Computational Hydraulics
ProfessorAnirban Dhar
Department of Civil Engineering
Indian Institute of Technology Kharagpur
Lecture 15
Finite Volume Method - IBVP

Welcome to this lecture number 15 of the course computational hydraulics.We are in module 2 numerical methods. In this particular lecture I will be covering unit 11, finite volume method, initial boundary value problem.

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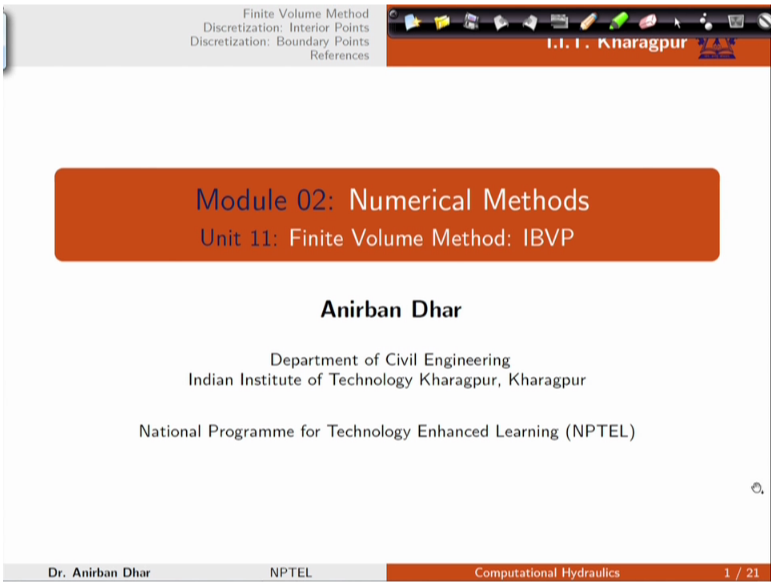

Learning objectives, our objective for this particular unit. At the end of this particular unit students will be able to derive the algebraic form for initial boundary value problem using finite volume method.

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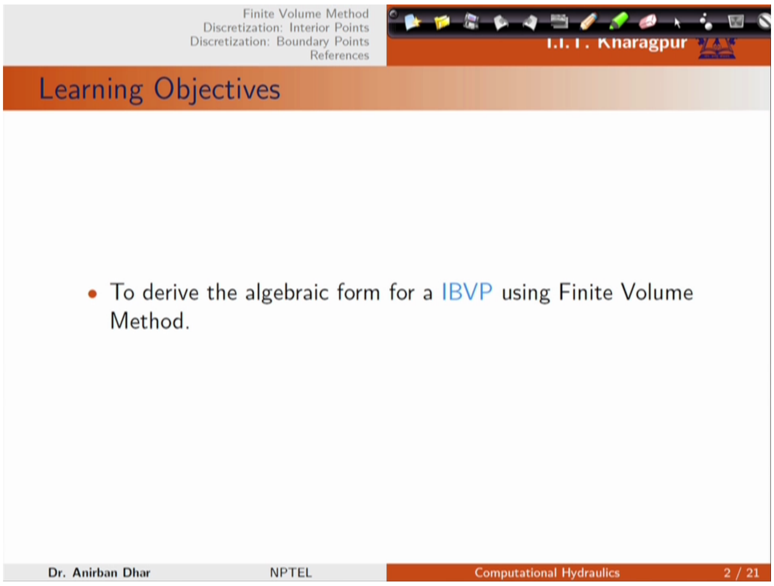

Let us consider our general equation in terms of general variable phi. we have solved the boundary value problem or BVP using this diffusing term and source sink term. Now in this particularlecturewe will be introducing this temporal term for IVBP problem.

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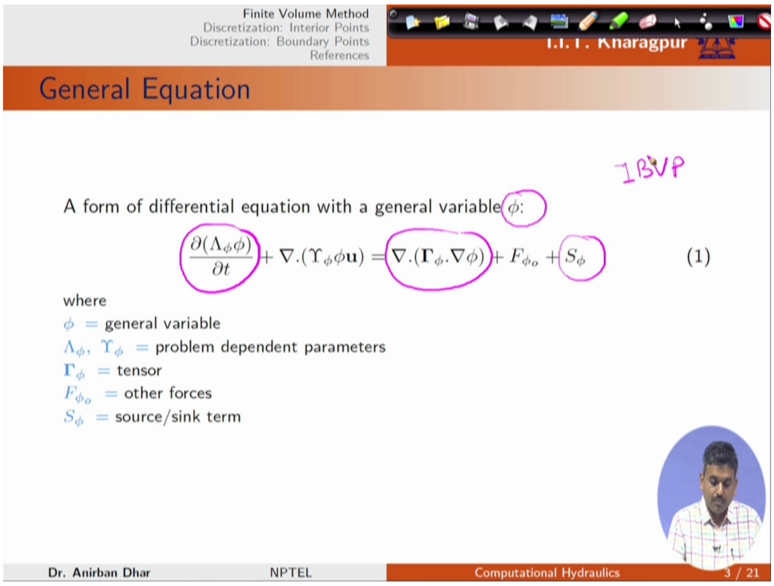

Now what is this problem definition? Like our finite difference case let us consider this lambda phi as coefficient for this temporal term. And gamma x, gamma y these are diffusioncoefficients. So this tensor, this is specifically two dimensional 0 0, gamma y. So we have only two terms.

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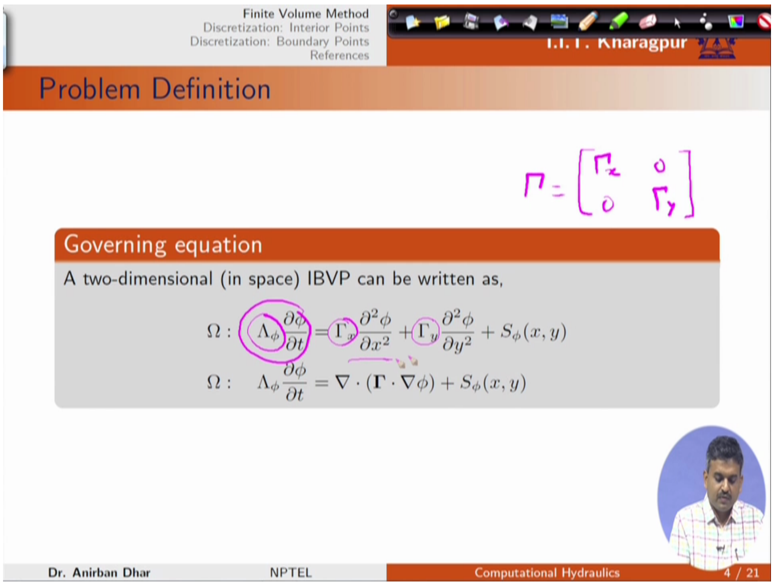

If we write this in terms of this del or divergence operator, we can write this one in terms of this. This is basically delphi or Dirac phi. This is equals to phiI, where I is the unit vector and direction xj is the unit vector in the direction y. And divergence of this will give you the other terms.

