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NPTEL
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Probability Methods in Civil Engineering

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Lecture – 23

Topic

Hypothesis Testing

Hello and welcome to this lecture this is basically third lecture of this module and in this lecture, we will be discussing about hypothesis testing you know that in the last lecture, we actually continued that hypothesis testing, we started that one so, in this lecture we will continue the same the hypothesis regarding.

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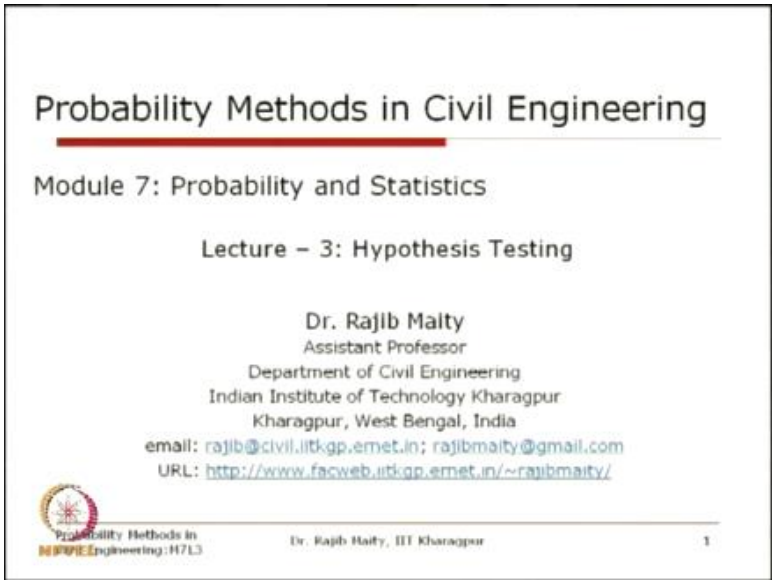

One mean.

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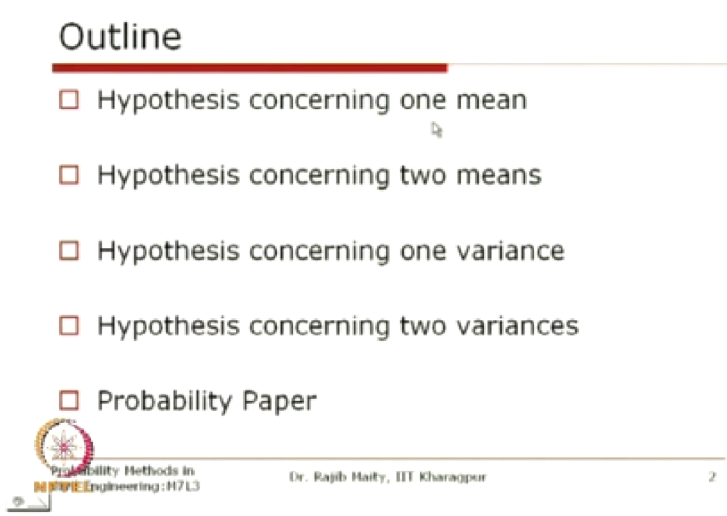

Or concerning one mean so one mean means here we are having one sample so we will do some testing basically this hypothesis testing are drawing some inference probabilistically from the sample mean with respect to the population mean those we discuss in this last lecture so our outline for today's lecture is hypothesis concerning one mean hypothesis concerning two means hypothesis concerning one variance and hypothesis concerning two variance and if time permits, we will also see the probability paper that we are talking about that once the data is available generally.

So far what we have done we have assumed that it is following some particular distribution. So this probability paper is basically that first step of for a graphical inspection of to test that what the data possibly could follow. So, this is basically the test of this sample data that what is the distribution that population may follow, so that if the time permits after this hypothesis testing we will do that one also well, so hypothesis concerning one mean

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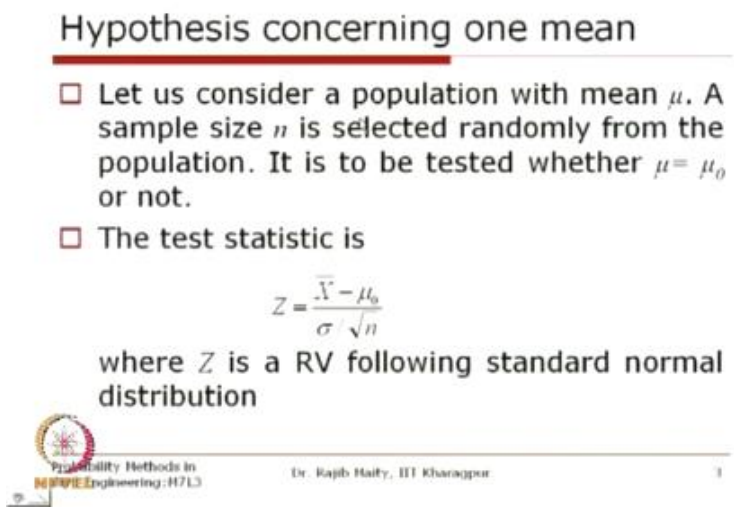

Let us consider a population with mean μ a sample size n is selected randomly from the population. It is to be tested whether this $\mu = \mu_0$ or not so this μ_0 is basically one threshold value that we need to test whether from the sample whatever the mean that we are estimating from this estimation technique that we discuss earlier whether that is having some relationship so that whether probabilistically, we can infer something about it some specific value.

So, that specific value is your μ_0, what is our test what is our goal to test this population mean. So here the test so you know that from our last lecture, we have to define some test statistics, so here the statistics is that $Z = \bar{x} - \mu_0 / \sigma / \sqrt{n}$. This $\sigma / \sqrt{n}$ that you know that is the standard deviation for the sampling distribution of this mean and this $\bar{x}$ is your mean and this μ_0 is the value which we are expecting that population mean might be this μ_0.

So, here you know that this is a reduce standard variant now so this z is a random variable which is following the standard normal distribution.

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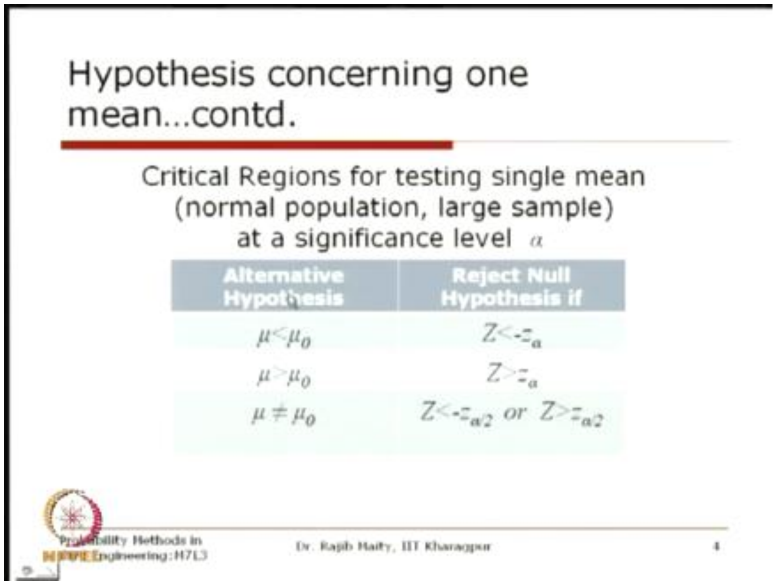

And the critical regions for testing this single mean and again here the assumption is that the population is normal and it is a large sample. This large means, here you can say that approximately if the sample size is more than 30, we can consider that this is a large sample. So, this critical region at the significance level α and the discussion on the significance level also we have covered in this last lecture.

So if it there are two possible case it can be one sided or it can be two sided for this one sided it can be the left hand side or the right hand side and reject the null hypothesis if this Z is less than -Z α. In this case when the alternative hypothesis is μ is > μ_0 then Z is < Z α. And if it is a two sided testing μ is not equal to μ_0 then, whether Z < - Z α / 2 or Z is >Z α /2 and you know that this Z α/ 2 is the quintile for which that remaining probability is your α / 2 towards the right hand side of this one that we have discussed while discussing that critical zone. So, if that test statistics falls in this critical zone then we have to reject the null hypothesis.

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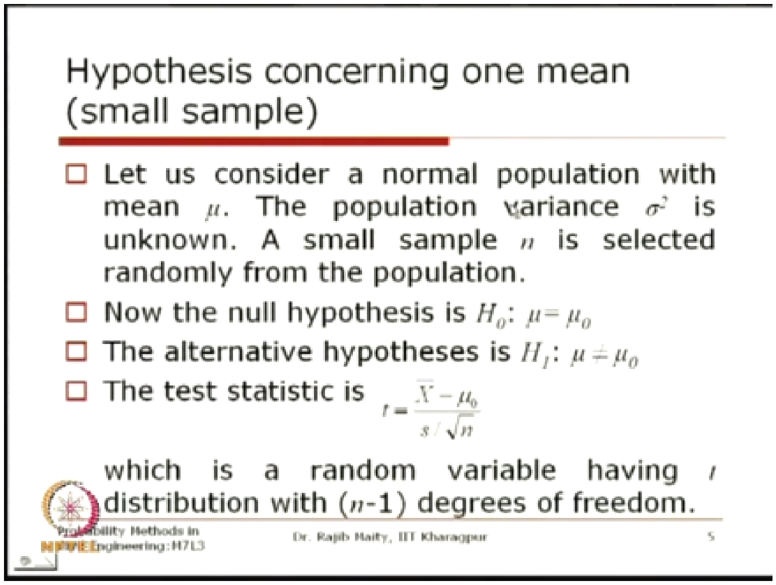

So let us consider a normal population with the mean μ the population variance σ^2 is unknown a small sample n is selected randomly from this population. Now in this case, what we are doing so far what we are telling that if this sample is large now we are discussing if the sample is small and as I was telling that if the sample size is less than 30 then we can we have to consider that this is a small sample and here if that the test statistics for this null hypothesis and the alternative hypothesis if we consider is to be this central one or in fact it is not only for the central it can be one sided also so, this quantity now which we were just estimating as a Z value that we have seen now this s is replaced has replaced that σ.

So, σ/ σ^2 n that σ was a standard deviation for the population here, it is a small sample so we do not, and the variance is unknown population variance is unknown so we have to estimate again this one from the sample. So, this s is basically the standard deviation estimated for this sample and so this quintile now is a random variable having t distribution with n - 1 degrees of freedom.

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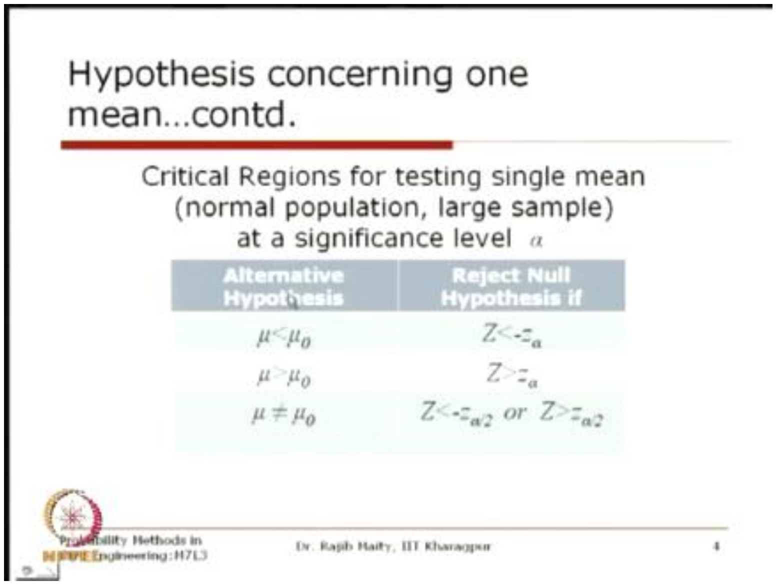

And for this case the critical regions will be for this one sided test it will be t $\leq$ - t σ and if it is right hand side then, it is t > t σ and for the central test, it is < -t σ / 2 or greater than t σ/ 2.So now you can see that for the large sample we are approximating it with the standard normal distribution and if it is an small sample we have the approximated to the t distribution with the degrees of freedom n - 1, where n is the sample size.

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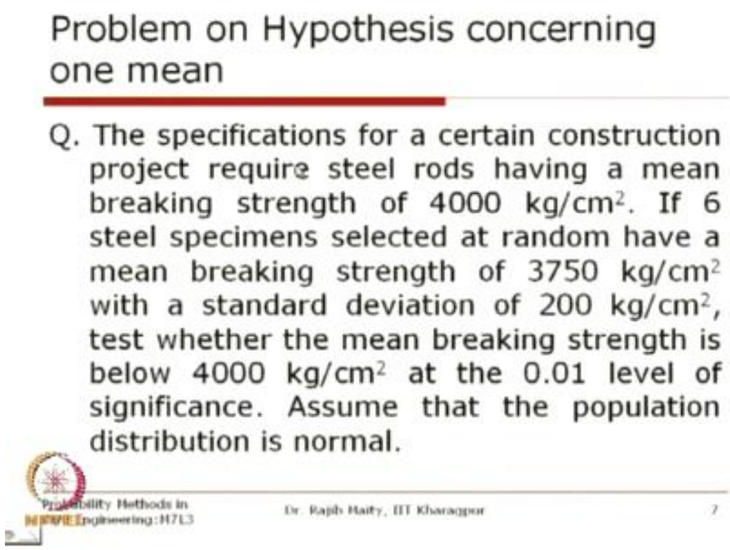

We well, take up one problem here the problem states the specifications for a certain construction project require the steel rods having a mean breaking strength of 4000 kg/ cm^2.If 6 steel specimen selected at random have a mean breaking strength of 3750 kg / cm^2 with a standard deviation of 200 kg/ cm^2 test whether the mean breaking strength is below 4000 kg/cm^{2}at the 0.01 level of significance.

