INDIAN INSTITUTE
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Probability Methods in Civil Engineering

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Lecture – 22

Topic

Sampling Distribution and Parameter Estimation (Contd.)

Hello and welcome to this second lecture of this module and you know that this module is on probability and statistics and we in the last class have started the discussion on the sampling distribution and parameter estimation and we have seen there are two different methods of this point distribution and we started that interval estimation and we completed that interval estimation for the mean with known variance. So basically in today's lecture we will continue from the same point and whatever was there for the sampling distribution for the other estimation that is for the variance and proportion and those we will see. Both for their point estimation and for their interval estimation.

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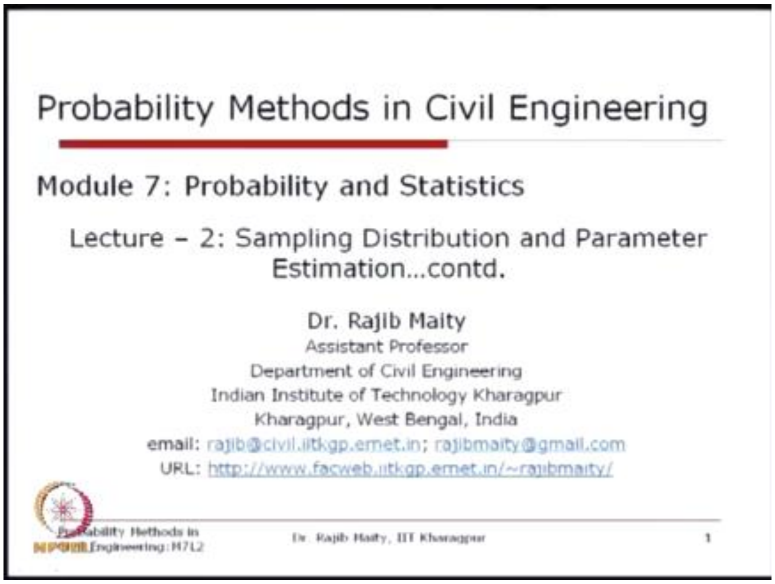

So, our today's.

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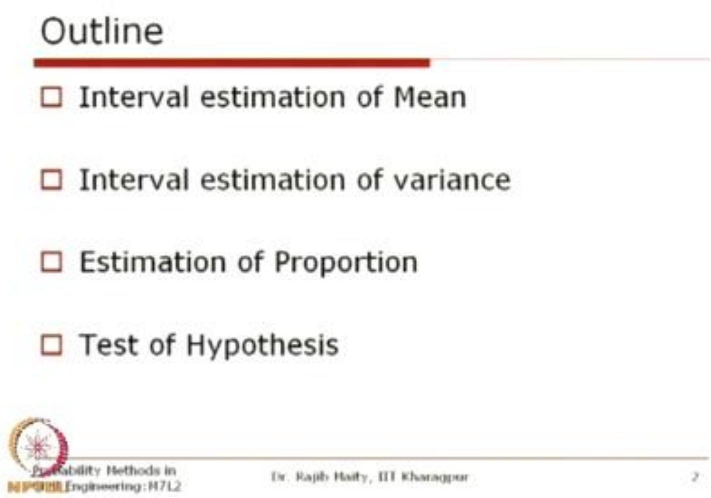

Lecture outline is that interval estimation of this mean basically this we started and started in the sense that we have used this for the certain when the standard deviation for the population is known in that case we have just seen and we have also discussed the theory when the standard deviation is not known and so we will start from here where in which case that the standard deviation for the population is not known we will start from that point and we will start with that example.

And after that we will see what is interval estimation for the variance and then we will see that both the point estimation as well as for the interval estimation for the proportion you know the proportion is generally used for some of this distribution for example the Bernoulli distribution that we use there so we will see those estimates and if time permits so we will also discuss about this test of the hypothesis which what we have mentioned in the last class that when we are interested about the two different samples and we want to infer something about their respective population.

Then this test of hypothesis is important so to start with we will start with this interval estimation of mean basically one problem where the standard deviation for the population is not known.

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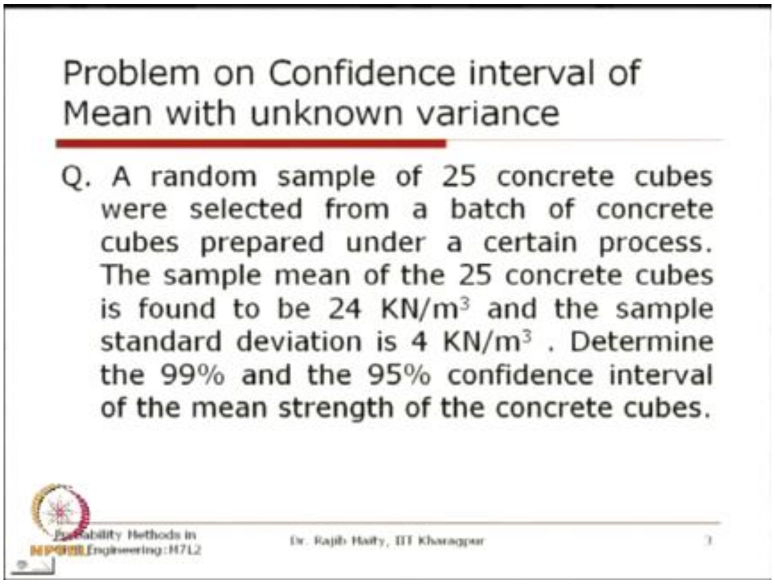

Random samples of 25 concrete cubes were selected from the batch of concrete cubes prepared under certain process the sample mean of 25 concrete cubes found to be 25, sorry 24 kN/m^3 and the sample standard deviation is 4 kN/m^3 determine the 99 % and 95 % confidence interval of the mean strength of this concrete cube so this is mean density of this concrete cubes .Now, if you just compare this problem with our last class problem then we will see that whatever the data is given that is whatever the sample mean and this standard deviation.

These are numerically same to the whatever the problem that we have taken in the last class, but only thing here is that this is both this is because in the last class also the mean was the sample mean but the standard deviation was the standard deviation for the population but here we are using it is to be the sample standard deviation because you can see that this random sample of 25 concrete cubes, earlier it was more than 30 so which can be considered to be the large sample.

And you know that we have discussed in the last class lecture that when it is more than 30 we can assume it to be the large sample particularly for the estimation for this mean and when it is less than 30 that is generally a small sample so you have to estimate the standard deviation from the sample itself and we have to use that standard t distribution with n – 1 degrees of freedom to calculate whatever the confidence interval for such cases so basically here as you can see that the 24 kN/m^3 and 4 kN/m^3,as this mean and standard deviation is given.

What can we have is that basically that 25 concrete cubes are there so 25 values are can be obtained from this one and from there we can calculate what this mean is and from the estimator of this mean that we discuss in the last class that is that $\bar{x}$ is equals to summation of all these values divided by n, n is here 25 so that we can estimate and similarly we can use the estimator for the standard deviation to get this one so here these values are already given just for this, because we are in this problem what we are mainly discussing is that to find out their confidence interval.

So basically the background to this data that is given to us this 24 as well as this 4, is basically obtained from this 25 different values that is obtained from the experiment.

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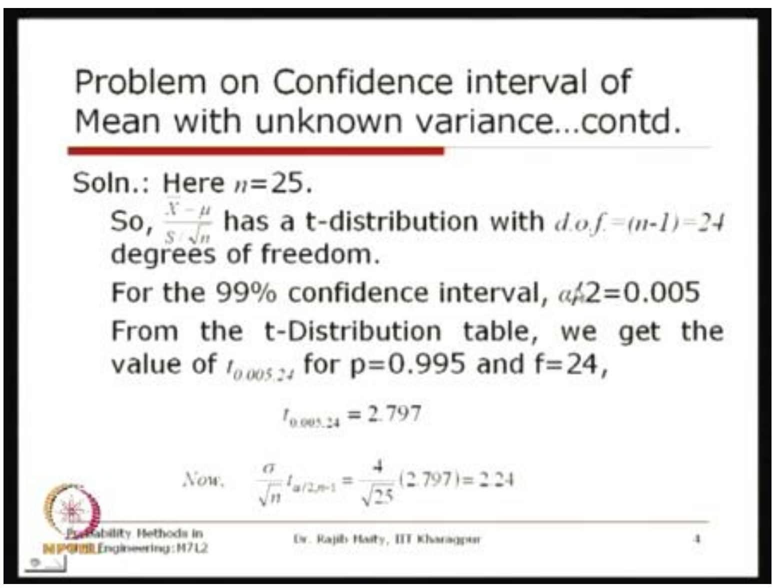

Well so here the n that is the sample size is 25 and we know from our last lecture is that $\bar{x}$ - this μ / s$\sqrt{n}$. Now, this is the s earlier, it was the population generally you know that when we use that

English character English letters then it is the sample, it is related to the sample and when we use that the Greek letters then this is generally to the population.

So, this s when we are using in earlier example we use the σ which is from the population and here is it is from the sample that denotes. So, this $\bar{x}$ minus μ / $s\sqrt{n}$ has a t-distribution with degrees of freedom n – 1 n is 25 here so n -1 is 24 degrees of freedom. Now, for the 99% confidence interval here the α / 2 is equals to 0.005, so this is where from we have discussed in this last class that where you are getting this value is that.

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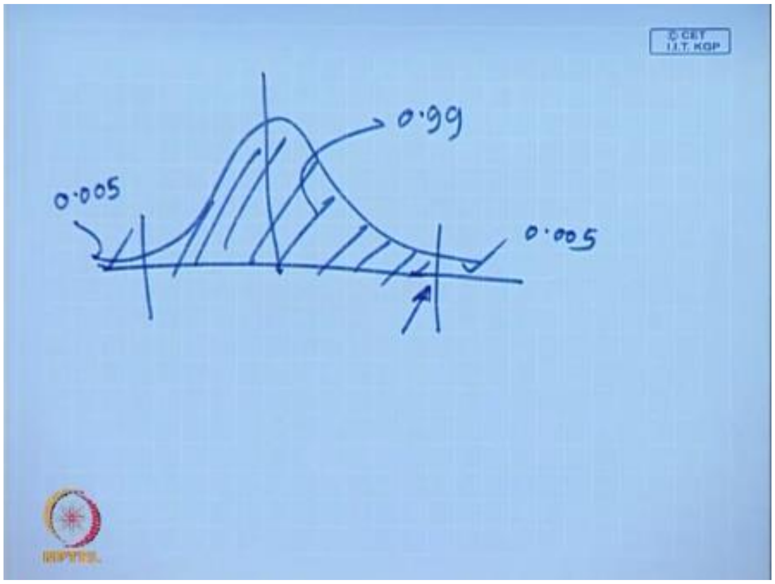

Whenever we are talking about this confidence interval so if this is the distribution table then so this area is your 99 % so 0.99 and so that whatever is remaining here and whatever is remaining here from the symmetry are same. So, this is your 0.005 and this is your 0.005 so that makes total equals to1. So, here we can have this value and that value multiplied by your / $s\sqrt{n}$ root and added to that one will give you the upper limit and subtracted from the mean will give you the lower limit.

