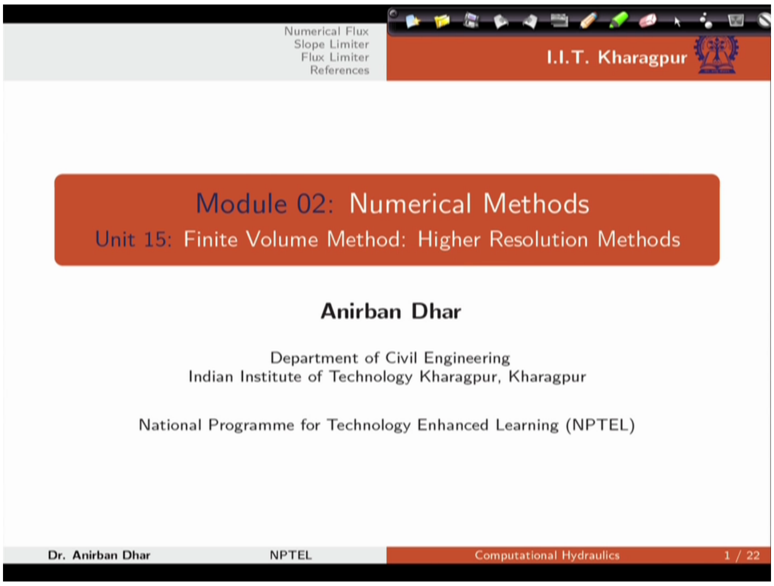
Computational Hydraulics
ProfessorAnirban Dhar
Department of Civil Engineering
Indian Institute of Technology Kharagpur
Lecture 19
Finite Volume Method - High Resolution Methods

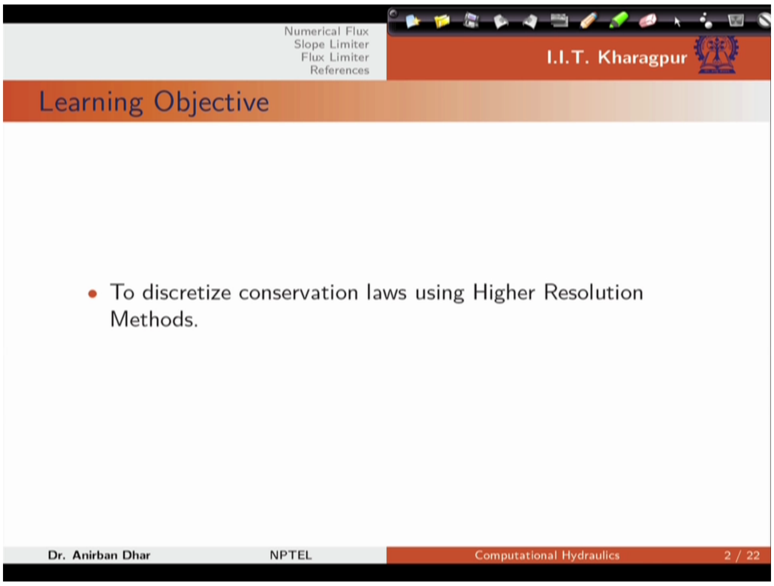
Welcome to of this lecture number 19 of the course computational hydraulics.We are in module 2, numerical methods. And in this particular lecture class I will be covering unit 15 and this is the last lecture class for finite volume method. And we will be discussing high resolution methods.

(Refer Slide Time 00:35)



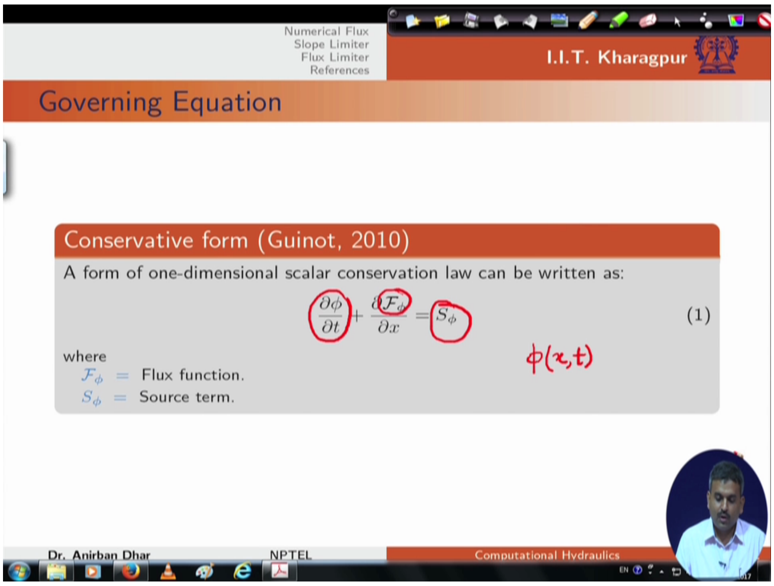
Learning objectivefor this particular unit. At the end of this unit students will be able to discretize conservation laws using high resolution methods.

(Refer Slide Time 01:00)



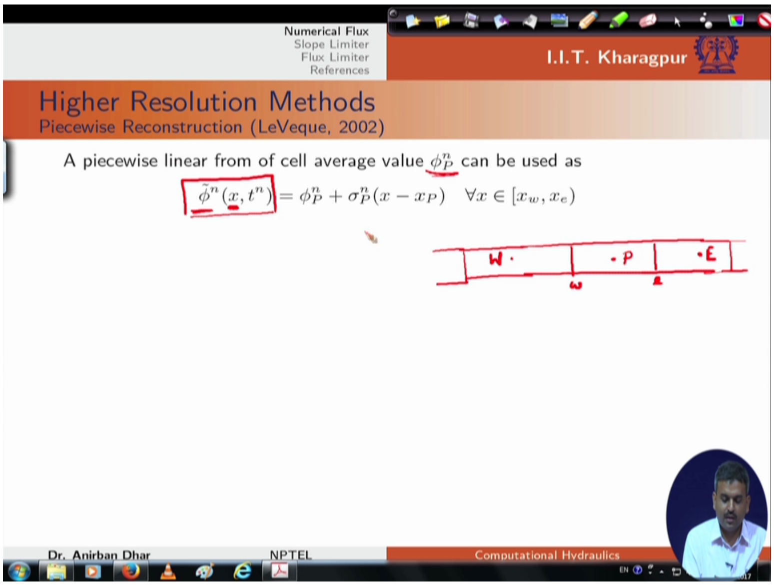
This is our well known governing equation that we have utilized for our earlier lecture classes. Phi is again general variable here and in one dimension we have only xt representation. F is our flux function. We can have different representations for this flux function. And S phi is our source sink term. So this is our one-dimensional conservation law.

(Refer Slide Time 01:46)



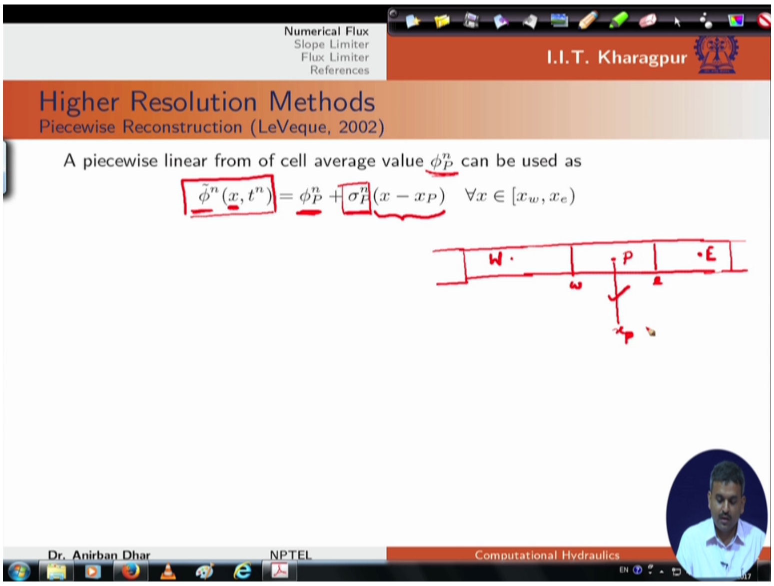
Now let us see that with high resolution methods, we will be utilizing the concept of piecewise reconstruction. Let us say that we have our general cell where in one dimension we have central cell as P. E cell is I plus 1 and west cell or let us say this is W cell. Now with this if we start, now we can have this east and west face for this one. Let us say that cell averaged value phi P can be used to construct this phi n tilde and this one is xtn which is dependent on x and tn that is present time level value.So here x is variable.

