

**Computational Hydraulics**  
**Professor Anirban Dhar**  
**Department of Civil Engineering**  
**Indian Institute of Technology Kharagpur**  
**Lecture 15**  
**Finite Volume Method - IBVP**

Welcome to this lecture number 15 of the course computational hydraulics. We are in module 2 numerical methods. In this particular lecture I will be covering unit 11, finite volume method, initial boundary value problem.

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The slide features a navigation menu at the top left with the following items: Finite Volume Method, Discretization: Interior Points, Discretization: Boundary Points, and References. The top right corner displays the I.I.T. Kharagpur logo. The main content area includes a large orange box with the text "Module 02: Numerical Methods" and "Unit 11: Finite Volume Method: IBVP". Below this, the presenter's name "Anirban Dhar" is listed, followed by his affiliation: "Department of Civil Engineering, Indian Institute of Technology Kharagpur, Kharagpur". The NPTEL logo is also present. The footer contains the text "Dr. Anirban Dhar", "NPTEL", "Computational Hydraulics", and "1 / 21".

Learning objectives, our objective for this particular unit. At the end of this particular unit students will be able to derive the algebraic form for initial boundary value problem using finite volume method.

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Finite Volume Method  
Discretization: Interior Points  
Discretization: Boundary Points  
References

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## Learning Objectives

- To derive the algebraic form for a **IBVP** using Finite Volume Method.

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Let us consider our general equation in terms of general variable phi. we have solved the boundary value problem or BVP using this diffusing term and source sink term. Now in this particular lecture we will be introducing this temporal term for IVBP problem.

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## General Equation

A form of differential equation with a general variable  $\phi$ :

$$\frac{\partial(\Lambda_{\phi}\phi)}{\partial t} + \nabla \cdot (\Upsilon_{\phi}\phi\mathbf{u}) = \nabla \cdot (\Gamma_{\phi}\nabla\phi) + F_{\phi\phi} + S_{\phi} \quad (1)$$

where

- $\phi$  = general variable
- $\Lambda_{\phi}, \Upsilon_{\phi}$  = problem dependent parameters
- $\Gamma_{\phi}$  = tensor
- $F_{\phi\phi}$  = other forces
- $S_{\phi}$  = source/sink term



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Now what is this problem definition? Like our finite difference case let us consider this lambda phi as coefficient for this temporal term. And gamma x, gamma y these are diffusion coefficients. So this tensor, this is specifically two dimensional 0 0, gamma y. So we have only two terms.

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### Problem Definition

$$\Gamma = \begin{bmatrix} \Gamma_x & 0 \\ 0 & \Gamma_y \end{bmatrix}$$

#### Governing equation

A two-dimensional (in space) IBVP can be written as,

$$\Omega : \Lambda_\phi \frac{\partial \phi}{\partial t} = \Gamma_x \frac{\partial^2 \phi}{\partial x^2} + \Gamma_y \frac{\partial^2 \phi}{\partial y^2} + S_\phi(x, y)$$

$$\Omega : \Lambda_\phi \frac{\partial \phi}{\partial t} = \nabla \cdot (\Gamma \cdot \nabla \phi) + S_\phi(x, y)$$


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If we write this in terms of this del or divergence operator, we can write this one in terms of this. This is basically delphi or Dirac phi. This is equals to phi I, where I is the unit vector and direction xj is the unit vector in the direction y. And divergence of this will give you the other terms.

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### Problem Definition


$$\nabla \phi = \frac{\partial \phi}{\partial x} \hat{i} + \frac{\partial \phi}{\partial y} \hat{j}$$

$$\Gamma = \begin{bmatrix} \Gamma_x & 0 \\ 0 & \Gamma_y \end{bmatrix}$$

#### Governing equation

A two-dimensional (in space) IBVP can be written as,

$$\Omega : \Lambda_\phi \frac{\partial \phi}{\partial t} = \Gamma_x \frac{\partial^2 \phi}{\partial x^2} + \Gamma_y \frac{\partial^2 \phi}{\partial y^2} + S_\phi(x, y)$$

$$\Omega : \Lambda_\phi \frac{\partial \phi}{\partial t} = \nabla \cdot (\Gamma \cdot \nabla \phi) + S_\phi(x, y)$$


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Now what are the boundary conditions or initial conditions? Initial conditions, let us say that initial value is specified for xy domain or omega domain or this one for all values of x and y, this phi 0 is given. Now left hand side like our previous problem it is a specified boundary or Dirichlet kind of boundary condition. Right hand side also Dirichlet kind of boundary condition. Bottom and top boundary, these are Neumann boundary conditions.

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### Problem Definition

subject to

**Initial Condition**

$$\phi(x, y, 0) = \phi_0(x, y)$$

and

**Boundary Condition**

$$\Gamma_D^1 : \phi(0, y, t) = \phi_1$$

$$\Gamma_D^2 : \phi(L_x, y, t) = \phi_2$$

$$\Gamma_N^3 : \frac{\partial \phi}{\partial y} \Big|_{(x, 0, t)} = 0$$

$$\Gamma_N^4 : \frac{\partial \phi}{\partial y} \Big|_{(x, L_y, t)} = 0$$

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Now domain discretization. Like our boundary value problem we can divide this  $L_x$  in  $x$  direction,  $L_y$  in  $Y$  direction in number of cells. At the cell center we will consider the variables. And in this case new ones are interior cells, for representative one. This black ones are next to boundary. And red ones, these needs spatial treatment because one side is Dirichlet and another side is Neumann.

In physical system there can be situations where both the boundaries can be Neumann or Dirichlet. But in this case we have combination of boundary conditions. One side Dirichlet another side Neumann. And  $\Delta x$  is a cell size of the  $x$  direction.  $\Delta y$  in the  $y$  direction.

