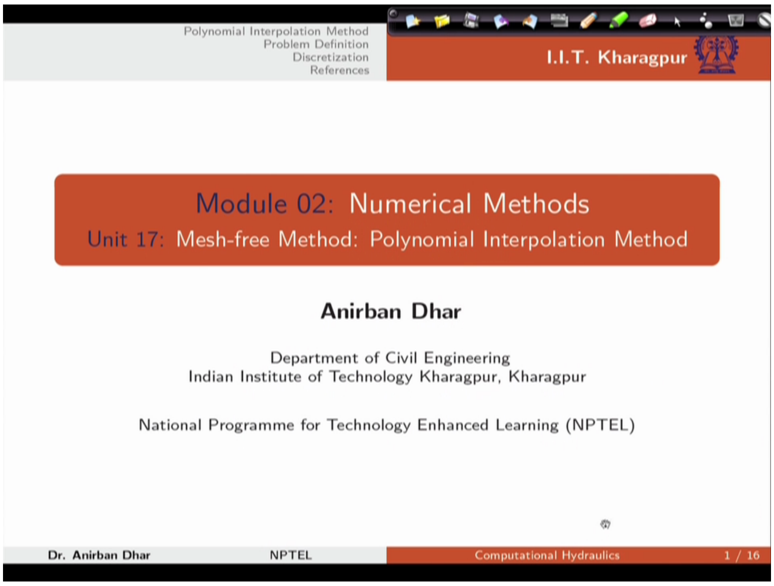
Computational Hydraulics
ProfessorAnirban Dhar
Department of Civil Engineering
Indian Institute of Technology Kharagpur
Lecture21
Mesh-Free Method: Polynomial Interpolation Method

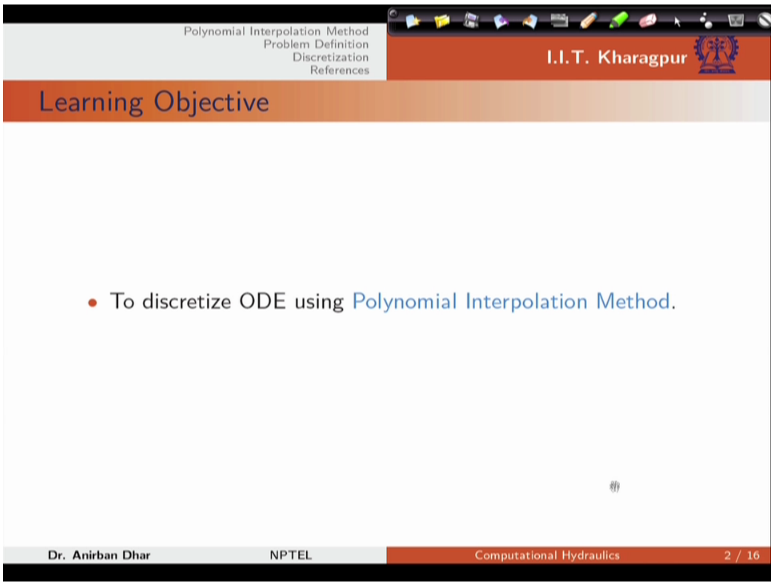
Welcome tolecture 21 of the course computational hydraulics.We are in module 2, numerical methods. And in this particular lecture class I will be covering unit 17 that is mesh free methodminus polynomial interpolation method.

(Refer Slide Time 00:30)



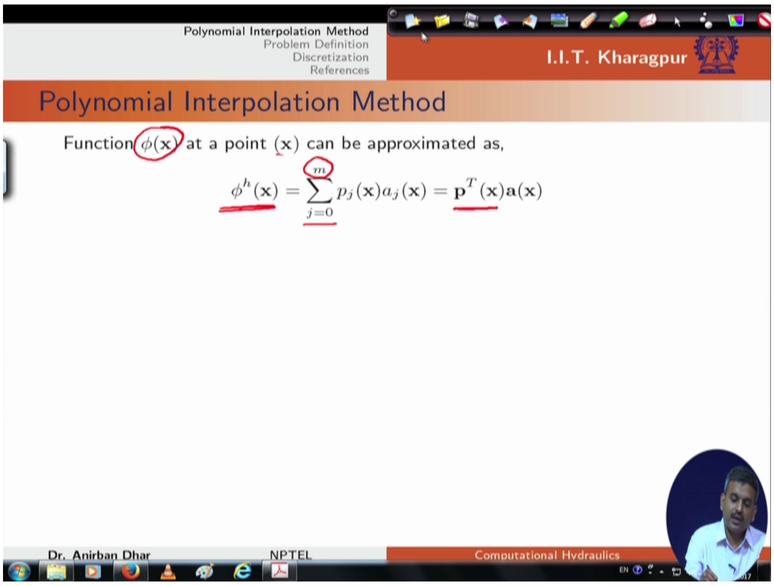
Learning objective of this particular unit. At the end of this unit students will be able to discretize ordinary differential equation using polynomial interpolation method.