Assume that the population is distribution normal so, you see that one small sample, which is sample size is 6 and it is showing that mean strength is 3750, which numerically if we test that it is below this4000 kg / cm^2 but this difference whether should we consider that whatever the specimens that is being supplied for that project is really less than the specification so that statistically we have to infer about that.

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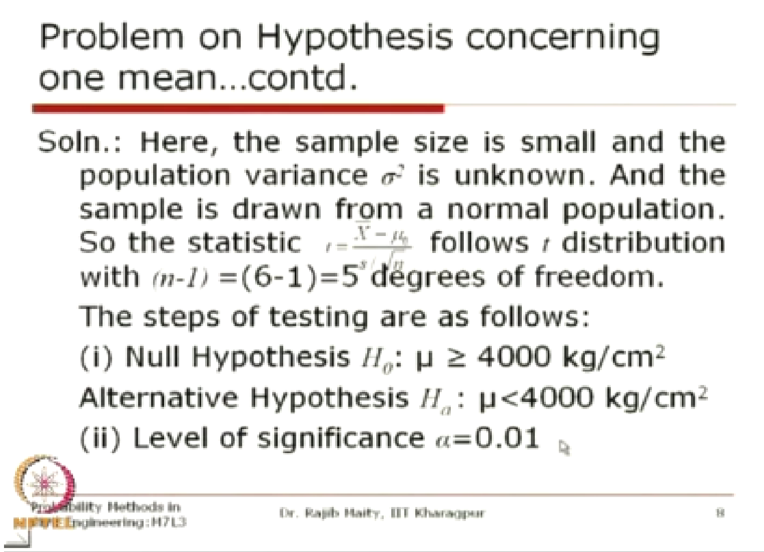

So, here the sample size is small and this population variance is unknown and the sample is drawn from a normal population. So the test statistics as we have discuss so far is this t equals to $\bar{x} - \mu_{0\,/}$ s/ $\sqrt{}$ n, which is following a t- distribution with n - 1,n = 6 years, so 5 degrees of freedom. So, our null hypothesis is, whether this is greater than or equal to this 4000 kg/ cm^2 which is our specification. So, we have to test that whether, it is below that our specification, so that is why in the alternative hypothesis, we put that μ is less than 4000 kg/ cm^2 and this α is equals to 1.01.

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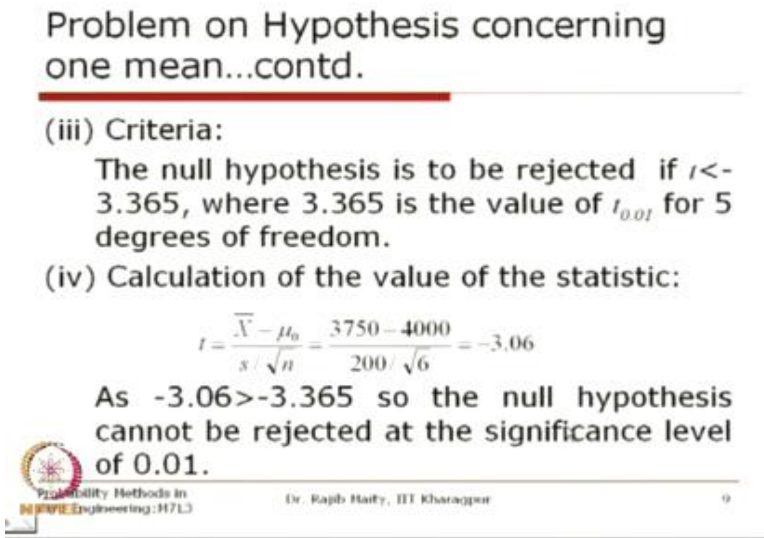

So, the criteria here is that as this depends on this, what is the level of significance ,so this t 0 α and this is an one sided test. So, this t 0 α for 5 degrees of freedom from the standard table from the standard textbook if you see, it is 3.365.So,that test statistics if it is greater than this then, we have to reject the null hypothesis if we calculate this one then, we see that so this is t is <- 3.365.So, this test statistics now from the data that is available X-bar is 3750 and s = 200 we are getting that it is - 3.06.

So as this - 3.06 > - 3.365, so the null hypothesis cannot be rejected at the significance level of 0.01. So we cannot claim we cannot we are failed to prove that this mean is really less than the specification which is 4000 kg per centimeter square. Even though the mean that we have seen for this 6 specimen is 3750 they take.

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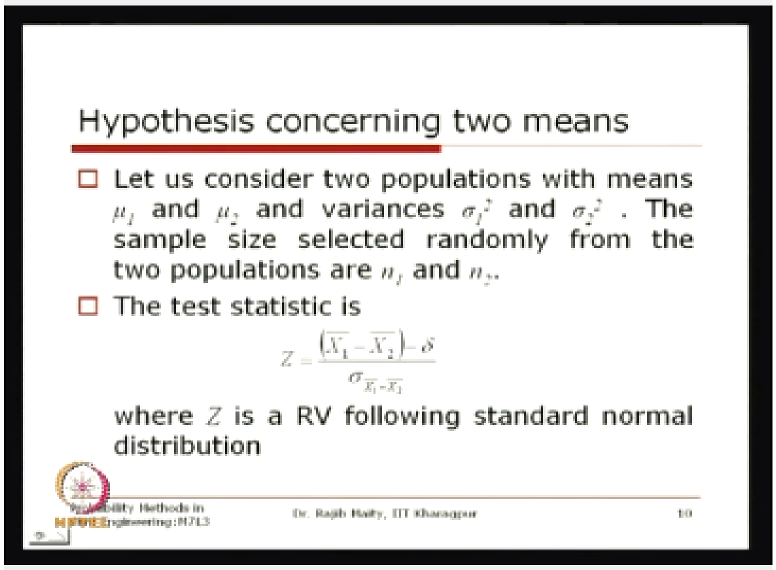

Well now we will take that hypothesis concerning two means, now this two means there are two samples are there and two sample means are there, so we can test whether many questions you can ask that whether both the sample is having the same mean even though that so far as the sample means should not be exactly same that we can test and then we can test that whether the one is really greater than the other one that also we can test.

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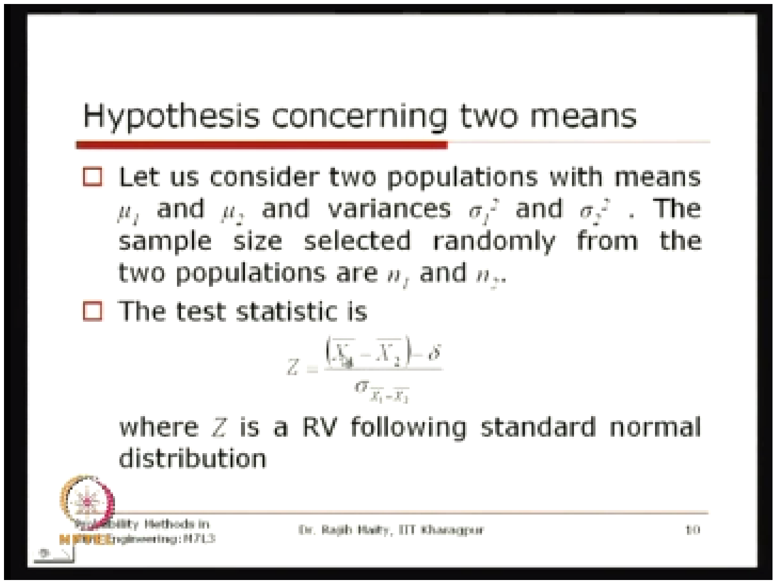

So here what we are taking is that basically the difference between this two means, that is X1 is the mean for the first population and the X2 is the mean for the second population. So if we take this difference then the mean basically now it is becoming a single random variable with that delta value. Now if I put that delta equals to 0 that means both the means are affectively what we are testing whether the means are same or not.

Or depending on what value of this delta that we are selecting depending on the whether we are testing that whether X1the mean for the first population is greater than the second population and so on. And this sigma this denominator sigma $\overline{x}$- $\overline{x}_2$ is the standard deviation for this difference and that this Z is now the random variable which is following again the standard normal distribution.

In case that this both this n1 and n2 which is the sample size are large enough. You can say that the if it is greater than 30 then we can say that it is a standard normal distribution. Once we get this one we can infer about this.

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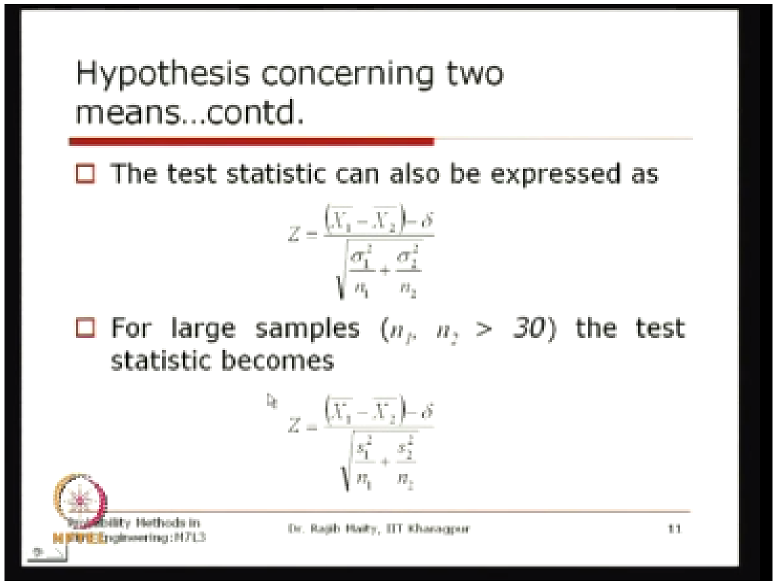

About their sample means, so that test statistics here can be the this one this $\bar{x}$- $\bar{x}_2$ -Δ and that standard deviation that we are getting for this difference is basically a $\sigma1^2/n1 + \sigma2^2/n2$ and their whole square root which is giving you this estimate for this standard deviation for this difference and for the large sample that I was telling that if they are greater than this 30 then also we can say that this can be easily replace by their sample standard deviation. Which is again that standard normal distribution we can say.

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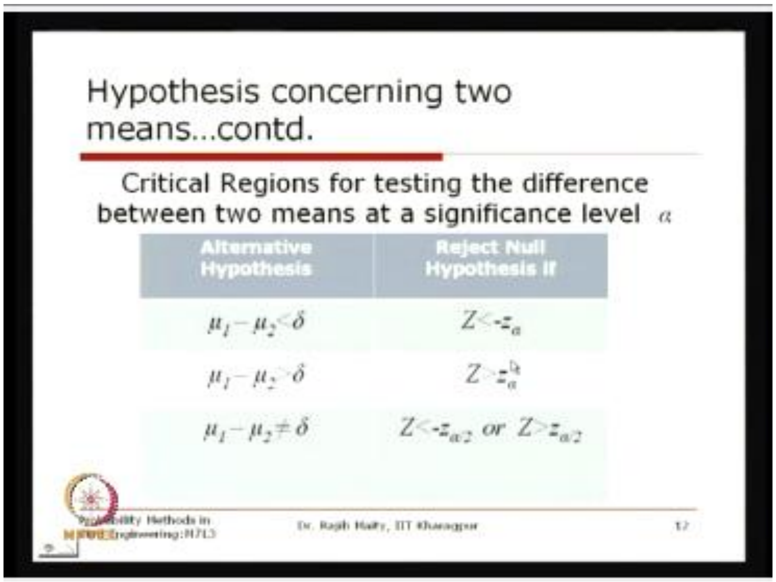

This is for this large sample as we have shown and then after this one we will also see that if it is the small sample. Now so far as this is large sample this will follow a standard normal distribution and the critical regions are now same for this are the different alternative hypothesis whether it is. Now this case is basically showing that whether $\mu1 < \mu2$ and this is showing that whether $\mu1 > \mu 2$.

And this is a two sided test whether $\mu1 \neq \mu2$. So these are the critical regions as same that we discuss earlier for this single sample mean few slides before.

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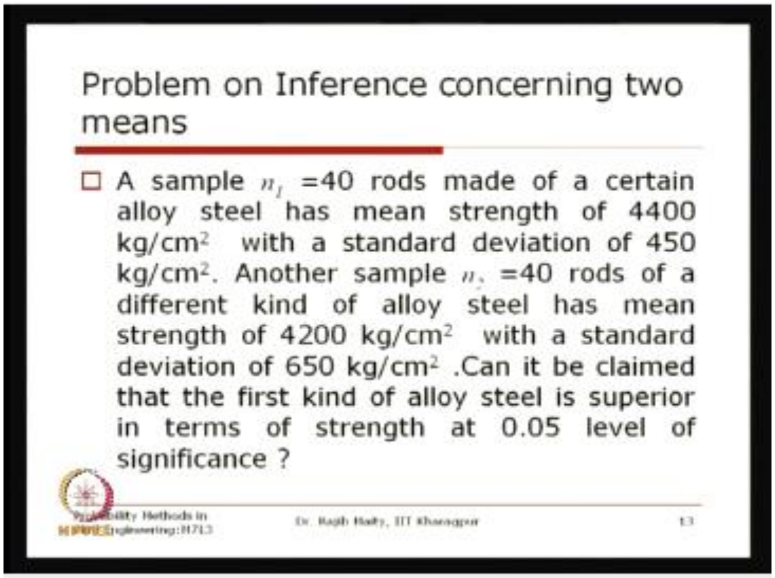

And the second thing is the first we will take one problem on this if the sample size is large. So a sample n1 = 40 rods made of the certain alloy steel has mean strength of 4400 kg per centimeter square with a standard deviation of450 kg per centimeter square. The another sample n2 = 40 rods having this different kind of alloy steel which is having a mean strength of 4200kg per centimeter square with a standard deviation of 650 kg per centimeter square.

