**Computational Hydraulics**
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**Lecture 20**
**Mesh Free Method: Overview**

We are in lecture 20 of the course computational hydraulics and this is module 2, numerical methods. In this particular lecture class we will be discussing unit 16 that is mesh free method, overview.

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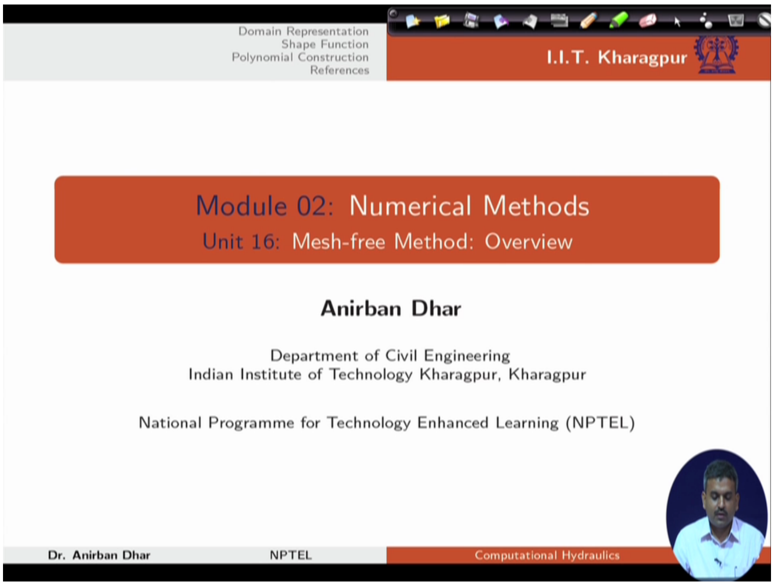


In our previous lectures we have covered finite difference method and finite volume method. Now we will cover this mesh free method. Learning objectives for this particular unit. At the end of this unit students will be able to define the shape functions andthey will be able to construct polynomial basis for function approximation.

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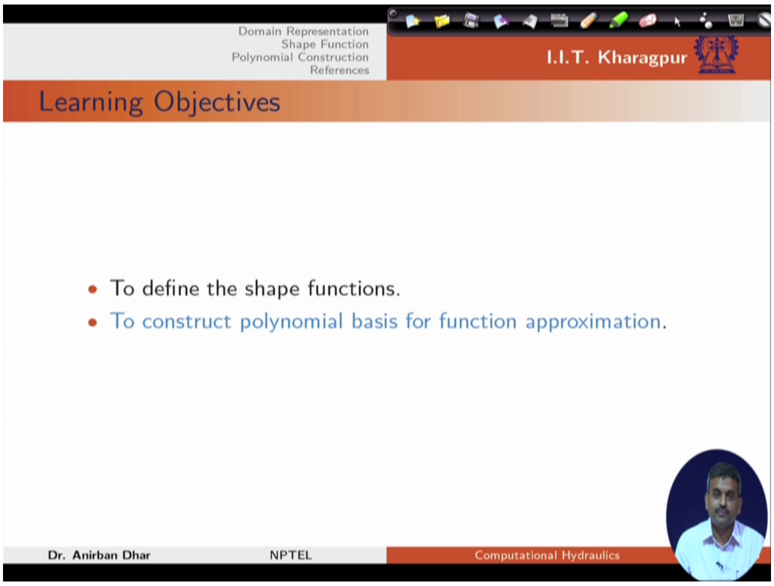


Domain representation. Let us consider a general domain where we have Neumann kind of boundary,Robin kind of boundary, Dirichletkind of boundary condition.

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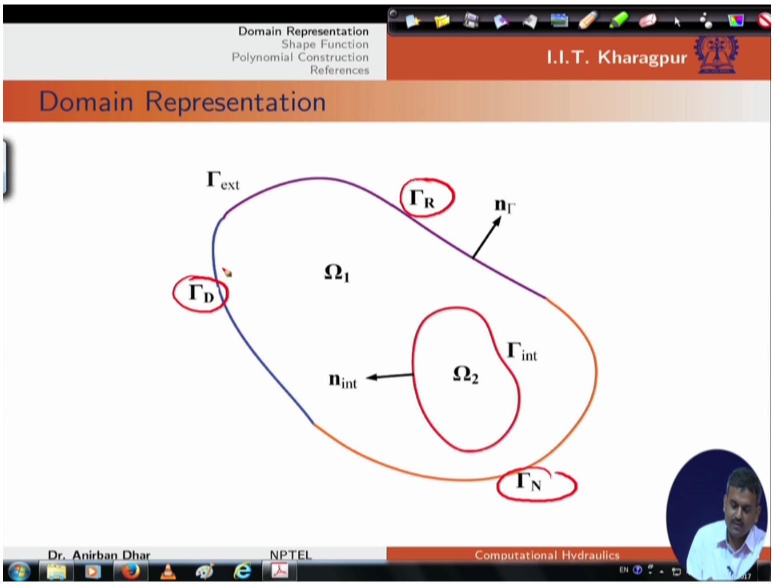


This is internal domain. Again phi 2 is another interior portion within the domain.

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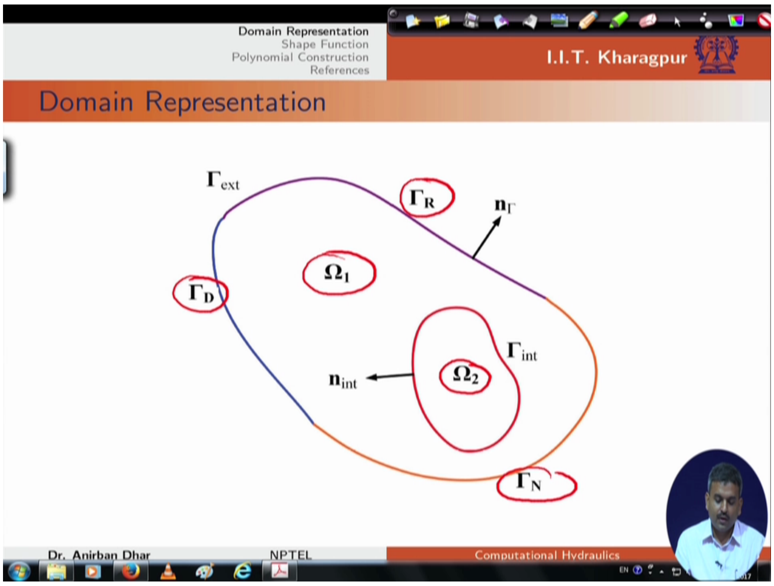


This is gamma interior and this is gamma exterior. Exterior is valid for exterior portion and interior is valid for interior boundaries.