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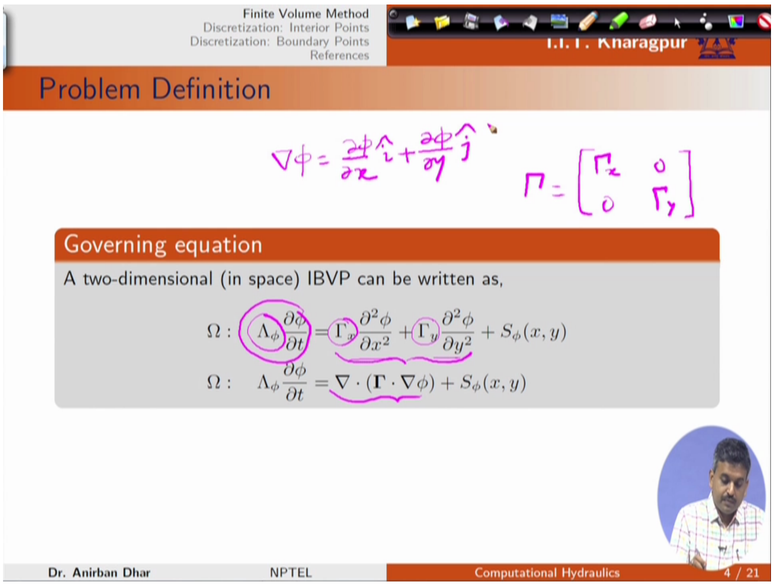

Nowwhat are the boundary conditions or initial conditions? Initial conditions, let us say that initial value is specified for xy domain or omega domain or this one for all values of x and y, this phi 0 is given. Now left hand side like our previous problem it is a specified boundary or Dirichlet kind of boundary condition. Right hand side also Dirichlet kind of boundary condition. Bottom and top boundary, these are Neumann boundary conditions.

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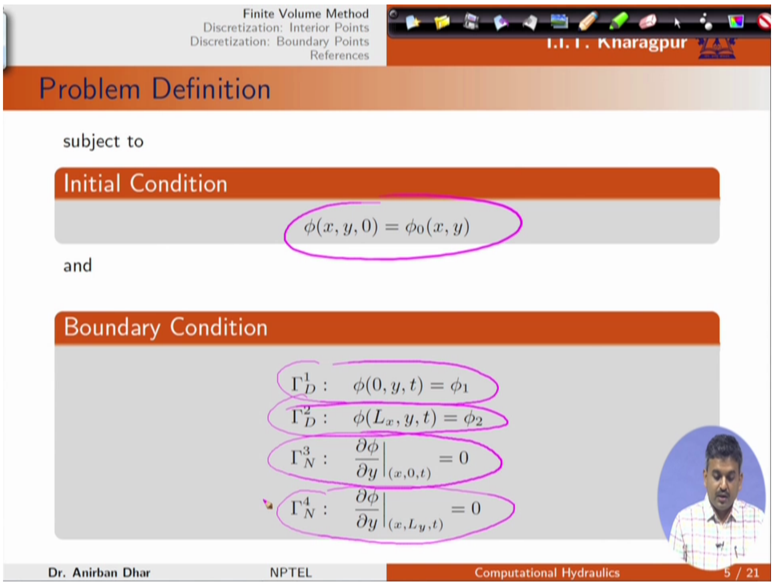

Nowdomain discretization. Like our boundary value problem we can divide this Lx in x direction, Ly in Y direction in number ofcells. At the cell center we will consider the variables. And in this case new ones are interior cells, for representative one. This black ones are next to boundary. And red ones, these needs spatial treatment because one side is Dirichlet and another side is Neumann.

In physical system there can be situations where both the boundariescan be Neumann or Dirichlet. But in this case we have combination of boundary conditions. One side Dirichlet another side Neumann. And del xis a cell size of the x direction. Del y in the y direction.

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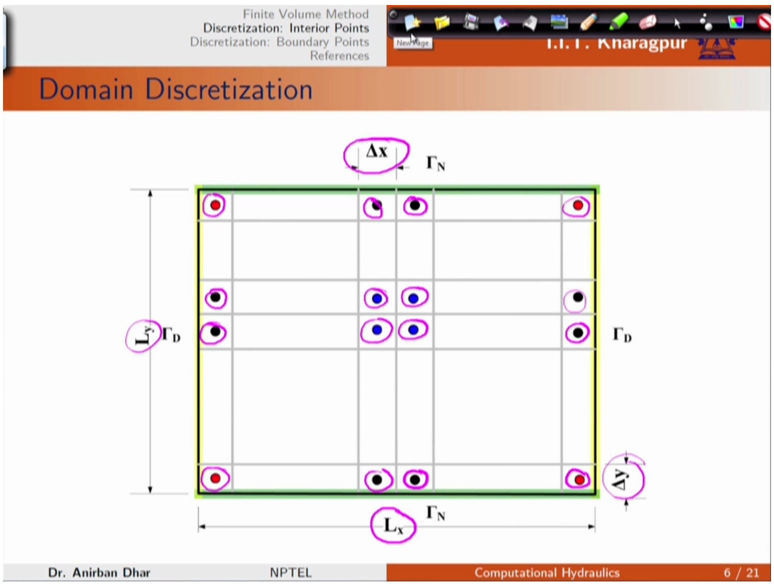

So with this we can start the discretization of the governing equation. If we consider any interior cell, then for any particular interior cell we have east, west, north, south, these four neighboring cells. For this one we can start the discretization. So in finite volume method, the governing equation is integrated over the elemental volume in space and time interval to form the discretized governing equation at node point P or cell center P.

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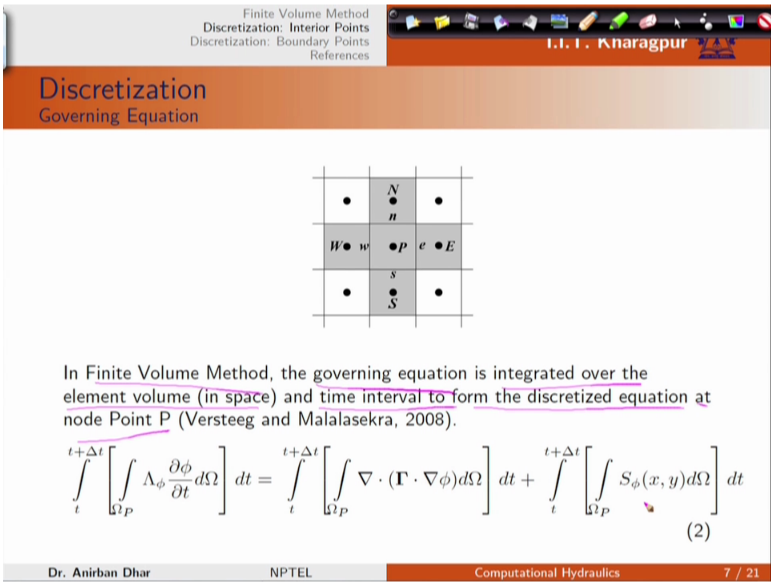

So in this case we are integrating from t to tplus delta t. That is time increment from present level t to future time level tplus delta t. And integration omega P, this is over elemental volume or element volume. This is actually element volume.

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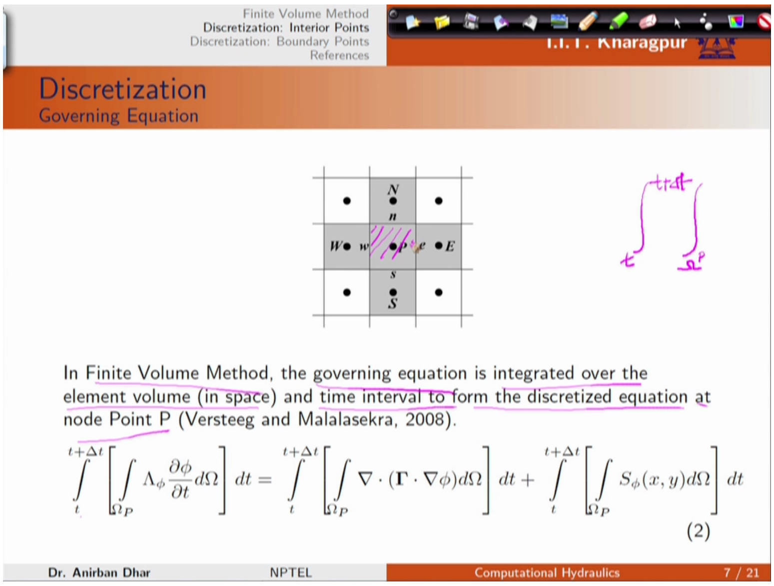

So if we integrate our original equation with respect to space and time then we can write like this. This is for temporal term, spatial diffusion term and source sink term.

