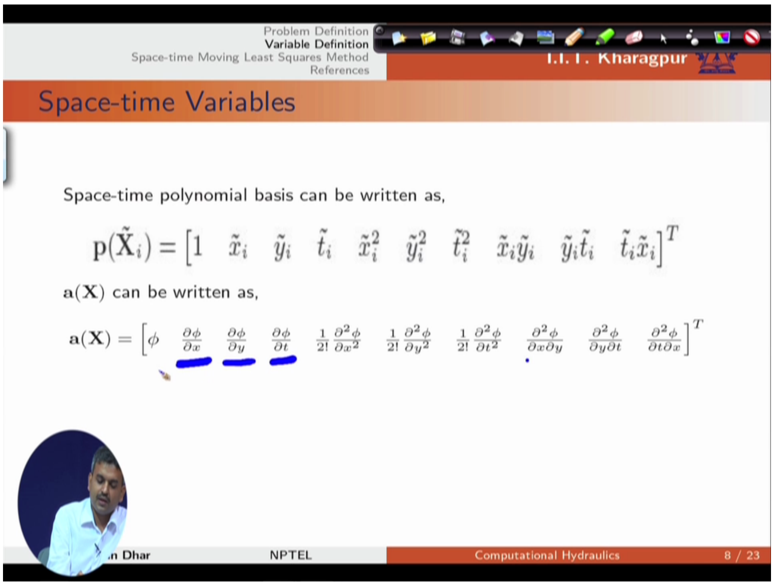

This is our function directly and this three are or rather this six individual second order derivative or mixed second order derivative, these can be obtained from our Asian matrix which is the second order derivative or the function with respect to three variables. Two in space and one in time.

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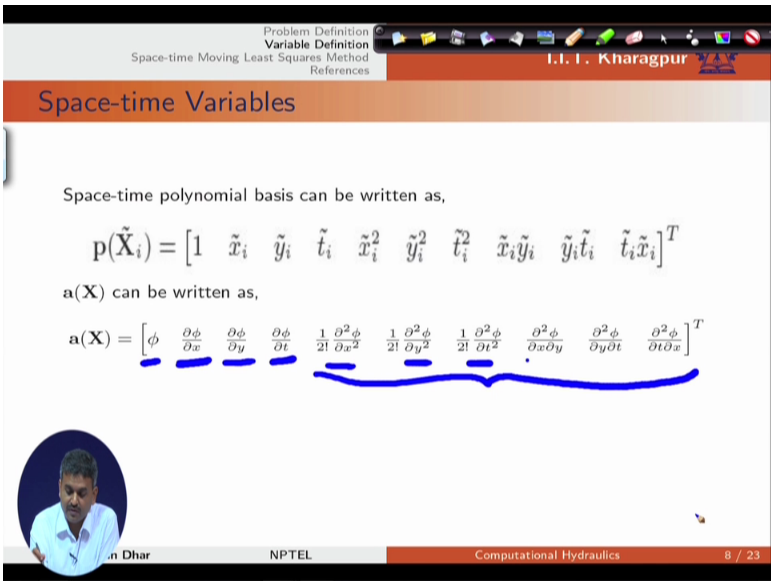

Now let us utilize this for construction of mesh free method. Now phican be estimatedfrom this one. So if you multiply this P transpose, this is actually our column vector. So this is actually column vector.

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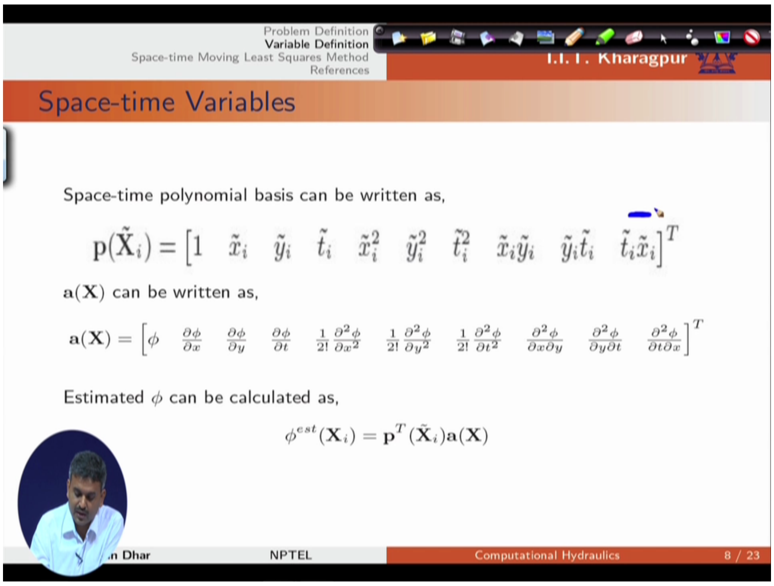

So P transpose will be here, row vector and individuallythis will be again a column vector in this case. So we can getthe mix terms and we can estimate this phi from this calculation.

