INDIAN INSTITUTE
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NPTEL
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Probability Methods in Civil Engineering

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Lecture – 17

Topic

Probability Models using Normal Distribution

Welcome to this course on probability methods in civil engineering. Today, we are starting a new module and this module is on common probability models and in this module we will take whatever the standard distribution that we discuss in module 4 we will take those models those distributions and we will show some practical application in the different fields of civil engineering. So, today that this lecture mainly we will be talking about the probability models using normal distribution and this distribution when we are taking the normal distribution, you know.

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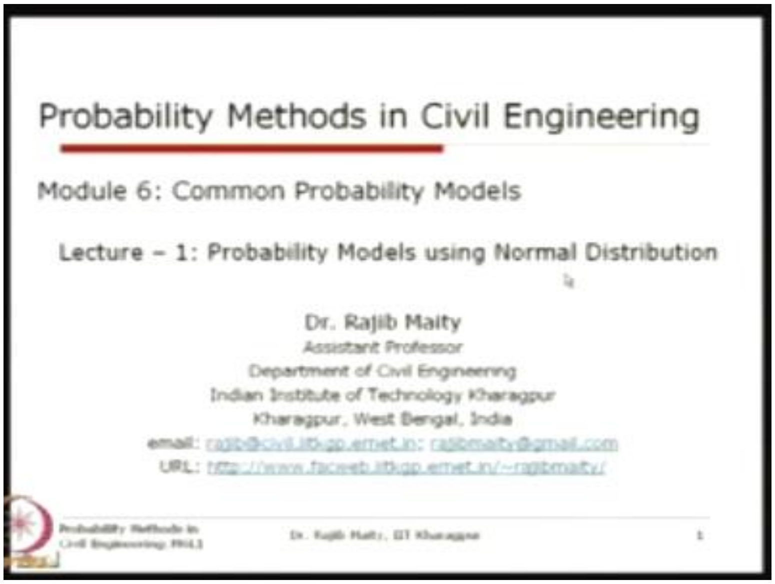

That its different, it is different properties and so far as this is pdf and CDF is concerned, we have discussed in earlier module and in this lecture also we will just briefly see those things and mainly our focus for today's lecture will be show some practical civil engineering related problem, in which this properties of this normal distribution has been used. That we will see. Basically in this module, we will take up both the discrete random variable as well as continues random variable, both and their standard distribution that we discussed earlier. So we are starting with the normal distribution because this is so far as the concept of that continues random variable will be easier for to start with this normal distribution. So the outline for today's lecture.

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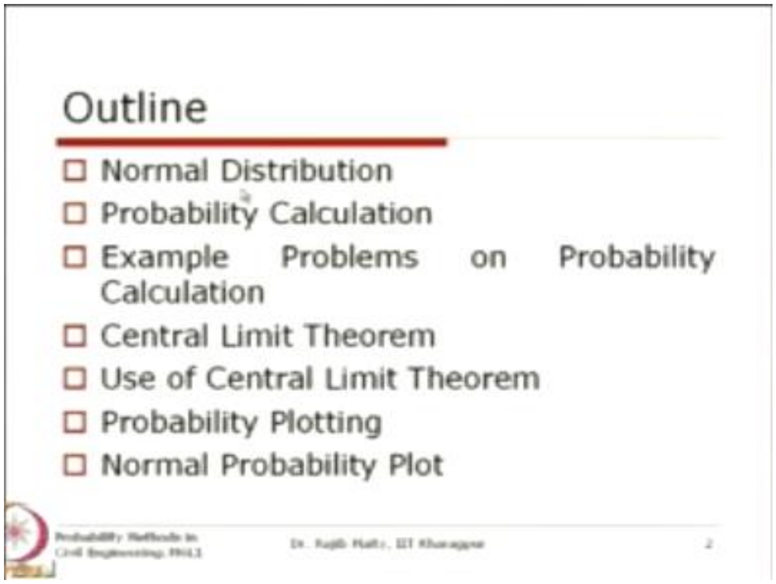

Will be, we will first briefly discuss about this normal distribution and how we calculate the probability because you can recall from our earlier lecture that for this normal distribution, the calculation of probability or the integration the integration form is not is not available in the closed forms. So, we have to go for some numerical integration and there are some standard tables that we introduced. So those tables have to be used. So, basically after that we will discuss about some of the problems related to the civil engineering gradually we will also see the other which are some of the very important theorem in the context of this normal distribution for example the central limit theorem.

That theorem we will discuss maybe not in this lecture but the subsequent lectures of this module. And after that we will take up this probability plotting basically when we see that probability plotting that in that probability plotting what we do we generally test. This is not specific to the normal distribution this can be for any distribution and probability plotting means we generally plot the probability on a special paper just to investigate whether the data that we are handling with whether that is following a particular distribution or not.

Basically in this in this lecture while discussing those problems we generally assume that that data set is following a normal distribution so that means the problems are started with the assumption of this normal distribution so the question that remain unanswered is that how do you test that whether really the data that we are talking about is following a particular distribution in this case the normal distributions.

So that is generally tested through different statistical test and those test we will take up in this next module and in this one what we will do we will show some the concept of this probability plotting and so through which we can test that from which distribution the data set is drawn. So here in the context of this normal distribution we will we will focus on this normal probability plot but this will be taken in the subsequent lectures today, we will specifically concern about that normal distribution at their some of the specific application in the different field of civil engineering.

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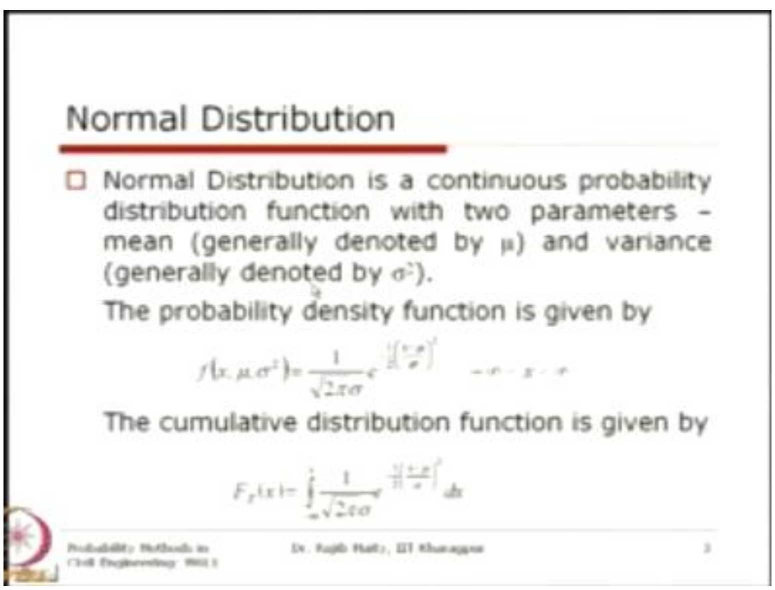

So, as you know from the earlier lecture that this normal distribution is having two parameters one you know, that is the μ and another parameter is called the variances so using those two parameters this is a bell set curve and the pdf of that normal distribution is from this one where these two parameters are there μ and σ^2so and this is a continuous probability distribution you know that and this continuous probability density function with two parameters one is that mean, which is denoted by μ and other one is variance which is denoted by this σ^2.And now, this probability density function is given by in this form, that is $f_x = f_x\ \mu = 1 / \sqrt{}\ 2\ \pi\ \sigma\ \sigma$ is outside this root and then ($e^{-\frac{1}{2}}$ x- μ / σ $)^2$ and this is a support of this distribution which is from - ∞ to + ∞

And, if we want to know that that cumulative distribution function which is CDF at any specific for a specific value, which is x, you know that this is a integration from this - ∞ to that specific point and this integral from. And this is the integration which, for which that any close from solution or the analytical solution is not available.

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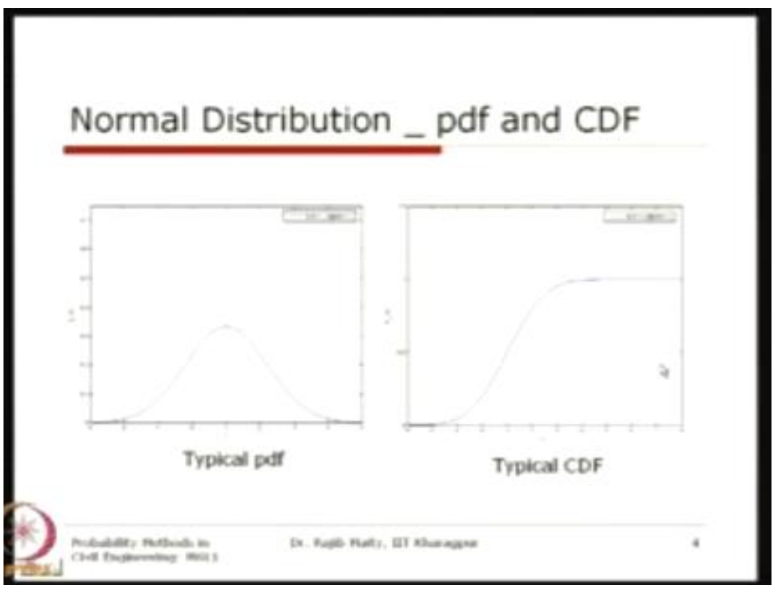

And we have also discussed that, there are some different that the numerical integration is done and that is presented in terms of some tables that table also we will see in a minute. Before that these are you know that this is a typical pdf of a normal distribution how the pdf looks like this is a this is bell shape curve and it is symmetrical with respect to its mean and its typical CDF that is distribution that cumulative distribution function which is starts from 0 at - ∞ and go gradually increases and it is asymptotic to 1 at + ∞.

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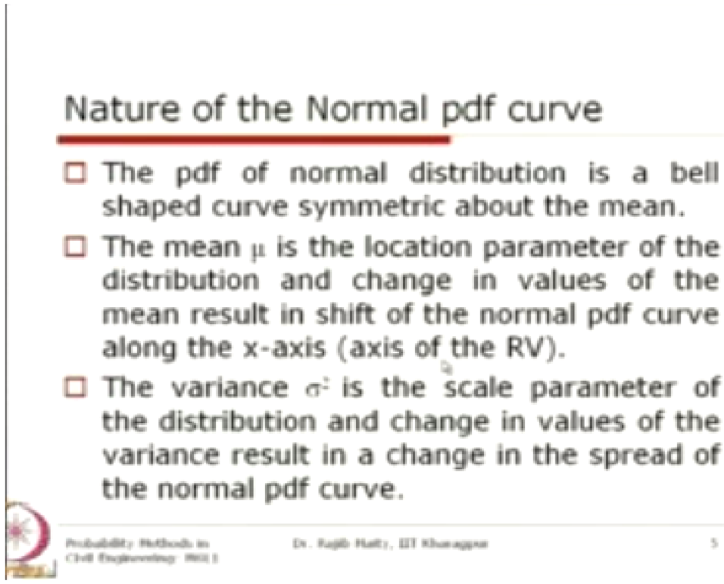

So now the pdf of the normal distribution is a bell shaped curve that you have seen that it is symmetric about its mean and the mean μ is the location parameter of the distribution and the change in the values of the mean results in the shift of this normal pdf curve along the x axis. So, that what happens is that?

