**Computational Hydraulics**
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**Lecture 22**
**Mesh - Free Method: Moving Least Squares Method**

Welcome to lecture number 22 of the course computational hydraulics. We are in module 2, numerical methods. And this is unit 18, mesh free method, moving least squares method. Learning objective, to discretize ordinary differential equation usingmoving least squares method.

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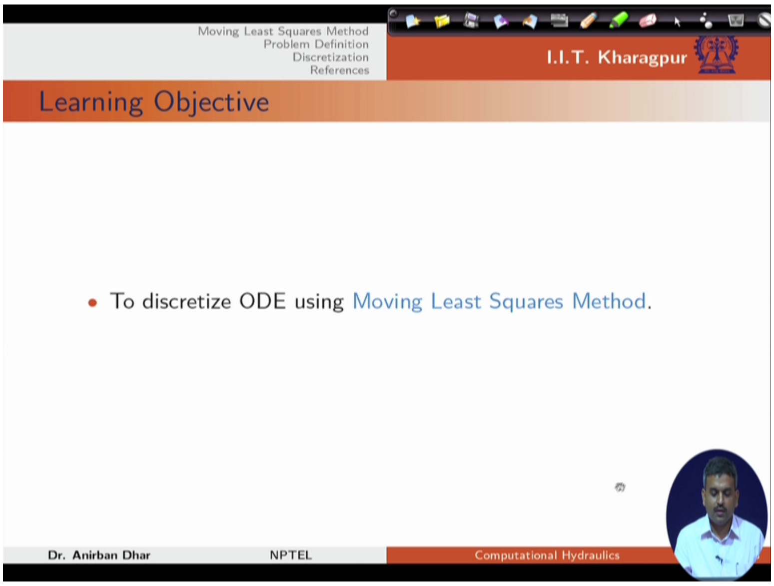


What is this moving least squares? In ourpolynomial interpolation we have represented our function in terms of P transpose A. And depending on the basis we have different kind of approximations. And we can utilize the A values which we can get from the inversion of the polynomial matrixbased onavailable function values for construction of weight functions.

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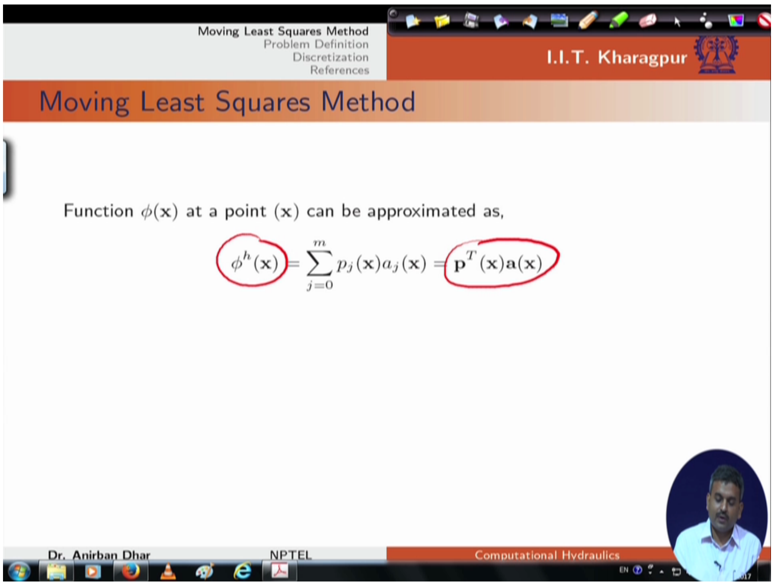


Now we can use this first step and then again we can use our basis for moving least squares.

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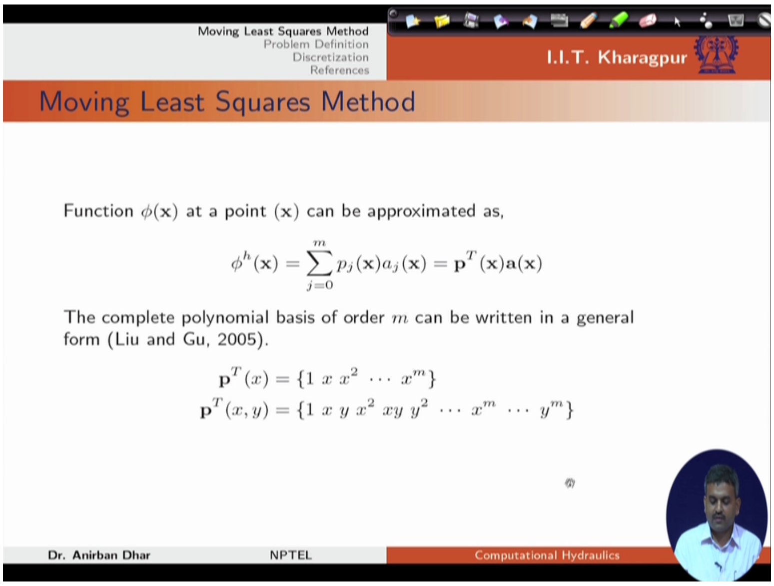


Now in local domain for arbitrary point x, we can have this phi x, xi where xi is for this polynomial and Ax is our originalvector, A vector. Now in this case MLS approximation or moving least squares method approximation is based on minimization of weighted residual for variable Ax. So Ax is variable. Previously we have got the Ax value based on inversion of matrix. Now we can represent it as minimization problem.