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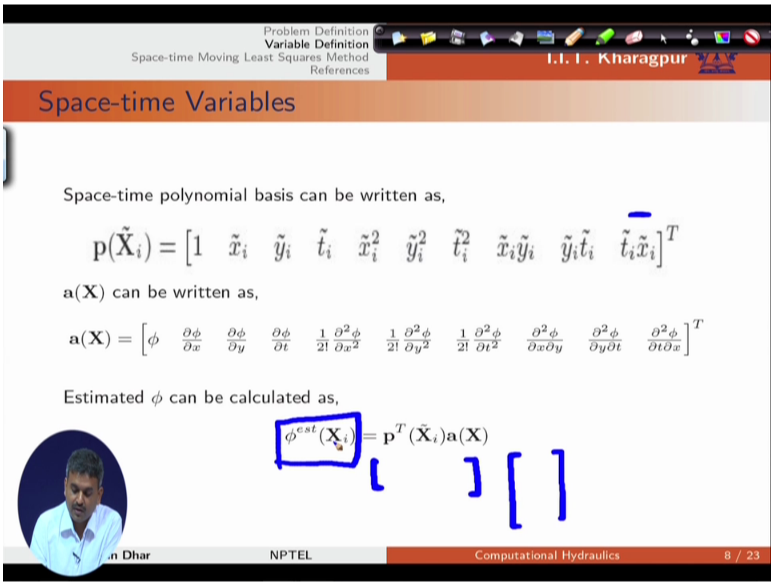

Now similar to our moving least squares method we can define weighted residual and Ns is the number of points in the support domain. It considers both spatial and temporalpoints at different levels. And this omega, this one considers the weight function. So we have to calculatethe estimated value and the actual value at phi. So we can get the difference from here.

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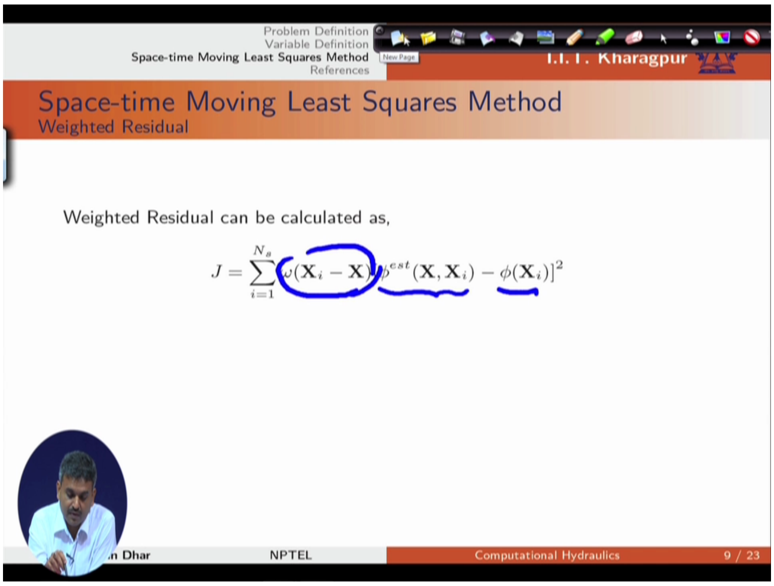

So if we further write it in terms of our polynomial basis, x,phi xi and this is our weighting function.

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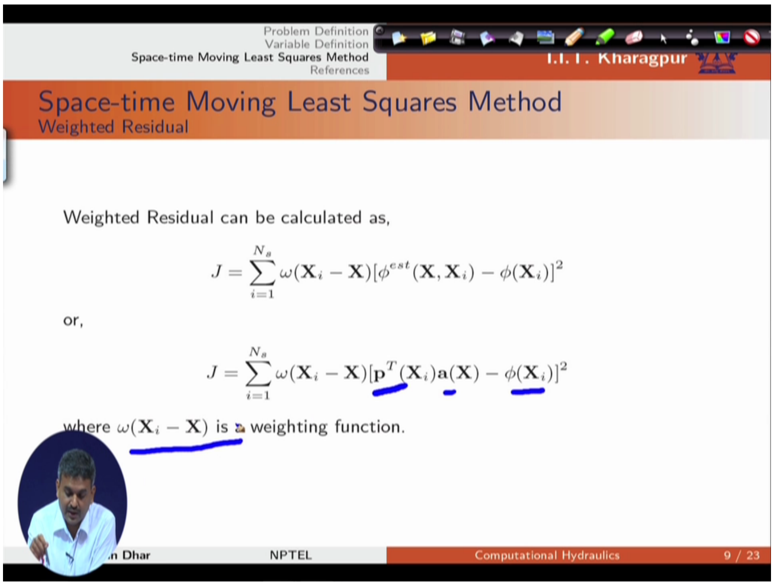

Now let us say this Jin matrix form this can be written with the help of P. Now we need to construct this P with the help of polynomial basis. A, we already know that is our derivative related function. And phi transpose isagain our phi1, phi2 to phi Ns, number of points will be there.