(Refer Slide Time 03:22)



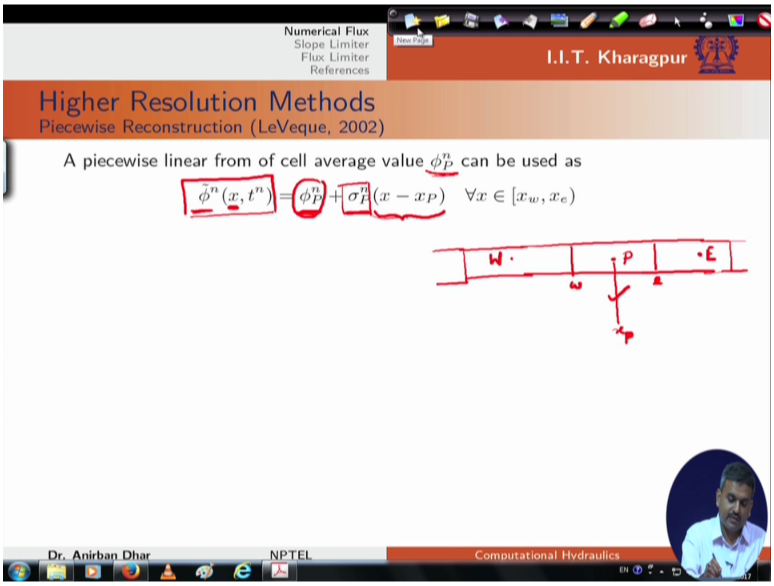
We have phi n value and this sigma Pn which is calculated at the present time step for cell P, we will be utilizing this thing. And we have this difference x. X is arbitrary any distance and xp is our co-ordinate for this Px cell.

(Refer Slide Time 03:57)



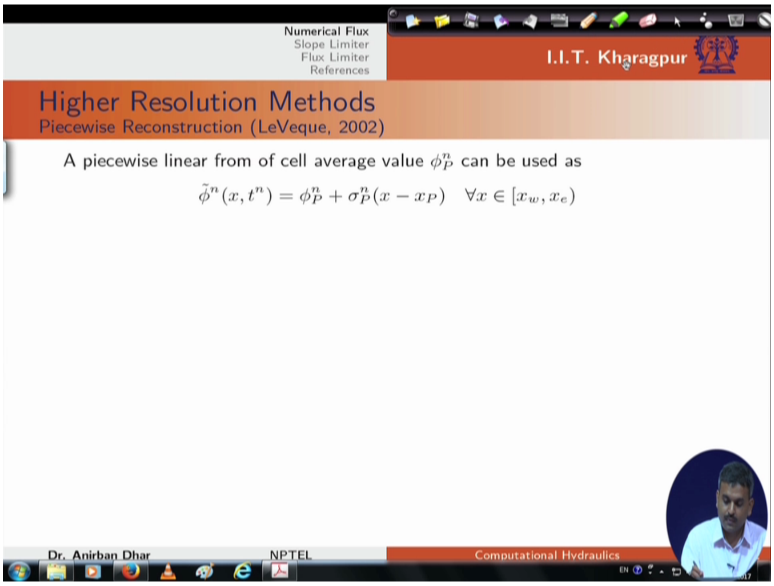
So in one dimension we have represented this phi tilde xtn in terms of cell centered value. Some differencewhich is with respect to our cell centered valueof the co-ordinate. And sigma Pn is another coefficient.Let us see what is this coefficient?

(Refer Slide Time 04:29)



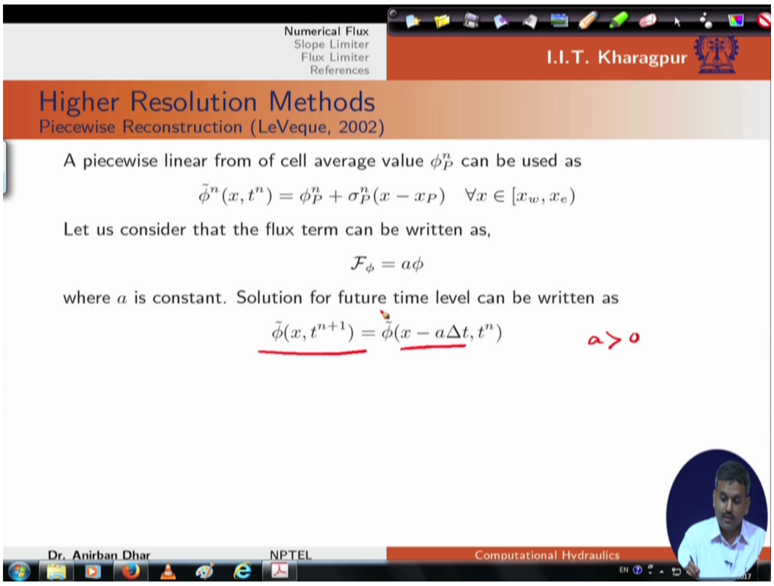
And this is validobviously for this range, xw and xe. That means within P cell only this approximation is valid. This is obviously a linear approximation.

(Refer Slide Time 04:54)



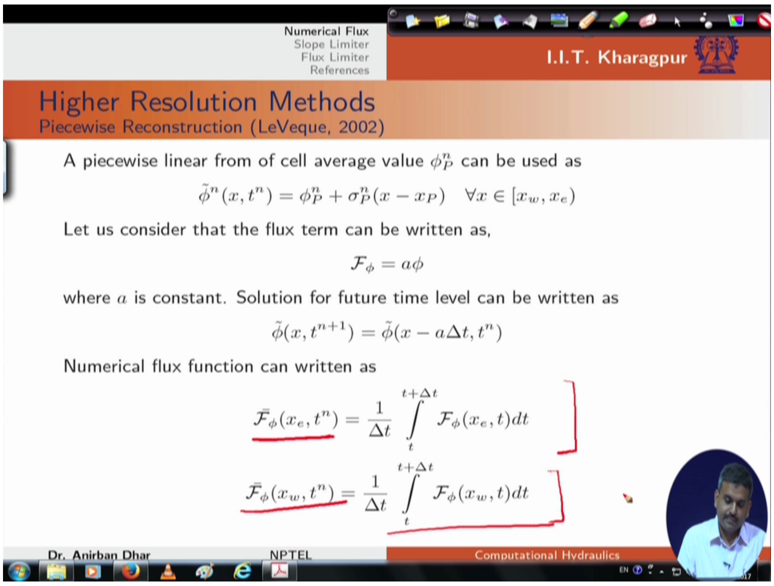
Now let us consider our simplified flux term where A phi is the flux function and A is constant. Phi is our again general variable. With this if we proceed, solution for future time level that is tilde x tn plus 1 can be represented in terms of tilde x A minus A t, provided that A is positive. So we can get the future time level value for a particular cell based on the previous time level value at any arbitrary location x tn plus 1.

(Refer Slide Time 05:57)



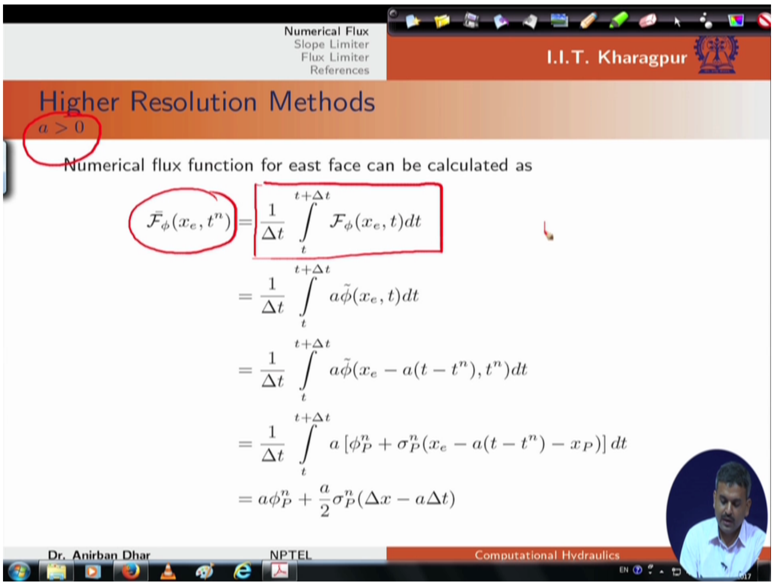
Now with thiswe can calculate our numerical flux function. Calculation of numerical flux function is the most important thing for this exercise of high resolution methods. We know that flux functions are calculated at the interface only or that means east face xe and xw for the west face. So average value we can calculate considering the time interval t to t plus delta t. That is tn to tn plus 1.Similarly for west face also we can calculate the same thing.

(Refer Slide Time 07:00)



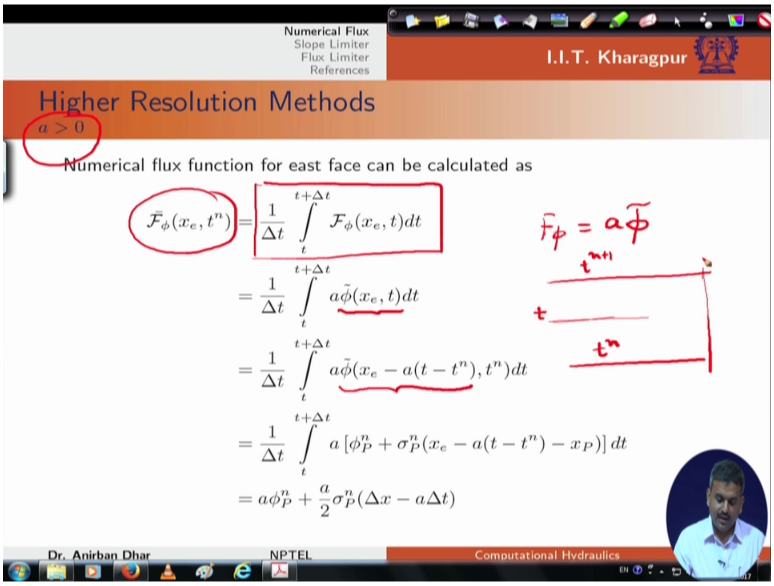
Now approximation is important in this case. So let us consider the situation where we have A greater than zero. That means A is positive and phi, the average value of the fluxat the face or east face, we can represent like this as per our definition.