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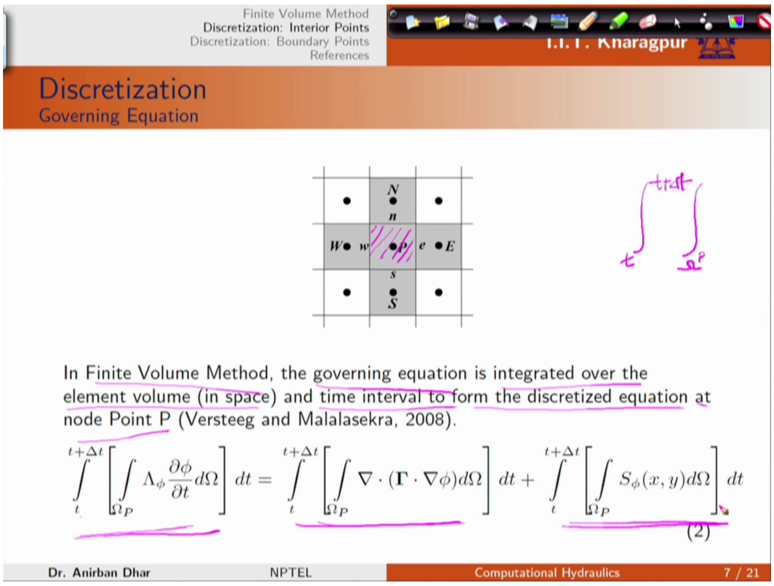

Now let us consider the discretization for individual terms. In the first case we have this temporal term. This quantity is not varying with time. Let us consider this is constant. So we can take out this term out of this integration. So we can write this term, this spatial derivative spatial integration with respect to temporal derivative. We can take this integration sign inside. And let us integrate this phi over this element volume.

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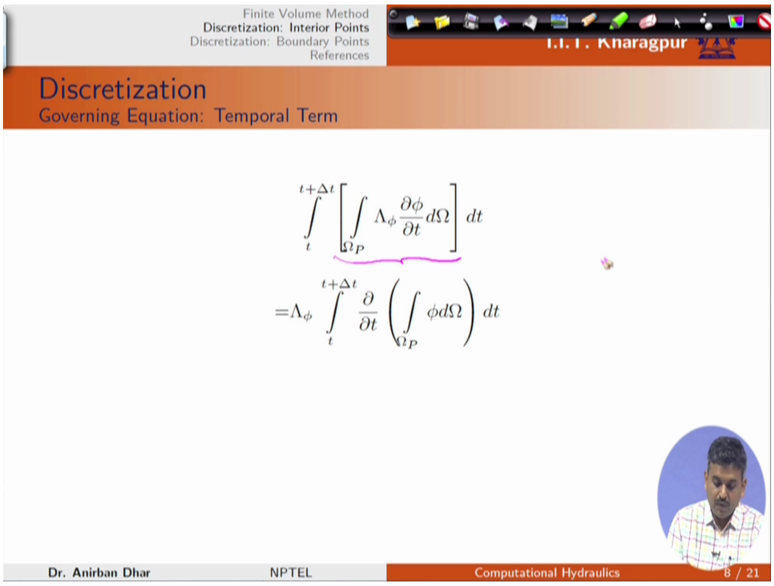

With this in the next step we can approximate this. This approximation is nothing but this phi P is the approximated value for this domain if we consider this spatial averaging. So this is actually spatially average value within this P cell. Now let us integrate it from t to tplus delta t. So if we do that then we can write this one to t this phi P Lplus 1 minus phi L. L represents the t time level and Lplus 1 represents tplus delta t time level. And this one is nothing but the volume of the cell P.

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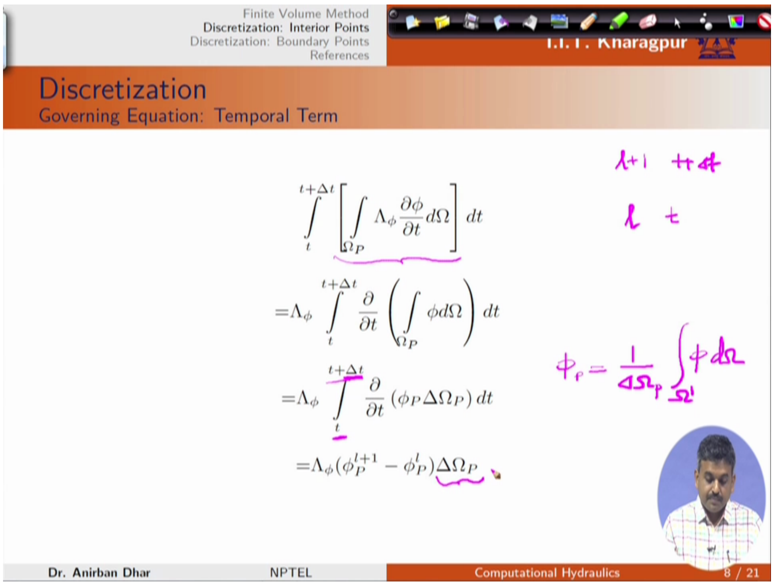

Next step we can simply write, this is del x into del y and virtually on the other direction we can consider the width as 1. So this del omega P or volume, we can directly write as del x del y into 1.

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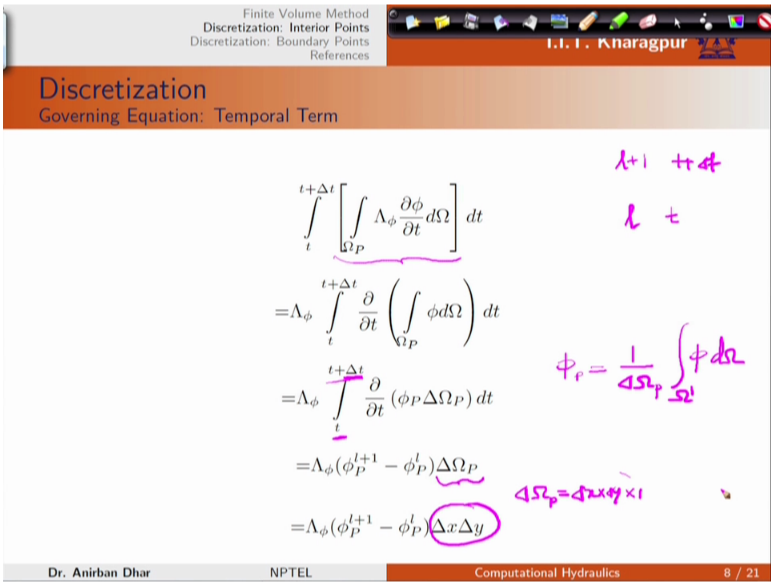

Now let us see the integration of the spatial or diffusing term. Spatial term, we can write this quantity in terms of vector like this.