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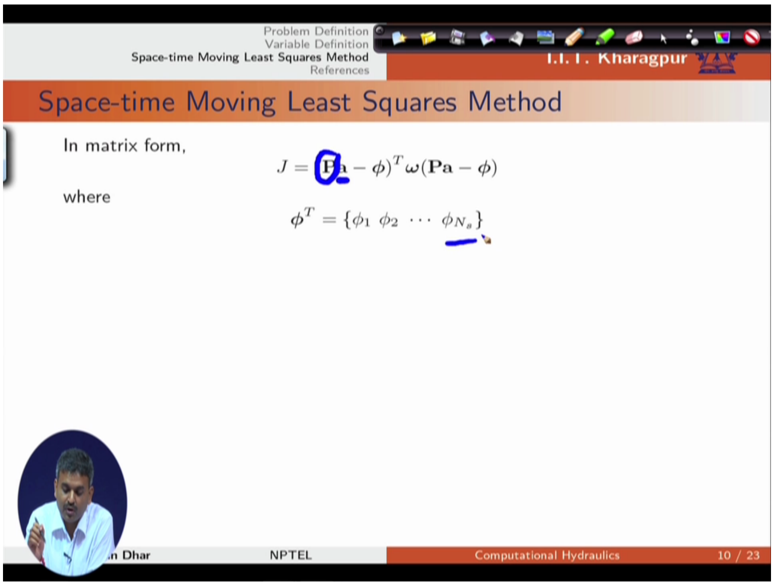

Now if we consider P, so we need to consider n number of points including the zero. So zero plus m,som plus 1 number of basis things will be there. And we have n number of points in this case or Ns is the total number. So in this case we can write our P explicitly in terms of that tilde variable that is p0 is essentially constant 1, p1 is x0, p2 will be y0, p3 will be t0 tilde, like that.

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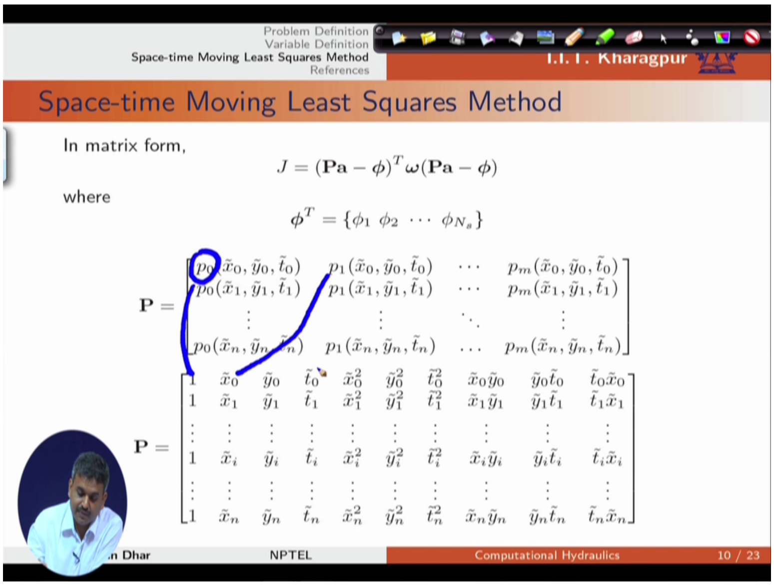

Now we can utilize this for our calculations. And in this case the weight function thingthat will be a diagonal matrix.

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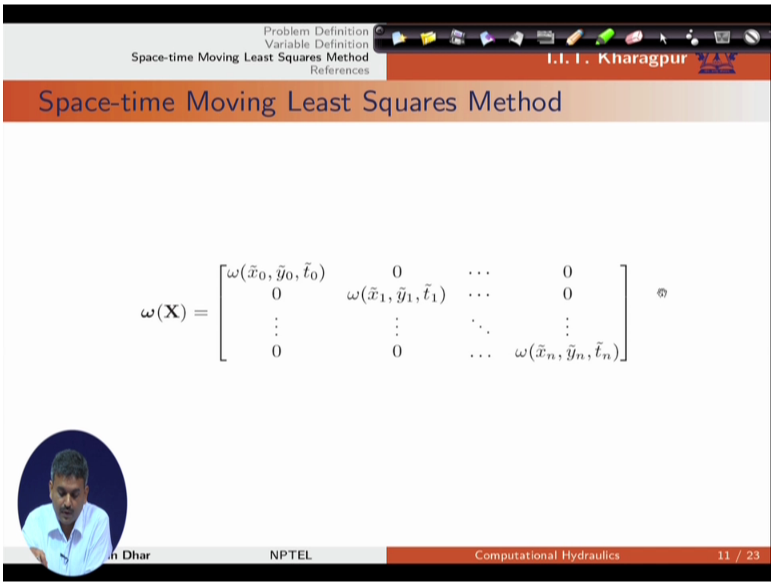

Soin this casewe will havethis J, Ax, A,Bx phi where Ax, A,Bx phi we need tocalculate. And A and B can be calculated based on omega and P value. And B again this can be calculated from P transpose this omega x. And further this Ax can be calculated directly from A inverse B, from this particular equation or expression.