(Refer Slide Time 07:37)



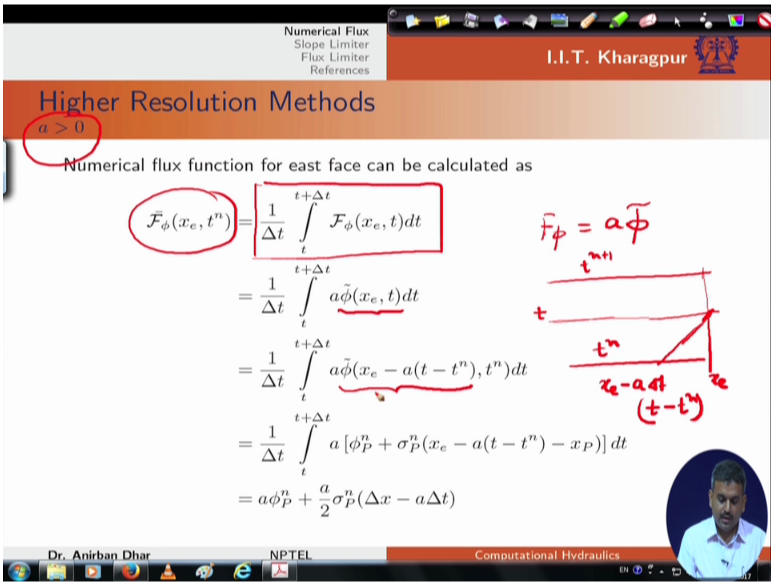
Now we already know that our Fphi equals to A phi. Now let us use our tilde values which is the constructed value. Now if we replace this phitilde with our approximation that meansfor this one we can get our phi tilde for phi xe A t minus tn. That means we have any arbitrary time level t which is in between tn plus 1 and tn, we are considering this. So obviously if we are considering the future time level value,we should consider this variation within tn to tn plus 1.

(Refer Slide Time 09:02)



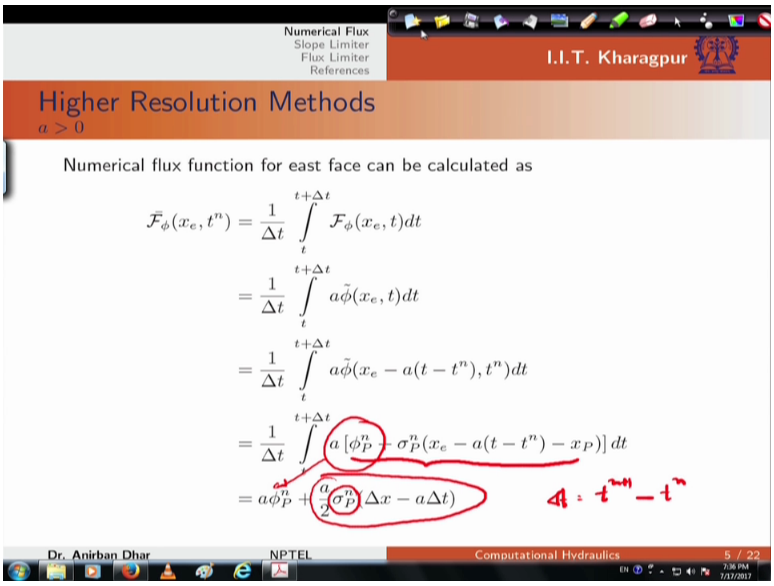
Now at xe t, we have some value. So obviously we should consider del t. That means if this is xe, this co-ordinate should be xe minus A delta t. In this case delta t is t minus tn. So we can replace this and we can calculate this derivative.

(Refer Slide Time 09:41)



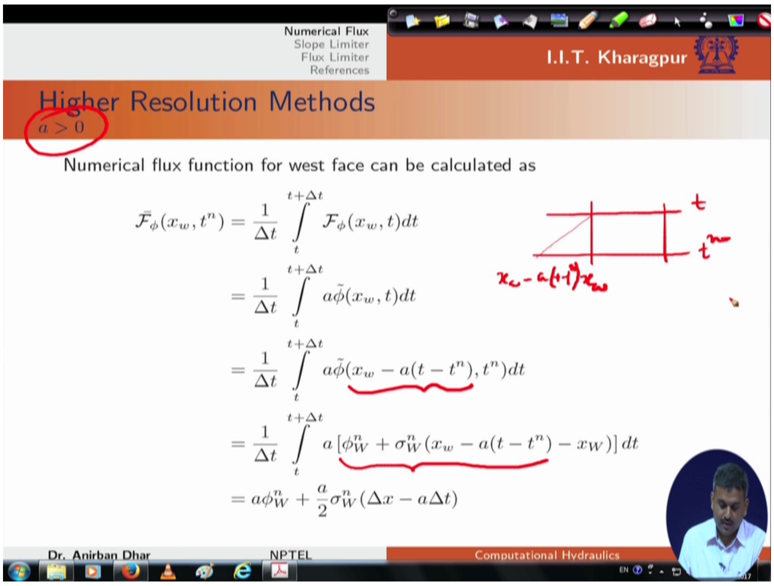
So in this particular casewe can replace our linear approximation for this one. So obviously this is xe minus A delta t or in this case delta t is t minus tn minus xp. So this is general x. In this case, x minus xp and phi pn. Now if we simplify thisfrom the first term we are getting, A phi pn. And from the second term we are getting this second part which is in this case obviously delta t means we are taking variation between t n plus 1 and tn. So this phi p or sigma pn, this term is important here.

(Refer Slide Time 10:59)



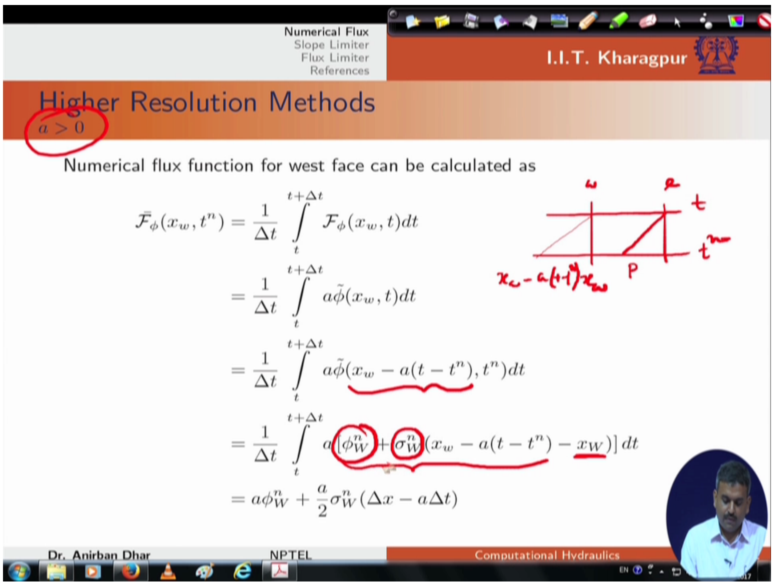
Similarly if we calculatethe numerical flux function for the west face. Again for A greater than zero or positive case we will have again t level and this is tn level. So obviously in this case if this is xw, we can calculate this one based on xw minus A into t minus tn. And this term have replaced here. And in this case approximation is in terms of west cell value.

(Refer Slide Time 12:02)



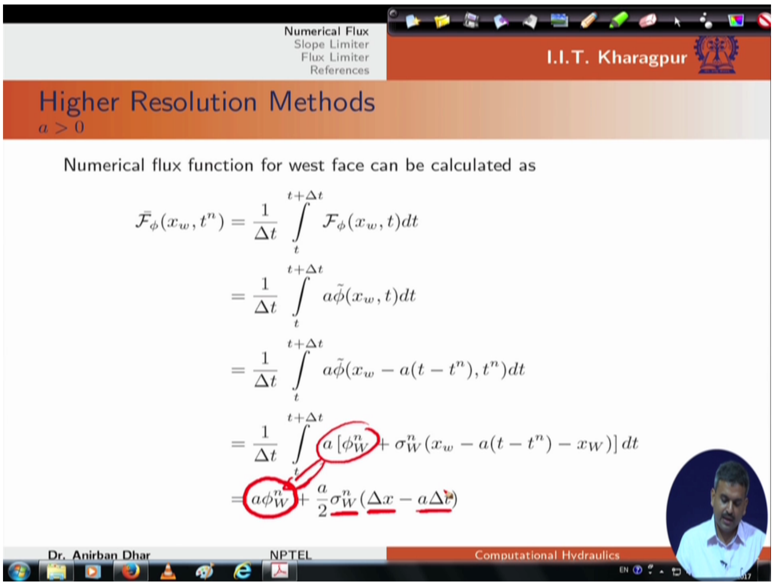
In previous case we have seen thatour calculations are based on P cell only. Because variation we have considered within cell and flow of information from that cell to this face. But in case of west face, the flow of information is from west cell. So we are considering phi wn, sigma wn, these two values. So obviously piecewise approximation should be in terms of xw which is the cell centered value of the W cell.

