Computational Hydraulics
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Lecture 14
Finite Volume Method - BVP

Welcome to this course, computational hydraulics.We are in lecture 14. In this particular lecture I will be covering unit 10 of module number 2, finite volume method, boundary value problem.

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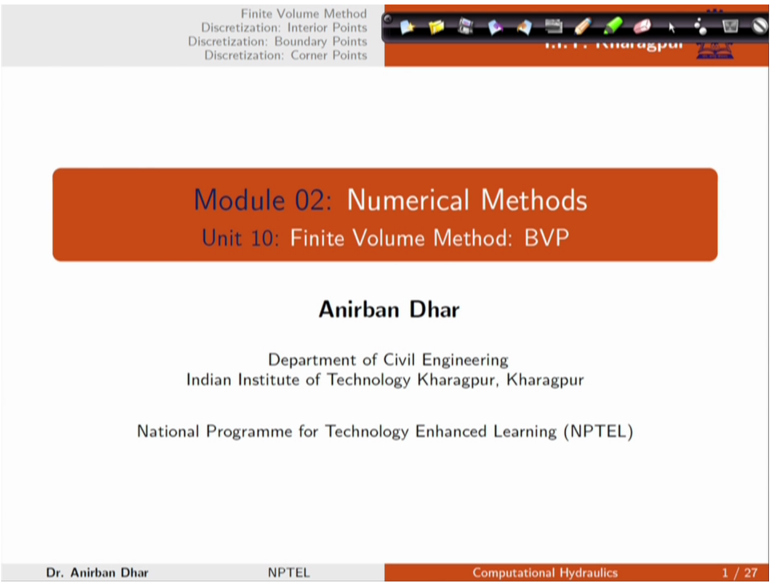

What is the learning objective for this particular unit?At the end of this unit students will be able to derive algebraic form for a boundary value problem using finite value method.

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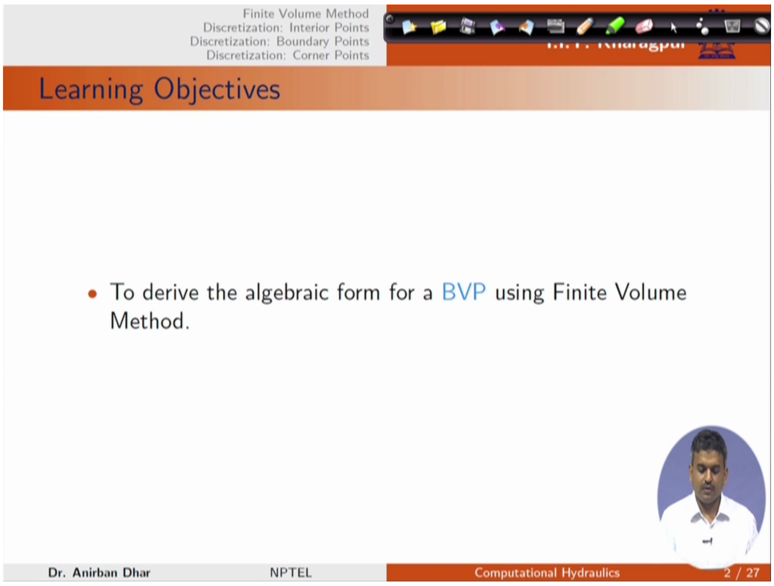

Let us consider our general equation. In finite difference method we have used the first term on the right hand side and the source term to define the boundary value problem using partial differential equations and boundary conditions.

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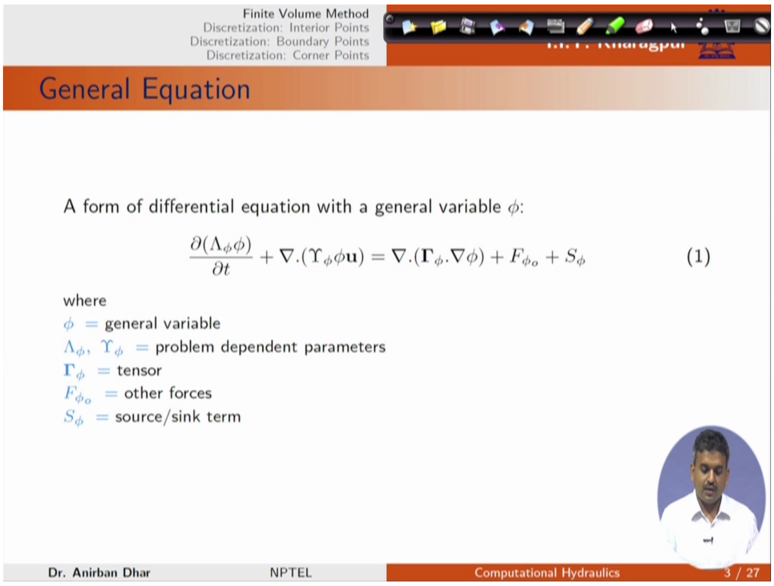

So let us use that for our case, problem definition. In this case this is our diffusion term where this gamma, this is nothing but gamma x 0, 0 gamma y. Now this form is suited for our finite volume method because this is written in (ja) divergence form. And inside one is a vector. So with boundary conditions on the left and right side as Dirichlet, bottom and top side as Neumann or zero Neumann, we can start this problem.

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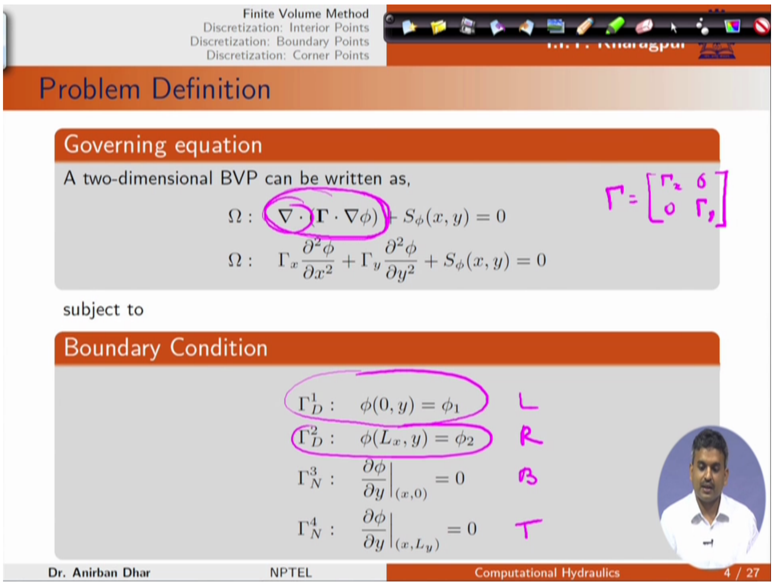

Domain discretization is somewhat different from our finite difference approach. Finite difference case, we have considered these as nodes. But in this case we will consider cell centers for discretization of the domain.

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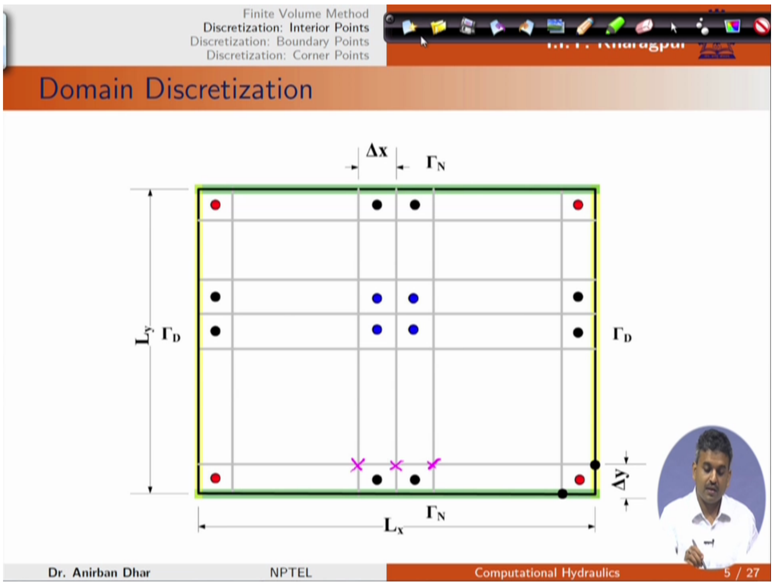

We can divide it into number of cells and cell size in X direction and Y direction, we can represent it with del x and del y. This is Lx, Ly for X and Y directions. These two are Dirichlet and top and bottom ones are Neumann boundary condition.