(Refer Slide Time 00:55)



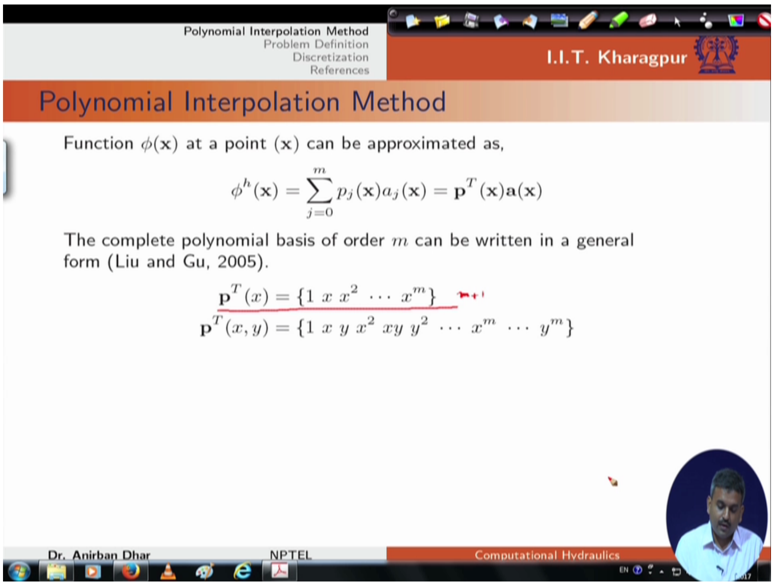
Polynomial interpolation method. Function phi can be represented at a point x with the approximationphi h starting from j is equal to zero to m. This is Pj x aj x. And we can represent it in terms of vectors like P transpose a.

(Refer Slide Time 01:38)



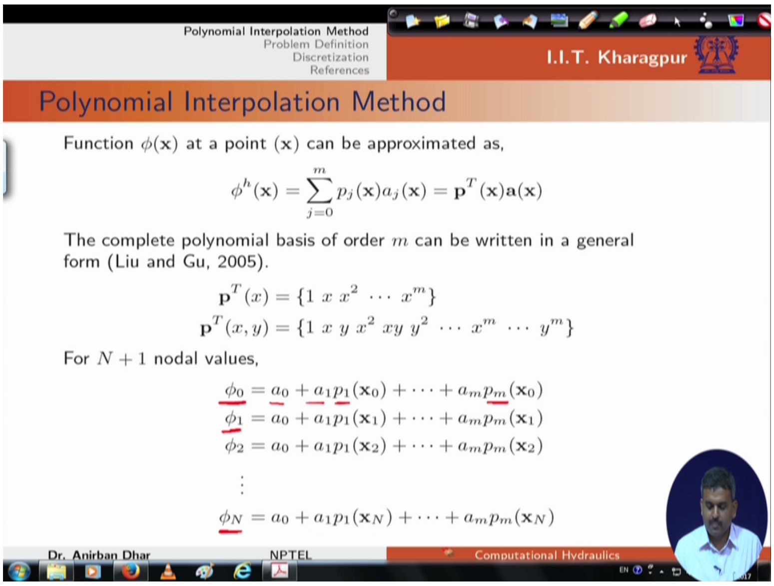
So in this casewe can use the concept that we have learnt in our previous lecture class.This is P transpose which is nothingbut for one dimension this is 1 x squareup to x to the power m. That means you will have mplus 1 number of terms here.

(Refer Slide Time 02:12)



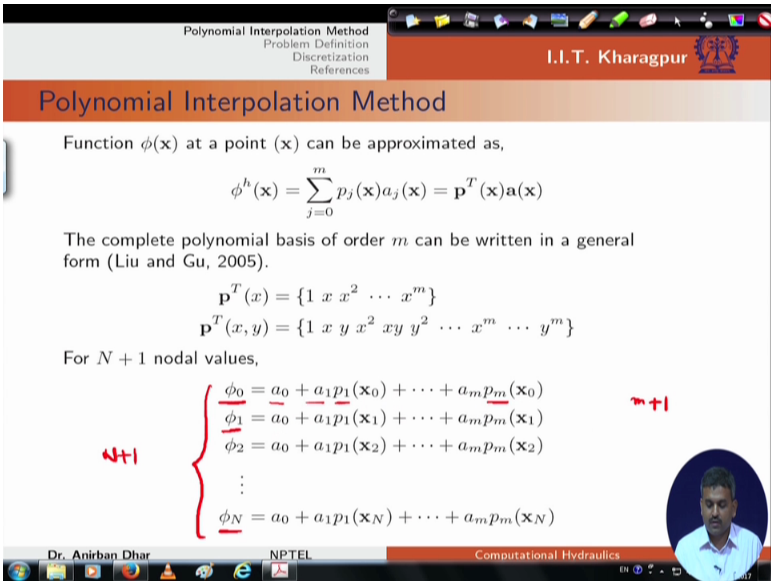
Similarly in this case you have xy 1 x, x square, x y, y square. This expansion is there. So for m plus 1 nodal valuesfor phi0 we can have a0, a1, p1 to pm. And phi1 this is phiN. Similarly we can represent.