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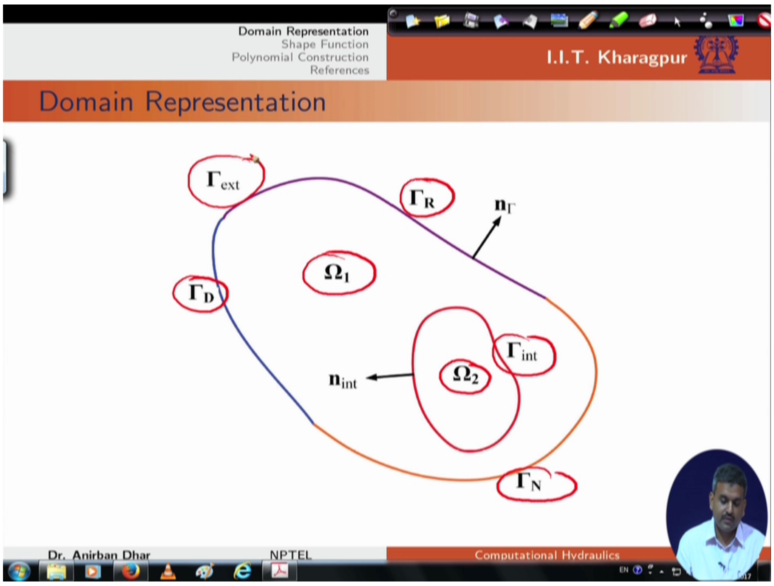


Now we can discretize this particular domainusing finite difference method. If we have finite difference method then we can divide this into different parts and we can use Taylor series to approximatethefunction of values at the nodal points. If we have finite volume, we will concentrate on the cell approach and at the cell center what is the value? We will try to utilize that for our calculations.

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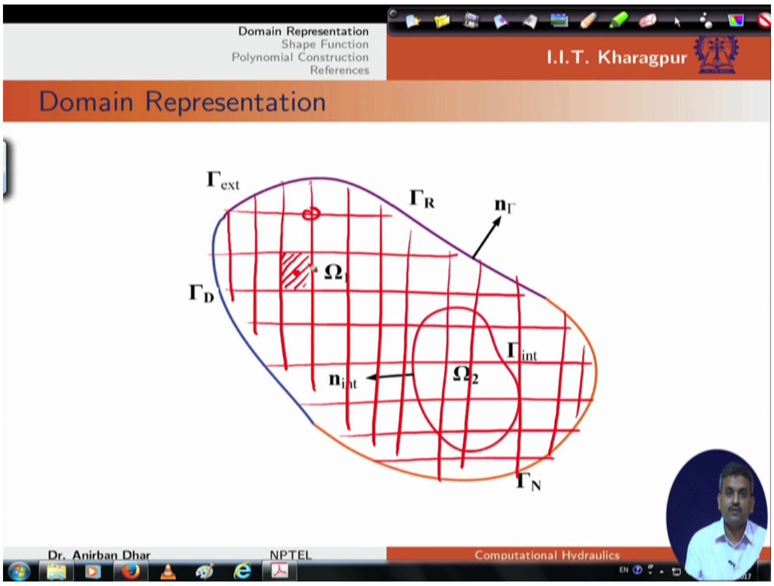


Now in these twoapproaches we need support of this grid lines. Now if we say that without the support of gridlines we want to solve the same problem, what should be our first step? Our first step should be to represent the same domain with number of discrete points. Internal portion, we can discretize with number of individual points. Againour boundary portion, we can divide it into number of segments for two-dimensional case. And number oflinear surface for three-dimensional case.

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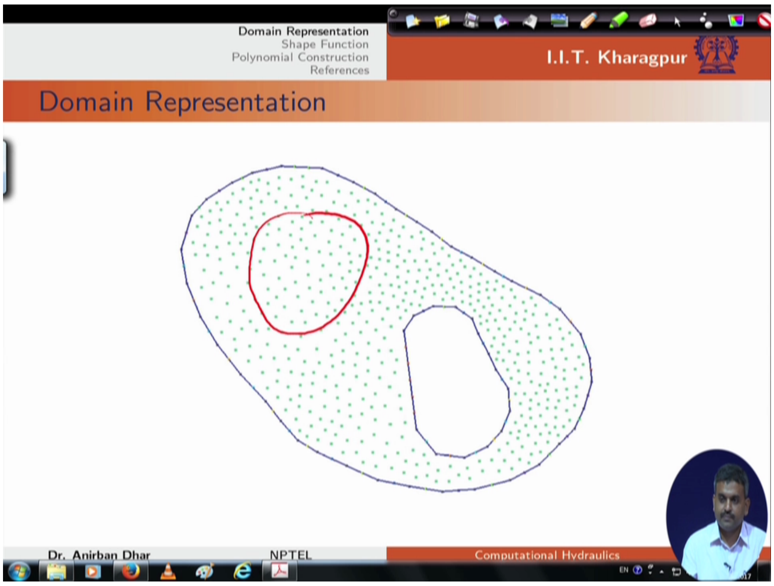


So with thisif we start then we can get some kind of approximation of the function. But in this case our generation or points can be structured or unstructured. Unstructured means the points can be randomly located or the points can be evenly distributed or equidistance between the domain.

