

**Computational Hydraulics**  
**Professor Anirban Dhar**  
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**Lecture 21**  
**Mesh-Free Method: Polynomial Interpolation Method**

Welcome to lecture 21 of the course computational hydraulics. We are in module 2, numerical methods. And in this particular lecture class I will be covering unit 17 that is mesh free method minus polynomial interpolation method.

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The image shows a presentation slide with a white background and a dark blue header. The header contains a navigation menu on the left with the following items: 'Polynomial Interpolation Method', 'Problem Definition', 'Discretization', and 'References'. On the right side of the header, it says 'I.I.T. Kharagpur' next to the institute's logo. The main content area features a large blue rounded rectangle with the text 'Module 02: Numerical Methods' and 'Unit 17: Mesh-free Method: Polynomial Interpolation Method'. Below this, the name 'Anirban Dhar' is displayed, followed by his affiliation: 'Department of Civil Engineering, Indian Institute of Technology Kharagpur, Kharagpur'. At the bottom of the slide, it mentions 'National Programme for Technology Enhanced Learning (NPTEL)'. The footer of the slide includes 'Dr. Anirban Dhar', 'NPTEL', 'Computational Hydraulics', and '1 / 16'.

Learning objective of this particular unit. At the end of this unit students will be able to discretize ordinary differential equation using polynomial interpolation method.

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Polynomial Interpolation Method  
Problem Definition  
Discretization  
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### Learning Objective

- To discretize ODE using Polynomial Interpolation Method.

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Polynomial interpolation method. Function  $\phi$  can be represented at a point  $x$  with the approximation  $\phi^h$  starting from  $j$  is equal to zero to  $m$ . This is  $P_j \times a_j \times x$ . And we can represent it in terms of vectors like  $P^T \times a$ .

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### Polynomial Interpolation Method

Function  $\phi(x)$  at a point  $(x)$  can be approximated as,

$$\phi^h(x) = \sum_{j=0}^m p_j(x) a_j(x) = \mathbf{p}^T(x) \mathbf{a}(x)$$

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So in this case we can use the concept that we have learnt in our previous lecture class. This is  $P^T$  transpose which is nothing but for one dimension this is  $1 \times \text{square up to } x \text{ to the power } m$ . That means you will have  $m+1$  number of terms here.

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### Polynomial Interpolation Method

Function  $\phi(\mathbf{x})$  at a point  $(\mathbf{x})$  can be approximated as,

$$\phi^h(\mathbf{x}) = \sum_{j=0}^m p_j(\mathbf{x}) a_j(\mathbf{x}) = \mathbf{p}^T(\mathbf{x}) \mathbf{a}(\mathbf{x})$$

The complete polynomial basis of order  $m$  can be written in a general form (Liu and Gu, 2005).

$$\mathbf{p}^T(x) = \{1 \ x \ x^2 \ \dots \ x^m\}$$

$$\mathbf{p}^T(x, y) = \{1 \ x \ y \ x^2 \ xy \ y^2 \ \dots \ x^m \ \dots \ y^m\}$$

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Similarly in this case you have  $xy$ ,  $x$ ,  $x$  square,  $x$ ,  $y$ ,  $y$  square. This expansion is there. So for  $m$  plus 1 nodal values for  $\phi_0$  we can have  $a_0$ ,  $a_1$ ,  $p_1$  to  $p_m$ . And  $\phi_1$  this is  $\phi_N$ . Similarly we can represent.

