Computational Hydraulics
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Lecture 18
FVM - Godunov Approach

Welcome to thislecture number 18 of the course computational hydraulics.We are in module 2, numerical methods.In this particular lecture I will be covering unit 14,finite volume method and Godunov approach.

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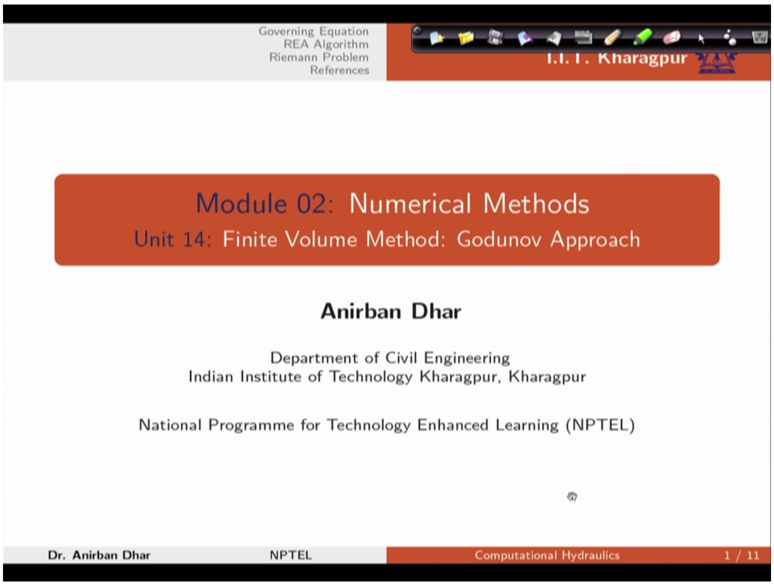

Learning objectives for this particular unit. At the end of this unit students will be able to discretize conservation laws using Godunov method.

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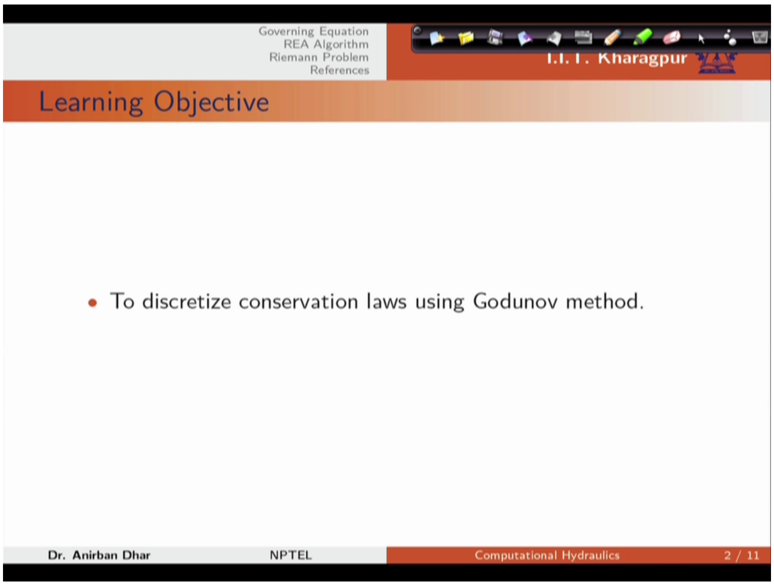

We already know this is our conservative form in terms of phi. F phi is the flux function. S phi is the source sink term.Let us consider from our general case one simple approximation. That is, flux can be written in terms of A phi, where A is constant, phi is the scalar variable.

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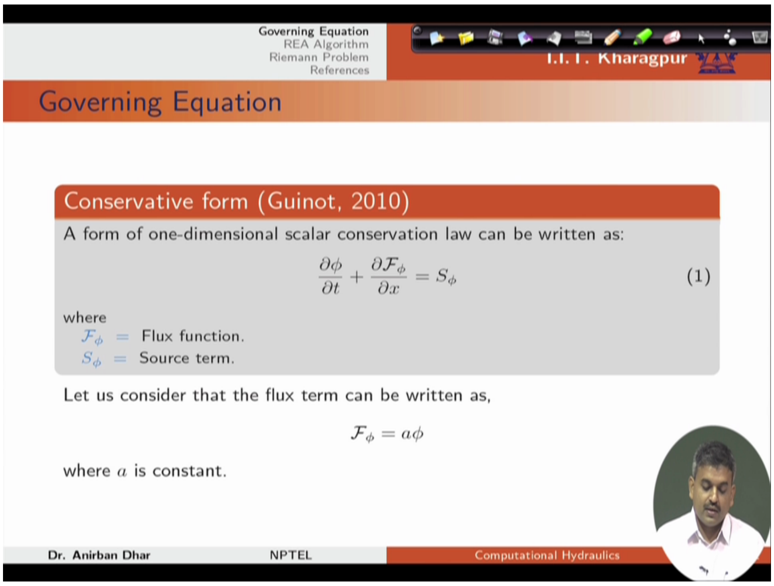