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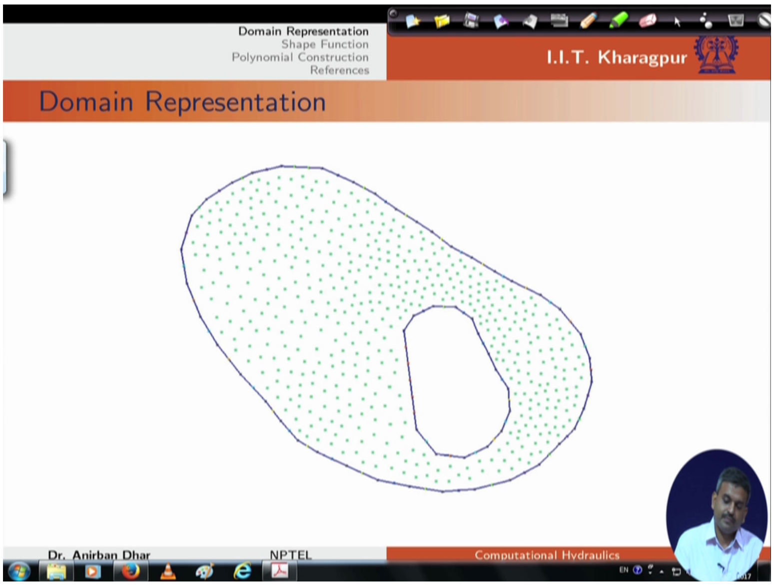


Now if we use this domain representationlike our finite difference approach where in two dimension if our central node is ij, this is ij minus 1, I plus 1j, this is I minus 1j, this is Ij plus 1. So for the calculation at the central nodal point, we have utilized the information from these 4 neighboring nodes in two dimension. That is ij minus 1, I plus 1j, Ij plus 1 and I minus 1j.

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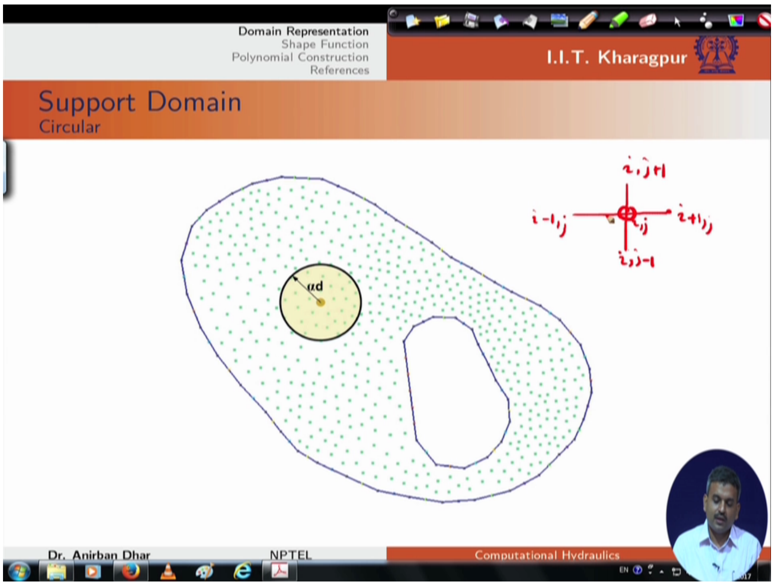

If we extend this for mesh free methods, we can define a general support domain in this case. Let us say initially this domain can be circular. So within this circular domain for any central point we can have number of particles. These circular domain is called as support domain. And the center particle isrepresented in terms of the function values available at this neighboring particles within the support domain.

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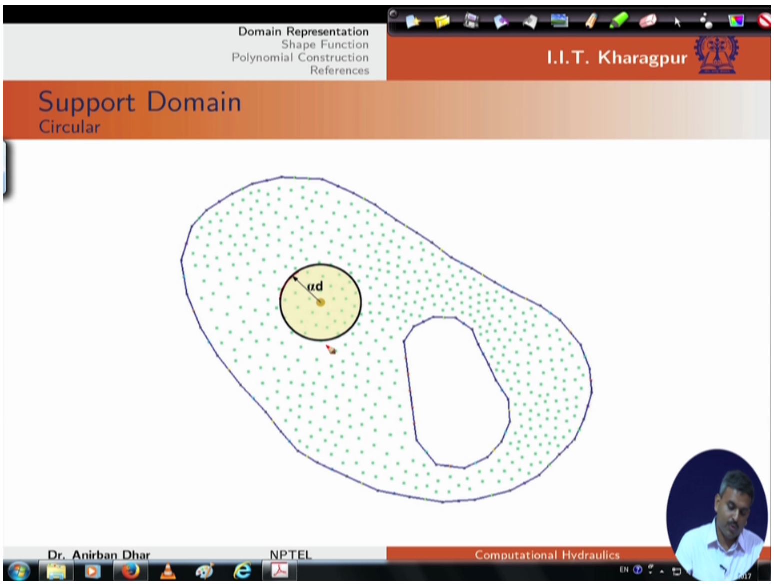


Now in this case we have seen that circular support domain. We can have rectangular support domain also. Let us say in X direction or Y direction we can havedifferent length values, lx ly. And depending on that we can consider a different number of points in different directions.

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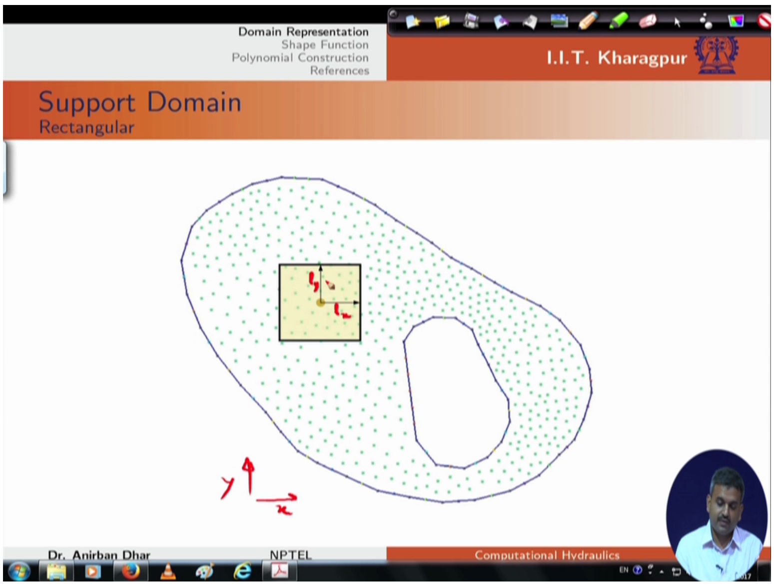

So we can see that with the change in thesupport domain, your representation will change. So support domain accuracy of interpolation depends on the nodes present in the support domain. And dimension of the support domain Ds can be written in terms of dimensionless size alpha s of the support domain and node spacing Dc near the point under consideration.