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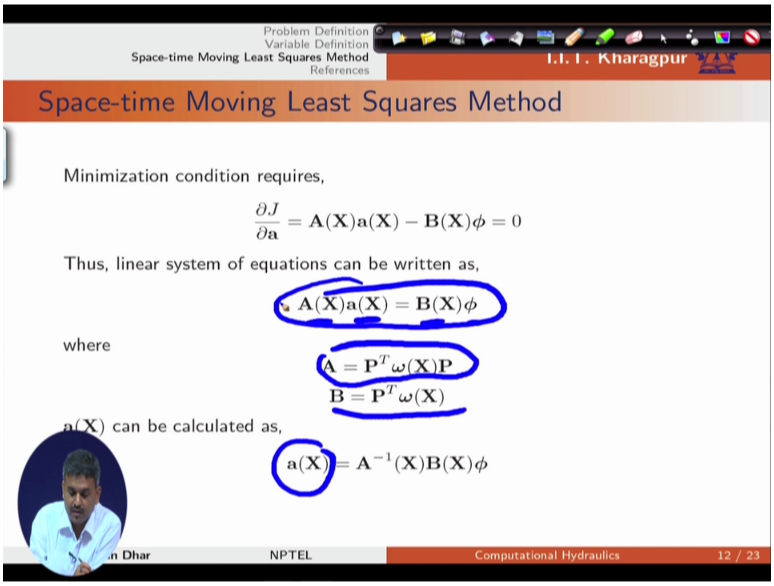

So minimization condition, we need to equate it with a zero. From that we are gettingthe next level expression which can be further calculated explicitly with polynomial basis. And Ax A inverse Bx phi, this can be calculated with the help of A and B.

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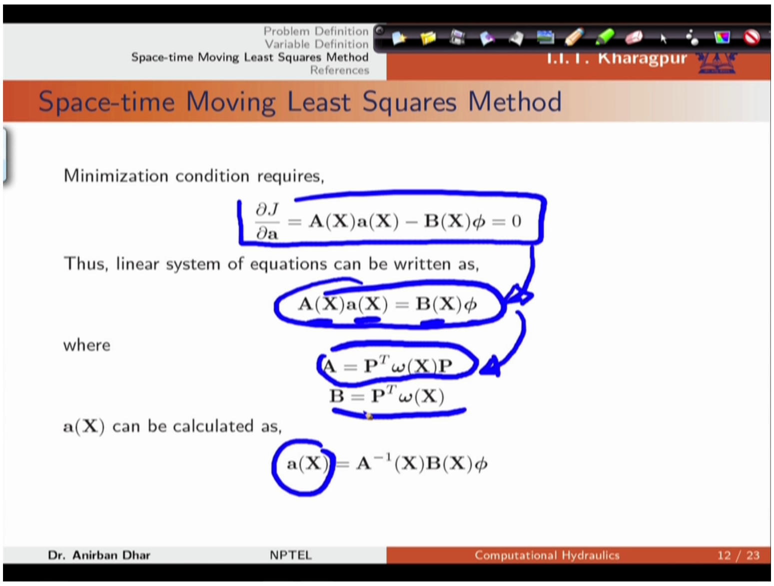

Now in this one space time moving least squares method, we have expressed A in terms of A inverse B phi. So left hand side we have derivative and right hand side we have our defined terms.Now with the help of this we can get the derivative values.

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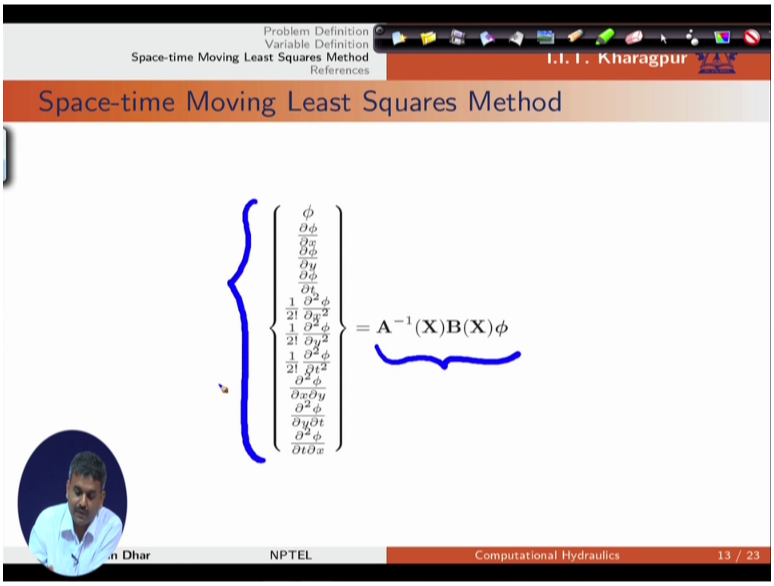