Basic structure of Godunov method is with REA algorithm. REA means, reconstruct, evolve and average. In this case we reconstruct a piecewise polynomial from cell average value phi P n. As phi tilde this phi tilde n, x, tn, that is present time level. This is with the present time level value and where x is local.

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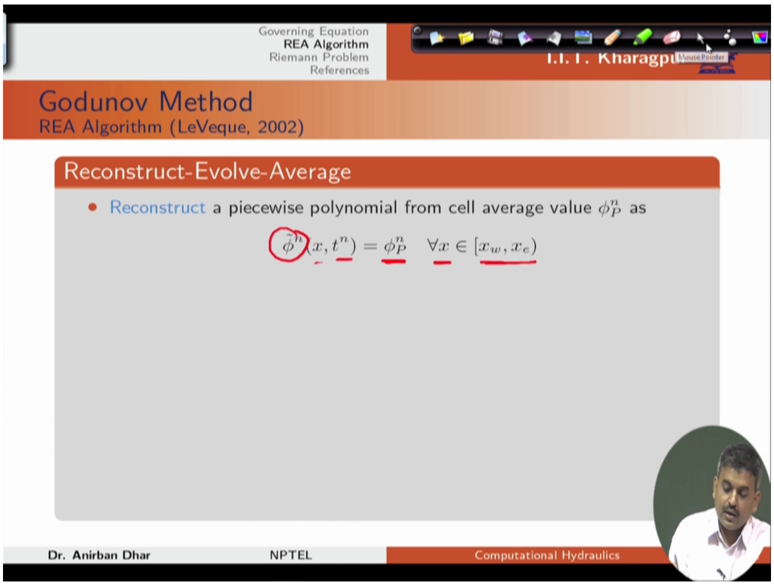

So in the next step we evolve the hyperbolic equation with base condition. Base condition or initial condition to obtainthe future time level value tilden plus1. And at thislevel we use our existing knowledge or previous methods to evolve this hyperbolic equation with base condition. Then in the last stagewe take average of the polynomial function at cell level to obtain cell average value at future time level. Soit has got 3 steps. One is reconstruct, second is evolve and third one is average or taking average of the value.

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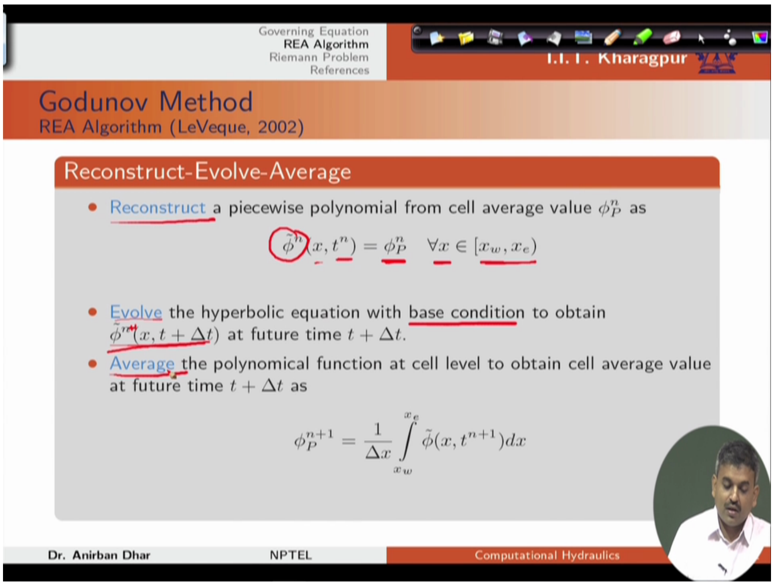

And in this case, this phi tilde n x tn or tn, this is subscript or superscript, is constant over time interval tn. Where t is within n to n plus 1. And within this time interval,this tilde phi function is constant.