(Refer Slide Time 12:50)



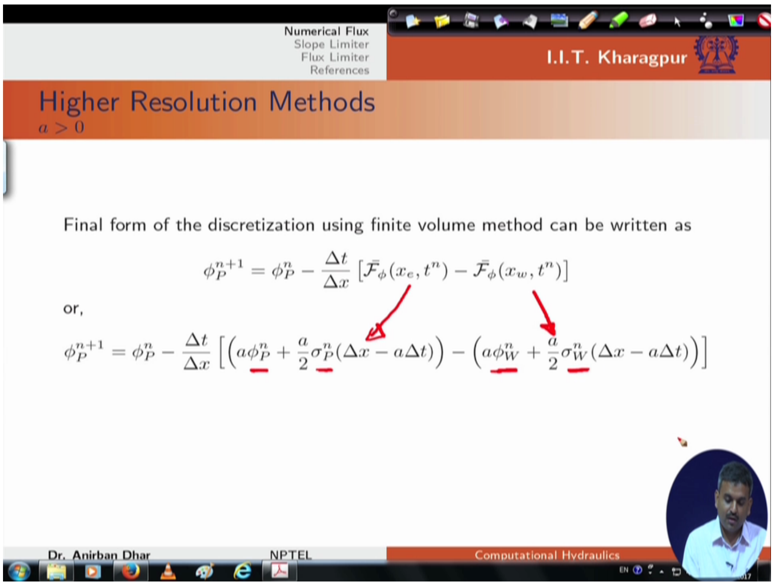
Now from the first term we can get this approximation directly. And from the second term we can get this A by 2 sigma w and delta x minus A delta t. Obviously delta t means the change in time level from tn to tn plus 1.

(Refer Slide Time 13:18)



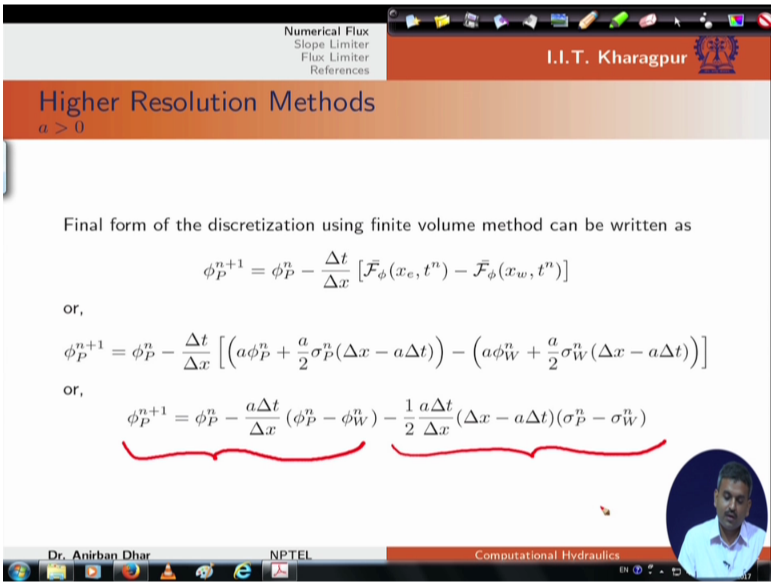
Nowlet us considerour original finite volume equation. If you want to calculate phi P n plus 1, we need information about phiPn, del t by delx. And these two are numerical flux functions. So numerical flux functions,we have calculated in terms of phi P and phi w. Now we can replace this in this particular expression. Now you can see that for east face we have phi P, sigma p and for west face we have phi w and sigma w.

(Refer Slide Time 14:22)



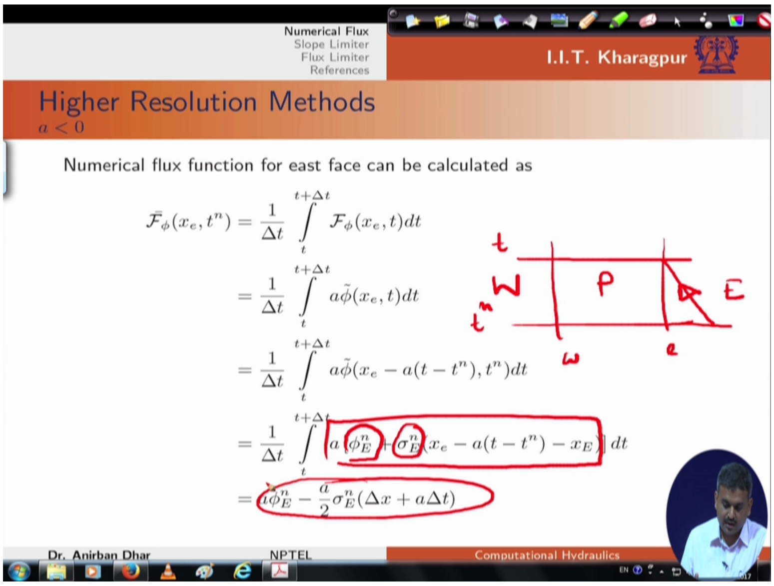
Now if we (re) rearrange this one, we can write it like this. So this is our original expression where we have directly replaced this A phi p value minus A phi w n value. And this term is extra. This term is extra in this case. Now we need to interpret this particular term physically.

(Refer Slide Time 15:03)



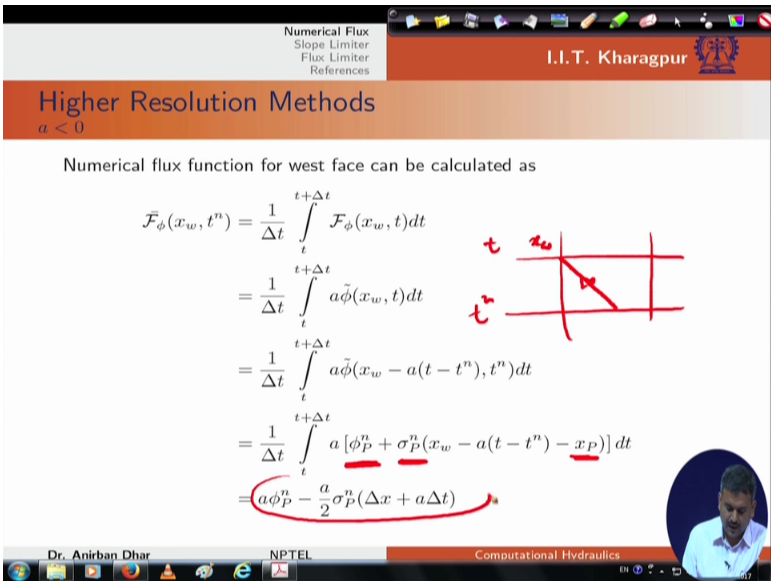
Next thing is if we consider A less than zero. In case of A less than zero that means again this is east face, this is west face and this is t level, this is tn. So obviously the flow of information for this particular level is from east cell. So we are considering phiE sigma E in this case. This is E cell, this is P cell, this is W cell. So flow of information is from east cell. So we have calculated all the values in terms of phi E and sigma E. Now finally we can get the value of the integration.

(Refer Slide Time 16:11)



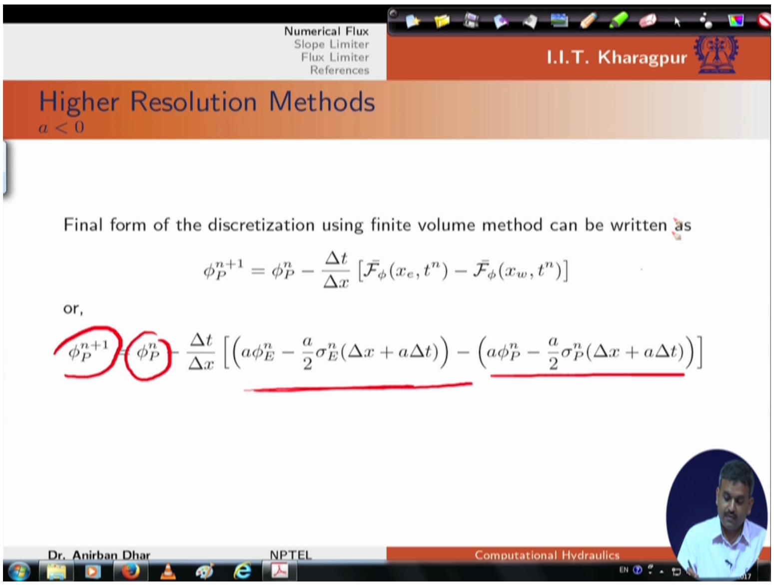
Similarly for west face. Now for west face again, this is t, this is tn. So information will be travelling from our P cell to xw. So we can replace this phi P sigma P and xp, in this case. Now this is our final form of the numerical flux.