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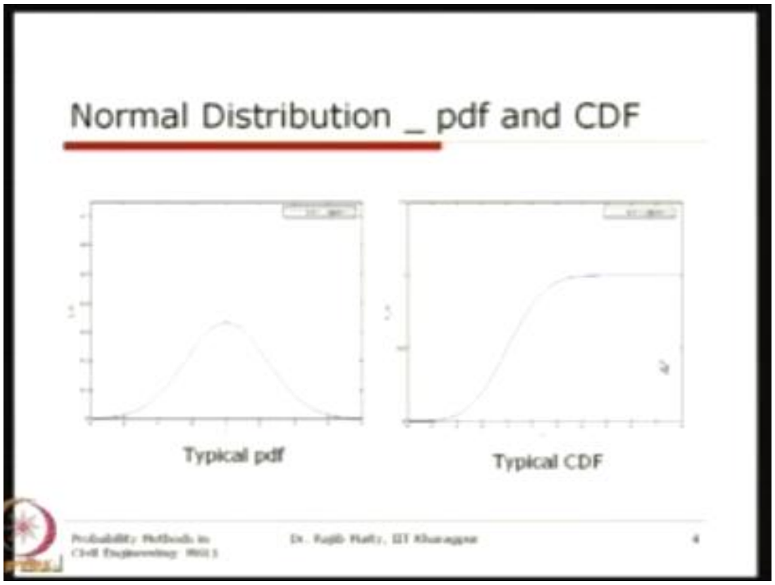

So here you can see that may be the mean is around so mean is at 1 so if we just change it so at this mean this is the maximum you can say that the value of this probability density function is maximum at this means so if we do not change this σ so this over all shape of this curve will not change only that that depending on what the mean is this curve will be will be relocated or this will be shifted.

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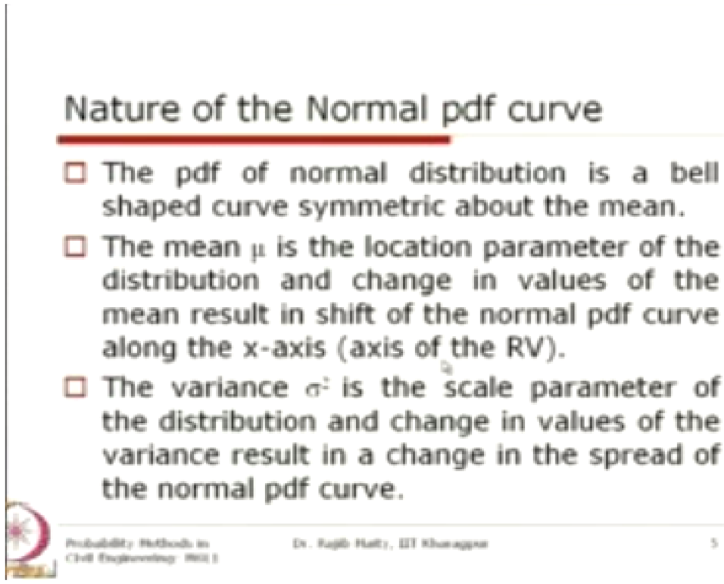

So that is why this μ is known as your that location parameter and whereas that this variance that is σ^2, this σ^2 is the scale parameter why this scale parameter because it is generally controls that how what is the shape of that distribution and if we change this parameter that is if the variance is changed then the that it will result in the change in the spread of the normal pdf curve.

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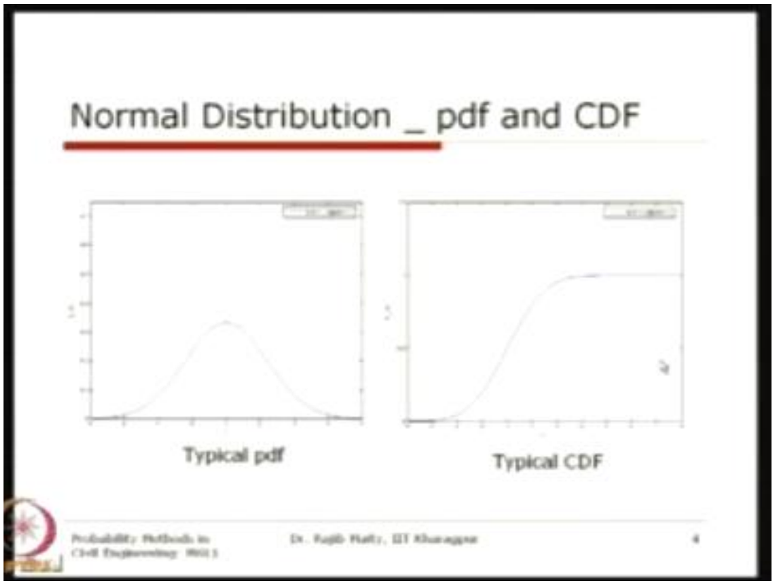

So the change in the spread of this normal pdf curve means it will be so if the mean is same so, it is maximum value of this pdf will be same at this location only but its spread will be changed. So, you have seen it earlier so the curve will look like this so if this.

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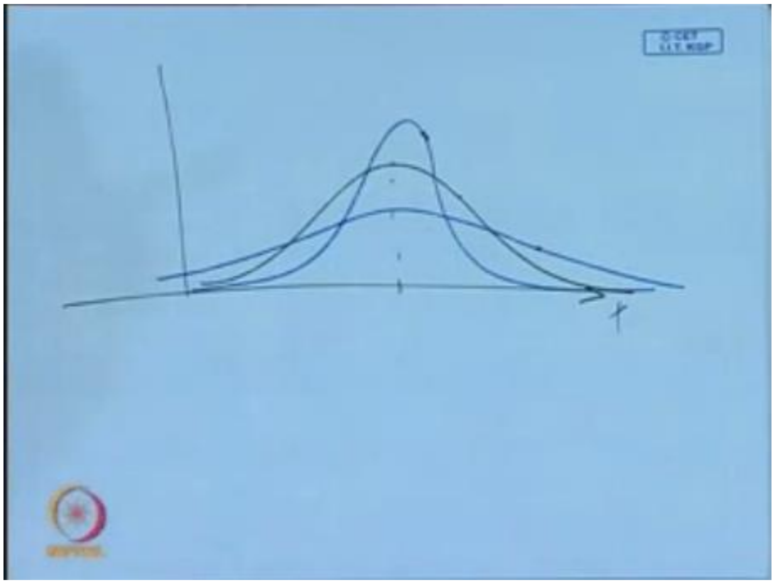

Is your x axis then if the 1normal distribution looks like this and if this its mean and if the scale parameter changes then the then depending on whether it is increasing or decreasing this will be either, it will be flatter or it may be even the even be more the spread will be much lesser so, this is for the minimum among these 3 curves this is for the minimum variance and this is for the medium and this is the maximum variance is here but all for all these the mean is same the location parameter is same.

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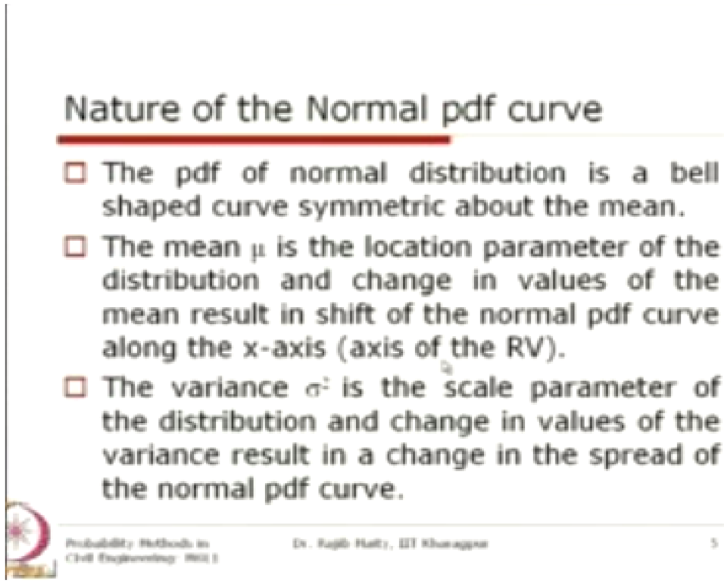

So this is what is explained here that variance $\sigma^2$is the scale parameter of this distribution and change in values of the variance, will result in a change in the spread of the normal pdf curve.

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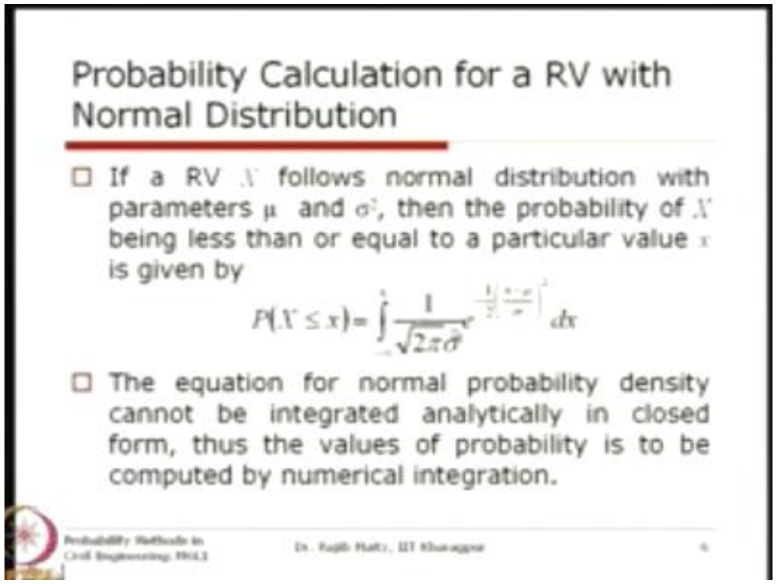

Now, as we have shown earlier also that for this random variable x which follows a normal distribution with parameter mean is μ and the and other one is σ^2, then the probability of x being less than or equal to a particular value x is given by the probability x less than equals to that particular value, which is the integration from this left extreme, that is $-\infty$ to x and from 1 by this form, that pdf form, $(1 /\sqrt{}\ \pi\ \sigma\ e - 1/2\ x - \mu\ \sigma)^2 x$. So, this integration if we do it for any specific value, putting any specific value x, then we will get it is CDF that is cumulative distribution function.

Now, this equation for this normal probability density cannot be integrated analytically as I told in the in the close form. Thus the value of the probability is to be computed by numerical integration so once this μ, these parameters are known that is the μ and this σ is known then using this one for a specific value of this x if we do this integration numerically then we will get what is the probability from minus infinity to that particular to that specific value.

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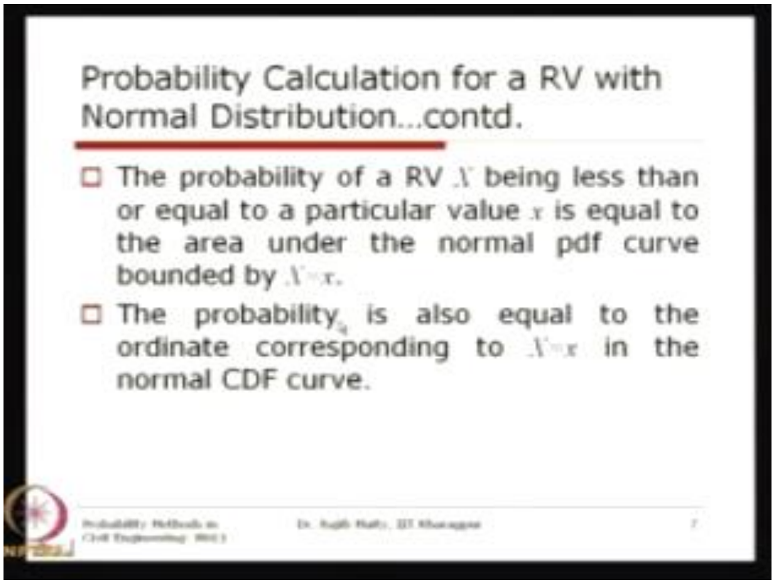

So every time the numerical integration instead of doing that one there are some standard tables are also available that we will discuss in a minute. So we can make use of that table but that table is generally for the standard normal distribution and we can use that standard normal distribution for the calculation of this probability and that we will discuss while we are applying to a different problems.

