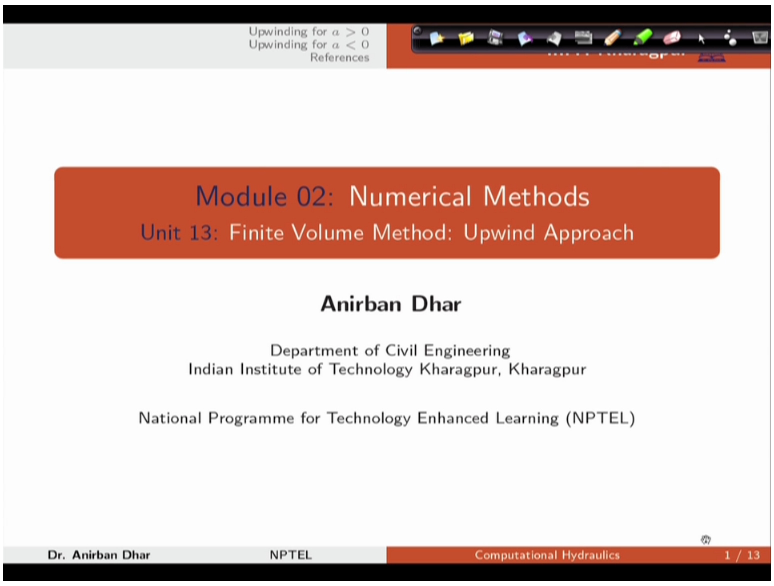
Computational Hydraulics
ProfessorAnirban Dhar
Department of Civil Engineering
Indian Institute of Technology Kharagpur
Lecture 17
FVM - Upwind Approach

Welcome to lecture number 17 of the course computational hydraulics. We are in module 2, numerical methods. And this is unit 13, finite volume method, upwind approach.

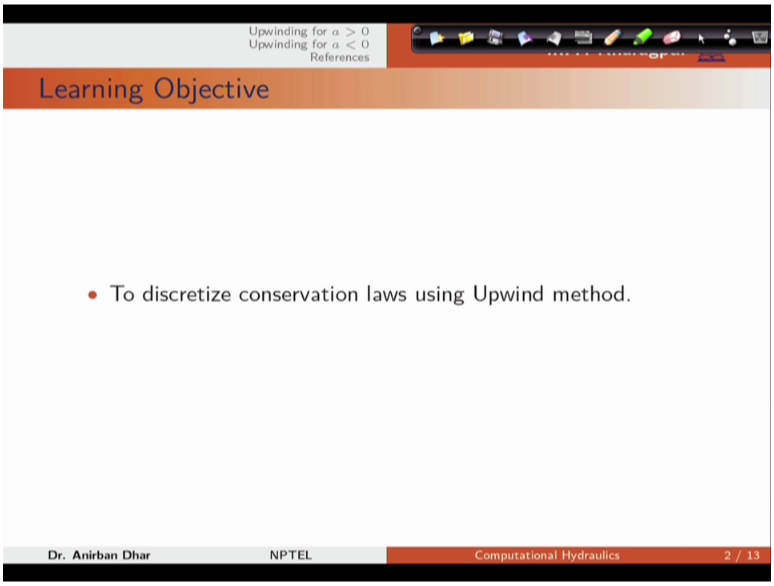
(Refer Slide Time 00:25)



In our previous lecturewe have seen that numerical flux function approximation is important. We cannot use arbitrary approximation of the numerical flux function. If we use averaged value of the flux up to adjacent cellsfor the interface, we are getting unstable schemes. At the same time adding some extra term or virtual terms in the equation givesstability to the numerical discretization. So in this particular lecture we will be concentrating on upwind approach.

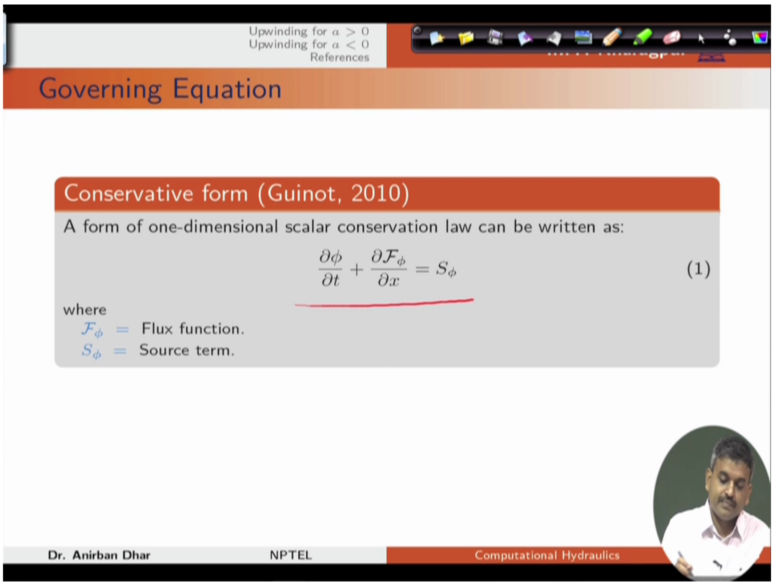
Learning objective.At the end of this unit students will be able to discretize the conversation law using upwind method.