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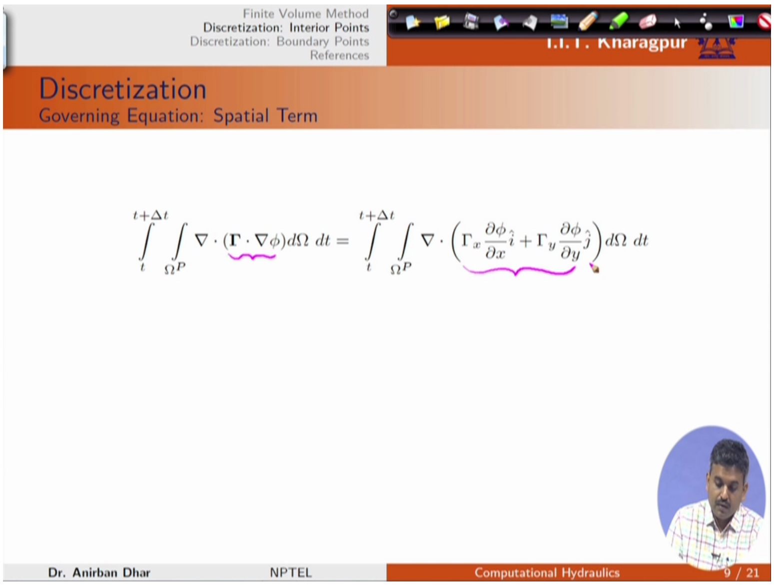

Now in the next stepif we see this one, this is divergence of this vector. As per Gauss divergence theorem, if you have divergence of any vector A, then we can write it in terms of surface integral. So in this case we can first divide it into two parts. One is t plus theta delta t. This is intermediate time level and from tplus theta delta t to t plus delta t. Like our theta scheme in finite difference, this is intermediate time level, this is basically zero level, this is theta and this is basically zero level. This is theta, this is 1, this is t plus del t, tplus delta theta and t only.

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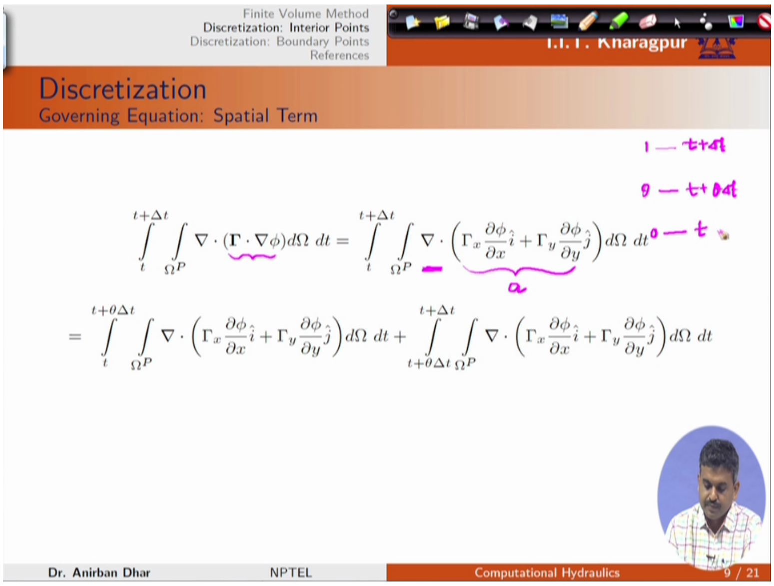

So in this case if we consider this one we can use the Gauss divergence theorem. And from Gauss divergence theorem, we can simply write it in terms of this step. Intermediate step will be there. That intermediate step is that actually t t plus theta delta t. And this will be over sP or surface. And only gamma x del phi del x I plus gamma y del phi del y j dot n cap into ds. This is for first term.

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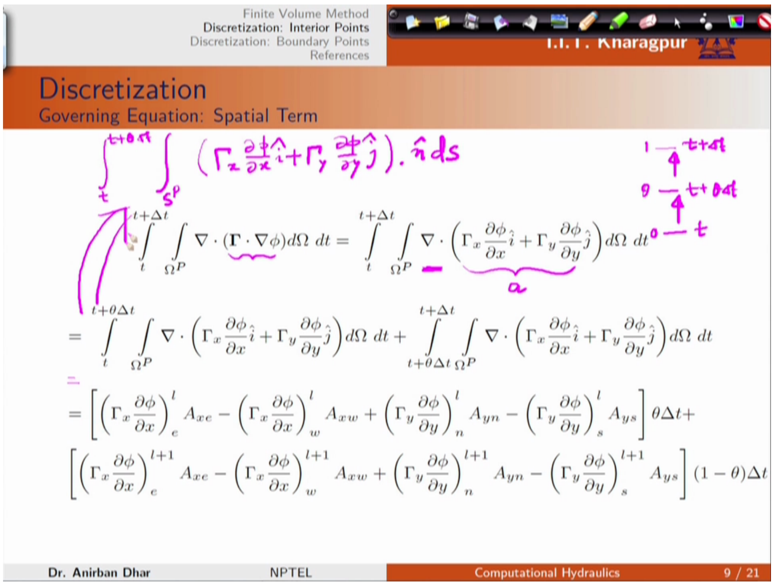

Similarly we can write the other term tplus theta delta t. This is tplus delta t and again spatial derivative for this one, this is for delta t and sP, that is for surface P. Now in this case we can see that n, this unit vector is always outward positive. Outward positive means for any case if we consider for our cell P in east face outward positive normal is in the direction positive i and in our west face we have this negative i direction. North face I have positive j and south face I have negative j.

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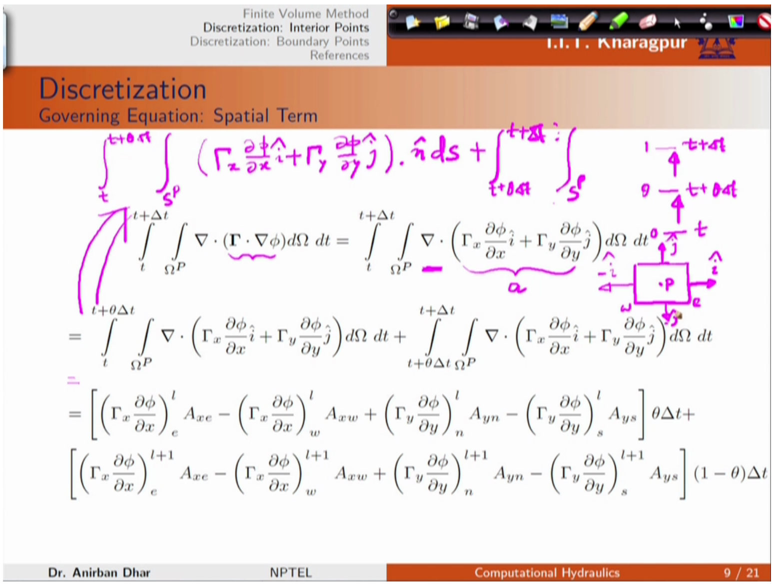

So these are the directions for outward normal. Individually we can consider this effect. So projected area in that direction is actually A xe. So for east face Idot this A xe, this is positive is considered. So if you multiply this only I dot I, that term will be 1. there will be y related term.

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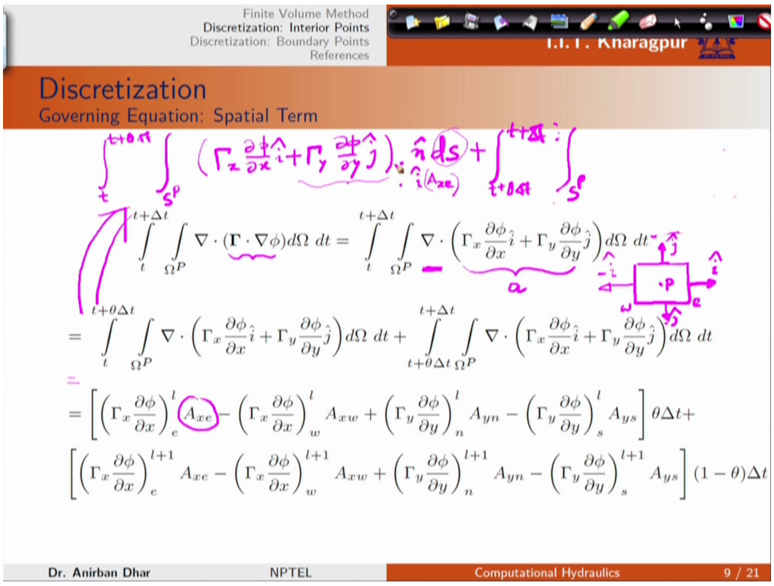

Similarly if we consider west face, this negative sign will be there. North face positive sign and south face there will be again negative sign and this theta delta t, this is basically value for this first integral.