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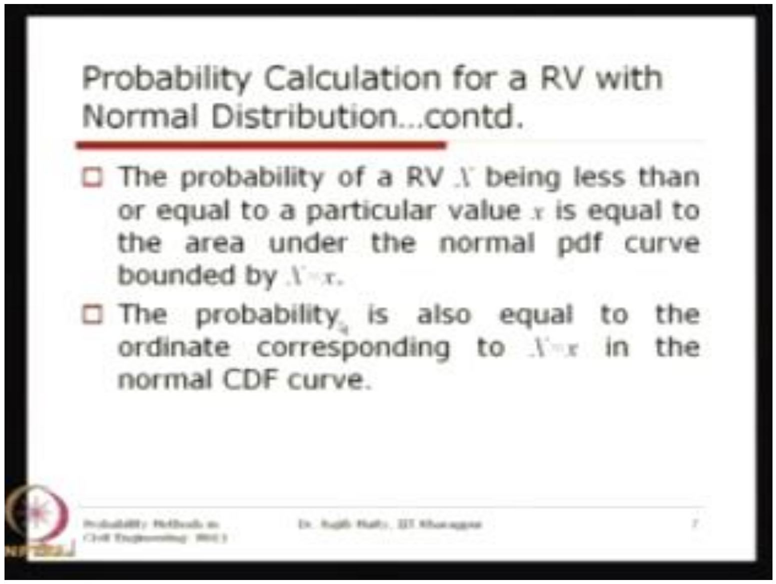

So the probability of a random variable that is x being less than or equal to a specific value x is equal to the area under the normal pdf curve bounded by x equals to that specific value x and the probabilities also equal to the ordinate of the corresponding to the x equals to this x in the normal cdf curve. So what is mean is that so this is the total area that we are talking about from this integration.

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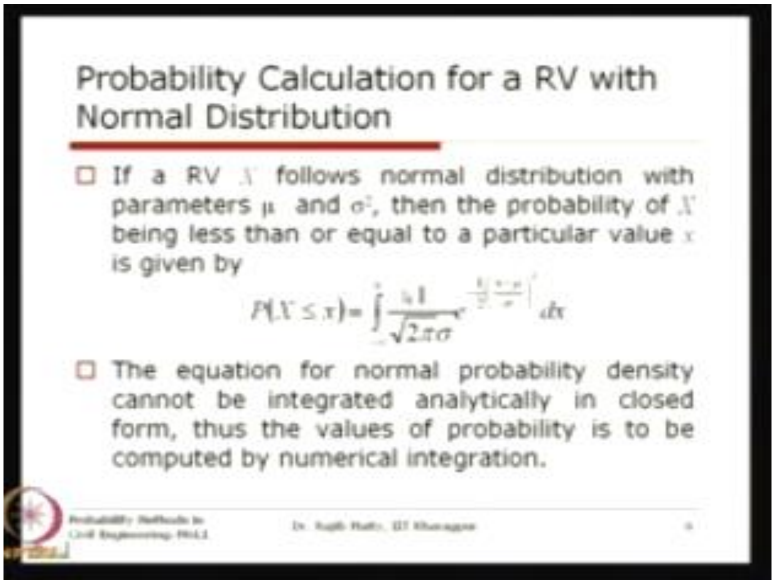

That what we are getting.

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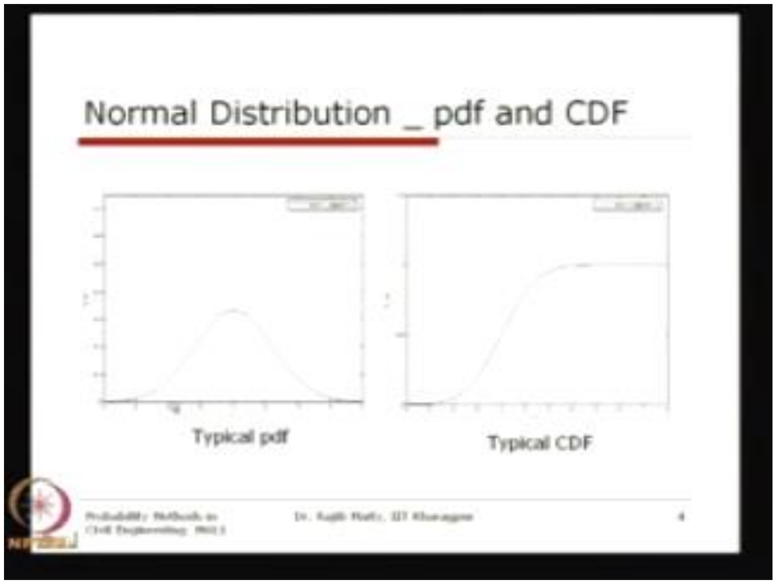

If we just refer to the those curves that is so this is so up to any specific point whether it is from any specific value if we come so it is the total area up to that point below this curve and so and that for that specific point whatever the area is obtained that is the probability which is indicated here by its ordinate value. So what we are referring here is that is that this area.

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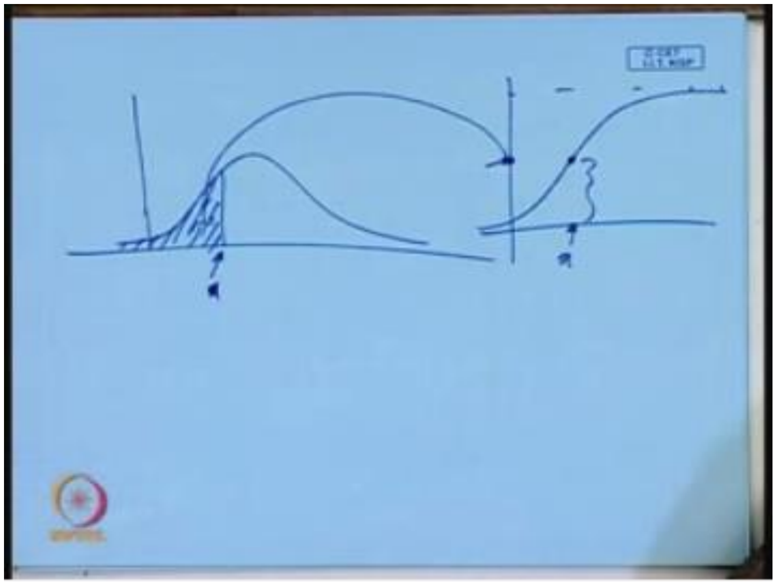

That is for say this is your pdf. So any specific value that we are talking about that x so up to this whatever the total area is covered here so this area for this value x will be reflected in the cdf so if this x corresponds to here this x now the cdf curve that we are that we have obtained is so this ordinates so this value this value here this height or this value is nothing but equal to this area. So this is what how you know that this pdf and cdf is related to.

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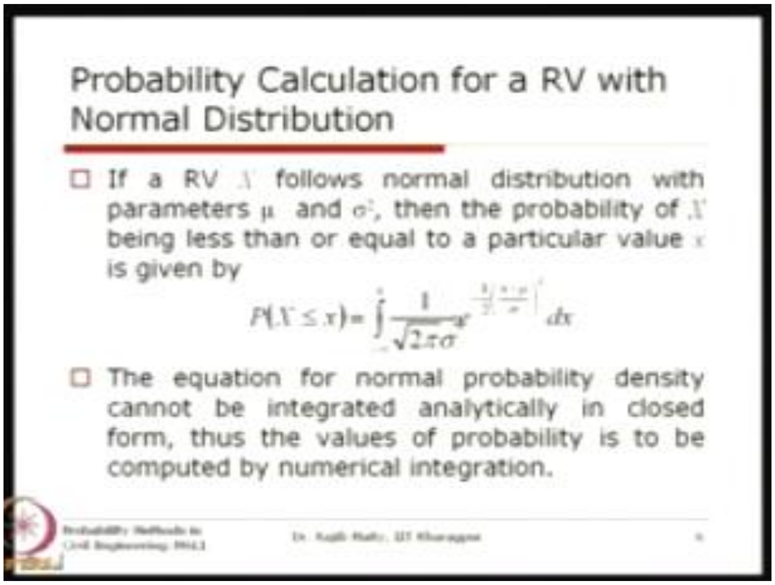

So this integration when we are talking about this integration when are talking about this integration is basically referred to the area and that the curves. So far as pdf is concerned and the ordinate value in case of the cdf.

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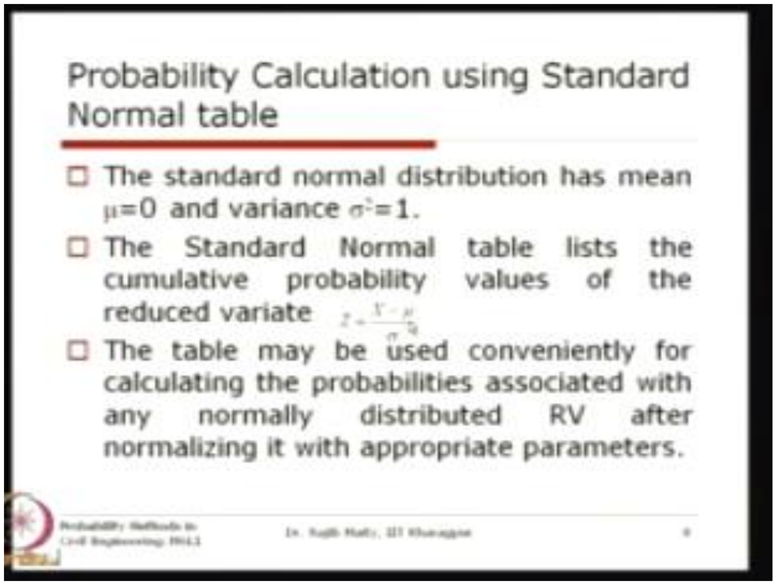

So now we will see that that a standard normal distribution. So you can you can now understand that even though this integration it can be done numerically now depending on the choice of this μ and σ so that curve will change. So once so there can be some many means infinite numbers of the combinations so where for a specific value of μ and the specific value of σ^2, so depending on this mean and σ^2 the curve will change.

So every time you are supposed to do that numerical integration if you want to do that integration for the specific normal distribution. So instead of that if we use the concept of this standard normal distribution then for the standard normal distribution thus mean is 0 and the variance is 1, so for that specific curve if we can do this integration and if the value is available then for any such distribution we can use the same table to know what is that integral value or what is the probability corresponding probability.

So this is what is explained here this standard normal distribution as you know has the mean μ=0 and variance σ^2= 1.

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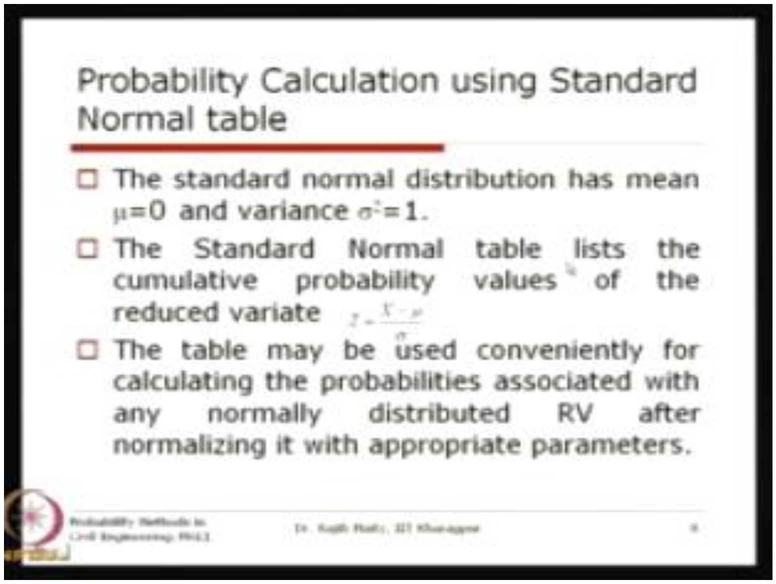

So the standard normal table lists the cumulative probability values of the reduced variate z = x-μ/σ. So this is important that when so if this x is following a normal distribution which is having the parameter mean μ and variance σ^2 now you we have also seen in this last module also in the function of random variable you can refer to that this is a 1to1 transformation and this is a linear transformation as well.