(Refer Slide Time 01:41)



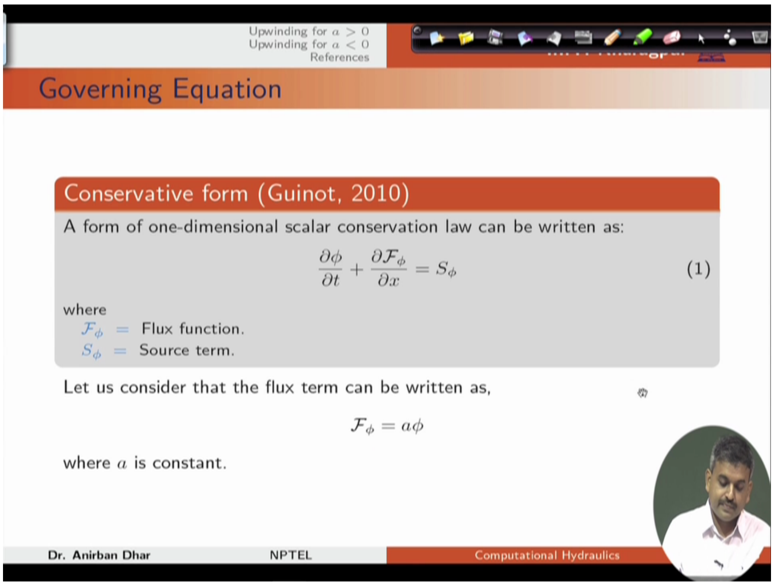
We already know this is our conservative form uh in terms of phi,F phi and S phi.

(Refer Slide Time 01:54)



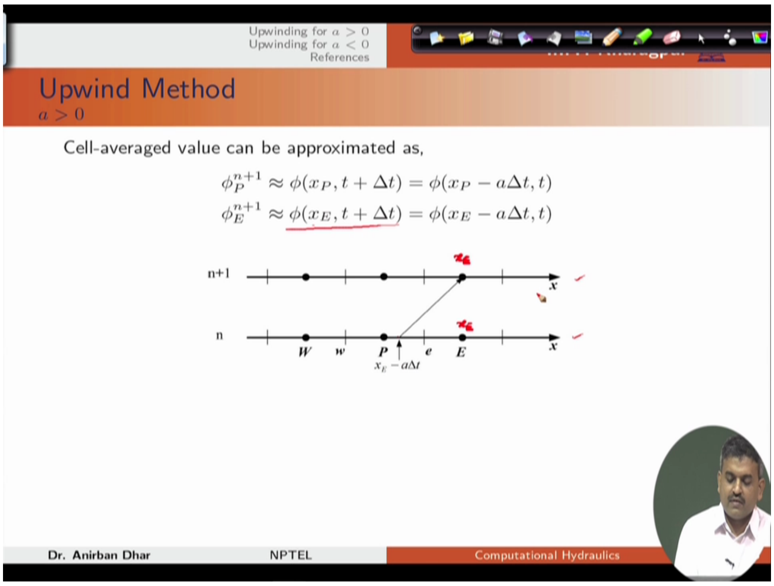
Let us consider the simpleapproximation of the flux for linear case where A phi is a linear approximation.

(Refer Slide Time 02:09)



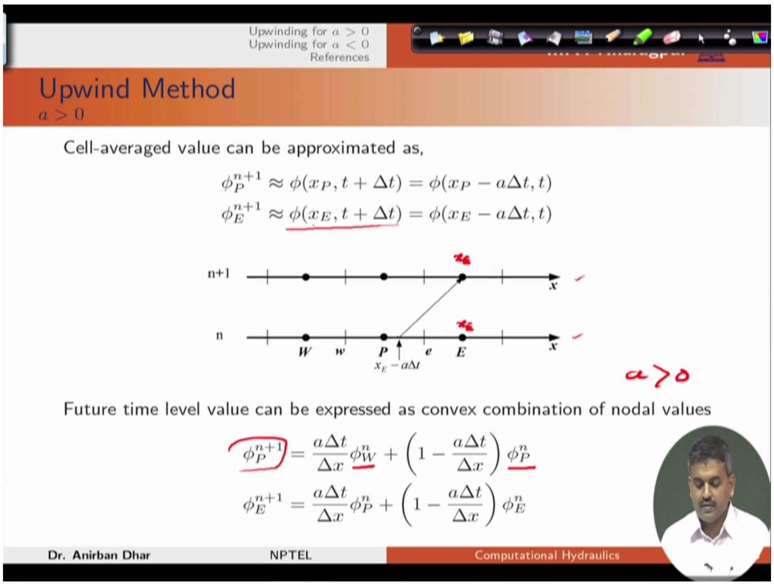
Now cell average valuecan be approximated for this case, where the information istravelling. Let us say this is X E point, this is x axis, this is point XE. This is X E. So information is travelling at certain speed. So in this casethis line is linear or this is characteristic line. So information which is travelling from level n to n plus 1, in this case phi e t plus delta t, that is from phiE minus A delta t. A delta t is the distance travel in this case.