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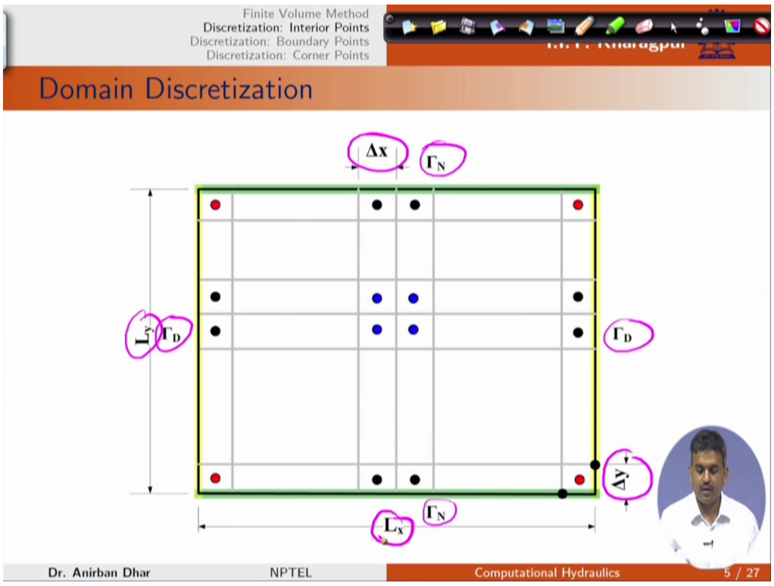

We need to have spatial treatment for corner points in this case. Because for corner cells we have one side Dirichlet, other side Neumann condition. In this case Dirichlet, Neumann condition.

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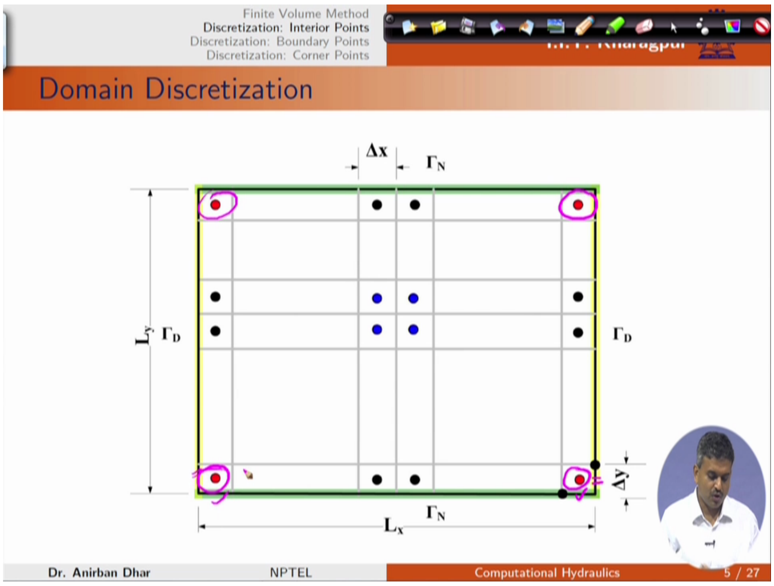

So let us consider the discretization of the governing equation for interior nodes. So for interior nodes there will be interior node p, there will be 1 north cell, 1 south cell, 1 west cell and east cell. With east, west, north, south these faces.

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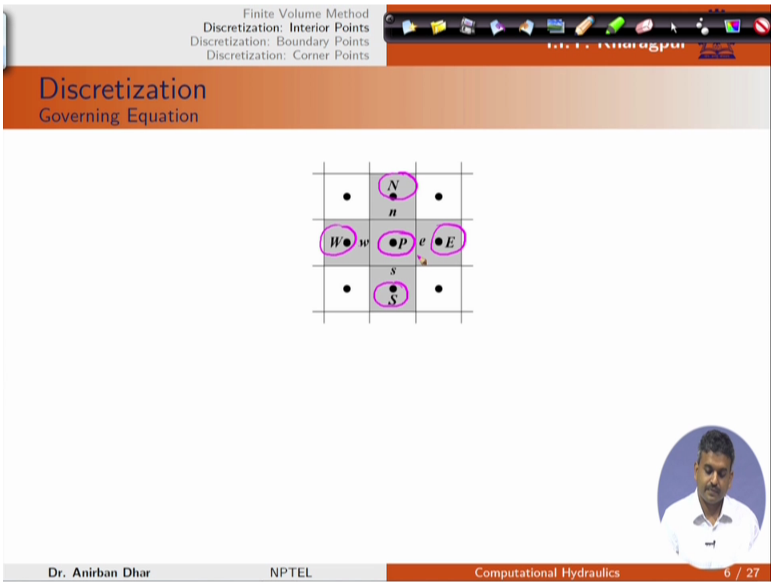

Now in finite volume method governing equation is integrated over elementvolume to form the discretized equation at node point p which is cell centered. With this one we can start and we can use our Gauss divergence theorem on this particular problem.

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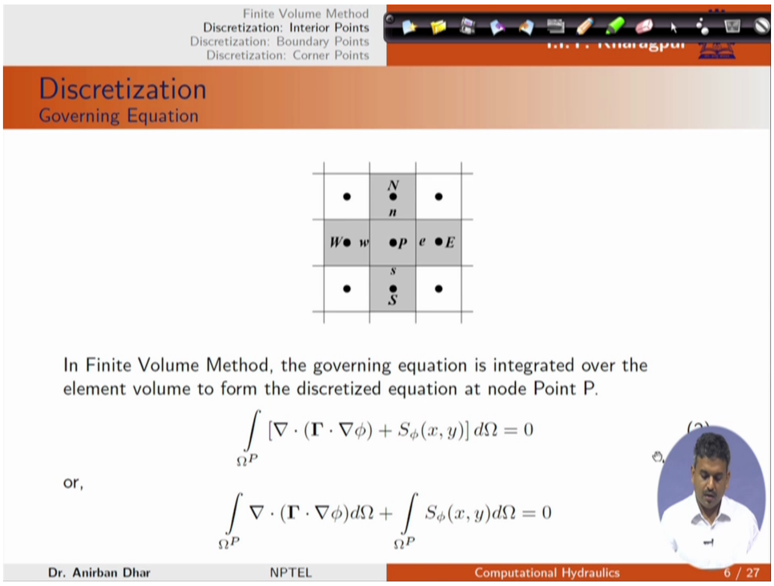

If we use our Gauss divergence theorem, this is our volume integral. We can write this volume integral as gamma x d phi by d x i, gamma y d phi by dy j. Ij are unit vectors in X and Y directions. This is corresponding to X this is corresponding to Y. So inside one is a vector. This is written in terms of dot products. So now with the surface integral we can reduce it and this n is outward normal. We can further simplify it and write it in terms of face values or face values in this case. So howthis step is coming from this integral equation.

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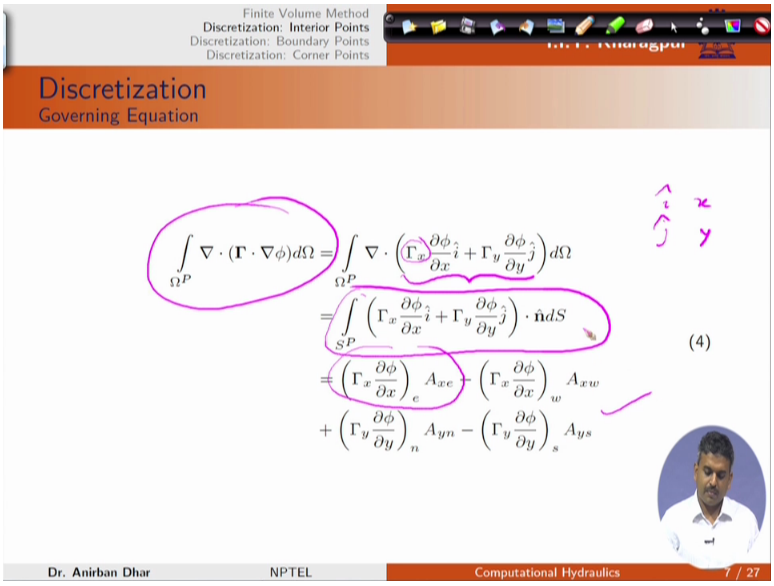

Let us consider that we have our p cell and east face. Now east face, this is normal to Y direction and this is same as X direction. So we can write the area in terms of i vector. So area in this case is del y and vertical perpendicular to this direction, we can consider unit area or the unit distance.