So if we do this type of transformation then this is also another random variable which is denoted here z and is generally known as their reduced variate now this z will have will also follow a normal distribution which will have your that mean is equals to 0 and the variance is equals to 1.So once we from any random variable if it follows the normal distribution now if we do this type of transformation that is the transformation is that x – μ/ σ.

Then the resulting random variable will have a will also have the normal distribution but its parameter will be mean 0 and variance 1. Now the table may be used conveniently for calculating the probabilities associated with any normally distributed random variable after normalizing it with the appropriate parameters. Now with the appropriate parameters what is mean is that. So with the mean of the original random variable that is x and divided it by the standard deviation of the original random variable x. So before you discuss further on this we will just see.

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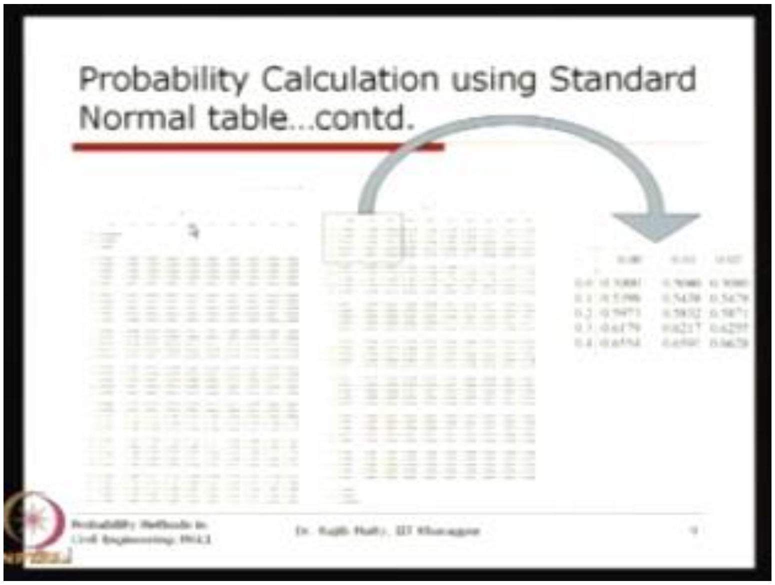

For the standard normal distribution how the integral values or the standard statistical table is available to us. This we have we have discussed that if so this is the bell shaped curve this is the pdf of this normal distribution basically this is for the standard normal distribution and this point is indicating that this is 0. So which is the mean of this of this distribution and this for this standard normal distribution you know how this forms look like. So this is the cumulative form that is shown here which you can see which you know that that is your fz.

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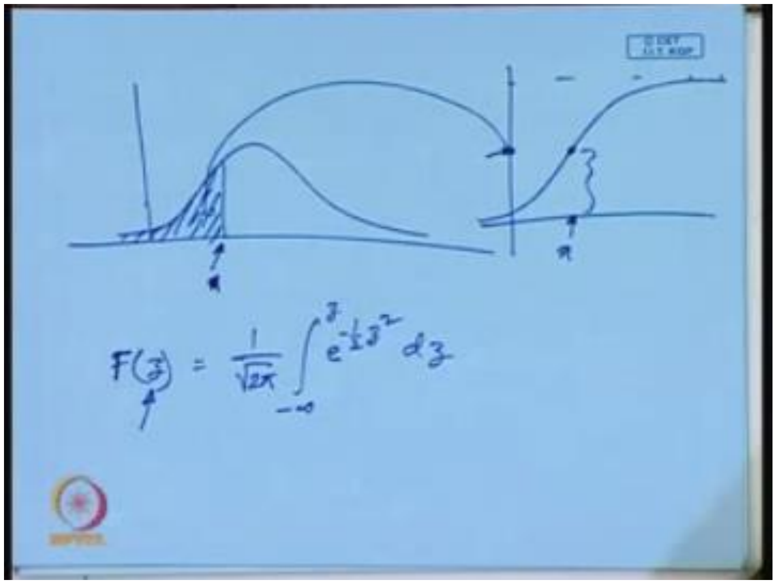

That is a reduced variate z and this is capital F that means we are referring to that the cumulative distribution function for this standard normal distribution which is equals to 1/ √2Π we are supposed to multiply it by σ but σ you know that this is 1. So now this is integration, integration from the left extreme which is minus infinity up to the specific value of that z and the integral form will be that $e^{1/2}$.

I can write half then that this is that $(z-0/\sigma)^2$. So σ is 1 μ = 0. So it will be only z^2 dz. So this is the form what is the value is shown in this table. Now for a specific value of z this value refers to this area if this parameter is μ =0 and standard deviation equals to 1.

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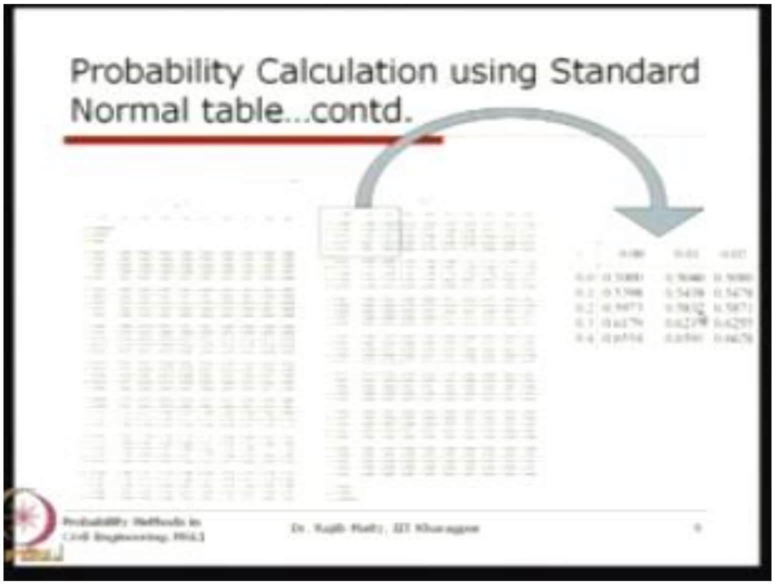

So here that probabilities have shown here you can see that this z values are starting from this minus 1. So this is shown here for so this is actually the entire table which is actually starting from -5 to +5. So here some portion is shown for the easy visibility. So here you can see that when the z = 0. So this column will just read that 1 that is z = 0 and this row is for the second decimal.

So this point corresponding to so 0.5832 that you can see here this is basically corresponds to z = 0.2 1. So this is the second decimal point. So at z = 0.2 1 the value is 0.5832. So this is the value is given.

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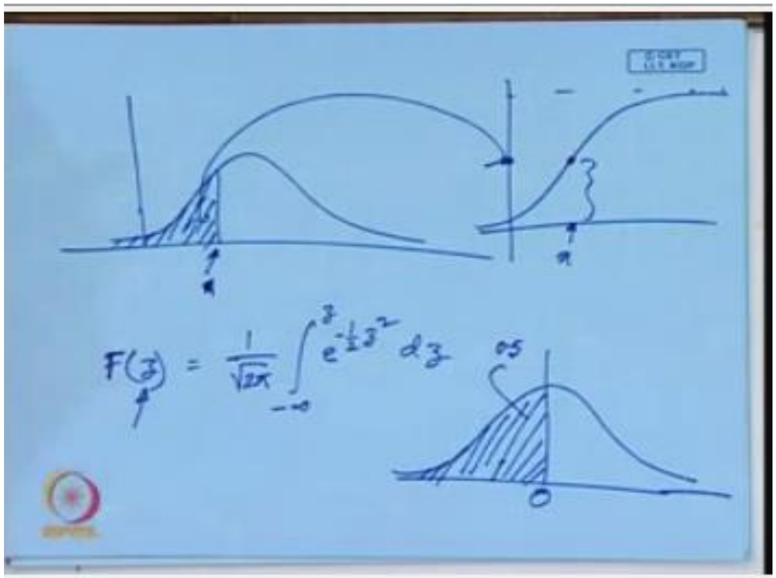

Now you know from the symmetry if this is the standard normal distribution. So this is your 0 and as this is symmetric you know that due to the symmetry, so what should be this area up to this 0 means the total area under this curve is 1. So up to this 0 it would be 0.5.

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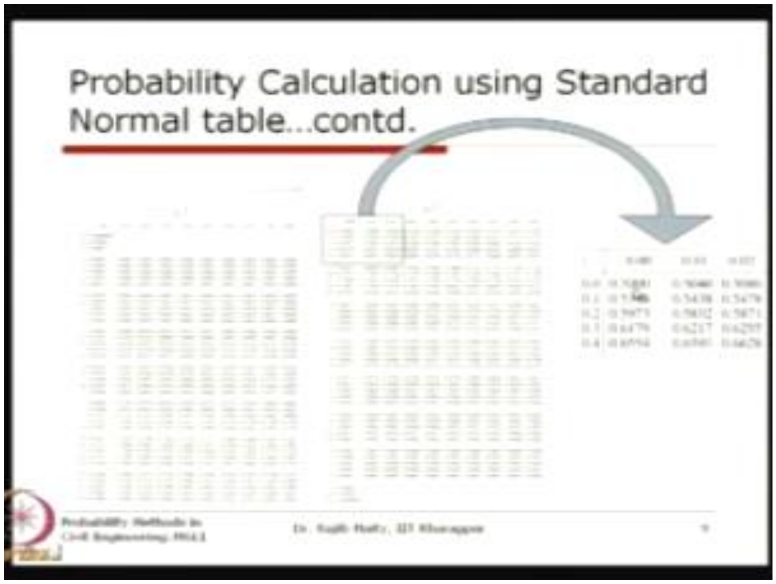

This is what is also shown in this curve that is when z = 0 the integral value is 0.5.

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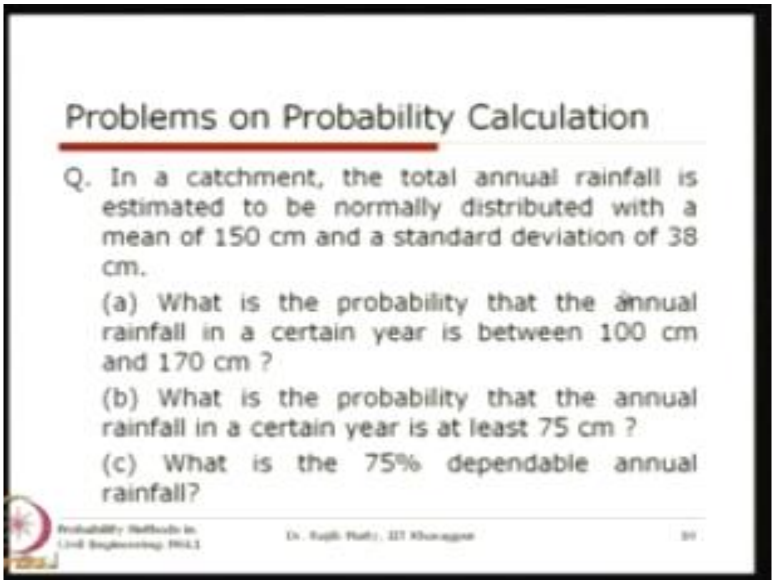

So we will now see a specific problem on this how we can utilize this property is of this normal distribution for some specific problem in civil engineering. So the first problem in a catchment the total annual rainfall is estimated to be normally distributed with a mean 150 centimeter and a standard deviation of 38 centimeter, now here lies the lies the question that we just started with this class that whenever we are starting this in this lecture whenever the problem we are starting with we are stating that a particular random variable is following a normal distribution.