(Refer Slide Time 03:22)



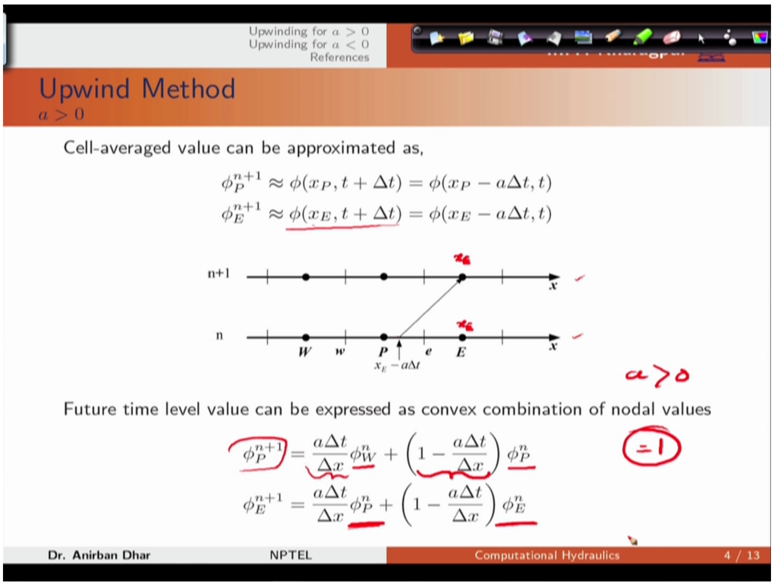
Soessentially we are using the previous time level value for calculation of future time level values directly, in this case. future time level values can be directly expressed in this form, where this phi por phi p m plus 1can be writtenin terms of convex combination of phi w and phi p. Convex combination means,in this case our consideration is A is greater than zero. That is why it is travelling on the right side.

(Refer Slide Time 04:23)



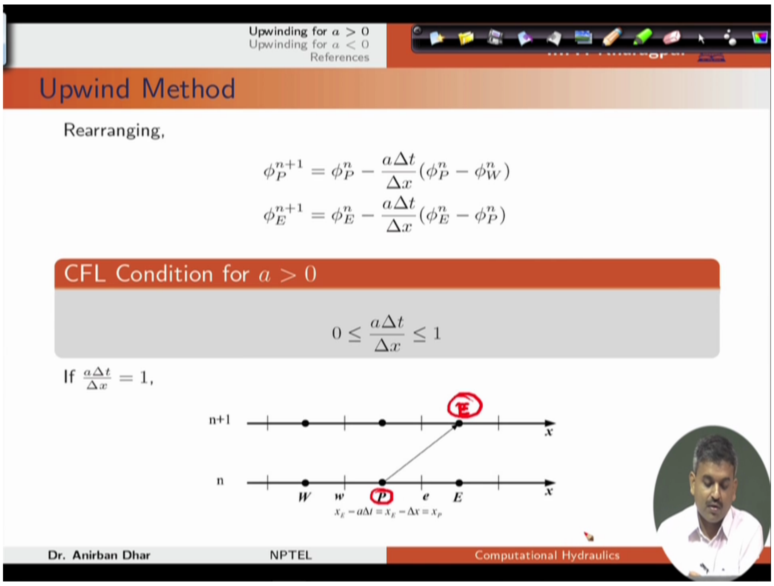
So this weight functionsif we add this to weight functions, the submission will be equal to 1. So phi p, we have written in terms of phi w and phi p. And phi e n plus 1, we can write it in terms of phi e. And phi p, phi e and phi p again convex combination for this one.