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### Domain Discretization

$\Delta x$   $\Gamma_N$

$\Gamma_D$   $\Gamma_D$

$L_x$   $\Gamma_N$   $\Delta y$

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So with this we can start the discretization of the governing equation. If we consider any interior cell, then for any particular interior cell we have east, west, north, south, these four neighboring cells. For this one we can start the discretization. So in finite volume method, the governing equation is integrated over the elemental volume in space and time interval to form the discretized governing equation at node point P or cell center P.

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Finite Volume Method  
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### Discretization Governing Equation

In Finite Volume Method, the governing equation is integrated over the element volume (in space) and time interval to form the discretized equation at node Point P (Versteeg and Malalasekera, 2008).

$$\int_t^{t+\Delta t} \left[ \int_{\Omega_P} \Lambda_\phi \frac{\partial \phi}{\partial t} d\Omega \right] dt = \int_t^{t+\Delta t} \left[ \int_{\Omega_P} \nabla \cdot (\Gamma \cdot \nabla \phi) d\Omega \right] dt + \int_t^{t+\Delta t} \left[ \int_{\Omega_P} S_\phi(x, y) d\Omega \right] dt \quad (2)$$

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So in this case we are integrating from t to t plus delta t. That is time increment from present level t to future time level t plus delta t. And integration omega P, this is over elemental volume or element volume. This is actually element volume.

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### Discretization Governing Equation

In Finite Volume Method, the governing equation is integrated over the element volume (in space) and time interval to form the discretized equation at node Point P (Versteeg and Malalasekera, 2008).

$$\int_t^{t+\Delta t} \left[ \int_{\Omega_P} \Lambda_\phi \frac{\partial \phi}{\partial t} d\Omega \right] dt = \int_t^{t+\Delta t} \left[ \int_{\Omega_P} \nabla \cdot (\Gamma \cdot \nabla \phi) d\Omega \right] dt + \int_t^{t+\Delta t} \left[ \int_{\Omega_P} S_\phi(x, y) d\Omega \right] dt \quad (2)$$

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So if we integrate our original equation with respect to space and time then we can write like this. This is for temporal term, spatial diffusion term and source sink term.

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### Discretization

Governing Equation

In Finite Volume Method, the governing equation is integrated over the element volume (in space) and time interval to form the discretized equation at node Point P (Versteeg and Malalasekera, 2008).

$$\int_t^{t+\Delta t} \left[ \int_{\Omega_P} \Lambda_\phi \frac{\partial \phi}{\partial t} d\Omega \right] dt = \int_t^{t+\Delta t} \left[ \int_{\Omega_P} \nabla \cdot (\Gamma \cdot \nabla \phi) d\Omega \right] dt + \int_t^{t+\Delta t} \left[ \int_{\Omega_P} S_\phi(x, y) d\Omega \right] dt \quad (2)$$

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Now let us consider the discretization for individual terms. In the first case we have this temporal term. This quantity is not varying with time. Let us consider this is constant. So we can take out this term out of this integration. So we can write this term, this spatial derivative spatial integration with respect to temporal derivative. We can take this integration sign inside. And let us integrate this phi over this element volume.

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### Discretization

Governing Equation: Temporal Term

$$\int_t^{t+\Delta t} \left[ \int_{\Omega_P} \Lambda_\phi \frac{\partial \phi}{\partial t} d\Omega \right] dt$$

$$= \Lambda_\phi \int_t^{t+\Delta t} \frac{\partial}{\partial t} \left( \int_{\Omega_P} \phi d\Omega \right) dt$$

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With this in the next step we can approximate this. This approximation is nothing but this phi P is the approximated value for this domain if we consider this spatial averaging. So this is actually spatially average value within this P cell. Now let us integrate it from t to t plus delta t. So if we do that then we can write this one to t this phi P L plus 1 minus phi L. L represents the t time level and L plus 1 represents t plus delta t time level. And this one is nothing but the volume of the cell P.

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Discretization  
Governing Equation: Temporal Term

$$\int_t^{t+\Delta t} \left[ \int_{\Omega_P} \Lambda_\phi \frac{\partial \phi}{\partial t} d\Omega \right] dt$$

$$= \Lambda_\phi \int_t^{t+\Delta t} \frac{\partial}{\partial t} \left( \int_{\Omega_P} \phi d\Omega \right) dt$$

$$= \Lambda_\phi \int_t^{t+\Delta t} \frac{\partial}{\partial t} (\phi_P \Delta\Omega_P) dt$$

$$= \Lambda_\phi (\phi_P^{l+1} - \phi_P^l) \Delta\Omega_P$$

$l+1 \quad t+dt$   
 $l \quad t$

$\phi_P = \frac{1}{\Delta\Omega_P} \int_{\Omega} \phi d\Omega$

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Next step we can simply write, this is del x into del y and virtually on the other direction we can consider the width as 1. So this del omega P or volume, we can directly write as del x del y into 1.

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Discretization  
Governing Equation: Temporal Term

$$\int_t^{t+\Delta t} \left[ \int_{\Omega_P} \Lambda_\phi \frac{\partial \phi}{\partial t} d\Omega \right] dt$$

$$= \Lambda_\phi \int_t^{t+\Delta t} \frac{\partial}{\partial t} \left( \int_{\Omega_P} \phi d\Omega \right) dt$$

$$= \Lambda_\phi \int_t^{t+\Delta t} \frac{\partial}{\partial t} (\phi_P \Delta\Omega_P) dt$$

$$= \Lambda_\phi (\phi_P^{l+1} - \phi_P^l) \Delta\Omega_P$$

$$= \Lambda_\phi (\phi_P^{l+1} - \phi_P^l) \Delta x \Delta y$$

$l+1 \quad t+dt$   
 $l \quad t$

$\phi_P = \frac{1}{\Delta\Omega_P} \int_{\Omega} \phi d\Omega$

$\Delta\Omega_P = \Delta x \Delta y \times 1$

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Now let us see the integration of the spatial or diffusing term. Spatial term, we can write this quantity in terms of vector like this.