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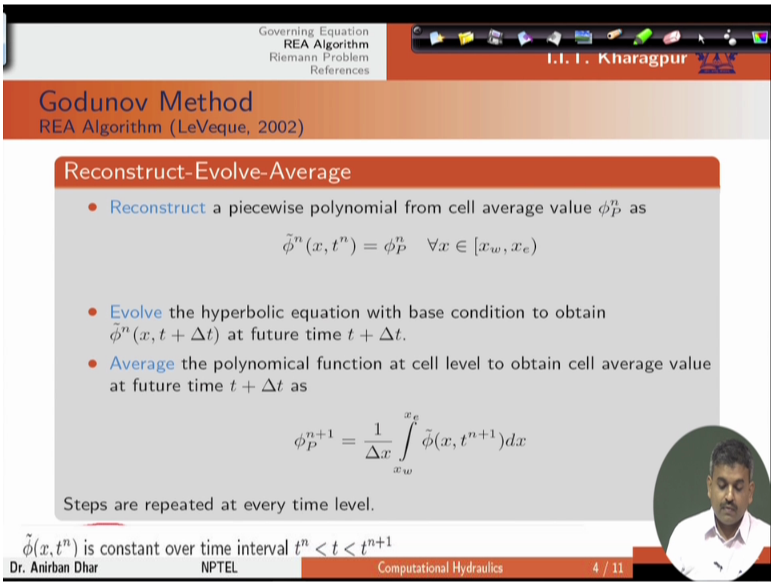

Now we already know from our Riemann problem that at interface there is discontinuity in the information on the each side.On the left side we have p cell and this side we have e cell. So there is discontinuity in the value.

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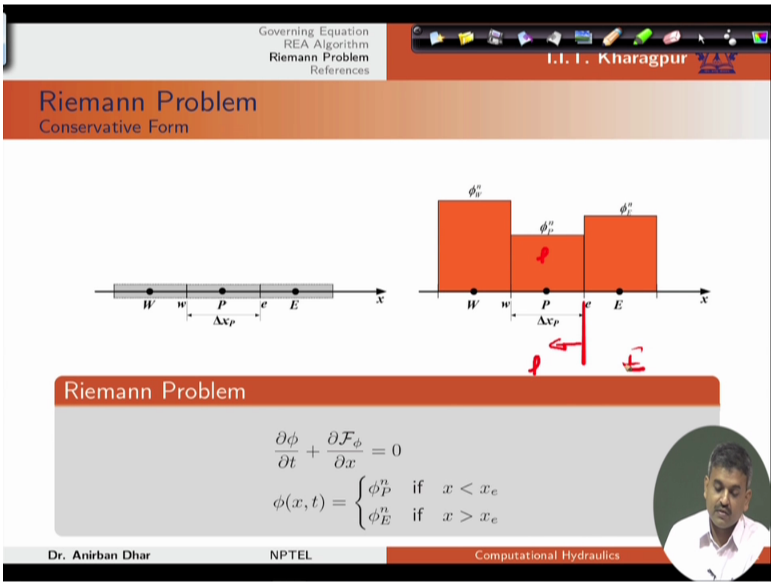

Now at this e face we can construct Riemann solution. For construction of Riemann solution we can define single variable. So the flux at cellface depends on the exact solution of this tilde x tof the Riemann problem along with the access along the axis. Considering along the t axis. Considering local coordinates, we have this tilde x t. Where in the east face we have x minus xe, t minus tn. That means we are taking this point as a starting point.

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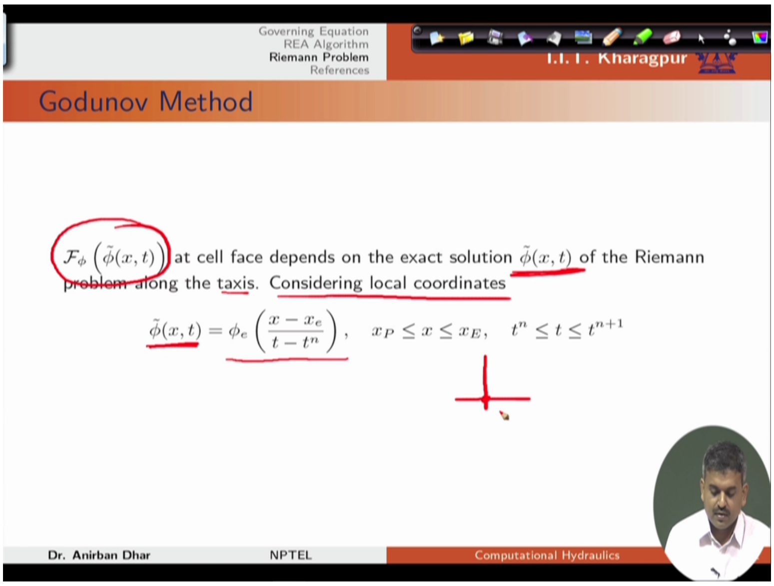

We have p cell, this is e face, this is east cell, west cell. At interface we are considering the Riemann problem at e face at this point only. So x in this case varies from xp. This is xp to xe. That means center of the cell e and p. And the time, this is again, if this is level n, this is n plus 1. So time interval will beconsidered in this case. So everything varies between this n and n plus 1 time level. And we have defined one single variable. Single variable, we can also write like this xi, x minus xe divided by t minus tn. So this is only definition for e face.