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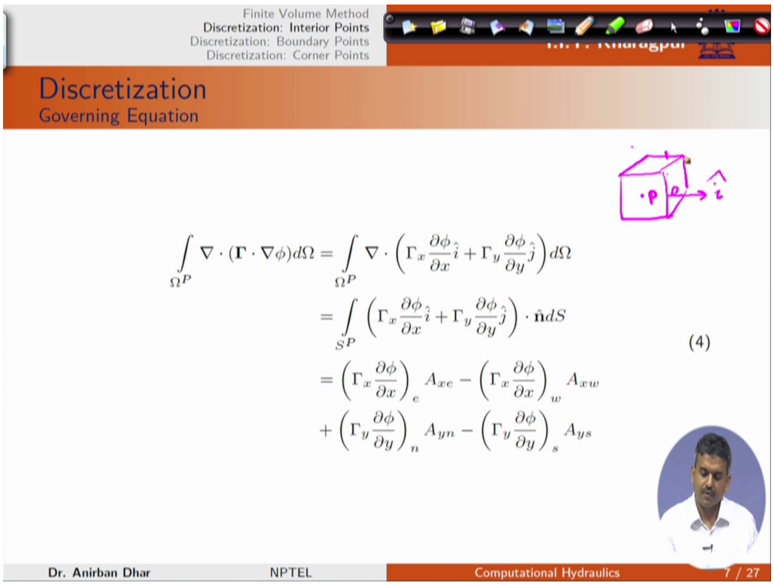

So with this we can write it as del y into 1. This is the magnitude of the area and direction of the area is i.For east face, this is the area

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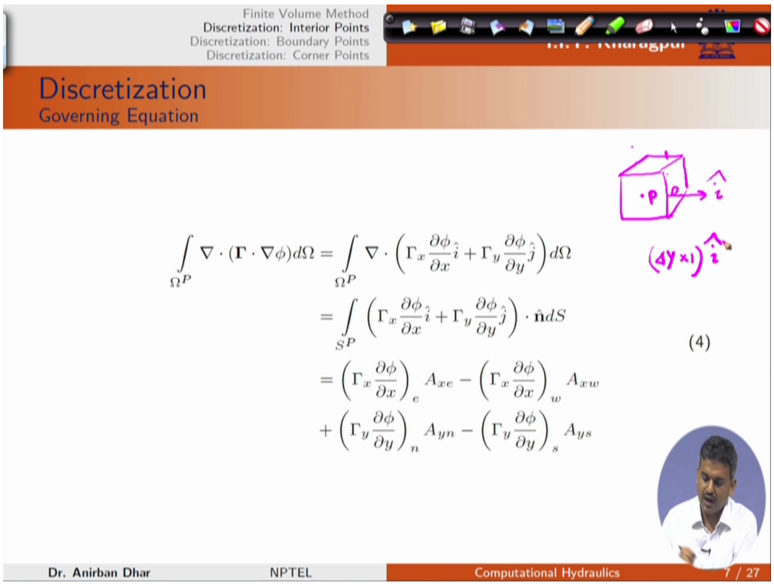

And for vector we have two components, i and j. We need to integrate it over face. That means all faces east, north, west, south. So if we write it as submission then we will be exactly solving this integral equation.

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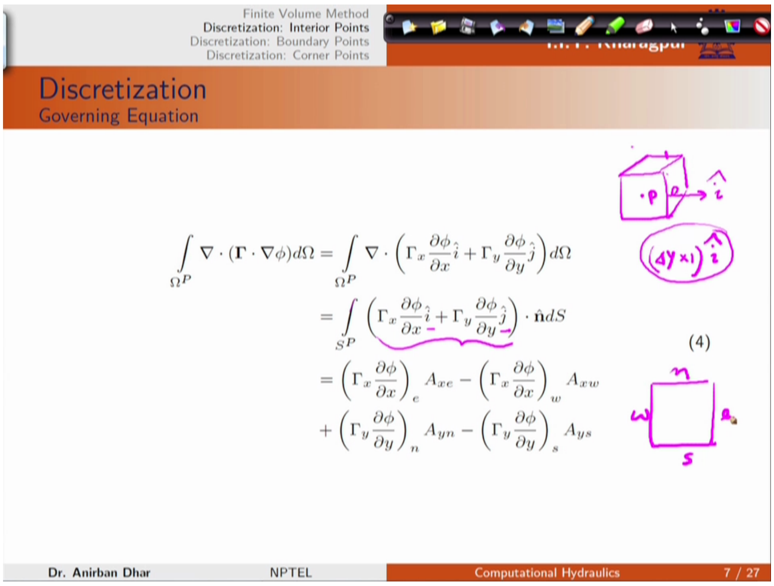

So for east face we have del phi by del x, i plus y del phi by del y j, dot A xe. A xe is the area on ith direction. So this is for face e. Now if we take dot product for this particular face then we will get gamma x del phi by del x. This is for east face and A xe multiplied. A xe is nothing but del y or magnitude of the area. Similarly this is for east direction. If we consider west direction, the outward normal is pointing towards negativeX direction. So we have used negative sign here.

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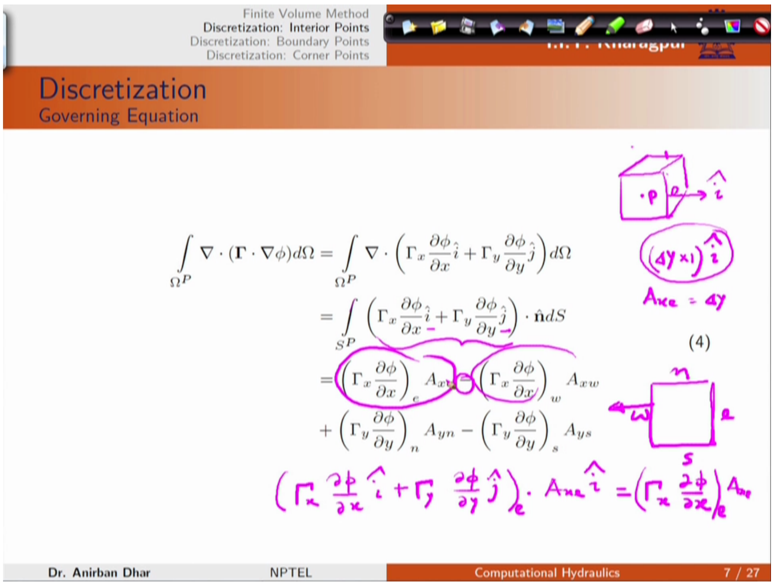

And similarly we are getting only X component. For north direction we will get only Y component and south direction also with j component or y del component, but with a negative sign.

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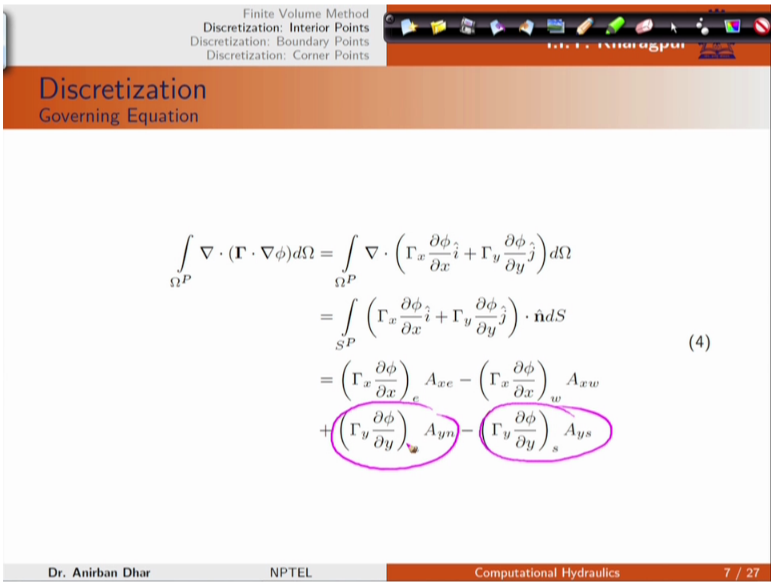

In uniform grid system we can write this east face derivative as gamma xe and gamma evaluated at e face, phie minus phi p divided by del x. West face, this is phi p minus phi wdivided by del x. North face, this is our phi n minus phi p divided by del y. And South face we have this gamma ys phi p minus phi s divided by del y.