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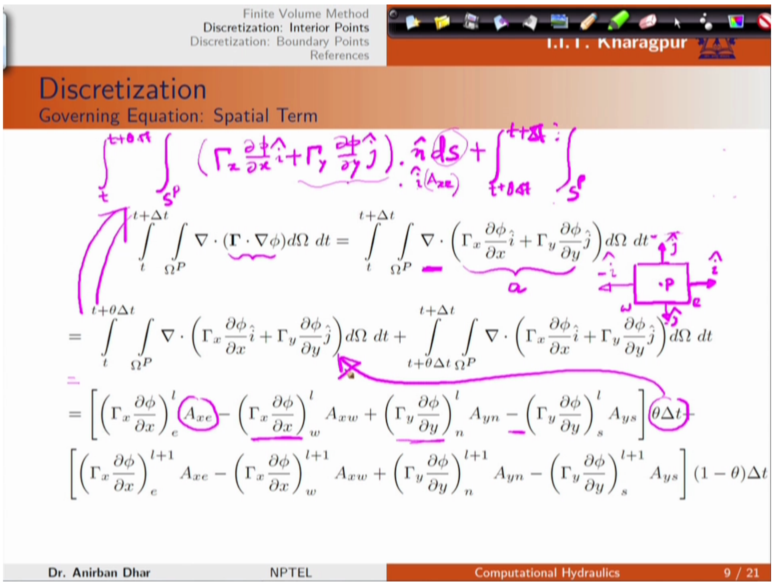

And 1 minus theta delta t, which is basically tplus delta t minus our tplus theta delta t. This is nothing but 1 minus theta delta t.

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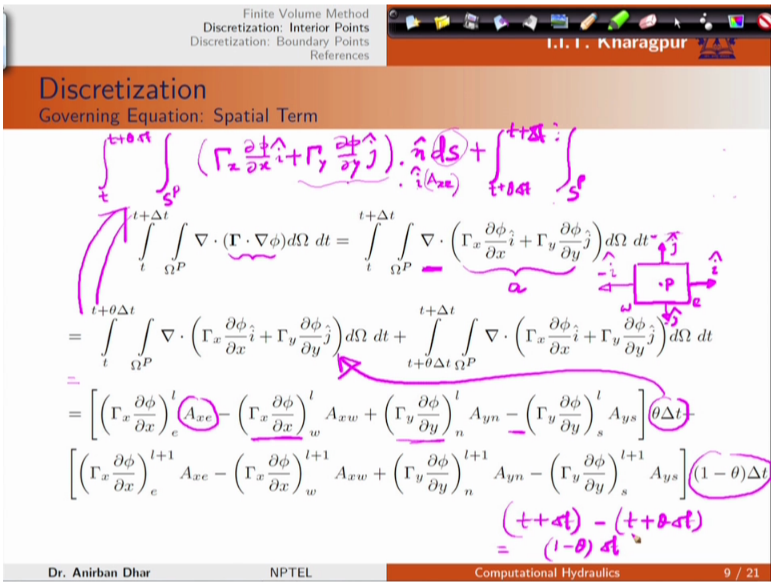

So we have two different levels. At the present level we have this L, Lth level. And future we have Lplus 1 level. So with these two levels we have expanded this space integrals and we have averaged it for this time interval. So for two different time intervals initial one, t to t plus theta delta t and next one is tplus theta delta t to tplus delta t, we have this multiplication.

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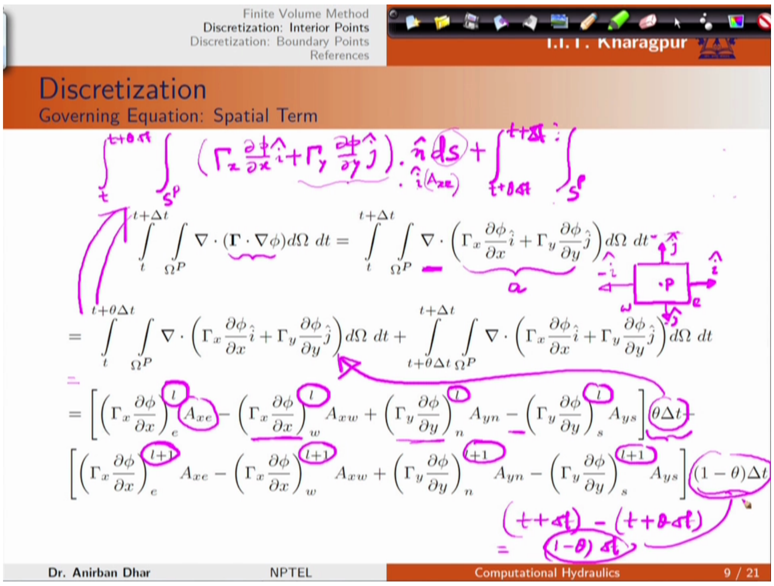

So with this if you proceed for further discretization, in uniform grid system we can say that for our uniform grid system, this is our P cell. This is east, west and this is our east face. So for our east face this phi Eminus phi P, this is at Lth level because time level is L. For Lplus 1 level again we can write the same discretization. For L plus 1 time level.

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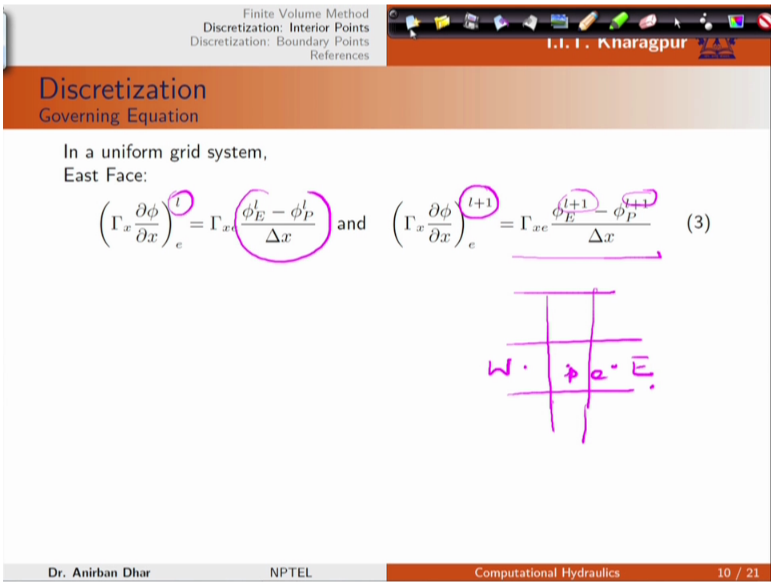

And similarly we can write for west face where phi P minus phi W is required. North face phi N minus phi P. South face phi P minus phi S is required. So if you substitute these values within our original governing equation, we can get the discretized form.