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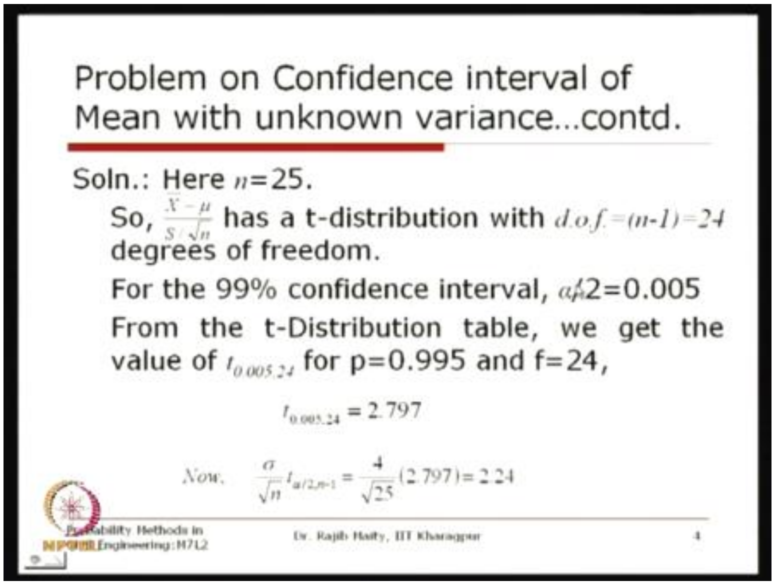

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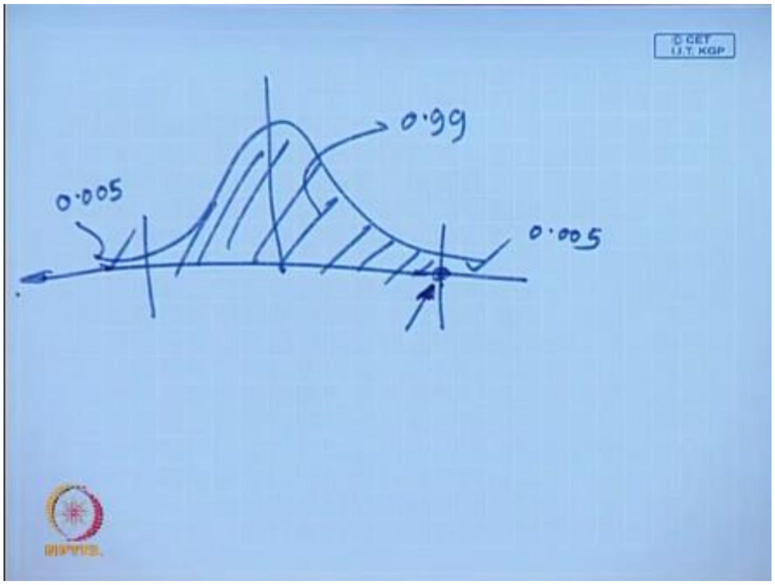

So here that 99 % confidence interval the α / 2 is equals to 0.005 from the t- distribution able we get the values of t 0.005, 24. This value when we are taking that cumulative probability equals to 0.995, and for f equals to, which is degrees of freedom equals to 24. So, this p = 0.995 means if you refer to this one that is from the - ∞ this to support of t-distribution also you know that -∞ + ∞ from - ∞ to this point the total probability that is covered is0.99 + 0.005 that is 0.995.

So that this value we are interested what is this value so this is the value at which the cumulative probability is 0.995, so this is the value that we are looking for.

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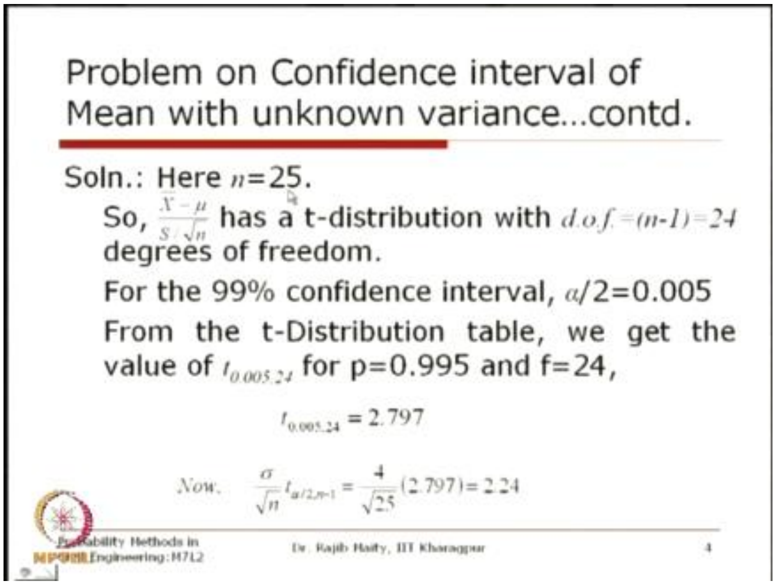

And this t-distribution safe changes with respect to f degrees of freedom so for the degrees of freedom 24, this one this value is equals to 2.797. Now, if you recall that from our earlier example that if it is a standard normal distribution then at this point 99 % confidence interval this value was 2.575.So, this is changed and this change that you can see this is basically for these degrees of freedom and that is 24.

So the lesser the sample size lesser the degrees of freedom and this difference from this standard normal to this t will be more. Now, if this is increased and it goes beyond 30 then we can say that this value and the corresponding value for this standard normal distribution are essentially same, that is why that the 30 can be used as a judicial cut off to declare that greater than 30 is the large sample and less than 30 is the smaller sample for which we have to use that t-distribution and that is you have done here. So now this will be s as just now I discussed that this is a sample estimate, so the $s/\sqrt{n} \times t_{\alpha/2, n-1}$, so this quantity now become 4 by square root 25 multiplied by this value will give you that of 2.24.

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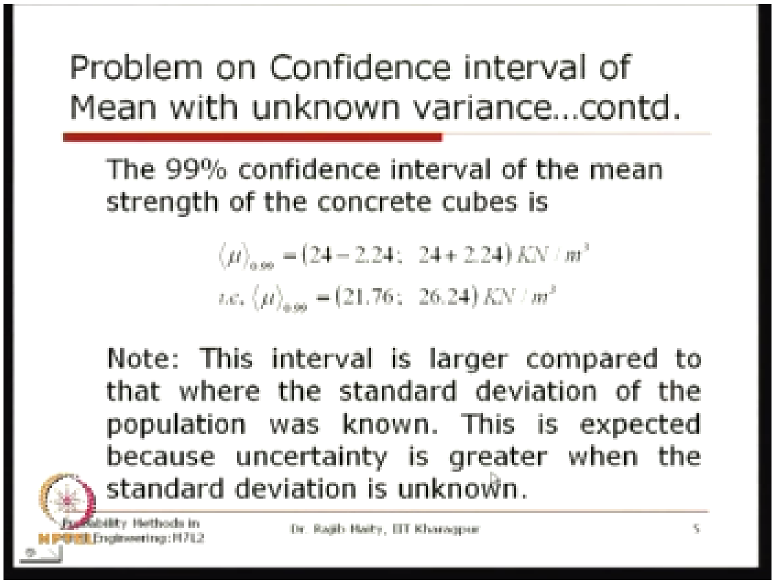

If we just take that so this 99 % confidence interval will be the difference between whatever the sample mean which is 24. So 24 - 2.24 and 24 + 2.24 that is 99% confidence interval is 21.76 and 26.24.Note that this interval is larger when compared to that where the standard deviation of the population was known, this is expected because uncertainty is greater when the standard deviation is unknown. So this was also we are mentioning in the last class when we are talking about this large sample and this small sample.

So you can see that if we have this if the sample size is small that means that time your confidence to tell your confidence infer something will be lower. So as this is lower so you have to specify a wider interval to declare that, okay at this confidence level whatever the actual value of this parameter can be captured. Now we are comparing and if we compare now the same confidence level one is then. So both are 99% confidence interval that we are talking, once we are using it for the large sample which is greater than 30 and we have used that normal standard normal distribution, there that confidence interval if you will see and if you compare it with that what we have seen now for the 25 samples size.

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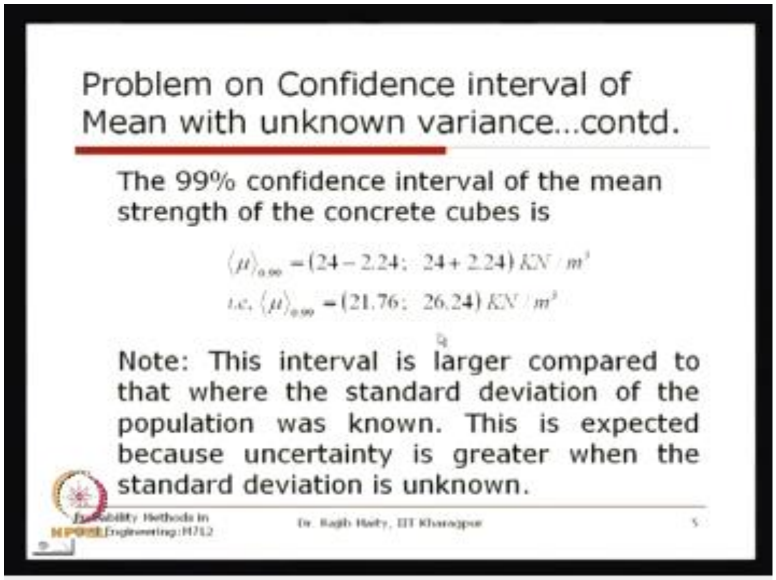

So this interval is more than what we got in the earlier and this is because that we are having the less sample so our determination is less so we have to give a wider range to take care that with the same probability level, this can be that actual value and can be captured, if the sample size even becomes lesser so at the same confidence level this interval will be even wider.

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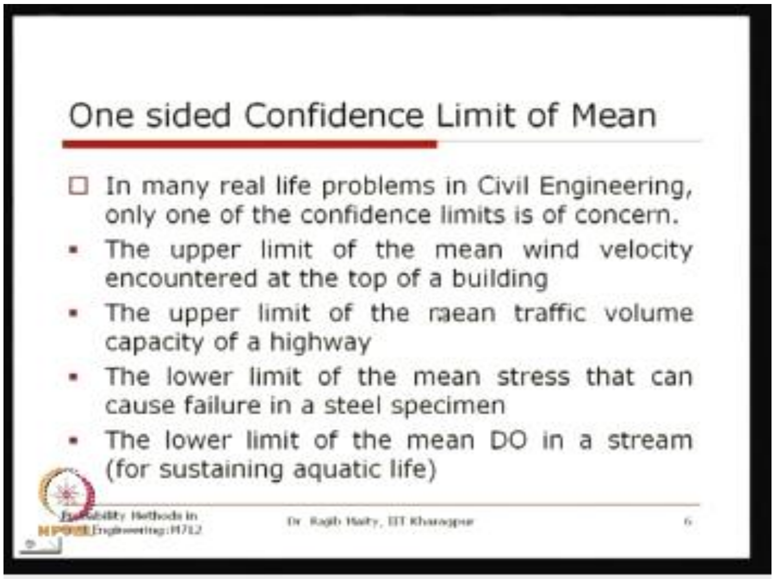

Now so far what we have seen is that basically that is called the two-sided confidence limit of the mean, so what is there that we are talking here.

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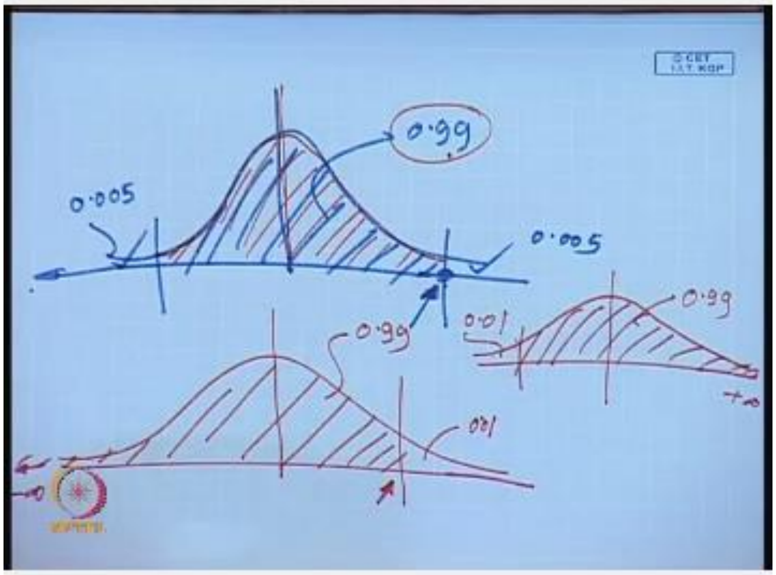

Is that whenever we are talking about the confidence interval and then we are just comparing with respect to this central value and whatever the confidence level that we have just put is basically what is symmetrical with respect to the central value. So this area what we have declared as my confidence and this is equated to what is my desire confidence level. Now in many cases we may not be interested of this both the sides of this confidence interval.

I just I may be interested and particularly there are many example that we will discuss now is that we are interested only one side of this confidence. So now if you want to know that what is the upper limit of the confidence level and at this confidence on the lower side I am not interested. So only the upper side and what is this one? That is called the one sided confidence.

So the same thing what we have to present here is that I need to know what is that point if I use that same confidence level, which is from this minus infinity. So from this minus infinity this is my confidence zone where it is 0.99. Now this one this area in earlier case it was divided both sides, so it is 0.005 here and 0.005here, in this case when it is one side it is 0.1, when we are interested for the upper limit.