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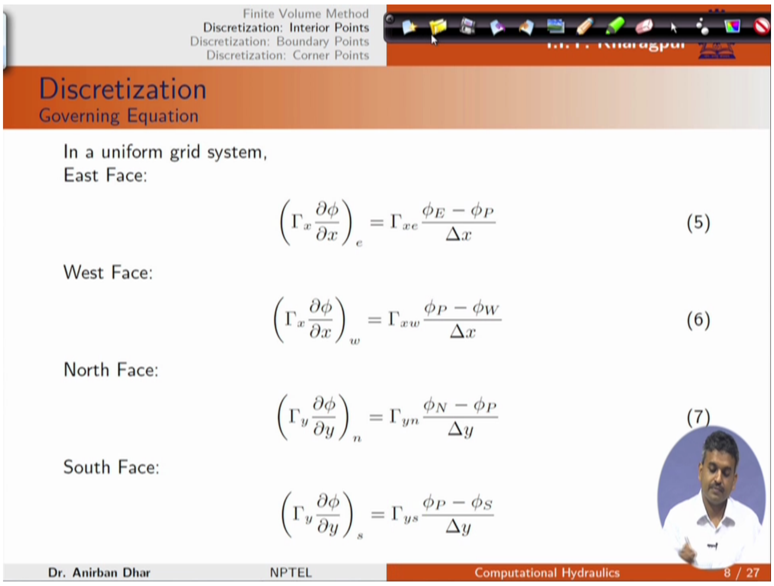

We can consider the uniform grid system where for X and Y directions we have fixed value for intervals del x and del y.

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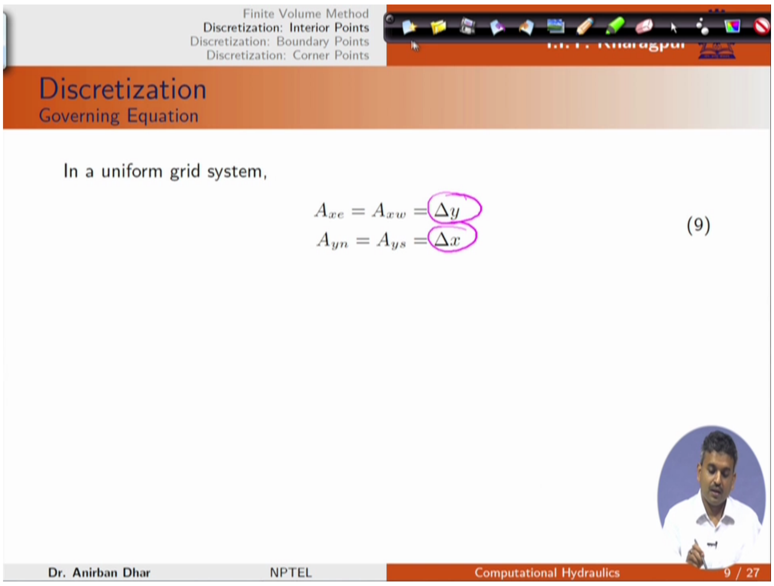

Now we can utilize this for our case and for source term we can approximate it withour approximated value at the center point into volume. Del x into del y into 1 is the volume of this particular thing. So this is approximation of the source term. Now we can write this compact form of the equation considering all derivatives at different faces. And we can write thiswith the source termeast face, west face, north face and south face.

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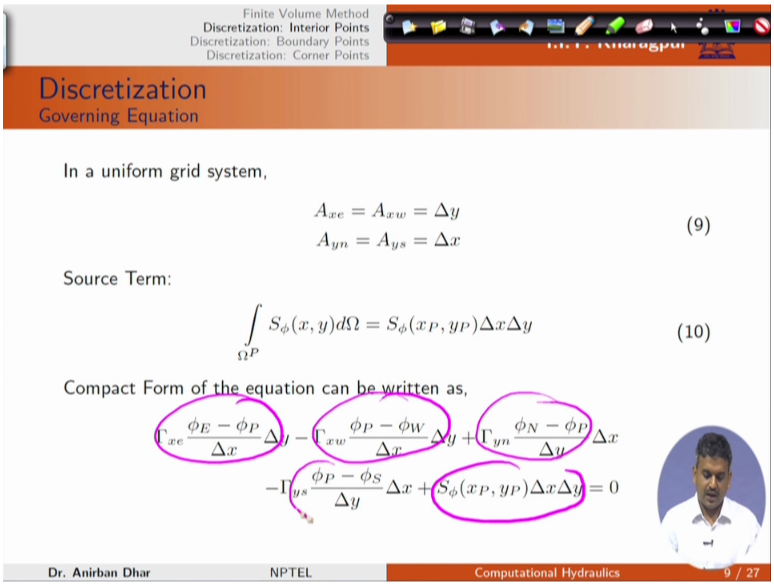

With consideration that east face value west face value, this is gamma x and yn, ys, y s, this equal to gamma y. And xe minus xw, this is del x.

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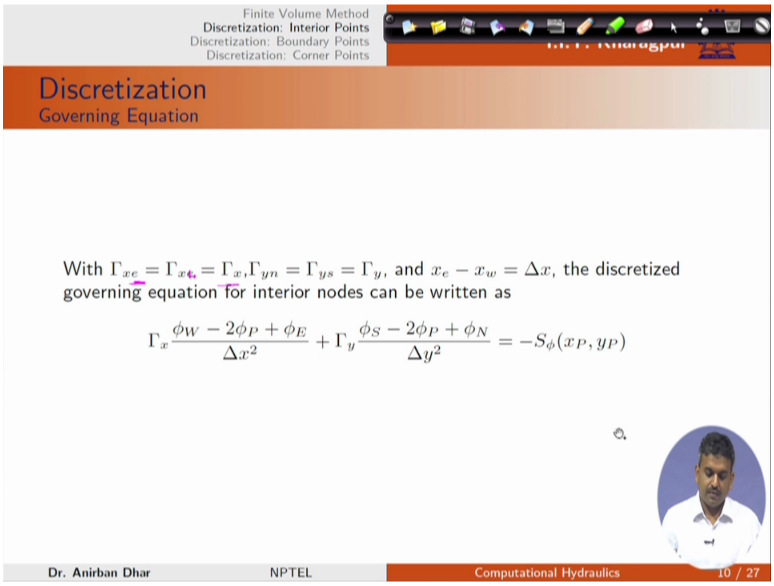

We can approximate this and we can write our governing equation simply. if we can compare this thing with ouroriginal finite difference equation, then we can see that this is nothing but, gamma x into phi i minus 1j minus 2 phi ij plus phi i plus 1j divided bydel x square, plus gamma y phi ij minus 1, 2 phi ij plus phi ij plus 1 divided by del y square equals to minus phi xi y or ij evaluated at ij. So in this casewe can see that there is equivalence in the finite difference and finite volume discretization.

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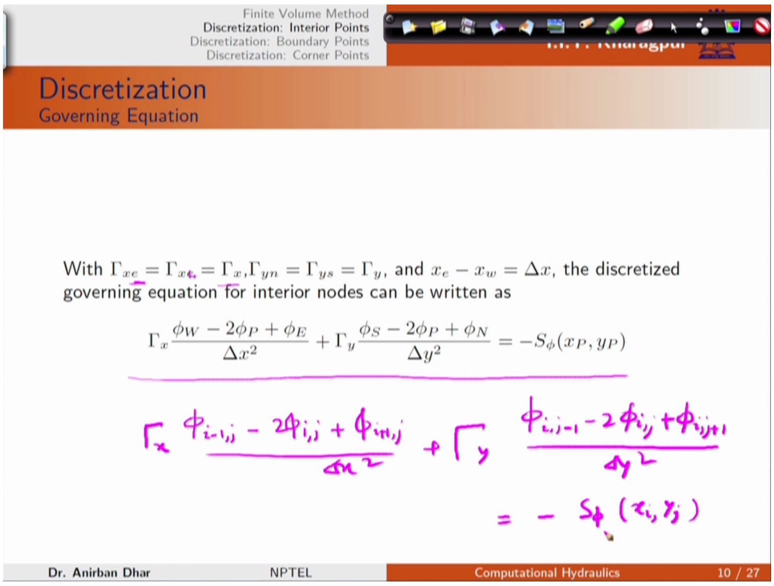

Now we need to consider the left boundary. Left boundary main point is that at west face we have specified value. So if we consider our original equation, we need to change this derivative. Otherwise we have north-southeast cells available. So we can use our usual derivatives like our interior points.