Now we need to define our weight function, this weight function can be calculated orfrom the exponential function. This sigma X, this is our weight function support. So beyond this support domain this value is zero. Otherwise this iscontinuouswithin the support domain and it is varying within the support domain.

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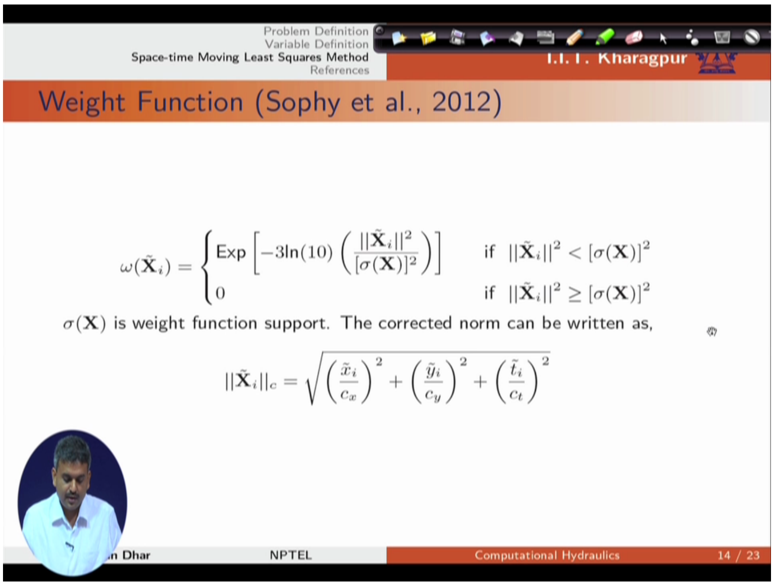

Now we can furthercorrect the norm. This is directly calculated norm but the problem is, due to the order changes in x and x, y and t, it may create problem in the weight function definition. So we can scale it with Cx, Cy and Ct values and we can correct the norm. Andwe can utilize this particular norm for ourweight function calculation.

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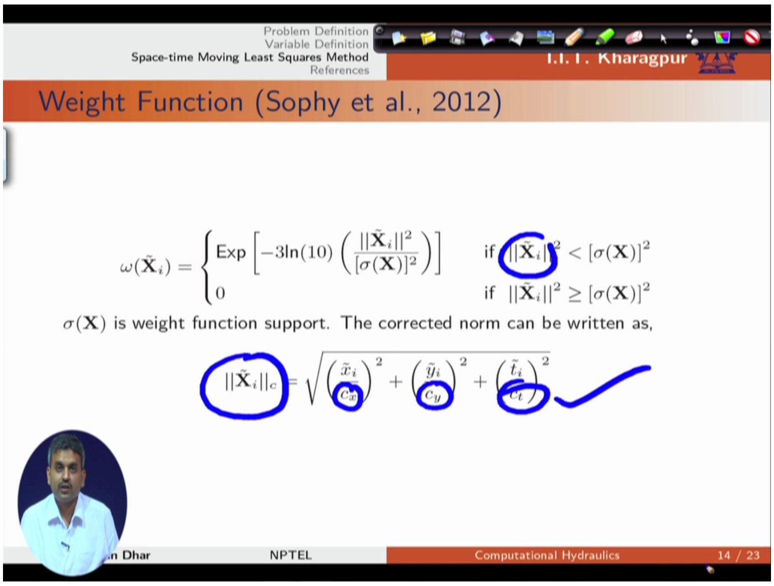

So we need to consider the influence domain. Let us say we have xt domain where it is one dimensional in space, one dimensional in time. So if we consider a particular time level which is xt domain, we can see that to calculate that we are considering multiple points. If we move on the left ward side that is with change in space values. Right ward side change is only space values. But if we move upward or downward there is change in the time values. So different time levels are incorporated in this calculation process.