And if you are interested for the lower limit then the same thing we can plot and we just interested to know so this side from this one to this plus infinity up to here, so this is your 0.99 and this area is your 0.01 sorry this is only 1, so this is the lower limit and this is the upper limit and this thing is common.

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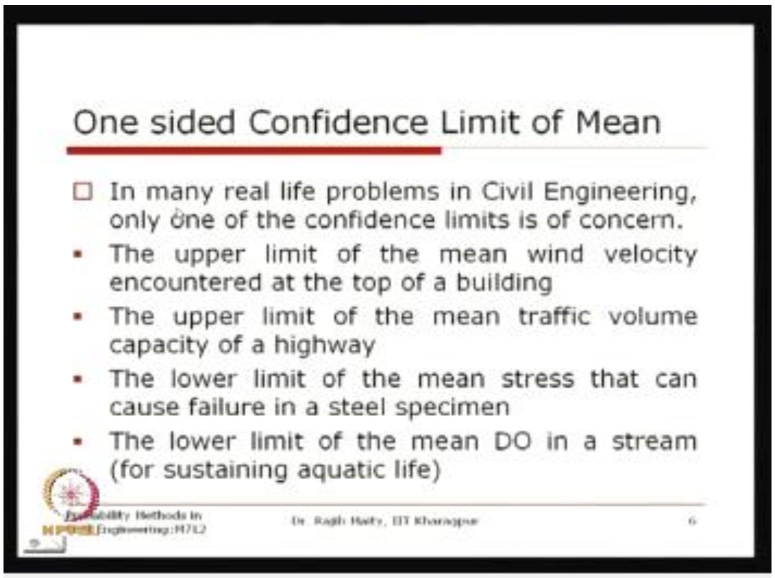

In almost all the application field including civil engineering. In many real life problems in civil engineering only one of the confidence limit is of concern, Say for example the upper limit of the mean wind velocity encountered at the top of this building, so I may not be interested what is the lower limit of this wind velocity. So I want to know the upper limit of the mean wind velocity. Because I need the maximum load what the building can face.

And that is the upper limit of the mean traffic volume capacity of a highway. So this mean traffic volume if it is the lower side then obviously is not of my interest I want to know that what should be the maximum mean traffic volume that can be expected over a highway, the lower limit of the mean stress that can cause the failure in the steel specimen. So in this case when the failure is my concern.

So I just want to know that what is the mean stress, now what is the lower low limit at which the specimen can fail, so here we are interested only in the lower limit upper limit is of no interest to us. The lower limit of the mean dissolve oxygen DO in a stream for sustaining aquatic life, so here the lower limit of this DO that is whether the minimum requirement for this aquatic life is maintained in the stream or not that I am interested.

So the upper limit of DO in such case is not of my interest. So I need only one side of this confidence interval. So these are some of this example where we are interested to know that what should be their limits either I am interested in the upper limit or in the lower limit. So what is the change in this respective concept is that.

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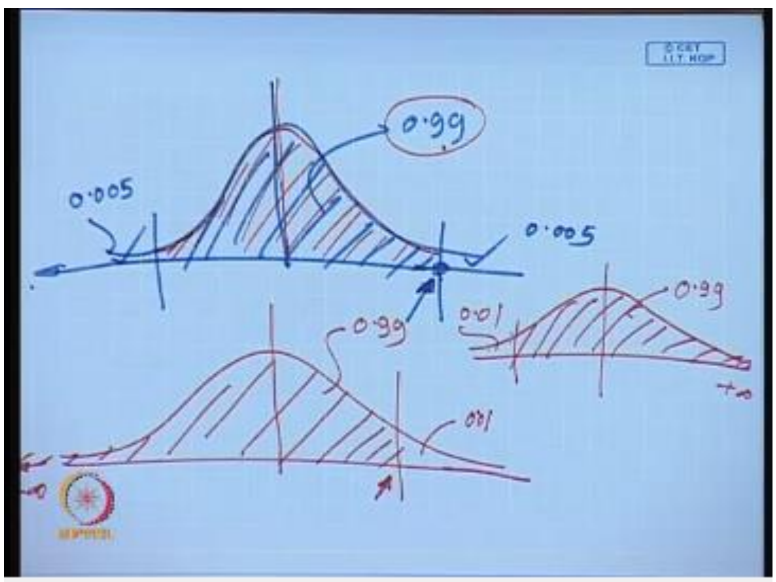

Only thing is that from whatever if you are looking for this upper limit then you have to start from this minus infinity to that point that is my confidence level, so that remaining part is 0.0 this one minus this confidence level, similarly for the lower limit. As contrast to the earlier case when that confidence interval is symmetrical with respect to the central value, so where this the remaining probability is equally divided in the upper side as well as from the lower side in the earlier cases.

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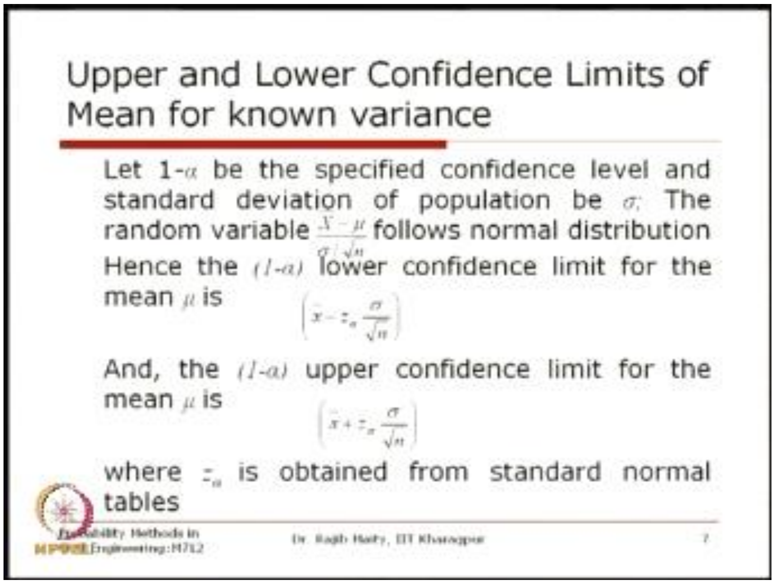

So we will just see for such cases how we can get those confidence limits. Let that 1-α be the specified confidence level and the standard deviation of the population be σ, the random variable x-bar – μ/σ √ n follows the normal distribution, hence this 1-α is lower confidence limit of this mean μ is z - z α σ /√ n. So you see that here also we know that standard deviation that is standard deviation of the population is known, which is sigma.

So It will follow that standard normal distribution and instead of using that z α/ 2 we are using the α and the upper limit if you are interested then that mean plus that z α σ /√ n, as we have just now shown through a pictorial representation, where this z α is obtain from the standard normal tables.

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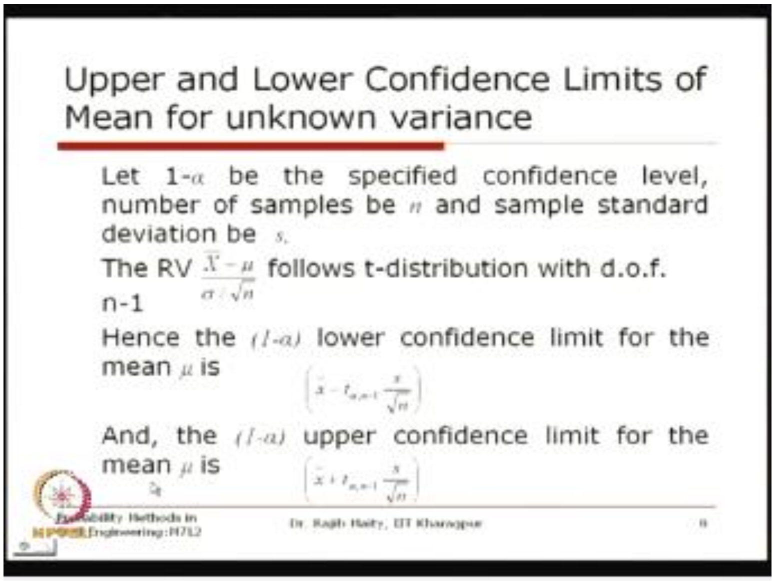

And if the variance is unknown and this is basically for this smaller sample then this1 – α be the specified confidence level, The number of samples be n and the sample standard deviation is this s. Then this x-bar – μ/ $\sqrt{}$ n obviously this is from the sample estimate hence s by $\sqrt{n}$ follows t-distribution with degrees of freedom n - 1 and that 1- σ the lower confidence limit of the mean μ is x bar - t σ n - 1 s by $\sqrt{n}$ here you can see that it is instead of in the earlier case we are using this σ by 2 so it is t σ and with this n - 1 are the degrees of freedom similarly for the upper confidence limit is that x bar plus that this quantity t σ n minus1s /$\sqrt{n}$.

So when this t σ n- 1 is in both of this expression from here this t σ n - 1 and t σ n- 1 here that is used this is basically is this value here that you can see if it is the t- distribution with degrees of freedom.

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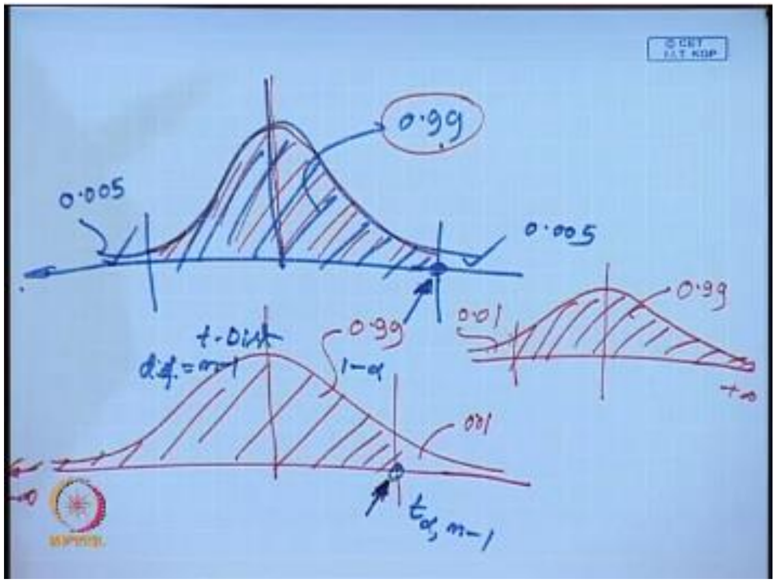

n - 1 df degrees of freedom equals to n – 1 so this we are looking for this value is your t σ n - 1 and this one is basically 1 - σ this 0.99 so this value we are interested so this one we are just taking this deducted from this mean and this is added to the mean.

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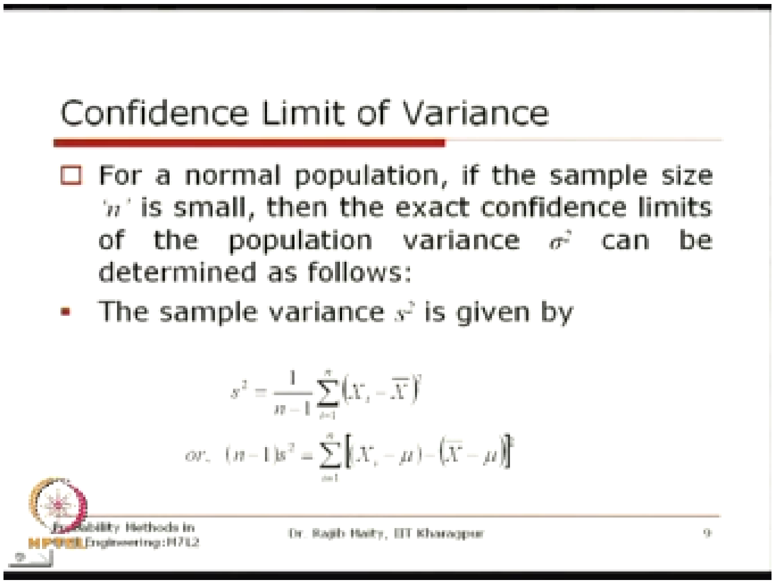

Now the confidence limit of this variance so I hope that this one that we have discussed for the upper limit or lower limit just as the small change that we are doing that you can see and the respective values should be picked up from this standard table then we can determine whether the upper limit or the lower limit whatever is desired that we can pick up next we will move to the confidence interval of the variance so mean we have discussed now the variance is also once we are getting from the sample.

