INDIAN INSTITUTE
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NPTEL
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Probability Methods in Civil Engineering

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Lecture – 06

Topic

Concept and Definition of Random Variables

Hello and welcome to the course probability methods in civil engineering. Today, we are starting a new module, module 3.

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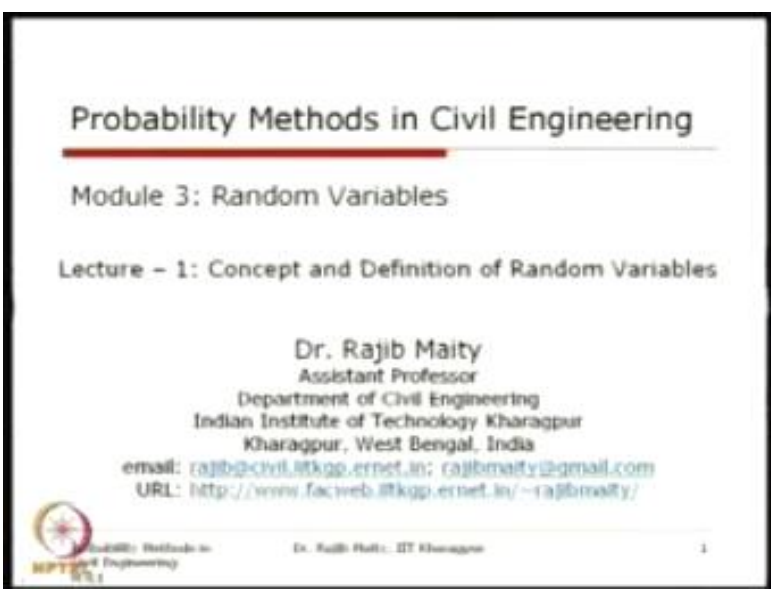

In this module, we learn about random variables and there will be couple of lectures and in this first lecture, we learn the concept of random variables and their definitions.

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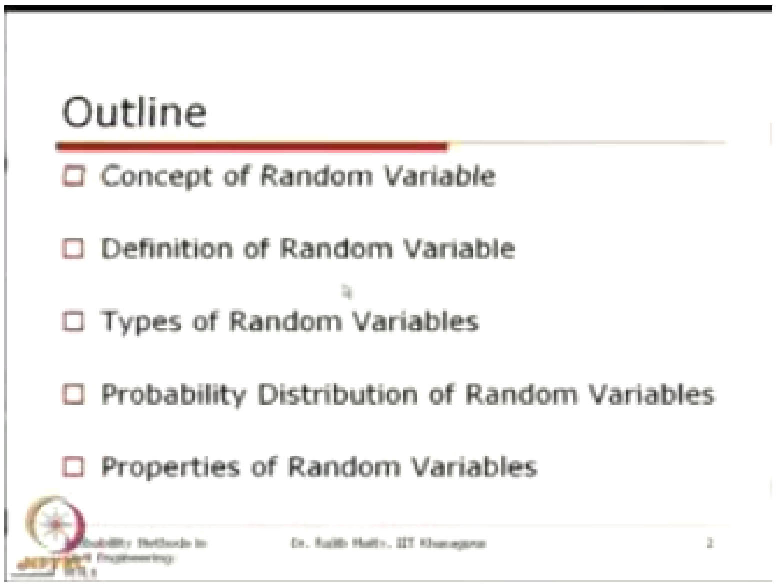

So our outline for today's lecture is arranged broadly like this. First we will discuss about concept of random variable and then, we will understand the definition of random variable. Then, we will discuss different types of random variables and their probability distributions. This random variables are important in the sense, that this is the, this concept is important to understand how it is used in the probability theory and this probability theory for this random variable is generally assessed through their distributions.

So which is known as the probability distribution of random variables and after that, we will learn different properties of random variables.

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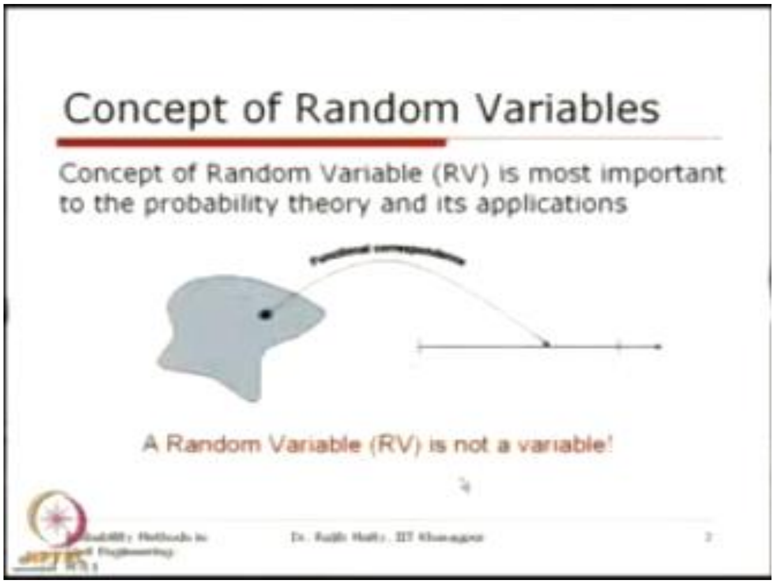

So as just what we are telling, that this concept of this random variable, which is abbreviated as RV, is most important to the probability theory and its application. Now if we see that, how what actually this random variable mean, if this shaded area what you see here is the is the sample space, then this sample space consist of all feasible outcome of one experiment. Now this feasible outcome of this experiment can be continuous or can be discrete points from through this sample space.

Now from the sample space we have to, for the mathematical analysis, we have to map the outcome of the experiment, of one experiment to the, to some number according to our convenience. Now to map this each and every output of a random experiment to this real line, is generally through a functional correspondence and this correspondence, this functional correspondence is one random variable.

So what is important and what is, what we want to understand from this slide is that, this random variable even though this variable is shown here in the name of the random variable is not a variable. So which is generally, what you want to say is a functional correspondence or is a function, that function generally map between the sample space of one experiment to the real line. Now this mapping is done based on our convenience and that we will see in a minute.

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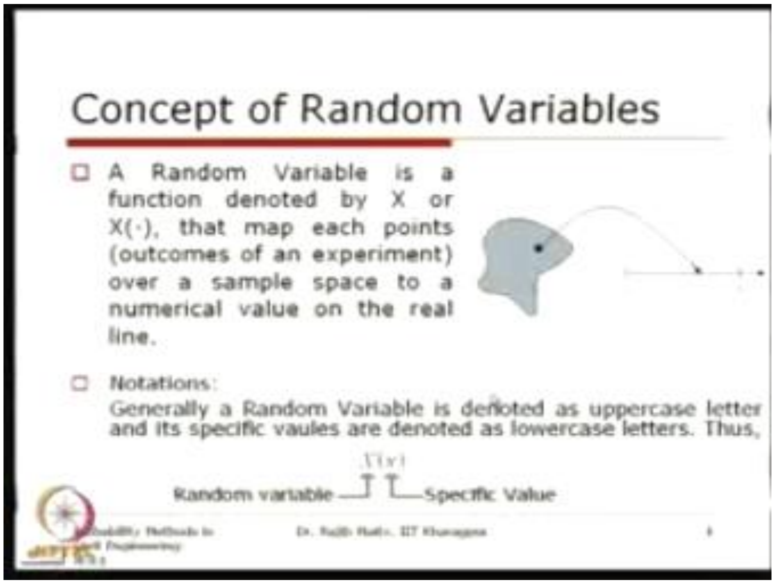

Thus, if we say the definition if we say, then we will say that a random variable is a function denoted by x or x some specific number, it can be denoted by any letter generally, these things are denoted by this small lower case of this random variable. These random variables are generally expressed in terms of the uppercase letters. So this random variable x is a function that map each points, that is a outcome of an experiment over a sample space to a numerical value on the real line.

What just we have shown in this figure in the last slide. Now before we discuss some of the further concept of this random variable, so there are different notations are being used to denote this random variable and the specific value. For this lecture or you will see in some of the classical text book, that this notation will be followed for this for this course. Generally, this random variable are denoted by the uppercase letters and its specific values are denoted as lowercase letters.

Thus in this notation, this uppercase letter is indicating the random variable which is denoted by this x and any specific value to this random variable is denoted by this small x, which is denoted, which is shown here. So this is the specific value of the random variable x.

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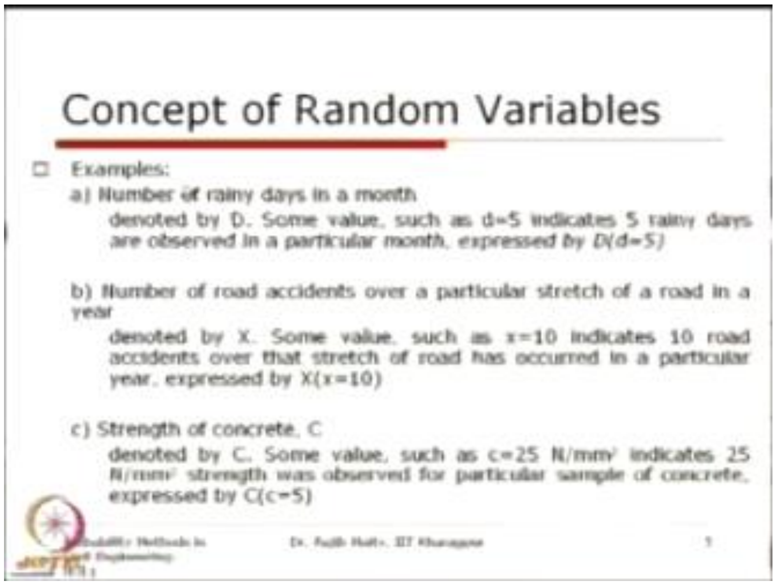

Now we will see some example of this random variable which we just discussed. Suppose that number of rainy days in a month, so, if we, you can say that this is a random variable and we do not know. So this value that number of rainy days in a month can vary and if we denote this random variable as letter D and some value say D=5, so this indicates that 5 rainy days are observed in a particular month.

Now which is, which can expressed by D in parenthesis D=5. Second example, say the number of road accident over a particular stretch of the road in a year. Suppose, this is denoted by x and some specific values, such as say x = 10 indicates that there are 10 road accidents over that stretch of the road has occurred in a particular year. Now this value 10 or this value 5 can change in the successive time step or in the success for in this case in the successive year that can change.

So this number of road accidents is my random variable, which for a particular year it takes a specific value x equals to 10, which is denoted by this. The last one say the strength of concrete if I designate is at C, so this is denoted by C and some specific value such as C=25 Newton/milli meter square indicates that 25 Newton/milli meter square strength was observed for a particular sample of concrete and which is expressed by C in parenthesis C=5, C is sorry this shall be C=25 sorry for this mistake this should be 25.