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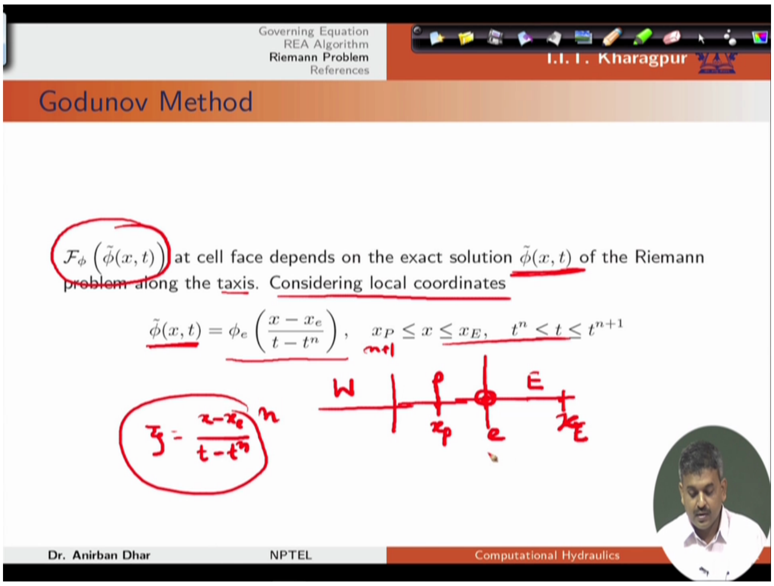

We can again define another Riemann problem for this face considering w and p.That will be bounded between xp again. We need to consider that. So in terms of local co-ordinate we can define number of Riemann problem and we can individually solve them to get the values at future time level. So tilde xt. This is w again, x minus xw divided by t minus tn.

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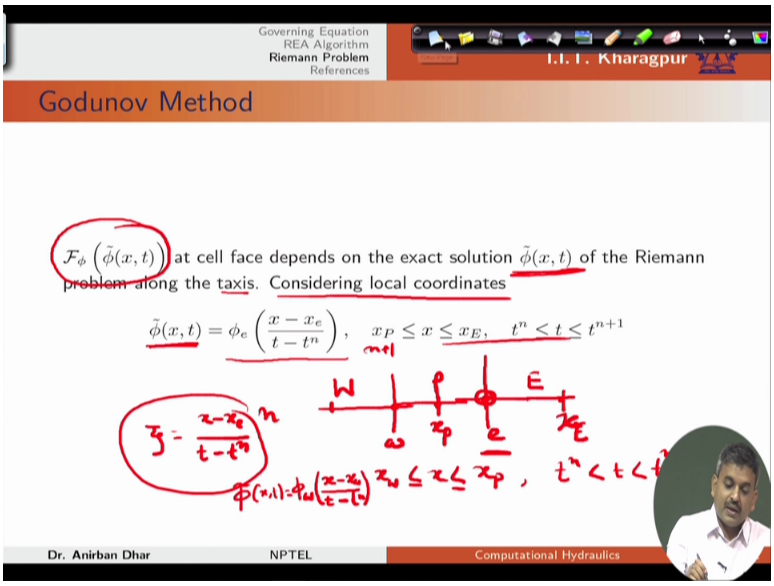

So with this basic knowledge we can start our finite volume discretization. From Riemann problems this is at w face. If we have w face obviously x will be xw, x equals to xw. So x equals to xw, this is essentially phi w0. Because we have define ournew variable like that. Xi so this is equal to zero.