So that also should follow some distribution we should know what distribution it follows and should know what should be that confidence limit for the variance as well for a normal population so this is the background assumption that you can say that for a normal population if the sample size n is small then the exact confidence limit of the population variance σ^2can be determined as follows.

So the sample variance s^2 is you know that this is estimated that we have discussed last class that is s^2 = 1 / n minus1,i equals to 1 to n, xi minus x bar square this x bar is the sample mean and in xi there are n samples are there x1, x2, x3, up to xn this is the way we get that underestimate of this sample variance so this one as we get and if we just take this n - 1 just after algebraic manipulation if we do then we can see that n-s 1of s^2 = square is equal to ∑ of these two quantity which is xi - μ - x bar - μ whole square after some.

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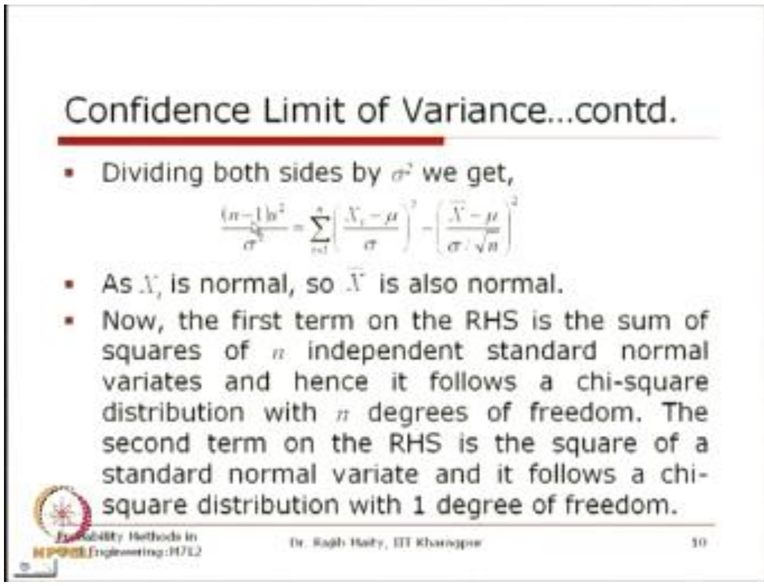

Step that we can express that this n - 1of s^2 divided by σ^2 can be expressed through this that is i equals to 1 to n xi - μ by σ whole square - x bar - μ divided by σ by √n whole square as xi is normal so this x bar is also a normal distribution so you know that this we discuss in the last class that is x bar is also one random variable which follows a normal distribution with mean and this standard deviation of this σy√ n even though this xi is not normal then also this distribution of this x bar is approximately correct.

But in case of the variance that assumption is more crucial that is the background distribution of the population is normal so here you can see that this xi is normal so x bar is also normal if you just see this right hand side so this is the left hand side of this expression and this right hand side there are two component what is the first term if you just see the first term is the sum of the square of n independent standard normal variants so xi is the normal distribution with mean μ and σ is the standard deviation.

So this x minus μ by σ is a standard normal distribution hence this is square and we are summing up for this n such standard normal distribution we have seen in the earlier module of this function of this random variable that summation of this normal distribution if it is square then and we sum then for the n distribution then the resulting distribution is a chi square distribution with n degrees of freedom the second term again is also the square of the standard normal variate and it follows a chi - square distribution with one degrees of freedom.

Now the summation of two chi square distribution having two different degrees of freedom is also another chi square distribution so what we can say from here is that this full quantity is one chi square distribution with degrees of freedom equals to n – 1 that is why this n - 1 s^2 square by σ^2 where s is the sample estimate of the variance and σ^2 is the population variance their ratio multiplied by this n - 1 is a chi– square distribution with degrees of freedom n - s 1.

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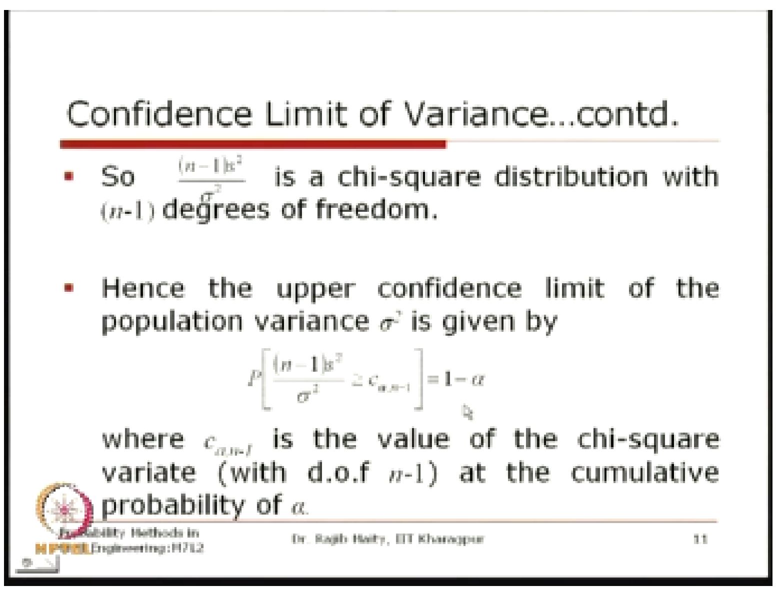

This is what is written here so this n - 1 s^2/σ^2 is a chi – square distribution with n - 1 degrees of freedom once we know the distribution then whatever the confidence limit and all we should get it from the chi- square distribution hence if you are interested to know the upper confidence limit of this population the variance of σ^2 is given by n - 1s^2/σ^2 should be greater than equal to c σ n -

1 = 1 - σ now you see here this 1 minus σ this is the confidence that we are looking for and this 1 this c σ n – 1 is the value of chi- square variate with degrees of freedom n - 1 at cumulative probability σ.

These values we can get from these standard chi – square distribution tables which is available in most of the text book.

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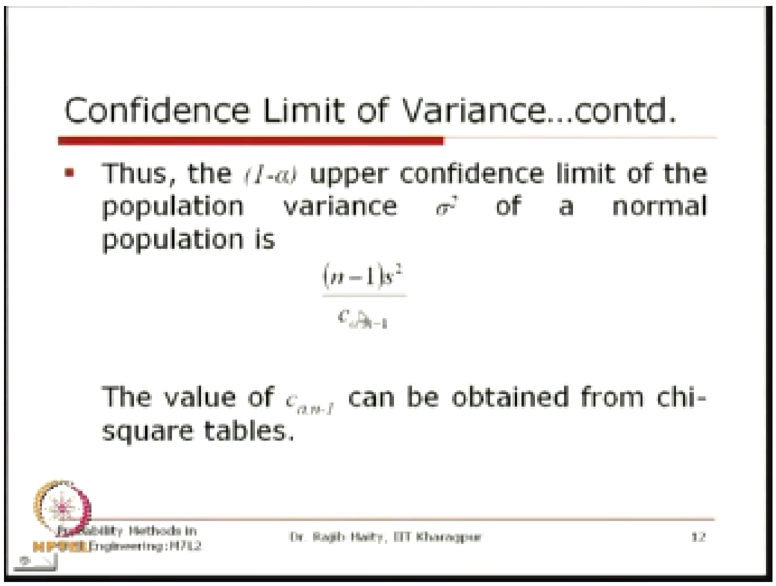

Thus the 1 - σ upper confidence limit of the population variance σ square of a normal population is n - $1s^2$ divided by c σ n - 1 so once we get this value from this chi- square table then we can and this is from the sample estimate so we can get this value which is the upper confidence limit at 1 – σ confidence level.

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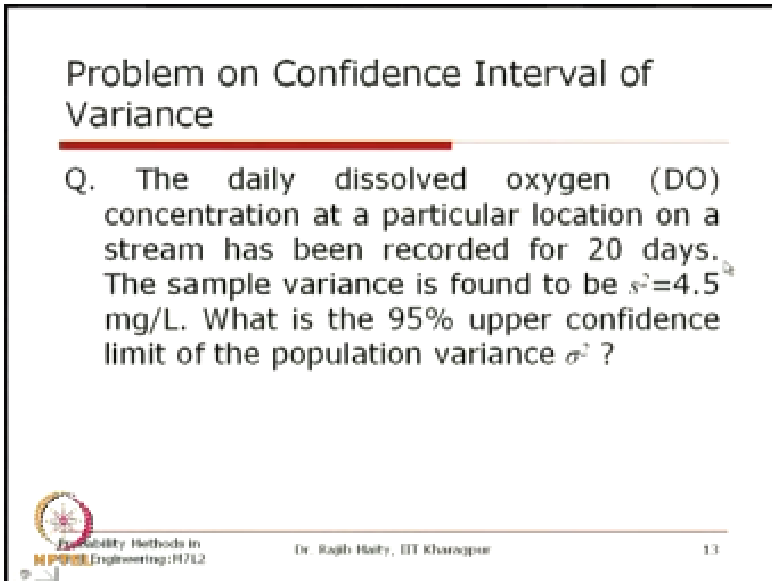

We will take one example that is of the daily dissolved oxygen DO concentration at a particular location on a stream has been recorded for20 days the sample variance is found to be s^2 square = 4.5 milligram per liter what is the 95% upper confidence limit of the population variance σ so the sample is known from the sample we have estimated the variance and that estimation is 4.5 milligram per liter the sample size is 20 days here so remember one thing here before I go to this solution that when we discuss that this is the mean and.

We have shown that this is more than the sample size more than 30 can be treated to be the large sample when we are talking about this variance even the 30 is not sufficient to declare that this is a large sample so generally for even the sample size is more than 30 also here the degree of freedom is always will be associated to this any way this is the chi - square distribution which we should get with respect to that respect to those degrees of freedom.

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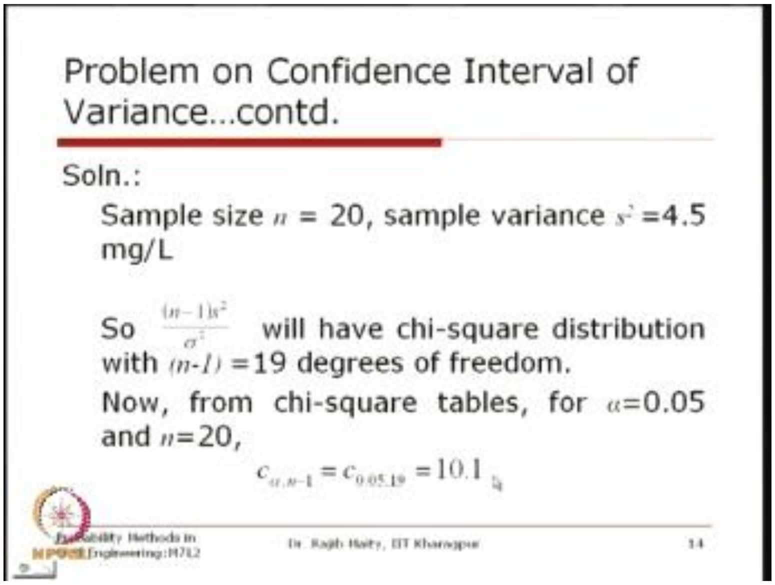

Here the sample size n equals to 20 as we have seen in this example problem and the sample variances square is the 4.5milligram per liter so this n - 1 s^2 /σ^2 will have chi- square distribution with n minus1 is equals to 19 degrees of freedom so we have to refer to the table of this chi square where this degrees of freedom generally the tables are provided with different degrees of freedom starting from 0 to at least some values up to50 or so we have to refer to that particular distribution for this degrees freedom equals to 19 now for which cumulative probability level that you are interested that depends on what confidence level that you are looking for.

So here the 95% confidence that we are looking here the σ value is equal to 0.05 and the n equals to 20 so this degrees of freedom is19 so for this one so c σ n – 1 that is c is 0.0519 if we see it from table we will see that it is value is 10.1.