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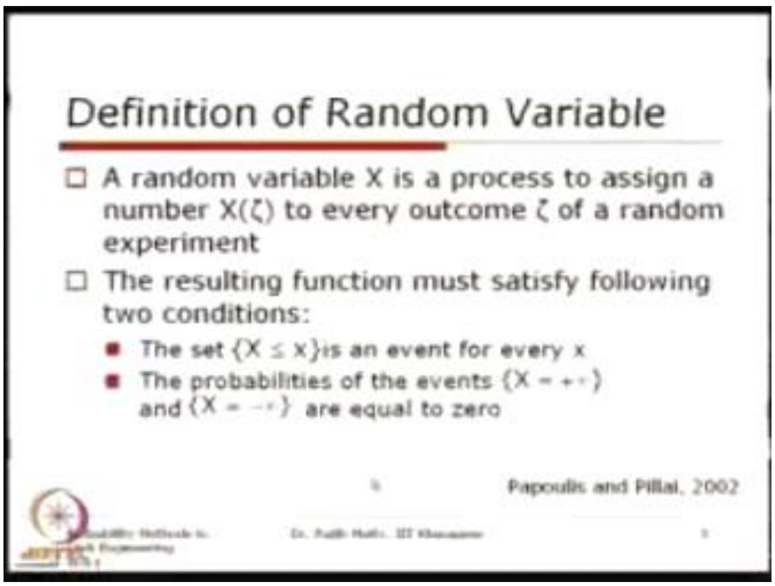

So a little bit formal definition of random variable if we see, that a random variable x is a process to assign a number, which is a real number x ζ to every outcome ζ of a random experiment. So if we just take one, take that thing, so suppose that we are taking.

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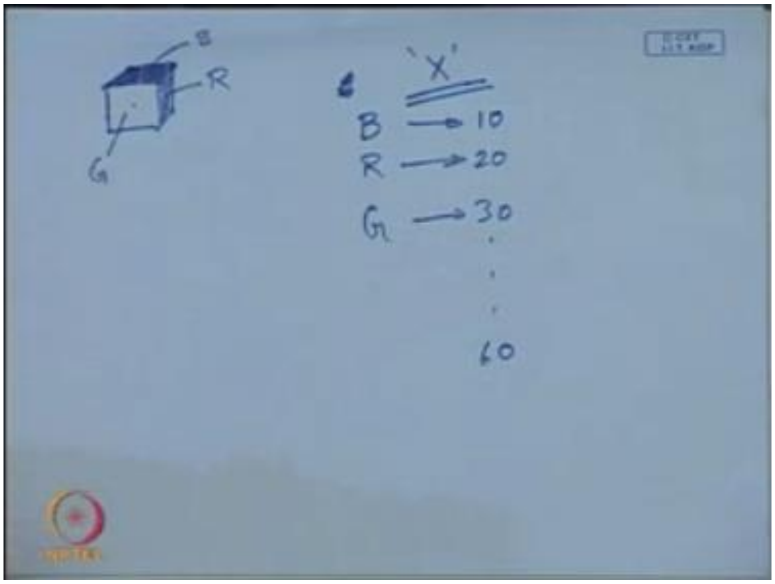

That example of that of tossing a tossing a die then we know that whatever the numbers that generally comes, if it comes 6 on the toss then we say that the outcome that is coming, it is my 6. So similarly 6 different surfaces can have 6 different numbers and that can be happened. Now, see that in this experiment if I just change this one so this is quite straightforward that whatever number of dots that we are usually used to know. Suppose that instead of having this kind of dice if I just have a have a dice which is having different color suppose that this is having a blue color this side is having may say red color this side is having green color and similarly 6 different faces having 6 different colors.

Then, what I am trying to do is that this outcome of this is a top surface the color of the top surface if I just want to say that this is my outcome of this of this experiment then what I can do is that I can take different color code and I can assign to some number none necessarily 1 to 6, it can be of any number. Say that B I am giving some number 10 R giving some number 20 green giving some number 30 and this way say up to 60 this number is given so these mapping from this outcome to some number this mapping is your random variable.

So we will come after this how to assign the probabilities now this is very straight forward so each and every outcome if this dice is fair then the outcome of each and every of this outcome will be equal so everything will be 1 / 6. So, that is again another process how we will assign the probability to that particular random variable if it is x, so that will be how we assign the probability now when we are defining this random variable itself so this random variable is nothing but is a process to assign a number to each outcome of that random experiment. This is exactly what we are calling it as a formal definition of.

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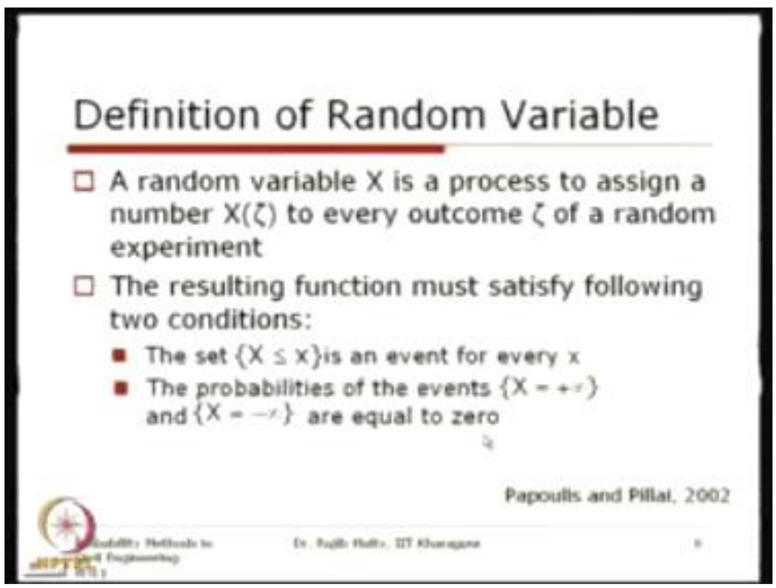

This random variable is that A random variable X is a process to assign a number X (ζ) to every outcome ζ of a random experiment. The resulting function must satisfy the following two conditions so even though we are saying that this is a function so this function of mapping from this outcome experimental outcome to some numbers should follow two such two conditions. The first condition is the set X ≤ x. Now, you see here this is the capital letter which is your random variable and this is the specific value.

Now this random variable below some specific value is an event for every x. So, whatever the x you take so this should constitute and event for that random variable. For that for this X, now other things of probabilities of the events, that X= + ∞ and X= -∞ = 0.Why this things we are just saying is that even though means mathematically this X can take any value on the real line but still we say that so the extreme that is+∞ and -∞ this probabilities if I take this probabilities this would be should be equals to 0.

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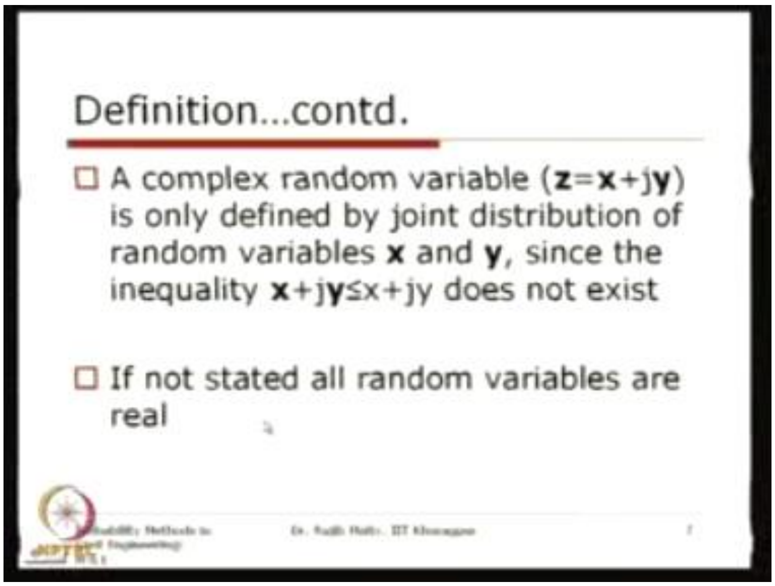

A complex random variable so just we will to complete this definition part if we do not say this one it will not be completed so this is there is another type this is the real random variable which just we discussed is now it can be the complex random variable as well which you say z= x+jy. So this random variable is only defined by their joint distribution so if I want to define the complex random variable z this should be defined by their joint distribution this joint distribution will be discussed later in this in this in this course.

So but for the time you can say that this joint distribution is generally obtained from that distribution of this x and y both so this both this distribution should be taken care to find the joint distribution to define the distribution of the complex random variable of this of the random variable of individual x and y. Since this inequality because so this kind of inequality say this are the random variable and this is the less than equal to that number does not exist. But for this course if it is not stated specifically all the random variables are real we should follow this norm.

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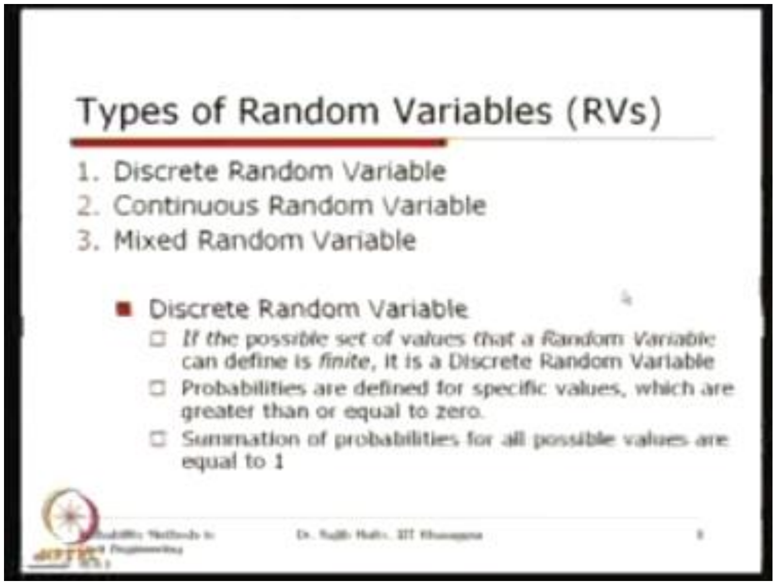

Now with this concept we will just see different types of random variables there are basically two types of random variables one is the discrete random variable and other one is continuous random variable but there is there are some times in particularly in the civil engineering application we have seen there are some distributions which can be treated as the mixed random variable as well.

So we will discuss this different types of random variable one after another now. The first we take this discrete random variable if the possible set of values that a random variable can define is finite it is a discrete random variable now so just to tell this thing here that.

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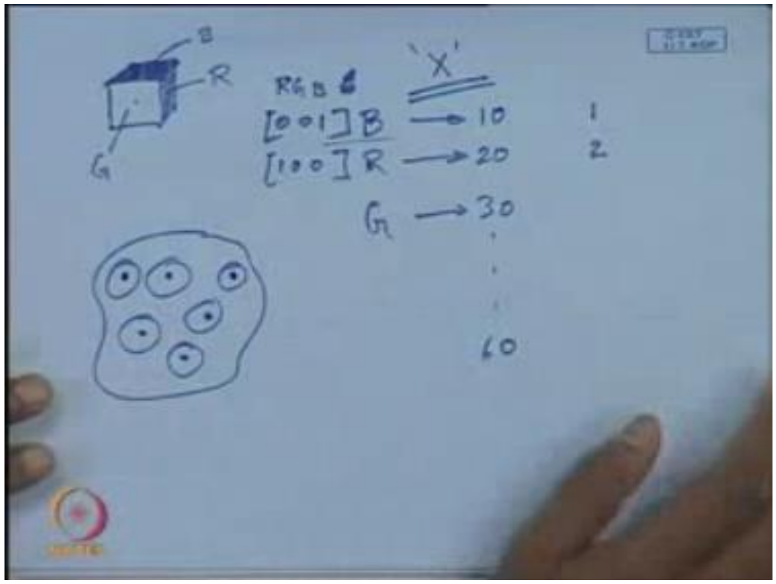

Suppose that this is my this is my sample space now if I say that these random variable is defined for some specific value and rest of the part it is not defined so it takes some specific some specific value so those values only it can it can define so in that case what we say that this is this is a discrete case so in the in the previous example if we just see here that it can define that some specific outcome of this of this random variable it cannot it is not that continuous that a range cannot be cannot be defined here.