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### Polynomial Interpolation Method

Function  $\phi(\mathbf{x})$  at a point  $(\mathbf{x})$  can be approximated as,

$$\phi^h(\mathbf{x}) = \sum_{j=0}^m p_j(\mathbf{x}) a_j(\mathbf{x}) = \mathbf{p}^T(\mathbf{x}) \mathbf{a}(\mathbf{x})$$

The complete polynomial basis of order  $m$  can be written in a general form (Liu and Gu, 2005).

$$\mathbf{p}^T(x) = \{1 \ x \ x^2 \ \dots \ x^m\}$$

$$\mathbf{p}^T(x, y) = \{1 \ x \ y \ x^2 \ xy \ y^2 \ \dots \ x^m \ \dots \ y^m\}$$

For  $N + 1$  nodal values,

$$\phi_0 = a_0 + a_1 p_1(\mathbf{x}_0) + \dots + a_m p_m(\mathbf{x}_0)$$

$$\phi_1 = a_0 + a_1 p_1(\mathbf{x}_1) + \dots + a_m p_m(\mathbf{x}_1)$$

$$\phi_2 = a_0 + a_1 p_1(\mathbf{x}_2) + \dots + a_m p_m(\mathbf{x}_2)$$

$$\vdots$$

$$\phi_N = a_0 + a_1 p_1(\mathbf{x}_N) + \dots + a_m p_m(\mathbf{x}_N)$$

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So starting with this we have  $n$  plus one nodal values. Similarly we have  $m$  plus 1 polynomial function values or  $A$  in this case.

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### Polynomial Interpolation Method

Function  $\phi(x)$  at a point  $(x)$  can be approximated as,

$$\phi^h(x) = \sum_{j=0}^m p_j(x) a_j(x) = \mathbf{P}^T(x) \mathbf{a}(x)$$

The complete polynomial basis of order  $m$  can be written in a general form (Liu and Gu, 2005).

$$\mathbf{P}^T(x) = \{1 \ x \ x^2 \ \dots \ x^m\}$$

$$\mathbf{P}^T(x, y) = \{1 \ x \ y \ x^2 \ xy \ y^2 \ \dots \ x^m \ \dots \ y^m\}$$

For  $N + 1$  nodal values,

$$\begin{cases} \phi_0 = a_0 + a_1 p_1(x_0) + \dots + a_m p_m(x_0) \\ \phi_1 = a_0 + a_1 p_1(x_1) + \dots + a_m p_m(x_1) \\ \phi_2 = a_0 + a_1 p_1(x_2) + \dots + a_m p_m(x_2) \\ \vdots \\ \phi_N = a_0 + a_1 p_1(x_N) + \dots + a_m p_m(x_N) \end{cases}$$

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So let us use this for our calculation. We have represented these  $m + 1$  nodal values in terms of  $A$  and  $P$ . Now if we write it in the form of matrix, we can write this like  $\phi$  which is column vector. Again  $P$  is a matrix. This is our size of  $m + 1$  into  $1$  and  $P$  is of size  $(m + 1) \times (m + 1)$ . And we have this  $A$  again. This is  $m + 1$  into  $1$ . So with this we can start our calculation.

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### Polynomial Interpolation method

The set of equations can be written in matrix form as,

$$\begin{Bmatrix} \phi \end{Bmatrix}_{(N+1) \times 1} = \begin{bmatrix} P \end{bmatrix}_{(N+1) \times (m+1)} \begin{Bmatrix} a \end{Bmatrix}_{(m+1) \times 1}$$

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We can get for  $n + 1$  equals  $m + 1$ , we can invert this matrix. So in this case we can get  $\phi$  which is a field variable in terms of  $P$  transpose  $x$ . So  $P$  transpose, if we replace this  $A$  from this equation, we can get  $P$  inverse  $\phi$ .

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### Polynomial Interpolation method

The set of equations can be written in matrix form as,

$$\phi = P_{m+1} a$$

In PIM,  $N + 1 = m + 1$

$$a = P_{m+1}^{-1} \phi$$

The approximated form can be written as,

$$\phi(x) = p^T(x) a(x) = p^T(x) P_{m+1}^{-1} \phi = W^T \phi$$

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So we can represent this first portion  $P^T P^{-1}$  as  $W^T$  which is our weight function. And  $\phi$  is individual function values.