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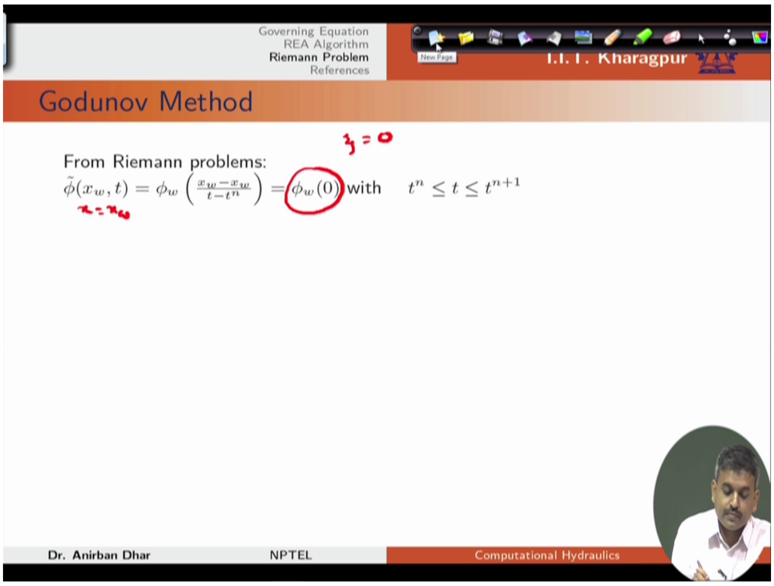

Similarly for east face again this will be 0.

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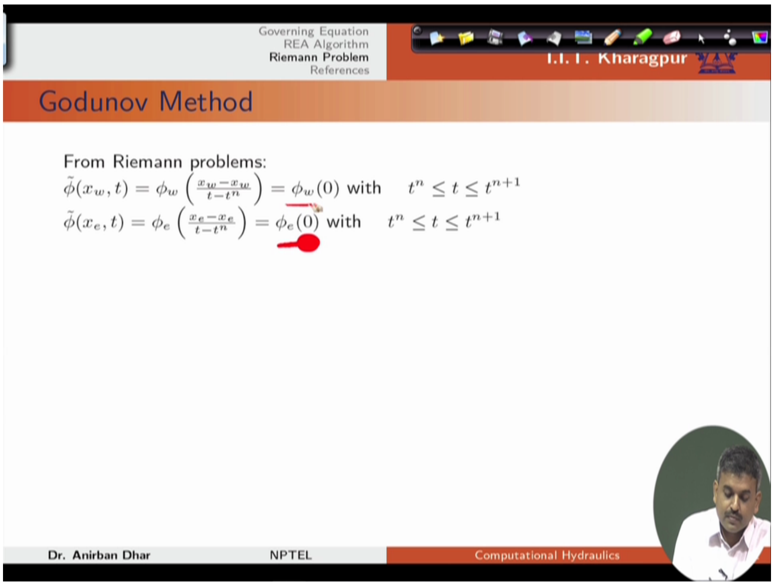

So with that information we can say that the information is travelling from w face and e face either on the right hand side or left hand side through this constant line or any chord line or characteristics line. But the main thing is that, within this time interval there should not be intersection of thesecharacteristic lines. So this should not intersect each other.

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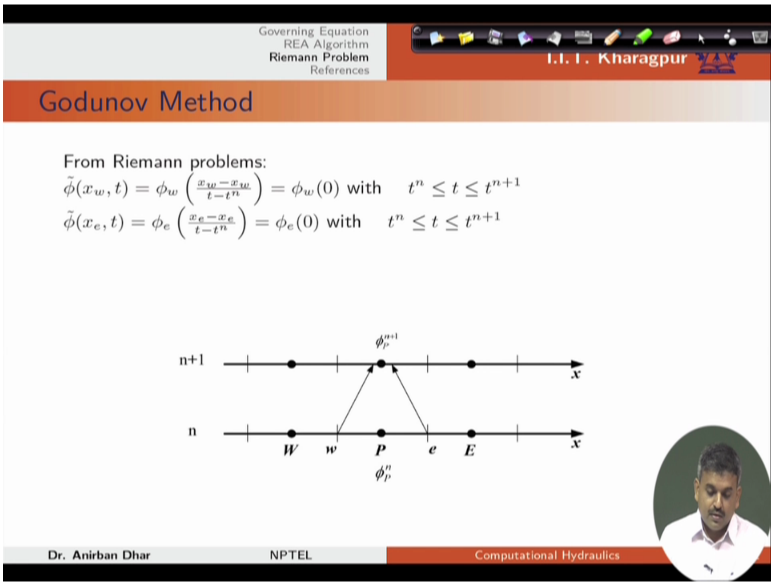

Now, in this case the basic thing for the Godunov method is that with that approximate tilde value we can approximate our numerical flux. So numerical flux in this case can be written like this, where this flux function. We know thatour original thing was phi n plus 1 equals to phi pn minus delta t by delta x. This was f phi, xe t minus f phi xw t, without considering the source sink terms.