So, if it is if it is dots then we can say that it is defined for the outcome 1 defined for the outcome 2. But it is not defined for the outcome anything any number in between this two. Similarly if we just see from this remote sensing concept the color code then we can say that it is defined for the blue for this colored surface it is defined for the blue it is defined for the red but in between mixed I think you know this color code.

This stands for this blue if it is the color are RGB are the primary colors and this 100 is for the red. Now you can just change this number to some other to get some in between color so that is not defined there so what we want the say for this discrete is that these random variables are described only for some specific outcome of this experiment. It is not defined for this all. So, those random variables whose outcome who can map who can define the outcome of the of the random experiment for some specific values.

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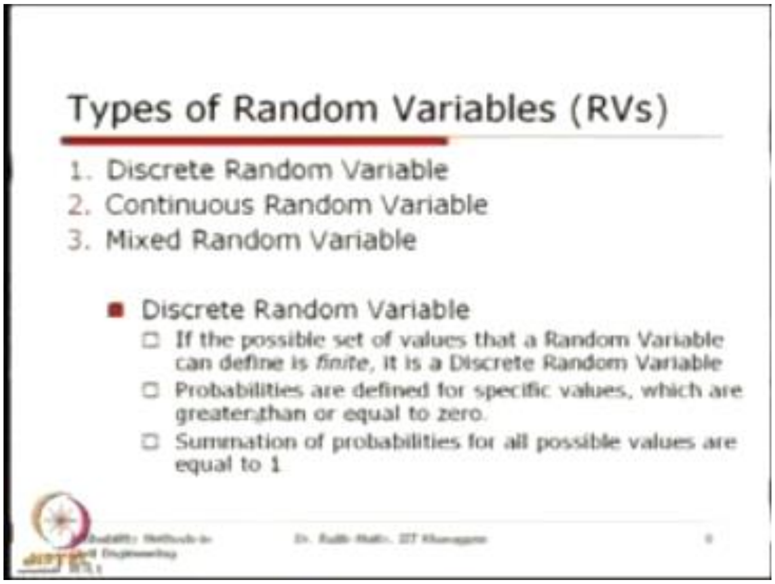

Those are known as the discrete random variable thus if the possible set of values that a random variable can define is finite it is discrete random variable the probabilities are defined for the specific values ≥ 0 so this we will see in detail which are ≥ 0 but what we are trying to stress here is that the probabilities are defined for the specific value. So it is just like that for that point this probabilities are defined for the adjoining point this is not defined.

Summation of all the all the probabilities all the possible values are equals to 1, these two things are basically coming from the actions of the probability theory that we discussed in the previous classes. The next type is continuous random variable.

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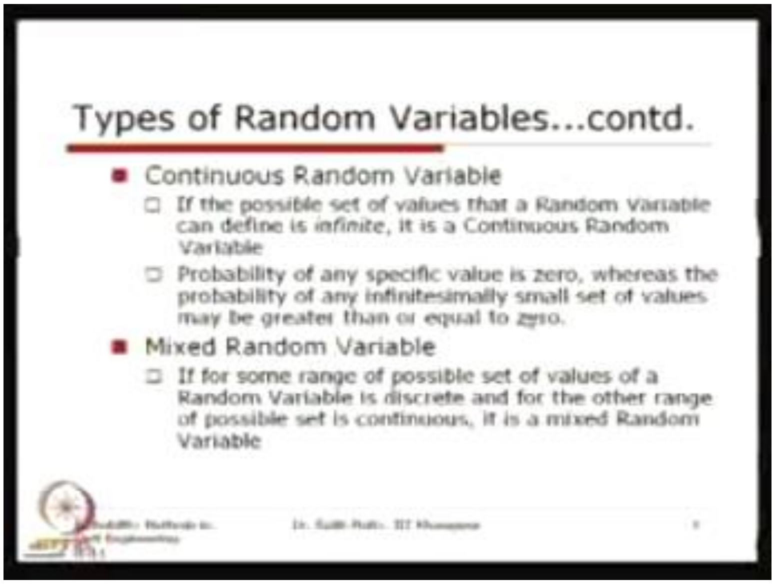

So on the other end in contrast to this to this discrete random variable if the possible set of the values that a random variable can define is infinite it is continuous random variable. So over a zone it is over a zone over a range this random variable is if it is also defined then this is known as the continuous random variable. The probability of any specific value is 0.Now this is coming from this density point of view.

Now once we are saying that this probability of any specific value if I just take then this specific value probability is 0.This is again coming from the actions of this probability. But even though we say that it is exactly specific value it is 0 but whatever small set of the values if we think that can be greater than or equal to 0.So whatever the small range if we consider then it will be 0. I think this will be more clear from this pictorial view here.

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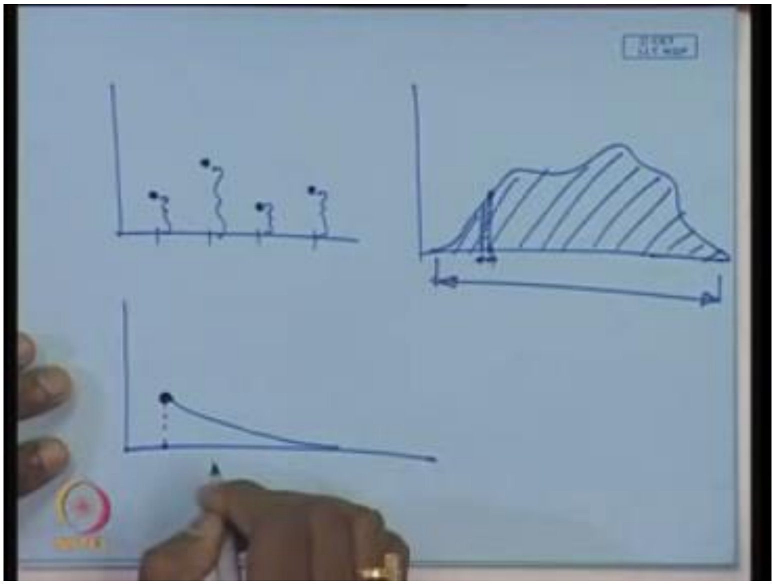

So suppose what we are trying to say that if there are some specific outcome and those probabilities are assigned like this whatever may be the probabilities if it is not these are like the concentrated mass for that specific outcome. So this is this is your discrete random variable. Now for this continuous random variable what is defined is that it looks like this, so over this zone from this to this zone.

For any specific value this is nothing but the density of this zone, now following the actions of this probability if we just adapt with this axis is your probability so if I add up all this values to what we just discussed, this will be equal to 1. But in this case when we are talking about this is one continuous distribution then what happens for a specific point the probability is 0.

Because this is basically a density. Now whatever the small area that we will consider then this area is giving you the probability for this small zone. Now for the full zone if I just take then the total probability that is total this is your feasible range. So the total probability in this feasible range should be equals to 1 that comes from the actions of this probability theory. Now again here the next type what comes is the mixed distribution.

So mixed distribution sometimes what happens for specific value it is defined some probability some probabilities concentrated here as a mass and for the rest of some range it can be it can be continuous and goes up to some level or up to the end of this or up to the infinity. So now this for this particular point this probability mass is concentrated here and for the rest of the region it is having some continuous things. In such cases we say that these kind of random variables are mixed random variable.

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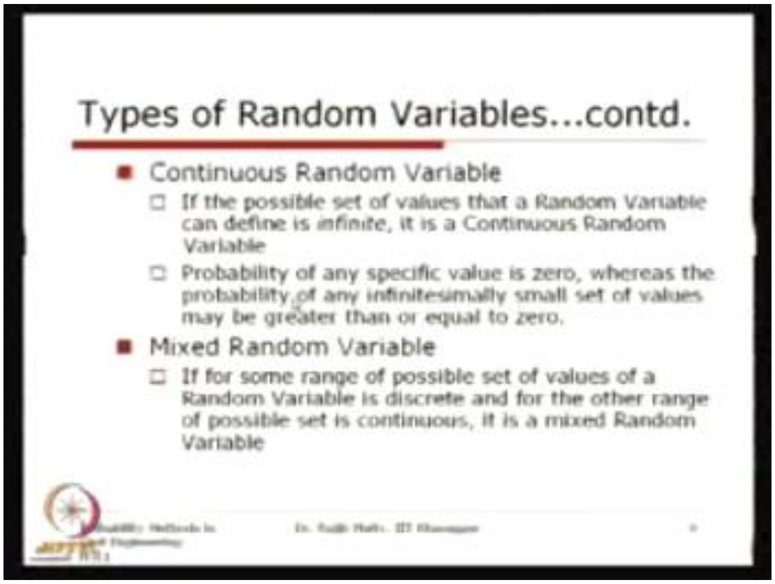

So this is the third thing, what we are seen that. So mixed random variable says if for some range of possible set of the values of a random variable is discrete and for the other range of possible et it is continuous it is a mixed random variable. So now with these three type of random variables some example in the civil engineering context we will see now.

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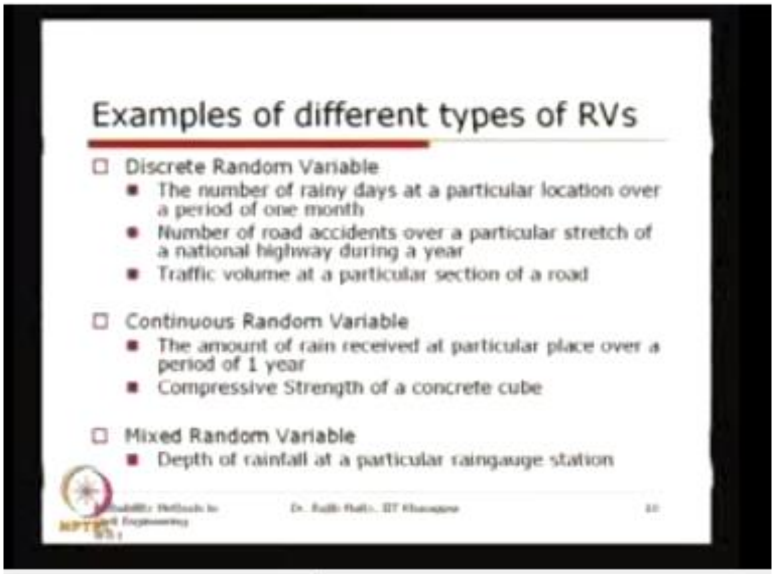

The first is that we discuss is the discrete random variable say that this number of rainy days. Now number of rainy days at a particular location over a period of one month say so it can take either the specially the integer value either 1 day or 2 days 3 days in this way up to 30 days or 31 days whatever may be the case. But it can never take any value in between two integers. So as this so as this can take only some specific values.