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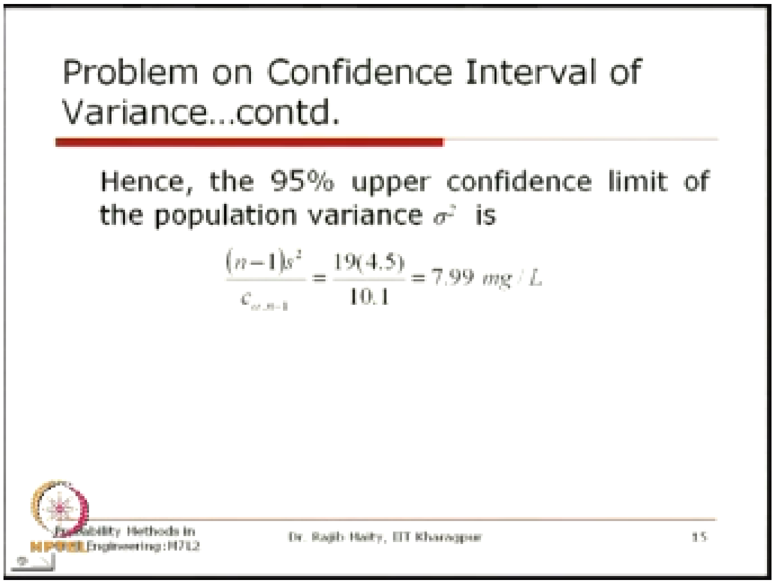

So, if we use this value then we can see that this $(n-1)s^2/c_{\alpha n-1}$=19(4.5)/10.1 so7.99 mg/L is the 95% confidence limit this is upper confidence limit of this population variance is 7.99mg. So, if we are now interested for this lower confidence level for this 95% confidence level, then this value will change.

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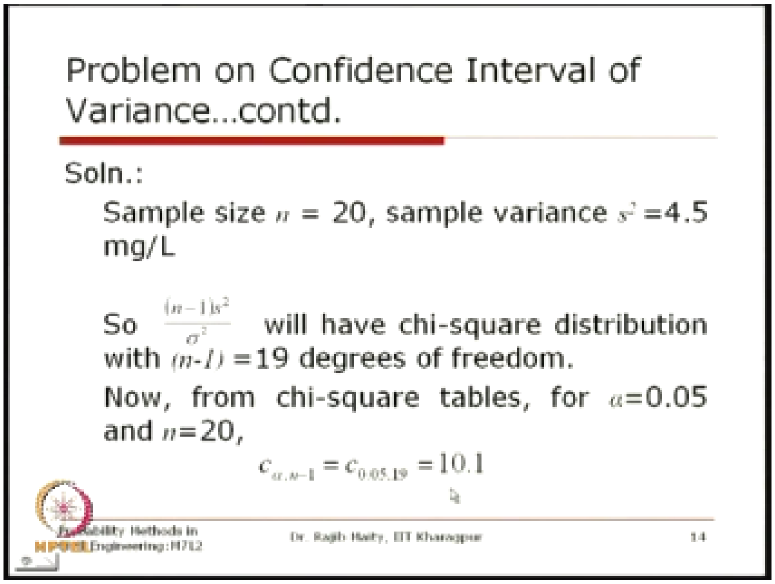

And you have to find out the chi-square value for the same degrees of freedom at the cumulative probability equals to 0.95 and this will be obviously higher than this10.1.

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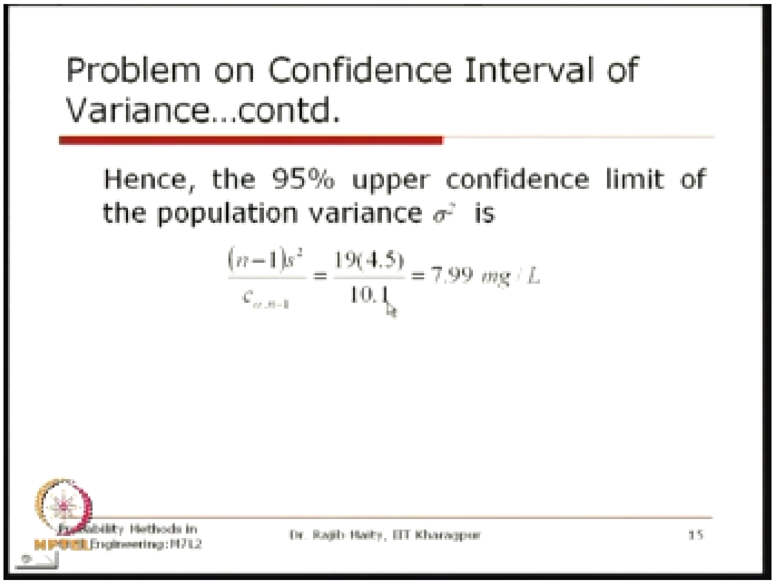

So that the lower limit will be the lower than is the, sorry the lower limit will be the lower than this value what we have seen from this sample estimate and this value will be higher than whatever we have seen it for the.

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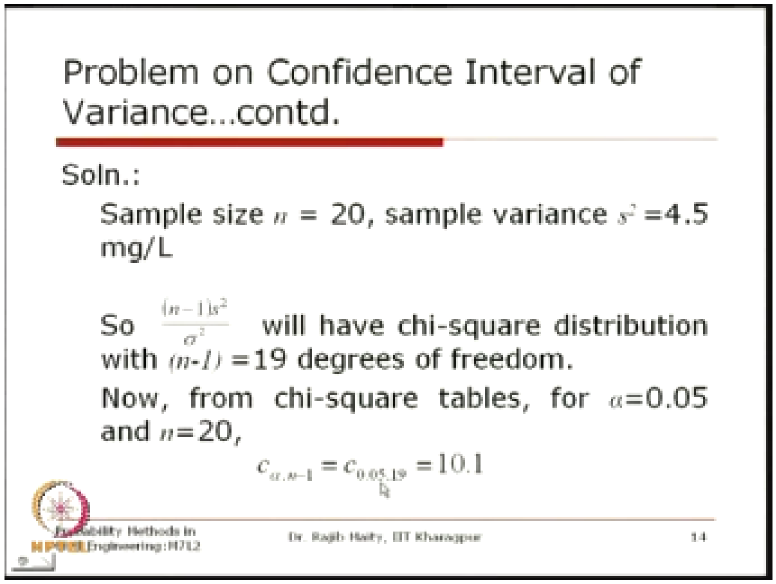

Now for this 0.05 cumulative probability level.

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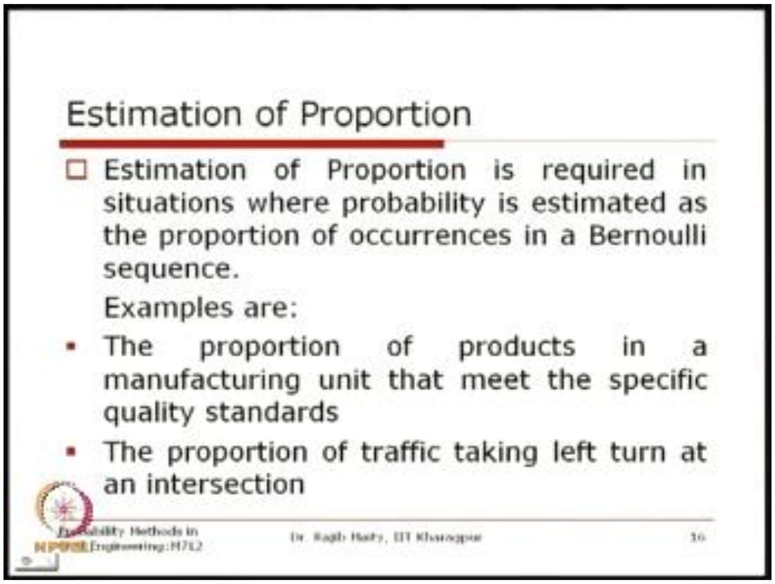

Well, so next one, the next one of this parameter that is also very useful in many distributions is the proportion. So estimation of proportion is required in the situation where the probability is estimated as the proportion of the occurrence in a Bernoulli sequence. We have seen that in the Bernoulli distribution, this one the parameter that we use is the probability of success and that is denoted as p. So, there we need to know what the proportion here the examples are the proportion of the product in a manufacturing units that meet the specific quality standard.

So what should be the proportion that I can say okay, this much percentage of this product are pass in the quality test. Second is that the proportion of the traffic taking the left turn at a particular intersection. So there are total numbers of vehicle you can count and out of which how many vehicles are taking the left turn, so in that way we can estimate what should be the proportion that is taking the left turn at a particular intersection. Just to design the traffic at that particular junction.

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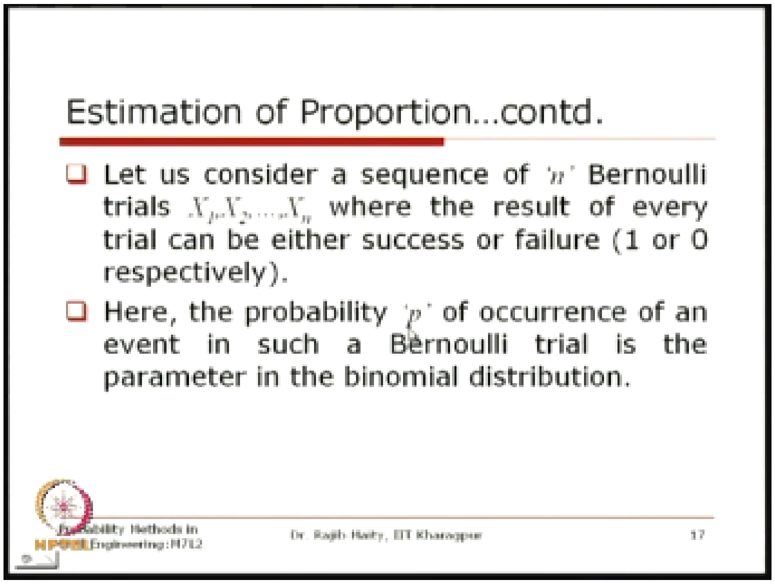

Now, let us consider that a sequence of n Bernoullis trials X_1,X_2 up to X_n, where the resultsof every trial can be either success or failure that is 1 or 0 respectively. Here the probability p of occurrence of an event in such a Bernoulli trial is the parameter in the Bernoullis distribution, binomial distribution this p if we have to estimate.

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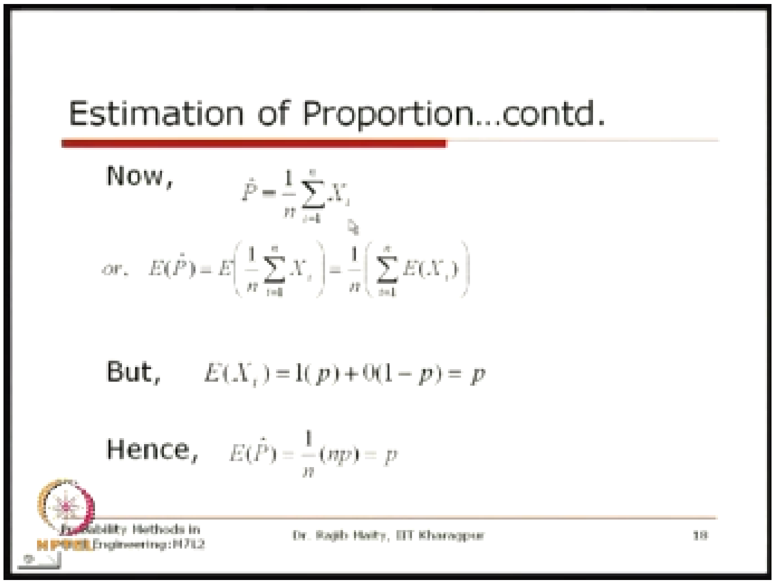

So that estimate that is x_i and can take the values as.

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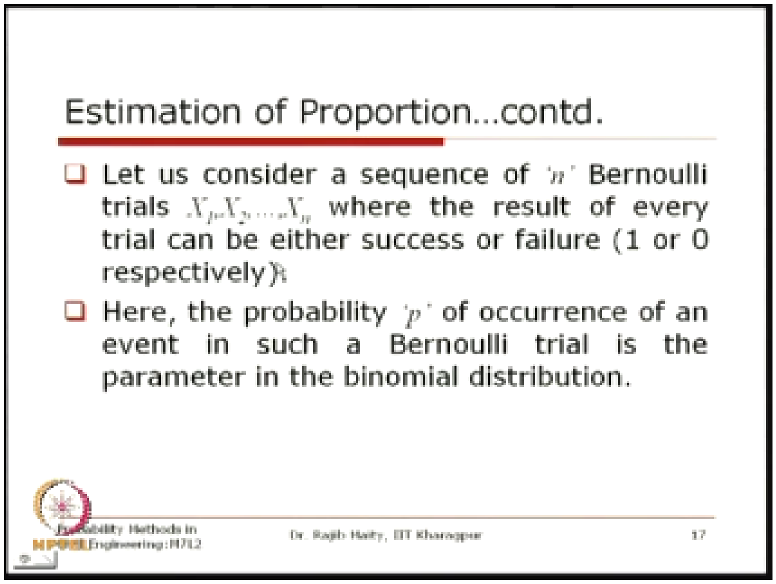

You can take the either 1 or 0, so that whichever is success that we are denoting as 1, and which one is failure we are denoting as 0.So, here again that I mentioned many times earlier that success and failure are the arbitrarily selected. So even that tossing a coin head may be the success and tail may be the failure, so that when we are calculating the proportion it is basically nothing but, of that sequence that 1, 0 that this binary numbers that arithmetic mean of those numbers will give you the proportion.