So in this case we have a definite structure for this omega or weight function for minimization. This h is the approximated value based on polynomial approximation. And xi is the original value for that point. So if we consider any arbitrary point, so obviously if this isour original function value and this is constructed polynomial, so obviously we will try to reduce the difference between these two values.

Now in this minimization problem the first condition is that if we differentiate this J which is the objective function for this minimization problem with respect to variables, the first order derivative should be zero.

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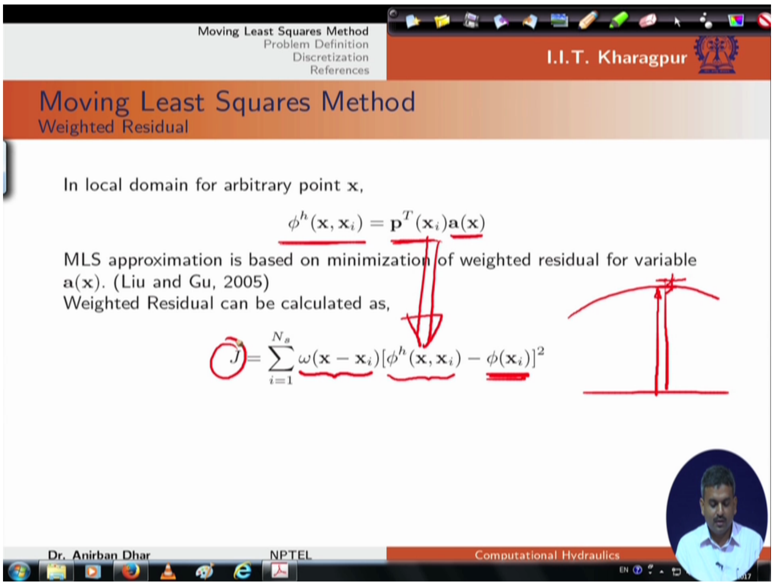


So in this caseif we replace our original form from this equation then we can write this equation as P transpose xi, Ax. And this is our again function value for phi xi. And given this omega x minus xi which is given function for this particular problem.

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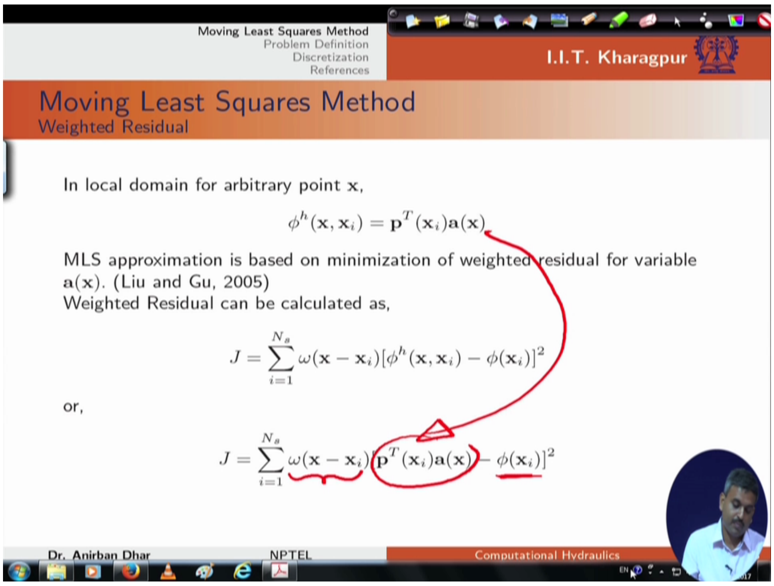


Now minimization condition, J should be differentiated with respect to A. A means for all A values. Now in this case if we differentiate with respect to A1, A2, then we will get system of equations. Because individually if we equate with zero so we will get number of terms for this particularderivative.

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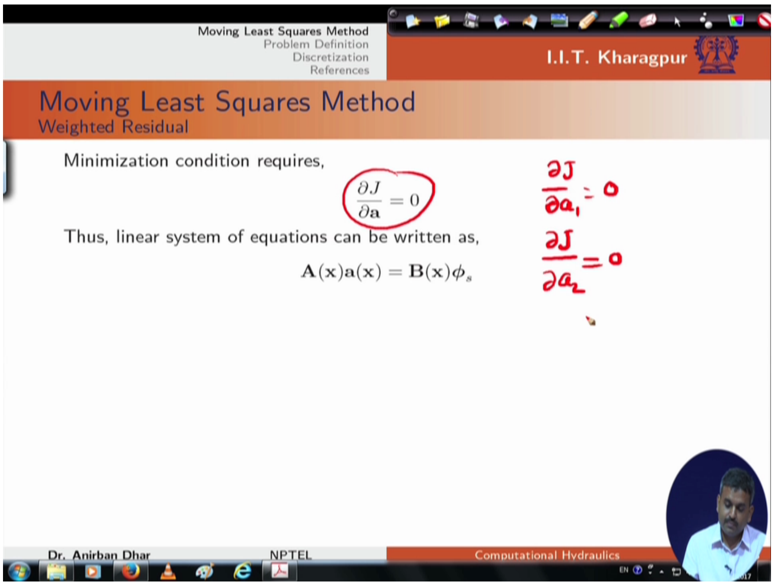


So this linear system is Ax into Ax where your A is again represented in terms of wx minus xi,P xi and P transpose xi. So in this particular casewe arewriting this and B on the right hand side, this is B1, B2 to BNs. Ns again number of points in the support domain. And Bi is our omega x minus xi, P xi. So this is for a one particular B. That means Bi if we write B1, so obviously in this case this is first point xminus x1, P x1, like that. But P is again vector in this case.