So this is an example of discrete random variable. Second is a number of road accidents over a particular stretch of a national highway during a year. So again these road accidents can be either 1, 2, 3, 4 in this way up to infinity but it can never take any value in between two integers. So this also one example of discrete random variable. That is the traffic volume at a particular location of a road.

So this is the number of vehicle that is crossing that particular section of the road again this can take only the integer values for that one.

So this is also on example of the discrete random variable. Now the example of continuous random variable on the other end that amount of rain received at particular place over a period of one year. So annual rainfall for that session for the say it is the number of rainy days which is a discrete. Now if I just say that total depth of the rain received as a particular point that can vary from 0 to any possible any possible number.

So this is one example of the continuous random variable. Compressive strength of a concrete cube, now there are different ways there are different criteria maintaining different design criteria if we prepare the concrete cube and then we test what is this compressive strength that can actually take any value between certain range. So this one also one example of the continuous random variable.

Now coming to the mixed random variable so here say that depth of rainfall at a particular rain gauge station. So this depth of rainfall at a particular rain gauge station which is gives this one that is as a continuous random variable here. Now if you see that there if there are station where most of the days if it is the 0 rainfall for the station, then how the how it will take. So for a 0 suppose that out of 365 days in a year.

Suppose that there are 220 days are 0 rainfall. So there is a probability mass that is concentrated at a particular value of depth of rainfall 0a and for the non zero value it can have some distribution, so that distribution should have shown the first part at particular value that x=0 that is the depth equal to 0 some mass is concentrated there some probability mass is concentrated there.

And rest of the probability is distributed over the positive side of the axis. So this is an example of the mixed random variable.

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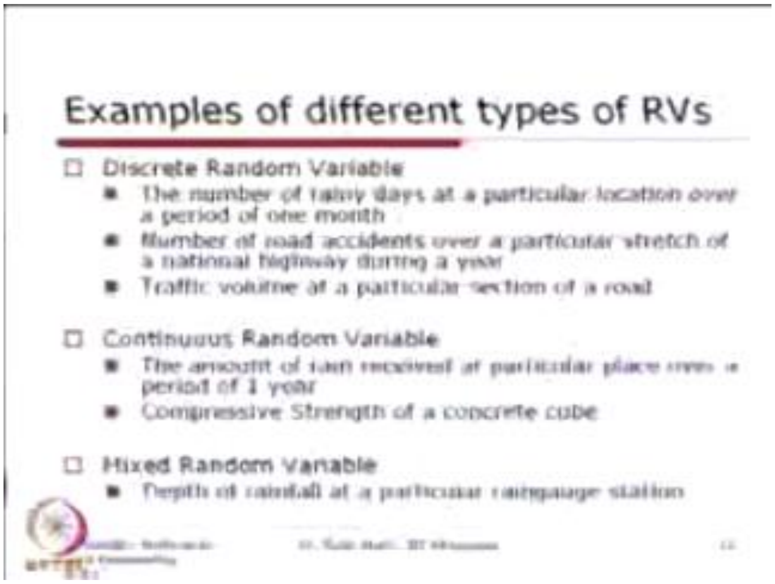

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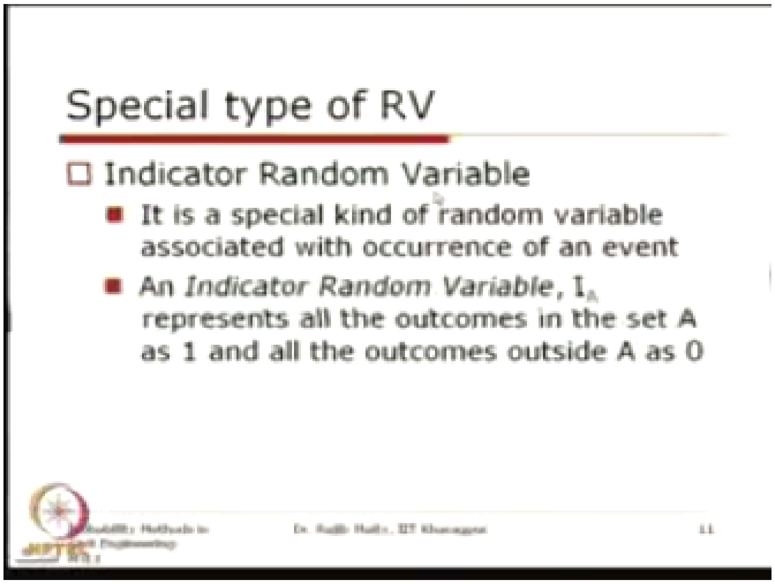

So saying these three types of random variable it is important to note that another special type of random variable which is known as the indicator random variable so it is a special kind of random variable associated with the occurrence of an event so this random variable which is says that whether the a particular event is occurred or not say an indicated random variable IA is represents all the outcomes in a set A as 1 and all the outcome outside A as 0 so it is like that whether yes or no if I say that again.

If I take the road accident whether there is a road accident If there is road accident his random variable in fixed in such a way that is that functional correspondence that we are talking about again here that is it is function is that it will just see whether the accident has taken place or not if it is yes it will say 1if it is no it will say 0 say the each day I am just taking this taking this measurement and just preparing that series so that series consist of either 1 or 0 again for the rainy days I will just see every day whether it is raining or not.

I am not interested in this random variable if it is designed in such a way that it is not interested what is depth of the rainfall only the interested is that whether it is a rainy day or not if it is not a rainy day then it will return 0 if it is rainy day it will return 1 so these types of random variable just indicate the occurrence of one particular event then the first case whether the occurrence of the event is the accidents and another one is that occurrence of the rainfall or not so if it is yes that is it is within the outcome of the set.

That indicates 1 if it is outside then it is indicates 0 so this kind of random variable is known as indicator random variable.

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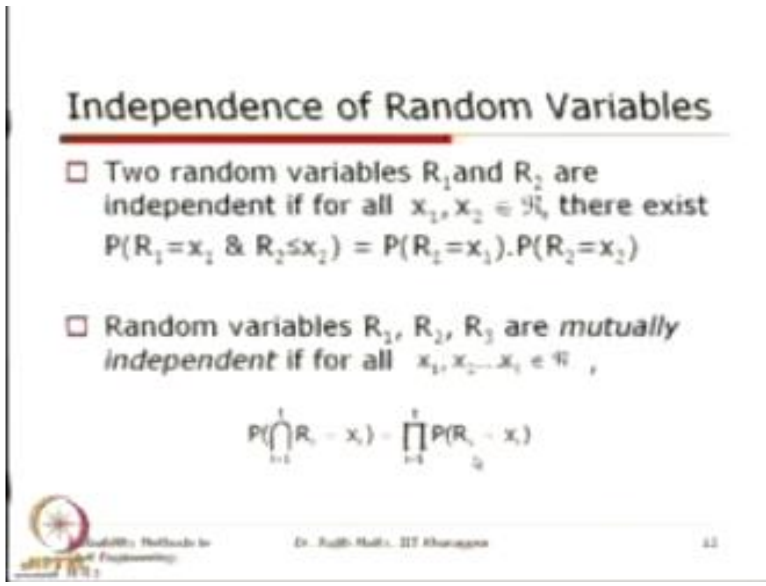

Independence of random variable this is another important concept to understand when we can say that the two random variables are independent the two random variables R_1 and R_2 are independent if for all X_1 X_2 belongs to this real line there exist that probability of $R_1 = X_1$ and the probability of R_2 less than X_2 = probability $R_1 = X_1$multiplied by probability of $R_2 = X_2$ then if this condition is satisfied we say that these two random variables are independent now there are other kinds of independents are also there which is known as the mutually independent.

And there are key wise or the pair wise in independence suppose that there are three more than two random variables which is the R1 R2 R3 if we will say that these are mutually independent if the probability of this particular one particular that is $R_1 = X_1$ and $R_2 = X_2$ $R_3 = X_3$ and all is equals to the multiplication of the individual probabilities then this can be this can be told that these random variables R_1 R_2 R_3 are mutually independent.

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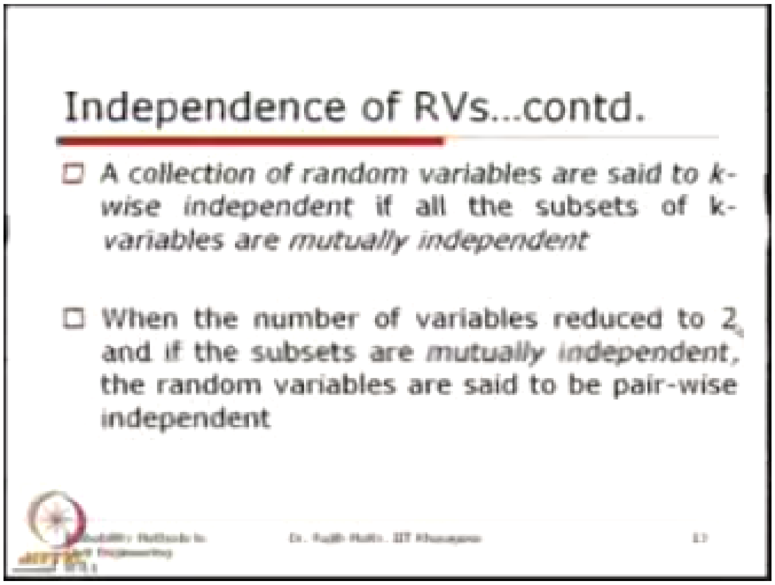

Then there are k wise independent random variables now out of this set suppose there are n random variables are there and in that out of that out of that n random variables it will say to be k wise independent if all the subsets of the k variables are mutually independent so obviously the k is less than n so whatever the total number of random variables are there if I now pickup randomly k random variables and if it satisfies that the mutually independence among this k random variables then we said those n random variable are k wise independent.

Now if this k becomes 2 then when this number of the variable reveals 2 out of n I take the 2 that if the subsets are in mutually independent then this random variable are said to be pair wise independent so what is happening out of n random variables I am picking up n random I am picking up only two random variables.

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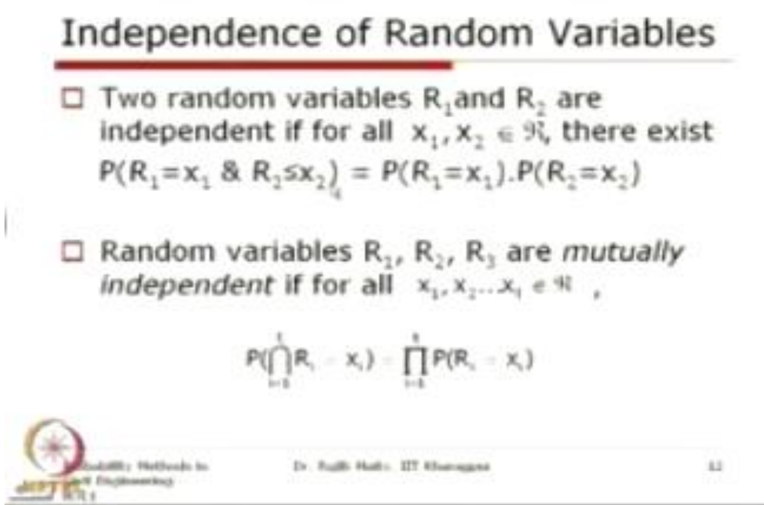

One small correction here is that what that R_2 is less than equals to X_2 was written so it should be $R_2 = X_2$.