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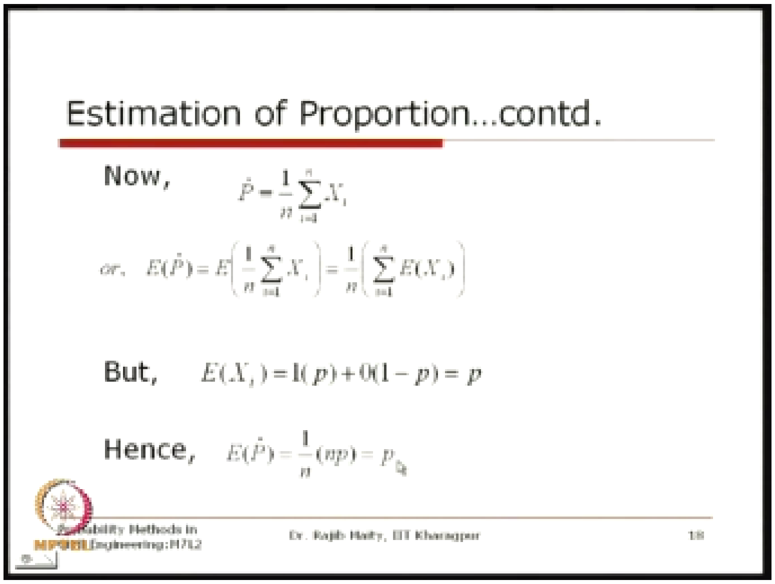

That is what is estimated here that is ∑ of all these x_i and divided by n will give you that estimate of this proportion. Now, the expectation of this proportion, this estimate is that expectation f 1/n and ∑ of these values, which we can write that expectation of these each and every outcome, which are also again a random variable and we know that for this Bernoullis trial each and every outcome is independent and having the same probability of the success so which we can just get that x_i is equals to again that p and which can be shown from this equation.

So, this is p and this expectation of this estimate of this $\hat{P}$ is 1/n(np) so there are n numbers of expectation, all the expectation of P, so this quantity is np so the expectation of this estimate also is equals to p.

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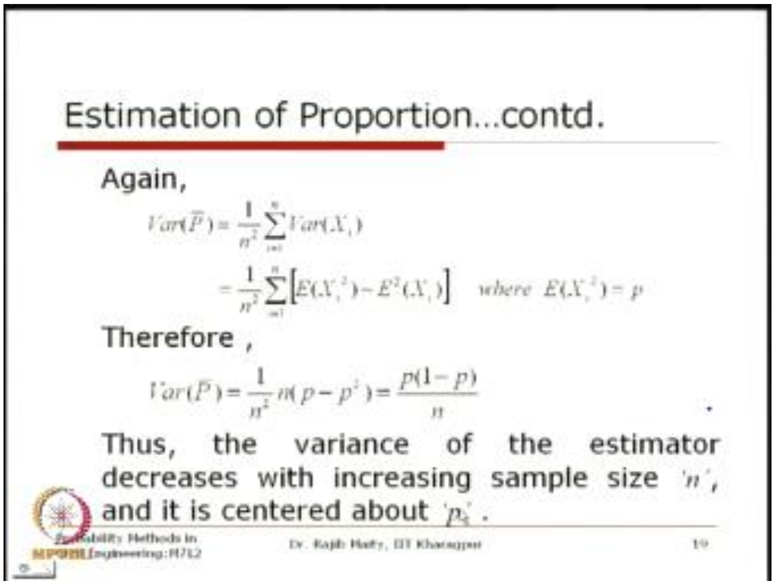

And the variance, if we calculate then $1/n^2$ of this variance of this x_i, and this expectation of X^2 is also that p. So, this is$1/n^2$ and this multiplied by this n of this one is p and this one is p^2. So, the p(1-p)/n, this is the variance of the estimate of this proportion. So thus, the variance of the estimator decreases with the increase in the sample size n that you can see. So, this is the variance of the $\hat{P}$, not the P bar, this is the proportion and it is centered about the population proportion p.

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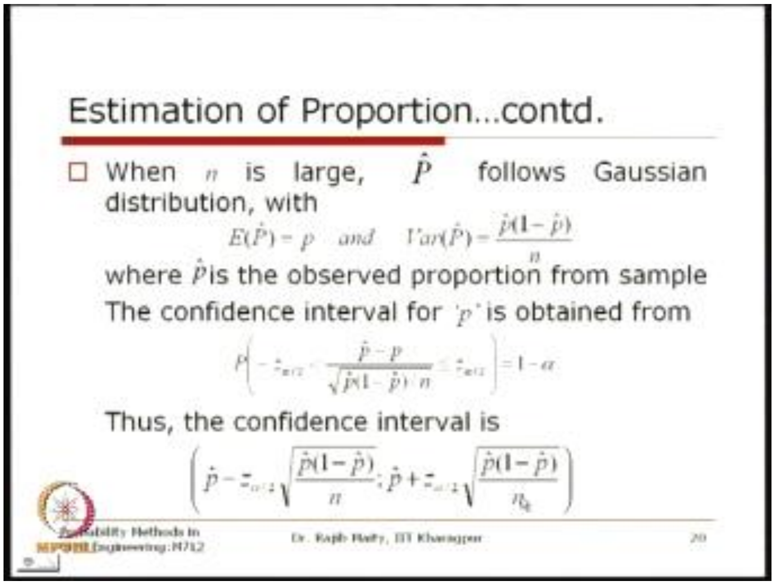

Now, when n is large that $\hat{P}$ follows the Gaussian distribution with mean p and variance is that p-(1- $\hat{p}$)/n so, we have that mean also, it is from this sample so this we can where this $\hat{p}$is the observed proportion from the sample and the confidence interval of this p and once we know that this is having that normal distribution then this $\hat{p}$ minus this mean, divided by its standard deviation, which is nothing but $\hat{p}$-(1- $\hat{p}$)/n, that is the square root of the variance and this should have the confidence limit of this $-z_{\alpha}/2$ to $z\alpha/2$ with the confidence level $1-\alpha$.

So, this we have discussed in the last lecture also. We just taken for the standard normal distribution and this value we are looking for that particular quintile value where at the confidence level, at the cumulative probability level is $1-\alpha/2$. So, thus the confidence interval of the proportion is just after from this thing we have to just see the confidence interval of this p then we will just multiply this quantity with this value and added to this one. So, we will get that $\hat{p}-z_{\alpha/2}$ square root of this standard deviation, the square root of this variance that is standard deviation and the upper limit is the $\hat{p}+z\alpha/2\sqrt{\hat{p}(1-\hat{p})/n}$.

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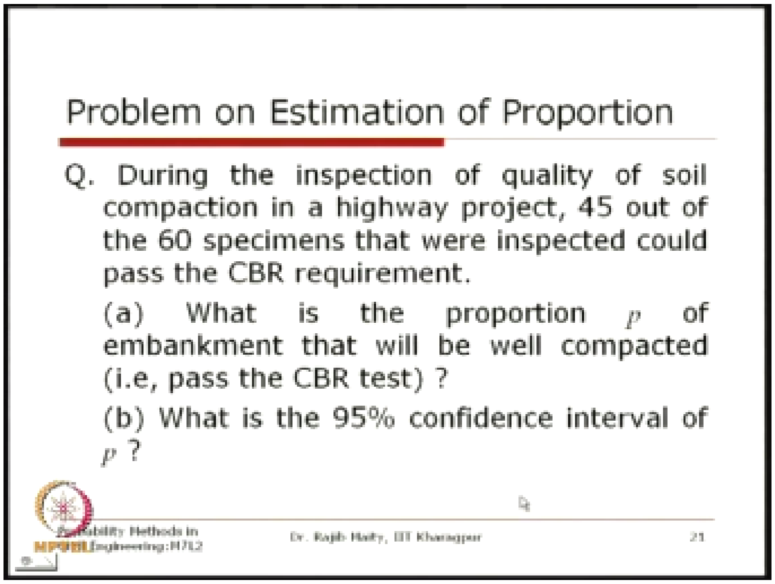

Now, you take one example of this proportion and its confidence interval during the inspection of quality of the soil compaction in a highway project 45 out of 60 specimens that were inspected could pass the CBR requirement. So, here again this is a Bernoulli process, where the one particular specimen may or may not pass the CBR test and out of 60 specimen 45 specimen is as passed. So, the first question is what is the proportion p of the embankment that will be well compacted that is pass the CBR test. So, this is a point estimation and that what you have seen the point estimation is just a ratio of whatever the number of success divided by total number of specimen.

So that we can estimate and the second thing is that what is the 95% confidence interval of that p. so that we will see and how we have to use this standard normal distribution.

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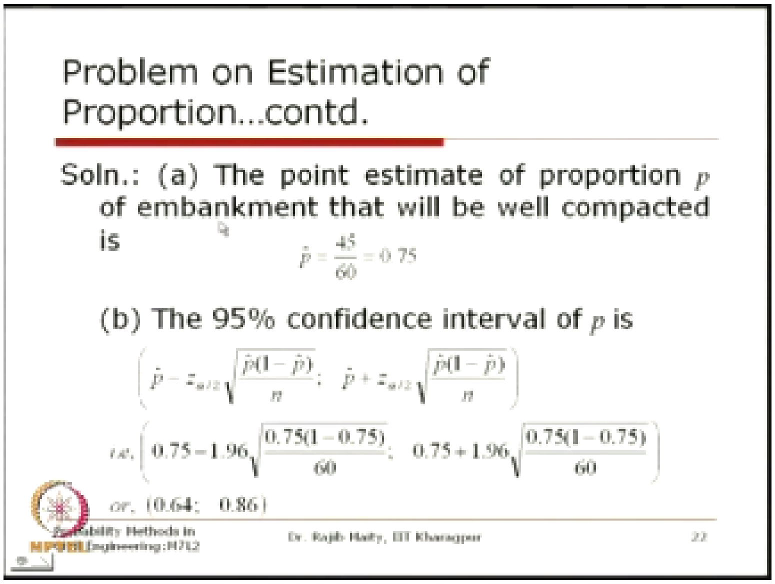

Now, you can see that the point estimate of proportion p of the embankment that will be well compacted that is p̂ I told that it should be the 45/ 60=0.75 this is straight forward. Now, the 95% confidence interval of this p is that this estimate minus this $z_{\alpha/2}$ into this their standard deviation and p̂+ $z_{\alpha/2}$ multiplied by this standard deviation. So, this α/2, in case of 95% confidence level that we have seen in this last lecture, also which is 1.96, so 0.75 -1.96 square root of this and 0.759+1.96 square root of this magnitude, which is the variance of this estimate.

So, if we do this one then we will get that this quantity becomes 0.64 and other one become 0.86.So, the confidence interval for the estimate of this proportion p̂ is 0.64 and 0.68, and that you can see that it is symmetrical with respect to that estimate 0.75 so more if we increase this confidence interval to for say for example, from 95% to 99% then it will go wider this one will be even become lower and this one even become higher.

So this at 95% confidence interval this confidence limits are 0.65 and 0.86. So far we have discussed about the point estimation and in this lecture we have discussed about this interval estimation of this parameter. Now basically what happens is this estimation when we are doing with the respect to one data set that is available to us and now suppose that there are two data sets are available to us.

We may sometimes interest to know that whether the mean of that population can be like this or it cannot. So or I can even say that whether I can say that two samples are coming from the same population or not that means the parameters that is associated with the population and what we are getting from those samples those are same or not. To answer this type of question what we need that is known as this Hypothesis Testing. So that is what our next interest that we will discuss now.

So this is your in this hypothesis testing, in real life decision making it is often necessary to decide whether a statement concerning a parameter of a probability distribution is true or false. This hypothesis test is used to check the validity of a possibility or guess means like that kind of English word that we are using which is basically is known as this hypothesis of one possibility. This just may happen like that about the population where the necessary decision can be taken depending on the test result.