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### Polynomial Interpolation method

The set of equations can be written in matrix form as,

$$\phi = P_{m+1} a$$

In PIM,  $N + 1 = m + 1$

$$a = P_{m+1}^{-1} \phi$$

The approximated form can be written as,

$$\phi(x) = p^T(x) a(x) = p^T(x) P_{m+1}^{-1} \phi = W^T \phi$$

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Now  $W$  is a vector of shape functions where  $W^T$ , this is our original thing. So  $w_0, w_1, w_2$  to  $w_m$ .

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### Polynomial Interpolation method

The set of equations can be written in matrix form as,

$$\phi = \mathbf{P}_{m+1} \mathbf{a}$$

In PIM,  $N + 1 = m + 1$

$$\mathbf{a} = \mathbf{P}_{m+1}^{-1} \phi$$

The approximated form can be written as,

$$\phi(\mathbf{x}) = \mathbf{p}^T(\mathbf{x}) \mathbf{a}(\mathbf{x}) = \mathbf{p}^T(\mathbf{x}) \mathbf{P}_{m+1}^{-1} \phi = \mathbf{W}^T \phi$$

where  $\mathbf{W}(\mathbf{x})$  is a vector of shape functions,

$$\mathbf{W}^T(\mathbf{x}) = \mathbf{p}^T(\mathbf{x}) \mathbf{P}_{m+1}^{-1} = \{w_0(\mathbf{x}) \ w_1(\mathbf{x}) \ w_2(\mathbf{x}) \ \cdots \ w_m(\mathbf{x})\}$$

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Now in case of our ground water flow in one dimension, this was the problem definition. We have seen the discretization using finite difference and finite volume technique. Now let's see how we can utilize the same problem and we can utilize our mesh free method to discretize the governing equations.

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### Problem Definition

Ground Surface  
Groundwater Table  
Unconfined Aquifer  
Leaky Layer  
Confined Aquifer  
Impervious Bottom  
Water Body  
Impervious

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So mathematical conceptualization is clear. We have  $d^2h/dx^2$ . And this is right hand source sink term. And Dirichlet boundary on the left hand side. Right hand side we have Neumann kind of boundary or zero Neumann boundary condition.

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### Mathematical Conceptualization

The differential equation describing the head distribution in the aquifer is given as ,

$$\frac{d^2 h}{dx^2} = \frac{C_{\text{conf}}}{T} (h - h_{wt}) \quad (1)$$

where,  
 $h$  = head,  
 $T$  = aquifer transmissivity,  
 $C_{\text{conf}}$  = hydraulic conductivity/thickness of confining layer,  
 $h_{wt}$  = overlying water table elevation ( $c_0 + c_1 x + c_2 x^2$ ).

#### Boundary Conditions

- Left Boundary is specified head/ Dirichlet boundary:  $h(x=0) = h_s$
- Right Boundary is impervious/ no-flow/ Neumann Boundary:  $\frac{dh}{dx} \Big|_L = 0$

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Domain discretization, within this straight line starting from zero to L, we can generate number of points  $x_1, x_i, x_{i-1}, x_{i+1}, x_n$ , these nodal points. Now we can use these nodal points for our calculation. Let us say that for  $x_i$  this is our support domain.

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### Domain Discretization

Ground Surface  
 Groundwater Table  
 Unconfined Aquifer  
 Leaky Layer  
 Confined Aquifer  
 Impervious Bottom

Water Body

Impervious Bottom

$x_0$   $x_1$   $x_{i-1}$   $x_i$   $x_{i+1}$   $x_{N-1}$   $x_N$

$\Delta x$   $\Delta x$   $\Delta x$   $L$

Support Domain  
 Nodes

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So we have  $x_{i-1}$  and  $x_{i+1}$  within the support domain. In case of numerical discretization using mesh free shape function,  $H$  can be approximated as  $h$  which is  $W$  transpose  $h$ . So in this case we can also approximate the derivative. So in calculation of the derivative we can differentiate the weight function of the shape function.