Can it be claimed that the first kind of alloy is superior in terms of this strength at 0.05level of significance. So the numerically if I just take this mean then I can say that this 4400 > 4200 but if we see their spread their standard deviation so this is 450 and here it is 650.So the statistically we have to test whether really that first alloy steel is superior than the second one.

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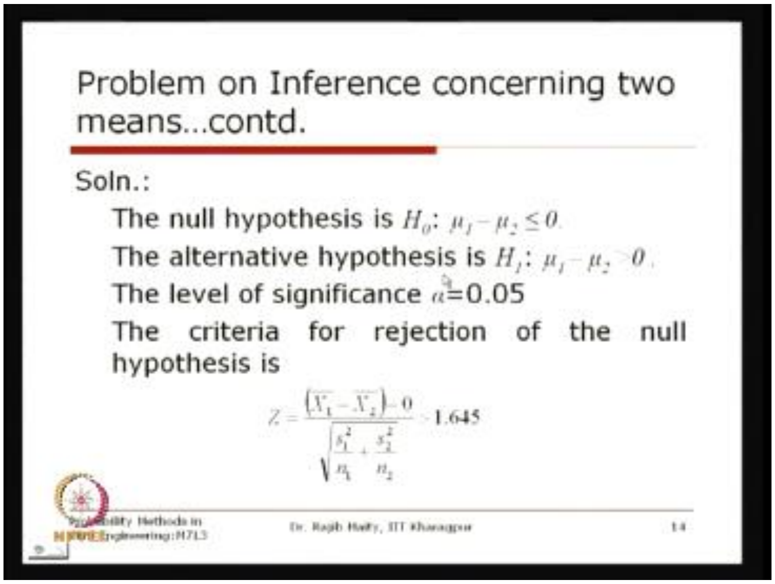

So at the null hypothesis is that whether $\mu1- \mu2 < 0$ here we place is delta equals to 0.And this alternative hypothesis basically what we are testing is this $\mu1 – \mu2 > 0$ that means we are testing whether $\mu1 > \mu2$ or not and the significance level is 0.05.The test statistics the criteria here it should be at 0.05 you know that this Z quintile is 1.645 at 95 % probability level for this standard normal distribution the value is 1.645.

So this is our critical value and greater than this value if we get that z then we should reject the null hypothesis.

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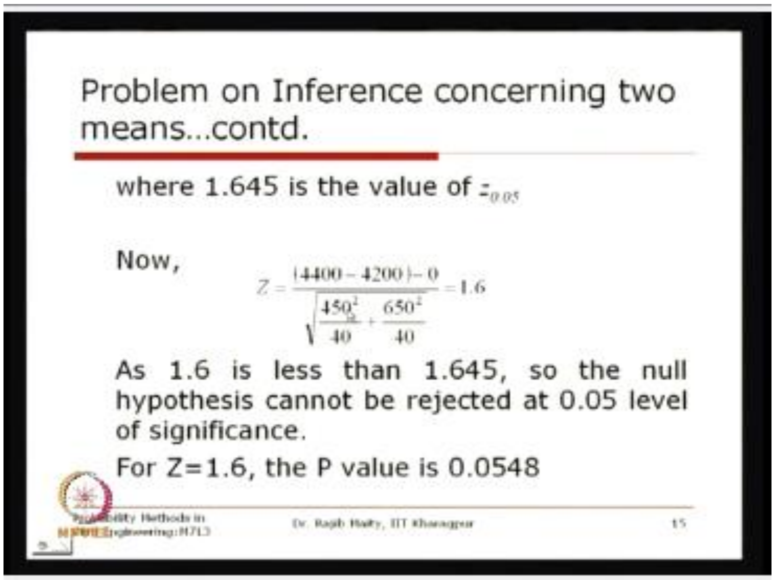

So this is the value of this Z at 0.05.So this Z now if we just use those the sample statistics and we will get that is 1.6 so which is 1.6 < 1.645 so the null hypothesis cannot be rejected at 0.05 level of significance. So we cannot say that whether the sample one is really superior than the sample two because we have seen that we failed to reject the null hypothesis. As I mention earlier also we should not say that null hypothesis is accepted.

We generally we should say that either the null hypothesis is rejected or it cannot be rejected with the sample data because we have to take then larger sample and for further test.

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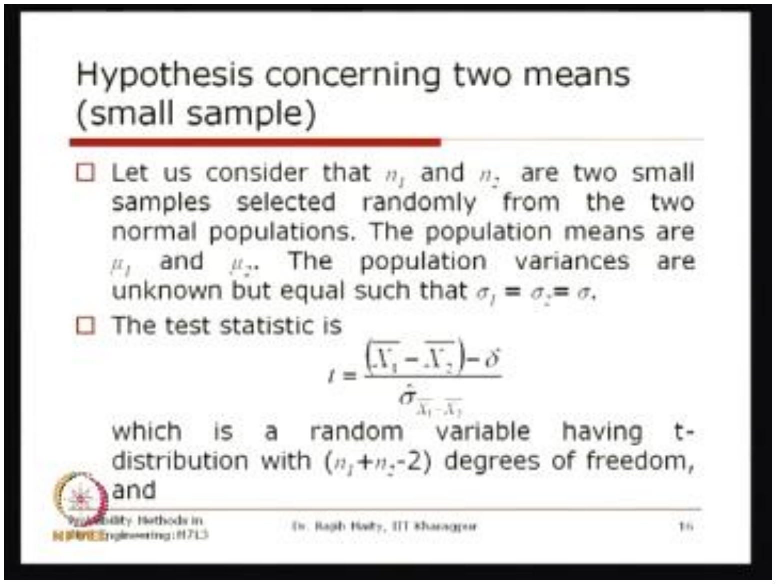

Well so regarding this two sample means again if either of these two sample the sample size is less than 30 then this should be consider as a small sample and this small sample as we have seen in this single mean also it follows a t- distribution. So here the statistics is your $\bar{x}$- $\bar{x}_2$ - Δ and this sigma hat $\bar{x}$- $\bar{x}_2$ that this is a estimate of this difference between two mean standard deviation. So which is a this test statistics is a random variable having a t- distribution with n1 + n2 - 2 degrees of freedom.

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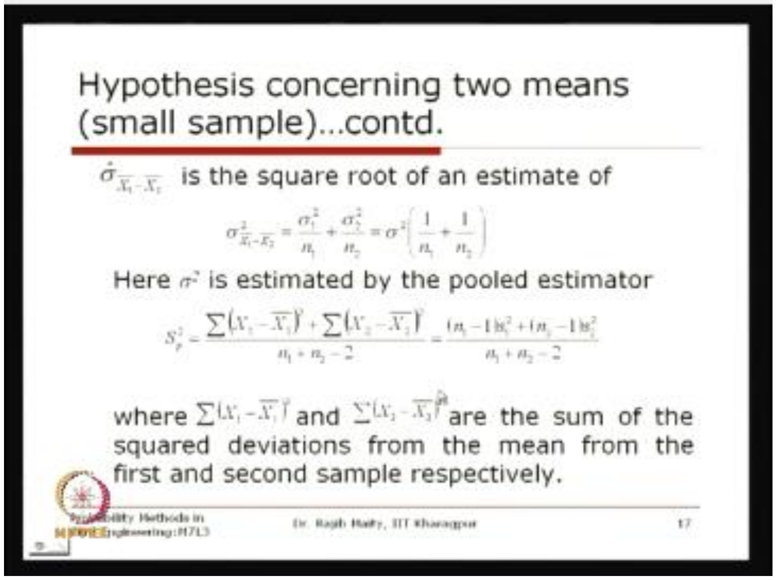

And this estimate for this standard deviation is a square root of an estimate of this one, so this variance is $\sigma 1^2$ / n1 + $\sigma 2^2$ /n2 like this. So if this population variance is known and if it is not known then it should be estimated by the pooled estimator it is called the pooled standard deviation or pooled variance Sp square which is we will get from this one. So X1 - $\bar{x}1^2$ + the summation of X2 - $\bar{x}2^2$/n1+n2/2.

Which is basically that n1 minus 1 times of this standard variance of this first sample and n2 minus 1 times the variance of second sample divided by n1 + n 2/2 where these two quantity as the sum of the square deviation from the mean of the respective samples.

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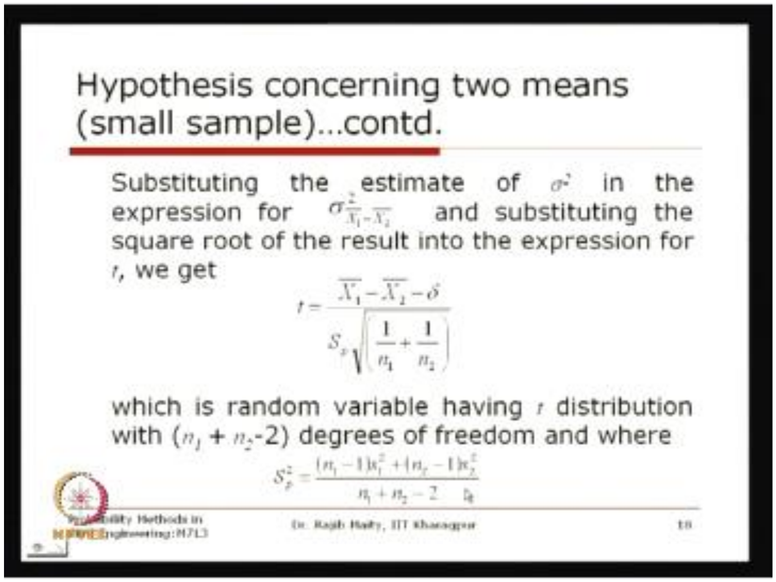

So if we substitute the estimate of this σ^2 in the expression of this standard this variance then we get that this $t = \bar{x}1 - \bar{x}_2 - \Delta / St \times \sqrt{1/n1 + 1/n2}$ which is a random variable having t-distribution with n1 + n2 - 2 degrees of freedom and this Sp can be obtained as this $Sp^2 = n1-1 \times s1^2 + n2-1 \times s2^2 / n1+n2-2$.

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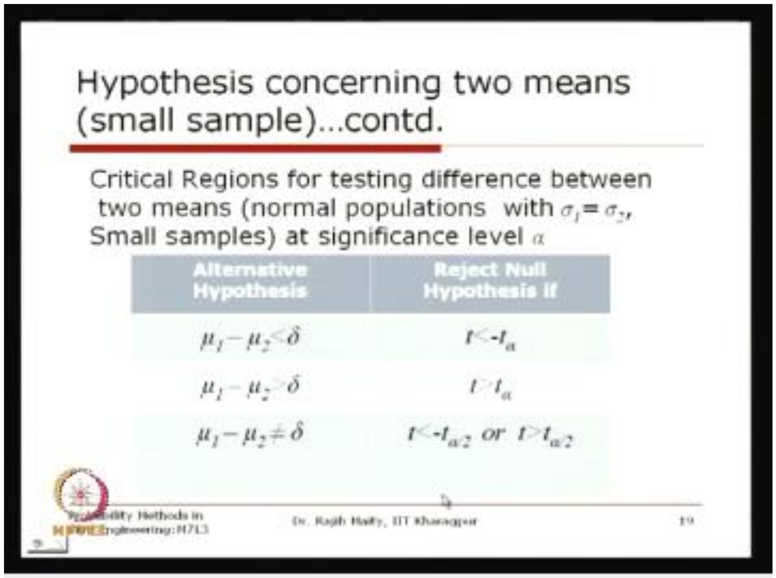

And the critical regions are these for that test statistics for this left sided test that is whether the $\mu 1 < \mu 2$ then this whether the $t < t\alpha$ then in this case $t > t\alpha$ and two sided test whether $t < -t\alpha/2$ or $t > +t\alpha/2$.

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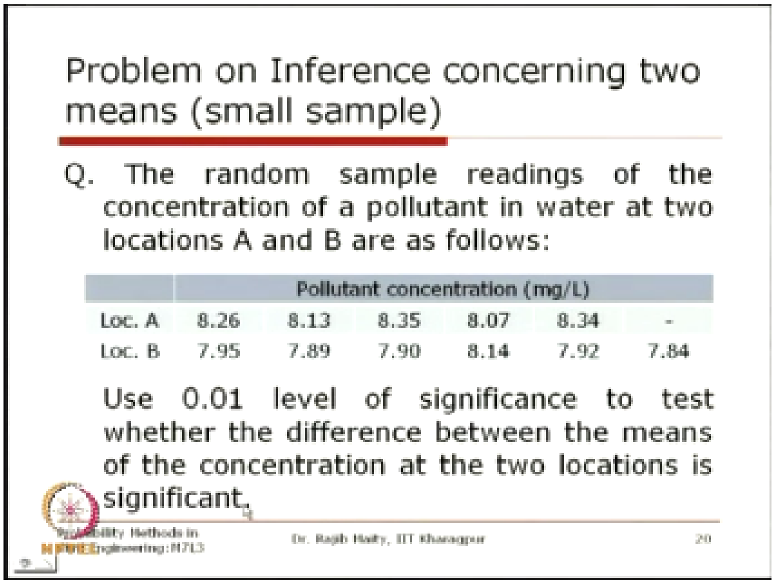

So we will take one problem on this small sample test is like this that the random sample readings of the concentration of the pollutant in the water at two locations A and B are as follows so there are two locations one is that location A and other one is this location B and for this location A we are having 5 samples and for this location B we are having 6 samples so this problem is taken because you see that it is not required that we should have this same length of this data so that n 1 and n_2 need not be same so for the sample one we are having 5 samples and for the sample two we are having 6 samples.

So use the 0.01 level of significance to test whether the difference between the means of the concentration at two locations is significant or not so then what we have to do.