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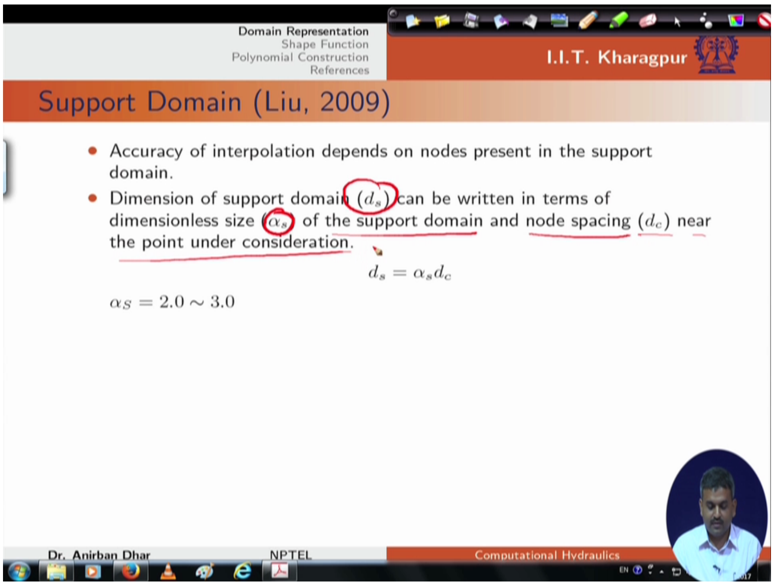


So we represent this Ds equals to alpha s intoDc. Where alpha s is 2 to 3 times or alpha equals to 2 to 3. That means the Ds equals to 2 to 3 times of Dc. Support domain is 2 to 3 times of node spacing.

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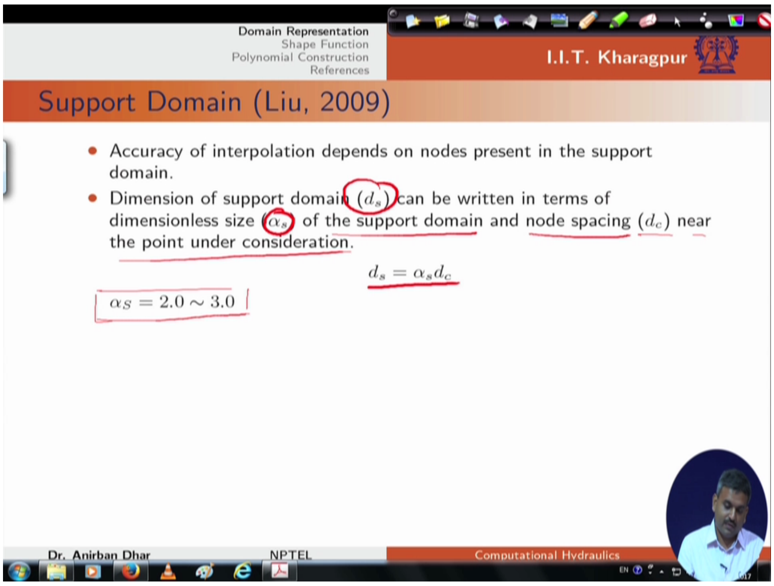

Node spacing Dccan be calculated for our one-dimensional, two-dimension and three-dimensional case also. In this case if nDs is a number of nodes present in the support domain and Ds is our support domain size,thennDs and nAs, nVs these are number of nodes present in the support domain. So Dc can be calculated based on that.

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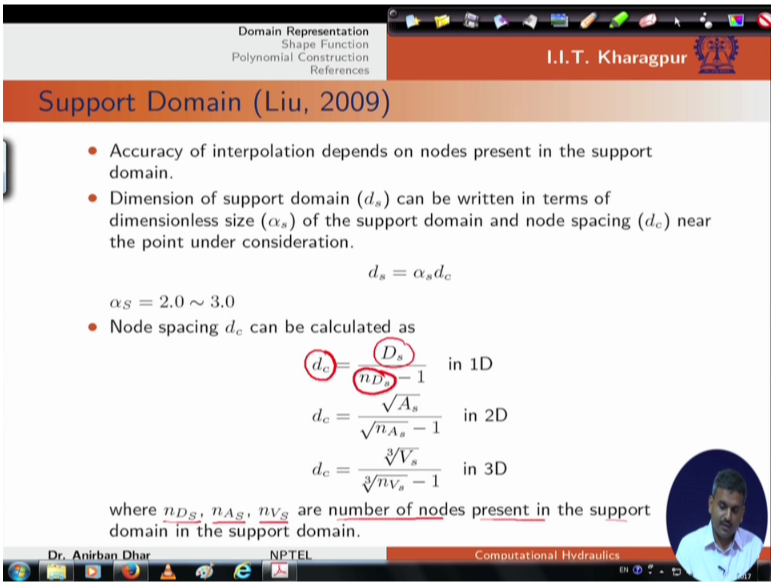


And interestingly this is based on Ds. this is As, which is Ds is length scale, As is area, V is volume. So approximate area, approximate value covered in this support domain.