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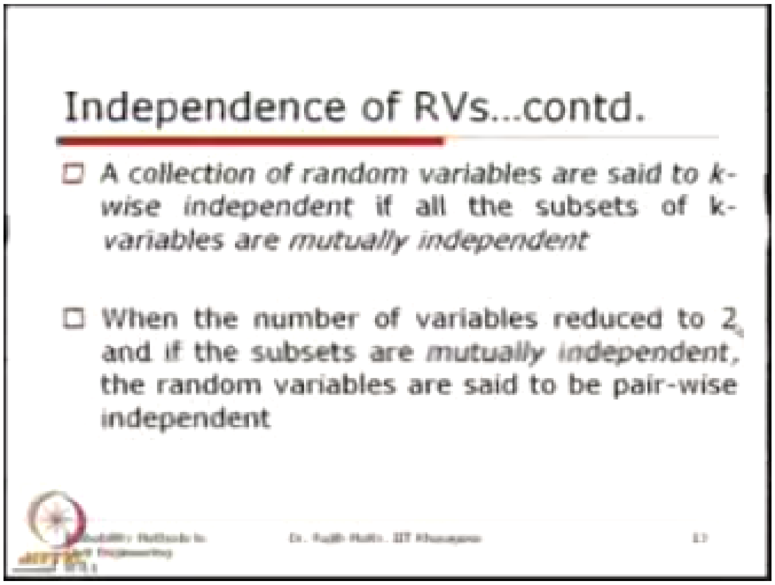

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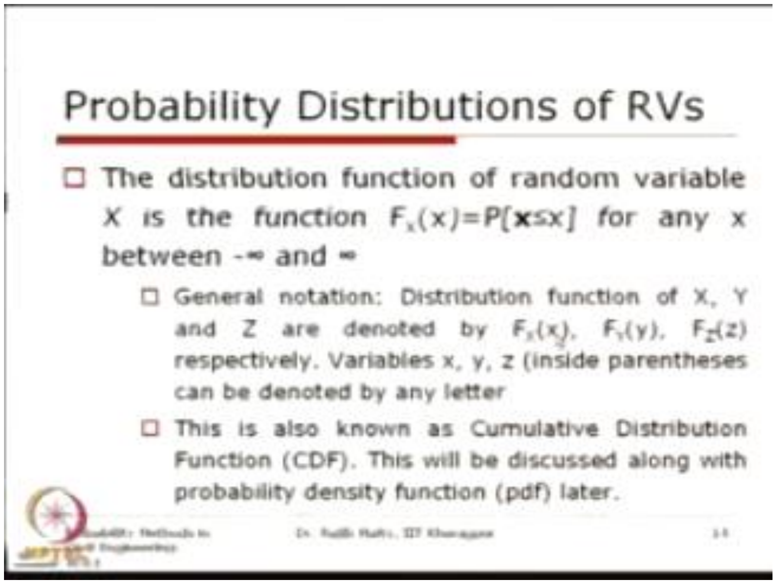

Now the most important thing why we just learn this random variable is that probability distribution of random variables the distribution function of a random variable x is a function which is denoted by this F subscript the capital letter of that random variable if it is x then this a capital letter of this X with some smaller case letter it may or may not be x which can which can take any letter so which is nothing but this inside this one it is nothing but as in few previous slides we have seen that this is nothing but the specific value of that random variable.

So this is denoted as the this is the distribution function of the random variable X which is nothing but is the probability of that random variable X when it is less than equals to that specific value of that random variable so this is the way we define the distribution function of the random variable X which is which is valid over the region from the minus infinity to the plus infinity so the entire real axis so here again the general notation says that the distribution function of X Y and Z are generally denoted by f subscript X any letter lower case letter f x x f y y, f z z respectively.

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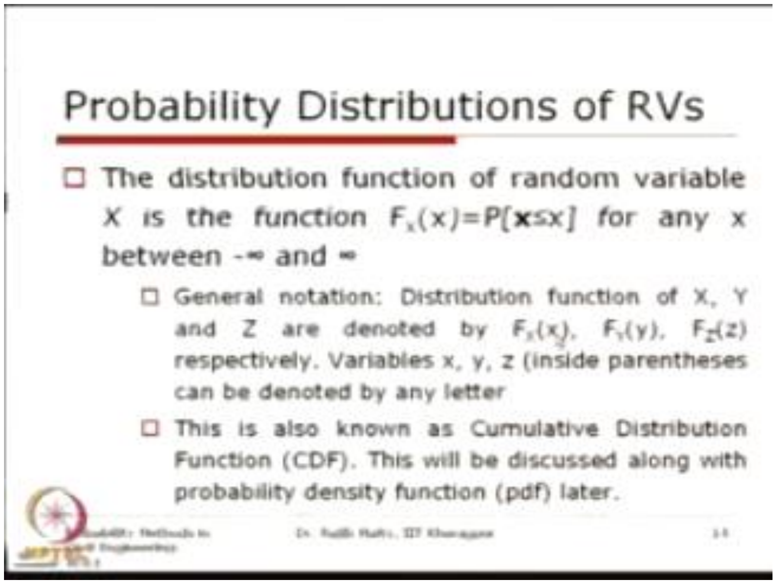

This variable that is a lower case letters that what I was just telling here is x inside y inside z inside the parentheses can be denoted by any letter as these are nothing but the specific value of that random variable this is also known as the cumulative distribution function when in the most probably in the next class we will just discuss about this cumulative distribution function this is actually the cumulative distribution function of the this one of this particular random variable this will be discussed along with probability density function pdf letter.

So what we want to tell is that for a particular random variable this random variable over this range some probabilities are assigned for the specific range and that that the how it is how it is distributed over this real axis is known as the distribution function of that particular random variable.

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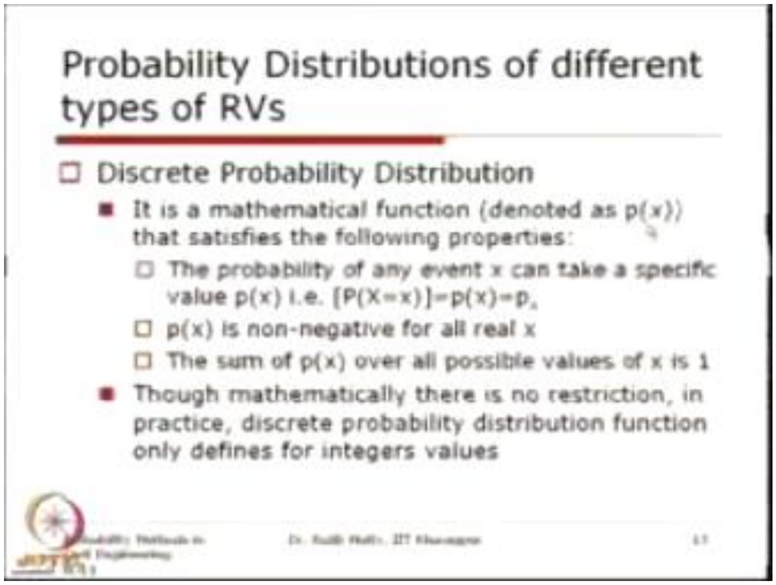

Now probability distribution of different types of random variables now we have discussed about three different types of random variables the first is the discrete random variable we will see how it is defined that it is the mathematical function denoted by generally for when it is discrete we denote by this p x this is also known as the probability mass function.

We will discuss this later. So this is denoted by p x that satisfy the following properties. The first one is that that the probability of any particular event x, because this is as it is discrete it can any take specific value all any means that those specific value, which is there in the feasible sample space.

So that particular specific value is denoted by p x which is nothing but, the probability of the random variable taking the specific value x, denoted by either p inside that specific value x or p subscript that value the specific value obviously from the axioms of this probability from the axioms of this probability this p x is a nonnegative for all the real x, it can either be 0or greater than 0.And the summation of this all this p x over this possible values of x is 1.This, again from the axioms of this probability.

Though mathematically there is no restriction in practice discrete probability distribution function only defines for integer values. So this is just when we have also seen in previous slide, that some example of this discrete probability distribution that this is generally take the integer values. But it is not specific it is not mathematically there is no restriction and it can take any specific value the way I'm defining that random variable.

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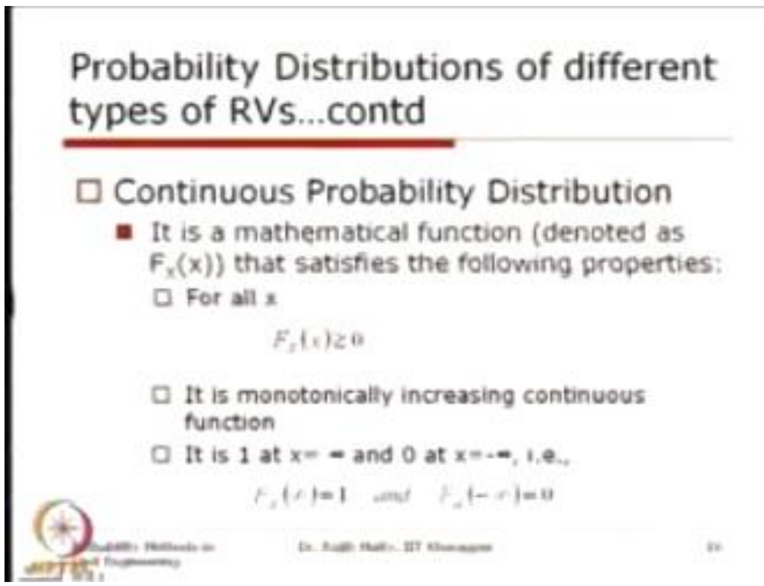

Second type of this random variable is the continuous random variable. So this continuous distribution function for that thing, it is a mathematical it is again a mathematical function which is denoted by this capital f subscript, that random variable or any lower case letter particularly this random variable lower case letter that satisfy the following property. The first is that for any specific value should be greater than equal to 0.

It is monotonically increasing and continuous function. So here it is monotonically increasing that means whenever it starts from and it will go and it will go on increasing and you know that this can go to maximum so generally for it is defined from 0to 1, so this it can go up it starts from 0and go up to 1 and it increases monotonically and it is then continuous function, as this random variable itself is continuous. So it is 1 at x is equals to infinity and 0at x is equals to minus infinity that is this probability at this x is 1 and probability x minus infinity is 0.