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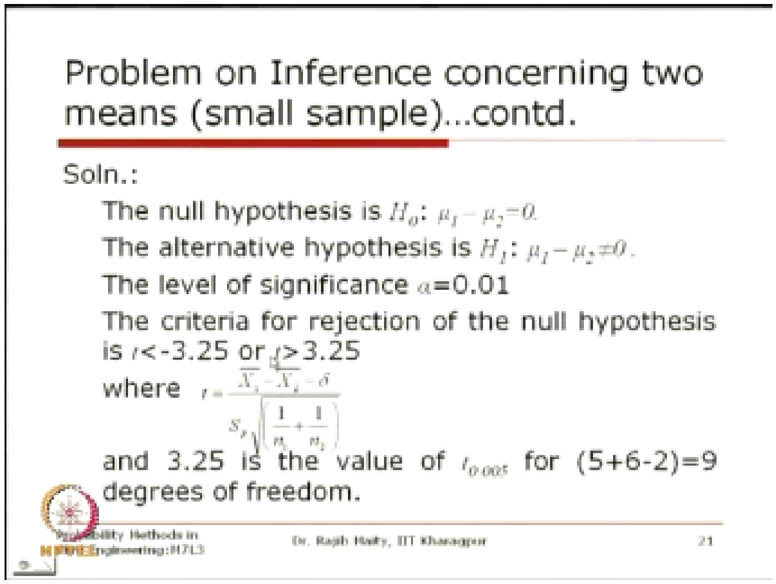

We have to first we formulate that hypothesis then the null hypothesis is that whether the difference is significant or not so this is a two sided test and the null hypothesis we put that μ 1 – μ 2 is equals to 0 because we are trying to test whether these are different or not μ1 - μ 2 is not equal to 0 that is what our test goal so this we have put in this alternative hypothesis and this mu 1 - μ2 equals to 0 is your null hypothesis and here the level of significance σ is your0.1 the criteria for rejection of the null hypothesis is that if this t is less than - 3.25 or t is greater than3.25.

So these two values are basically the statistics value at the significance level 0.01 and for the t distribution having the degrees of freedom equals to this sample lengths.

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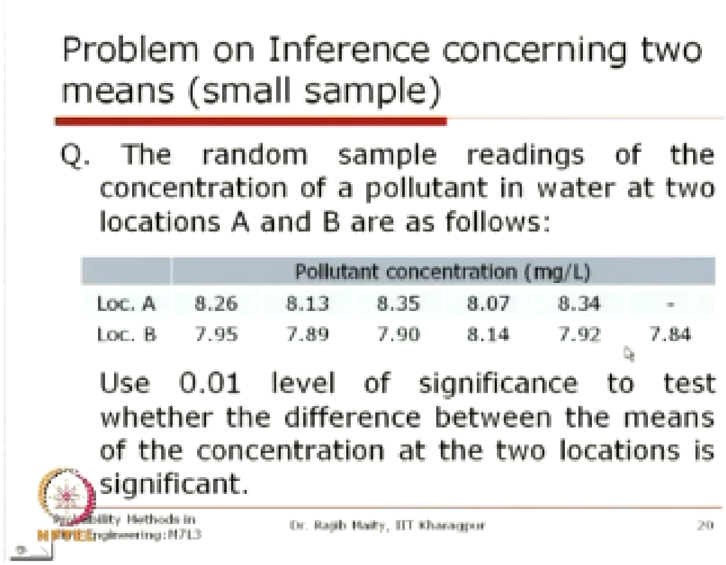

Say this is your 5 this is your 6 so 5 + 6 = 11 - 2 so 9 so with the degrees of freedom 9 at significance level 0.01 we get these two values that 3.25 and 3 - 3.25 and 3.25 so if our test statistics is falling in this region then we should reject the null hypothesis and the test statistics that is that X_1 bar - X_2 bar- σ divided by S_p √1 by n_1 +1 by n_2 so this X_1 bar is the sample mean for the first sampleX_2 bar is the sample mean for this second sample that we can get whatever the data that we are having and this Sp that is the pooled variance sorry pooled standard deviation we should also calculate whatever we have discussed so far.

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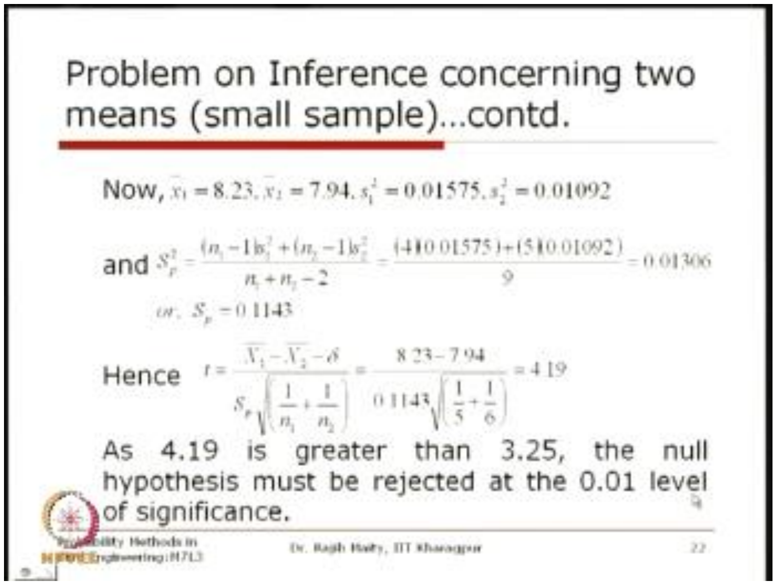

So here are the calculations this X_1 bar which is your mean for the first sample which is 8.23 and mean for the second sample is your 7.94 variance for this first sample is 0.01575 and variance for this second sample is .01092 so this sample estimates we have already learned so we can apply that one for whatever the data is given and we will get this sample estimates now this pooled variance is that n 1 - 1 times this standard deviation of this first sample sorry variance of this first sample multiplied by + that n_2 - 1 times of this variance of second sample divided by $n_1 + n_2 - 2$.

So we will just use this one so this first sample as that there are 5 samples that is 4 multiplied by it is variance and then second sample size is 6 so 6 - 1is 5times it is variance divided by 5+ 6 - 2 which is 9 so we will get that .01306 so the Sp here is equals to your 0.1143 hence the t is equal to X_1 bar - X_2 bar - σ divided by $S_p\sqrt{1}$ by $n_1 + 1/n_2$ so if we put this one we will get the statistics as 4.19 now this 4.19 which is greater than your 3.25 it is falling in this right side of their distribution and which is obviously greater than the critical value so the null hypothesis must be rejected at this 0.01 level of significance we can say so in this null you can say that it was there.

That this both this sample means are same so we should must reject this null hypothesis that means we can infer that the mean from both the locations are not same well.

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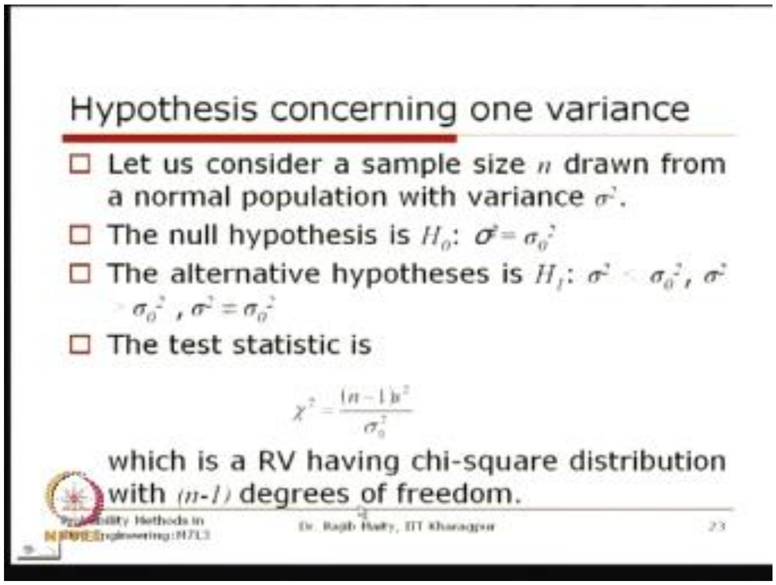

So far we have seen that what is the test for the one mean and two means and now we will see that the variance first we will start with this one variance and thus then we will go for the second variance and let us consider a sample size n drawn from a normal population with variance σ^2 and this null hypothesis is h_0 equals to $\sigma^2 = \sigma_0$ and the alternative hypothesis is H1can be say that less than σ^2 this is greater than σ_0^2 and this is naught equal to σ^2 so anything this can happen so and depending on this null hypothesis also will change.

So this is basically for this alternative hypothesis when this is naught equal to this and in our last lecture when we are doing this sampling distribution also we have seen that this variance that is n - 1 where n is the sample size that times they are variance divided by σ_0^2 this ratio is basically following a chi- square distribution so this chi square so this statistics we can use to test for the single sample variance so this is whether one variance and this σ_0 is your some threshold value for this which we are inferring from this population and this quantity this statistics is following which is having a chi square distribution with n - 1 degrees of freedom.

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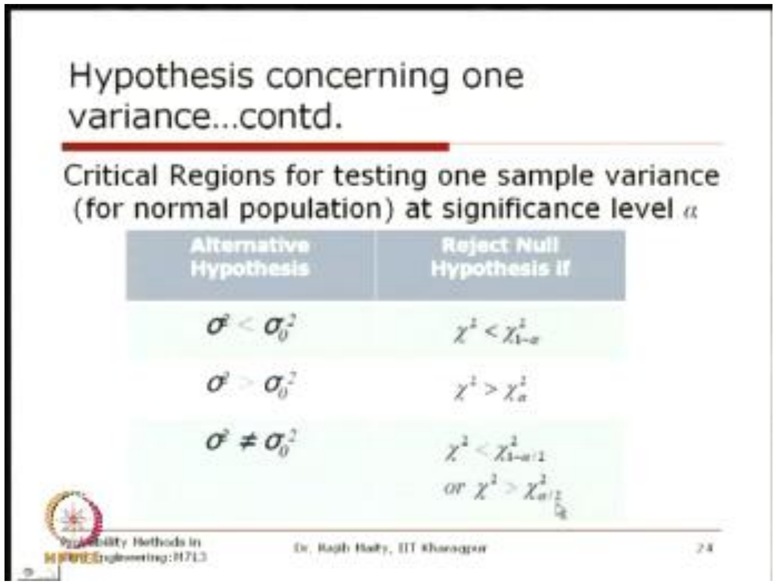

Now the critical region for testing this one sample variance and here the assumption is that this population from where the sample is drawn is having a normal distribution at this significance level α if it is one sided test and if it is left sided test then whether the chi square the statistics that we have discuss so far that is n - 1 time variance divided by sample variance divided by this population variance that statistics if it is less than this chi square value at 1 – α cumulative distribution if it is less than that then.

We should reject the null hypothesis and if it is for this right side test if it is greater than that chi square α then we will reject this null hypothesis and for the two sided test if it is less than1 - chi square 1 – α by 2 or greater than chi square α by 2 then we should reject this null hypothesis if we just see quickly that this typical shape of 1 chi square distribution.

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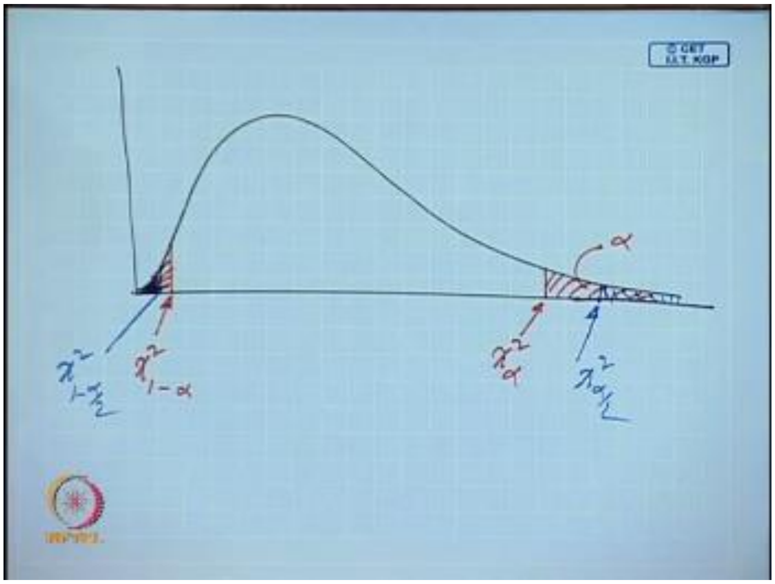

It looks like this so this is not symmetrical so here what we are referring to that this left side so this area is basically what we are telling so this value what we are referring to as this $X^2_{1-\alpha}$ and the same area for this that when we are doing for this right hand side test, suppose, that this is your this area if is your what is that α, then this value what we are saying that is X^2_{α}. So, here the $X^2_{1-\alpha}$ means, the total area below this curve is one, so this full area apart from this red one, this starting from the white area as well as this shaded area is your 1-α and these are for this one sided test.

Now at this significance level of this α if we go for this two sided test then obviously we are testing that 1-α/2 which is obviously will be little smaller than this one, so this one and this will be somewhat greater than here, so these two values which is your $X^2_{1-\alpha/2}$ and this is your $X^2_{\alpha/2}$ okay, and this area, this now what I am now, I am shading the blue area here plus the blue area should consists it should be equal to α and as this X^2 distribution is not symmetrical so here these two values that we are looking for this critical value will not be symmetrical, as we have seen so far in case of that standard normal distribution and as well as in the distribution.