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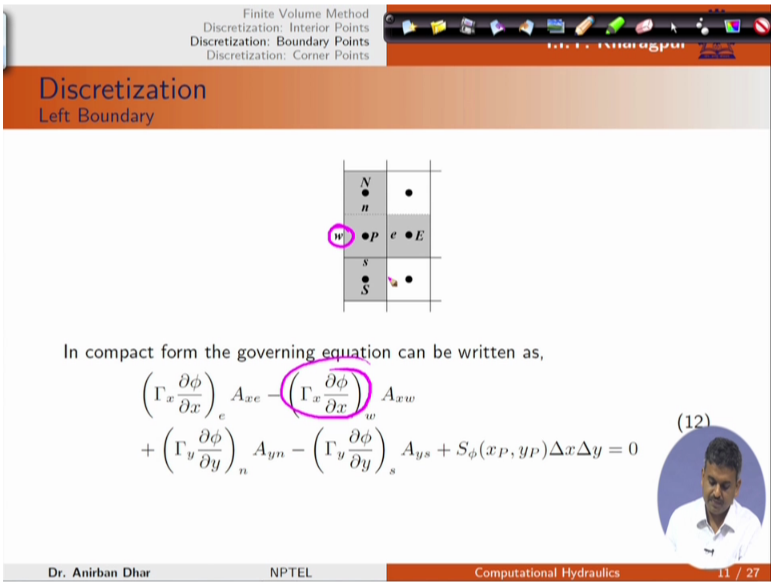

Now for west face we have half distance available. So this is phi p minus phi wdivided by del x by 2.

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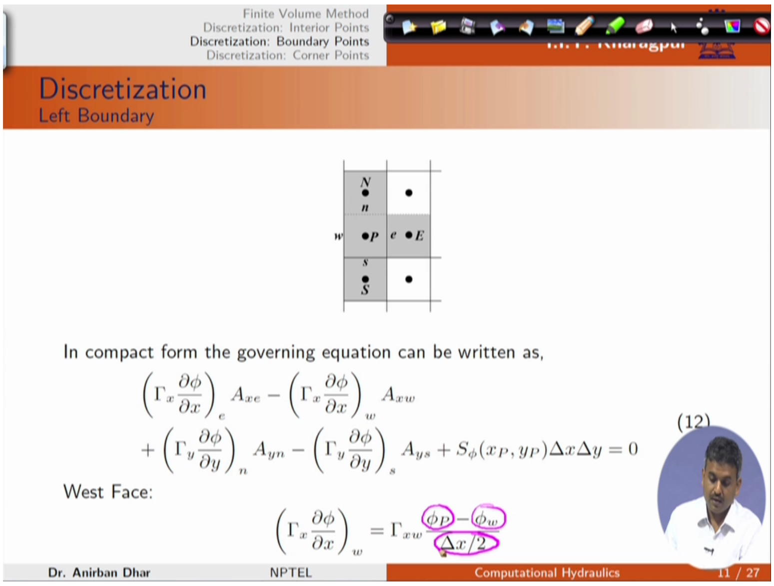

If we utilize this value in compact form.If we approximate this, this is the change. If we use this change value, we need to transfer this phi w into right hand side. Because this is known.

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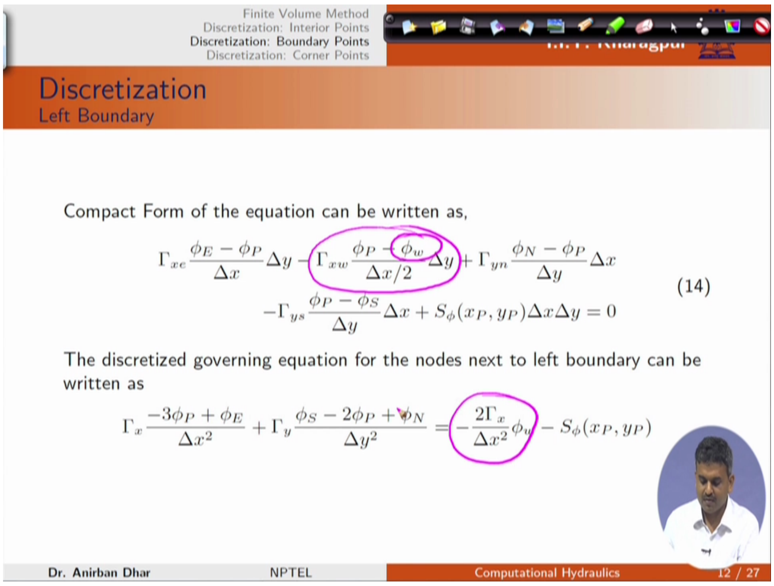

So this is the final form of the equation for left boundary.

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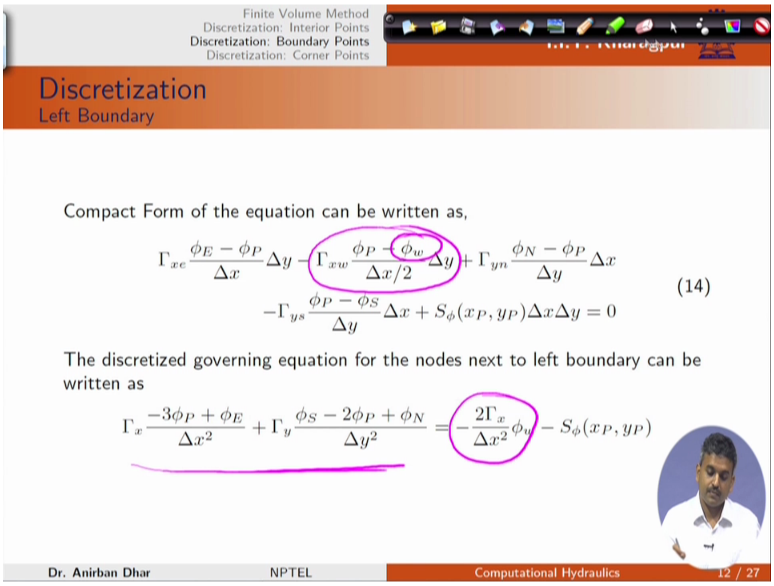

Now for right boundary east face, we have known value. So derivative that we need to change is this east face derivative. So east face derivative, this phi e minus phi p, this is del x by 2.

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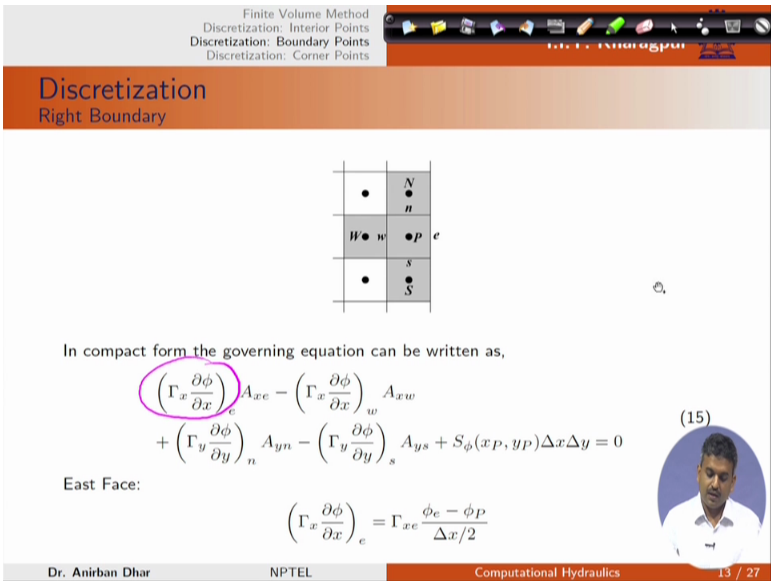

Now we can use thisto get our compact form of the equation. Only change is here.