(Refer Slide Time 02:56)



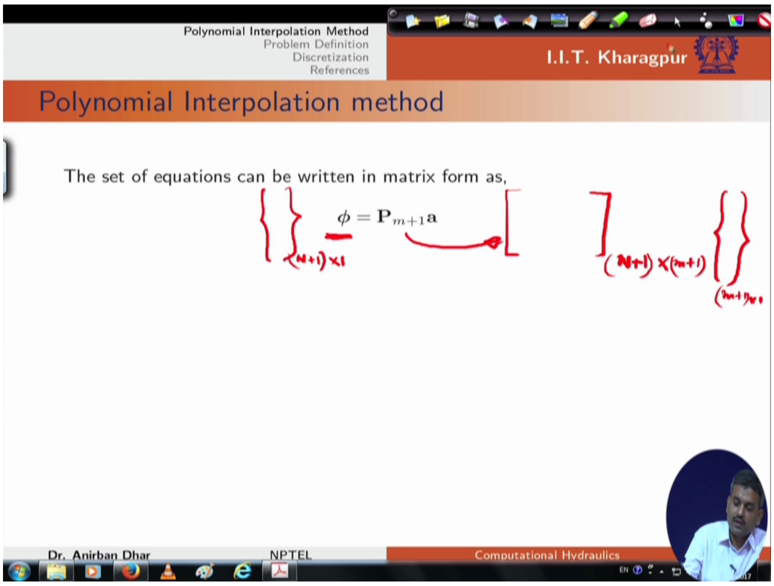
So starting with this we have n plus one nodal values. Similarly we have mplus 1polynomialfunction values or A in this case.

(Refer Slide Time 03:17)



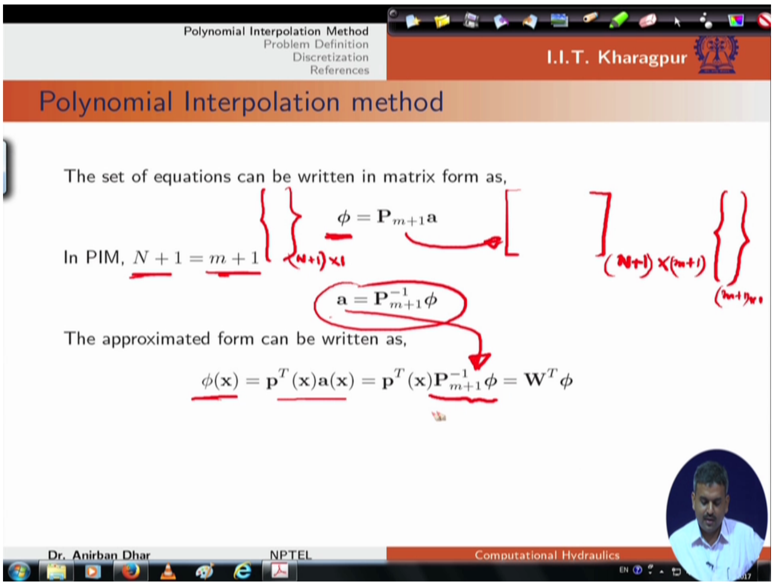
So let us use this for our calculation. We have represented these m plus 1 nodal values in terms of A and P. Now if we write itin the form of matrix, we can write this like phi which is column vector. Again P is a matrix. This is our size of m plus 1 into 1 and P is of size. This is our m n plus 1 into m plus 1. And we have this A again. This is mplus 1 into 1. So with thiswe can start our calculation.

(Refer Slide Time 04:51)



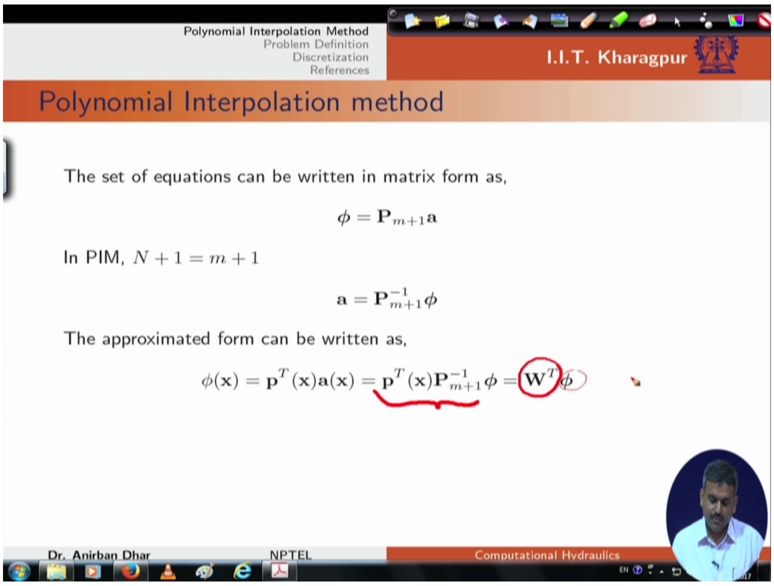
We can get for nplus 1 equals m plus 1, we can invert this matrix. So in this case we can get phi which is a field variable in terms of P transpose x. So P transpose, if we replace this A from this equation, we can get P inverse phi.