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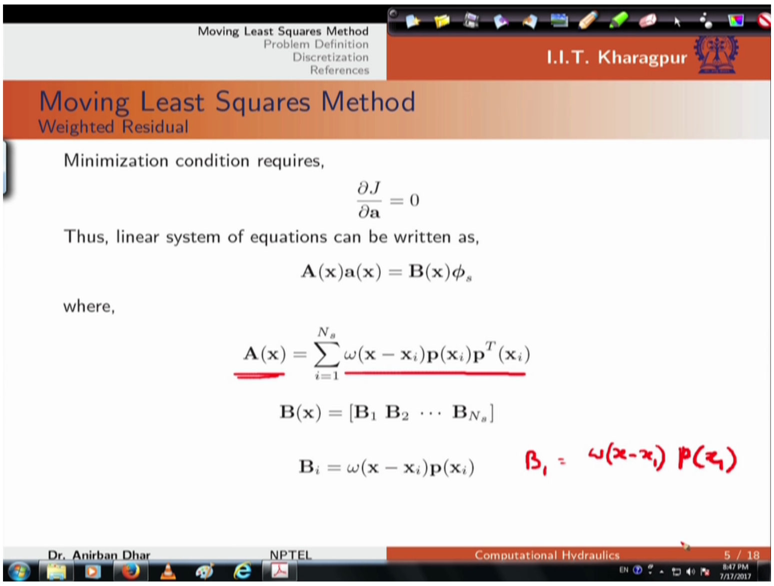


Now in this case this phi Siscolumn vector. So we have phi1, phi2 to phi Ns in this case.

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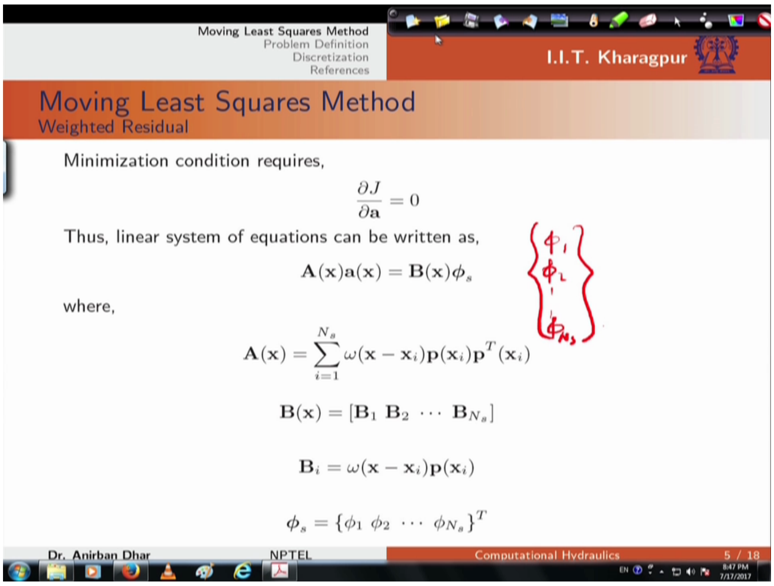


So with this we can calculate A. So we can take inverse of A into B. And phiS is the value which is available in the neighborhood of the given point. So in this case we can represent this omega i. This is x into phi i number of points in the support domain. And omega i, this can be represented as P transpose A Bi. Because we are considering only one i in this case. Sothe matrix form in this case is our W transpose x,phi s, w1 to wNs.

That means weight for all the points available in the support domain. So we can utilize thisconcept in case of our one dimensional problem.

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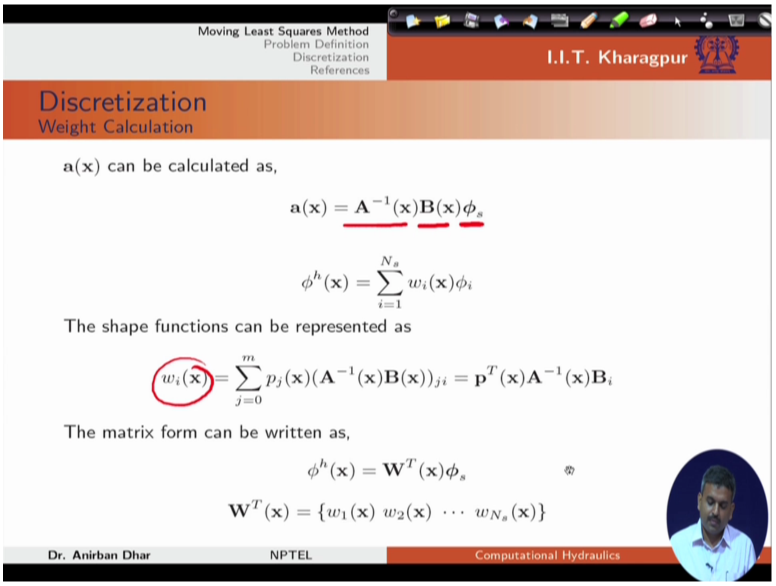


Butone problem is there. The problem is associated with the derivative calculation of W. In previous case of polynomial interpolation,the calculation of derivative is simple. But in this case this is somewhat different. So let us represent this W transpose comma x, P transpose x, A inverse B. And we need to differentiate this whole thing with respect to x.