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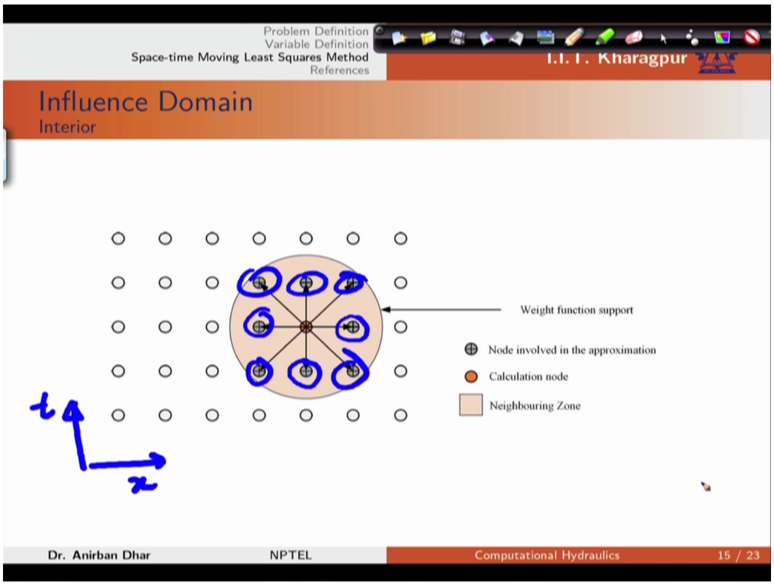

Now we need to define influence domain for boundary. If we have this xt problem where x is one dimensional and t is one dimensional then on t axis some point is there, sonodes involved in approximation will consider like this.

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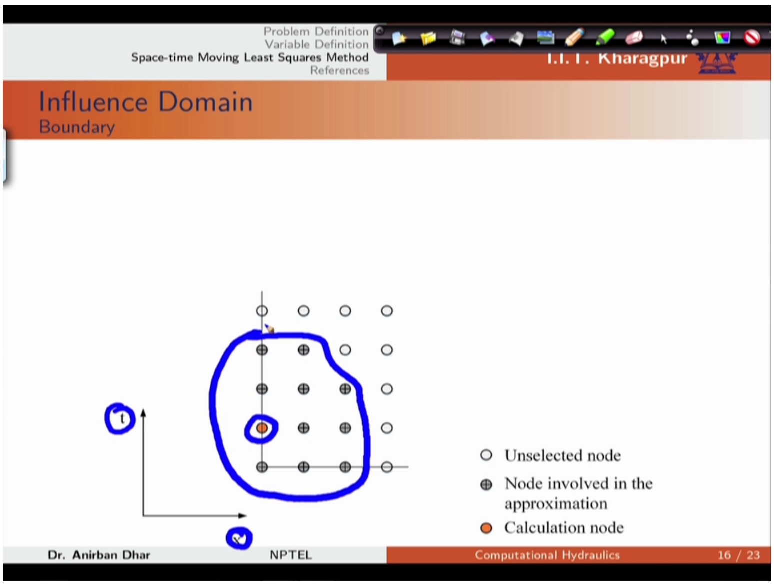

And these open circles are unselected nodes. So this orange color node is our calculation node.

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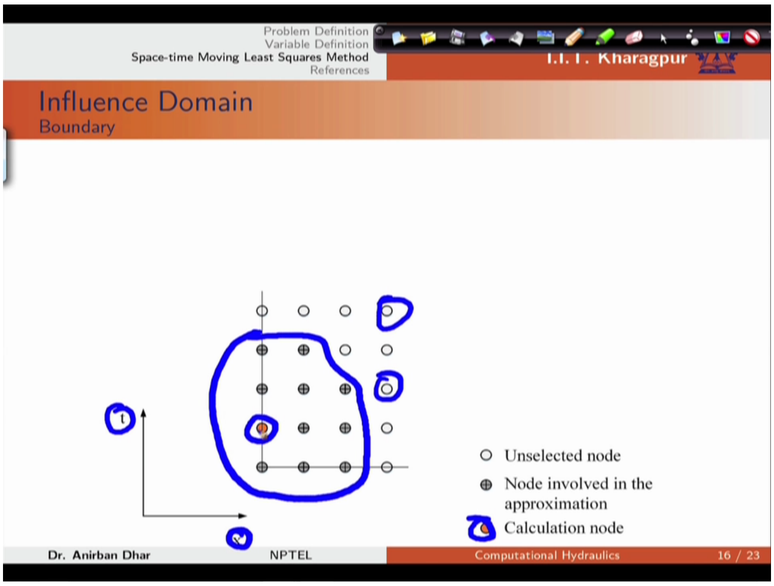

Similarlythis is the extreme case where we have a zero-zero point available here. Time is starting from zero and space is also starting from zero.