So once we say the normal distribution that means that parameters are also we have to know. So here the mean and standard deviation we know that for the normal distribution there are two parameters so those two parameters are also known so but the what is unanswered is that how to test that here it is referred to that annual rainfall how to know that vary in normal distribution or not.

So that question will be taken in the subsequent lecture how to test that its normality that we will see. But here we are starting the problem with the assumption that this follows a normal distribution with some parameter value. In a catchment the total annual rain fall is estimated to be normally distributed with a mean of 150 cm and a standard deviation of 38 cm so what is the probability that the annual rainfall in a certain year is between 100 cm and 170 cm second is what is the probability that the annual rainfall in a certain year is at least 75 cm and the third is what is the75 % dependable annual rainfall so there are 3 questions is put and the 3 specific goal is kept here the first question reads that what is the probability that it will be between 100 cm to 170 cm so may be for the policy making in some cases.

I need to know that what is the probability that it will be a normal year normal year means that the rainfall is near normal and what is that that probability so if I say that, that probability is very high then we will be more confident to state so that particular year the probability of this much rainfall that we get is having high probability and the second type of question what is thrown is that what is the probability that the annual rainfall in a certain year be at least 75cm the is kind of issues comes when we are interested about that minimum requirement of the catchment.

So if this is so some stake holders for the water resource there are a different stake holders for example the for the reservoir operation we are supposed to give the water for that multipurpose one may be the power generation or for the irrigation or for the industrial use so there are different users are there whatever the water is available now the question is that to sustain the whatever the need to the community sometimes we need to ask the question what is the probability that at least this much rainfall or at least this much water is available to me so this is of that type of question that at least 75 cm.

We can get from the thing and the last one is the what is the 75 % dependable annual rainfall that means that I want to be confident I want to be confident that the with the 75 % confidence level that I need to know that yes this much rainfall is certain at some probability level so at 0.75 probability level what is the value of this rainfall that I can expect so all these answers will or based on the assumption that at that point the rainfall is following certain probability distributions so we are first developing that probability e thing that probability model.

And from that whatever the parameter that we are estimated from that we have to answer these two questions in terms of a probabilistic manner so we will take these problems now one after another the first one is that 100 cm to 170 cm now again if you go back what is the information that is supplied to us the first is that it is mean is that 150 cm and this variance the standard deviation is given 38 cm so you know that for this one we do not have any statistical table if of this of any normal distribution.

Which is having mean 150 and this standard deviation is 38 what we are having is that we are having the table for this 0 mean and standard deviation equals to 1 means variance equals to 1 means that, that is standard deviation is also 1 so first fall s we are talking that we have to do the transformation we have to generate we have to obtain this reduce variate first for which we are interested to know the probability and after that we can make use of that table to give this type of answer so first question we are taking what is the probability that the annual rainfall in a certain year is between 100 cm and 170 cm and 170 cm.

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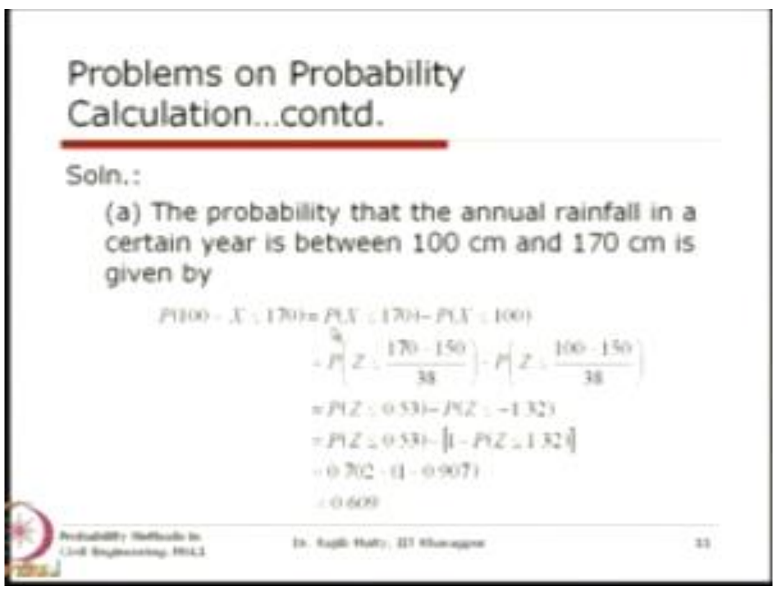

Now then so to find out this probability what basically we are looking for is the probability of this random variable x so that annual rainfall that we are getting in that catchment is we are the random variable is denoted by this x so this x will be between 100 to 170 so if we want to know this one then what we can say is that so if I just want to know that this is that.

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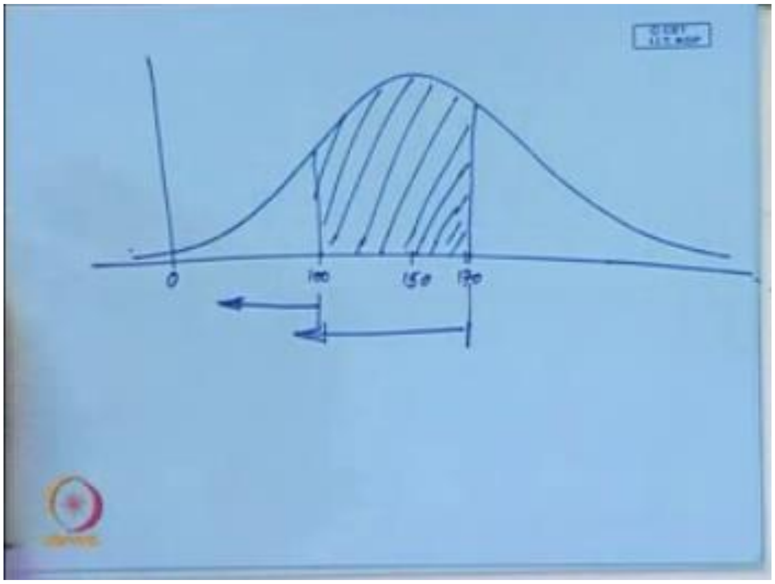

Distribution form so obviously if this is the origin I can say that this is 0 so the mean of this normal distribution which is 0 which is away from this origin away from this 0 so mean is shown to be that 150 so this is your 150 so what we are interested to know that this random variable between 100 and170 so that this is your 170 and this is your 100 so what we are interested to know is this area so what we can how to get this area what we cannot from the CDF point of view if you want to know.

That what is this area that is what we can get first we can get the total area up to this 170 which will be include this area as well minus this area this up to 100 so that difference will give you this area.

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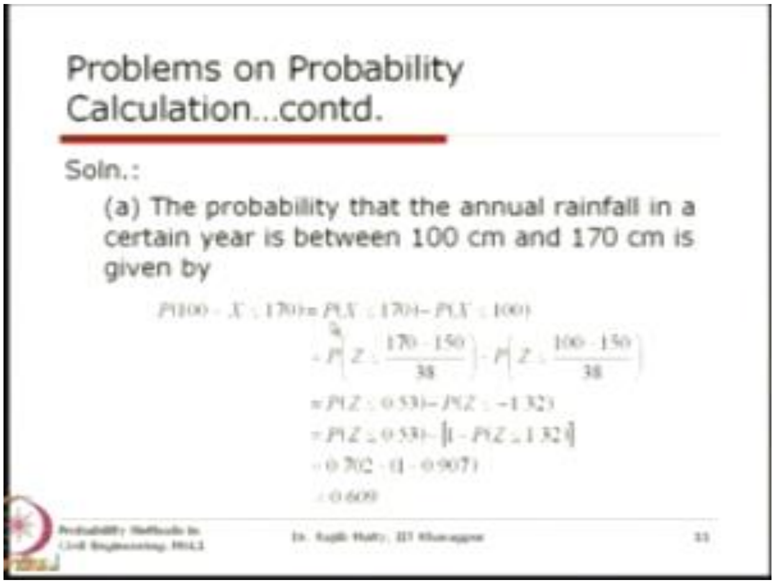

Which is exactly what is written here also that is probability x less than1 less than = 170 - probability x less than 100 now from here so for this type of normal distribution that numerical integrate numerically integrated values are not available to us we are having the standard normal distribution only so for that what we have to do we have to first convert it to this standard normal distribution and we have we know that what should be the conversion to declare that the new random variable will be having that 0 mean and standard deviation 1 so that conversion.

Is nothing, but the x that specific value that is 170 - mean and mean is 150 here divided by the standard deviation so that is what we are just mentioning that x - .

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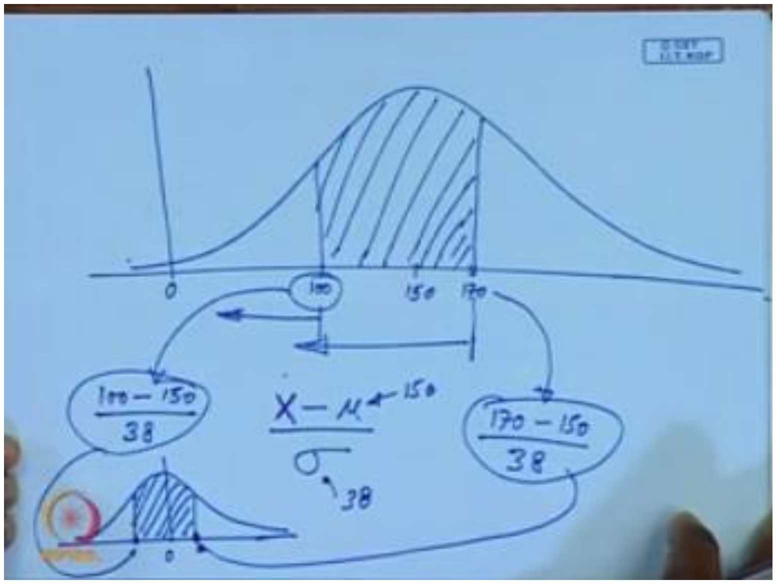

μ divided by σ now x can be of anything whatever is our interest and here μ we have defined to be 150 and σ is your 38 now for this 170 the reduced variate will be 170 - 150 divided by 38 similarly for the 100 this one the reduce variate will be 100 - 150 divided by 38 basically when we are getting these values so this values what basically what we are doing it we taking this full graph we are basically converting it to another normal distribution for which the mean is at 0 standard deviation is 1.

And we are here instead of referring to 170 this 170 has converted to this value and this value is somewhere here which is showing to be this point and similarly the 100 is converted to this one and this value is somewhere here which is a here so now we are interested to know that this area so this area is equals to this area to that original random variable scale so first of all we have converted to like this and because these values are available to us then we will be referring to this what the standard distribution is available to us then to get this answer that what is this area so here is done this one that is that we are reducing it from 175 -150 divided by 38 - probability of z 100 - 150 divided by 38.

So this corresponds to 0.53 and this corresponds to minus 1.32. now these values can also be directly obtained from the table that we have shown just now so first of all we will take this one we will just discuss more on this minus 13 basically we are using the symmetrical property so if the value is 0.53 then we have to we have to find out what is this value is corresponding to.