(Refer Slide Time 16:50)



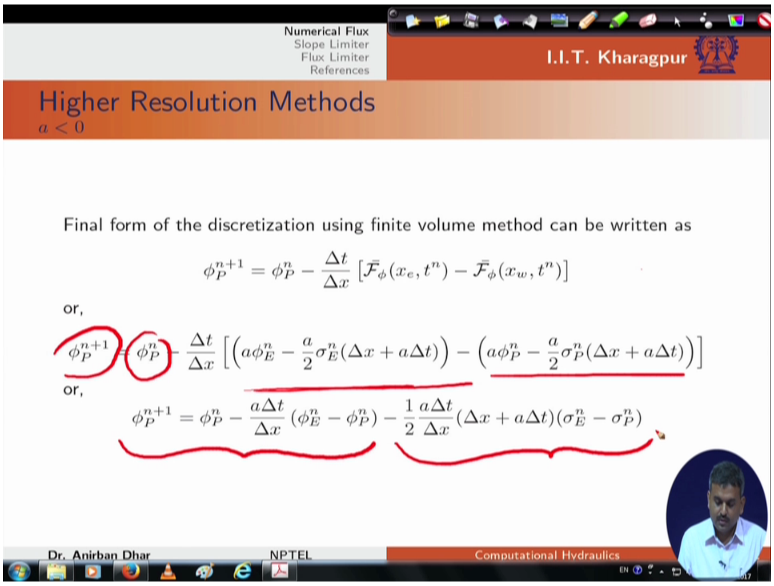
Now we can write similar expression for A less than zero by putting this integral values. Now in this case we can see that phi Pn,phi n plus 1 equals to phi Pn. Andwe have phiE and phi P combination. In case of A greater than zero, we have phi p and phi W combination.

(Refer Slide Time 17:38)



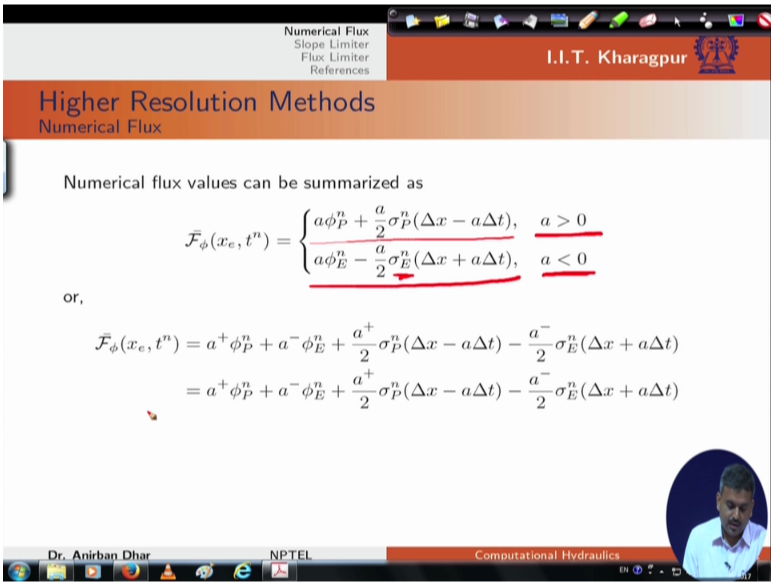
In this casewe have in natural terms, this is our original expression and this is extra one.

(Refer Slide Time 17:51)



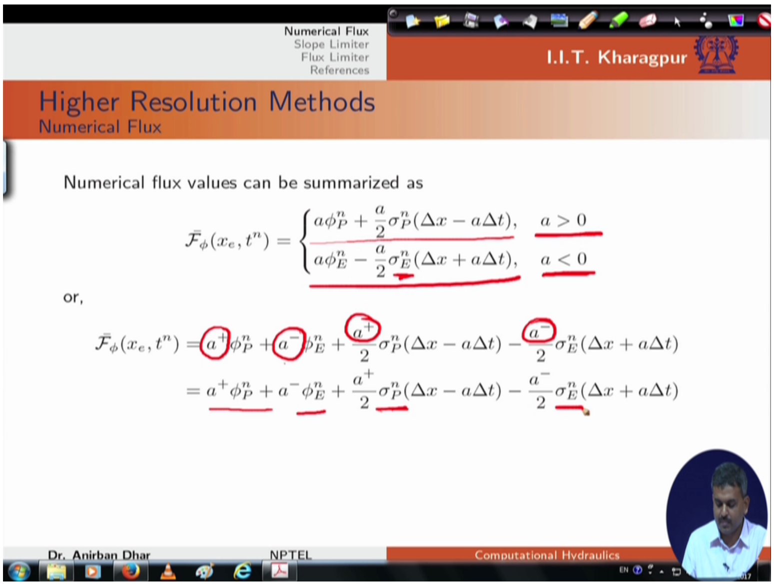
So in this case if we summarize both the things, so for east face we have A greater than zero. It should be represented in terms of phi P. If A less than zero, it should be represented in terms of phi E and sigma E.

(Refer Slide Time 18:24)



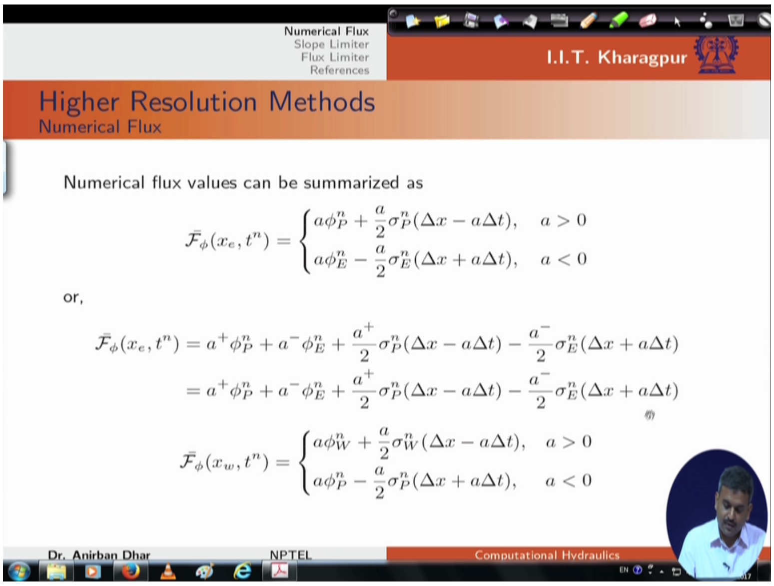
Now if we combine these two, we can easilyrepresent this one and in this casewe can use this expression where we have A plus and A minus that we have utilized in our previous lecture class. A plus, A minus obviously for A greater than zerothis is our expression. And we have just written it in terms of phi P, phi E, sigma P and sigma E, in general form.

(Refer Slide Time 19:22)



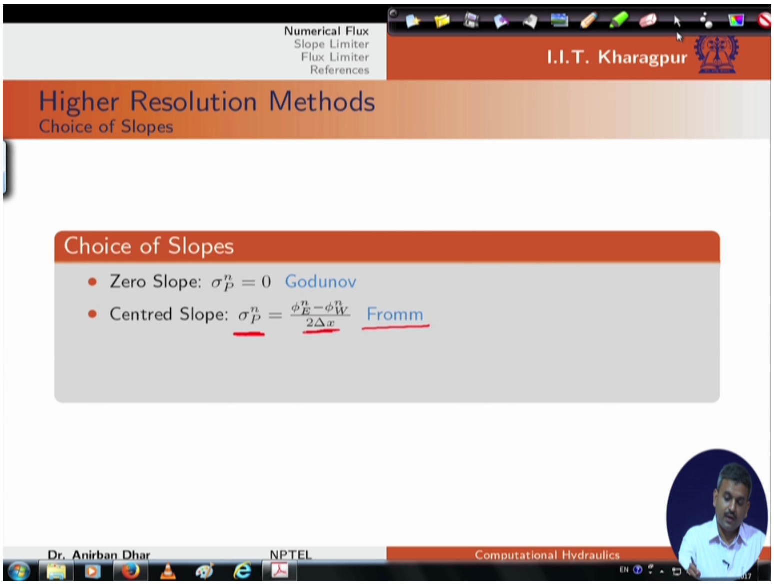
Similarly we can also write the numerical flux function in terms of this compact representation.