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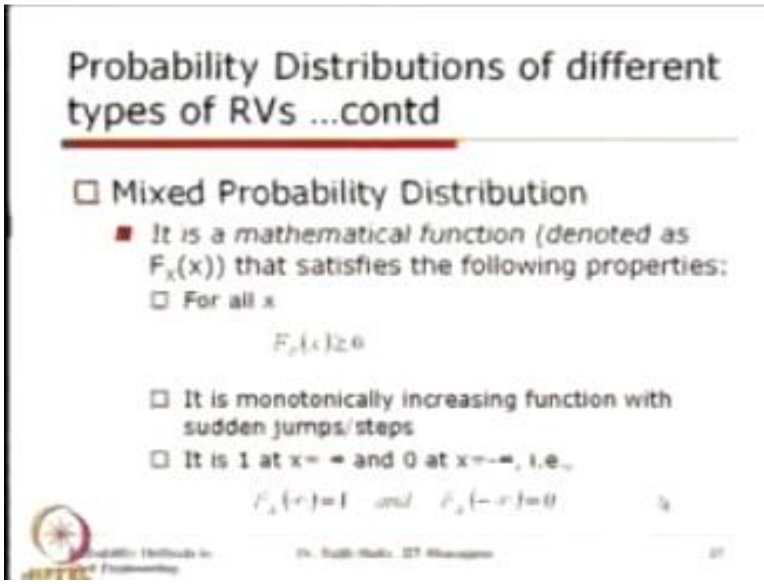

Then the last thing is this mixed probability distribution, which is which can take it for some range it can take that discrete values and some range can take continuous value. This also denoted by f x and keeping the all other properties all other conditions, that is the properties are same, which is obviously greater than equal to 0.

Though, with the only difference with this continuous is that, it is monotonically increasing function with sudden jumps or the steps and again the third one is again same. Now this one this lies the difference between this continuous probability distribution and this mixed probability distribution now if you just see it here.

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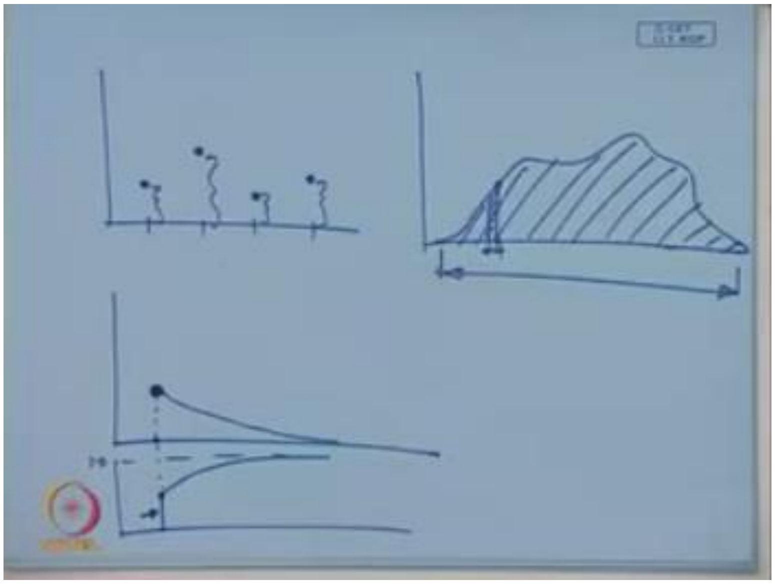

As we are just telling that there are at some point, where the probabilities concentrated now if I want to just see that how this cumulated accumulated over time and so, it starts from here and it will go and go up to maybe the way it is drawn it should be assumed 1. So this is the jump that we are talking about so this is the jump where the probability masses are concentrated here it is only once so it can be concentrated in some other range.

Then there also will be one jump so wherever the probability masses concentrated for some specific value so they are generally in this distribution function. We see that type of jump here which is the mixed probability distribution.

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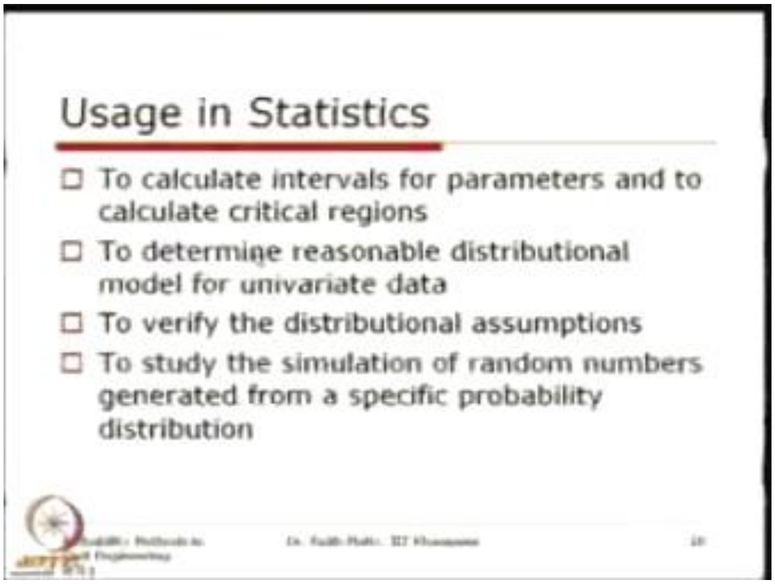

Now this concept the usage of this concept of this random variable in statistics in it is a it is a never ending list, I should say. So, just few examples are given that to calculate the intervals of the parameters and to calculate critical region we will see this, what is critical region and this interval of parameters in the subsequent classes to determine the reasonable distributional model for invariant data. Now, this data can be of any field of this civil engineering.

So now for this analysis the distributional model for those kinds of data this is useful to verify the distributional assumptions. We generally for any probabilistic model we have assumed some distribution. Now we have to verify whether that particular distribution is followed or not to study the simulation of random numbers generated from a specific probability distribution.

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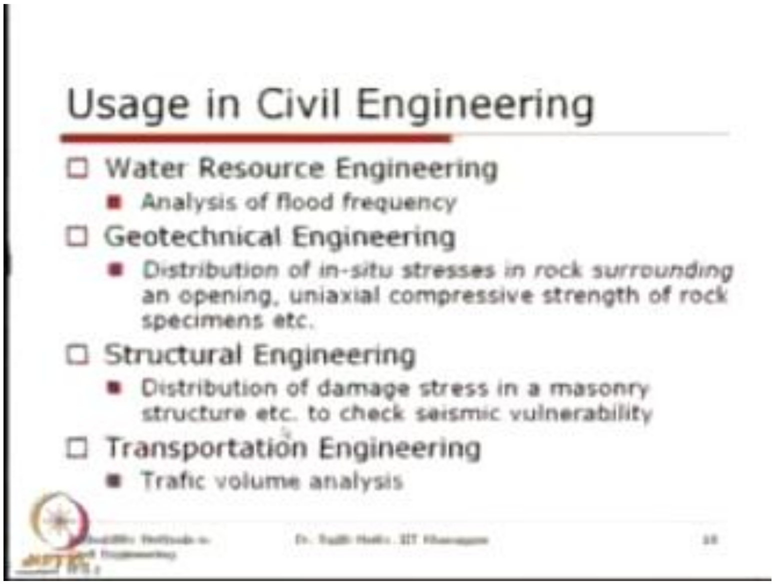

Coming to the specific civil engineering there are different facets of civil engineering, starting from water resource engineering, geotechnical engineering, structural engineering, transportation engineering, environmental engineering and there are many such. So in water resource, for one example the analysis of flood frequency this concept is used in geotechnical engineering, distribution of in situ stresses in the rock surrounding and opening or the any axial compressive strength of the rock specimen, etcetera.

In structural engineering, distribution of the damage stress in a masonry structure, etcetera, to check the seismic vulnerability of the structure transportation engineering for example the traffic volume analysis and this kind of thing in different application of this civil engineering.

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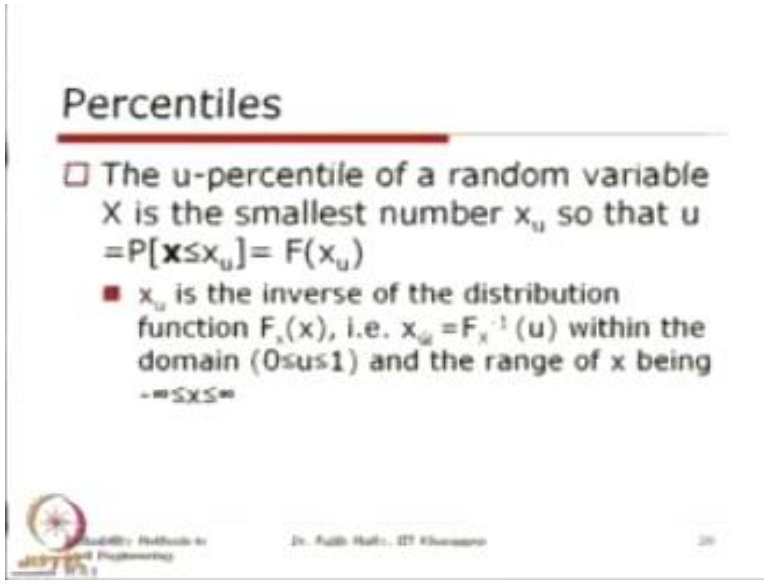

Now we will see the concept of the percentile for a random variable. The u percentile of a random variable x is the smallest number x u, so that u equals to the probability of x less than u, which is equals to probability of x u. So to determine what is the value of this x u, the x u is generally the inverse of the distribution function f x, that is, x u is equals to the f x inverse u, within the domain 0 to u to 1.So now, to know this thing, basically if the graphically if I just want to see it here now.

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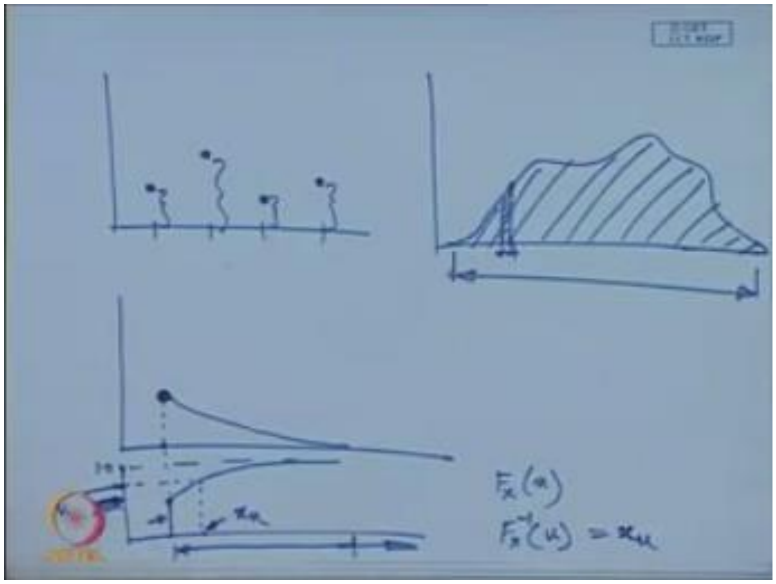

To show this is the feasible range of that particular variable suppose this goes on here so I now to calculate this percentile what some percentile if we just say some u, so here we can say where is the u. Now basically suppose this is here so basically we just go and see how much percentage is covered here. So this particular value is graphically representing this particular value is your u percentile of that random variable.

So generally what happens we generally see this particular value calculate its cumulative probability and get this one? So that is what is your mapping as this f x of any value x, that is your mapping. Now when you are coming from this side then what we are doing we are just giving this f x of u inverse will give you some value that x u, which is your x u here. So, this is that inverse function of that one to get that get that percentile.