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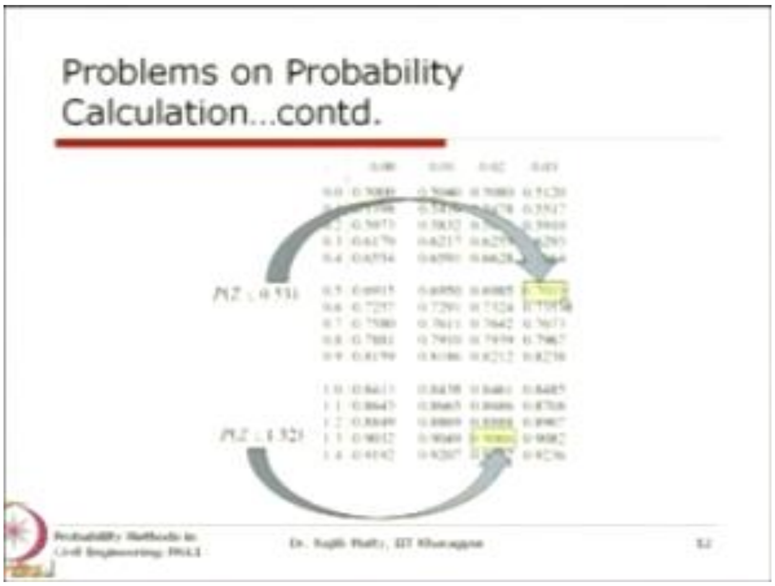

Now you see here so this is so 0.53 as we have telling that this column is the first decimal of the reduce variate z and this row is the second one so we have to go to this 0.5 and here we have to go to that003 so this is your 0.53 the value for this 0.53 which is 0.7019 so this is what we have written that probability z less than equals to 0.53 is equals to 0.702 now this 1 that z less than equals to - 1.32 can also be obtained from the table and can be directly used for what is this value.

Sometimes what happens in some text book you will find that using the symmetrical nature of this normal distribution table only one side and preferably the positive side values are shown so what happens.

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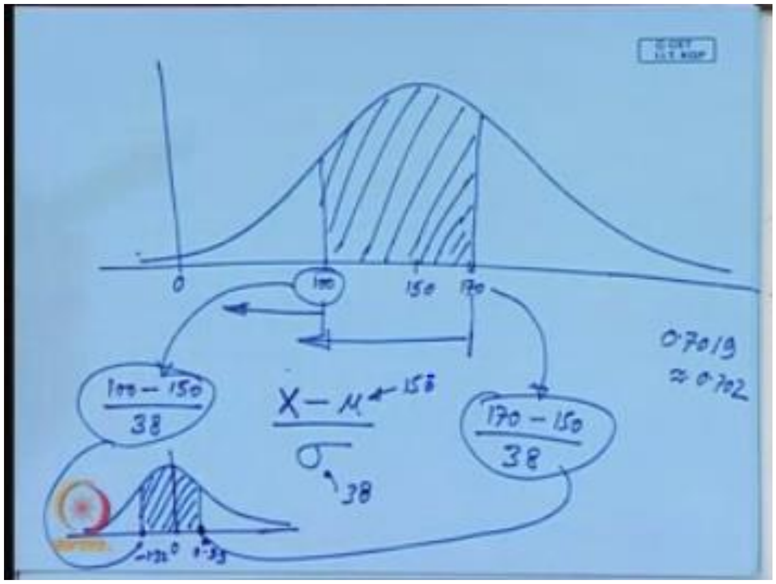

In those cases is done that from this 0 it starts and go up to infinity means here only 4 or5 and those values are given there so once we know this value that area then using this symmetrical property, it will be easier to know that what is the area that we are looking for. For example, this point for this problem what we have seen is referring to -1.32. Now, we are interested to know this area, now what is so, we are interested to this area. So, what we are doing is that so up to this point, that is 0.53, we have we came to know this from -∞ to this 0.53 that total area that we have seen is 0.7019 which is approximated to 0.702.

Now, what which area we have to, we have to deduct is this white area, that is from -∞ to point to 1.32 so this much area we have to deduct. So, if this is the negative side is not available to you then using simply that positive side also, we can do that analysis. So, how to do that one, so, we have to just replace this one that is 1.32 to this positive side, this area we have to know and this remaining this, whatever the white area is there, which is equals to this side area.

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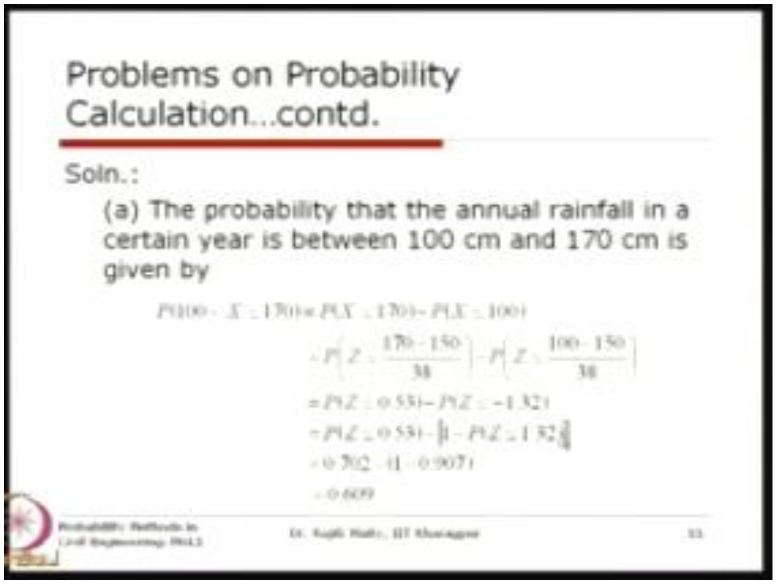

So, this is what is shown here that is probability of z less than equals to -1.32= 1-P(Z≤1.32) that is now we are using only the positive side of that value.

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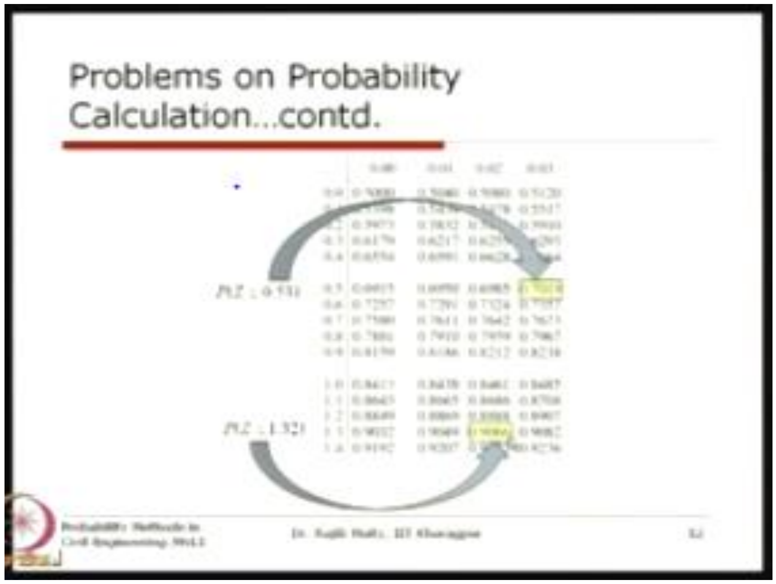

So, now that 1.32 value is also known from this table, that is this is z is 1.3 and second decimal is 2 which is 0.9066.

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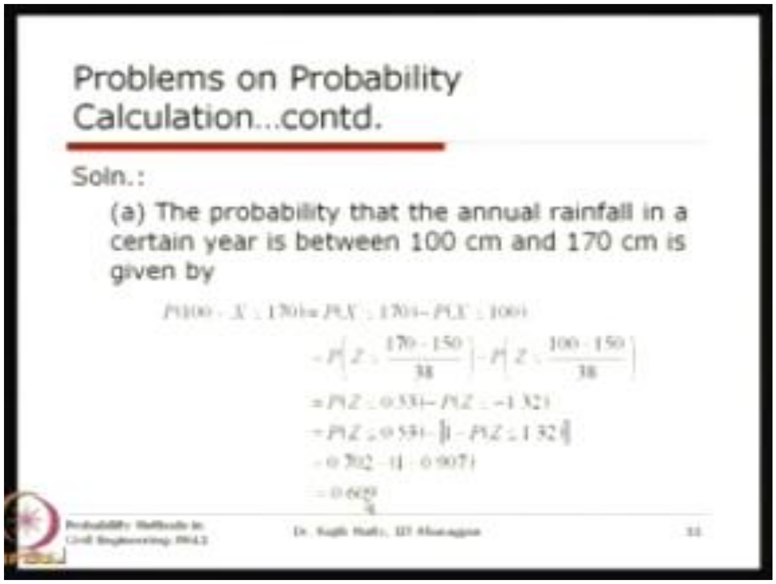

So that 0.9066 is approximate 0.907so this we calculate so we are getting the total area is 0.609.

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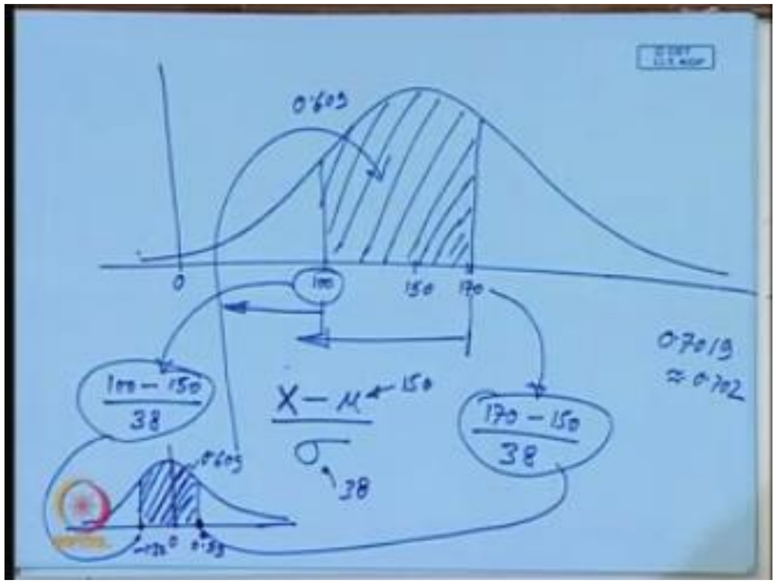

So, what is this total area that we are getting here from this reduced variate is 0.609, which is equal to this area as well. So, this is0.609 so, we can say that probability of the random variable x being between 100 to170 is equals to 0.609.

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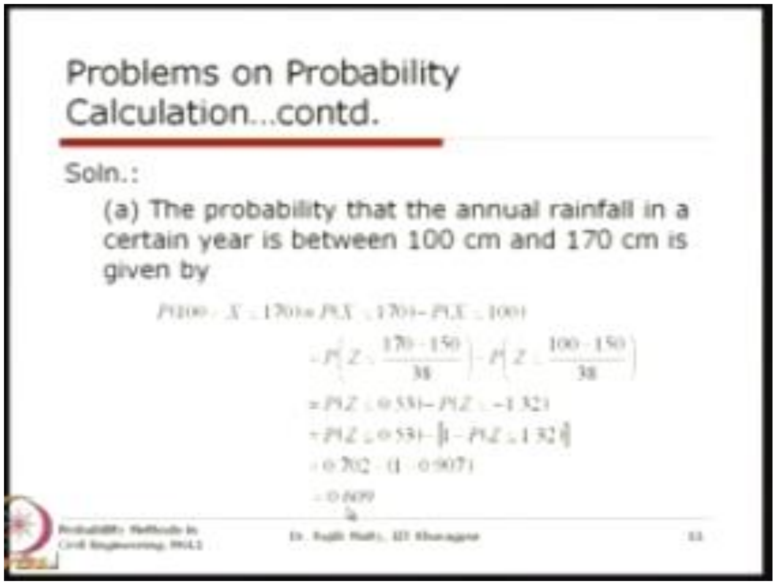

This is what is obtained here.