(Refer Slide Time 19:38)



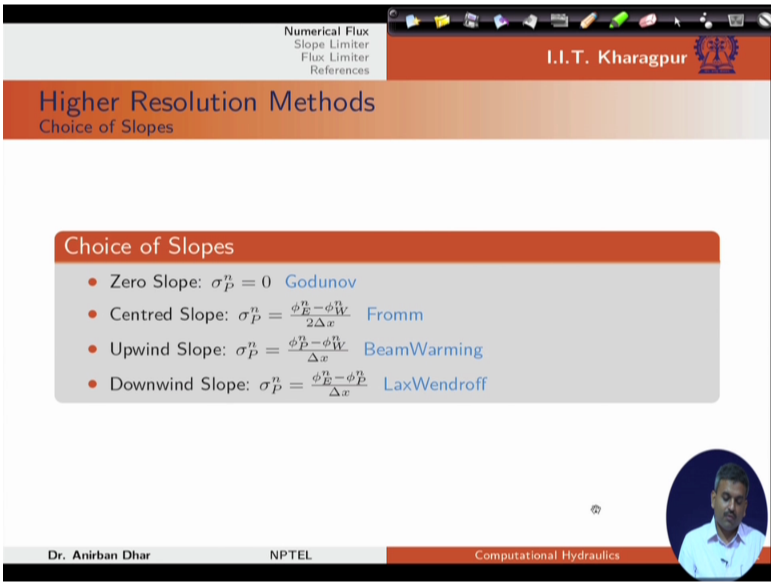
So what is this sigma P? Sigma P is nothing but this is slopeas per Godunov, zero slope. That means if we use sigma Pn equals to zero, then we will get our Godunov scheme that is the basic one. Now for higher order or higher resolution method we need to use different slope values. If we use this centered slope, that meansin this case this sigma P is dependent on phi E and phi W value (divi) and divided by 2 delta x that means the distance between them. Soif we use this, this is called as Fromm slope.

(Refer Slide Time 20:50)



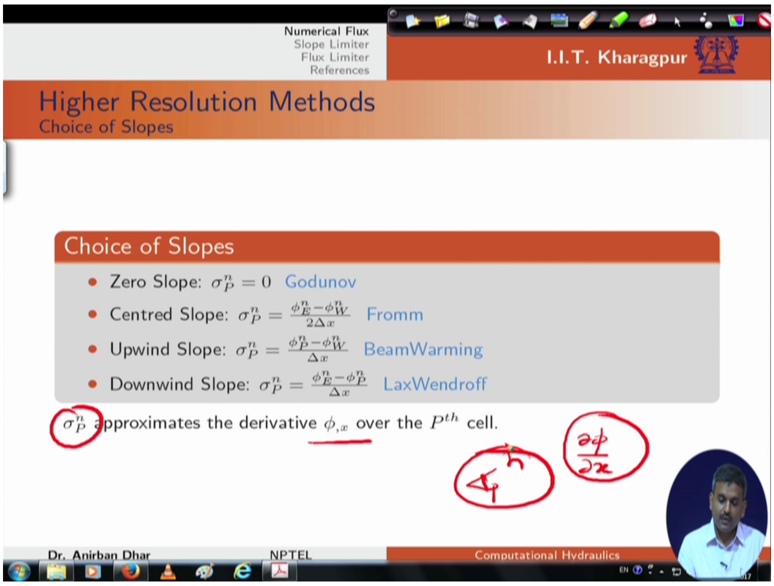
Next one is obtained for upwind.Depending on the direction we change the slope phi P minusphi W. And this is divided by delta x. So we areconsidering the adjacent cell values in this case.

(Refer Slide Time 21:31)



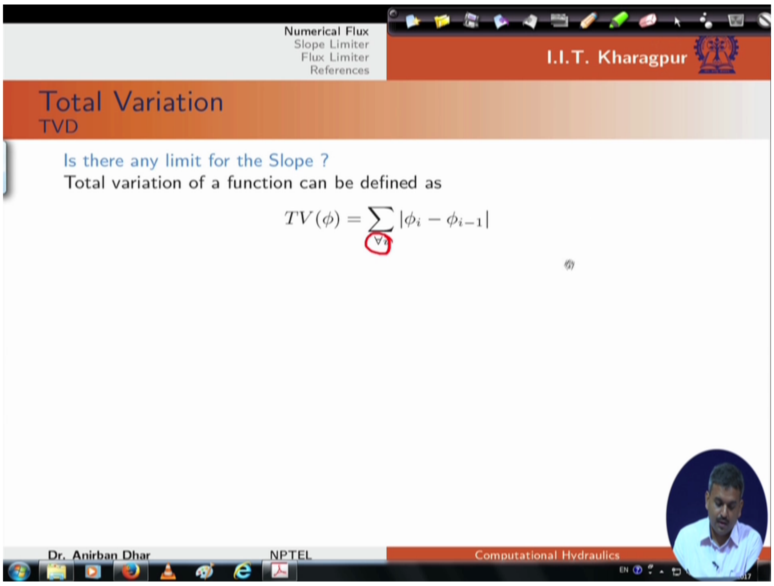
And downwind, this is phi E minusphi Pdivided by delta x. This is called as Lax Wendroff scheme. Now the choice of slope may create some problem or oscillation withinthe method. Sothis case we can see that the sigma P, physicallyit approximate the derivative phi comma x. That means del phi by del x for over the pth cell, if you are considering sigma Pn.

(Refer Slide Time 22:11)



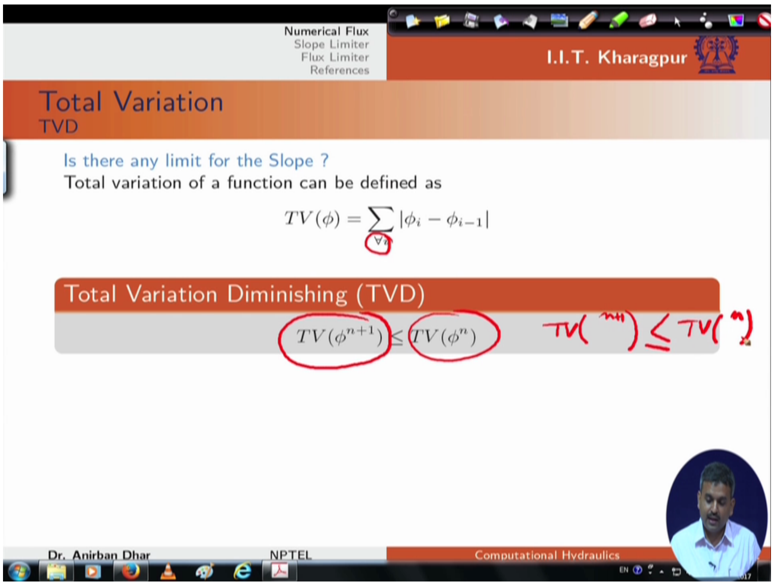
Now we need to see what can be done for this particular slope. Is there any limit for the slope? We can check the total variation. Total variation of function can be defined as submission over all I and this is phi I minus phiI minus 1. That is previous cell value.

(Refer Slide Time 22:47)



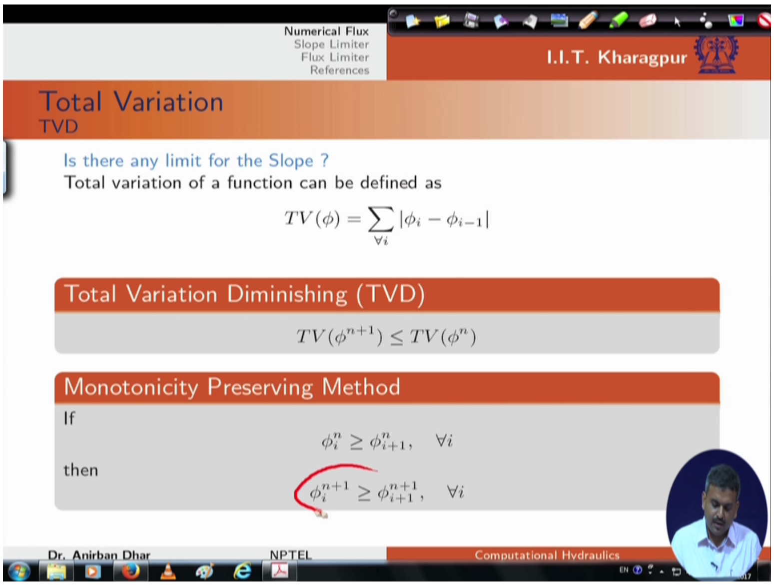
Now total variation diminishing scheme such that if we have total variation for nth n plus 1 level should be less than equal to total variation in phi or nth level. That means TV, total variation for n plus 1 level, this is less than equal to total variation at our nth level.

(Refer Slide Time 23:20)



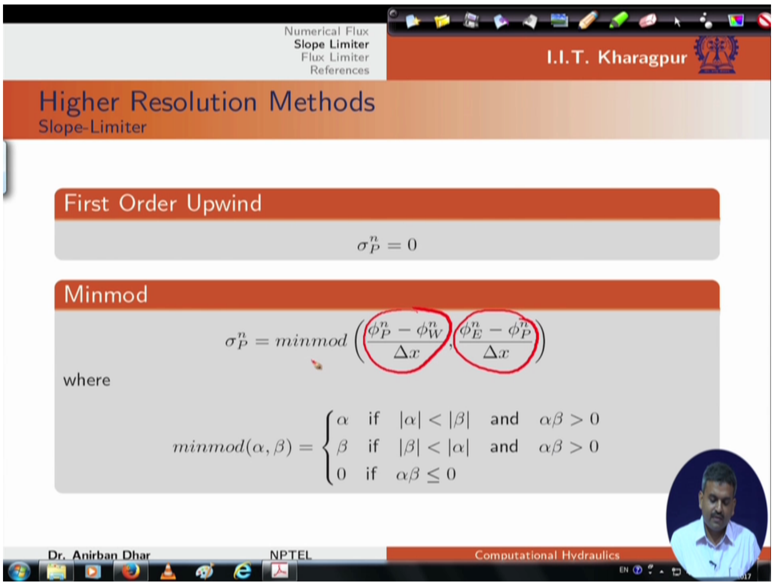
Now monotonicity preserving method. If phi In is greater than phi I plus 1n, thenphi I n plus 1 this should be satisfied. This is called as monotonicitypreserving method.