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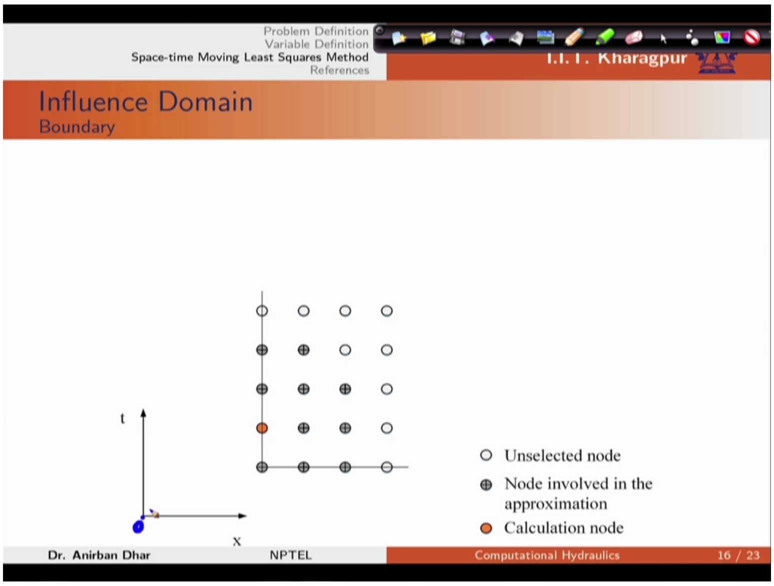

Now if we have another case where time is zero but our x is zero, but time is not zero. So it's on t axis. Butwe are considering multiple time levels. So for that calculationwe will consider only previous or past time levels and the present time level values, not at the future time level.

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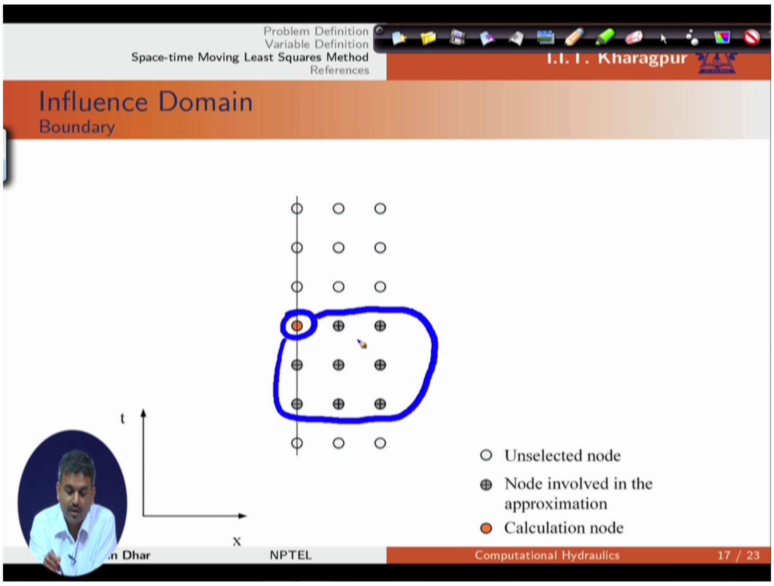

Now next level consideration. If you have zero time level and there is variation in x, so for a point which is near to boundary or space level and immediate next time level, we need to consider both future and present level time nodes.

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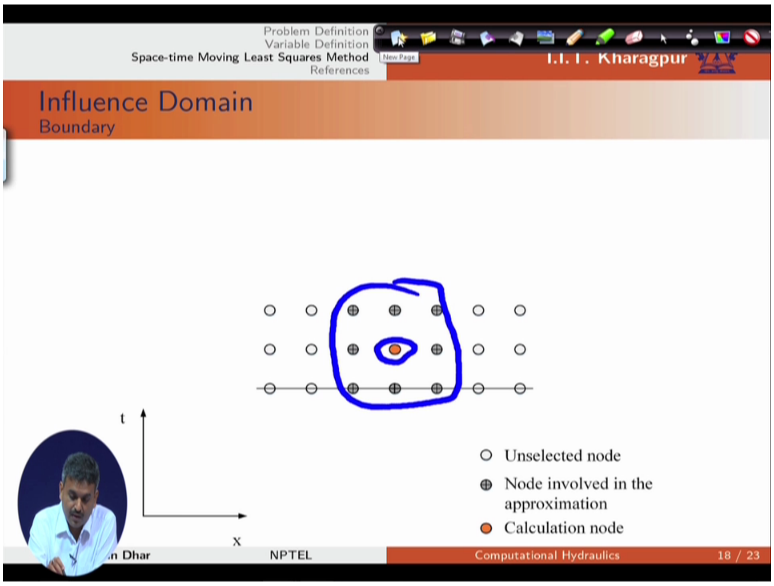

If we have in betweenother cases we can define our node like this. This triangular kind of configuration will be there for intermediate cases or internal cases.