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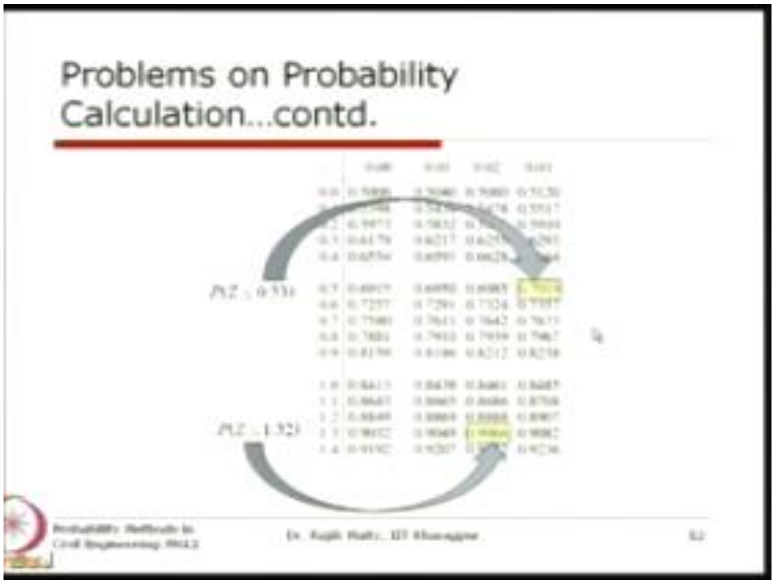

And we have explained that how we got this corresponding values.

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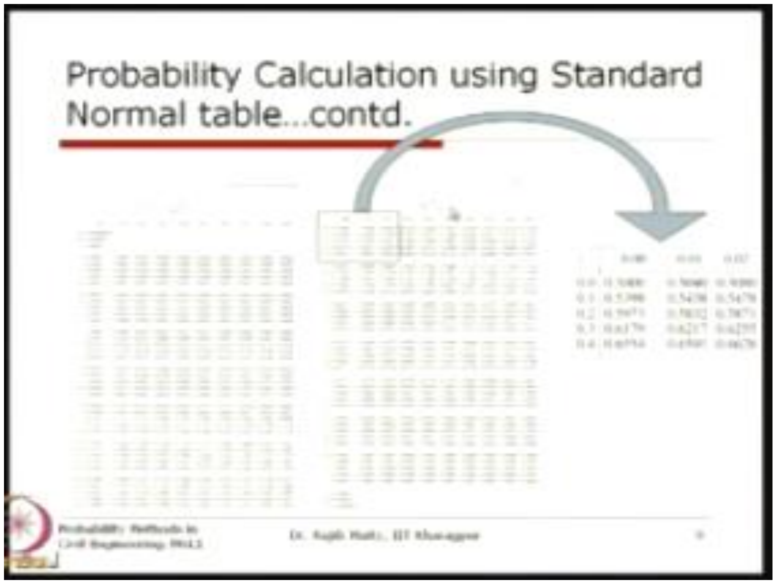

So, the table that we shown here we should know here that the table that we shown here is continuing from this $-\infty$ to $+\infty$ you can use, you can directly use this table also to know that what is the probability for this -1.32, that also you can use directly or if for some text book only the positive side is given, using that the property of this symmetry.

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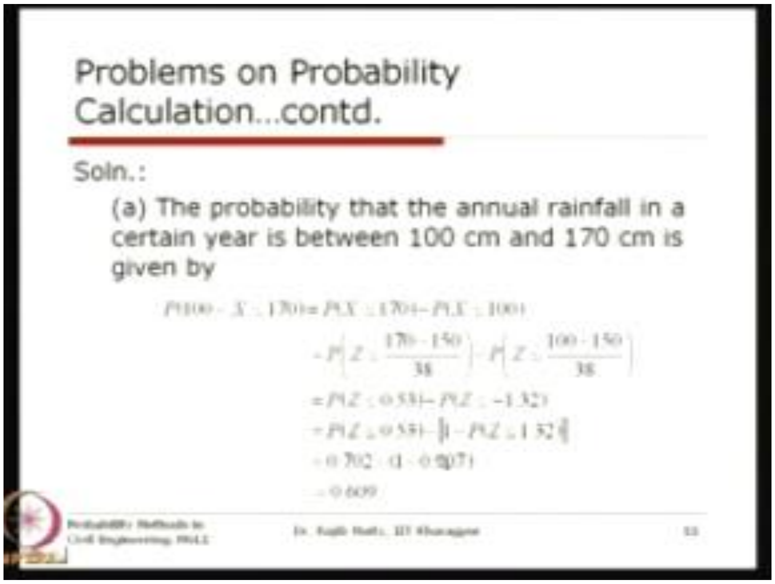

Then we can just convert it to this positive number, with the consideration of the fact that the total area is 1. So, then you can use this, that particular table as well.

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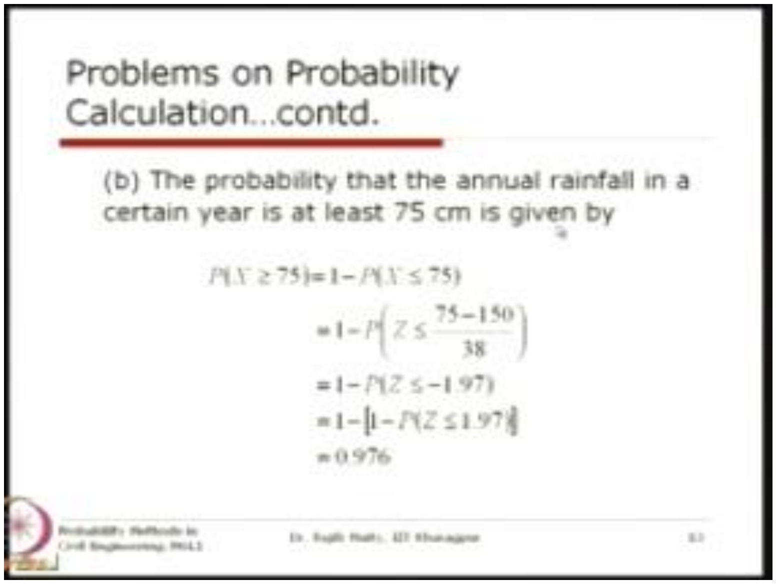

The second question is, what is the probability that the annual rainfall in the certain year is at least 75 centimeter. So, before I start that one, so the concept here is, that if you understand that, how we are we are obtaining the probability for the normal distribution after reducing it to this standard normal distribution, then all of this type of problem will be straight forward, may be for the rest of the problem that we are discussing, we will just discuss their inside, but how we are referring to this table and all we will not be discussing in so details.

So, I hope that you understood how we are getting the probability by using the standard normal table.

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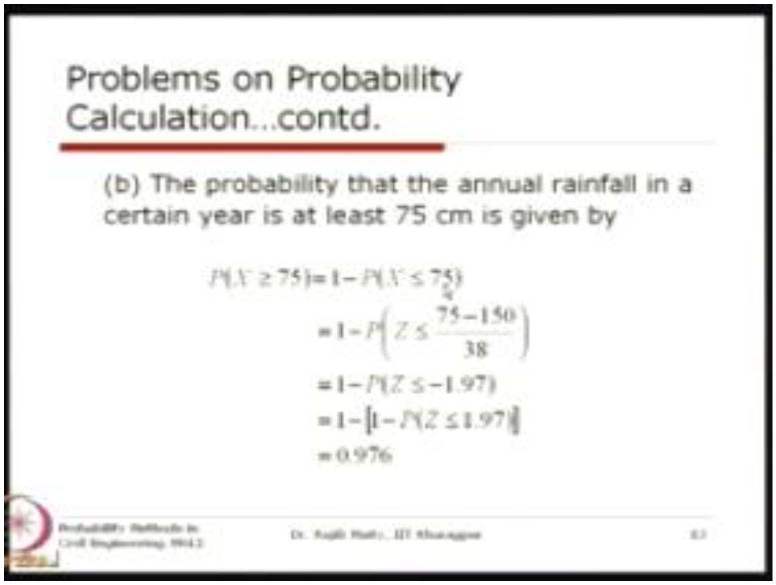

So, here the second question is, that the probability, that the annual rainfall in the certain year is at least 75 centimeters. So, what does it mean, at least 75 centimeters means, what we are looking for is the $P(X \geq 75)$. Now, this probability $X \geq 75$ can be, we are first converting it to the thing that $1-P(X \leq 75)$. So, why we are doing this one, you know that now the $P(X \geq 75) + P(X \leq 75)$, basically includes everything, all these all the area under the normal curve.

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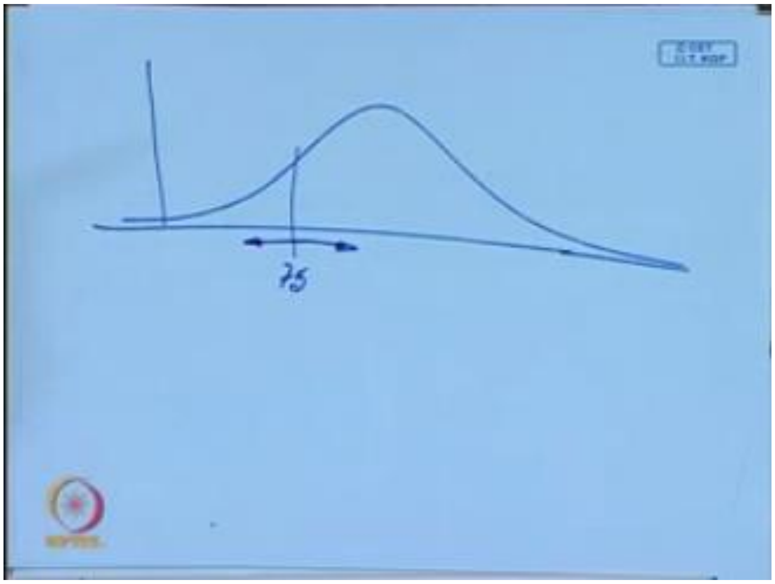

So, what we are talking here is that, if this normal distribution and if this is the mark for this 75, then the then the $P(X \geq 75)+P(X \leq 75)$ is covering the entire area below this curve, that is which is 1.So, that is why and the standard normal distribution you know that is available for the less than equal to values. So, as it is available for this less than equal to values.

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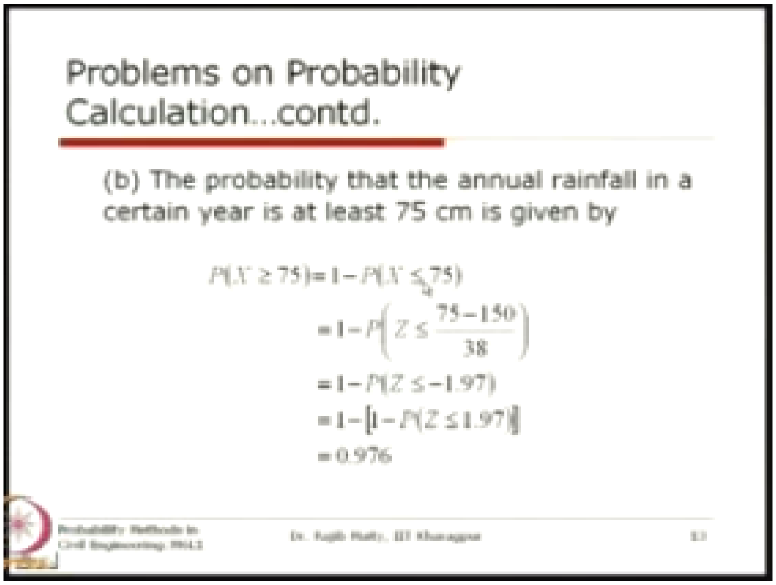

Then that is why we are interested to transfer it from this ≥ sign to this ≤ sign. Now, you know one thing is also important here, just to say you that, whether we will include the greater than equal to sign will be included in the both the side, this question may come to your mind. Basically in the mathematical concept, what we should not, we cannot include both the equality sign in both the side, either this should be equal to or that should be equal to.

But, remember that, so far as the continuous random variable is concerned, this is of no harm, because for the continuous random variable, if we want to know the probability of it is specific value, which is equal to the specific value is 0, that we discuss at the beginning of this course. And this is what and this normal distribution you know, this is a continuous random distribution, continuous probability distribution. So, for this one, if I want to know, what is the probability that X=75. That means, that probability X=75 is 0.