Suppose that for example if I take on this point one civil engineering real life scenario like suppose that I am estimating the strength of a concrete. And so what we are supposed to get is that I need say for example for a specific requirement I need the strength should be greater than some threshold value that we know. Now how to test that one, so we have to take some sample and we will get say that some sample mean.

Now this sample mean may or may not cross that particular threshold value or it may be the difference from whatever our expectation and what we are getting from this sample, obviously will not match as soon as we are changing the sample also that will change. So, depending on that now we have to test whether that difference that we are getting is it by chance or there is some problem in the construction itself. So, depending on that after this test result of this hypothesis testing that we have to decide on what is that; whether it is by chance that the difference that we got or there is some problem in this problem.

What we are targeting or what we are trying to test that is whether the strength can be assumed to exceed that threshold value. That possibility that guess is known as the hypothesis here. So that hypothesis we have to test and depending on that test result, we have to take some necessary decision whatever we can take in that. These decisions are basically based on this statistics and known as this the statistical inference that we can draw from whatever the sample data that we are having.

When a probabilistic model is developed to describe some process, say for example, we will take in these coming lectures, after few lectures, we will take that regression, which is one of the very important models. There if we just express those expressions then that model.

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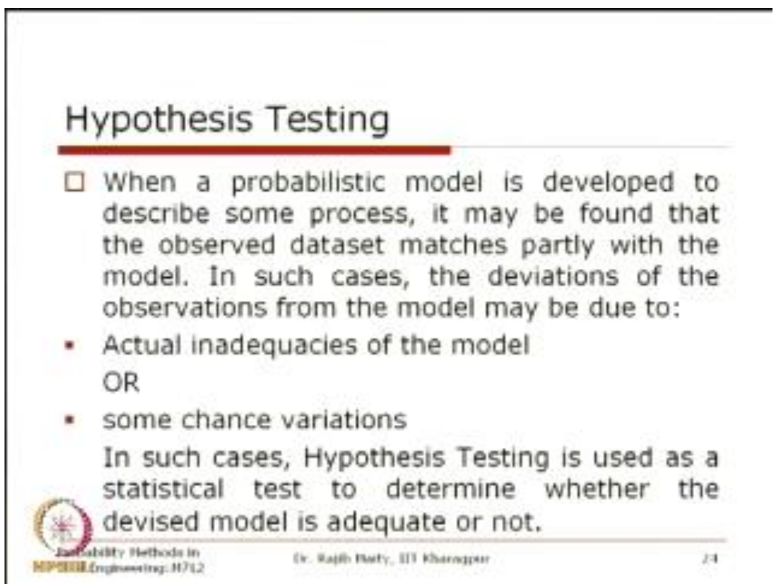

Whether that is exactly matching with this data or not, so that probabilistic model which we have developed it may be found that the observed dataset matches partly with the model. In such cases the deviations from the observations from the model may be due to two cases. One is that actual inadequacy of the model or some chance variation. So these two are important the actual inadequacy of the model if we declare this one, then we have to seriously think about that we have to change the model.

Those models should be changed so that we can better explain the process. On the other hand, even though the model is correct, it will never match with the observed data set. So that variation is called the chance variation. In such cases that is whether it is varying by chance or there are some real inadequacies there in the model. For that case, this hypothesis testing is used as statistical test to determine whether the devised model is adequate or not.

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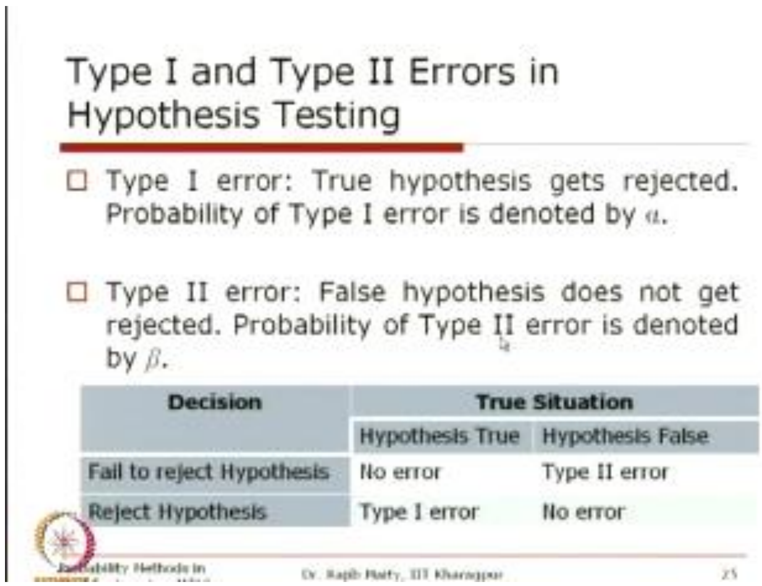

There are two types of errors in the hypothesis testing. These are known as the type one error and type two errors. So the true hypothesis gets rejected, so the hypothesis that we are first assuming say that is actually correct and through this hypothesis testing, if we rejected that means that we are facing one type of error and that is known as this type one error. The probability of this type one error is denoted by α.

Similarly when the hypothesis is false, but we fail to reject that hypothesis that time also, we are doing another error and that is known as this type two error. And probability of type two errors is denoted by β. So if I just see through this table that the true situation. The hypothesis can be true or the hypothesis can be false. The decision that is taken after the hypothesis testing that we fail to reject the hypothesis. So, if we fail to reject the hypothesis, incase when the hypothesis is true, then obviously there is no error. So, we have taken with the right thing that has occurred. Now, if we fail to reject the hypothesis and the hypothesis is false then that is the type two errors as we have explained here.

On the other hand if we reject the hypothesis and the hypothesis is true then also this is one type of error which is known as the type one error. If we reject the hypothesis and hypothesis is false, then also there is no error occurred and the probability of this type one error is α. It is denoted by α and probability of type two errors is denoted by β. Now you can see this type one error when the hypothesis is true and we reject the hypothesis.

Basically from the manufacturer point of view this type of error is very critical and we generally allow very low value for this type one error. After giving this one we try to minimize as much as we can for this type two error. The task of this hypothesis testing is there are the following systematic steps we can follow in this hypothesis testing, the formulation of a null hypothesis and appropriate alternative hypothesis, which is accepted if the null hypothesis has to be rejected.

So the definition of this null hypothesis such as what is null hypothesis and what is alternative hypothesis we will see in a minute. Only one point that we should note here that whatever we are putting in this null hypothesis, we can say that either we should reject that null hypothesis or we should say that based on the data available, we fail to reject the null hypothesis. Generally, it will be wrong to state that this null hypothesis is accepted.

So, as we cannot declare this null hypothesis is accepted, maximum what we can say that we fail to reject this null hypothesis. So on the other hand what you can say that null hypothesis is rejected. When you say that null hypothesis is rejected that time we can accept this alternative hypothesis. So, our in this hypothesis test whatever the hypothesis that we want to test is generally put in this alternative hypothesis.

So, at the end of this test, if the null hypothesis is rejected, we can say that the alternative what basically we are trying to test, we can accept that hypothesis. So, whatever our goal or whatever we are trying to test is generally put in the alternative hypothesis, specifying the level of significance. This is the second step. Once we have clearly defined what my null hypothesis is and what my alternative hypothesis, I have to define one level of significance.

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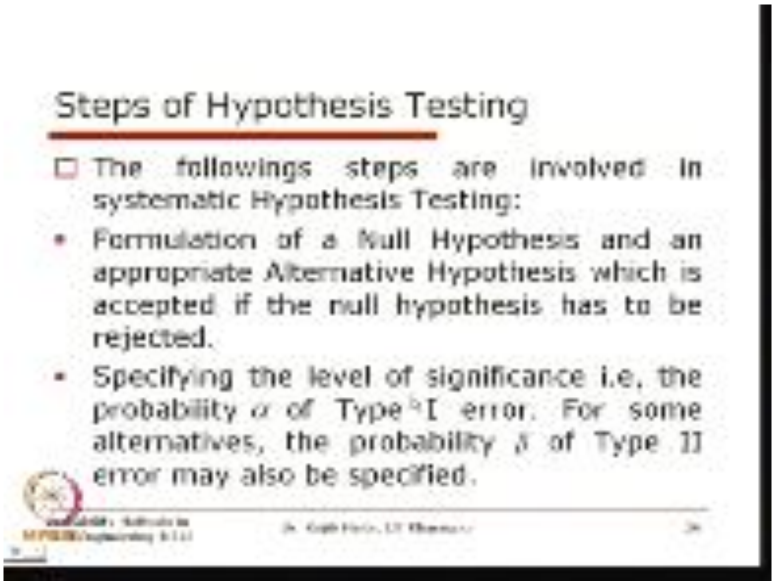

This is the second step. Once we have clearly defined what my null hypothesis is and what my alternative hypothesis, have to define one level of significance. That is the probability α of the type one error. As we just now told that our goal is that for this type one error, we should minimize as much as we can, so generally this α can assign the value of 5 percent or1 percent like that, so α equals to 0.05α equals to 0.1.Theseare the typical values of this alpha that we can fix.

Of course, in some alternatives, the probability of β that is probability β of type two error may also be specified. But, in general case what we do is the level of significance is generally predetermined before I go for this hypothesis testing.

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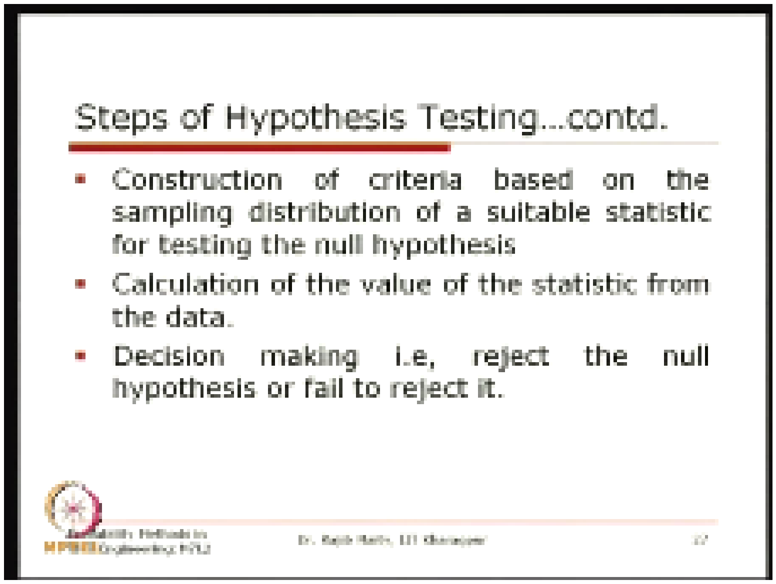

Then construction of criteria is based on the sampling distribution of a suitable statistic for testing the null hypothesis. So That suitable statistics that we have to develop and we should know what is the probabilistic nature of the statistics. In the earlier discussion, of this parameter estimation there also we have developed some statistics and we have seen that what the sampling distribution of different statistics is. Similarly, here also the suitable statistics for this test, we have to construct and depending on their probabilistic distribution we have to find out what is the critical value for that.

This suitable statistic has to be constructed. Then a calculation of this value of this statistics from the data, so whatever the statistics that you have decided, for the hypothesis test in hand. For that one whatever the observed value that we are having, based on which were testing this hypothesis that statistical value should be obtained. After that the final step is for this decision-making. That is rejecting the null hypothesis or we can say that fail to reject the null hypothesis.

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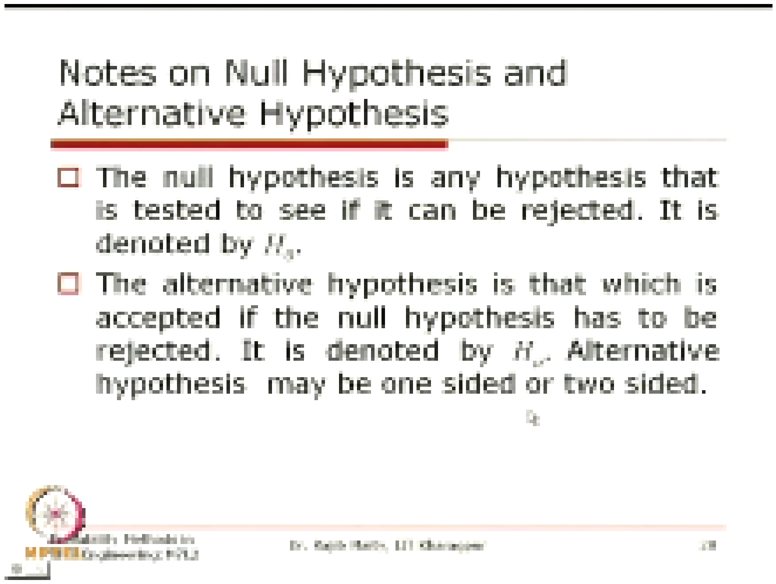

So, as we are telling that using the two words, this null hypothesis and the alternative hypothesis, the null hypothesis is any hypothesis that is tested to see if it can be rejected; it is denoted by H_0. The alternative hypothesis is that which is accepted, if the null hypothesis has to be rejected. It is denoted by Ha. Alternative hypothesis may be one sided or two sided. We will see what is one sided and what is two sided here. So, what we have just discussing that in case we can reject the null hypothesis then we accept this alternative hypothesis. We never say that the null hypothesis is accepted, so that is why whatever our goal is we generally put it in the alternative hypothesis.