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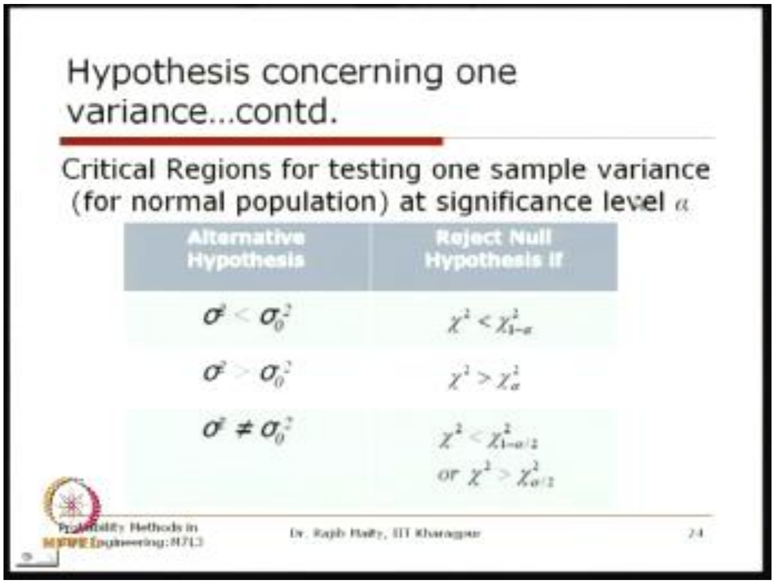

So, these are the critical regions for those for the one sample for the chi square distribution, for one sample variance.

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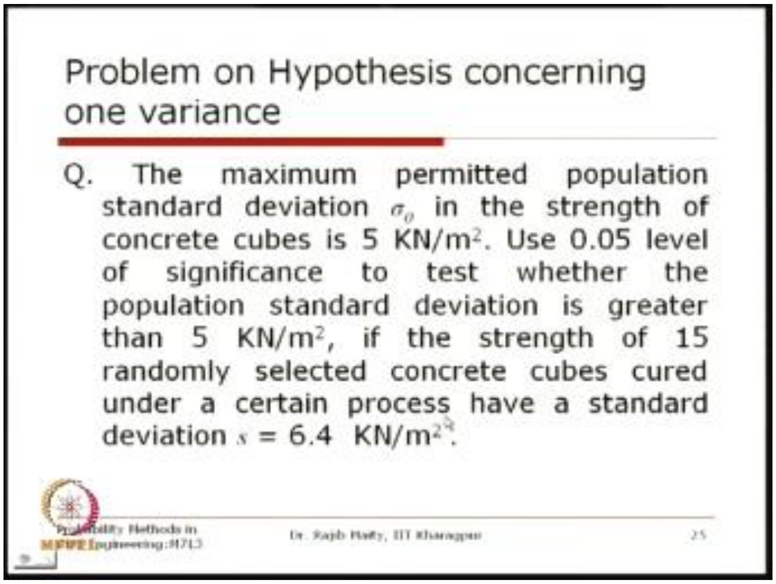

Well, take one example here, the maximum permitted population standard deviation σ_0 in the strength of a concrete cube is 5 KN/m^2, so this is our requirement use that 0.05 level of significance to test whether the population standard deviation is greater than this threshold value, that is,5 KN/m^2, if the strength of 15 randomly selected concrete cubes cured under a certain process have a standard deviation equals to 6.4 KN/m^2.

So, we have taken randomly 15 samples and that sample standard deviation is 6.4, if we just compare this 6.4 and 5KN/m^2 this is obviously greater than. Now, through this hypothesis testing for this variance, one variance we have to say that whether probabilistically we have to infer, whether this value is really greater than what is our that requirement.

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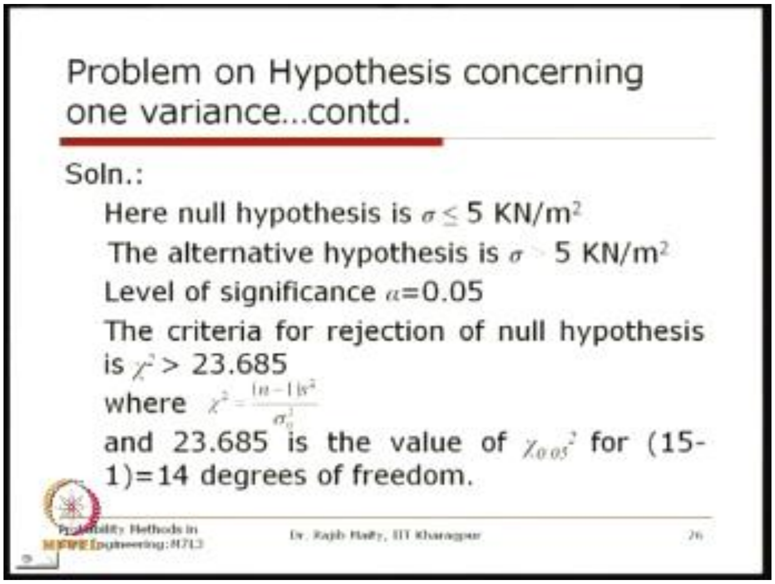

So our null hypothesis is that whether this $\sigma \leq 5$ KN/m^2, because we are targeting to test whether thus that population standard deviation from which the sample is taken is really greater than 5 KN/m^2 or not. So the level of significance here is that 0.05, the criteria for rejection of this null hypothesis is this $X^2 > 23.685$.Now, where from we got this 23.685 is the value for the α 0.05, where this chi square distribution having a degrees of freedom equals to (15-1)=14.

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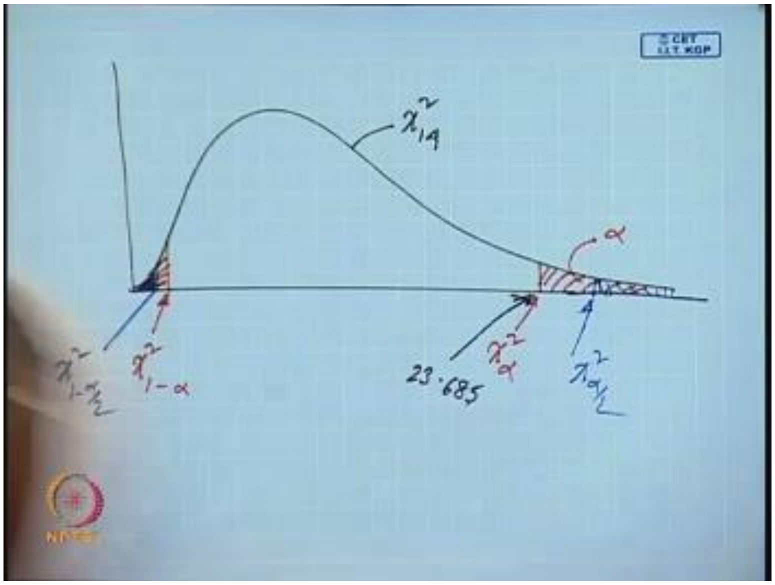

So, now, if we see that, if I say here that, this is your the X^2 distribution with 14 degrees of freedom then, that this $X^2\alpha$ that we are saying that, if we just say that this red area is your 0.05 then, this value is your 23.685, which you can get from any standard text book, these tables are this X^2 tables are given here. So, if my test statistics falls in this one you know that this is your critical zone, so if the test statistics fall in this zone, then we have to reject the null hypothesis, and if it is below this one so we cannot reject the null hypothesis with the sample data that is available with us.

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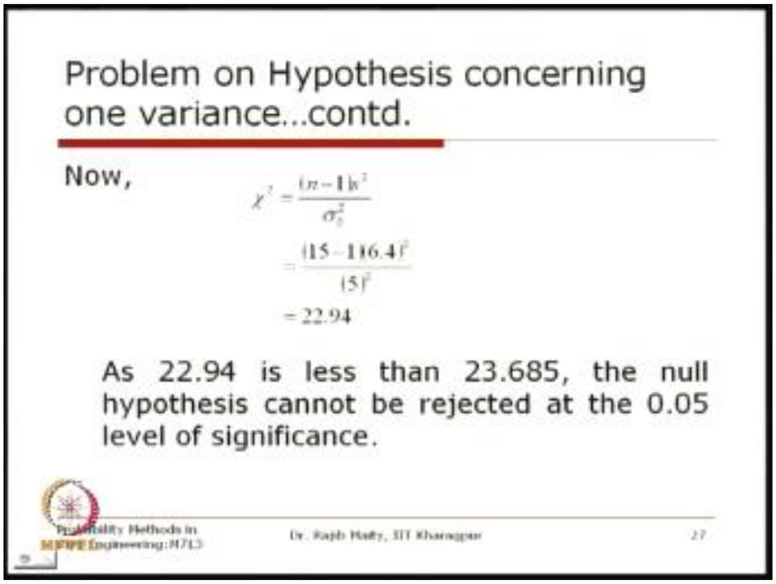

So, this is why this 23.685 is our critical value, and if we do this, if we calculate this test statistics then with this sample data, it is now n=15-1 and 6.4 and divided by this $|5|^2$, then we get that 22.94 which is definitely less than that our critical value so null hypothesis cannot be rejected at 0.05 level of significance that means we cannot say that whatever the sample data that we are having from that we cannot infer that the population standard deviation is greater than our what is our target that is, what is our requirement 5 KN/m^2 it is not greater than that.

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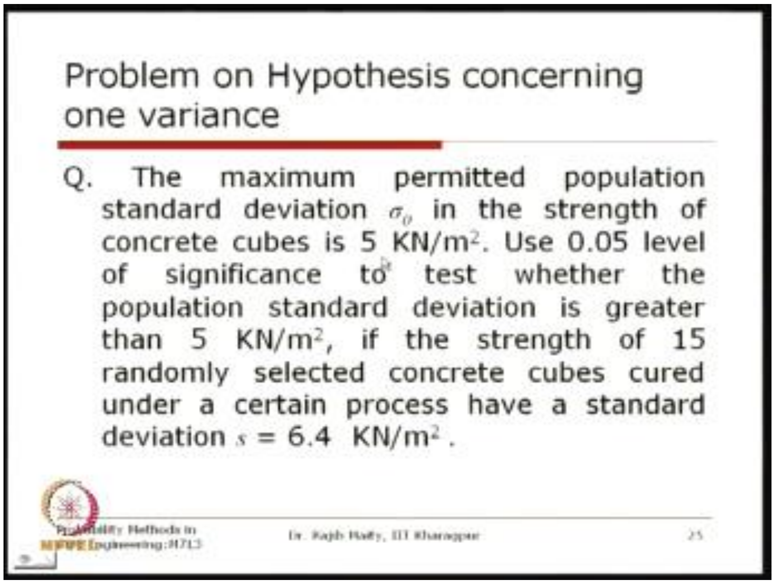

Even though we have seen that numerically, the sample standard deviation is something 6.4 which is clearly greater than this 5, but probabilistically we cannot infer that.

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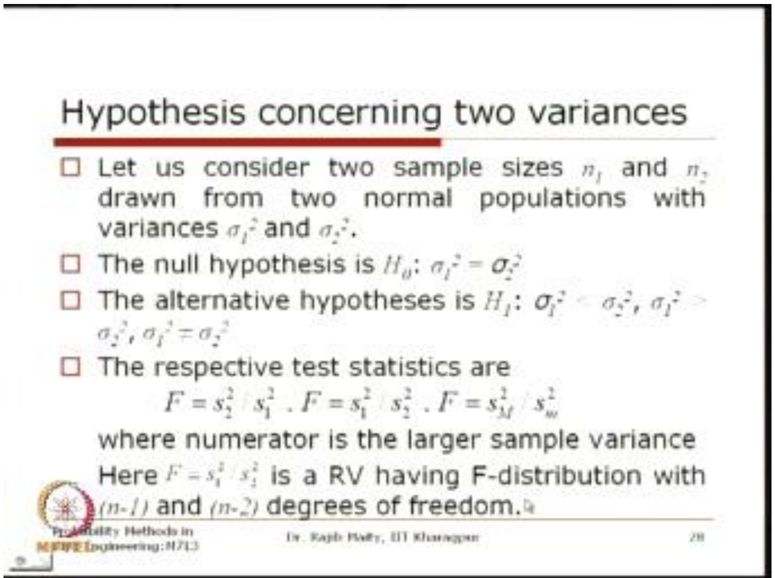

Well, now we will take that hypothesis concerning two variances let us consider two sample size of n_1 and n_2 drawn from two normal populations, so this is that requirement that whatever sample we are drawing it is from the normal population with their variances σ_1^2 and σ_2^2. So, here, the null hypothesis is can be like this, so σ_1^2 is equals whether they are equal to or 1 is less than or, so 1 is less than other like that.

So, here the test statistics are depending on what is your alternative hypothesis, whether the $\sigma_1^2 < \sigma_2^2$ or $\sigma_1^2 > \sigma_2^2$ or they are not equal depending on that the test statistics is s_2^2/ s_1^2 or s_1^2/ s_2^2 or $F= s_M^2/s_m^2$ where the numerator is the larger sample variance. So, you see, for the first test, when we are saying that this σ_1 testing, we are trying to test that whether σ_1^2 is less than σ_2^2 then, we have to test that the sample variance for the second sample divided by sample variance of the first sample, this statistics we should use.

On the other hand, we should use that σ the variance for the first sample divided by variance for the second sample, if the first sample is greater than the second one. And if we are testing the two sided one then, we have to find out which one is the greater so, that greater value should be in the numerator and this F is having a F distribution with n-1 and n-2, so this is basically, that this n when we are writing here, this is there is something typing mistake here, so this the first one is for the numerator.

So, whatever in this statistics whatever is there in the numerator that should be your first degrees of freedom and whatever is there in the denominator that should be your second degrees of freedom. Now, if I just want to take all these thing in a single statement, then whatever we are testing always the F statistics that we are calculating, it should be always we have to keep in mind that should be always greater than 1, that mean ,in the numerator always we are putting the greater value and this numerator and the denominator we are putting the smaller value.

So, if we can ensure that then we can say that F statistics is having a F distribution which is following the F distribution with the first degrees of freedom is the larger sample size -1 and second degrees of freedom is the smaller sample size -1, this is not 2, this is also 1. So, these two degrees of freedom with that is a F distribution.