(Refer Slide Time 05:42)



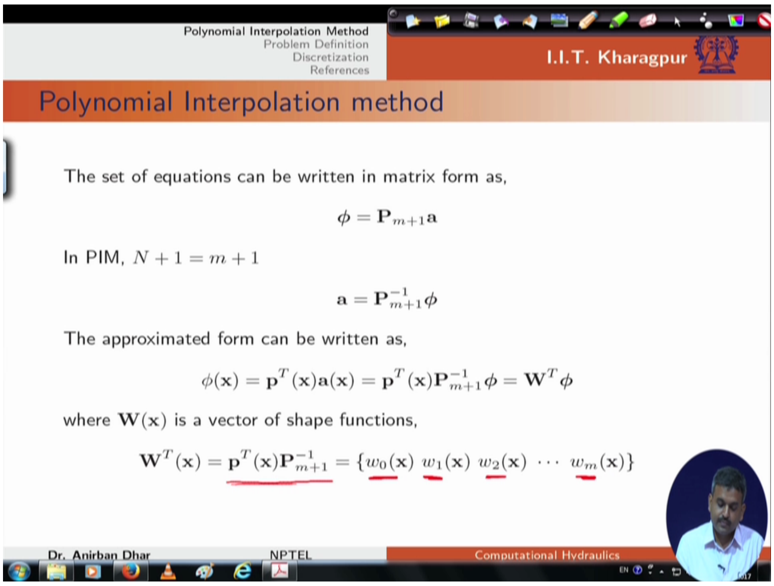
So we can represent this first portion P transpose P inverse mplus 1 as W transpose which is our weight function. And phi isindividual function values.

(Refer Slide Time 06:01)



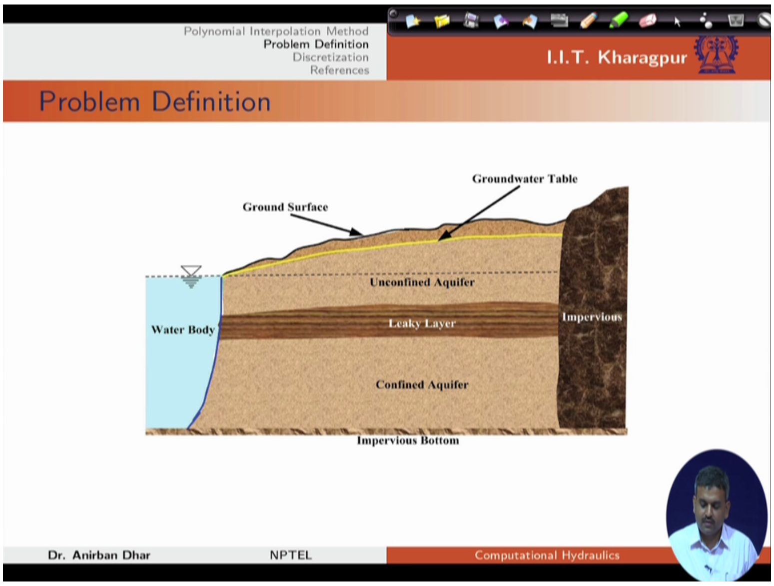
Now W is a vector of shape functions where W transpose, this is our original thing. So w0, w1, w2 to wM.

(Refer Slide Time 06:27)



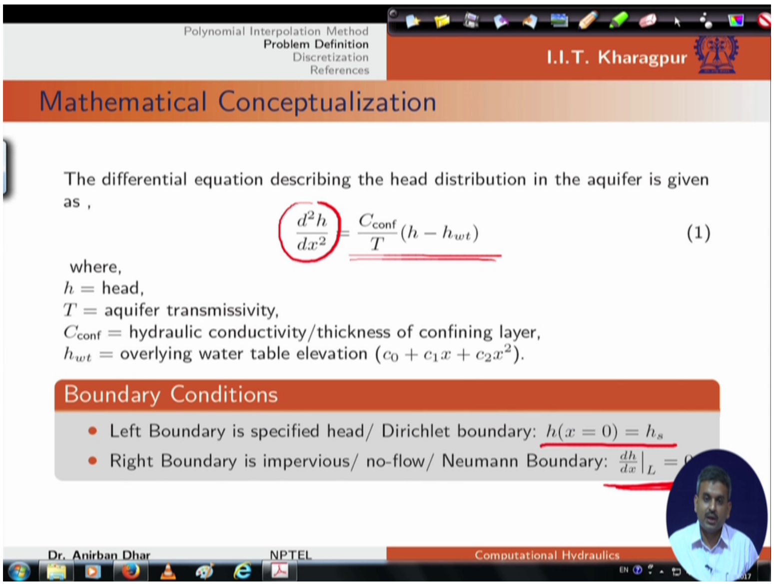
Now in case of our ground water flow in one dimension, this was the problem definition. We have seen the discretization using finite difference and finite volume technique. Now let's see how we can utilize the same problem andwe can utilize our mesh free method to discretize the governing equations.

(Refer Slide Time 07:01)



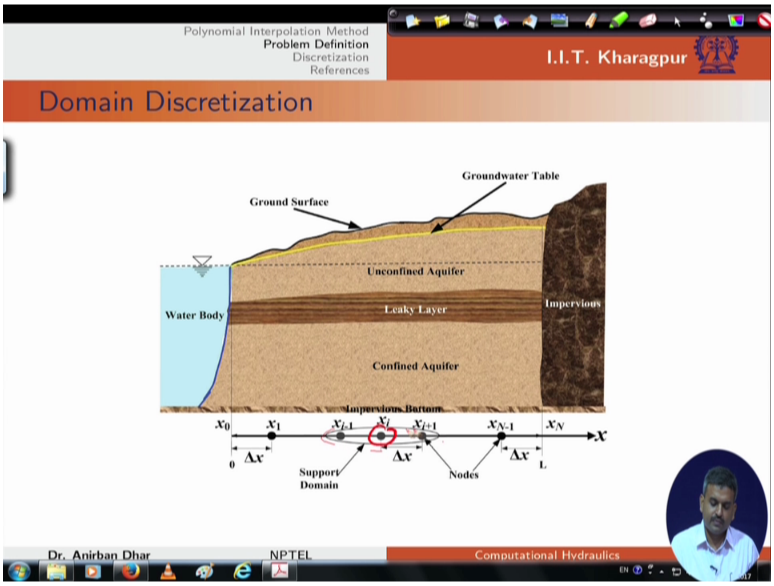
So mathematical conceptualization is clear. We have d2h dx2. And this is right hand source sink term. And Dirichlet boundary on the left hand side. Right hand side we have Neumann kind of boundary or zero Neumann boundary condition.