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Discretization  
Governing Equation: Spatial Term

$$\int_t^{t+\Delta t} \int_{\Omega^P} \nabla \cdot (\Gamma \cdot \nabla \phi) d\Omega dt = \int_t^{t+\Delta t} \int_{\Omega^P} \nabla \cdot \left( \Gamma_x \frac{\partial \phi}{\partial x} \hat{i} + \Gamma_y \frac{\partial \phi}{\partial y} \hat{j} \right) d\Omega dt$$

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Now in the next step if we see this one, this is divergence of this vector. As per Gauss divergence theorem, if you have divergence of any vector A, then we can write it in terms of surface integral. So in this case we can first divide it into two parts. One is t plus theta delta t. This is intermediate time level and from t plus theta delta t to t plus delta t. Like our theta scheme in finite difference, this is intermediate time level, this is basically zero level, this is theta and this is basically zero level. This is theta, this is 1, this is t plus del t, t plus delta theta and t only.

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Discretization  
Governing Equation: Spatial Term

$$\int_t^{t+\Delta t} \int_{\Omega^P} \nabla \cdot (\Gamma \cdot \nabla \phi) d\Omega dt = \int_t^{t+\Delta t} \int_{\Omega^P} \nabla \cdot \left( \Gamma_x \frac{\partial \phi}{\partial x} \hat{i} + \Gamma_y \frac{\partial \phi}{\partial y} \hat{j} \right) d\Omega dt$$

$$= \int_t^{t+\theta\Delta t} \int_{\Omega^P} \nabla \cdot \left( \Gamma_x \frac{\partial \phi}{\partial x} \hat{i} + \Gamma_y \frac{\partial \phi}{\partial y} \hat{j} \right) d\Omega dt + \int_{t+\theta\Delta t}^{t+\Delta t} \int_{\Omega^P} \nabla \cdot \left( \Gamma_x \frac{\partial \phi}{\partial x} \hat{i} + \Gamma_y \frac{\partial \phi}{\partial y} \hat{j} \right) d\Omega dt$$

1 - t+\theta\Delta t  
2 - t+\theta\Delta t  
3 - t

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So in this case if we consider this one we can use the Gauss divergence theorem. And from Gauss divergence theorem, we can simply write it in terms of this step. Intermediate step will be there. That intermediate step is that actually  $t + \theta \Delta t$ . And this will be over  $s_P$  or surface. And only  $\gamma_x \frac{\partial \phi}{\partial x} i + \gamma_y \frac{\partial \phi}{\partial y} j \cdot \hat{n} \cap ds$ . This is for first term.

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### Discretization

Governing Equation: Spatial Term

$$\int_t^{t+\theta\Delta t} \int_{s_P} \left( \gamma_x \frac{\partial \phi}{\partial x} i + \gamma_y \frac{\partial \phi}{\partial y} j \right) \cdot \hat{n} ds$$

$$\int_t^{t+\Delta t} \int_{\Omega^P} \nabla \cdot \left( \gamma_x \frac{\partial \phi}{\partial x} i + \gamma_y \frac{\partial \phi}{\partial y} j \right) d\Omega dt = \int_t^{t+\Delta t} \int_{\Omega^P} \nabla \cdot \left( \gamma_x \frac{\partial \phi}{\partial x} i + \gamma_y \frac{\partial \phi}{\partial y} j \right) d\Omega dt$$

$$= \int_t^{t+\theta\Delta t} \int_{\Omega^P} \nabla \cdot \left( \gamma_x \frac{\partial \phi}{\partial x} i + \gamma_y \frac{\partial \phi}{\partial y} j \right) d\Omega dt + \int_{t+\theta\Delta t}^{t+\Delta t} \int_{\Omega^P} \nabla \cdot \left( \gamma_x \frac{\partial \phi}{\partial x} i + \gamma_y \frac{\partial \phi}{\partial y} j \right) d\Omega dt$$

$$= \left[ \left( \gamma_x \frac{\partial \phi}{\partial x} \right)_e^t A_{xe} - \left( \gamma_x \frac{\partial \phi}{\partial x} \right)_w^t A_{xw} + \left( \gamma_y \frac{\partial \phi}{\partial y} \right)_n^t A_{yn} - \left( \gamma_y \frac{\partial \phi}{\partial y} \right)_s^t A_{ys} \right] \theta \Delta t +$$

$$\left[ \left( \gamma_x \frac{\partial \phi}{\partial x} \right)_e^{t+1} A_{xe} - \left( \gamma_x \frac{\partial \phi}{\partial x} \right)_w^{t+1} A_{xw} + \left( \gamma_y \frac{\partial \phi}{\partial y} \right)_n^{t+1} A_{yn} - \left( \gamma_y \frac{\partial \phi}{\partial y} \right)_s^{t+1} A_{ys} \right] (1-\theta) \Delta t$$

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Similarly we can write the other term  $t + \theta \Delta t$ . This is  $t + \theta \Delta t$  and again spatial derivative for this one, this is for  $\Delta t$  and  $s_P$ , that is for surface  $P$ . Now in this case we can see that  $\hat{n}$ , this unit vector is always outward positive. Outward positive means for any case if we consider for our cell  $P$  in east face outward positive normal is in the direction positive  $i$  and in our west face we have this negative  $i$  direction. North face I have positive  $j$  and south face I have negative  $j$ .