So let us reduce this full expression into two parts. That is first term, let us write it as gamma transpose. That means gamma transpose equals to P transpose A inverse. Now if we take transpose of this one, so obviously this will be A inverse and this is P.

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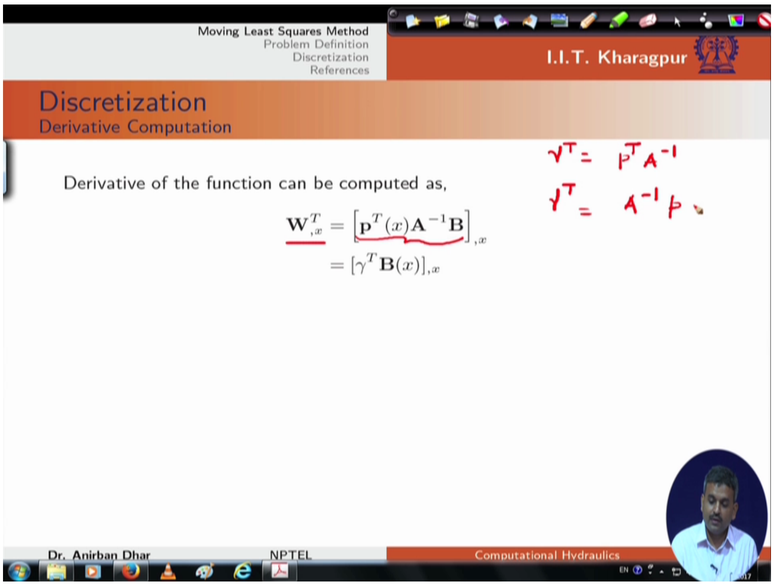


In this case we will have gamma and there will be change of side. So we can write this A gamma equals to P. So this is our form.

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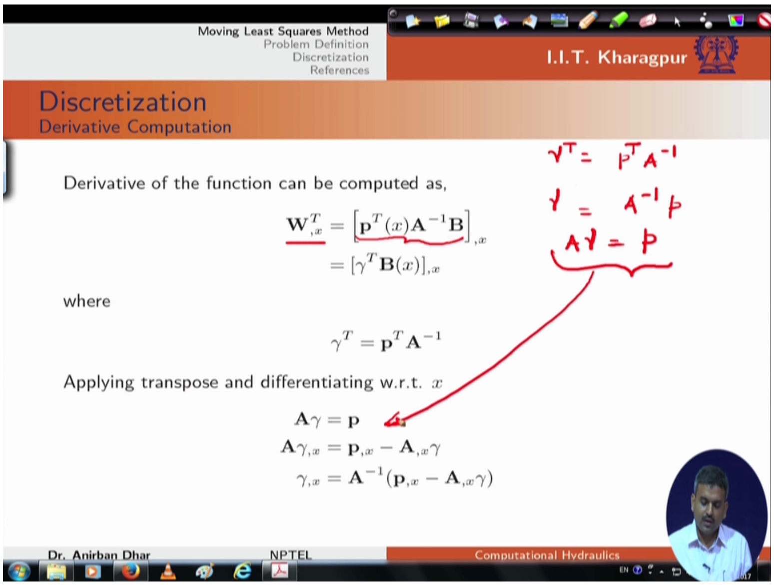


Now we know that how to calculate derivative of this. So first part this is A gamma x and this is p comma x. This is represented as A comma x gamma y. So this is nothing but full derivative. And we can write it in two parts.Plus A comma x into gamma. And we can directly equate it with P comma x and we can get this expression.

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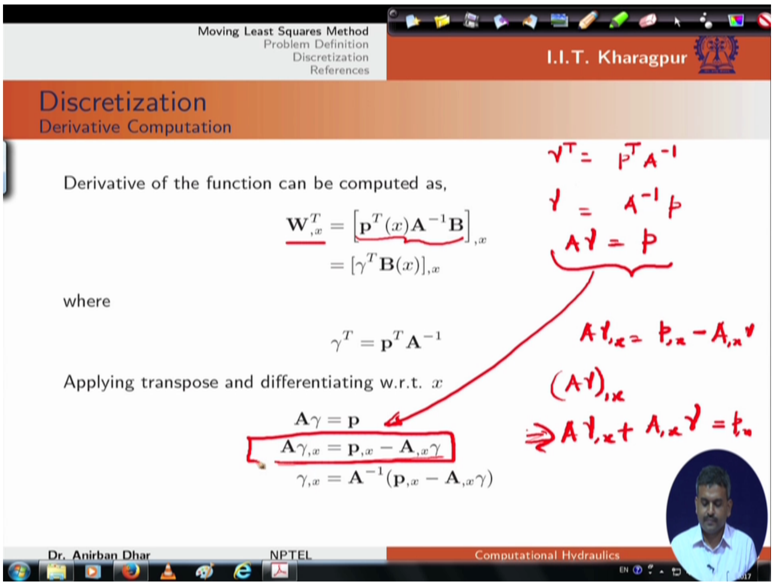


Now gamma x or first derivative of gamma can be directly calculated with A inverse, this whole thing, P comma xminus A comma x into gamma. So this is all about first order derivative of gamma, that is gamma comma x.