(Refer Slide Time 04:59)



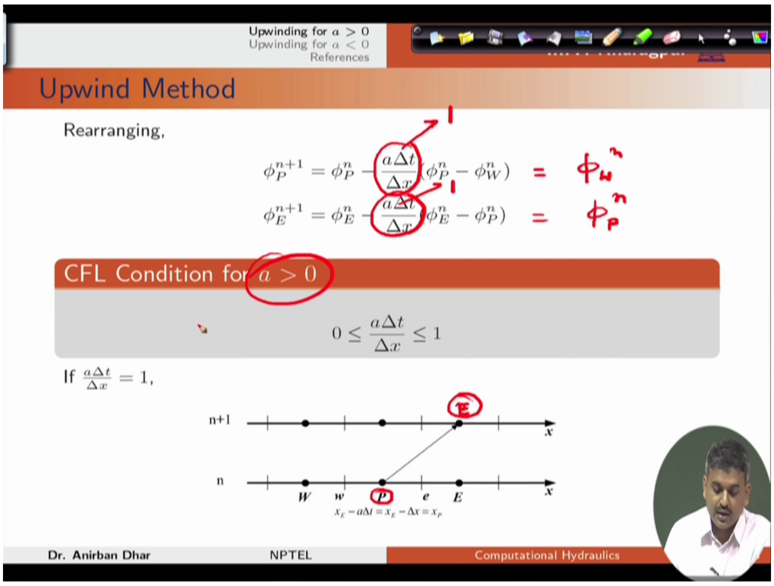
If we simplify this with our approximation thing, we can get these forms. In this case CFL condition for A greater than zero is, A delta t delta x. We have seenthis derivation of the CFL condition from stability criteria for our finite differenceapproximations. A delta t equals to 1, A delta t deltax equals to 1 or Courant number equals to 1. That means the information which is travelling from the nth level and reaching to the n plus 1 level, this is exactly for point e. Whatever is there at the point p, that is directly getting transferred to point e.

(Refer Slide Time 06:22)



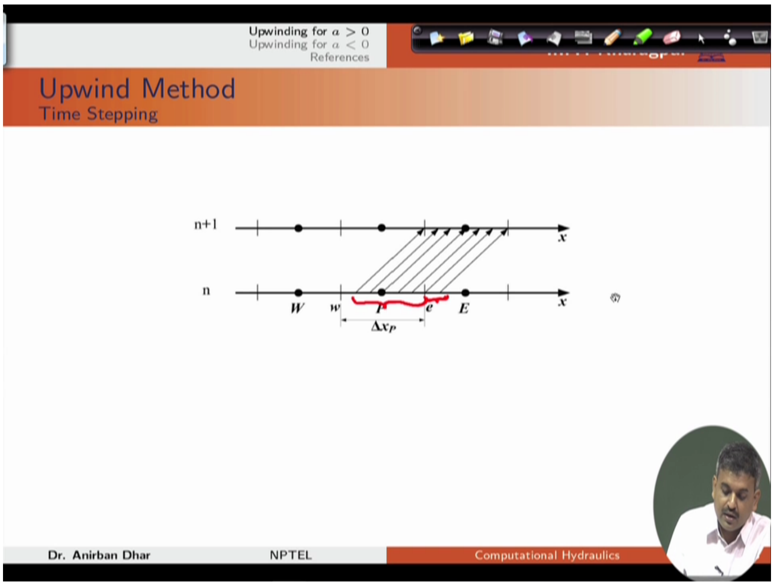
Or in this case if you have e equals to 1, so obviously this is phi w n. Or in this case if this is equals to 1, again this is phi p n. That means if A greater than zero. The information of the previous cell is directly getting transferred to the right ward cell, in the future time level.

(Refer Slide Time 06:59)



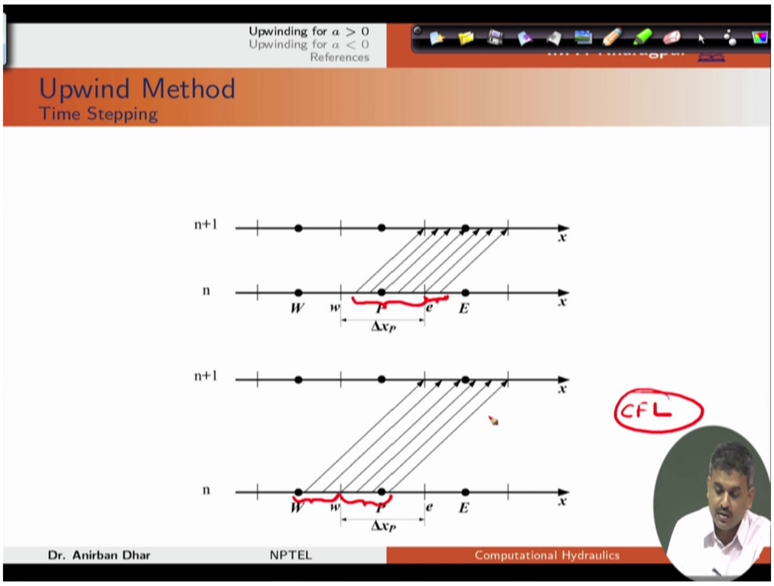
In upwind, time stepping is important. So (fo) in this case information is travelling from p cell to e cell or e cell to. This part is information is travelling from p cell to e cell. In this case information is travelling from e cell to E cell.