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Finite Volume Method  
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### Discretization

Governing Equation: Spatial Term

$$\int_t^{t+\Delta t} \int_{\Omega^P} (\Gamma_x \frac{\partial \phi}{\partial x} + \Gamma_y \frac{\partial \phi}{\partial y}) \cdot \hat{n} ds + \int_t^{t+\Delta t} \int_{\Omega^P} \nabla \cdot (\Gamma \cdot \nabla \phi) d\Omega dt = \int_t^{t+\Delta t} \int_{\Omega^P} \nabla \cdot (\Gamma_x \frac{\partial \phi}{\partial x} + \Gamma_y \frac{\partial \phi}{\partial y}) d\Omega dt$$

$$= \int_t^{t+\Delta t} \int_{\Omega^P} \nabla \cdot (\Gamma_x \frac{\partial \phi}{\partial x} + \Gamma_y \frac{\partial \phi}{\partial y}) d\Omega dt + \int_{t+\theta\Delta t}^{t+\Delta t} \int_{\Omega^P} \nabla \cdot (\Gamma_x \frac{\partial \phi}{\partial x} + \Gamma_y \frac{\partial \phi}{\partial y}) d\Omega dt$$

$$= \left[ \left( \Gamma_x \frac{\partial \phi}{\partial x} \right)_e^l A_{xe} - \left( \Gamma_x \frac{\partial \phi}{\partial x} \right)_w^l A_{xw} + \left( \Gamma_y \frac{\partial \phi}{\partial y} \right)_n^l A_{yn} - \left( \Gamma_y \frac{\partial \phi}{\partial y} \right)_s^l A_{ys} \right] \theta \Delta t +$$

$$\left[ \left( \Gamma_x \frac{\partial \phi}{\partial x} \right)_e^{l+1} A_{xe} - \left( \Gamma_x \frac{\partial \phi}{\partial x} \right)_w^{l+1} A_{xw} + \left( \Gamma_y \frac{\partial \phi}{\partial y} \right)_n^{l+1} A_{yn} - \left( \Gamma_y \frac{\partial \phi}{\partial y} \right)_s^{l+1} A_{ys} \right] (1-\theta) \Delta t$$

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So these are the directions for outward normal. Individually we can consider this effect. So projected area in that direction is actually  $A_{xe}$ . So for east face  $\hat{i} \cdot \hat{i}$  this  $A_{xe}$ , this is positive is considered. So if you multiply this only  $\hat{i} \cdot \hat{i}$ , that term will be 1. there will be y related term.

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### Discretization

Governing Equation: Spatial Term

$$\int_t^{t+\Delta t} \int_{\Omega^P} (\Gamma_x \frac{\partial \phi}{\partial x} + \Gamma_y \frac{\partial \phi}{\partial y}) \cdot \hat{n} ds + \int_t^{t+\Delta t} \int_{\Omega^P} \nabla \cdot (\Gamma \cdot \nabla \phi) d\Omega dt = \int_t^{t+\Delta t} \int_{\Omega^P} \nabla \cdot (\Gamma_x \frac{\partial \phi}{\partial x} + \Gamma_y \frac{\partial \phi}{\partial y}) d\Omega dt$$

$$= \int_t^{t+\Delta t} \int_{\Omega^P} \nabla \cdot (\Gamma_x \frac{\partial \phi}{\partial x} + \Gamma_y \frac{\partial \phi}{\partial y}) d\Omega dt + \int_{t+\theta\Delta t}^{t+\Delta t} \int_{\Omega^P} \nabla \cdot (\Gamma_x \frac{\partial \phi}{\partial x} + \Gamma_y \frac{\partial \phi}{\partial y}) d\Omega dt$$

$$= \left[ \left( \Gamma_x \frac{\partial \phi}{\partial x} \right)_e^l A_{xe} - \left( \Gamma_x \frac{\partial \phi}{\partial x} \right)_w^l A_{xw} + \left( \Gamma_y \frac{\partial \phi}{\partial y} \right)_n^l A_{yn} - \left( \Gamma_y \frac{\partial \phi}{\partial y} \right)_s^l A_{ys} \right] \theta \Delta t +$$

$$\left[ \left( \Gamma_x \frac{\partial \phi}{\partial x} \right)_e^{l+1} A_{xe} - \left( \Gamma_x \frac{\partial \phi}{\partial x} \right)_w^{l+1} A_{xw} + \left( \Gamma_y \frac{\partial \phi}{\partial y} \right)_n^{l+1} A_{yn} - \left( \Gamma_y \frac{\partial \phi}{\partial y} \right)_s^{l+1} A_{ys} \right] (1-\theta) \Delta t$$

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Similarly if we consider west face, this negative sign will be there. North face positive sign and south face there will be again negative sign and this  $\theta \Delta t$ , this is basically value for this first integral.

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### Discretization

Governing Equation: Spatial Term

$$\int_t^{t+\theta\Delta t} \int_{\Omega^P} \nabla \cdot (\Gamma \cdot \nabla \phi) d\Omega dt + \int_{t+\theta\Delta t}^{t+\Delta t} \int_{\Omega^P} \nabla \cdot (\Gamma \cdot \nabla \phi) d\Omega dt = \int_t^{t+\Delta t} \int_{\Omega^P} \nabla \cdot \left( \Gamma_x \frac{\partial \phi}{\partial x} i + \Gamma_y \frac{\partial \phi}{\partial y} j \right) d\Omega dt$$

$$= \int_t^{t+\Delta t} \int_{\Omega^P} \nabla \cdot \left( \Gamma_x \frac{\partial \phi}{\partial x} i + \Gamma_y \frac{\partial \phi}{\partial y} j \right) d\Omega dt + \int_{t+\theta\Delta t}^{t+\Delta t} \int_{\Omega^P} \nabla \cdot \left( \Gamma_x \frac{\partial \phi}{\partial x} i + \Gamma_y \frac{\partial \phi}{\partial y} j \right) d\Omega dt$$