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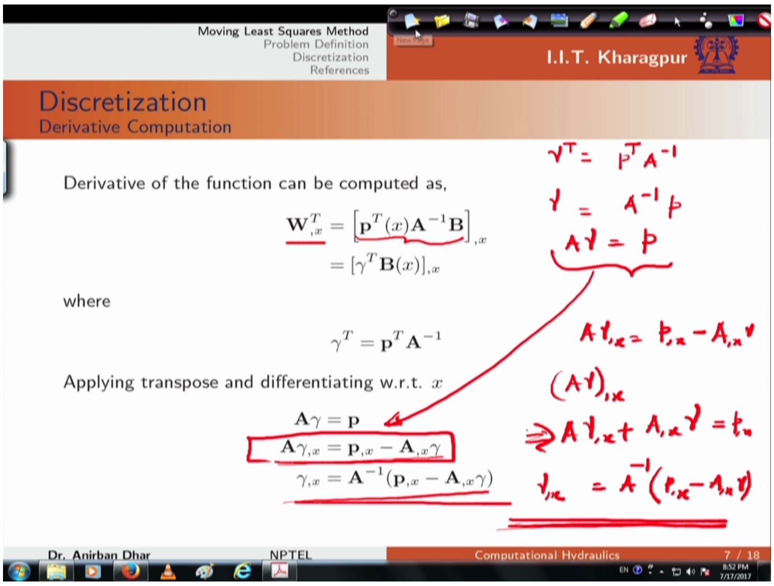


Now similarlywe can write the same thing. So if we have A gamma comma x equals to P comma x, A comma x into gamma. Now in this casewe can again differentiate that means if we have A gamma twice, differentiate the same thing. So this is A comma x into gammaplusA gamma comma x. And this whole thing is again comma x. This commais our first derivative.

So this is A comma double x, this is gamma plus A comma x gamma comma x plus, from the second term we get A comma x gamma comma x plus A gamma comma double x equals to P comma xx. So from this one we can simplify A comma xx gamma plus 2 A comma x gammacomma x plus A gamma comma double x equals to P comma xx. So from this one we can directly calculate the second order derivative of gamma xx, like this.

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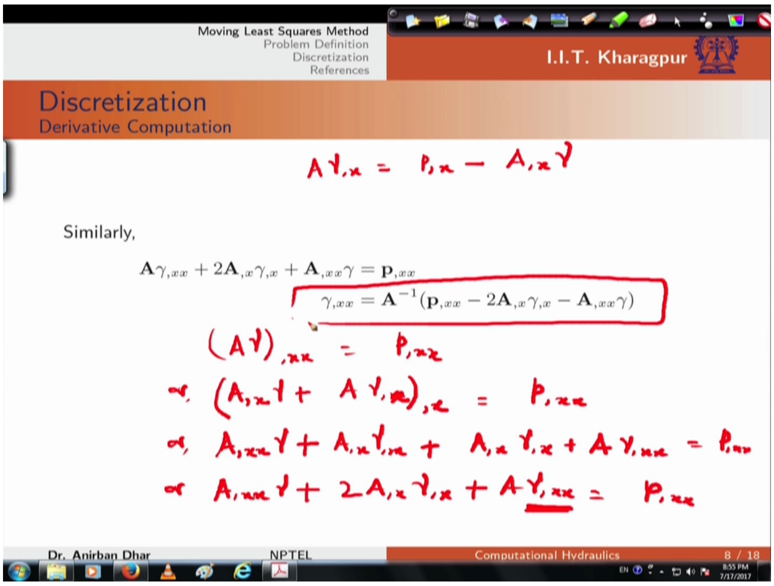


So this is all about calculationby indirect method. We are not directly differentiating the W with individual terms. We have transformed it with a new variable gamma. And we are calculating the derivative of that function. So first and second order derivatives can be calculated like this. So now the full thing is gamma transpose comma x.This is our first derivative and this is second derivative.

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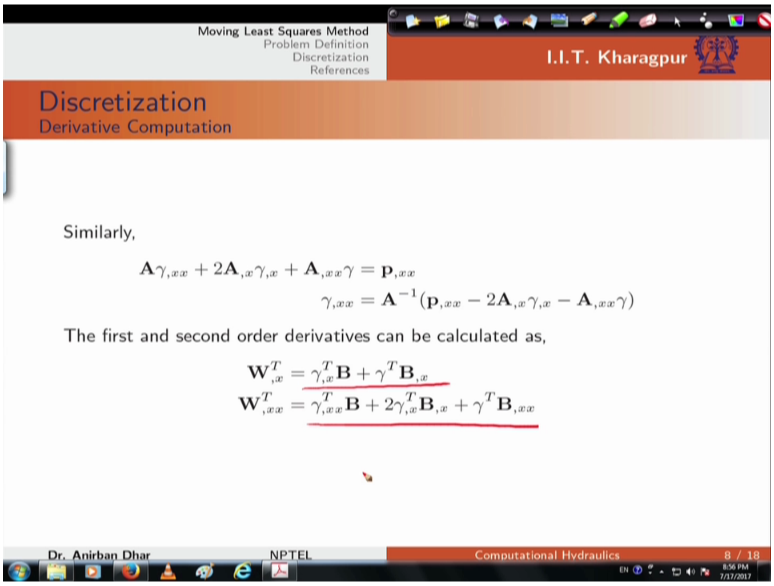


We already know the value of gamma comma x and gamma comma xx. We can utilize that value for calculation of first and second order derivative of our weight functions.