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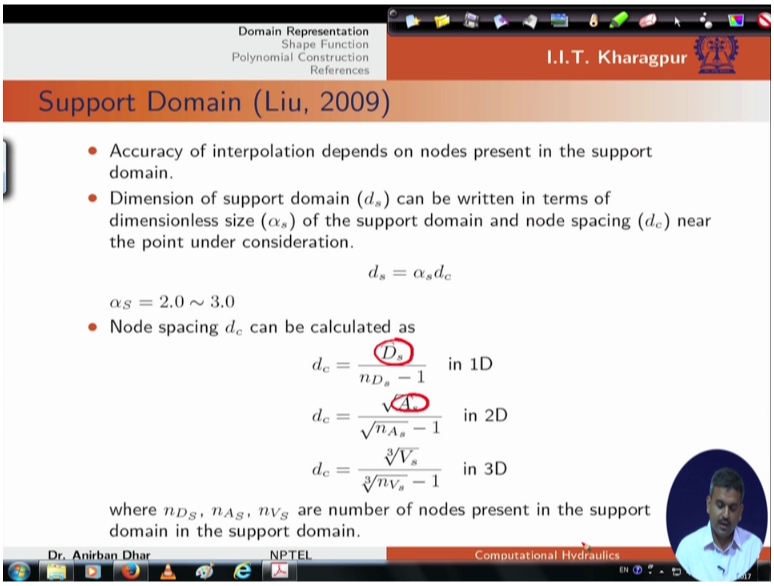


Field interpolation. Mathematically field variable phi can be represented at any point Xi within problem domain in terms of all node values within the support domain of Xi. So phi

Xi equals to Wij into phi Xj. J equals to 1 to n. So n is the number of points present in the support domain, Wij is the weight function at node J with respect to Xi.

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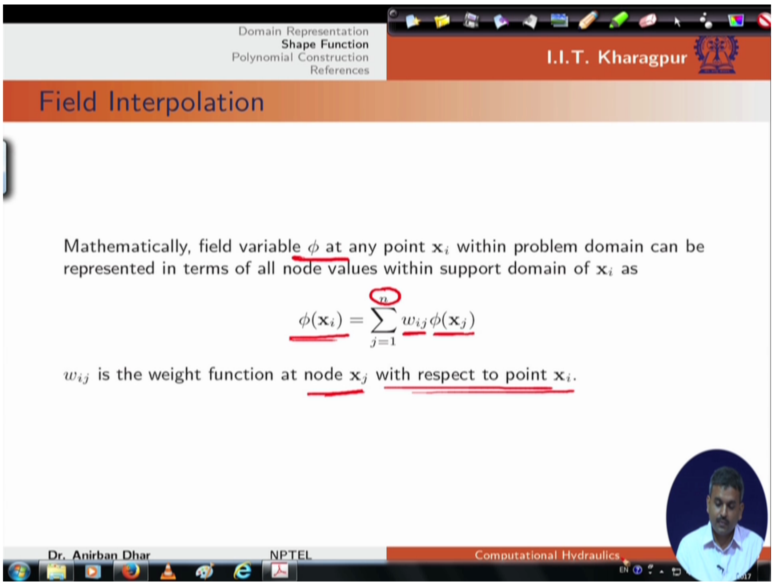


So we need to construct the shape functions. So shape functions can be constructed by satisfying basic requirements,as per Liu.Suitable for arbitrary distributed nodes.Consistency criteria should be satisfied. Consistency is closely related to reproducibilityin case ofmesh free methods. Soit should be reproducible. In case of our finite difference we have seen that consistency means if you reduce our size of discretization, it should converge to the original solution.

Obviously that means the (repress) reproducibility is there. So consistency is similar criteria here. Suitable for compact support domain and Kronecker delta property should be satisfied. That means if the function value is available and if we are considering the same node, it should be directly satisfied.

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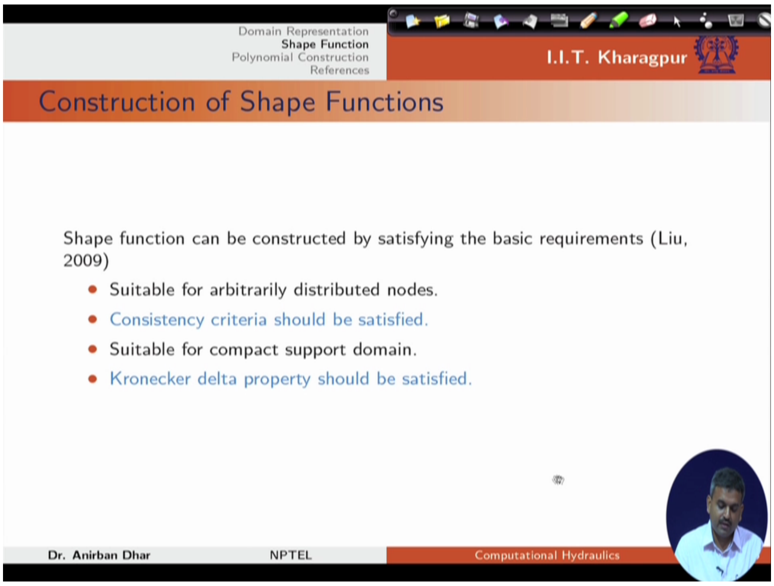


So Kronecker delta property. If Xj equals to I equals to J. This is 1 or weight function is 1. And of I is not equal to J, this is 0.

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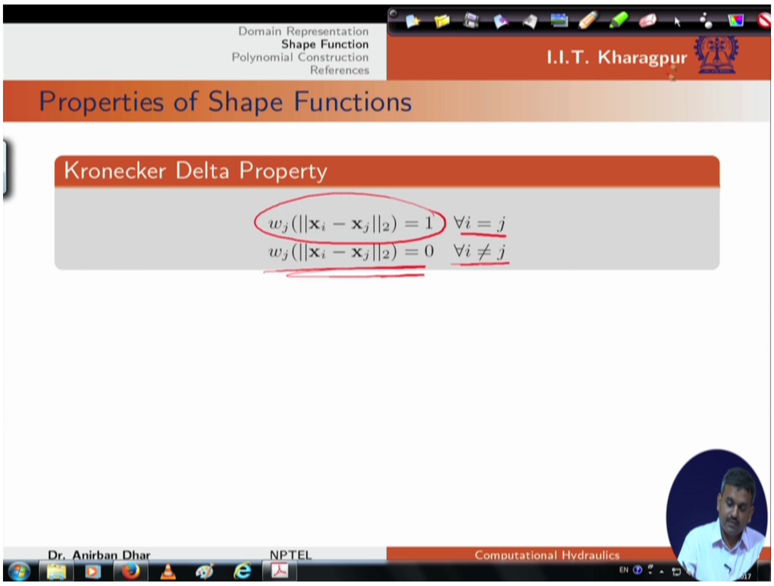


So in this case partition of unity. That means within support domain, your submission of this weight function this should be 1.