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## Numerical Discretization

Using mesh-free shape function,  $h$  can be approximated as,

$$h^h(x_i) = \mathbf{W}_i^T \mathbf{h}_S$$

Similarly, derivatives can be approximated as,

$$\left. \frac{dh}{dx} \right|_i = \frac{d\mathbf{W}_i^T}{dx} \mathbf{h}_S$$

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Again we can differentiate this  $\mathbf{W}$  transpose or the vector of shape function.  $\mathbf{h}_S$  is the vector that collects the nodal values of the unknown functions.  $\mathbf{W}_i$  transpose in general it could cover all the points. And  $\mathbf{h}_S$  also it should cover all the points. But we have limited our support domain to adjacent nodes only. So  $n + 1$  number is a nodes used in this case.

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## Numerical Discretization

Using mesh-free shape function,  $h$  can be approximated as,

$$h^h(x_i) = \mathbf{W}_i^T \mathbf{h}_S$$

Similarly, derivatives can be approximated as,

$$\left. \frac{dh}{dx} \right|_i = \frac{d\mathbf{W}_i^T}{dx} \mathbf{h}_S$$

$$\left. \frac{d^2h}{dx^2} \right|_i = \frac{d^2\mathbf{W}_i^T}{dx^2} \mathbf{h}_S$$

where  $\mathbf{W}_i$  is the vector of shape functions, and  $\mathbf{h}_S$  is the vector that collects nodal values of the unknown function, .i.e,

$$\mathbf{W}_i^T = \{w_0 \ w_1 \ w_2 \ \dots \ w_{i-1} \ w_i \ w_{i+1} \ \dots \ w_N\}$$

$$\mathbf{h}_S^T = \{h_0 \ h_1 \ h_2 \ \dots \ h_{i-1} \ h_i \ h_{i+1} \ \dots \ h_N\}$$

where  $N + 1$  is the number of the nodes used.

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So  $N_s$  is a number of nodes present in the local support domain. So we can have 1 into  $N_s$ . And for interior point  $x_i$ , governing equation can be obtained like this. So this is our differentiation or first term. We can directly write  $h$  as  $h_i$  for the center node. At right hand side we have  $h_{wt}$  which is again varying with  $x_i$ .



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## Numerical Discretization

### Governing Equation

If  $N_s$  number of nodes are present in the local support domain, then

$$\mathbf{W}_i^T = \{ \dots w_{i-1} w_i w_{i+1} \dots \}_{1 \times N_s}$$

$$\mathbf{h}_{S_i}^T = \{ \dots h_{i-1} h_i h_{i+1} \dots \}_{1 \times N_s}$$

For interior nodes at  $x_i$ , the discretized governing equation can be obtained by simple collocation at  $x_i$ .

$$\left. \frac{d^2 \mathbf{W}_i^T}{dx^2} \mathbf{h}_{S_i} - \frac{C_{\text{conf}}}{T} h_i \right|_i = - \left. \frac{C_{\text{conf}}}{T} h_{wt} \right|_i$$

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Now in this case we can define our polynomial basis of order 2 where we have 3 terms, 1, x and x square. This is one dimensional problem that's why we are only using x in this case. So with this we can calculate the weight function. So this is nothing but P transpose x. This is d2/dx2 of this. And this is P inverse Ns points in the support domain.

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## Numerical Discretization

### Governing Equation

The polynomial basis of order 2 can be used

$$\mathbf{p}^T(x) = \{1 \ x \ x^2\}$$

The weight function can be calculated as,

$$\left. \frac{d^2 \mathbf{W}_i^T}{dx^2} \right|_i = \left. \frac{d^2 \mathbf{p}^T(x)}{dx^2} \mathbf{P}_{N_s}^{-1} \right|_i$$

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So with this we can write that this is 1 which is coming from here. Again x which is again from support domain. X square from here it is coming. Now we have to differentiate it twice to calculate this one and we are writing this inverse thing. This is p0, p1 and p2. So we have 3 points within the support domain including the central node. Because if we have i, i minus 1

and  $i \pm 1$ , then we will have 3 rows and 3 columns here. Because we have considered 3 terms in the polynomial basis.