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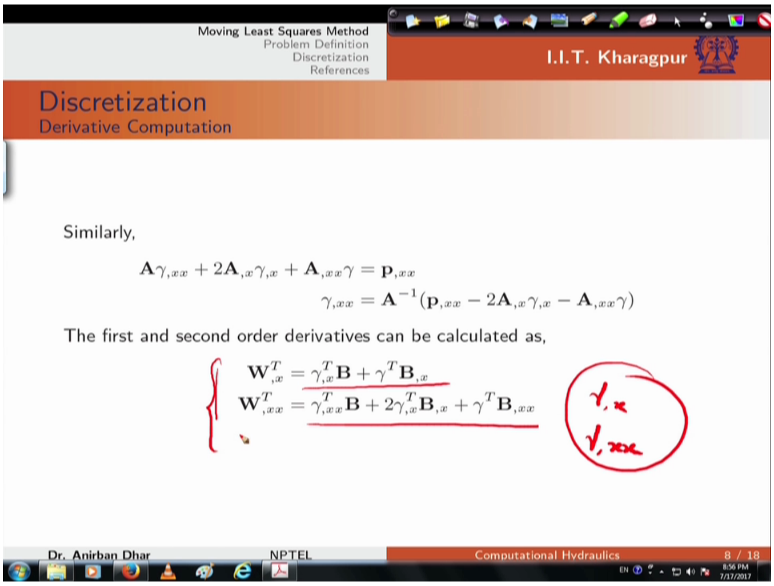


Now in this caseagain let us consider our original problem. Original problem is one dimensional groundwater flow equation.Mathematical conceptualization is in terms of second order derivative. And a support domain we can have2-3 or number of points in this case. Using mesh free shape function, we can represent it like this.

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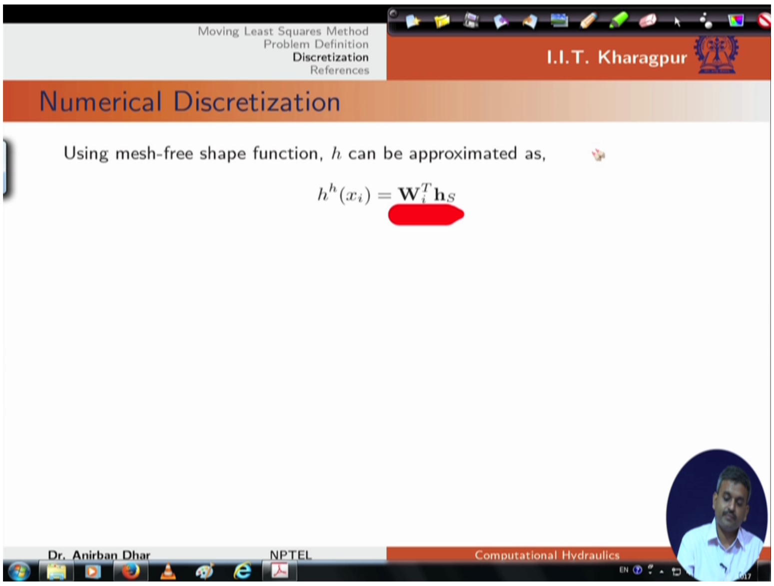


Now in this case derivative can be approximated like this. Now derivative calculation we have already performed usingthat by introducing new variable gamma.If we have Ns number of nodes present in the local support domain, obviouslywe should include all points.

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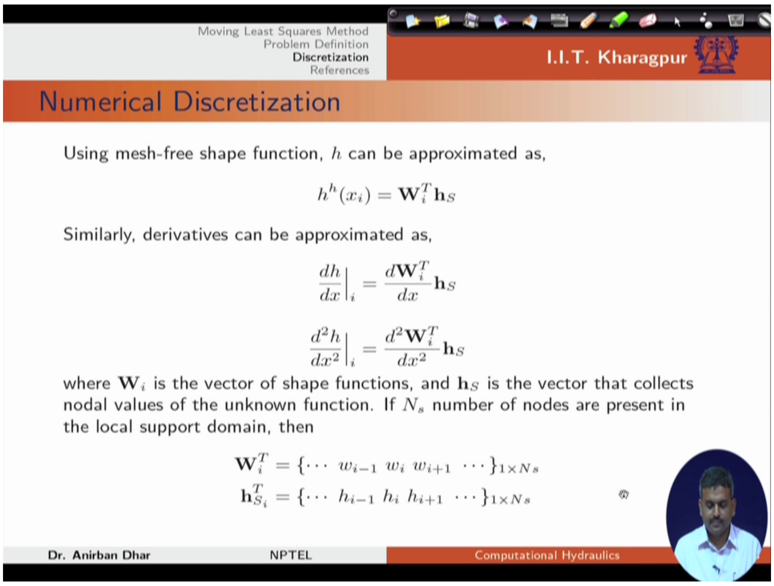


Now in this case our calculation is not straight forward like our finite difference of mesh free method of polynomial interpolation. If you have Ns number of nodes present in the local support domain, for interior nodes equation is notsimplified form. Onlything is thatproblematic term is this one.Because it involvesthe calculation of our shape functions.