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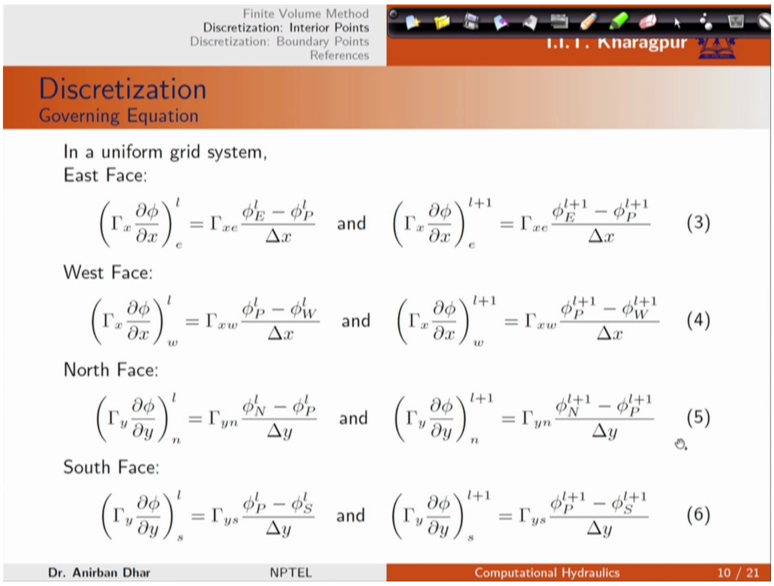

In uniform grid we can consider that area for east face, this is our P, this is east face, this is west face, south and north. A xe magnitude is del y and A yn magnitude is delx.

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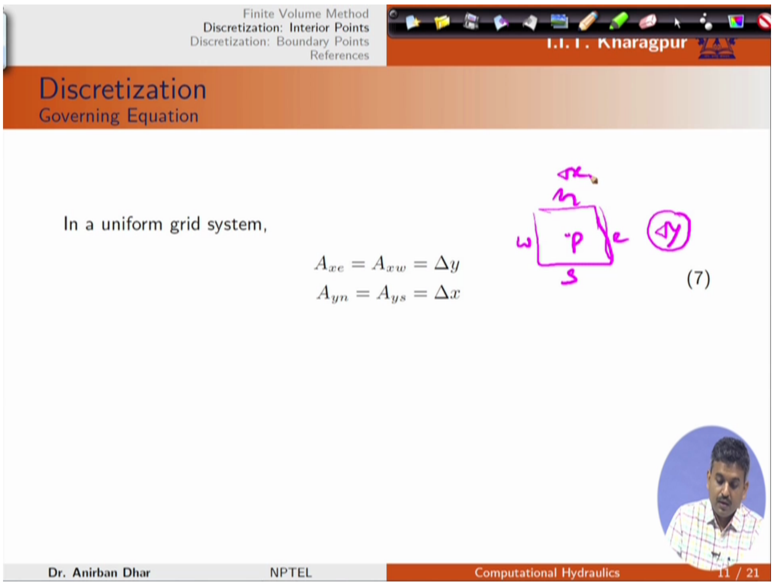

So now we need to see what is this source term? Source term, we can write it as average. Average of present time level value and future time level value with weights theta, delta t and 1 minus theta delta t and combined manner we can write like this.

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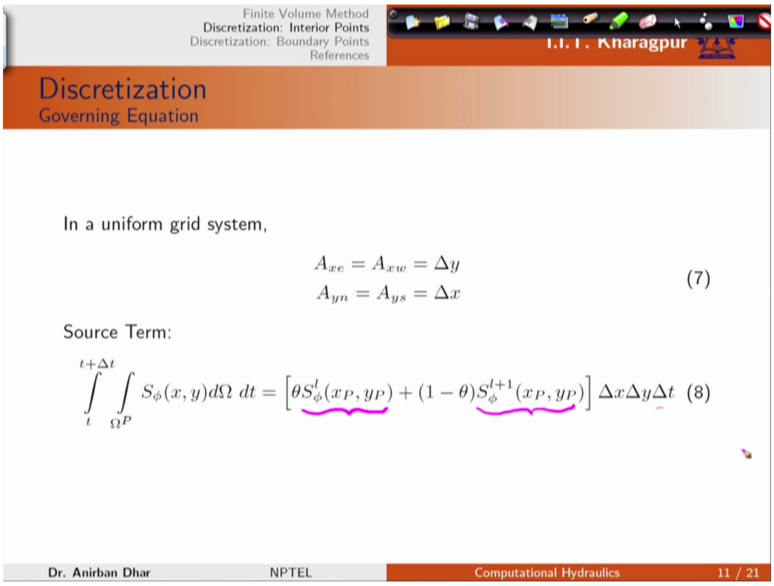

Now let us utilize all these discretize form to get the final discretized form of our governing equation. In the left hand side we have lambda phi, phi P Lplus 1, L, del x del y for volume of phi omega P or element P. Now for east and west face we have theta delta t. Like north and south face we have theta delta t. East and west face, 1 minus theta delta t. North and south face 1 minus theta delta t. So with this if we divide both sides with dot and, let us substitute individual values of these derivatives.

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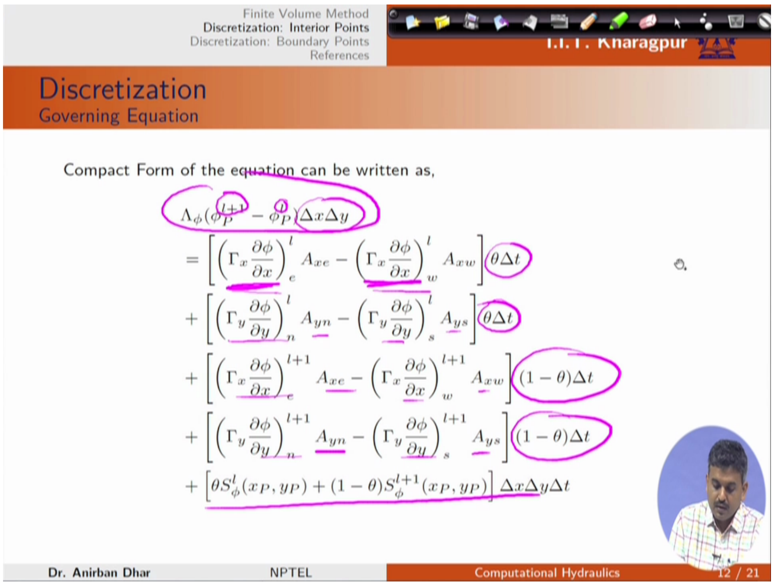

So with this spatial derivative evaluated at present time level, for this four and future time level for this box forms.

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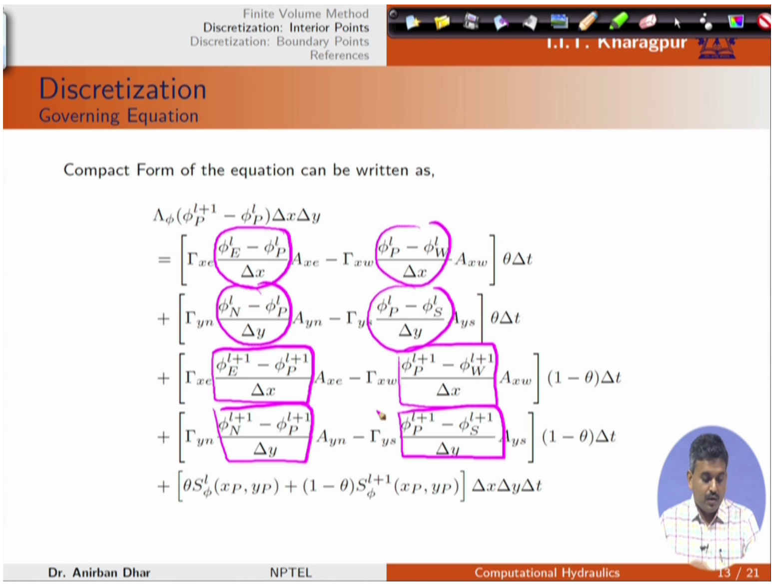

Now we can rearrange this thing with the consideration that gamma xe equals to gamma xw equals to gamma x. And gamma yn equals to gammays equals to gamma y.