$$= \left[ \left( \Gamma_x \frac{\partial \phi}{\partial x} \right)_e^l A_{xe} - \left( \Gamma_x \frac{\partial \phi}{\partial x} \right)_w^l A_{xw} + \left( \Gamma_y \frac{\partial \phi}{\partial y} \right)_n^l A_{yn} - \left( \Gamma_y \frac{\partial \phi}{\partial y} \right)_s^l A_{ys} \right] \theta \Delta t$$

$$+ \left[ \left( \Gamma_x \frac{\partial \phi}{\partial x} \right)_e^{l+1} A_{xe} - \left( \Gamma_x \frac{\partial \phi}{\partial x} \right)_w^{l+1} A_{xw} + \left( \Gamma_y \frac{\partial \phi}{\partial y} \right)_n^{l+1} A_{yn} - \left( \Gamma_y \frac{\partial \phi}{\partial y} \right)_s^{l+1} A_{ys} \right] (1-\theta) \Delta t$$

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And 1 minus theta delta t, which is basically t plus delta t minus our t plus theta delta t. This is nothing but 1 minus theta delta t.

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### Discretization

Governing Equation: Spatial Term

$$\int_t^{t+\theta\Delta t} \int_{\Omega^P} \nabla \cdot (\Gamma \cdot \nabla \phi) d\Omega dt + \int_{t+\theta\Delta t}^{t+\Delta t} \int_{\Omega^P} \nabla \cdot (\Gamma \cdot \nabla \phi) d\Omega dt = \int_t^{t+\Delta t} \int_{\Omega^P} \nabla \cdot \left( \Gamma_x \frac{\partial \phi}{\partial x} i + \Gamma_y \frac{\partial \phi}{\partial y} j \right) d\Omega dt$$

$$= \int_t^{t+\Delta t} \int_{\Omega^P} \nabla \cdot \left( \Gamma_x \frac{\partial \phi}{\partial x} i + \Gamma_y \frac{\partial \phi}{\partial y} j \right) d\Omega dt + \int_{t+\theta\Delta t}^{t+\Delta t} \int_{\Omega^P} \nabla \cdot \left( \Gamma_x \frac{\partial \phi}{\partial x} i + \Gamma_y \frac{\partial \phi}{\partial y} j \right) d\Omega dt$$

$$= \left[ \left( \Gamma_x \frac{\partial \phi}{\partial x} \right)_e^l A_{xe} - \left( \Gamma_x \frac{\partial \phi}{\partial x} \right)_w^l A_{xw} + \left( \Gamma_y \frac{\partial \phi}{\partial y} \right)_n^l A_{yn} - \left( \Gamma_y \frac{\partial \phi}{\partial y} \right)_s^l A_{ys} \right] \theta \Delta t$$

$$+ \left[ \left( \Gamma_x \frac{\partial \phi}{\partial x} \right)_e^{l+1} A_{xe} - \left( \Gamma_x \frac{\partial \phi}{\partial x} \right)_w^{l+1} A_{xw} + \left( \Gamma_y \frac{\partial \phi}{\partial y} \right)_n^{l+1} A_{yn} - \left( \Gamma_y \frac{\partial \phi}{\partial y} \right)_s^{l+1} A_{ys} \right] (1-\theta) \Delta t$$

$(t+\Delta t) - (t+\theta\Delta t) = (1-\theta)\Delta t$

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So we have two different levels. At the present level we have this L, Lth level. And future we have L plus 1 level. So with these two levels we have expanded this space integrals and we have averaged it for this time interval. So for two different time intervals initial one, t to t plus theta delta t and next one is t plus theta delta t to t plus delta t, we have this multiplication.

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Finite Volume Method  
Discretization: Interior Points  
Discretization: Boundary Points  
References

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### Discretization

Governing Equation: Spatial Term

$$\int_t^{t+\Delta t} \int_{\Omega^P} \nabla \cdot (\Gamma \cdot \nabla \phi) d\Omega dt = \int_t^{t+\Delta t} \int_{\Omega^P} \nabla \cdot \left( \Gamma_x \frac{\partial \phi}{\partial x} i + \Gamma_y \frac{\partial \phi}{\partial y} j \right) d\Omega dt$$

$$= \int_t^{t+\Delta t} \int_{\Omega^P} \nabla \cdot \left( \Gamma_x \frac{\partial \phi}{\partial x} i + \Gamma_y \frac{\partial \phi}{\partial y} j \right) d\Omega dt + \int_{t+\theta\Delta t}^{t+\Delta t} \int_{\Omega^P} \nabla \cdot \left( \Gamma_x \frac{\partial \phi}{\partial x} i + \Gamma_y \frac{\partial \phi}{\partial y} j \right) d\Omega dt$$

$$= \left[ \left( \Gamma_x \frac{\partial \phi}{\partial x} \right)_e^t A_{xe} - \left( \Gamma_x \frac{\partial \phi}{\partial x} \right)_w^t A_{xw} + \left( \Gamma_y \frac{\partial \phi}{\partial y} \right)_n^t A_{yn} - \left( \Gamma_y \frac{\partial \phi}{\partial y} \right)_s^t A_{ys} \right] \theta \Delta t$$

$$+ \left[ \left( \Gamma_x \frac{\partial \phi}{\partial x} \right)_e^{t+\Delta t} A_{xe} - \left( \Gamma_x \frac{\partial \phi}{\partial x} \right)_w^{t+\Delta t} A_{xw} + \left( \Gamma_y \frac{\partial \phi}{\partial y} \right)_n^{t+\Delta t} A_{yn} - \left( \Gamma_y \frac{\partial \phi}{\partial y} \right)_s^{t+\Delta t} A_{ys} \right] (1-\theta) \Delta t$$

$$= \left( \frac{t+\Delta t}{1-\theta} - \frac{t}{\theta} \right) \Delta t$$

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So with this if you proceed for further discretization, in uniform grid system we can say that for our uniform grid system, this is our P cell. This is east, west and this is our east face. So for our east face this phi E minus phi P, this is at Lth level because time level is L. For L plus 1 level again we can write the same discretization. For L plus 1 time level.