(Refer Slide Time 07:20)



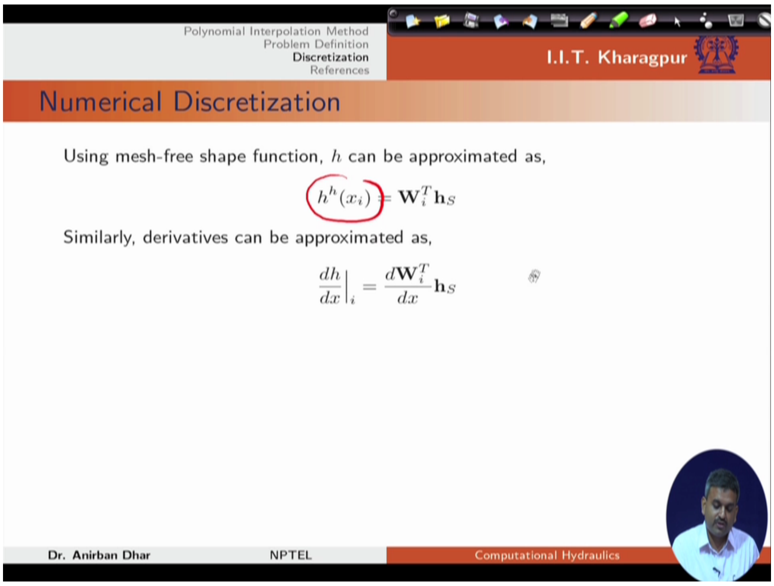
Domain discretization, within this straight line starting from zero to L, we can generate number of points x1, xi, x minus i, x plus i, xn, these nodal points. Now we can use these nodal points for our calculation. Let us say that for xi this is our support domain.

(Refer Slide Time 07:59)



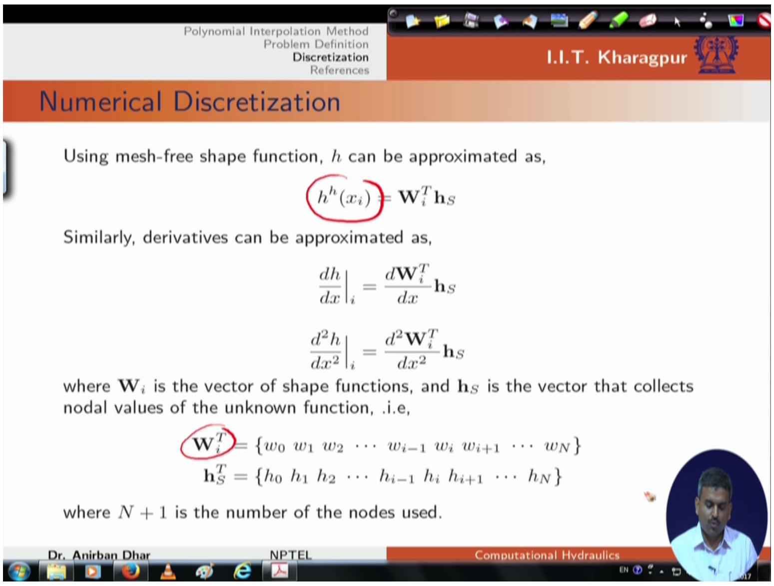
So we have xi minus 1 and xi plus 1 within the support domain. In case of numerical discretization using mesh freeshapefunction, H can be approximated as h which is W transpose h. Soin this case we can also approximate the derivative. So in calculation of the derivative we can differentiate the weight function of the shape function.

(Refer Slide Time 08:46)



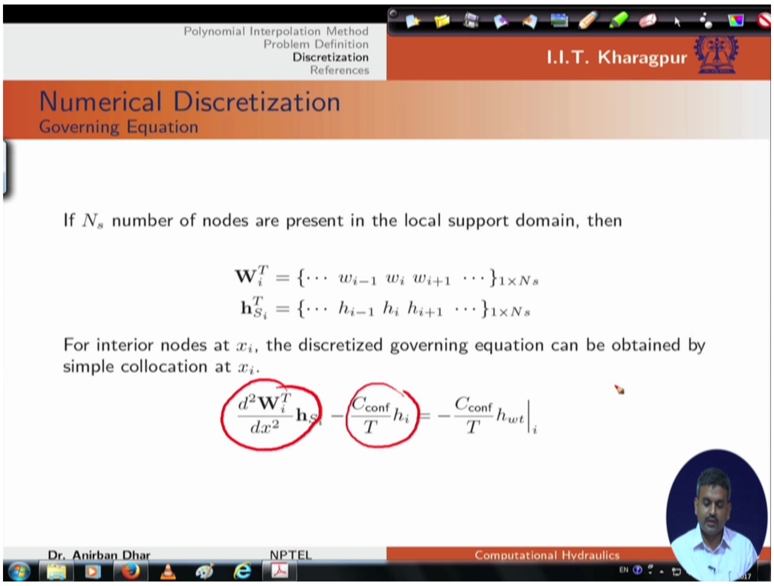
Again we can differentiate this W transpose or the vector of shape function. Hs is the vector that collects the nodal values of theunknown functions. W i transpose in general it could cover all the points. And Hs also it should cover all the points. But we have limited our support domain to adjacent nodes only. So n plus 1 number is a nodes used in this case.