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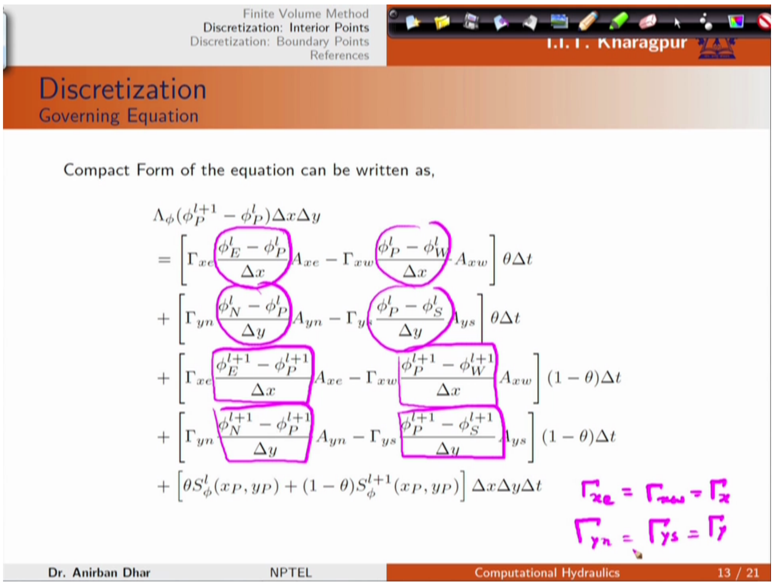

Now with this and dividing both sides with delta x, delta y and delta t, we can write this. Left hand side, this is similar to our finite difference discretization for any arbitrary node ij. In this case we have cell centered P Lplus 1 to L. And further as per our assumption we can consider this values equal these values equal. So we can combine north-south, east-west terms.

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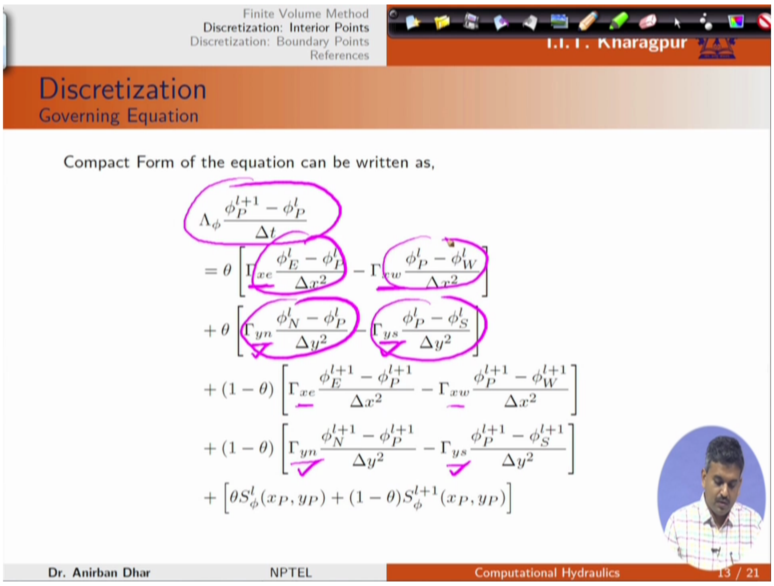

With this if we combine this east-west term, then this is basically phi E evaluated at Lth time step minus 2 phi Pplus phi W into theta. And north, minus 2 phi Pplus S, this is evaluated for south cell. So theta and 1 minus theta, these two weighted combinations are there. And theta minus 1 minus theta.

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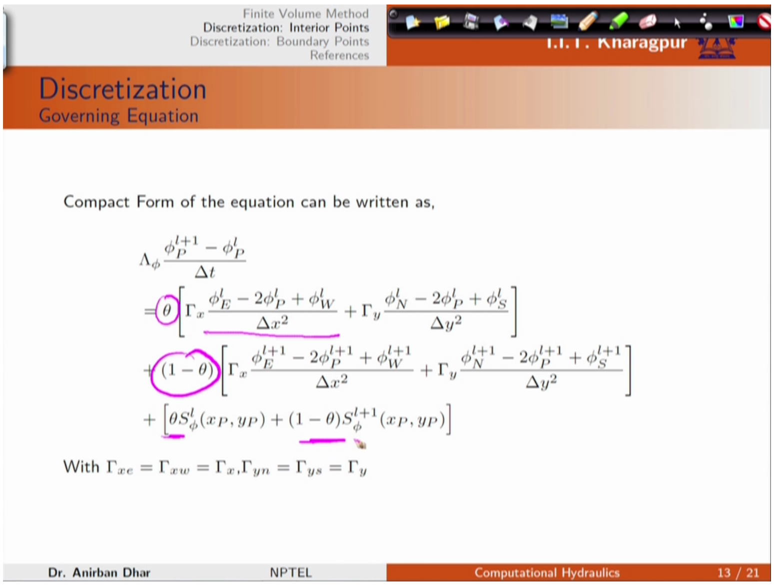

Now if we consider explicit scheme then we can consider theta equals to 1. And then the L plus 1 level is zero. Theta equals to zero. Then we have implicit scheme. Only L plus 1 level values are there for special derivative.

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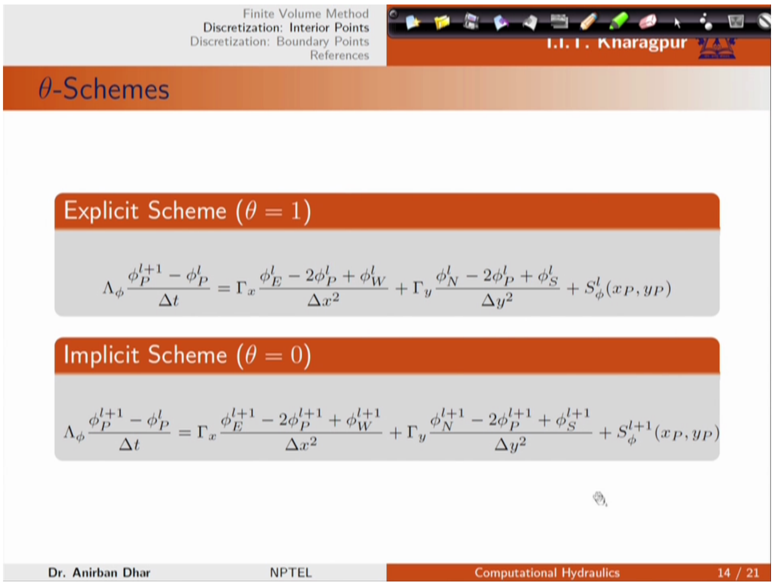

And if we consider theta is equals to half, we will have combination of L and L plus 1 level values. And I have transferred this half to the right hand side, so this 2 is multiplied here.