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Finite Volume Method  
Discretization: Interior Points  
Discretization: Boundary Points  
References

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### Discretization

Governing Equation

In a uniform grid system,  
East Face:

$$\left( \Gamma_x \frac{\partial \phi}{\partial x} \right)_e^l = \Gamma_x \frac{\phi_E^l - \phi_P^l}{\Delta x} \quad \text{and} \quad \left( \Gamma_x \frac{\partial \phi}{\partial x} \right)_e^{l+1} = \Gamma_x \frac{\phi_E^{l+1} - \phi_P^{l+1}}{\Delta x} \quad (3)$$

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And similarly we can write for west face where phi P minus phi W is required. North face phi N minus phi P. South face phi P minus phi S is required. So if you substitute these values within our original governing equation, we can get the discretized form.

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Finite Volume Method  
Discretization: Interior Points  
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References

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### Discretization

#### Governing Equation

In a uniform grid system,

East Face:

$$\left(\Gamma_x \frac{\partial \phi}{\partial x}\right)_e^l = \Gamma_{xe} \frac{\phi_E^l - \phi_P^l}{\Delta x} \quad \text{and} \quad \left(\Gamma_x \frac{\partial \phi}{\partial x}\right)_e^{l+1} = \Gamma_{xe} \frac{\phi_E^{l+1} - \phi_P^{l+1}}{\Delta x} \quad (3)$$

West Face:

$$\left(\Gamma_x \frac{\partial \phi}{\partial x}\right)_w^l = \Gamma_{xw} \frac{\phi_P^l - \phi_W^l}{\Delta x} \quad \text{and} \quad \left(\Gamma_x \frac{\partial \phi}{\partial x}\right)_w^{l+1} = \Gamma_{xw} \frac{\phi_P^{l+1} - \phi_W^{l+1}}{\Delta x} \quad (4)$$

North Face:

$$\left(\Gamma_y \frac{\partial \phi}{\partial y}\right)_n^l = \Gamma_{yn} \frac{\phi_N^l - \phi_P^l}{\Delta y} \quad \text{and} \quad \left(\Gamma_y \frac{\partial \phi}{\partial y}\right)_n^{l+1} = \Gamma_{yn} \frac{\phi_N^{l+1} - \phi_P^{l+1}}{\Delta y} \quad (5)$$

South Face:

$$\left(\Gamma_y \frac{\partial \phi}{\partial y}\right)_s^l = \Gamma_{ys} \frac{\phi_P^l - \phi_S^l}{\Delta y} \quad \text{and} \quad \left(\Gamma_y \frac{\partial \phi}{\partial y}\right)_s^{l+1} = \Gamma_{ys} \frac{\phi_P^{l+1} - \phi_S^{l+1}}{\Delta y} \quad (6)$$

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In uniform grid we can consider that area for east face, this is our P, this is east face, this is west face, south and north. A xe magnitude is del y and A yn magnitude is delx.

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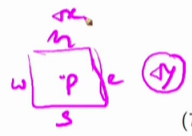

Finite Volume Method  
Discretization: Interior Points  
Discretization: Boundary Points  
References

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### Discretization

#### Governing Equation

In a uniform grid system,

$$\begin{aligned} A_{xe} &= A_{xw} = \Delta y \\ A_{yn} &= A_{ys} = \Delta x \end{aligned} \quad (7)$$



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So now we need to see what is this source term? Source term, we can write it as average. Average of present time level value and future time level value with weights theta, delta t and 1 minus theta delta t and combined manner we can write like this.

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Finite Volume Method  
Discretization: Interior Points  
Discretization: Boundary Points  
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## Discretization

### Governing Equation

In a uniform grid system,

$$\begin{aligned} A_{xe} &= A_{xw} = \Delta y \\ A_{yn} &= A_{ys} = \Delta x \end{aligned} \quad (7)$$

Source Term:

$$\int_t^{t+\Delta t} \int_{\Omega^P} S_\phi(x, y) d\Omega dt = \left[ \theta S_\phi^l(x_P, y_P) + (1 - \theta) S_\phi^{l+1}(x_P, y_P) \right] \Delta x \Delta y \Delta t \quad (8)$$

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Now let us utilize all these discretize form to get the final discretized form of our governing equation. In the left hand side we have  $\lambda \phi_P$ ,  $\phi_P$ ,  $\lambda + 1$ ,  $L$ ,  $\Delta x \Delta y$  for volume of  $\phi$   $\Omega^P$  or element  $P$ . Now for east and west face we have  $\theta \Delta t$ . Like north and south face we have  $\theta \Delta t$ . East and west face,  $1 - \theta \Delta t$ . North and south face  $1 - \theta \Delta t$ . So with this if we divide both sides with dot and, let us substitute individual values of these derivatives.