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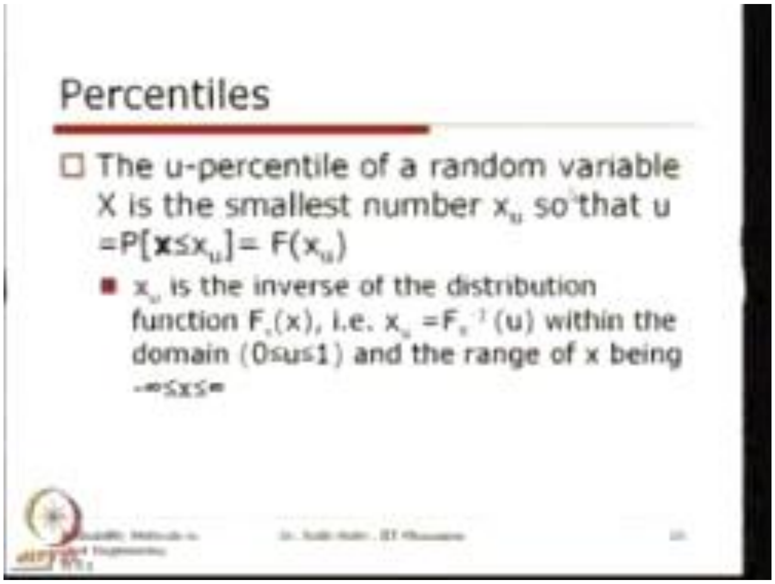

So, so the u percentile of a random variable x is the smallest number xu so, that u equals to probability of x less than equals to XU. So this is obtained that XU is equals to inverse of that of the distribution, of that particular number, particular random variable for that percentile U. And obviously, the u have the range from 0to 1 and so it is expressed in percentages taken from 0 to 1 range, and the range of x being whatever hat range of particular random variable.

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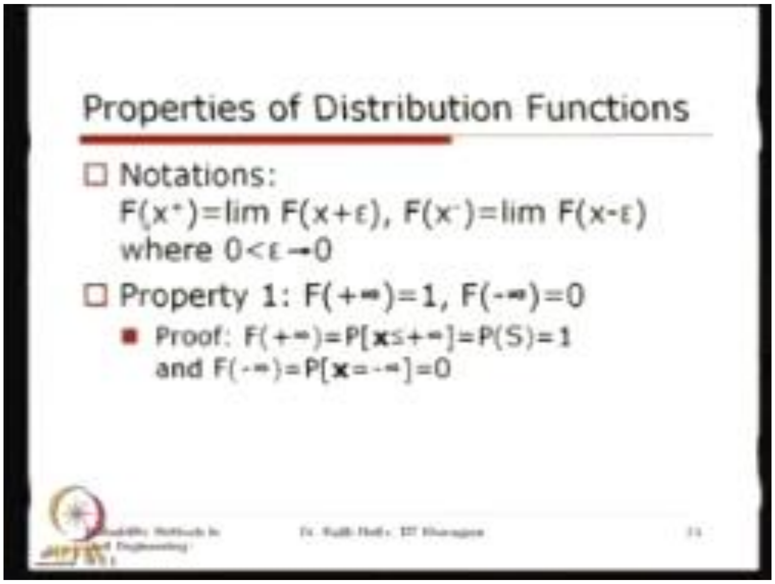

There are different properties of this distribution functions. So we will see one by another, one by one of these properties. In this discussion, we will follow some notation this is taken mostly from the Papoulis book. So, this is that f(x) plus of this random variable x ,of course, is equals to limit f that particular value x plus epsilon and minus means the limit of this one ,when this epsilons are greater than 0 but, it is tending to 0.So very small number ,so this f x plus means, just right side of that x and f X minus is the just left side of that of that X. So, property one, the first property that is, if I take this distribution function for this infinity is equals to 1,

So that the right extreme of this real x and left extreme of the real x, it is starts from 0.So, it always starts from 0 ends at 1.So, to put this 1,that is f plus infinity is equals to f(x) less than equals to plus infinity. So, if is f x is less than equals to plus infinity, that means it is encompassing the full sample space. So this is nothing but, that probability of the full sample spaces S and we know that full sample space from the axioms of the probability, that this is equals to 1

Similarly, so if minus infinity is nothing but, the probability of x equals x less than equals to, basically, less than equals to minus infinity, which is a, which is basically, basically null set, so probability of the null set is equals to 0.

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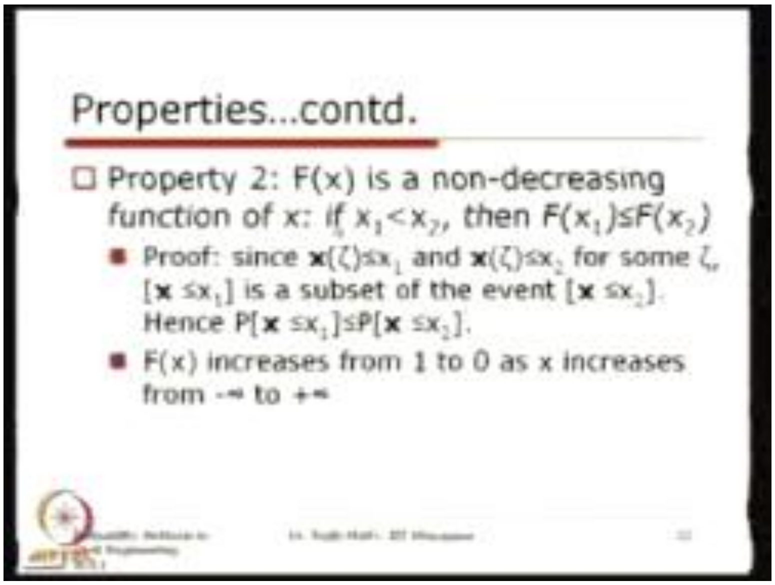

Second property, f(x) is a non-decreasing function of x. That is, this non-decreasing function or monotonically increasing function, which is the word that we used in few slide previous. So, it says that, if that x1 is less than x2, then always this f(x)1, the value of this distribution function at x1 is less than equals to fx2.So, it can never decrease. So, it will always, it will either be same or it will it will increase.

So that is basically what is known as this monotonically increasing. Proof of this one is that, if x xi is less than equal to x1 and x, xi less than equal x2, for some outcome of this xi, then x random variable less than equals to x1 is a is a subset of the event x less than equals to x2.So what, so this x1 is always there within this x less than equal to x2.

If this is greater than, if this x1 is less than x2,so that is why if it is a subset of this one, then obvious the probability this should be less than equals to probability of this x less than x2.,Fx increases from Fx increases from 0 to 1.Sorry for this mistake. Fx increases from 0to 1 as x increases from minus infinity to plus infinity this will be 0 to 1 as x increases from minus infinity to plus infinity.

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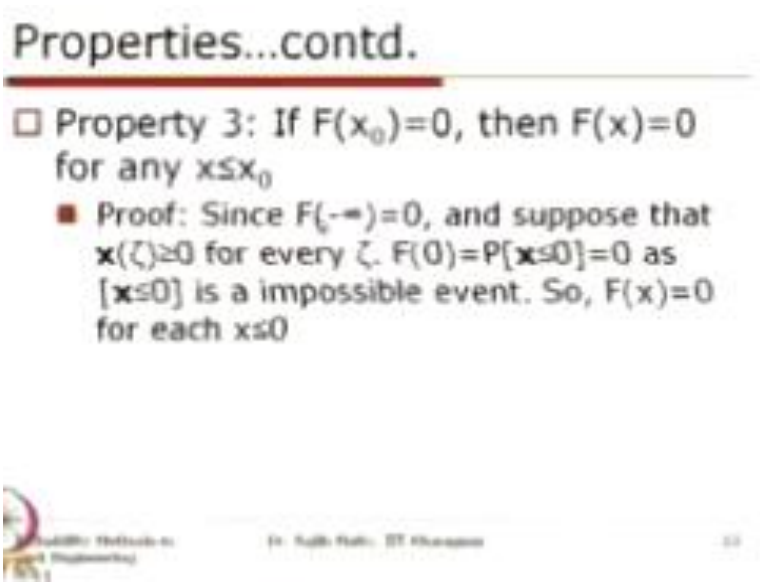

Third property says that, if fx not is equals to 0, then fx equals to fx equals to 0 for any, which is which is less than this x_0. So, for a specific value, if the fx equals to 0, anything which is lower than this x_0 obviously will be 0.This is basically the same concept of that it is non-decreasing function.

So if it is some portion, if it is 0, left side of that in the real accession context that is lower than any level of this x_0 obviously that will also be 0 proof since f -∞ =0 suppose that x sigh greater then equal to 0 for every sigh f0 equals to probability, x less then equals to 0 as x less then equals to 0 is an impossible event so f(x)=0 for each x less than x_0 .

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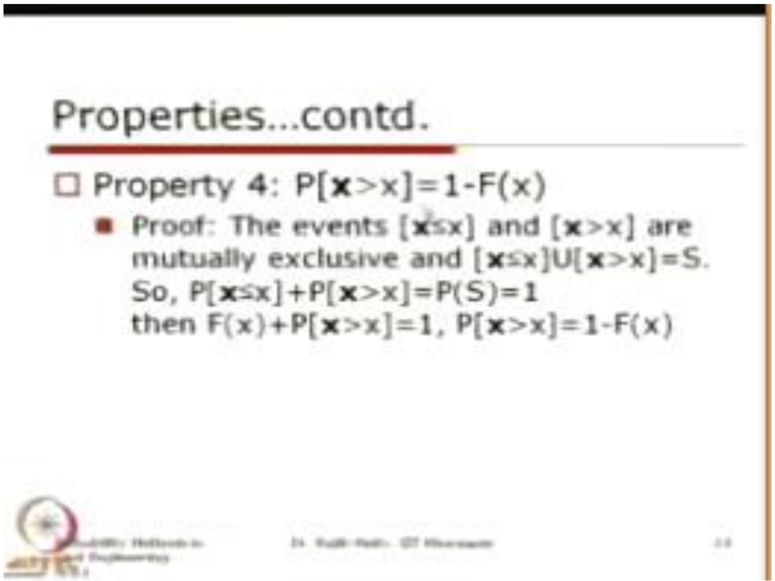

Property four probability x greater then x=1-f(x) probability x less then equals to f(x) is equals to F(x) is designated in this way now we know that total probability is always 1 so if I just want to know what is the probability of X>x obviously this the rest of this properties, so which is 1-f(x) proof says the invent f(x) <=to one and x greater then x are mutually x.

And X greater than x are mutually exclusive. So, if these two events mutually exclusive and collectively exhaustive, so collectively exhaust this x less than equals to x union x greater than x is equals to full sample space s. So that, probability of this less than x and greater than x, is nothing but, probability of the total sample space. That is, the s equals to equal to 1 and now this is denoted by f (x), which is this one equals to 1. So probability of x greater than x results to 1 minus f (x).