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### Numerical Discretization

Governing Equation

The polynomial basis of order 2 can be used

$$\mathbf{p}^T(x) = \{1 \ x \ x^2\}$$

The weight function can be calculated as,

$$\frac{d^2 \mathbf{W}_i^T}{dx^2} \Big|_i = \frac{d^2 \mathbf{p}^T(x)}{dx^2} \mathbf{P}_{N-1}^{-1} \Big|_i$$

$$\frac{d^2 \mathbf{W}_i^T}{dx^2} \Big|_i = \begin{bmatrix} d^2 \mathbf{1} & d^2 x & d^2 x^2 \end{bmatrix}_i \begin{bmatrix} 1 & x_{i-1} & x_{i-1}^2 \\ 1 & x_i & x_i^2 \\ 1 & x_{i+1} & x_{i+1}^2 \end{bmatrix}^{-1}$$

Diagram showing nodes  $i-1$ ,  $i$ , and  $i+1$  on a line.

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So with this if you proceed interestingly we can see that the second order derivative is zero. Only coefficient that we are getting is 2. Now  $x_i - 1$ , this can be represented as  $x_i - \Delta x$ , considering we have uniformly spaced nodes available within the domain. Then  $x_i$  equals to  $x_i$  and  $x_i + 1$  equals to  $x_i + \Delta x$  here. Now with this simplification if we use in this case we will simply get this  $1$  by  $\Delta x^2$ .

This  $d^2 W$  transpose  $dx^2$ , this will be  $1$  by  $\Delta x^2$  square minus  $2$  by  $\Delta x^2$  square,  $1$  by  $\Delta x^2$  square. Interestingly whatever we have got for our finite difference and finite volume case for this particular problem, for interior node discretization we have exactly got the same thing.

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### Numerical Discretization

Governing Equation

The polynomial basis of order 2 can be used

$$\mathbf{p}^T(x) = \{1 \ x \ x^2\}$$

The weight function can be calculated as,

$$\frac{d^2 \mathbf{W}_i^T}{dx^2} \Big|_i = \frac{d^2 \mathbf{p}^T(x)}{dx^2} \mathbf{P}_{N_s}^{-1} \Big|_i$$

Handwritten notes:  $x_{i-1} = x_i - \Delta x$ ,  $x_i = x_i$ ,  $x_{i+1} = x_i + \Delta x$

$$\frac{d^2 \mathbf{W}_i^T}{dx^2} \Big|_i = \begin{bmatrix} \frac{d^2 1}{dx^2} & \frac{d^2 x}{dx^2} & \frac{d^2 x^2}{dx^2} \end{bmatrix}_i \begin{bmatrix} 1 & x_{i-1} & x_{i-1}^2 \\ 1 & x_i & x_i^2 \\ 1 & x_{i+1} & x_{i+1}^2 \end{bmatrix}_i^{-1}$$

$$\frac{d^2 \mathbf{W}_i^T}{dx^2} \Big|_i = \begin{bmatrix} 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & x_i - \Delta x & (x_i - \Delta x)^2 \\ 1 & x_i & x_i^2 \\ 1 & x_i + \Delta x & (x_i + \Delta x)^2 \end{bmatrix}_i^{-1}$$

$$= \begin{bmatrix} \frac{1}{\Delta x^2} & \frac{-2}{\Delta x^2} & \frac{1}{\Delta x^2} \end{bmatrix}$$

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$N_s$  is equal to 3. Now we have symmetric support domain. Now this is interior equation for  $i$ th node. And governing equation is used for only interior points.