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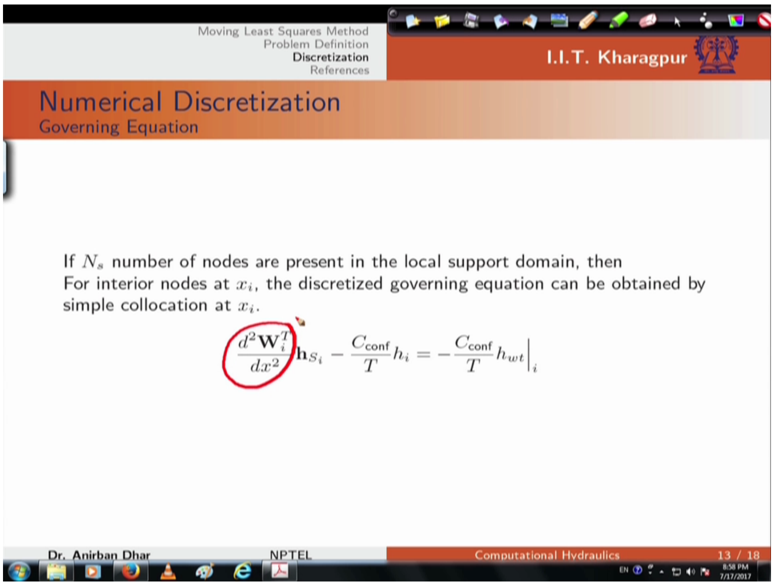


So in this casepolynomial basis of order 2 can be used. So if we use polynomial basis of order 2 that means we are using in this case 1, x, x square. So let us say that L is the index for support domain or points in the support domain. So L equals to 1 to Ns. That means all points in the support domain. We can directly write thisor if you multiply this two terms, we can directly write this thing. Problem is with the structure of this omega xminus xL.

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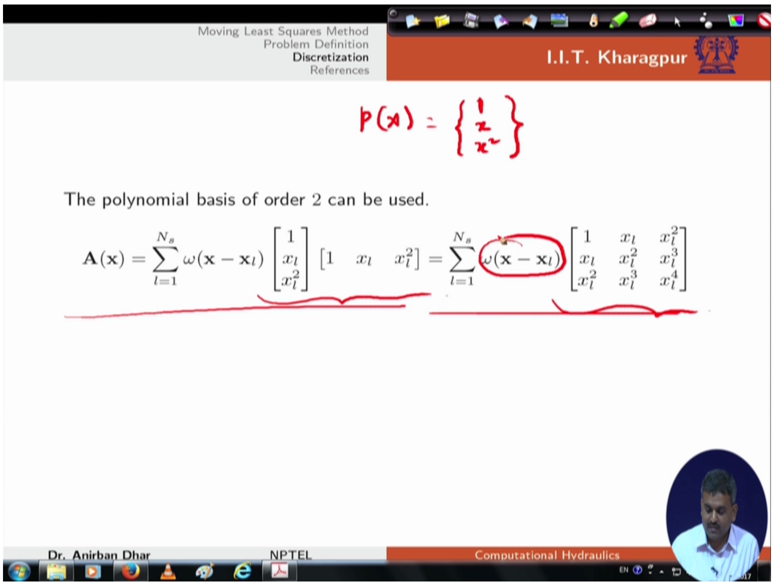


Now we need to define the structure of this one. Because this is a given function uh for our problem. Only A was a variable. So BL again, structure of BL for any arbitrary L. There is x minus xL, 1, xL, xL square.

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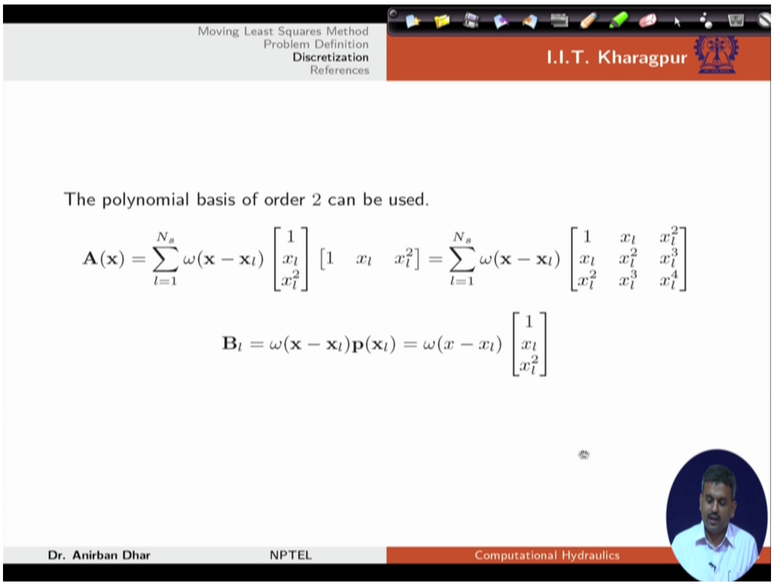


Now characteristic of this weight function, omega x minus xi should be greater than zero within support domain.Should be zero outside support domain andis sufficiently smooth. That means smooth function is required for this weight calculation. And this omega x minus xi, this one is monotonically decreasing from point of interest towards boundary. So if we have x minus xi by xr, this is our radius.