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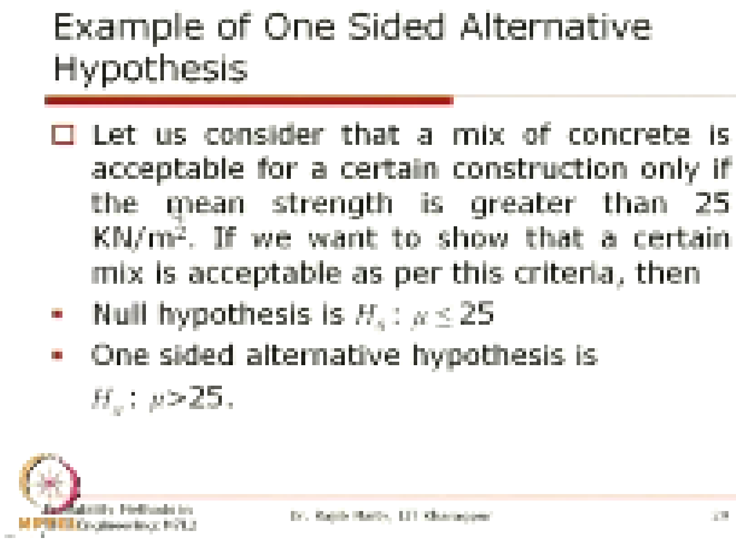

Now, the example of this one sided and two-sided that we will see is let us consider that a mix of concrete is acceptable for a certain construction, only if the mean strength is greater than 25kilo newton per meter square. So, if this is the threshold value and I say that only if it can be greater than this one then only it is accepted. Whether based on this sample data, we should decide or we should accept that yes the strength is greater than 25kilo newton meter square or not that we will see. So, that is why in the alternative hypothesis, we put that μ is greater than 25.So, the μ is greater than25, is in this alternative hypothesis.

So, in the null hypothesis, the remaining thing that is less than equals to 25 is put therein this null hypothesis. Remember that here it is acceptable only if the mean strength is greater than 25 kilo newton per meter square is written. If it is written that it is greater than equal to 25kilo newton meter square then in this one side alternative hypothesis, the equality sign we put here that is μ greater than equal to 25.Here, in this null hypothesis, we write that μ less than25.

So, depending on this statement what we are trying to test that will be exactly written in this alternative hypothesis.

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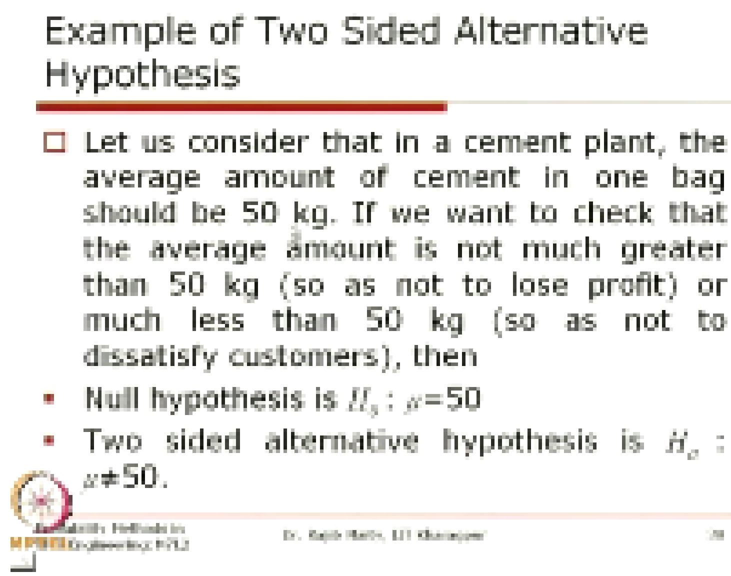

Similarly, the example of two sided alternative is the let us consider that in a cement plant, the average amount of cement in one bag should be 50 kg. Now, if we want to check whether the average amount is not much greater than25 kg, so if it is more than 25 kg, this will be the loss for the manufacturer. It should not be much less than 50 fifty kg as well because this will dissatisfy the customers. Then what we are trying to test is that whether we can accept that this check is whether per bag the amount is 50 kg or not. So, in this null hypothesis, it is written that μ is equals to 50 and two sided alternative hypothesis that we write is the μ is not equal to 50.

So, in this case, if we fail to reject that null hypothesis then the job is done, in the sense that manufacturer is happy and yes the μ equals to 50 kg that is 50 kg per bag whatever the sample that we have taken cannot be rejected.

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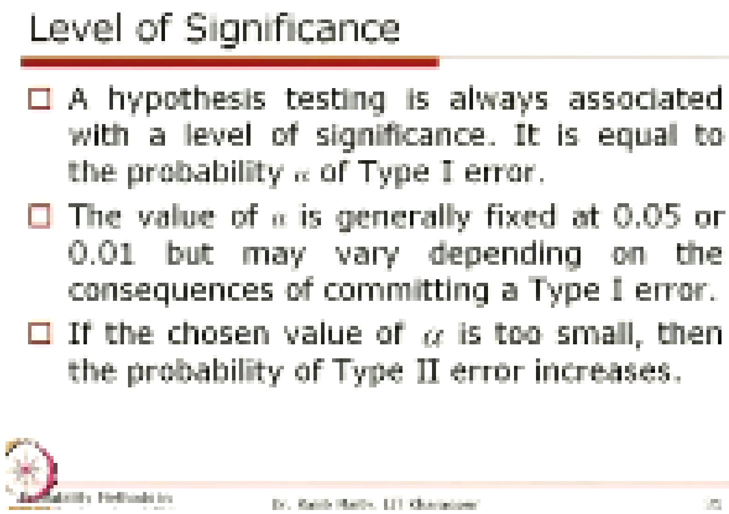

Level of significance says hypothesis testing is always associated with the level of significance. It is equal to the probability α of this type one error, so as we have discussed earlier. This value of this α is generally fixed at0.051or 0.01, but may vary depending on the consequence of committing the type one error. If the chosen value of α is too small then the probability of type 2 error is generally increases.

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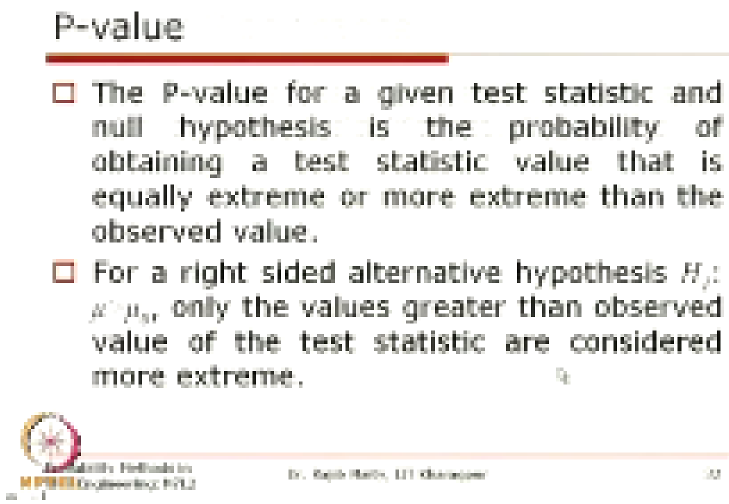

The P-value for a given test statistics and the null hypothesis is the probability of obtaining test statistics value that is equally extreme or more extreme than the observed. For a right side alternative hypothesis H_1equals to say μ is greater than some threshold value μ_0, only the values greater than the observed values of the test statistic are considered more extreme.

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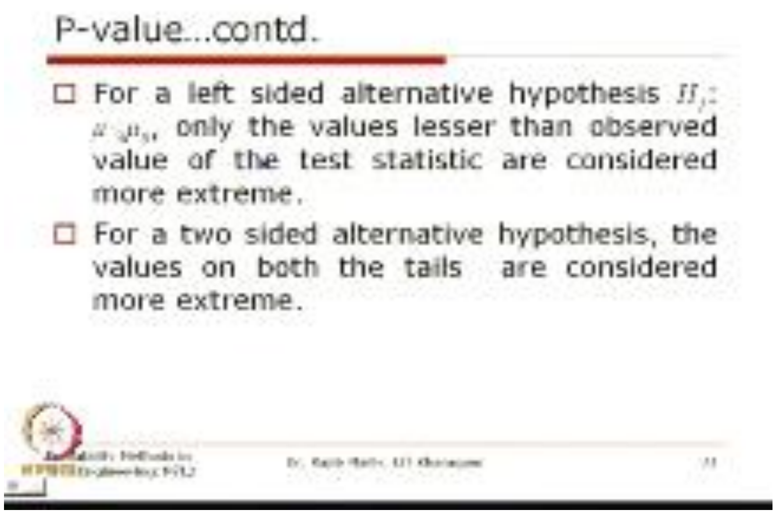

And For a left sided alternative hypothesis, when μ is less than some threshold value, only the values lesser than observed values for the test statistics are considered more extreme. For a two sided alternative hypothesis the values on both the tails are considered more extreme. So, basically if you see this P- value, suppose this is the distribution of this statistics that we are considering.

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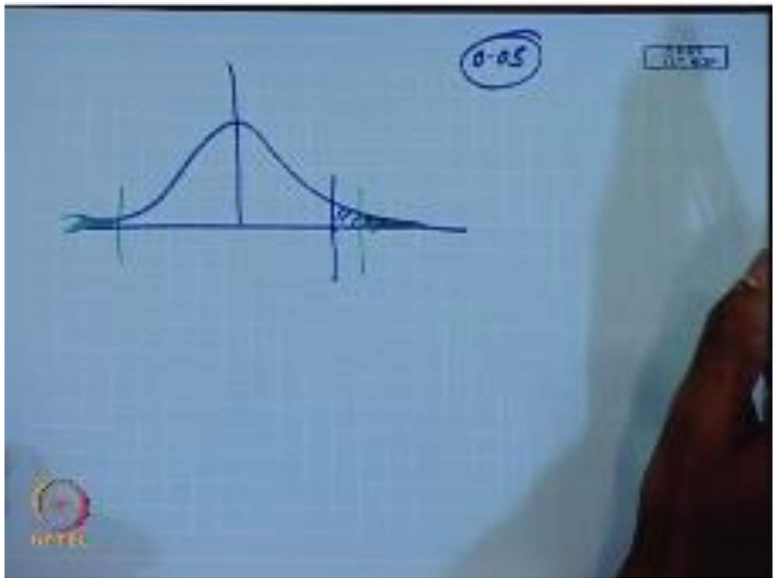

Now in this if you just see that the test statistics comes to this value then this is basically the test of significance at this level. So, which we generally fixed as 0.05 or 0.01, as the case may be. Now, suppose that the statistic is falling in this zone, then the P-value whatever is remaining here for the one sided test, the P-value is this green highlighted area. So, this area is known as this P-value. This is for this one sided. Obviously, the upper side, similarly, it can happen for this lower side also. Or if it is the two sided test then the symmetrical value we have to take here and this area also should be included for that for the calculation of this P-value.

So, once we set this significance level, then we get what is the zone. So, if the test statistics fall in this zone then we should reject the null hypothec is. If it does not fall in this critical region, the blue highlighted is the critical region for this any significance level then we cannot reject this null hypothesis. We will continue with this one, with this discussion of this hypothesis testing in the next lecture. Also we will take up some problems also and various hypotheses concerning two means, one variance or two variance. This type of problems we will take in the next lecture.

End of Lecture 36

Next: "Hypothesis Testing "in Lecture 37

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