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### Numerical Discretization

Governing Equation

For  $N_s = 3$ ,  $i - 1$ ,  $i$  and  $i + 1$  nodes are used for symmetric support domain of  $i^{th}$  node.

$$\frac{1}{\Delta x^2} h_{i-1} - \left( \frac{2}{\Delta x^2} + \frac{C_{cont}}{T} \right) h_i + \frac{1}{\Delta x^2} h_{i+1} = -\frac{C_{cont}}{T} h_{wt} \Big|_i$$

The governing equation is used only for the interior points and the boundary conditions only for the boundary points.

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Now let us consider for right boundary. Right boundary, again we have  $n$ ,  $n - 1$ ,  $n - 2$ . So with this if we calculate that means in polynomial basis we have 3 members. And in this case also we have 3 nodes that is  $i$ ,  $i - 1$ ,  $i - 2$  or  $n$ ,  $n - 1$ ,  $n - 2$ . So in this case again  $x_i - 1 = x_i - \Delta x$ ,  $x_i - 2 = x_i - 2 \Delta x$ . So if we use this approximation for our case then and this is again zero, this is zero and this is 2.

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### Numerical Discretization

Right Boundary

$$\left. \frac{dh}{dx} \right|_N = 0$$

The weight function can be calculated as,

$$\left. \frac{d\mathbf{W}_i^T}{dx} \right|_i = \left. \frac{d\mathbf{P}^T(\mathbf{x})}{dx} \right|_i \mathbf{P}_{N_i}^{-1} \Big|_i$$

$$\left. \frac{d\mathbf{W}_i^T}{dx} \right|_i = \begin{bmatrix} \frac{d(1)}{dx} & \frac{d(x)}{dx} & \frac{d(x^2)}{dx} \end{bmatrix}_i \begin{bmatrix} 1 & x_i & x_i^2 \\ 1 & x_{i-1} & x_{i-1}^2 \\ 1 & x_{i-2} & x_{i-2}^2 \end{bmatrix}^{-1}$$

$x_{i-1} = x_i - dx$   
 $x_{i-2} = x_i - 2dx$

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With this approximation the coefficients this is 1 because this is first order derivative. This is zero, this is 1 and this is  $2x_i$  for  $i$ th node. So if we simplify this one, this inverse thing then we can get  $3$  by  $2 \Delta x$  minus  $2$  by  $\Delta x$ . If you multiply this with  $2$ , this is  $4$  into  $2$ . Interestingly whatever boundary condition we have used for one sided finite difference approximation for first order derivative, for second order accuracy this is same thing. Same thing we are getting from mesh free method.

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### Numerical Discretization

Right Boundary

$$\left. \frac{dh}{dx} \right|_N = 0$$

The weight function can be calculated as,

$$\left. \frac{d\mathbf{W}_i^T}{dx} \right|_i = \left. \frac{d\mathbf{P}^T(\mathbf{x})}{dx} \right|_i \mathbf{P}_{N_i}^{-1} \Big|_i$$

$$\left. \frac{d\mathbf{W}_i^T}{dx} \right|_i = \begin{bmatrix} \frac{d(1)}{dx} & \frac{d(x)}{dx} & \frac{d(x^2)}{dx} \end{bmatrix}_i \begin{bmatrix} 1 & x_i & x_i^2 \\ 1 & x_{i-1} & x_{i-1}^2 \\ 1 & x_{i-2} & x_{i-2}^2 \end{bmatrix}^{-1}$$

$$\left. \frac{d\mathbf{W}_i^T}{dx} \right|_i = \begin{bmatrix} 0 & 1 & 2x_i \end{bmatrix} \begin{bmatrix} 1 & x_i & x_i^2 \\ 1 & x_i - \Delta x & (x_i - \Delta x)^2 \\ 1 & x_i - 2\Delta x & (x_i - 2\Delta x)^2 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} \frac{3}{2\Delta x} & -\frac{2}{\Delta x} & \frac{1}{2\Delta x} \end{bmatrix}$$

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Now we can directly use this approximation for right boundary. So  $3h_N$  minus  $4h_{N-1}$  plus  $h_{N-2}$ . And for left boundary we have directly  $h$  zero is equal to  $H_s$  or specified value in this case.