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Finite Volume Method  
Discretization: Interior Points  
Discretization: Boundary Points  
References

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## Discretization

### Governing Equation

Compact Form of the equation can be written as,

$$\begin{aligned} & \lambda \phi_P^{l+1} - \phi_P^l \Delta x \Delta y \\ &= \left[ \left( \Gamma_x \frac{\partial \phi}{\partial x} \right)_e^l A_{xe} - \left( \Gamma_x \frac{\partial \phi}{\partial x} \right)_w^l A_{xw} \right] \theta \Delta t \\ &+ \left[ \left( \Gamma_y \frac{\partial \phi}{\partial y} \right)_n^l A_{yn} - \left( \Gamma_y \frac{\partial \phi}{\partial y} \right)_s^l A_{ys} \right] \theta \Delta t \\ &+ \left[ \left( \Gamma_x \frac{\partial \phi}{\partial x} \right)_e^{l+1} A_{xe} - \left( \Gamma_x \frac{\partial \phi}{\partial x} \right)_w^{l+1} A_{xw} \right] (1 - \theta) \Delta t \\ &+ \left[ \left( \Gamma_y \frac{\partial \phi}{\partial y} \right)_n^{l+1} A_{yn} - \left( \Gamma_y \frac{\partial \phi}{\partial y} \right)_s^{l+1} A_{ys} \right] (1 - \theta) \Delta t \\ &+ \left[ \theta S_\phi^l(x_P, y_P) + (1 - \theta) S_\phi^{l+1}(x_P, y_P) \right] \Delta x \Delta y \Delta t \end{aligned}$$

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So with this spatial derivative evaluated at present time level, for this four and future time level for this box forms.


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Finite Volume Method  
Discretization: Interior Points  
Discretization: Boundary Points  
References

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### Discretization Governing Equation

Compact Form of the equation can be written as,

$$\begin{aligned} & \Lambda_\phi (\phi_P^{l+1} - \phi_P^l) \Delta x \Delta y \\ &= \left[ \Gamma_{xc} \frac{\phi_E^l - \phi_P^l}{\Delta x} A_{xc} - \Gamma_{xw} \frac{\phi_P^l - \phi_W^l}{\Delta x} A_{xw} \right] \theta \Delta t \\ &+ \left[ \Gamma_{yn} \frac{\phi_N^l - \phi_P^l}{\Delta y} A_{yn} - \Gamma_{ys} \frac{\phi_P^l - \phi_S^l}{\Delta y} A_{ys} \right] \theta \Delta t \\ &+ \left[ \Gamma_{xc} \frac{\phi_E^{l+1} - \phi_P^{l+1}}{\Delta x} A_{xc} - \Gamma_{xw} \frac{\phi_P^{l+1} - \phi_W^{l+1}}{\Delta x} A_{xw} \right] (1 - \theta) \Delta t \\ &+ \left[ \Gamma_{yn} \frac{\phi_N^{l+1} - \phi_P^{l+1}}{\Delta y} A_{yn} - \Gamma_{ys} \frac{\phi_P^{l+1} - \phi_S^{l+1}}{\Delta y} A_{ys} \right] (1 - \theta) \Delta t \\ &+ \left[ \theta S_\phi^l(x_P, y_P) + (1 - \theta) S_\phi^{l+1}(x_P, y_P) \right] \Delta x \Delta y \Delta t \end{aligned}$$


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Now we can rearrange this thing with the consideration that gamma xe equals to gamma xw equals to gamma x. And gamma yn equals to gamma ys equals to gamma y.

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Finite Volume Method  
Discretization: Interior Points  
Discretization: Boundary Points  
References

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### Discretization Governing Equation

Compact Form of the equation can be written as,

$$\begin{aligned} & \Lambda_\phi (\phi_P^{l+1} - \phi_P^l) \Delta x \Delta y \\ &= \left[ \Gamma_{xc} \frac{\phi_E^l - \phi_P^l}{\Delta x} A_{xc} - \Gamma_{xw} \frac{\phi_P^l - \phi_W^l}{\Delta x} A_{xw} \right] \theta \Delta t \\ &+ \left[ \Gamma_{yn} \frac{\phi_N^l - \phi_P^l}{\Delta y} A_{yn} - \Gamma_{ys} \frac{\phi_P^l - \phi_S^l}{\Delta y} A_{ys} \right] \theta \Delta t \\ &+ \left[ \Gamma_{xc} \frac{\phi_E^{l+1} - \phi_P^{l+1}}{\Delta x} A_{xc} - \Gamma_{xw} \frac{\phi_P^{l+1} - \phi_W^{l+1}}{\Delta x} A_{xw} \right] (1 - \theta) \Delta t \\ &+ \left[ \Gamma_{yn} \frac{\phi_N^{l+1} - \phi_P^{l+1}}{\Delta y} A_{yn} - \Gamma_{ys} \frac{\phi_P^{l+1} - \phi_S^{l+1}}{\Delta y} A_{ys} \right] (1 - \theta) \Delta t \\ &+ \left[ \theta S_\phi^l(x_P, y_P) + (1 - \theta) S_\phi^{l+1}(x_P, y_P) \right] \Delta x \Delta y \Delta t \end{aligned}$$

$\Gamma_{xc} = \Gamma_{xw} = \Gamma_x$   
 $\Gamma_{yn} = \Gamma_{ys} = \Gamma_y$

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Now with this and dividing both sides with delta x, delta y and delta t, we can write this. Left hand side, this is similar to our finite difference discretization for any arbitrary node ij. In this case we have cell centered P Lplus 1 to L. And further as per our assumption we can consider this values equal these values equal. So we can combine north-south, east-west terms.