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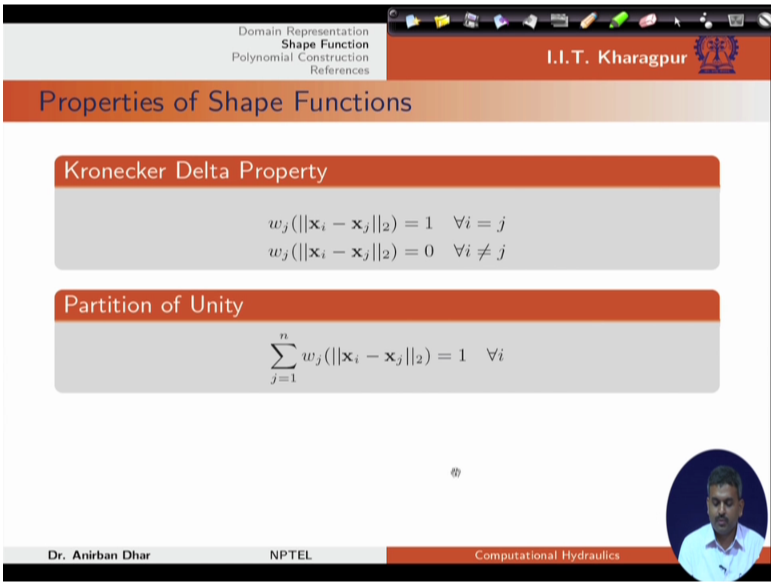


And linear field reproduction. This is for linear case. For in general nonlinear case also it should be reproducible. That means with multiplication of Xj within the support domain and if you take submission it should exactly represents Xi.

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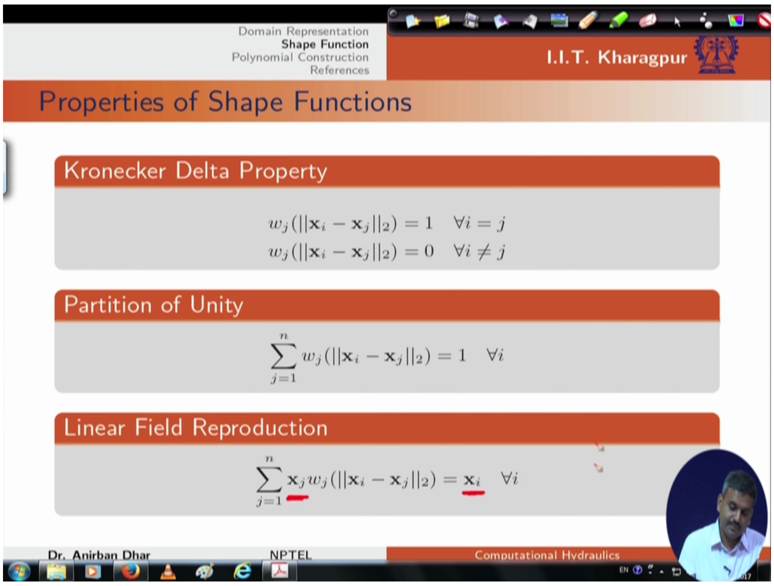


Construction of shape functions. Finite series representation methods, point interpolation method, moving least square or MLS method. Fx is any general function. This can be represented as A0 plus A1 P1x, A2 P2x plus other terms. So function can be approximated in terms of polynomial.

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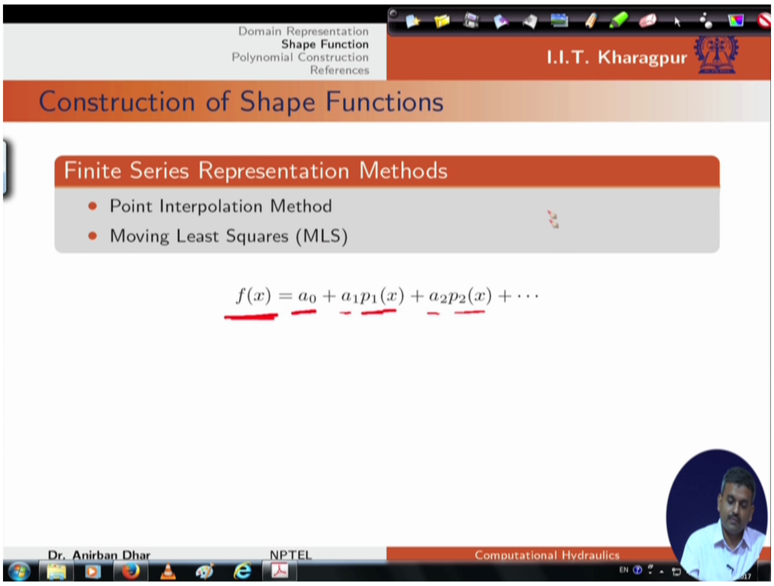


Now polynomial basis of order n can be written as P x. Where x to the power alpha such that modulus of alpha, this is less than equals to n and modulus of alpha this is submission I equals to 1 to d. D is the dimension and alpha I. And x vector to the power alpha, this is x1 to the power alpha 1, x2 to the power alpha 2, x3 to the power alpha 3, xd to the power alpha d.

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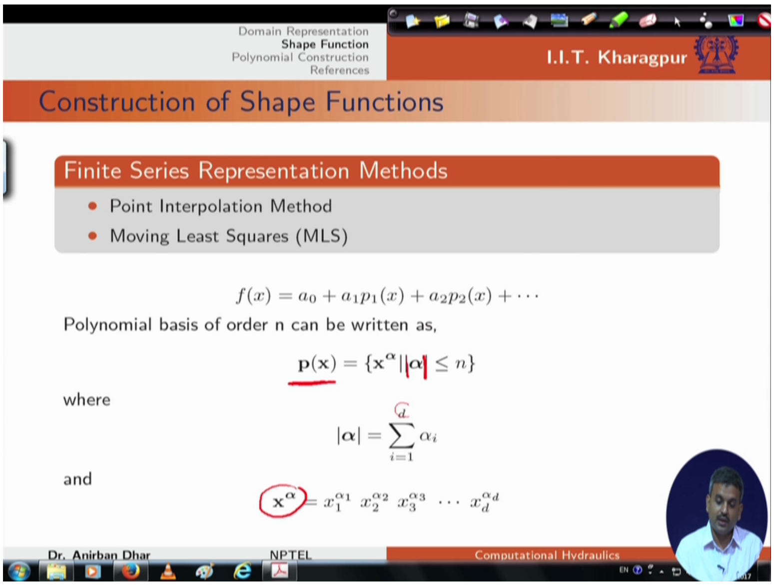


So if we consider one dimensional case, d equals to 1, n equals to 1. So obviously we have only 0 and 1. That means x to the power 0 and x to the power 1. We have 1 and x.