(Refer Slide Time 23:44)



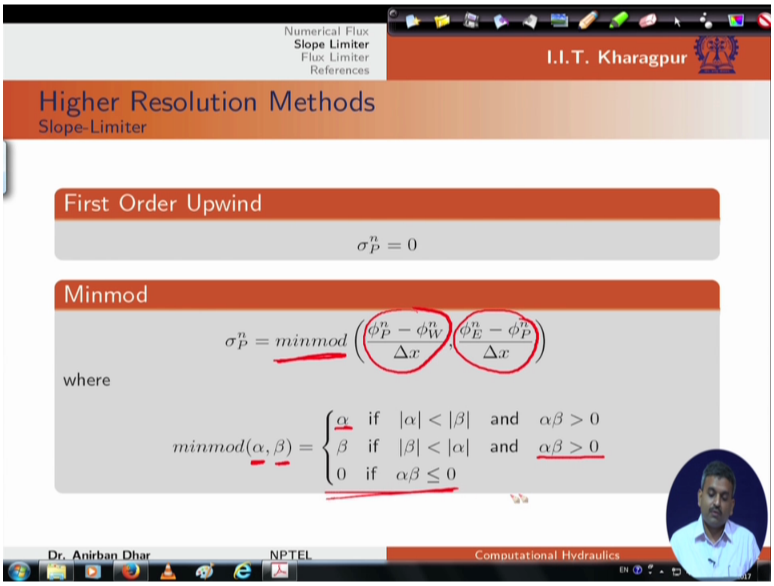
Now TVD is monotonicity preserving method. Now slope limiters are required for high resolution method to check the oscillation. So first order upwind, sigma P equals to zero. That is our Godunov method. Minmod slope, in Minmod slopewe generally useagain the first order derivative of the functionfrom both the sides and we use this Minmod function.

(Refer Slide Time 24:35)



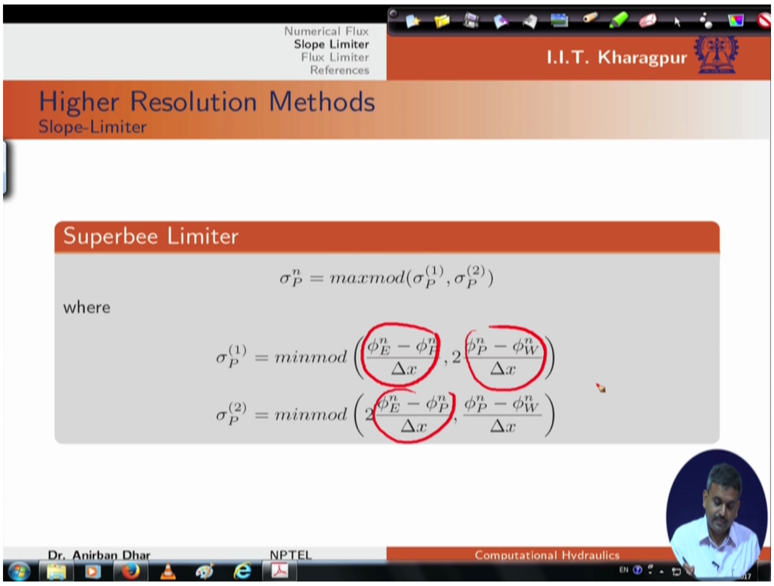
Interestingly if we have alpha and beta in Minmod function, this is equal to alpha if mod alpha is less than mod beta. And alpha beta greater than zero equals to beta if mod beta is less than mod alpha and alpha beta greater than zero. And equals to zero if alpha beta less than equals to zero.

(Refer Slide Time 25:07)



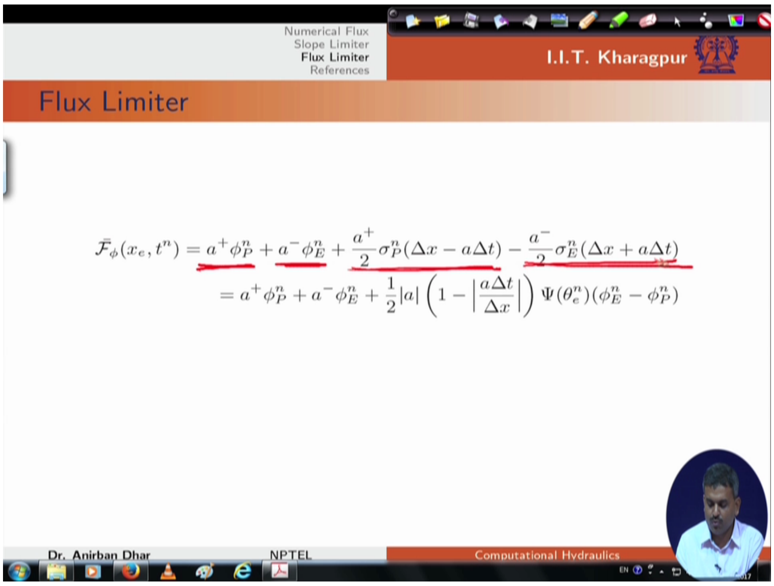
Another slope limit is Superbee limiter, where sigma P1 is Minmod. You can see that in this case only the derivative value and twice from the westside. And second one is twice the east side and this is single. So this is Maxmod function.

(Refer Slide Time 25:56)



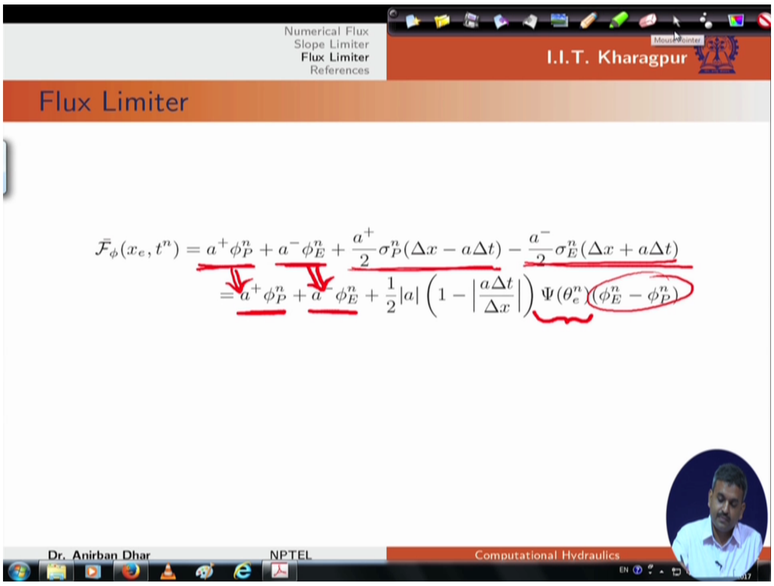
We have flux limiter and what is this flux limiter? In our previous case we have seen thatpositive values and negative valuescan be represented like this. And slope related terms are these two terms.

(Refer Slide Time 26:25)



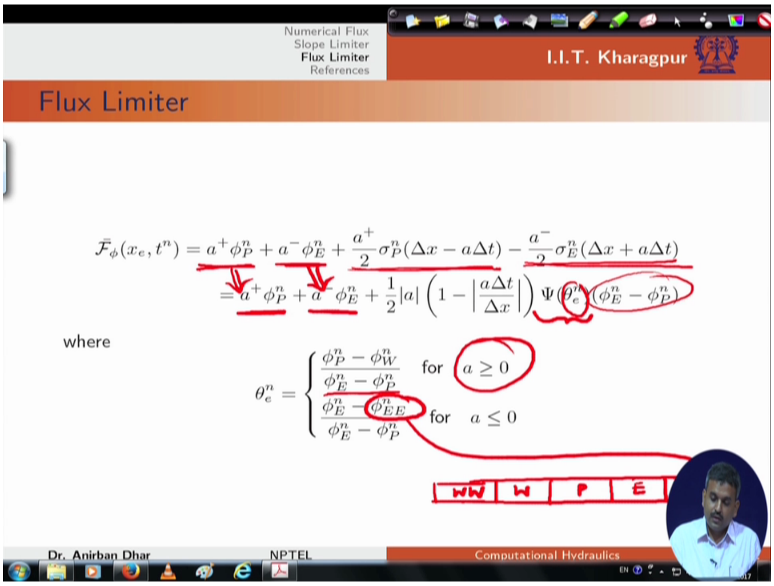
Now in this particular case if we write this with first two terms directly and we approximate the second term. That means we are using some kind of limit for the slope. But in this case we are considering sigma E this is phiE minusphi P and this psi function is used. Now we need to define different forms of this psi for flux limiters.