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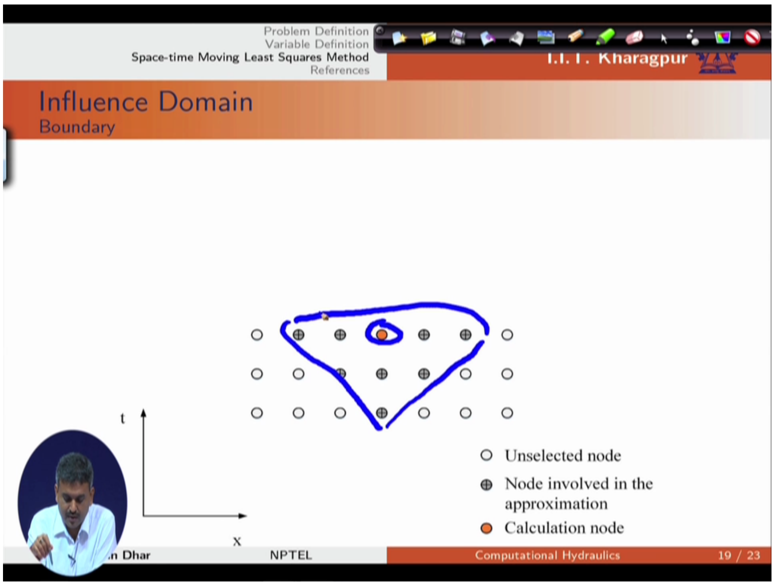

Again this is for one boundary. We can select all boundary points and justimmediatelynext level time values for this calculation.

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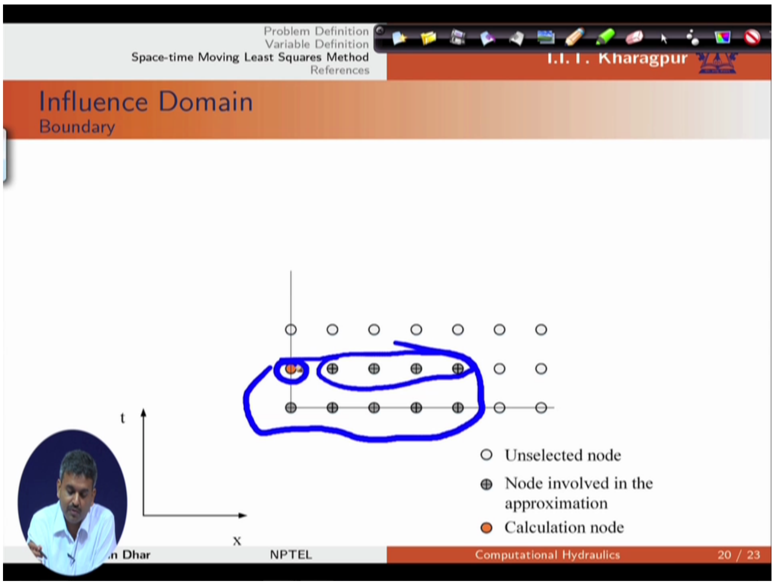

So you can see that influence domainis important in case of space time moving least squares.We need to define the nodes that we need to consider during our calculation process. So depending on thatthere will be change in the size of matrix.It may be the banded matrix or exclude one or it's a full matrix. Not a full matrix because multiple number of points will be there which are unselected nodes. So we need not to consider the influence of those points.

So with this we can start the approximation and we can solve the problem using mesh free. With this lecturewe are finishing our discretization process. Now next lecture onwardswe will consider our solution process that is solution of our A phi equals to r matrix that we have generated from our finite difference, finite volume and mesh free approximations.

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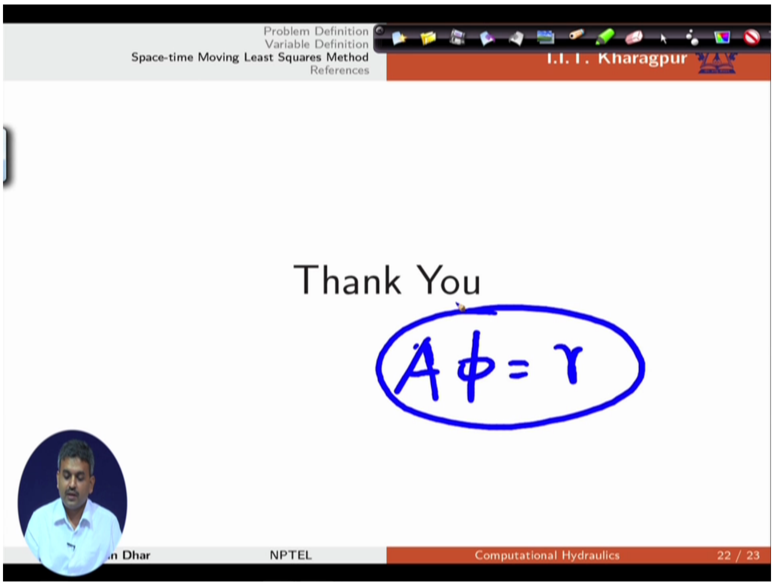

And we will try to solve thoseproblems using different numericalalgorithms. Thank you.