(Refer Slide Time 07:34)



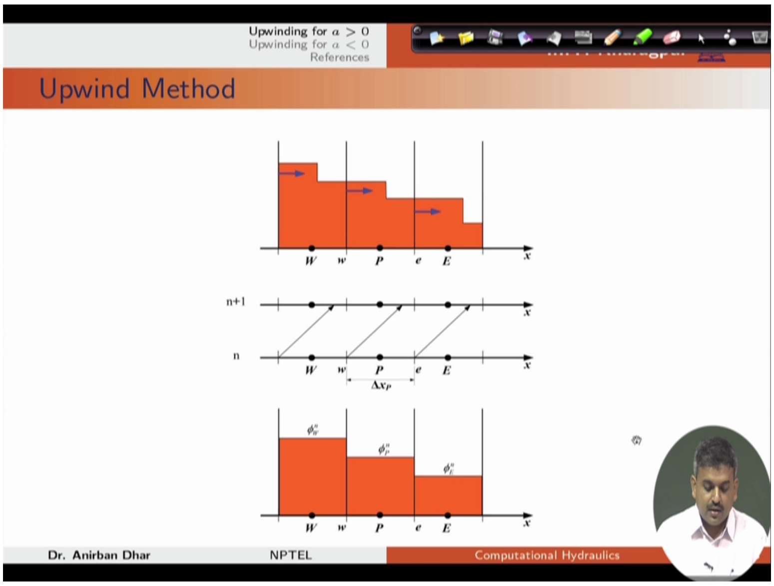
But if we increase this time step, then we can see that information transfer will be there from w cell and p cell. So time stepping is important. This iscontrolled by CFL condition or Courant Friedrich Lewy condition.

(Refer Slide Time 08:00)



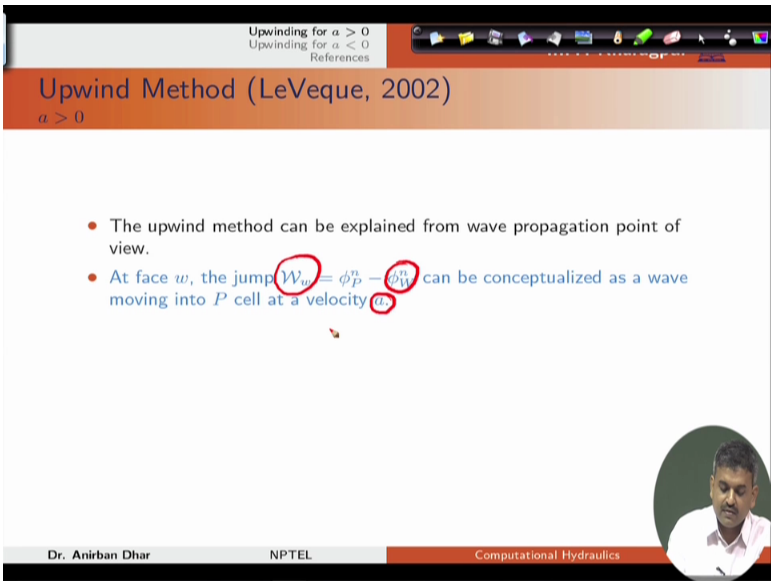
Upwind method, let us say that we are considering A greater than zero condition. Here P cell, W cell, E cell, we have different values. Now informationat the cell interfacethat is getting transferred to the next cell. And we are getting the average values.

(Refer Slide Time 08:37)



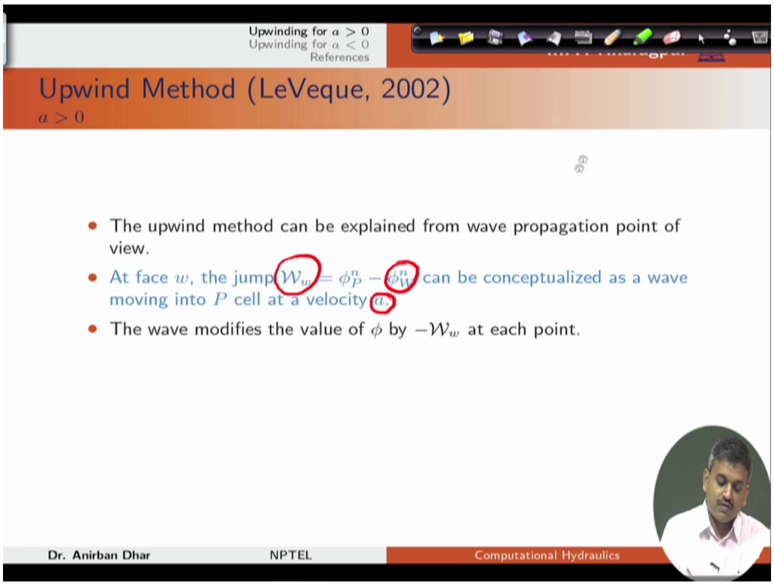
So upwind method can be explained from the wave propagation point of view. At w face , we can see that Ww is the jump from this phi w to phi pcan be conceptualized as wave moving into P cell at velocity a, in this case.