(Refer Slide Time 27:14)



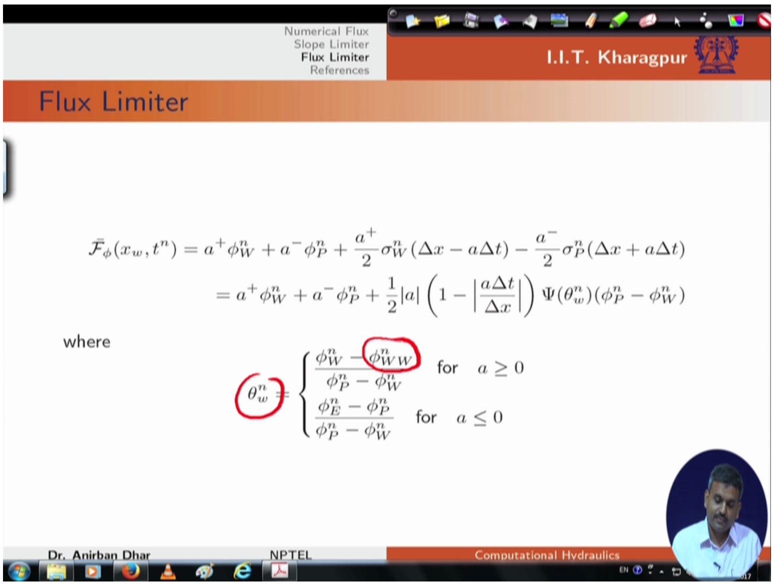
In this case, this theta En, this is defined as phi P minus phi W divided by phi E minus phi P for A greater than zero. And this is phiE minusphi EE. EE means east-east. If we have central cell as P, this is E, this is W, and extreme left we will have WW and extreme right we will have our east-east cell. This is actually east-east cell value. And this is for A less than zero.

(Refer Slide Time 28:28)



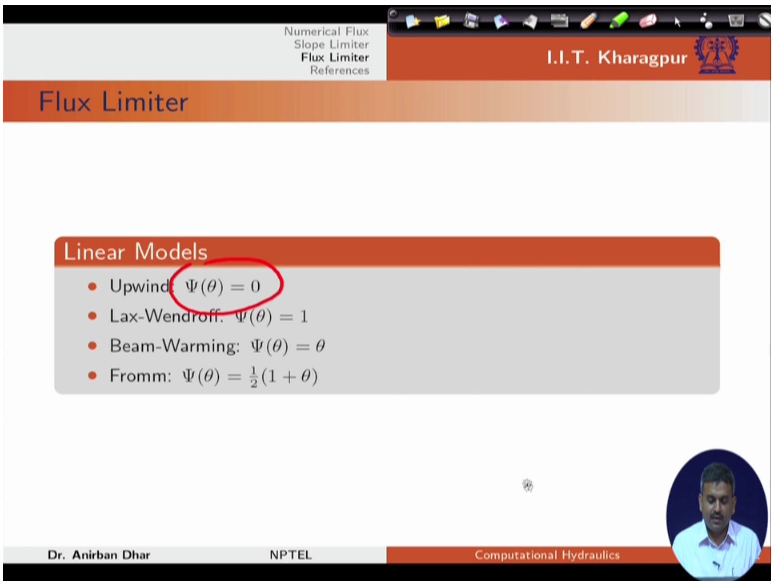
Now if we consider the west face, again we can approximate the flux for the west facewith this theta. And in this case we are utilizing this WW. That means west-west cell value or extreme west cell value.

(Refer Slide Time 28:58)



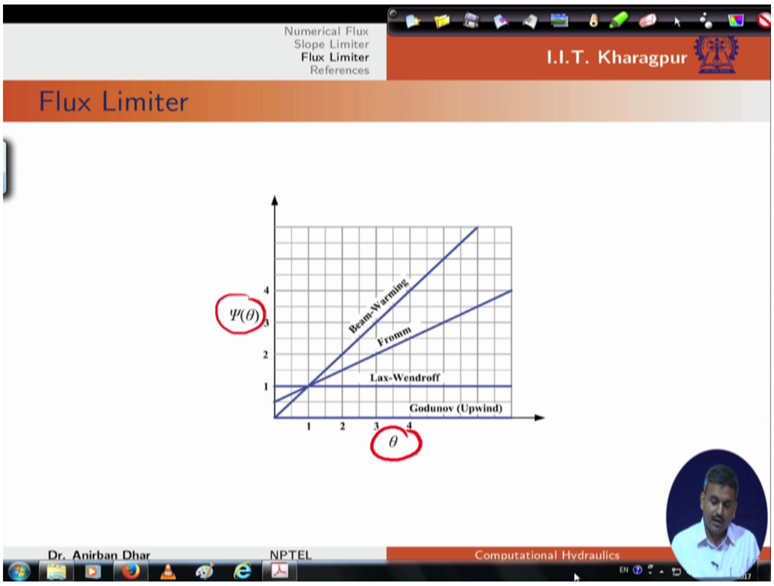
Now what can be this flux limiter? For upwind scheme we have psi theta equals to zero. These are linear modals.Lax Wendroff, we have psi theta equals to 1. Beam-Warming, this is psi theta equals to theta. And Frommslope, psi theta equals to half 1 plus theta.

(Refer Slide Time 29:32)



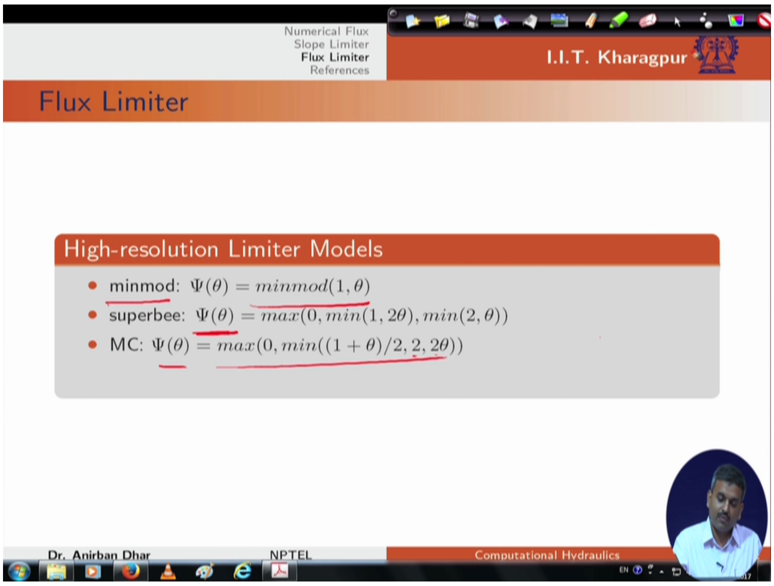
In this casethese are the different slopes. With variation of theta, what is the change in psi theta? We can see from this particular diagram.

(Refer Slide Time 29:54)



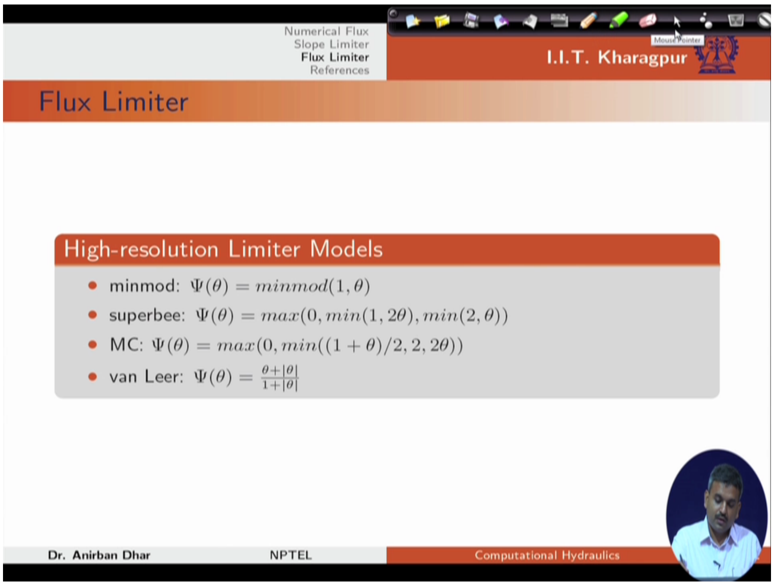
Now another kind of flux limiter can be therewhich are higher resolution limiter or nonlinear limiters. We can have other methods but we have men Minmod, Minmod of 1 theta. Then we have maximum of this is our Superbee. This is psi theta again written in terms of theta thing. And this isMC. This is psi theta again max, zero minimum 1 plus theta by 2 and 2 and theta. This is a minimum of that we can utilize.

(Refer Slide Time 31:00)



With this slope limiter and flux limiterwe can get different type of approximation for numerical flux and we can utilize these approximations for our cell centered value calculation for future time level. So another one is Van Leer, this one.

(Refer Slide Time 31:31)



Thank you.