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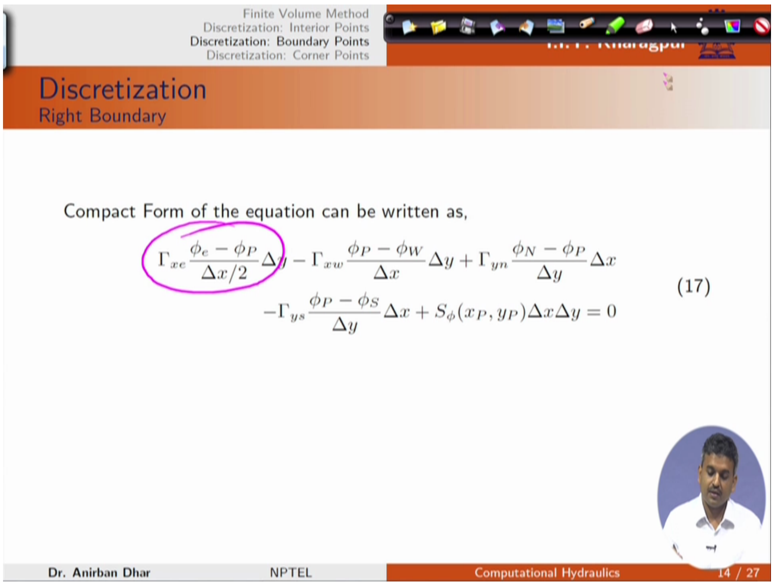

Now again we can get our discretized governing equation for nodes next to right boundary, like this. In this case phi e is known. So that's why it is in the right hand side.

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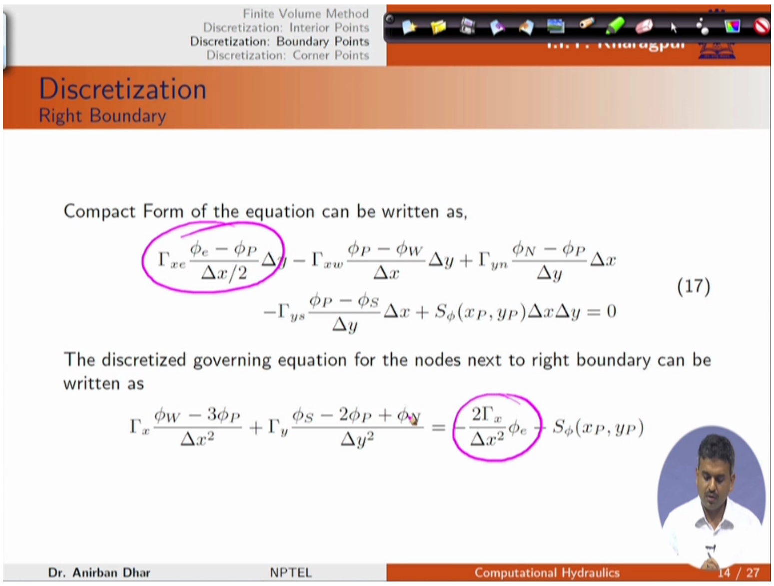

Bottom boundary, we need to change or we need to write this one as zero.Because south face, we do not have any value.

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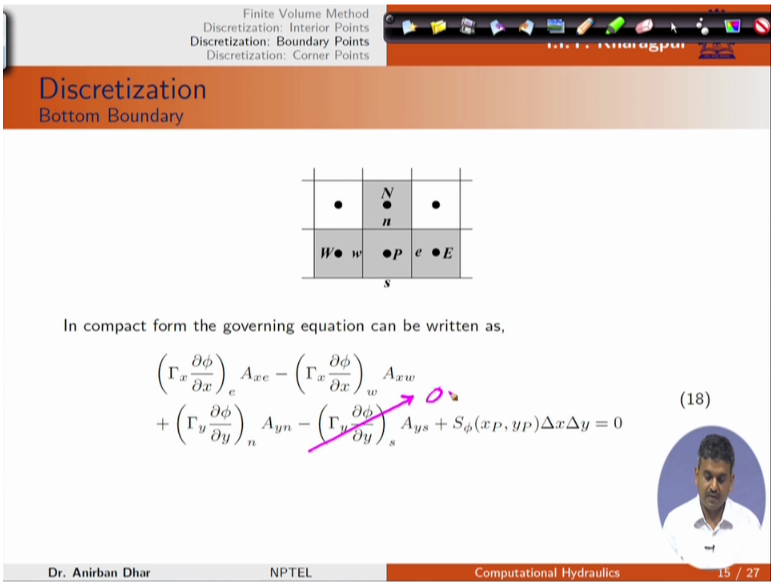

So without that derivative we can write our governing equation and only change is here. There is no phi s term.

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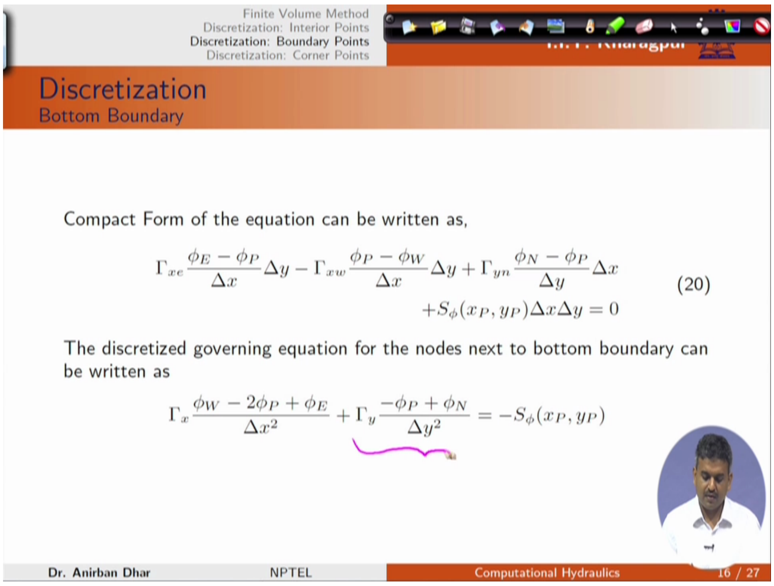

If we consider our top boundary, we need to eliminatethe derivative at the north boundary because that is Neumann boundary condition. Sothis is equals to zero. With this we can get the compact form.

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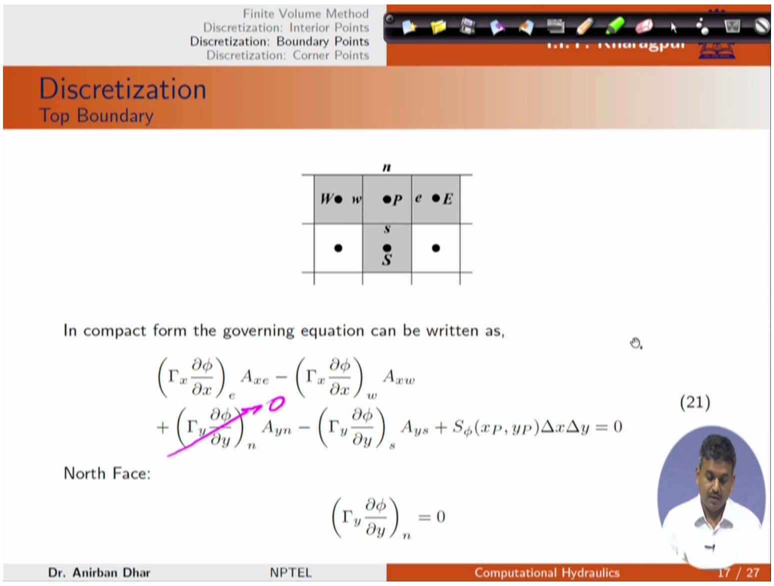

And in compact form we do not have the derivative at north face. So this is the reduced equation for nodes next to top boundary.

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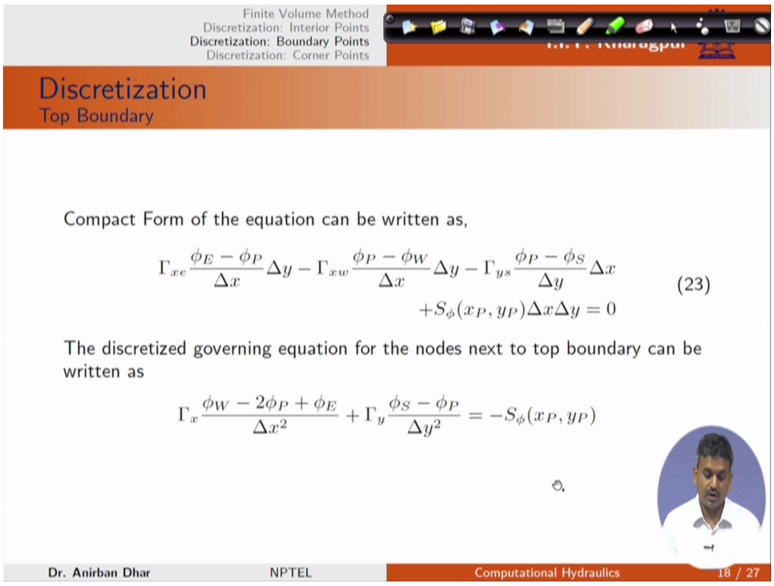

Now we need spatial treatments for corners. Let us consider the north-east corner.So north boundary is zero Neumann. East boundary is specified Dirichlet condition. So we need to change this derivative and north boundary we need toforce this one as zero.