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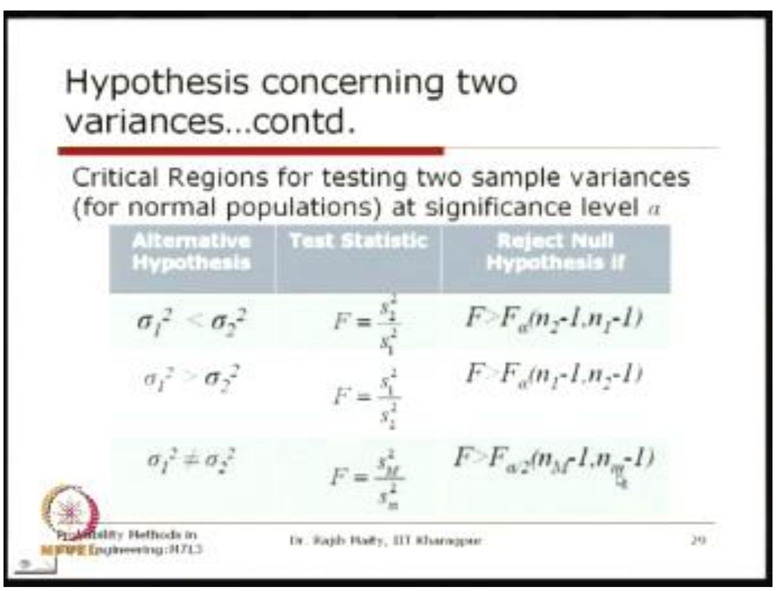

So there are again the three different cases and there are three different test statistics and we are, if we are just taking in this way whether with the first one is less than the second one or first one is greater than the second one or they are not equal then, here you can depending on that you have to find out what is your critical region here. And on the other hand, if you just want to just what I just now mention that, if you just simply want to use that which one is your larger variance, which one is your smaller variance, then always for all these cases you can calculate these F statistics, which is following that F distribution having this n_M -1 degrees of first degrees of freedom and second degrees of freedom is n_m-1.

Where n capital subscript M is the larger sample size and n subscript small m is a smaller sample size.

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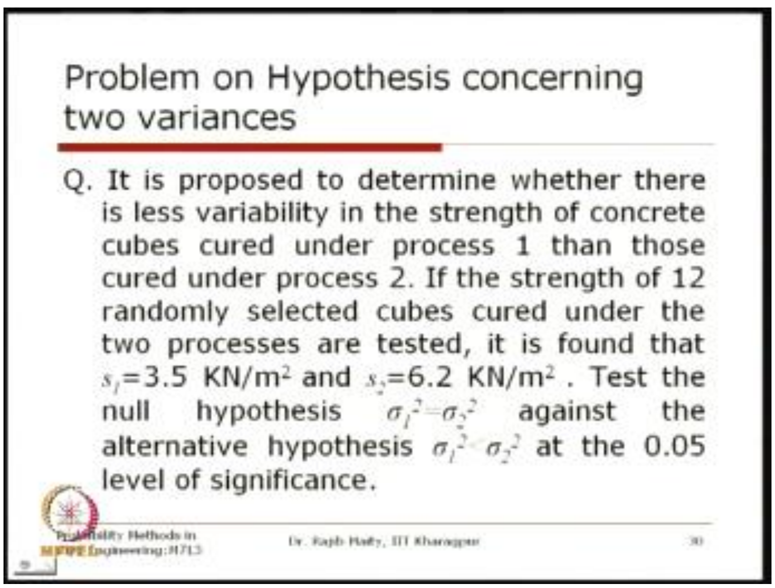

Well, take up one example here, it is proposed to determine whether there is less variability in the strength of concrete cubes cured under process one, than those cured under process two. So, there are two different process is identified and we have to test whether the strength of concrete in one process is different from the other process or here we are testing that whether in this process one what we are getting is the strength is less than this process two, before I can declare that we should follow the process two for curing the concrete, so that we can achieve that greater strength.

So sample data is taken, so like that there are 12 randomly selected cubes are there, under two processes are tested, it is found that this s_1 is 3.5KN/m^2 and s2 is 6.2 KN/m^2.Test the null hypothesis there are seen against that alternative hypothesis, whether the first one is lesser than the second one at the level of significance 0.105.

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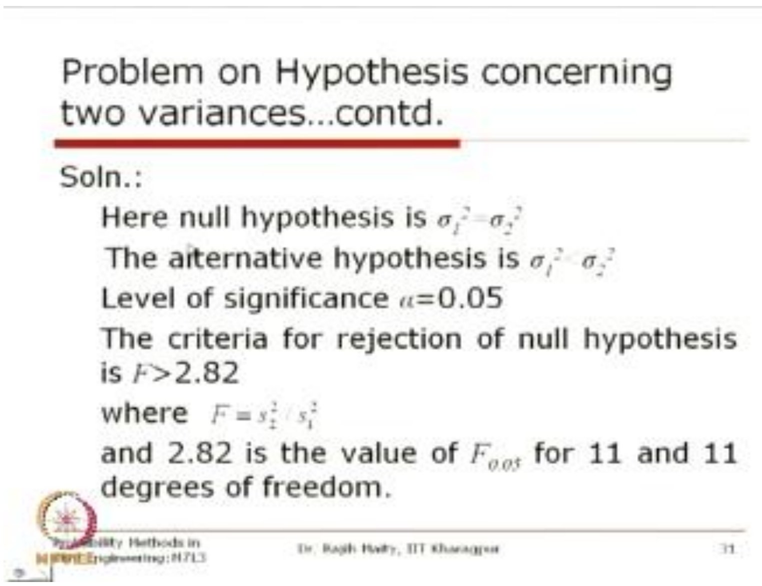

Here the null hypothesis is this and alternative hypothesis whether the σ1< σ2 square level of significance is 0.05.And this criteria for rejection of this null hypothesis whether if the statistics is greater than 2.42, so how we are getting this one. So, this is the f distribution having the degrees of freedom of the sample size.

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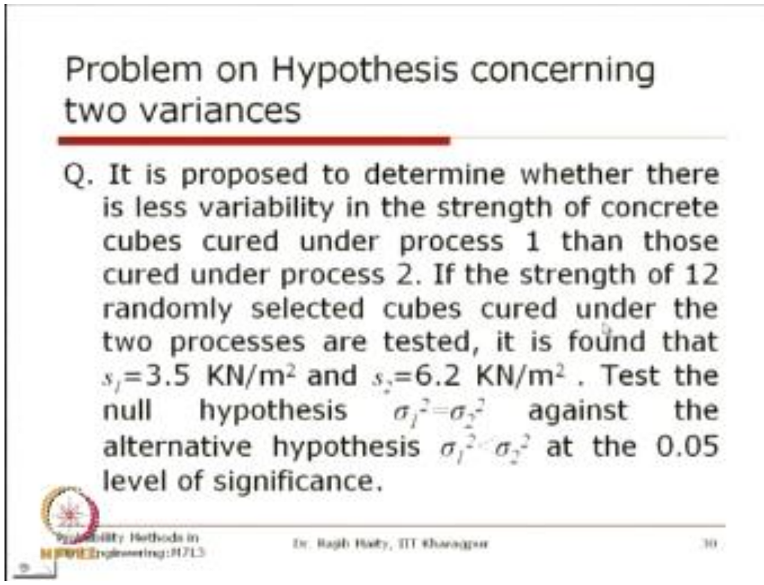

That is in both the cases the sample size is 12 here, so 12- 1, so 11; first degrees of freedom is11, second degrees of freedom is also 11.

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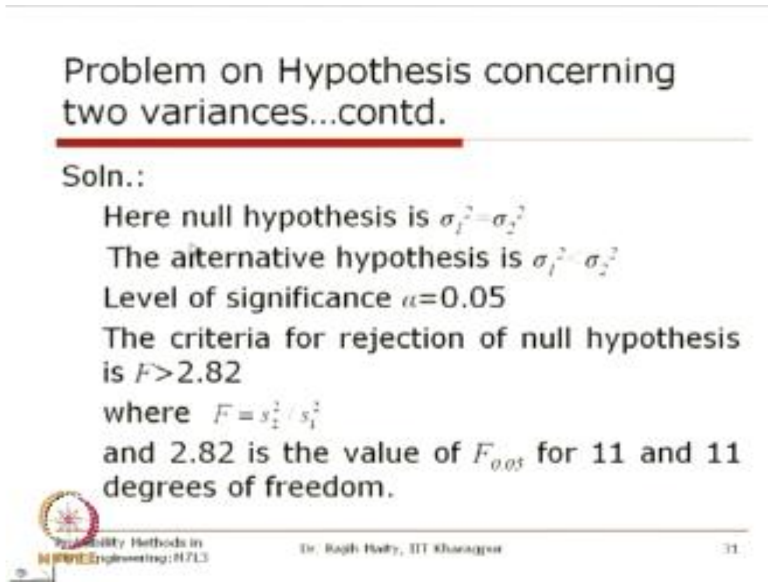

So where this F = s_2^2/s_1; s_1^2 and you have seen that this s_2 is your 6.2 and s 1 is3.5, so s_2 is greater than that, so if we use this test statistics s_2^2/s_1, so it is that anyway here in this case both the sample sizes are same.

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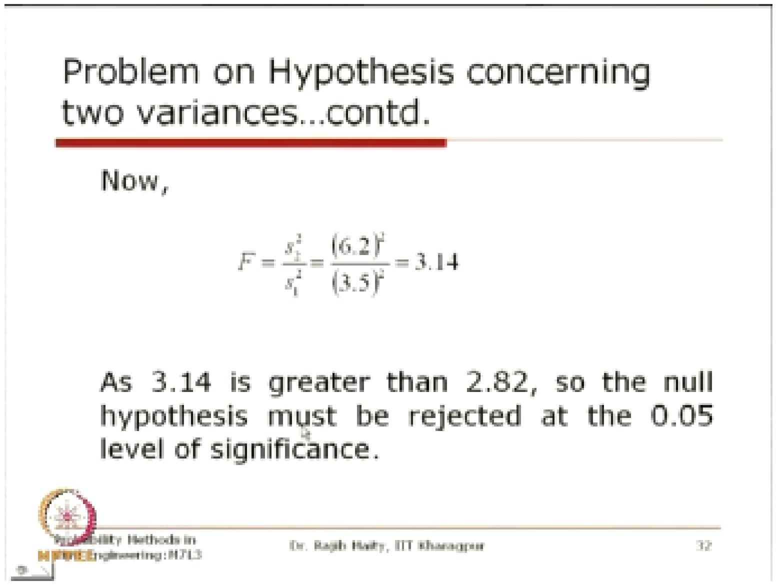

Here the test statistics counts to be 3.14as this 3.14 > 2.82, so the null hypothesis must be rejected at 0.05 level of significance.

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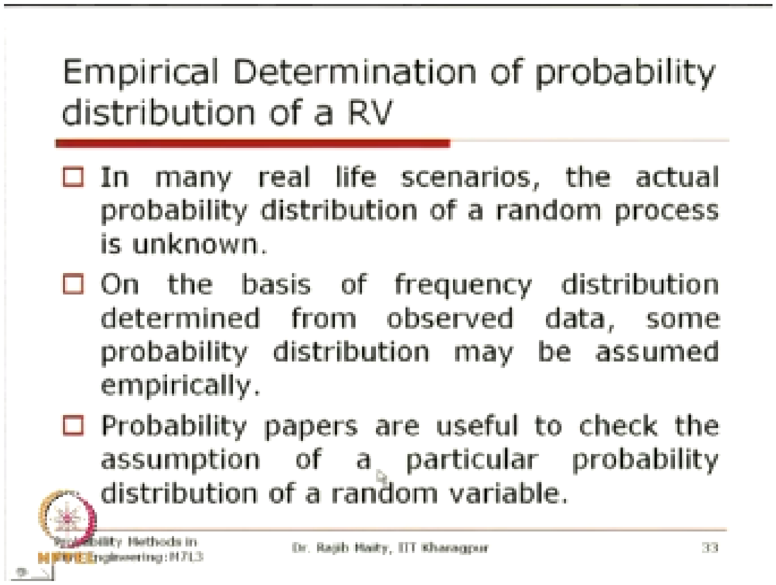

So far in this hypothesis testing what we have seen that, we have first found out in the earlier lecture, what are the sampling distribution of those test of those the that estimated value the sample estimation. And in this hypothesis testing, we are basically trying to infer something about the population from which the sample is wrong. Suppose, that we are having this some sample of size n and we estimate what is it is mean and if we want to infer something from that sample estimate and we want to infer something regarding the population.

Then, we have seen that, we have tested for this single mean, we have tested for the two means and again, we have taken the single variance, we have taken the two variances. Like this, we can test and there are several, we have even used different examples from this civil engineering. There should be some many examples like this, where you can use this theory to infer something about their population.

Because always the sample that we get is limited. So, before we can infer something about their population, we have to use this test properly, so that we can judge something. Even though we get some numerical values from the sample, just by this comparison of this numerical value can show you something else, but whether that is really significant, because always you take one sample and take the another sample from the same population, the statistics may not be same then, mean for both the samples may not be exactly same.

So, those differences whether they are really significant from the statistical point of view or not to test that one whatever the discussion we have done so far, with respect to the single mean, two means, single variance, two variances that we have to follow. Now, we will take some time to spend on that, because we have seen that in many times I mention that look at this data is following that distribution.

Now, we are having that data, now with the data how can we say that this data is coming from a from a population which is following this distribution. So, there are basically two things, one is that first we will do some graphical approach and there are some statistical test also for these two infers that whether, this is really following that population from which the sample is drawn is really following a particular distribution or not.

So, we will start that one and the first thing is the graphical representation which may be which we generally test through this probability paper. So, the construction of this probability paper and the testing that how, whether it is really following or not graphically that will see now.