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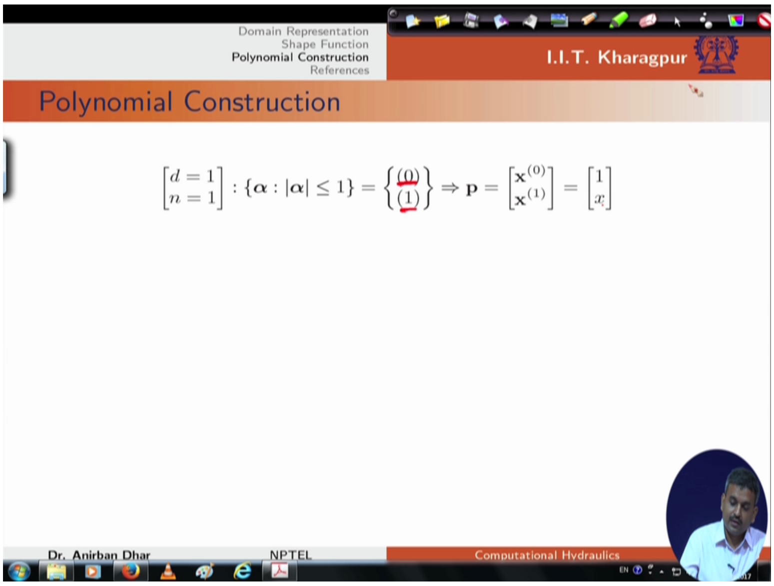


If we consider dimension equals to 1, d equals to 1, n equals to 2. That is less than equals to 2. That means 0, 1 and 2. So we have x to the power zero,x to the power 1, x to the power 2. This is 1, x, x square. Dimension equals to 2. That means 0 this mode alpha less than equals to 1,for n equals to 1. So in this case 01, 10, 10 and 00, 10 and 01. That means we are considering 1, x and y for this case.

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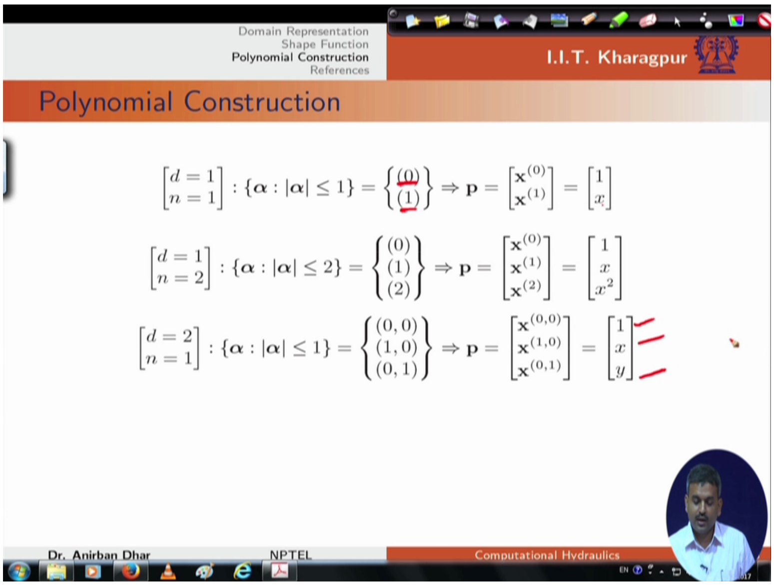


Similarly, for d equals to 2 and n equals to 2, we can have different combinations. So 1, x, y, x square, xy, y square like that.

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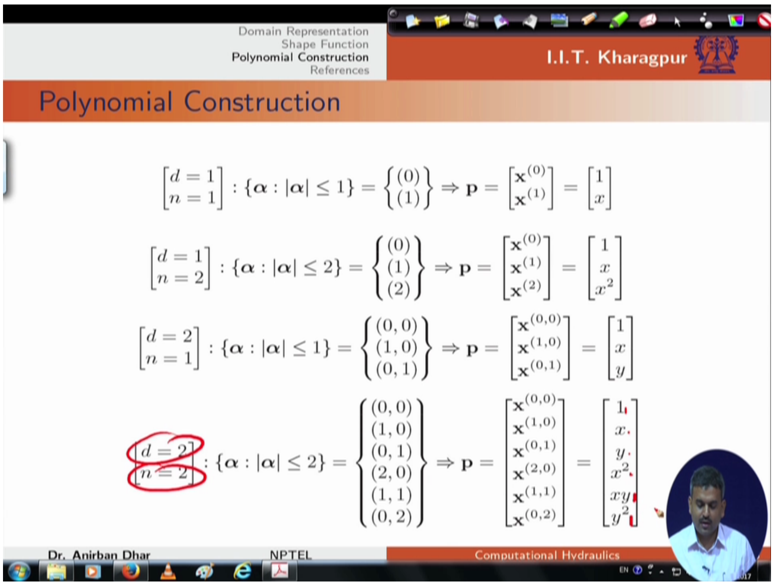


So by changing dimension and n value, we can get different polynomial representations. So in general polynomial construction can be as per Pascal's triangle. So we can have in one dimension 1, x. Then 1, x, x square, like that 1, x cube.