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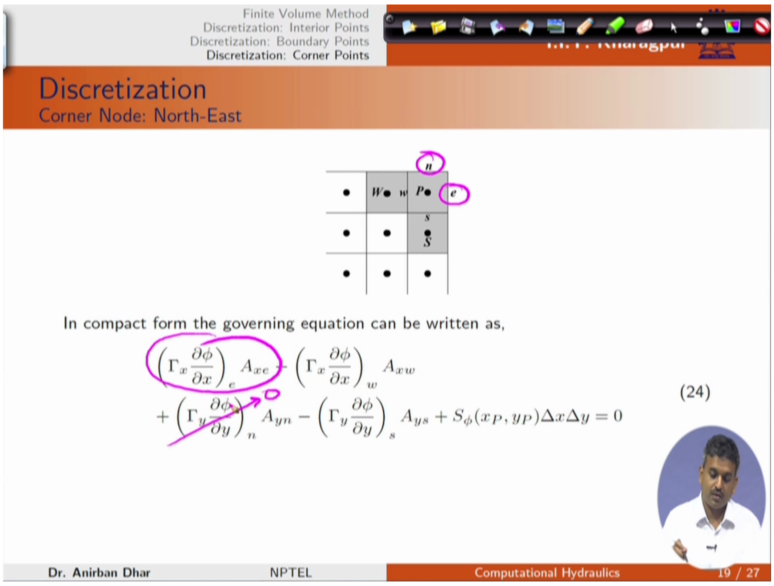

So with this we can write this one. Similarly, with the compact form we can get the final form of equation. Phi e is known value. So we have transferred it on the right hand side.

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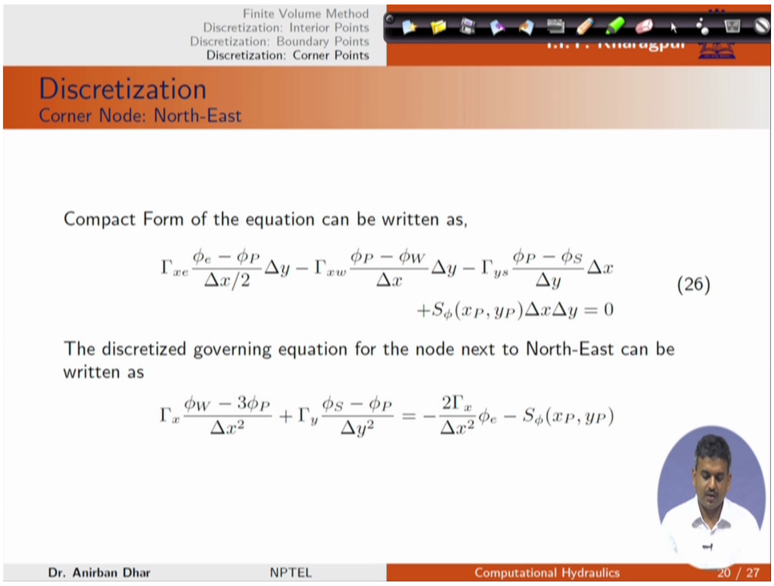

North-west corner, north boundary is zero Neumann w boundary, this is specified.

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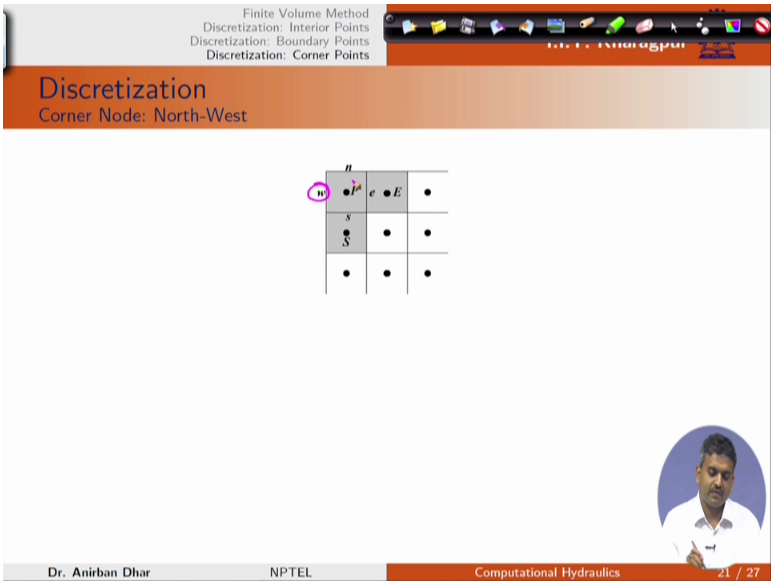

In this case we havenot derivative. This is zero. This phi w,needs to be changed. This one needs spatial treatment.

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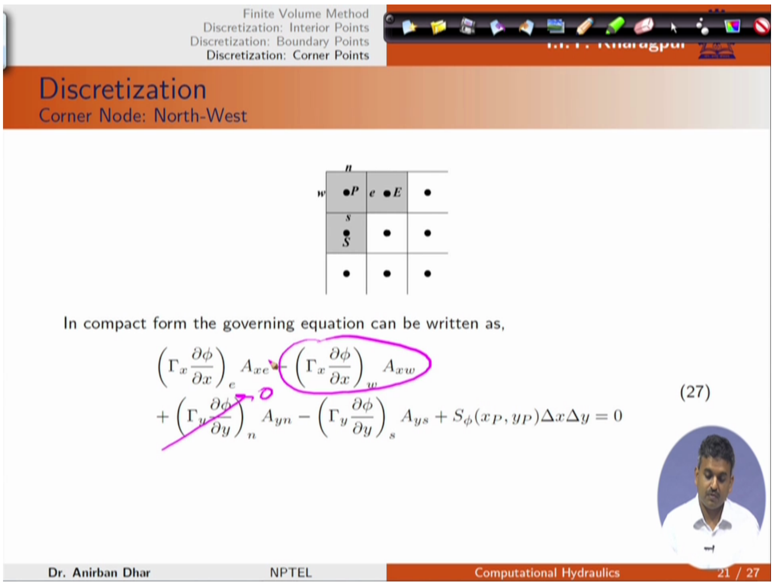

So with thiswe can change the derivative and we can get the final form of the equation. As phi w is known, so we need to write that phi w value related term on the right hand side.

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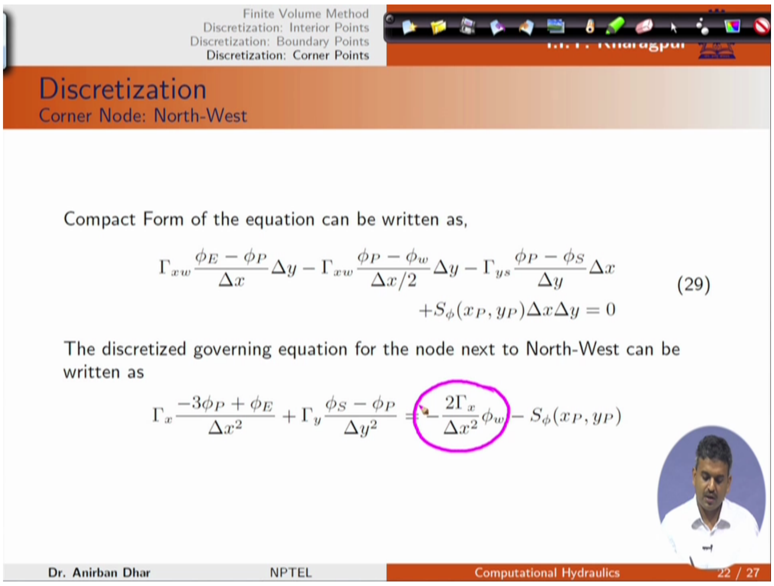

South-west south-east corner, again we have specified this one as Neumann condition.