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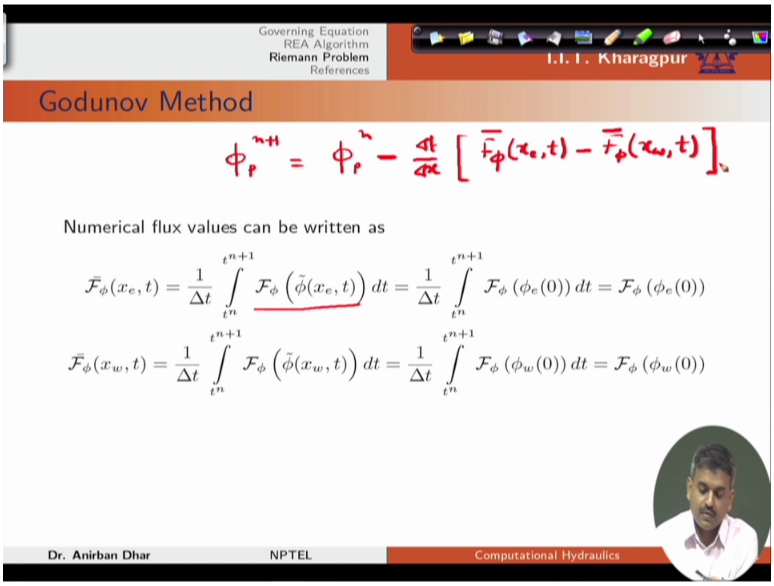

So in this case we need to approximate these two derivatives or numerical fluxes. So numerical fluxes essentially, these areaverage values within the time intervals. Now if we consider our case here, we already know that phi tilde xe t, this is phi e 0. And at west face we have w 0. We can use these two values directly within ourflux function calculation.

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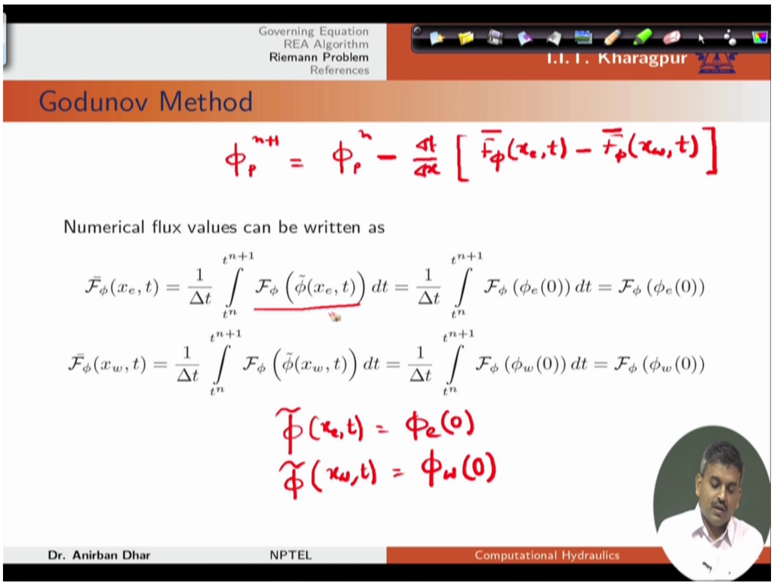

Obviously this f phi, this is again written as function of phi tilde e. And f phi, phi tilde xw t. This can be written with thisapproximation.

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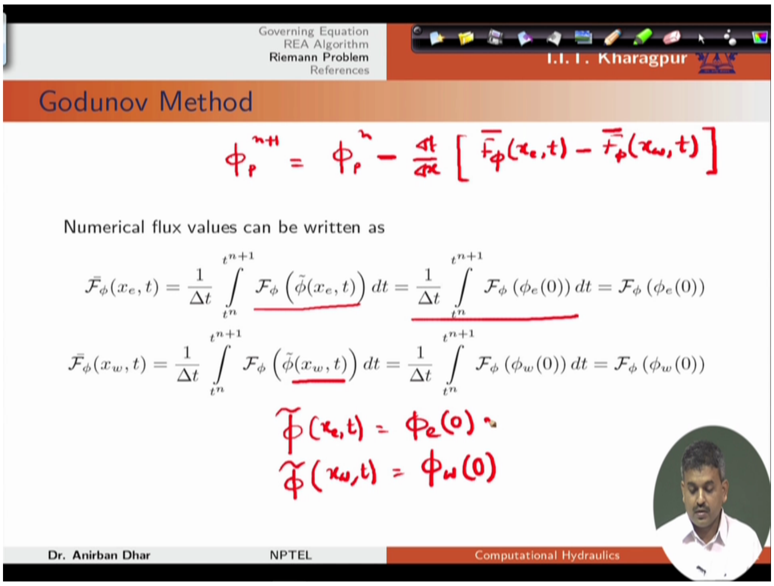

And for w again we can write it like this with this approximation that this is in terms of secondary variable, this is zero.