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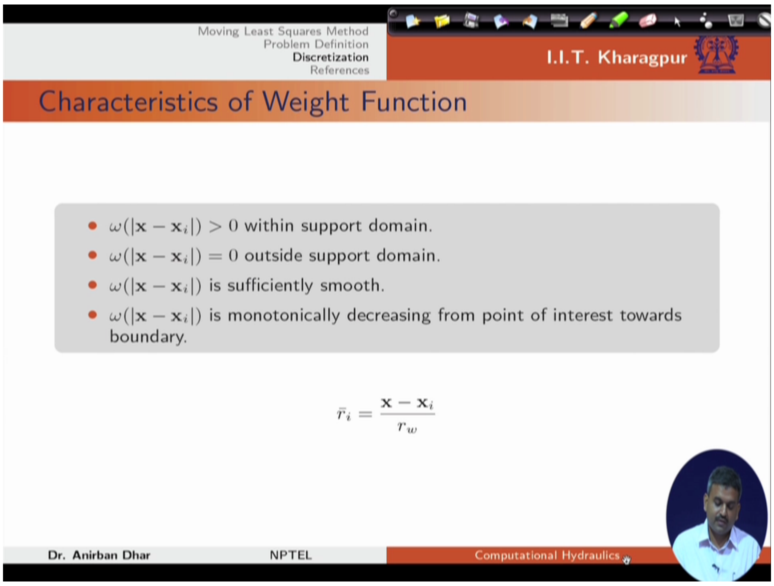


So we can use differentor weight functions. So in this caseour first representation is in terms of cubic spline. So if r greater than 1, r greater than 1 means outside support domain. So outside support domain it is zero. That is ri barequals tox minus xi by rw, this is normalized one. Radius starting from zero to 1.From 0.5 to 1 we have one variation and below this 0.5 we have another kind of variation.

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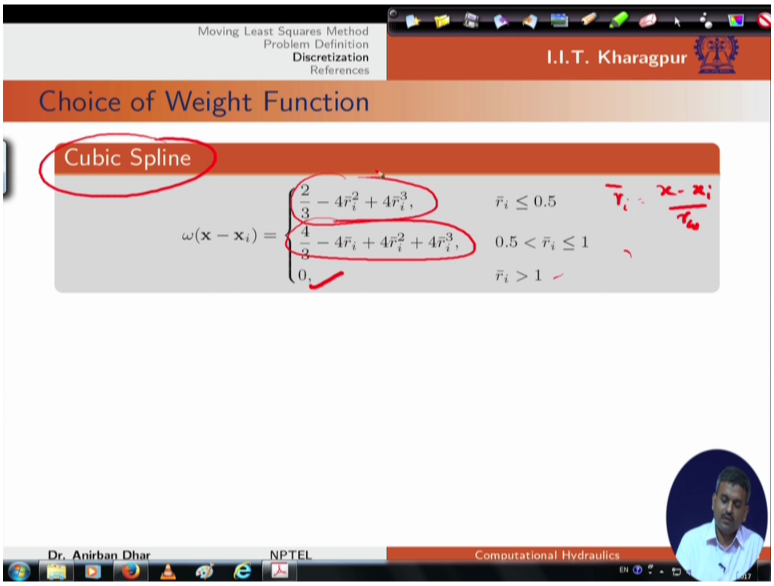


So this is continuous function.Again we can use this weight functions for calculation of our problems. We can have exponential function where alpha is some defined value.Ri again greater than 1, this iszero. And within this domain or ri less than equals to 1, we have this exponentially decaying function. That means at point of interest this is equal to1. A to the power zero is 1.

And then exponentially decaying value.If we have quartic spline.In this case again we have, within support domain it is defined in terms of some function which is varying with ri. And outside support domain again this is zero. So these three are typical example for choice of weight function or for the MLS approximation.

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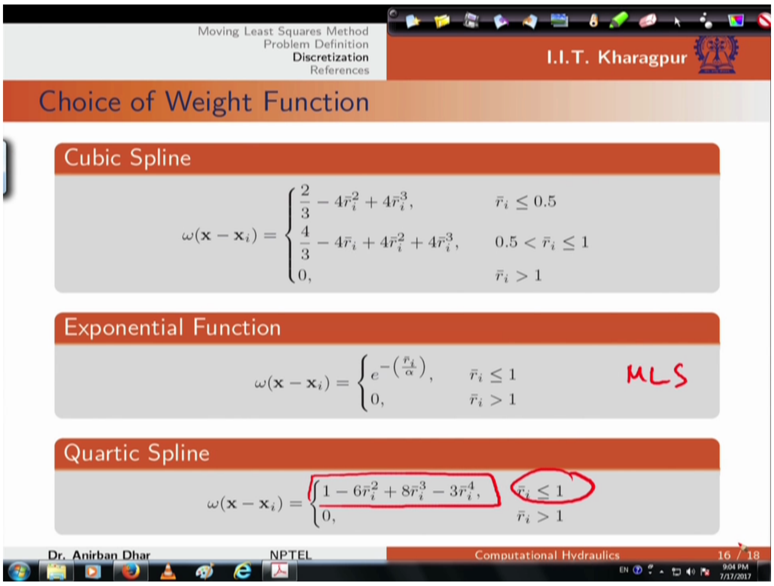


So we can see that in our MLS approximation, we are not getting any explicit formlike P polynomial interpolation. In case of polynomial interpolation we got our two things that is for interior points and for boundary pointstwo equations or expressions which are similar to finite difference method. Butin case of MLS we are not getting any explicit form of the equation. Indirectly we need to solve the equation and then only we can get the solution out of this method. Thank you.