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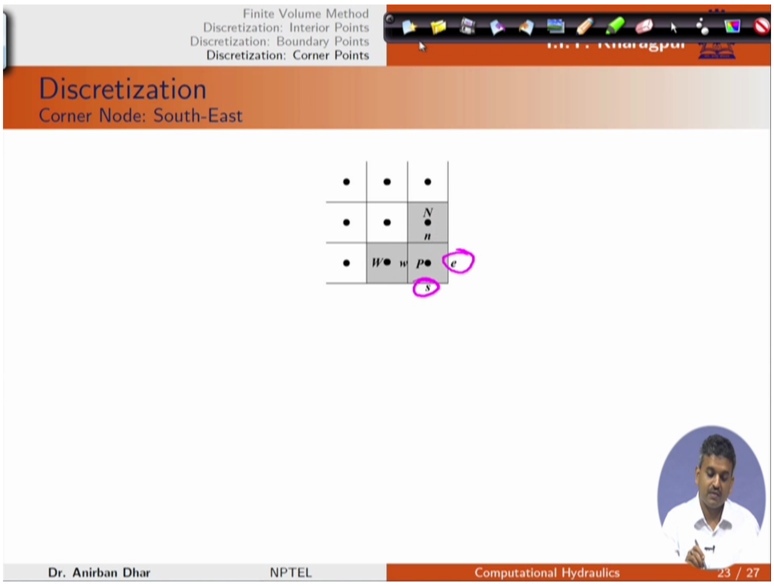

So zero value for derivative at south face.For east face we have derivative available.

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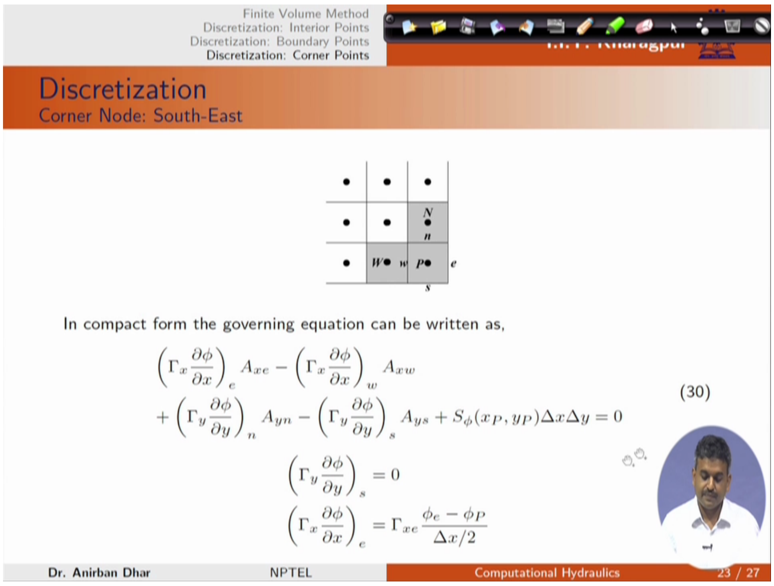

So in this case we need to utilize our south-east governing equation. We will have our different discretized governing equation for south-east corner.We can see that in this casethe east value is on the right hand side. The equation that is on the left hand side is, phi w phi p, phi w phi n.

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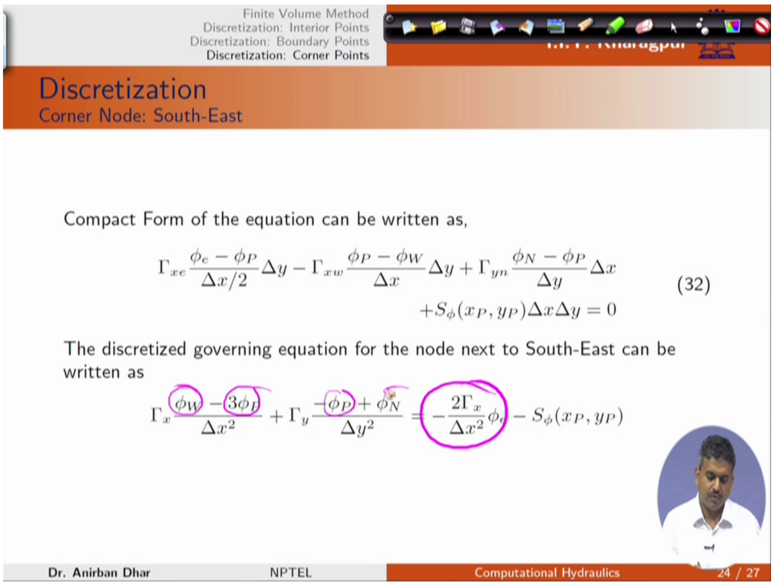

Similarly south-west we can write our equations In this case south face is force to zero.

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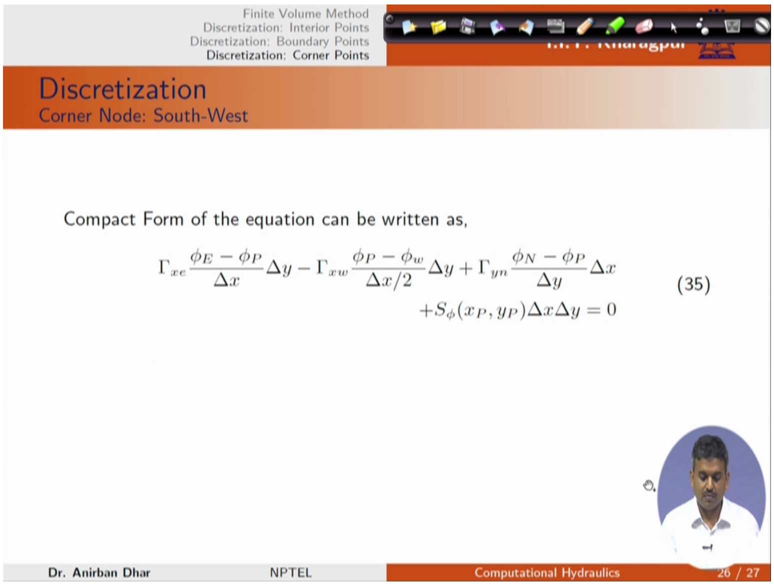

And we have only north face available.And west face value we can transfer on the right hand side. With this discretization we can get the interior point values and values near to our boundaries. But we cannot get the exactly boundary level values for Neumann boundary condition. In case of Dirichlet boundary condition, at boundary values are defined. But for Neumann boundary we need to use the internal values to get the value at the boundary.

Like our previousderivation, which was for one-dimensional case. And we have utilized derivative involving or utilized derivative involving three points. First one is thenode which is at the boundary. Next one is the cell center which is adjacent. And next to next point is again cell center at some interior cell. So we can get the values at the boundaries of the Neumann condition usingthe internal node values.

So in this casewe have represented our governing equation in terms of phi p, e, w, n, s. We are utilizing singleindex for this one. And whatever concept we have utilized for our finite difference case, we can use those conversion techniques to single index for this problem also.

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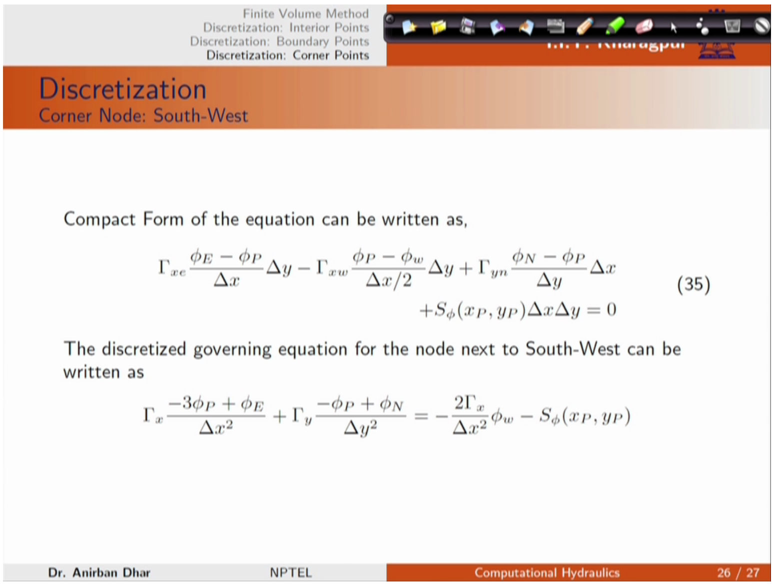

And we can solve our ultimate governing equation. Thank you.