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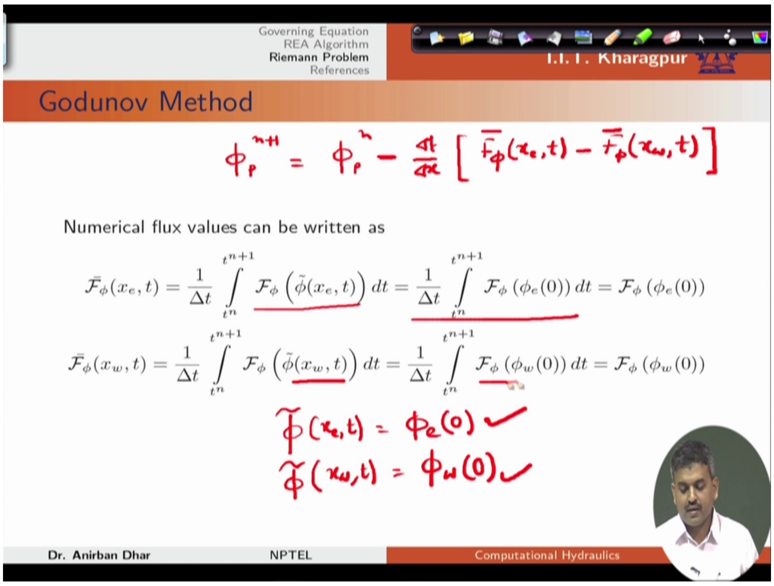

So essentially this tilde this thing is not varying within the time period or solution is not varying within that time interval. So we can directly write it as f phi, phi e 0 and f phi, phi w 0. This is from the basic definition here thatthe quantities are not changing within this time interval.

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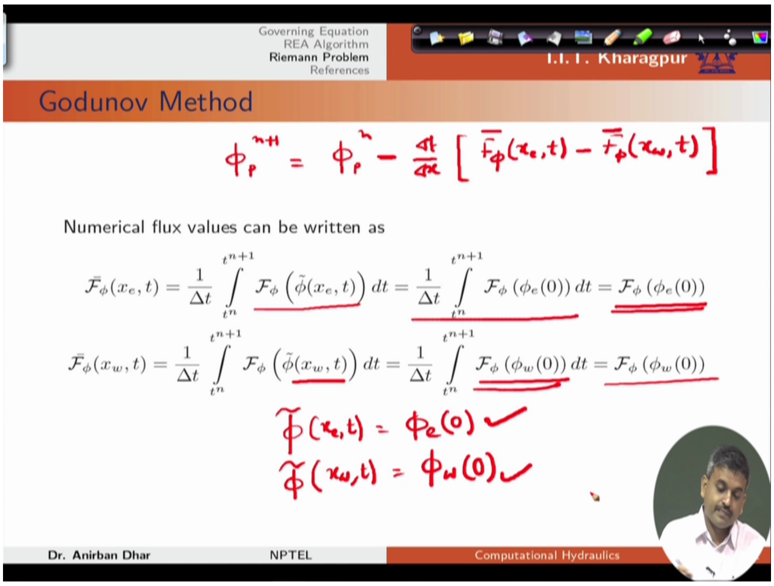

Now if we see our Godunov method that meanswe are solving this problem at this interface and westface where left hand side we have w, right hand side we have p cell. For east face we have east cell and p cell.

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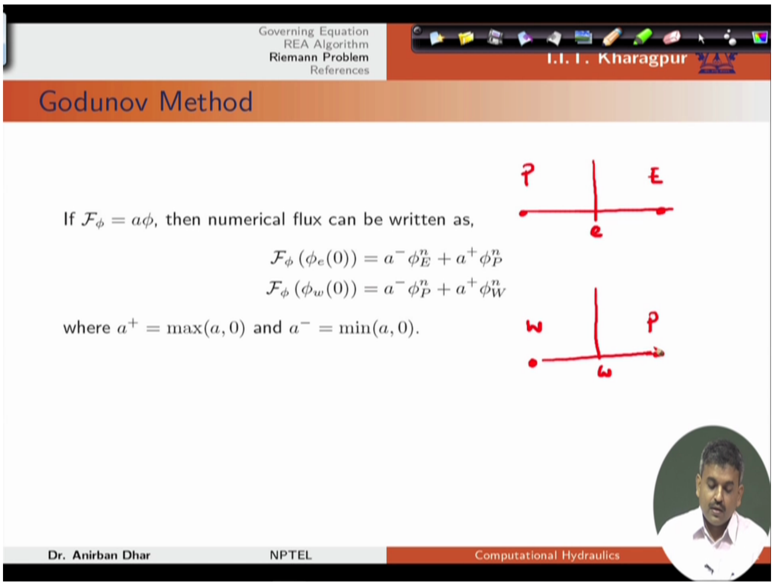

Again in this case depending on the nature of the characteristic line which is coming from this point.