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Finite Volume Method  
Discretization: Interior Points  
Discretization: Boundary Points  
References

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### Discretization

#### Governing Equation

Compact Form of the equation can be written as,

$$\Lambda_\phi \frac{\phi_P^{l+1} - \phi_P^l}{\Delta t} = \theta \left[ \Gamma_{xe} \frac{\phi_E^l - \phi_P^l}{\Delta x^2} - \Gamma_{xw} \frac{\phi_P^l - \phi_W^l}{\Delta x^2} \right] + \theta \left[ \Gamma_{yn} \frac{\phi_N^l - \phi_P^l}{\Delta y^2} - \Gamma_{ys} \frac{\phi_P^l - \phi_S^l}{\Delta y^2} \right] + (1-\theta) \left[ \Gamma_{xe} \frac{\phi_E^{l+1} - \phi_P^{l+1}}{\Delta x^2} - \Gamma_{xw} \frac{\phi_P^{l+1} - \phi_W^{l+1}}{\Delta x^2} \right] + (1-\theta) \left[ \Gamma_{yn} \frac{\phi_N^{l+1} - \phi_P^{l+1}}{\Delta y^2} - \Gamma_{ys} \frac{\phi_P^{l+1} - \phi_S^{l+1}}{\Delta y^2} \right] + \left[ \theta S_\phi^l(x_P, y_P) + (1-\theta) S_\phi^{l+1}(x_P, y_P) \right]$$

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With this if we combine this east-west term, then this is basically phi E evaluated at Lth time step minus 2 phi P plus phi W into theta. And north, minus 2 phi P plus S, this is evaluated for south cell. So theta and 1 minus theta, these two weighted combinations are there. And theta minus 1 minus theta.

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Finite Volume Method  
Discretization: Interior Points  
Discretization: Boundary Points  
References

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### Discretization

#### Governing Equation

Compact Form of the equation can be written as,

$$\Lambda_\phi \frac{\phi_P^{l+1} - \phi_P^l}{\Delta t} = \theta \left[ \Gamma_x \frac{\phi_E^l - 2\phi_P^l + \phi_W^l}{\Delta x^2} + \Gamma_y \frac{\phi_N^l - 2\phi_P^l + \phi_S^l}{\Delta y^2} \right] + (1-\theta) \left[ \Gamma_x \frac{\phi_E^{l+1} - 2\phi_P^{l+1} + \phi_W^{l+1}}{\Delta x^2} + \Gamma_y \frac{\phi_N^{l+1} - 2\phi_P^{l+1} + \phi_S^{l+1}}{\Delta y^2} \right] + \left[ \theta S_\phi^l(x_P, y_P) + (1-\theta) S_\phi^{l+1}(x_P, y_P) \right]$$

With  $\Gamma_{xe} = \Gamma_{xw} = \Gamma_x, \Gamma_{yn} = \Gamma_{ys} = \Gamma_y$

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Now if we consider explicit scheme then we can consider theta equals to 1. And then the L plus 1 level is zero. Theta equals to zero. Then we have implicit scheme. Only L plus 1 level values are there for special derivative.



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Finite Volume Method  
Discretization: Interior Points  
Discretization: Boundary Points  
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### $\theta$ -Schemes

**Explicit Scheme ( $\theta = 1$ )**

$$\Lambda_\phi \frac{\phi_P^{l+1} - \phi_P^l}{\Delta t} = \Gamma_x \frac{\phi_E^l - 2\phi_P^l + \phi_W^l}{\Delta x^2} + \Gamma_y \frac{\phi_N^l - 2\phi_P^l + \phi_S^l}{\Delta y^2} + S_\phi^l(x_P, y_P)$$

**Implicit Scheme ( $\theta = 0$ )**

$$\Lambda_\phi \frac{\phi_P^{l+1} - \phi_P^l}{\Delta t} = \Gamma_x \frac{\phi_E^{l+1} - 2\phi_P^{l+1} + \phi_W^{l+1}}{\Delta x^2} + \Gamma_y \frac{\phi_N^{l+1} - 2\phi_P^{l+1} + \phi_S^{l+1}}{\Delta y^2} + S_\phi^{l+1}(x_P, y_P)$$

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And if we consider theta is equals to half, we will have combination of L and L plus 1 level values. And I have transferred this half to the right hand side, so this 2 is multiplied here.


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Finite Volume Method  
Discretization: Interior Points  
Discretization: Boundary Points  
References

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### $\theta$ -Schemes

**Crank-Nicolson Scheme ( $\theta = \frac{1}{2}$ )**

$$\begin{aligned} 2\Lambda_\phi \frac{\phi_P^{l+1} - \phi_P^l}{\Delta t} &= \Gamma_x \frac{\phi_E^l - 2\phi_P^l + \phi_W^l}{\Delta x^2} + \Gamma_y \frac{\phi_N^l - 2\phi_P^l + \phi_S^l}{\Delta y^2} \\ &+ \Gamma_x \frac{\phi_E^{l+1} - 2\phi_P^{l+1} + \phi_W^{l+1}}{\Delta x^2} + \Gamma_y \frac{\phi_N^{l+1} - 2\phi_P^{l+1} + \phi_S^{l+1}}{\Delta y^2} \\ &+ S_\phi^l(x_P, y_P) + S_\phi^{l+1}(x_P, y_P) \end{aligned}$$


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Now up to this discretization IVBP is somewhat similar to finite difference discretization. Now we need to see what will be the boundary conditions. Left boundary again, we need to evaluate this west face derivative separately which is not common. And right boundary also, we need to evaluate the derivative at east boundary separately.

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Finite Volume Method  
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### Boundary Conditions

Left and Right Boundary

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For north also, this north boundary derivative of phi with respect to y, that will be zero. Similarly for south boundary or bottom boundary, at the south face derivative of phi with respect to y, that will be zero.

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Finite Volume Method  
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Discretization: Boundary Points  
References

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### Boundary Conditions

Top and Bottom Boundary

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Corners, west side we need to evaluate the derivative separately. North side the derivative will be zero. This north-east again east face derivative, north face derivative is zero.

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### Boundary Conditions

Corners

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South-west and south-east, similarly we can impose the corner boundary conditions. So with this discretization which are similar to our boundary value problem, we can get the discretize form of all the cell centered points. And we can solve the final matrix and get the value for all cells centered values for all cell centered positions. If you have Dirichlet boundary, your boundary values are specified. But for Neumann boundary we can utilize the nodes or cell centers next to our boundary to get information about the value at the boundary itself.

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### Boundary Conditions

Corners

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Thank you.