(Refer Slide Time 09:31)



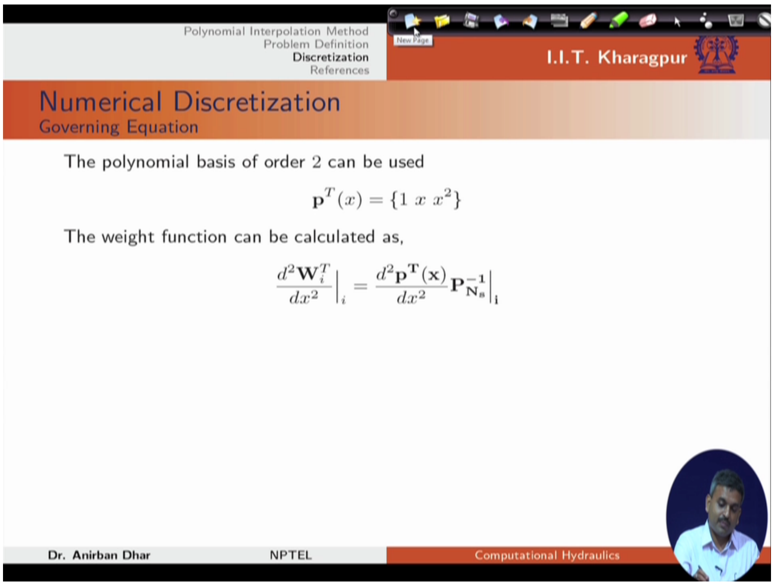
So Ns is a number of nodes present in the local support domain. So we can have 1 into Ns. And for interior point xi, governing equation can be obtained like this. So this is our differentiation or first term. We can directly write h as hi for the center node. At right hand side we have hwt which is again varyingwith xi.

(Refer Slide Time 10:14)



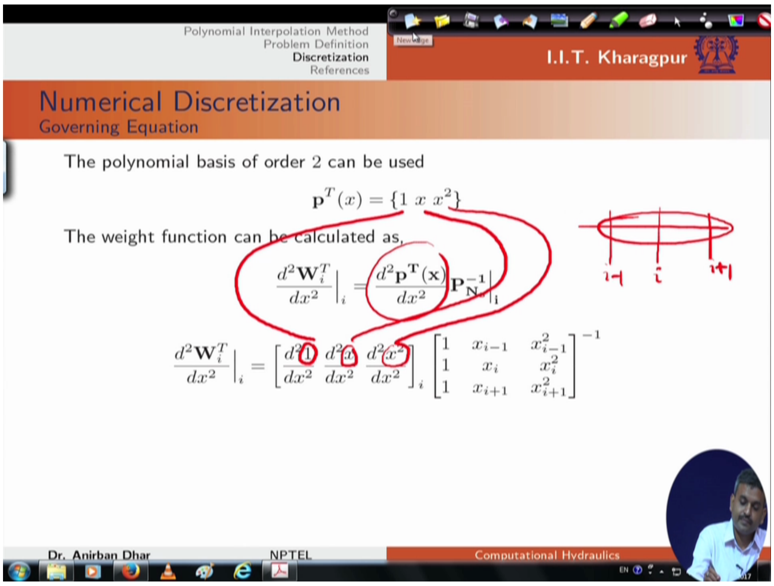
Now in this casewe can define our polynomial basis of order 2 where we have 3 terms, 1, x and x square. This is one dimensional problem that's why we are only using x in this case. So with this we can calculate the weight function. So this is nothing but P transpose x. This is d2, dx2 of this. And this is P inverse Ns points in the support domain.

(Refer Slide Time 11:02)



So with this we can write that this is 1 which is coming from here. Again x which is again from support domain. X square from here it is coming. Nowwe have to differentiate it twice to calculate this one andwe are writing this inverse thing. This is p0, p1 and p2. So we have 3 points within the support domain including the central node. Because if we have i, iminus 1 and iplus 1, then we will have 3 rows and 3 columns here. Because we have considered 3 terms in the polynomial basis.