(Refer Slide Time 09:18)



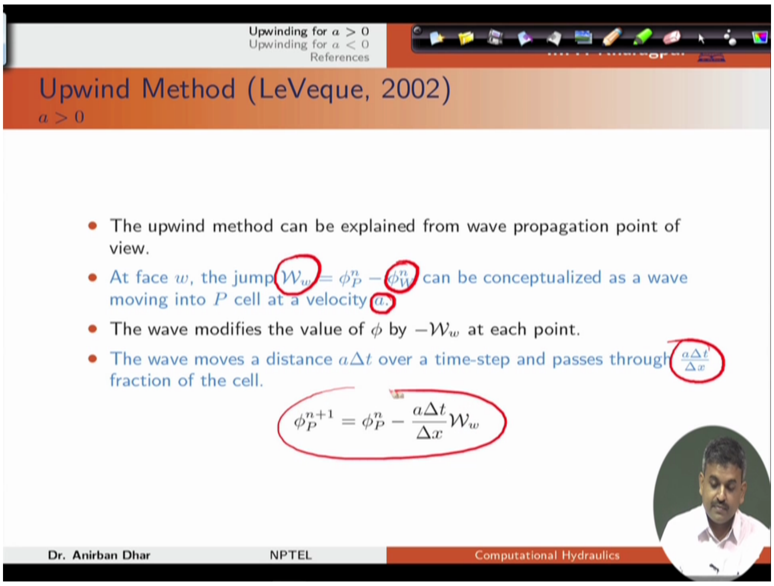
And this wave modifies the value of phi at each point by minusWw, this value.

(Refer Slide Time 09:36)



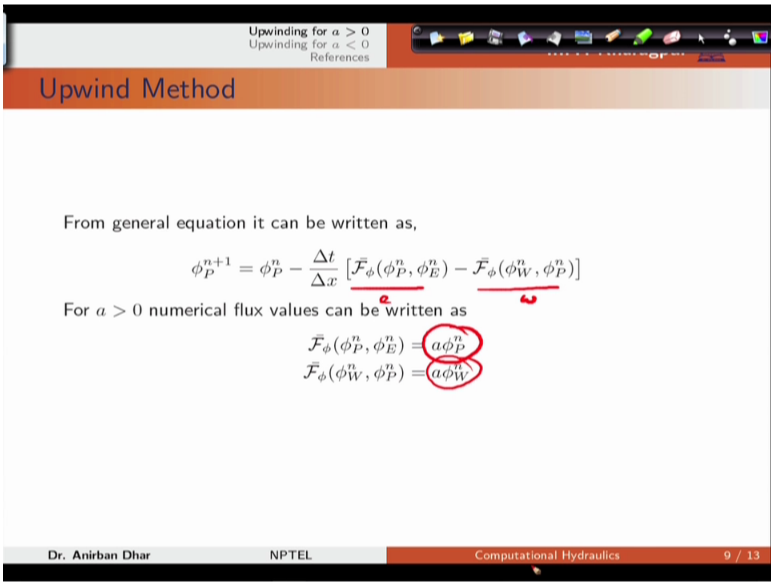
And wave moves at distance A delta t over time step and passes through A delta t by delta x fraction of the cell. So we can write it like this.

(Refer Slide Time 09:59)



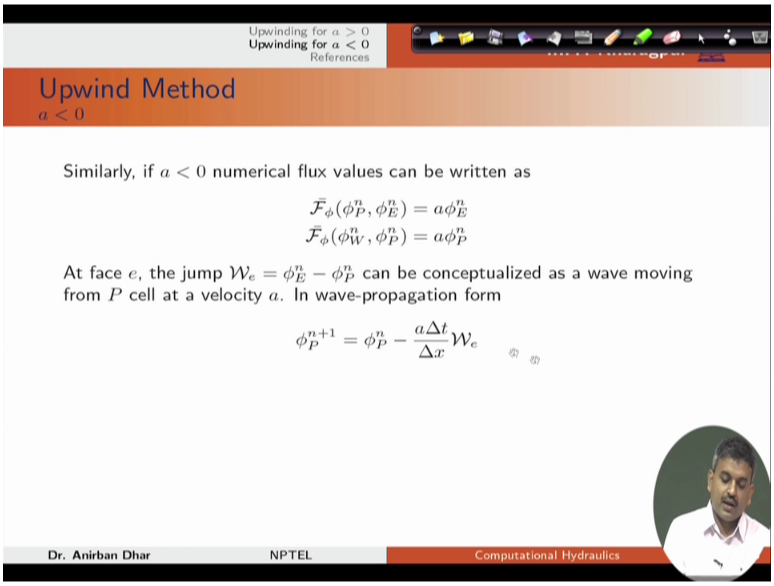
In this case we have general equation wherewe have general flux function for e cell and w cell. A greater than zero, we can approximate this numerical flux as a phi p. That means at the interface of the cell p and e information is travelling from p cell to ecell. Sowe can say thatthe approximate value of numerical flux will depend on this cell p which is up gradient cell. And for w facethis is w because information is travelling from w to p, for w face.