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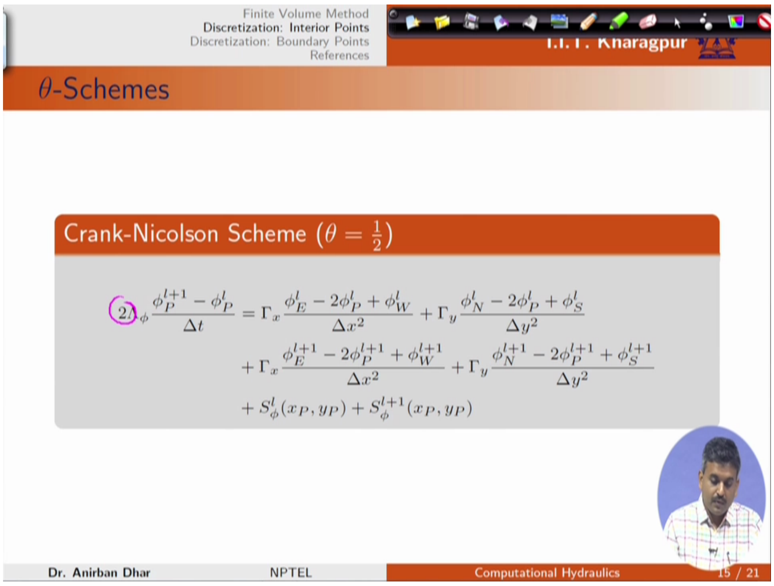

Now up to this discretization IVBP is somewhat similar to finite difference discretization. Now we need to see what will be the boundary conditions. Left boundary again, we need to evaluate this west face derivative separately which is not common. And right boundary also, we need to evaluate the derivative at east boundary separately.

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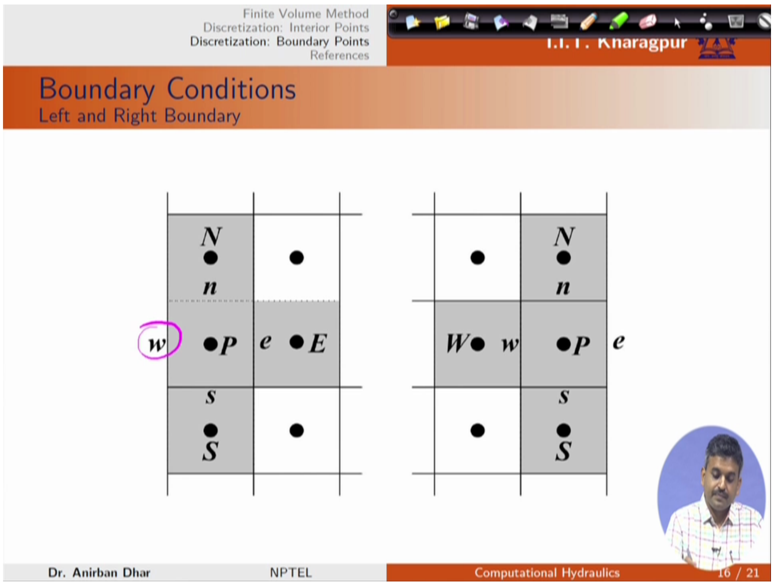

For north also, this north boundary derivative of phi with respect to y, that will be zero. Similarly for south boundary or bottom boundary, at the south face derivative of phi with respect to y, that will be zero.

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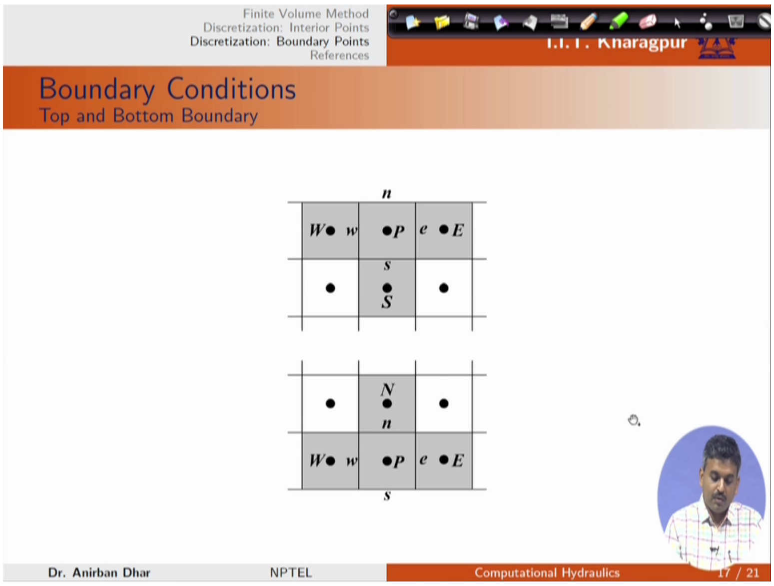

Corners, west side we need to evaluate the derivative separately. North side the derivative will be zero. This north-east again east face derivative, north face derivative is zero.

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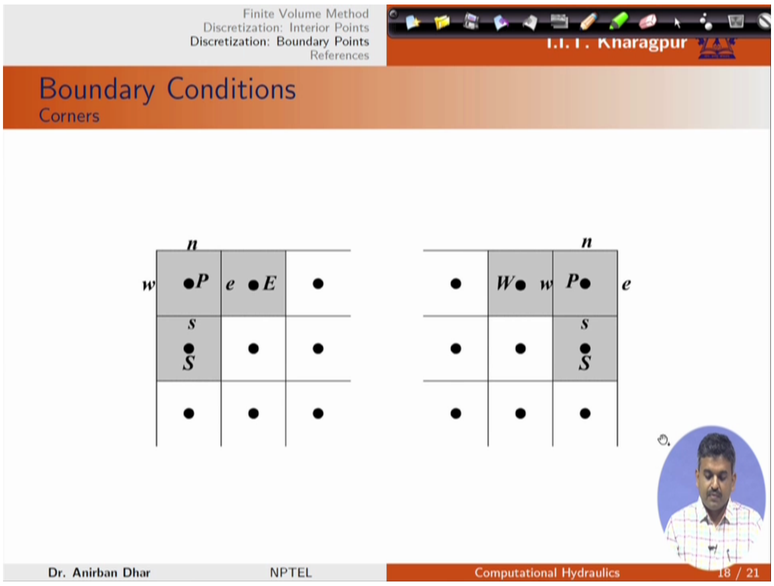

South-west and south-east, similarly we can impose the corner boundary conditions. So with this discretization which are similar to our boundary value problem, we can get the discretize form of all the cell centered points. And we can solve the final matrix and get the value for all cells centered values for all cell centered positions. If you have Dirichlet boundary, your boundary values are specified. But for Neumann boundary we can utilize the nodes or cell centers next to our boundary to get information about the value at the boundary itself.

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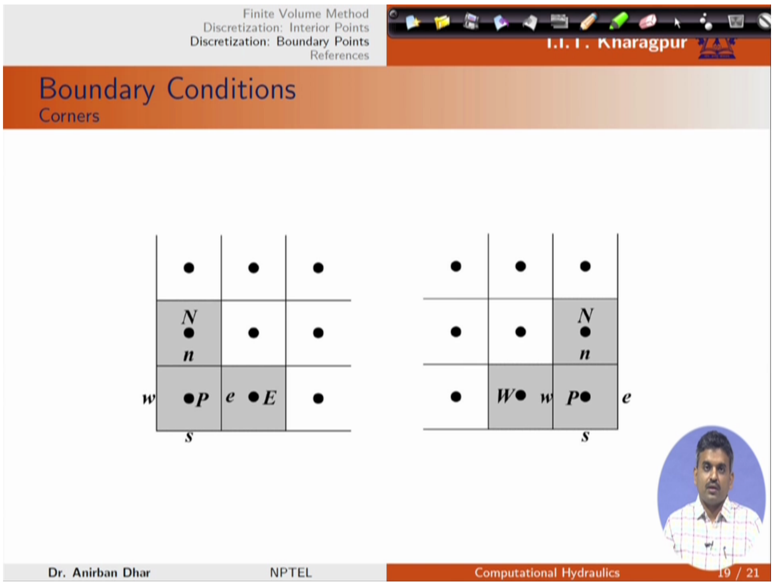

Thank you.