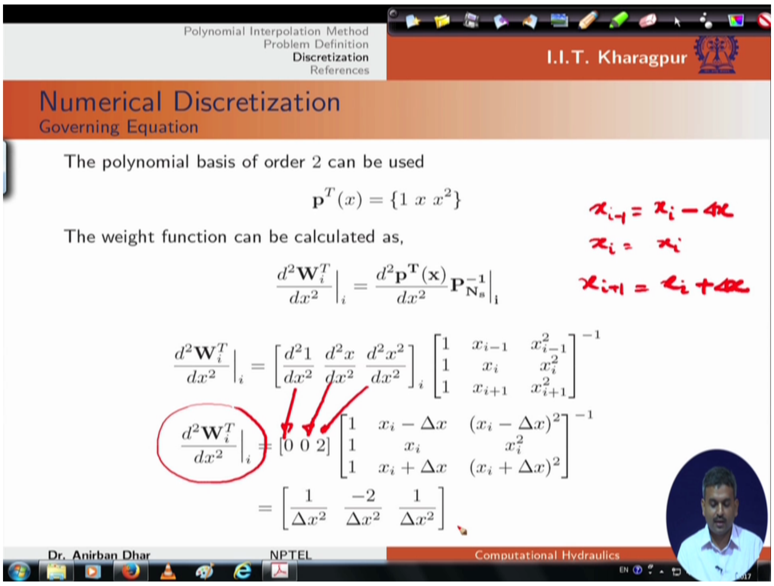
(Refer Slide Time 12:16)



So with this if you proceed interestinglywe can see that the second order derivative is zero. Only coefficient that we are getting is 2. Now xi minus 1, this can be represented as xi minus delx, considering we have uniformly spaced nodes available within the domain. Then xi equals to xi and xi plus 1 equals to xi plus delx here. Now with this simplification if we use in this case we will simply get this 1 by delx square.

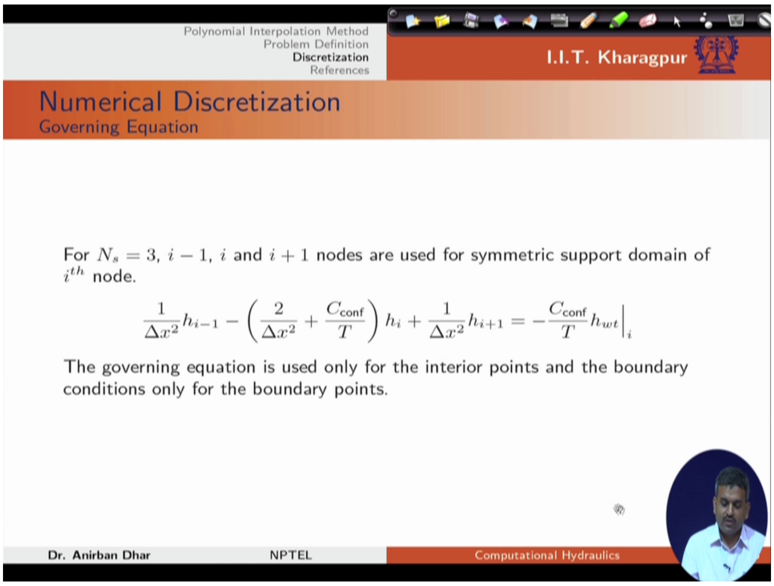
This d2 W transpose dx2, this will be 1 by delx square minus 2 by delx square, 1 by delx square. Interestingly whatever we have got for our finite difference and finite volume case for this particular problem, for interior node discretization we have exactly got the same thing.

(Refer Slide Time 13:42)



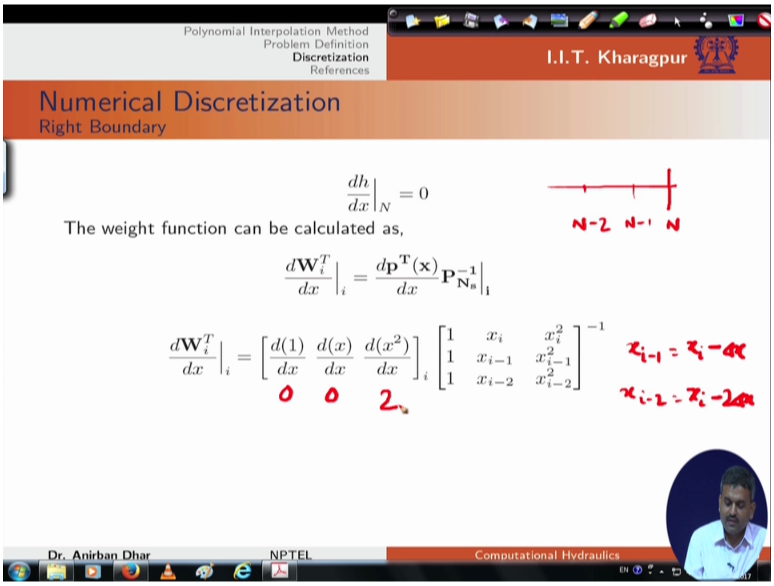
Ns is equal to 3. Now we have symmetric support domain. Now this is interior equation for ith node. And governing equation is used for only interior points.