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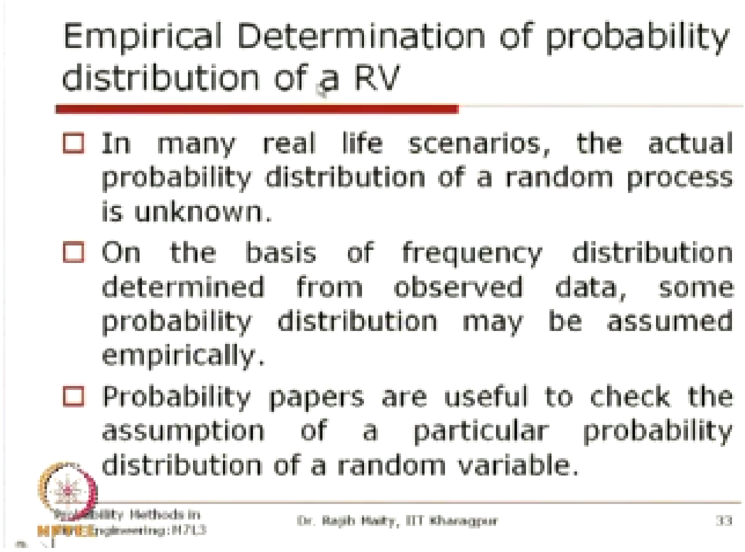

So, the empirical determination of the probability distribution of a random variable, so in many real life scenario the actual probability distribution of a random process is unknown. So on the basis of the frequency distribution of the sample data that we are having, so determine from this observed data which is the sample available to us some probability distribution may be assumed empirically.

Probability papers are useful to check the assumption of a particular probability distribution of a random variable. Say that, I have a sample I am saying that this sample is taken from a population which is distributed normally. Now, we have to use a normal probability paper and plot that data and then, we will just discuss how we can infer that or the visually how we can inspect that whether it is really following the normal distribution or not.

And again, we will take some statistical test to probabilistically infer whether really that is following the distribution or not.

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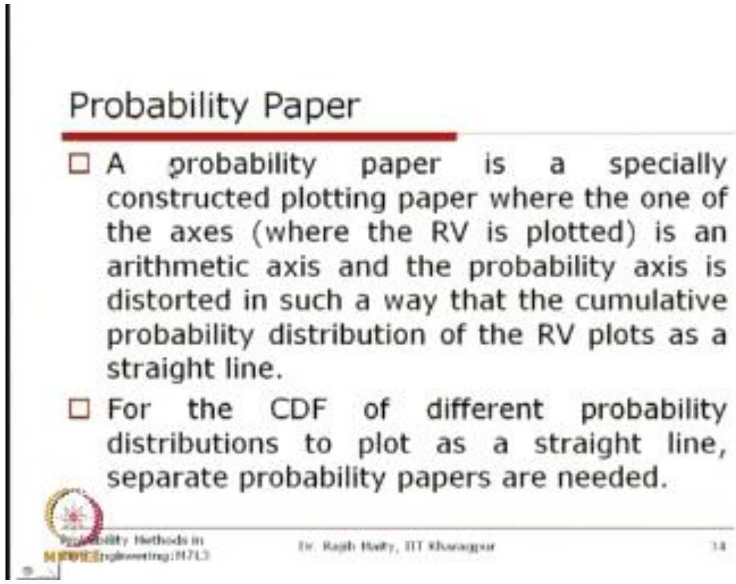

So, a probability paper is a specially constructed plotting paper, where the one of the axis, where the random variable is plotted is an arithmetic axis and the probability axis is distorted in such a way that the cumulative probability distribution of the random variable plots appears as a straight line. So, there will be one appears as a straight line, so for the CDF- that cumulative distribution function - of different probability distribution to plot as a straight line separate probability papers are needed.

So, if I want to test that whether it is following a normal distribution then, I have to use a normal probability paper, and if I want to test whether it is following a exponential distribution, so we have to use the different paper. Now, how these papers are constructed and how it is tested we will see now.

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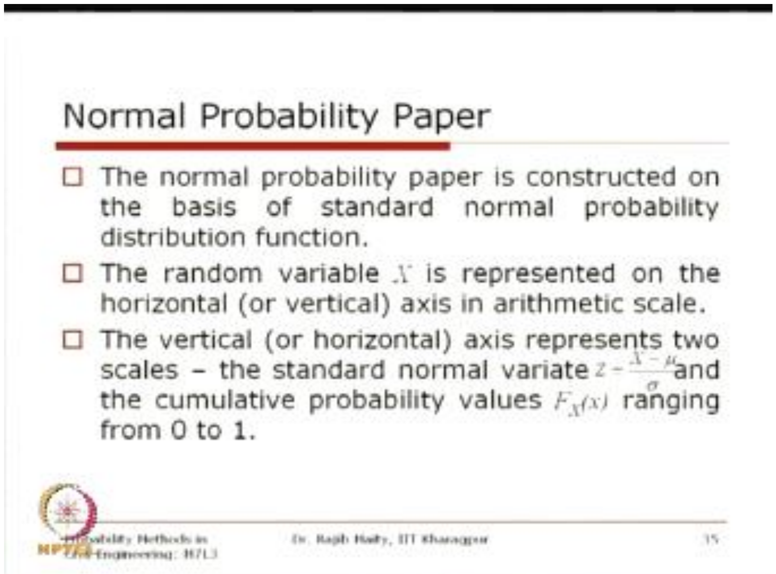

So first, we are taking this normal probability paper which is most widely used so to test that whether the sample data belongs to a population which is normally distributed. So, this normal probability paper is constructed on the basis of standard normal probability distribution function. The random variable X is represented on the horizontal or sometime in some cases vertical axis also, but mostly this random variable is generally represented on the in the horizontal axis and that axis is a arithmetic scale.

The vertical axis or horizontal, if I just reverse this one as I was telling that if this X is on the vertical end, this one will be the horizontal otherwise in most of the cases it is vertical axis. So, the vertical axis represents the two scales, the standard normal variate Z = X- μ/ σ and the cumulative probability values Fx ranging from 0 to 1.

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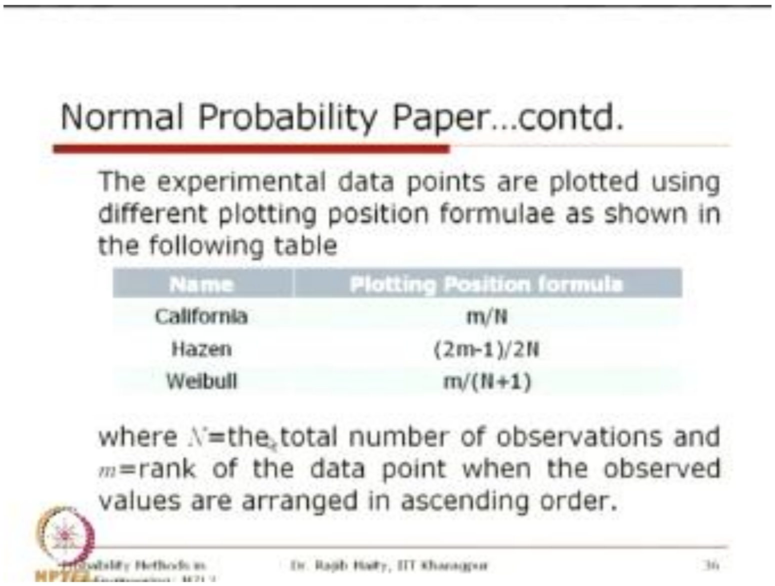

Now, before I go further, if I just see it graphically how these things, how the concept is taken, you know that if this.

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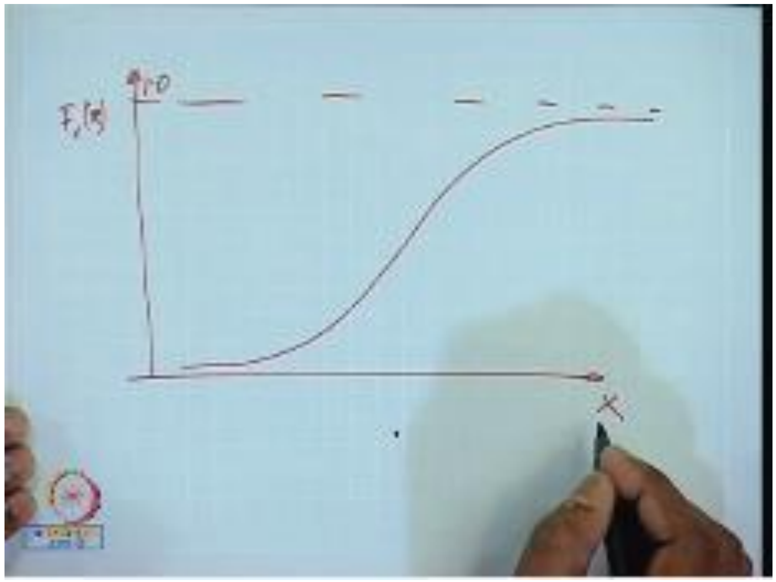

Is your that X this is the axis for your the random variable and if this is your that cumulative probability axis which is that FX for this specific value x, which is a cumulative distribution, you have seen earlier towards the beginning of this course this distribution is generally looks like this, so which is asymptotic to0 at this minus infinity and which is asymptotic to one at plus infinity. And we have also seen that it is from this, if it is a standard then, it is from this minus 3to plus 3 almost, that most of this probability is exhausted here.

Now, if I just take that this is you say that for this standard normal distribution, if we just see so here the mean is coming approximately say 0, and this is a -3andthis is a something +3 or in some computer application, sometimes even we can go that -5 to plus 5 almost very closed one probability is exhausted there.

So, what we generally want to do is that, basically, whatever the sample data that we are having now if I just watch now, that this is also that arithmetic scale and this is also the arithmetic scale .Now, if you plot that data, whatever the data that we are having with respect to their cumulative probability, how to get the cumulative probability, we will discuss. So, if we can plot that one and if it follow approximately this line then, we can infer that yet it is following this particular distribution, but by that I inspection, it is very difficult to say whether this particular shape is following or not.

Rather, we can say, if we can distort this axis in such a way that this will appear as a straight line then inspecting by I that whether, this is following a straight line or not that is easier than comparing that whether it is following this particular curve linear path or not. So, to make this axis distorted what we generally do is that, we generally take it straight-line between this. And now, let us create one new axis here such that, suppose, that this I am starting from 0.25, so I am starting from this line going up to this point.

And then, I am going to this straight line, and from this one I am just giving the name, giving the number as 0.25.Similarly, suppose, that I am starting from0.75 here, I will go to this one, first come to this straight line and go here, so that I write that 0.75.Similarly,from 0.05,0.1,0.15like this all these points whatever is there, if I just go there and distort this axis, basically, this axis I am just squeezing some part expanding some part, in such a way that this red curve linier path is becoming stretching to a straight line.

And in that axis - in that distorted axis- if I plot whatever the data that we are having that if it appears that straight line, then we can conclude that it is following the normal distribution. Now, with this x axis keeping in the same axis and this distorted probability axis, whatever the paper that we get that is known as this probability paper. Now, this example, that I have shown this is for the normal distribution and this similarly, this can be done for this other distributions as well. Now second question is, how we will get what is their cumulative probability?

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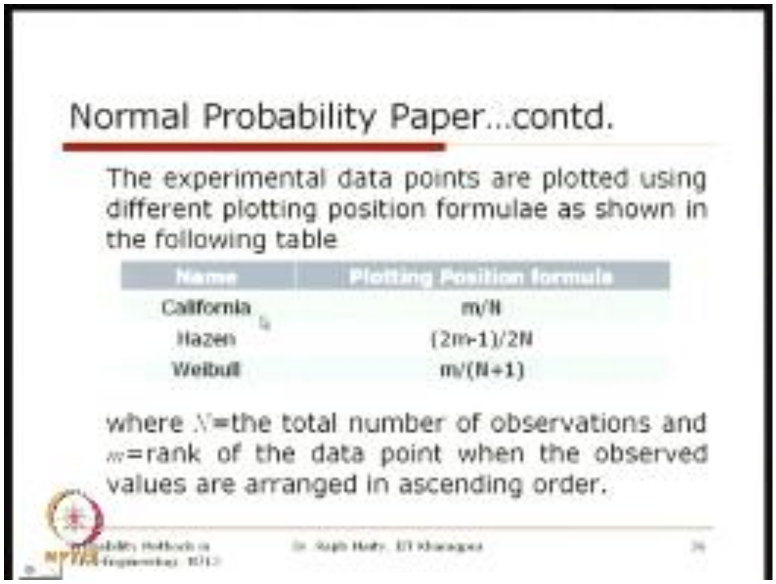

For that there are different plotting position formula is available; for example, the California gave that m / N, Hazen gave the 2 m -1 /2 N, Weibull gave the formula m by N plus 1.Where this N is the total number of observation that is the sample size and m is the rank of the data point when the observed values are arranged in ascending order.

So, this plotting from this plotting position, generally, this Weibull plotting position is mostly used, using this one we get this cumulative probability distribution, and we get that probability value and then, we plot it on this different probability paper. So, we can first plot it on this normal probability paper and check whether it is coming to be the straight line or not and other probability papers also from what it is following this straight line. We should conclude that this the population of this sample is following that particular distribution.