(Refer Slide Time 11:31)



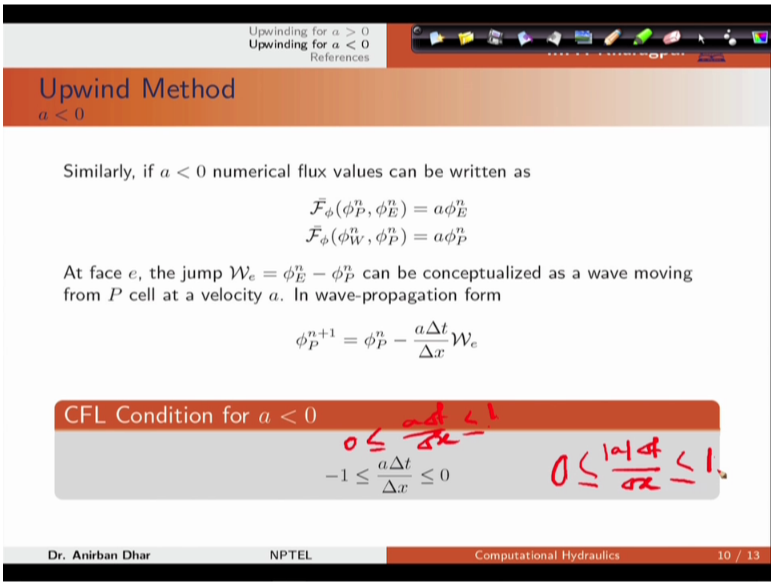
So if we consider A less than zero, this will be modified because at cell interface e, information will be travelling from e to p side, and at w face, from p to w side. So the corresponding values are utilized here. At e face the wave propagation again we can write it like this.

(Refer Slide Time 12:10)



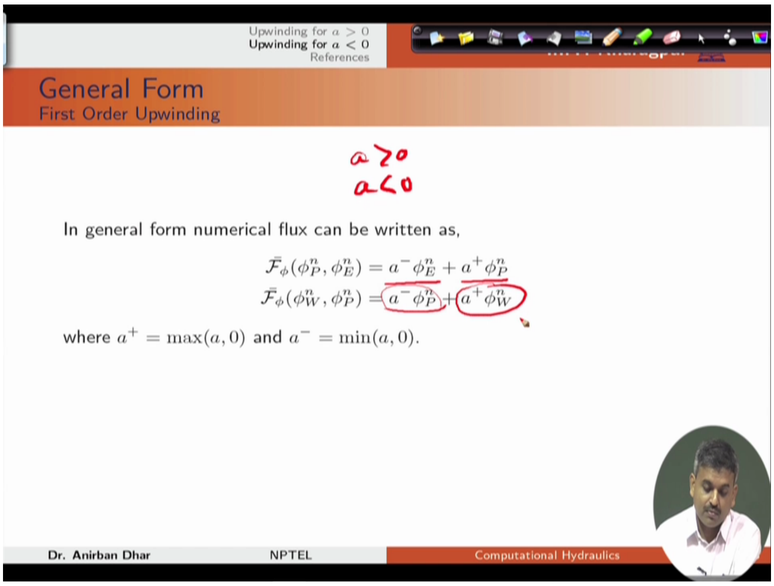
And CFL condition, if you do not consider absolute value of A, then this will be just opposite to our original condition. Original condition was A delta t bydelta x equals to 0. Sometimes the use combine at criteria where A delta t delta x is utilized. This will be always 1 in this case.

(Refer Slide Time 13:01)



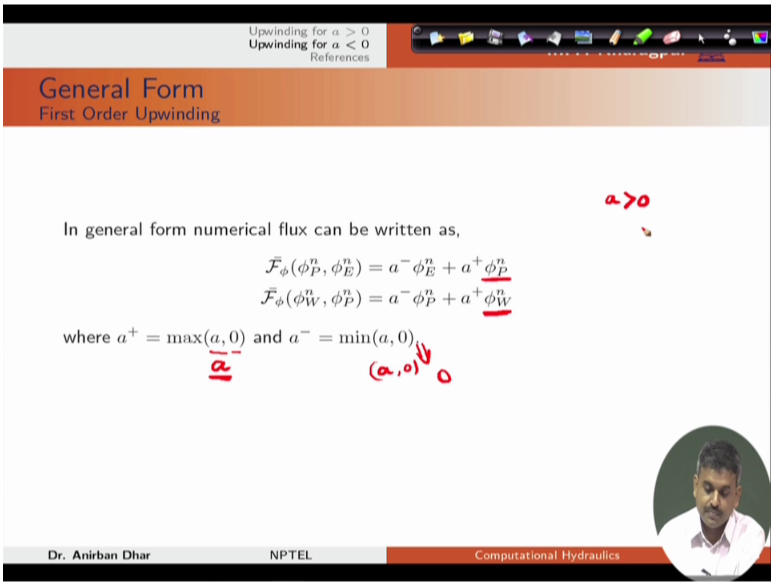
In general form, if we write, then this is written as considering both A greater than zero and A less than zero. A minus e and A plus p. At e cell if A or information is moving in thepositive direction or positive X direction, then we should consider p cell value or in negative direction e cell value. In this case if it is forw face. If it is ournegative direction, we should consider e cell value. At w face if we have positive direction thing then we should consider w cell value.