(Refer Slide Time 14:05)



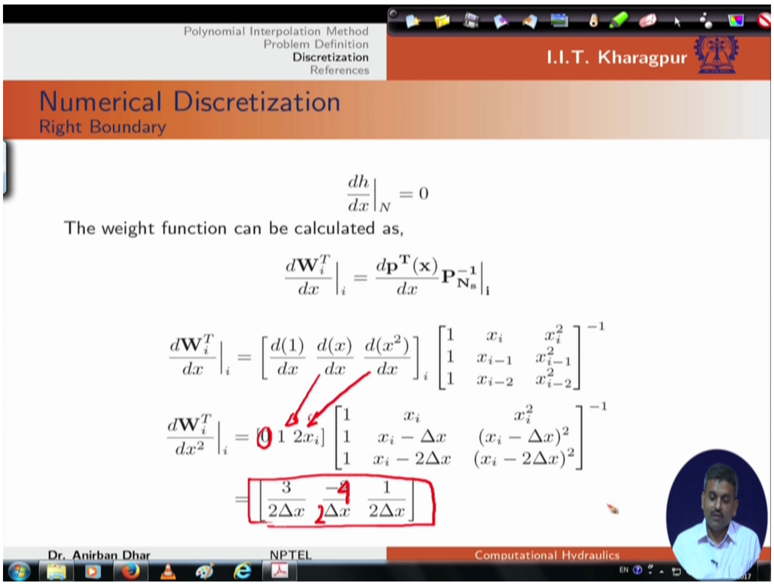
Nowlet us consider for right boundary. Right boundary, again we have n, n minus1, n minus 2. So with thisif we calculatethat meansin polynomial basis we have 3members. And in this case also we have 3 nodes that is i, iminus1, iminus 2 or n,n minus1, nminus2. So in this case again xi minus 1 equals to xi minus delx, xi minus 2 equals to xi minus 2 delx. So if we use this approximation for our case then and this is again zero, this is zero and this is 2.

(Refer Slide Time 15:22)



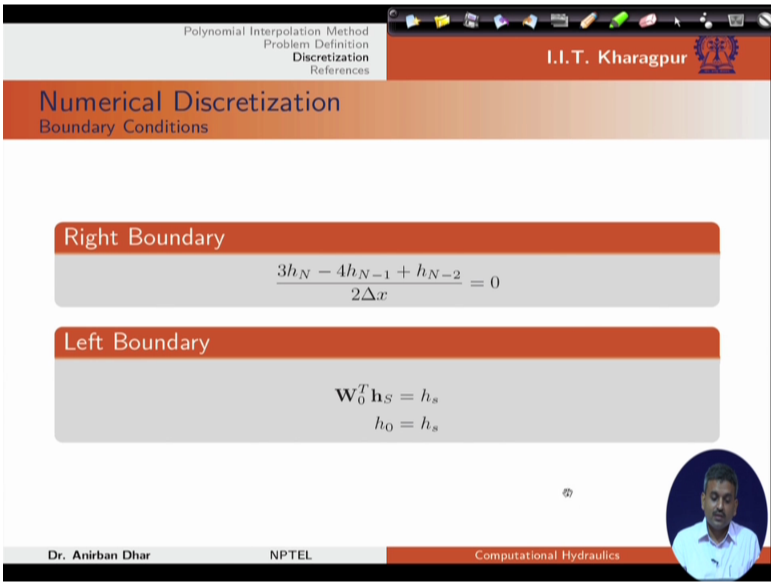
With this approximation the coefficients this is 1 becausethis is first order derivative. This is zero, this is 1 and this is 2xi for ith node. So if we simplify this one, this inverse thing then we can get 3 by 2 del x minus 2 by del x. If you multiply this with 2, this is 4 into 2.Interestingly whatever boundary condition we have used for one sided finite difference approximation for first order derivative, for second order accuracy this is same thing. Same thing we are getting from mesh free method.

(Refer Slide Time 16:22)



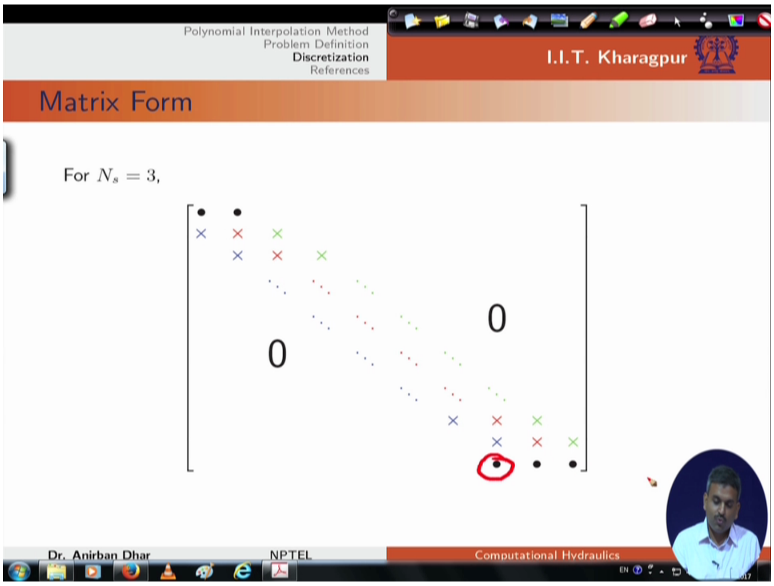
Now we can directly use this approximation for right boundary. So 3hNminus 4hN minus 1 plus hNminus 2. And for left boundary we have directly h zero is equal to Hs or specified value in this case.

(Refer Slide Time 16:55)



So we can construct our matrix for Ns equals to 3. Again that will be bended matrix. Butthis one dot will be extra because we haveextra term available in case of right hand boundary condition with our zero Neumann case. Sowe can use the full matrix or we can usefinite difference like approximation in this caseand we can include only two points near boundary for our calculations. But obviously in that caseour accuracy will reduce for inclusion of only 2 points during calculation.

(Refer Slide Time 18:03)



So we can directly invert the matrix and get thesolution from this calculation. Thank you.