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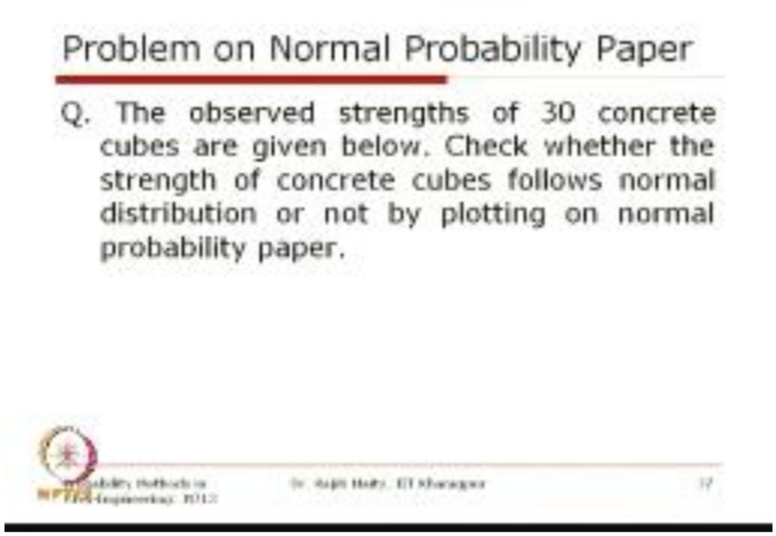

Well, take one example here, the observe strength of 30 concrete cubes is given below means, in this table next slide, check whether the strength of the concrete cube follows the normal distribution or not, by plotting on this normal probability paper.

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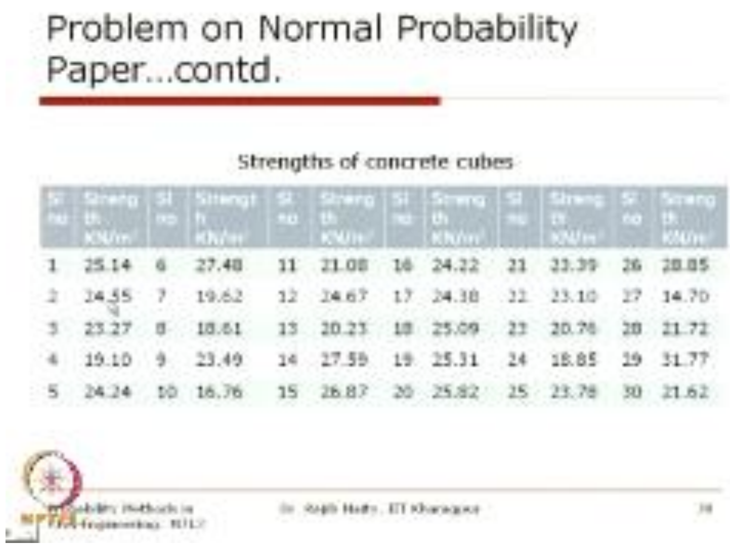

Problem on Normal Probability Paper…contd.

Strengths of concrete cubes

Sl no	Strength KN/m²	Sl no	Strength KN/m²	Sl no	Strength KN/m²	Sl no	Strength KN/m²	Sl no	Strength KN/m²	Sl no	Strength KN/m²
1	25.14	6	27.48	11	21.08	16	24.22	21	23.39	26	28.85
2	24.55	7	19.62	12	24.67	17	24.38	22	23.10	27	14.70
3	23.27	8	18.61	13	20.23	18	25.09	23	20.78	28	21.72
4	19.10	9	23.49	14	27.59	19	25.31	24	18.85	29	31.77
5	24.24	10	16.76	15	26.87	20	25.82	25	23.78	30	21.62

So, these are the strength of this 30 samples are given here,

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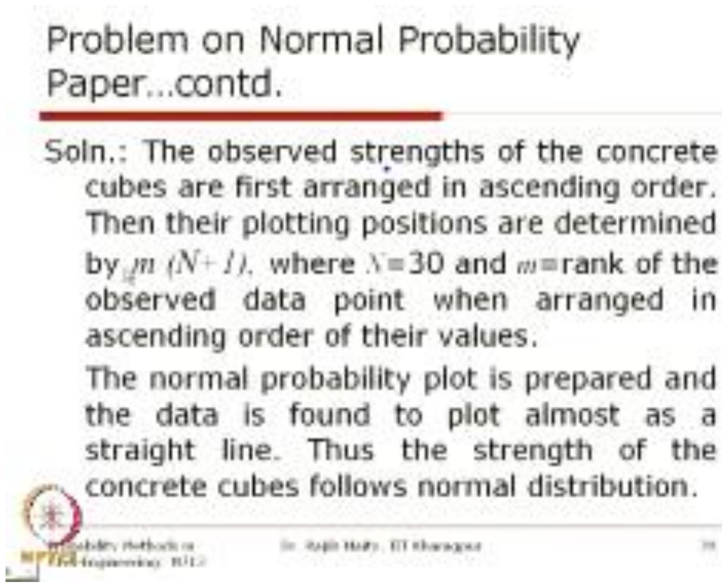

And after doing this using that Weibull formula for their plotting position we get all those we just arrange it in this ascending order and get their respective values.

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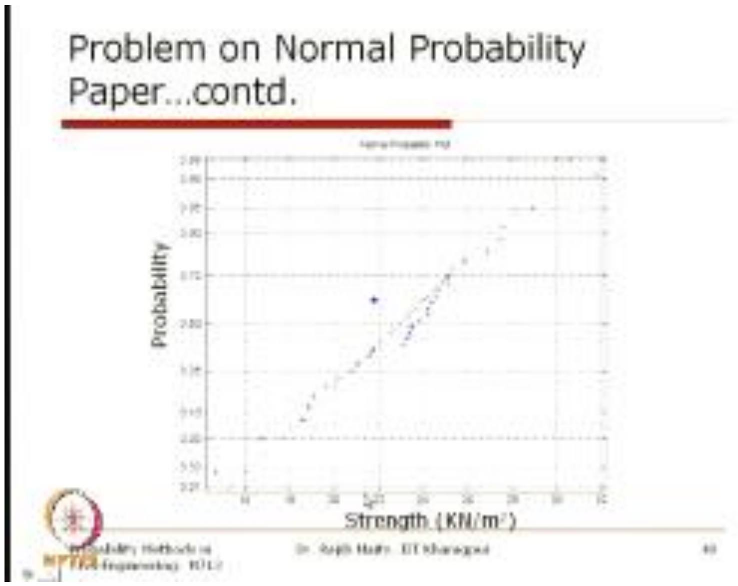

And then, we plot it here. Now, you see here, this x axis is your that actual strength, the pictorial representation that I have given it is for this standard normal distribution. Now, this axis can easily be, for this any axis that and so here that this actual axis is shown, so what it is the strength range. And this you can see that this axis is now distorted to get that what is their probability values.

Now, this one is coming, this blue plus sign are the data and now you have to buy your judgment you have to test whether this blue lines are your that this blue plus dots are really following a straight line or not. So, this is just by your Inspection and there are some test also, statistical test that also we will see next.

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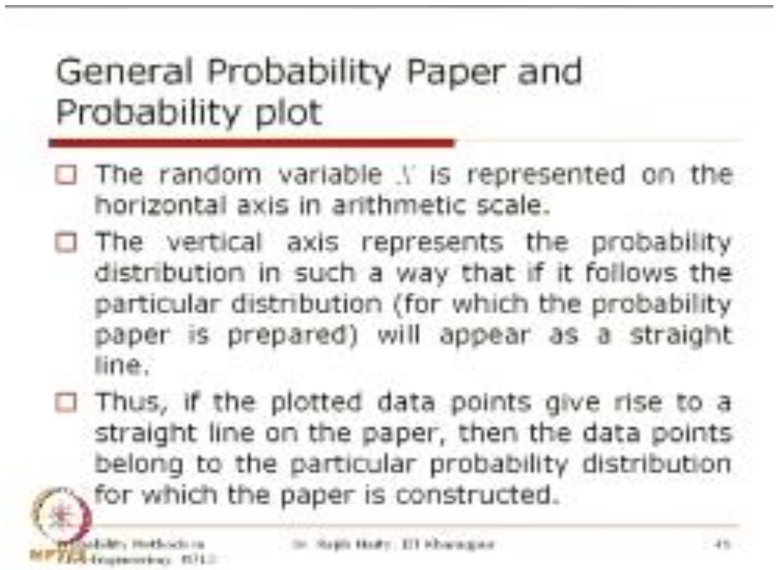

This is for this normal probability paper and that the general probability paper and the probability plot the random variable X is represented on the horizontal axis in the arithmetic scale. The vertical axis represents the probability distribution in such a way, that if it follows a particular distribution for which the probability paper is prepared, whether the probability paper is prepared for the normal distribution or gamma distribution or exponential distribution depending on that.

If it appears on that paper as a straight line then, we can say that this is following that particular distribution. Thus, if the plotted data points give rise to the straight line on the paper then, the data points belongs to the particular probability distribution for which the paper is constructed. Now, so this is for this general case, now say that, so this cumulative distribution that I have shown it here, this is that, this is for this normal distribution.

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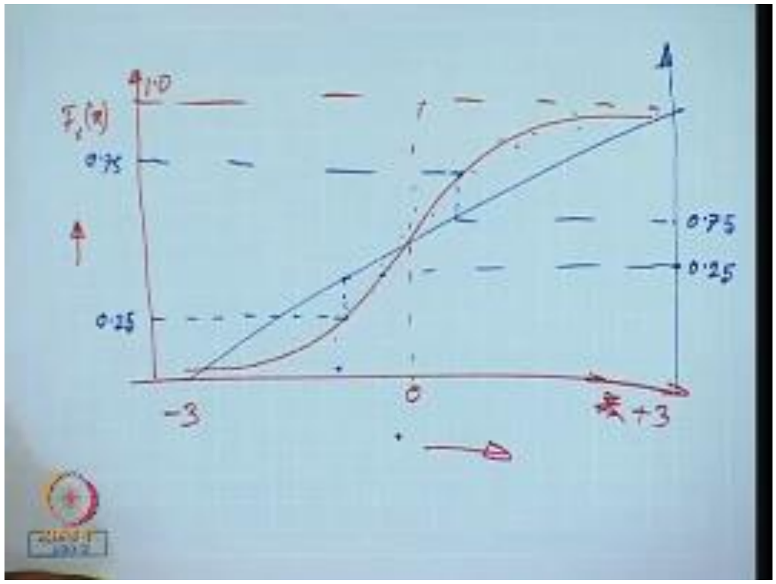

Now, I can take for this exponential distribution and this exponential distribution, also hat will be if I just take,

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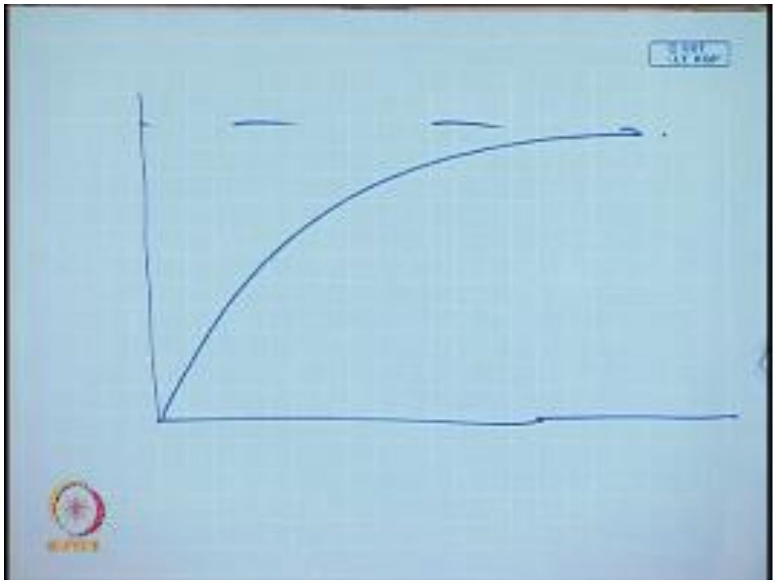

So this exponential distribution, the cumulative distribution looks like this, that now, this one also where this almost is approximately almost equals to one. So, from there if you have to draw a straight line and then, you have this probability axis, you have to distort thus within the same method, as I have discussed now. So, whatever the new axis you will get with respect to this actual axis this horizontal axis of this random variable you will get, the paper which is for this exponential distribution. And on that paper if it appears to be the straight-line then, we can say that this is following the exponential distribution.

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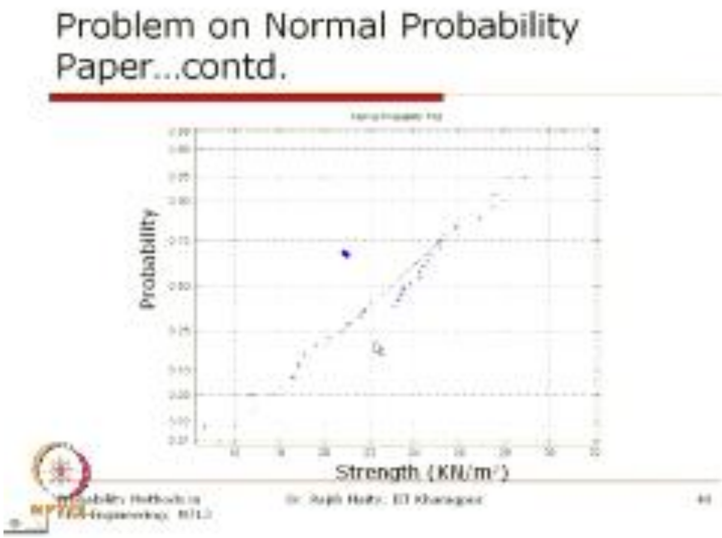

Now, one thing may ask that Now again, I am now going back to the problem we have discussed that. Now, this blue stars whether this is really a straight-line or not. So, this is now, whether you can say just by your Inspection you say whether it is really following the straight line or not .So, that is your, basically, a personal judgment that whether I can say that it is following the straight line or sometime we can say that no it is not following the straight line. also, what we need actually? We need one probabilistic test for which, basically, we can say, we can infer probabilistically at this significance level, this particular data sample or data sample from the population of that data sample is following that distribution.

So, there also we need the hypothesis testing, there are different test statistics are there, different tests are there, the chi square test, of test, through that test we can infer that which distribution that the population is following, and that we will take up in our next lecture; thank you.

Probability Methods in Civil Engineering

End of Lecture 37

Next: "Goodness – of - fit tests"

in Lecture 38

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