(Refer Slide Time 14:16)



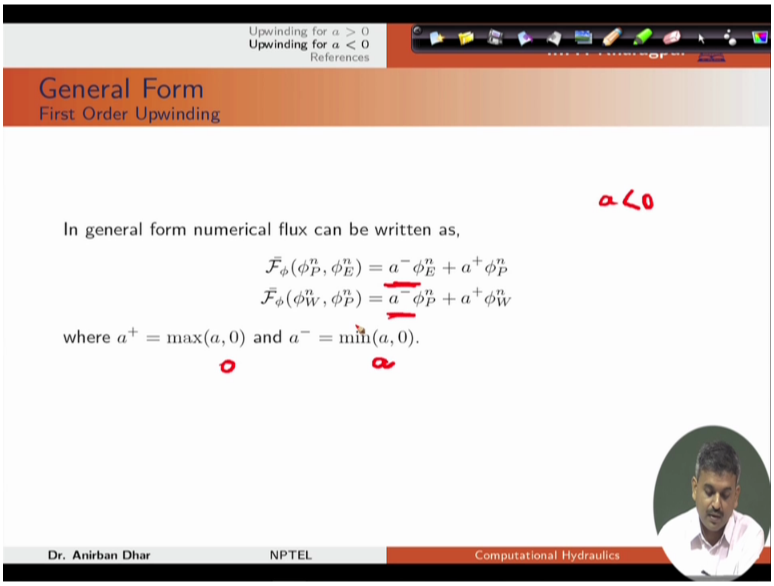
So this is general form. But what is the interpretation of this? If A greater than zero, some number. So obviously A plus will be maximum of A or 0. That means this will be A. And minimum of A 0. That means if A is positive here, this quantity will be 0, but this will be A.That means only p will be considered and w will be considered, if A greater than zero.

(Refer Slide Time 15:03)



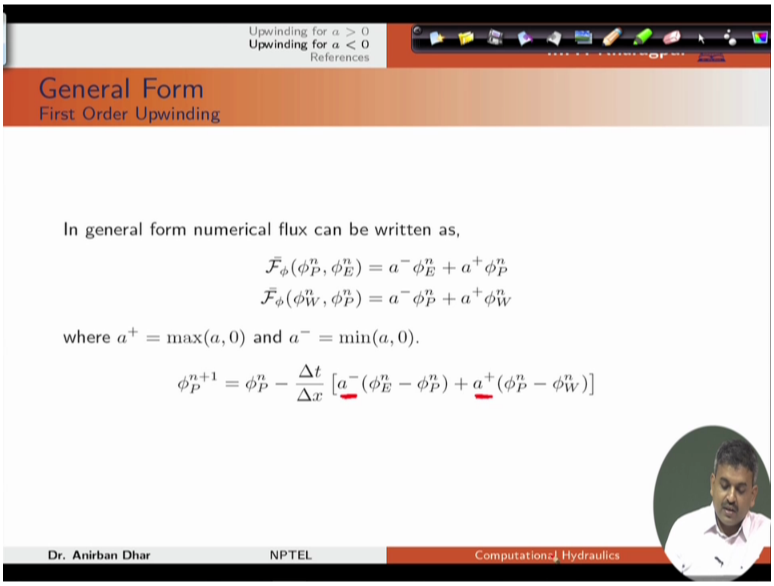
But if we have A less than zero, less than zero means in this case uh minimum will be considered. So this will be A and maximum of negative quantity and zero. So this will give zero value. So in this case we will have e and p terms or first terms.

(Refer Slide Time 15:36)



Now we can write our final form like this. We have just regrouped it with the coefficients A minus and A plus. Now you can see thatfrom our previous case we have unstable schemes and stability with some modification in the equation itself. But in this case we do not require any modification as such. But depending on the nature of the problem or characteristics of the problem, we are defining the interface values and at the same time we are defining the numerical flux.

(Refer Slide Time 16:23)



So this approach is morephysically consistent. And this is the basic thing for our upwind approach. This approach is called as first order upwind method. Thank you.