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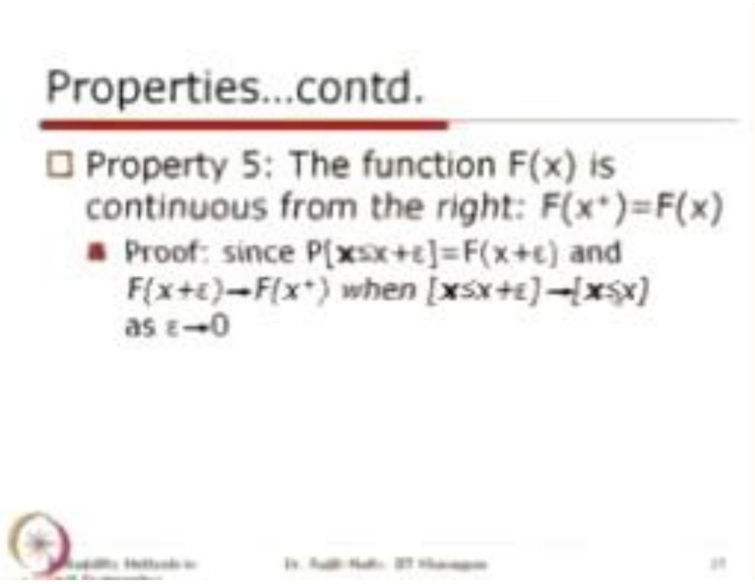

The fifth property says, the function f (x) is continuous from right. So, f (x) + , that is the from right is equals to f(x). Proof, since probability of x less than equals to x plus x sigh where this x sigh is nothing but very is standing to 0is equals to f x plus sigh f (x) plus x sigh tending to f x plus, when this f, this x is less than equals to x sigh tending to f (x) less than equal to x, as this sigh is tending to 0.

So, that is why, this is from this right hand side, if we say this is continuous from the right hand side of any specific value x.

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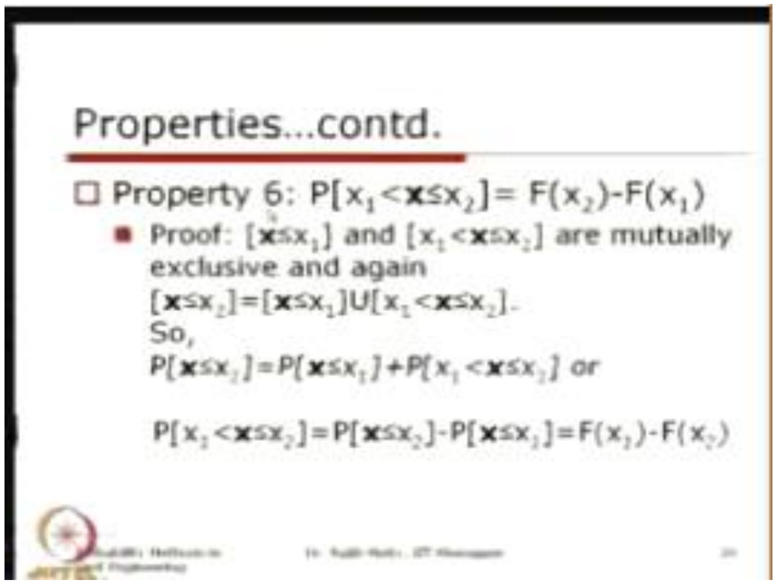

Then sixth property says, for a random variable, if it is bounded by this x1 and x2,theprobability of the of the value from starting from x1 to x2 is equals to the probability of x2 minus x1.So, graphically if I just see it again here, that is.

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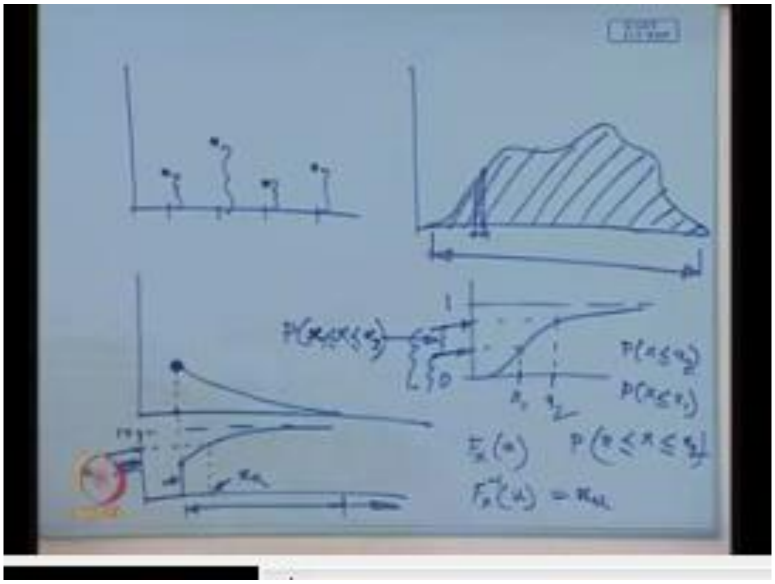

If I just take two values, if this is your say starts form 0and goes like this. So, if I want to know that, what if this is your x 1 this is x 2 then these probabilities that is, x 2 so probability that f(x) less than equals to x2 is nothing but, this particular value Now, probability of x less than equals to x 1 is nothing but, this particular value. Now if I want to know that if this x is in between these x2 and x 1, then this probability is nothing but, whatever the total probability this minus this probability. So this is the probability that we are talking about which is nothing but the probability of x1 less than equals to x less then equals to x 2.

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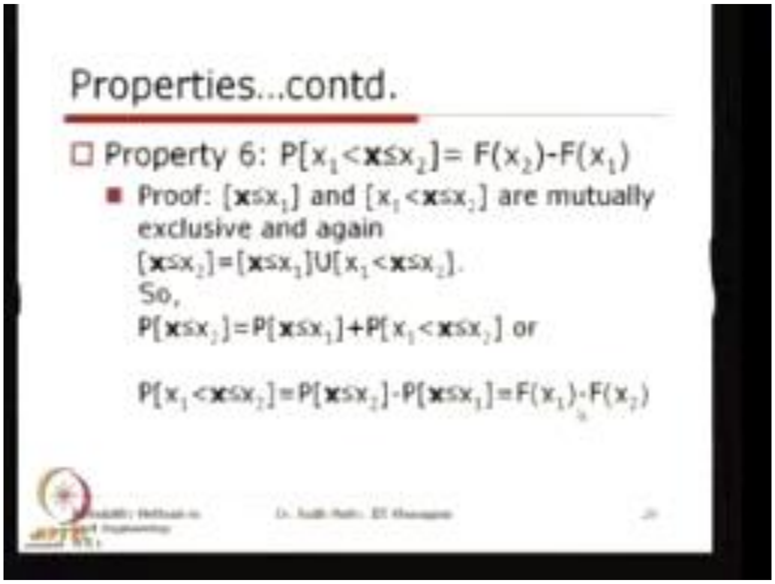

So this is how we get the probability for a range which are again this proof is proof says that x less than equals to x 1 and this x 1 less than x less than x 2 are mutually exclusive and again x less than equals to x 2is equals to x less than equals to x 1 union x 1 to 2 these. So the probability x less than equals to x 2 is probability x less than equals to x1 plus probability x in between x 1 to x 2 or if I just take this one here then probability of x1, this x random variable between x1 to x 2is equals to probability of x less than x 2 minus probability of xx 1which is again nothing but this f x 1f x2.

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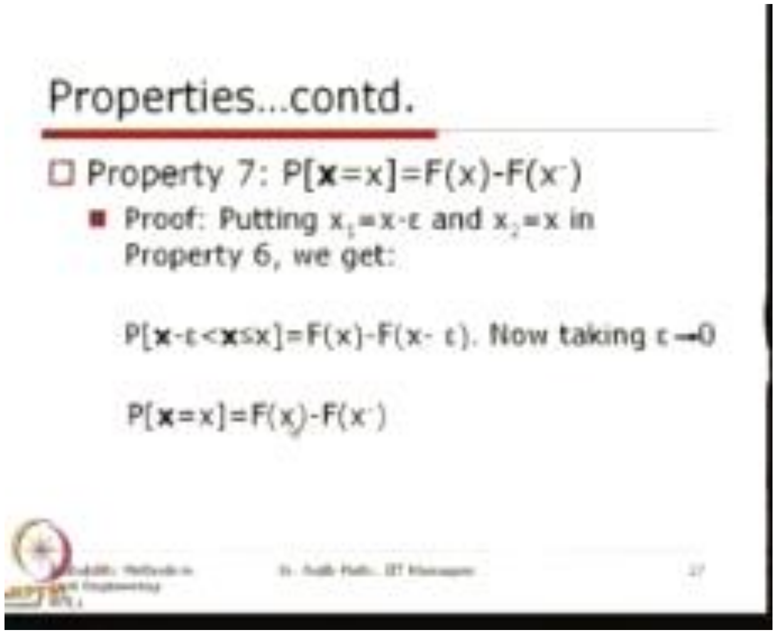

Now the property seven says, that if probability of x is equals to a particular value, x is equals to probability of x minus probability of x just left to that one. So at a particular point the probability there now these properties are in general for the discreet and continuous. So For a particular point the probability says that at that particular point and just left to that whatever the probability is there so it will be like this from that particular side to just to the left of this one proof putting that x 1 equals to x - ζ and x2 equal to x in this property six.

Then we can say that probability of x ζ less than x less than that x equals to probability x equal s to probability x - ζ now taking this ζ tending to 0 then we can say that probability at a particular specific value is that point probability at that point minus immediate previous value to that one that particular value. So if it is a discreet random variable then just left to this value this value comes to 0. So that for a discreet value at particular point the probability is equals to that that the functional value at that particular point.

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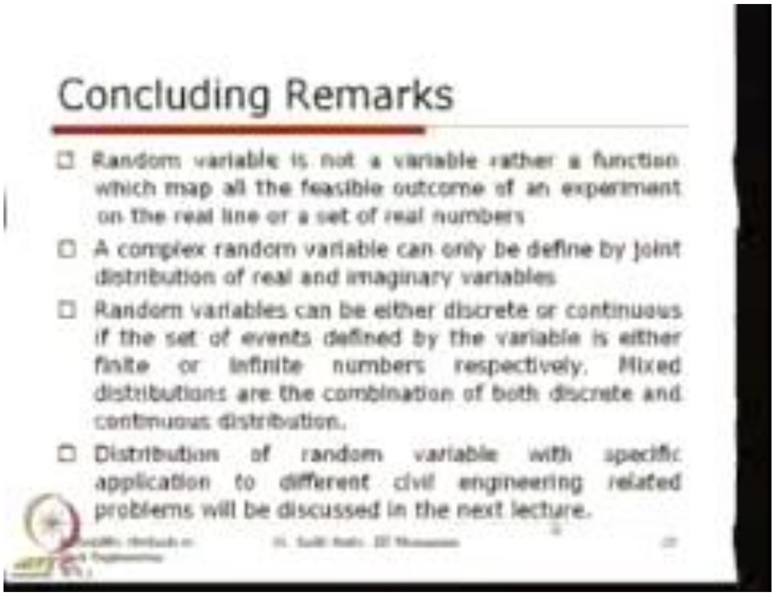

Okay finally in this lecture, we have seen that random variable is not a variable rather a function which map all the feasible outcome of an experiment on the real line or a set of real numbers. A complex random variable can only be defined by the joint distribution of the real and the imaginary variable. That is if that x + I y will be equal to that the joint distribution of this x and y both the random variables. Random variables can be either discrete or continuous if the set of events defined by this variable is either finite or in finite numbers respectively.

Mixed distributions are the combination of both discrete and continuous distribution. Distribution of random variable with specific application to this different civil engineering related problems will be discussed in the next lecture along with the concept of this probability density function and cumulative distribution function, thank you.

Probability Methods in Civil Engineering

End of Lecture 06

Next: "Probability Distribution of Random Variables" In Lec 07

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