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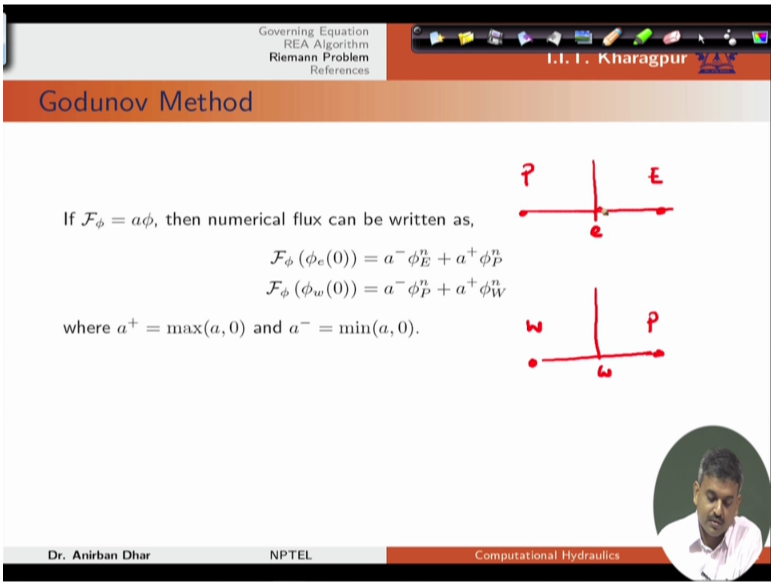

We need to consider the situation if it is in on the positive side or it is in the negative side. In this case also this is on the positive side or in the negative side. Essentially we have converted into the zero point and we are calculating the future time level values. Like our previous definition we have expressed it in terms of A minus, A plus. This is essentially our A plus line. This is A plus line. This is A minus line, this is A minus line.

That means information is travelling fromthis face to the positive side or the negative side. Depending on thatwe can write it like this. So this is again solution of the Riemannproblem that the solution at this interface e depends on the cell values on both sides, e and p. And if it is A plus, so this will be phi p. If it is negative then phi e. Again at w interface if we have A positive, then this is w. And if it is negative again this phi pn, this is coming because it is a negative one.

Andthe definition like our upwind approach this is similar. That means for A greater than zero only A plus will be A. A minus will be zero and A (great) less than zero will have A positive equals to zero and A negative equals to A.

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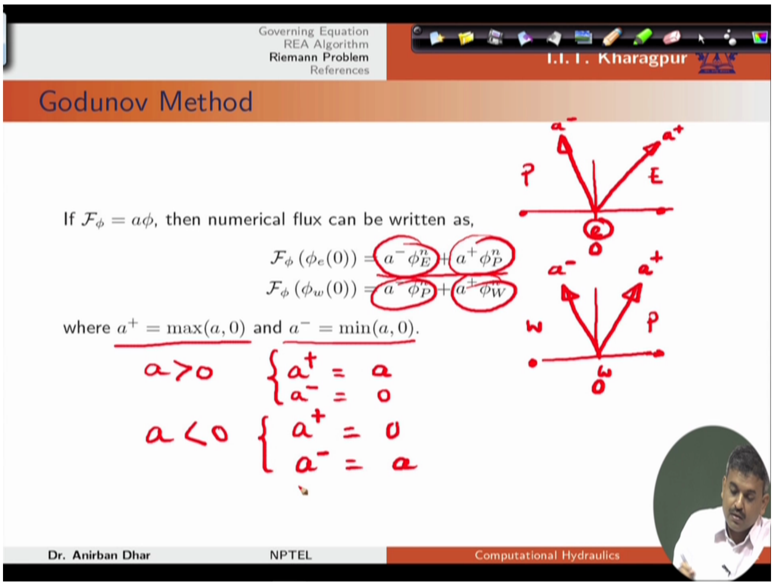

Now you can see thatthis issame as upwind approach. So we have used over approximations but stillwe are getting similar results like our upwind method. So Godunov method in basic form this is essentially first order upwind approach. Thank you.