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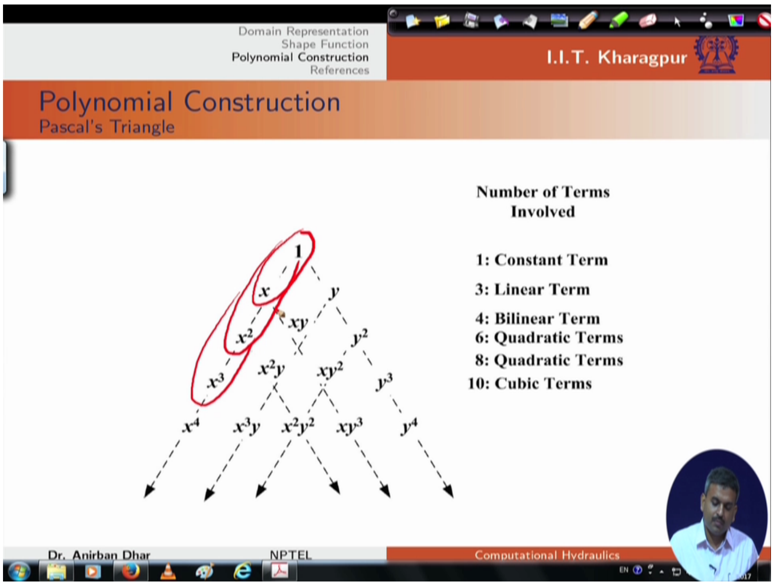


But if you are considering two-dimensional case, we will have 1, x, y, thencombination of like this.

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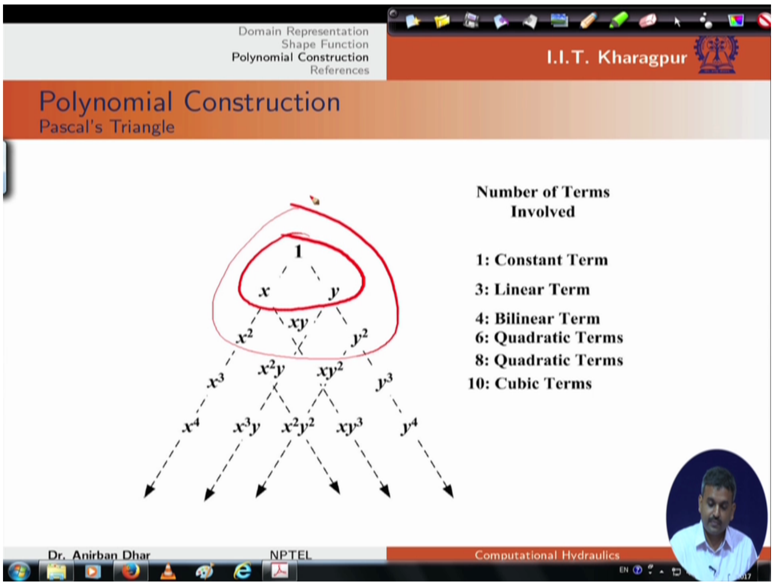


Now this is Pascal's pyramid. If you are consideringthree variablesin this particular case, thenwe can have 1, x, y, z. And on the other side we have x square, y square, z Square and xy, yz, xz terms.

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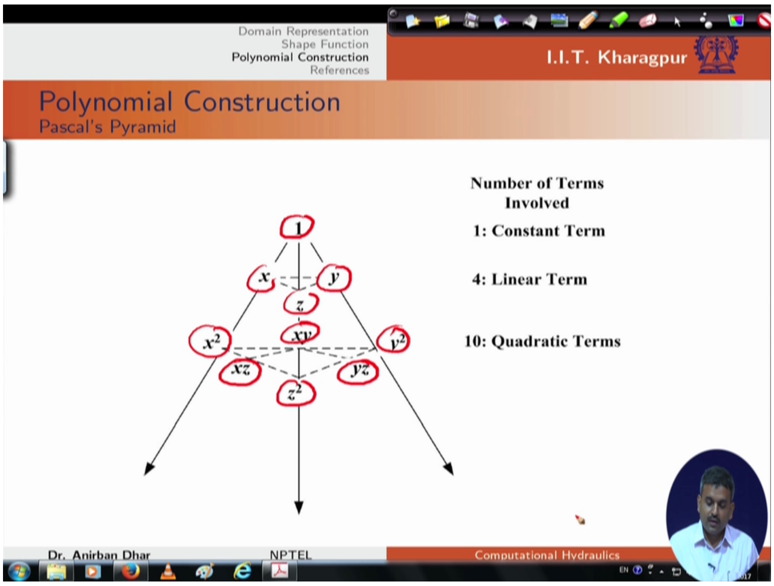


In our previous case we have seen thatwe can have constant terms with respect to 1, x, y. And in this case we have 1, x, y, z. So polynomial function, we have p0 x vector, this one equals to 1. P1 x, this is equals to x. P2 x, this is equal to y. In this case again p3 will be added, equals to z. So we can have constant terms, linear terms. Constant equals that means 1. Linear term, that means four terms, 1, x,y, and z. And quadratic, that means considering 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. Total ten terms.

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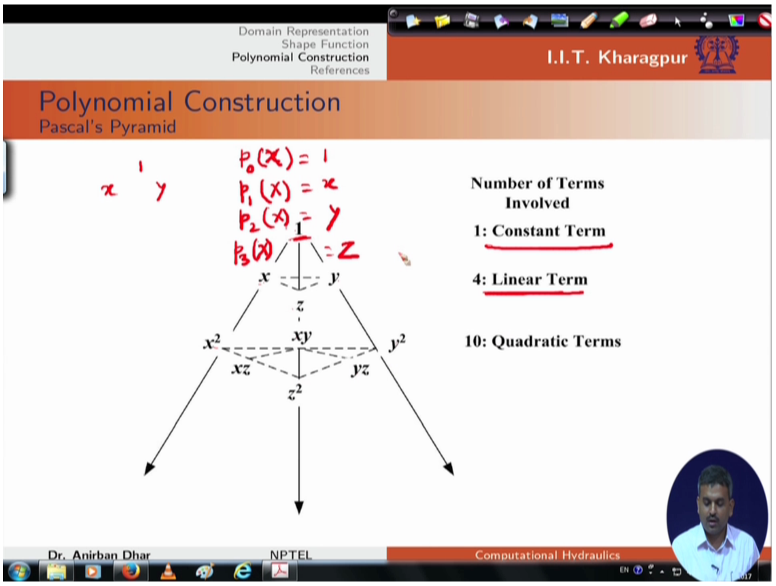


So with thispolynomial representation or basis representation we can represent a particular function in terms of series with a0, a1, a2 as coefficients with polynomial basis support. Thank you.