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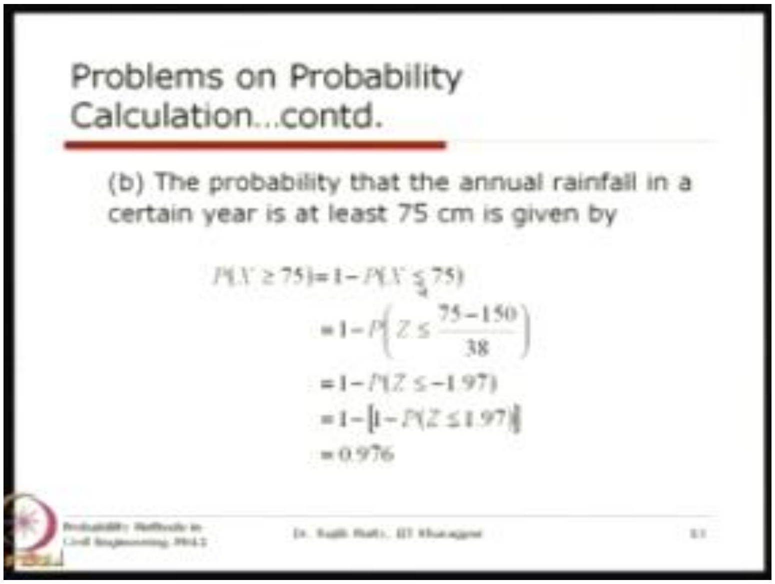

So, what we can, what we can think in this way, that $P(X \geq 75) = 1 - P(X \leq 75) + 0$ and that 0 is nothing, but probability $X=75$. So, whether you are including or excluding the equality sign, it does not matter mathematically, so far as the distribution is continuous. So, with this, now we will follow again the similar procedure, we will first we have to transfer it to get that reduce variate, which is your 75 minus the mean 150 divided by the standard deviation 38, which is equals to minus 1.97, this $Z \leq -1.97$ can be directly obtained from the table or as I told, if only the positive side is available, then also you can further transfer it to that $1-P(Z\leq)$ that $+1.97$.

So, using this one, if you now again if you get that the value of this Z for 1.97 and you do this calculation basically, this probability less than equals to this one will be equals to your 0.976. So, the probability that the annual rainfall in a certain year is at least 75 centimeter is your 0.976. So, for the catchment, that we are talking about, there is a high probability, that 75 centimeter of rainfall. So, getting 75 centimeter of annual rainfall is highly probable that is what we can infer.

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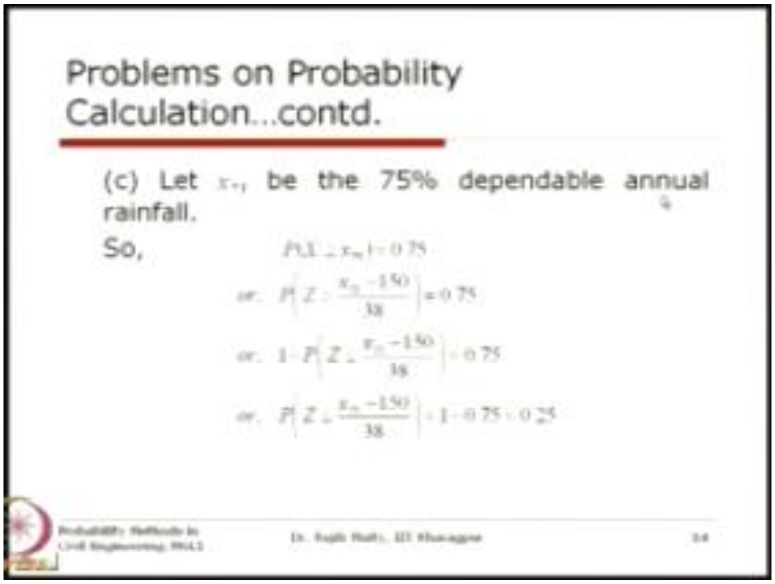

And the third thing, that the last question that we have seen that let that 75 percent dependable annual rainfall. Now, the 75 percent dependable annual rainfall, what we mean is that, let that X 75, this is value is denoted that 75 percent dependable annual rainfall. As I told that, this dependable rainfall means, I am assured that this much rainfall can be obtained at this station with the probability level 0.75.

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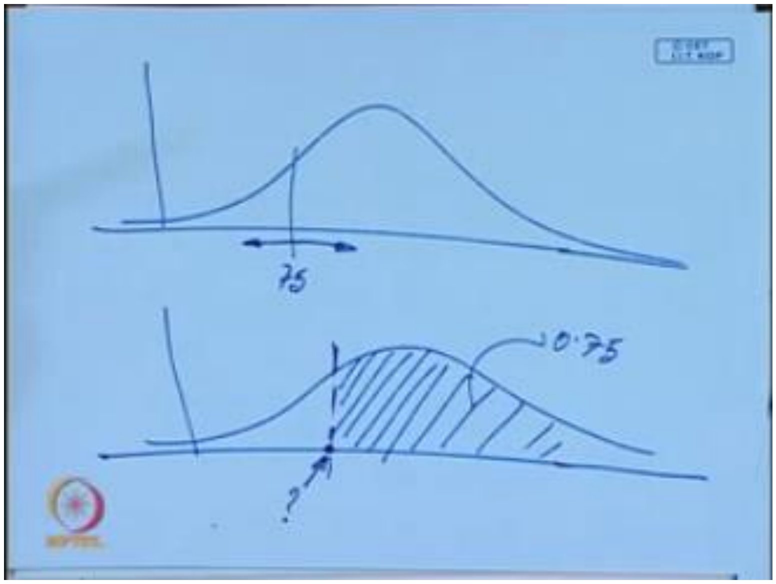

So, if that is the question then what I am looking from this one, so, if this is the distribution of this annual rainfall, then I want to know a specific value for which it is located in at some point. So, this value I am looking for, what is this value, in such a way that this is 75 percent dependable. This is 75 percent dependable means, whatever that probability on the higher side of that value is equals to your 0.75.

Now, from the cdf point of view, if you want to know, that means, what is the value up to which the total area covered under the pdf is equals to 0.25.

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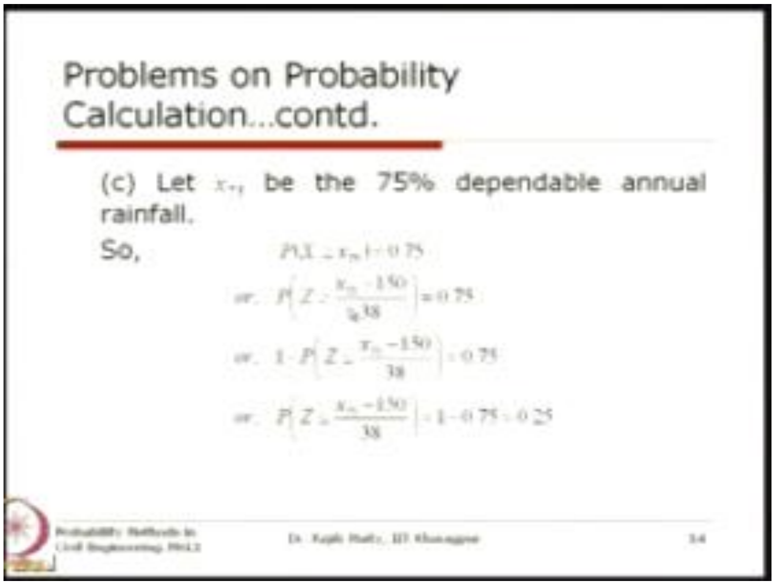

This is what is the concept and here now you can see that as we have denoted that X has 75 is that particular value, so that probability of X≥ that value is equals to 0.75. So, probability of Z, when we are doing that transformation, when we are getting that reduced variate that means, that X has 75 minus, that mean that150 divided by the standard deviation 38,that is the your reduced variate which is equals to 0.75.

The difference from this earlier two things, that we are discussed is that, there this probability we are looking for and this specific value was known here it is reversed that means, I know the probability, I want to know what is the specific value both the things can be done. Now, from that, so, if we just so, this is the ≥ sign, if you just refer to this one.

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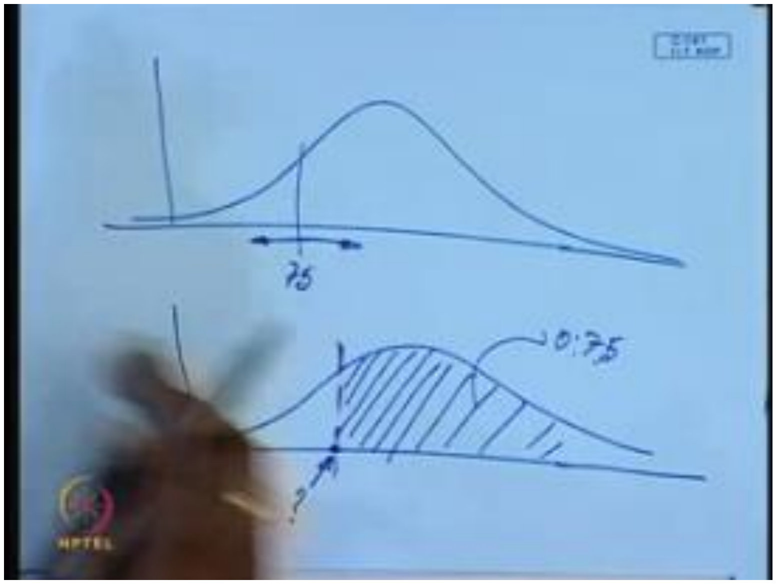

That means, this area I am just talking about the 0.75.Now, from the c d f we always know that this is the less than = to, that means, if we can convert into this side, then we can we can make use of that table.

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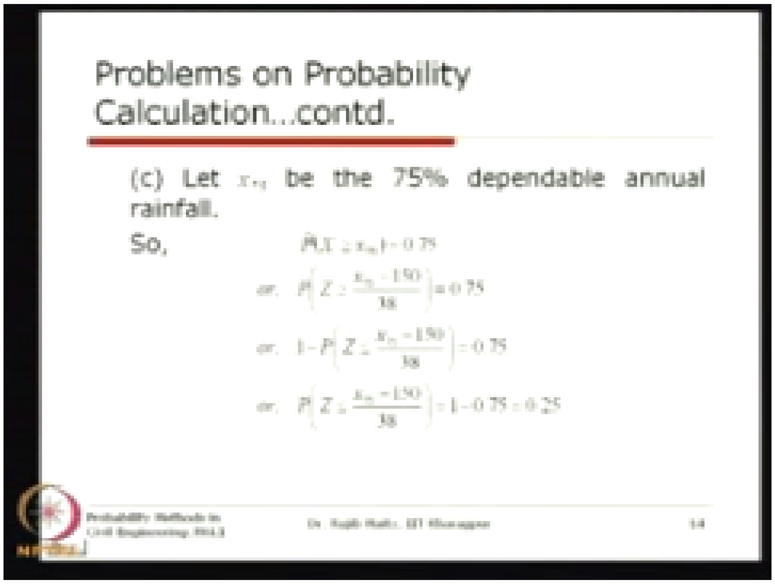

So, that is the interest. So, for that what we are doing is that, instead of using the greater than equal to sign, I am interested to know, what is the z less than = to this quantity, this reduced to variate, in that, what is happening to make this one this change, that we have to subtract it from the total area below this curve. So, total area or the total probability is 1. So, 1 -, so, this is replaced by 1 - probability z less than = to this value, which is = 0.75.Now, we are just doing this algebraic thing. So, this will be = to 0.25. So, this probability should be = 0.25.Now, how we will make use of the table.

So, now, how we will make the table is that, we do not know what is z, what we know, we know what is the probability covered. So, from the table, we have to find out the probability = 0.25 and corresponding to that what is the z quintile that we can get.

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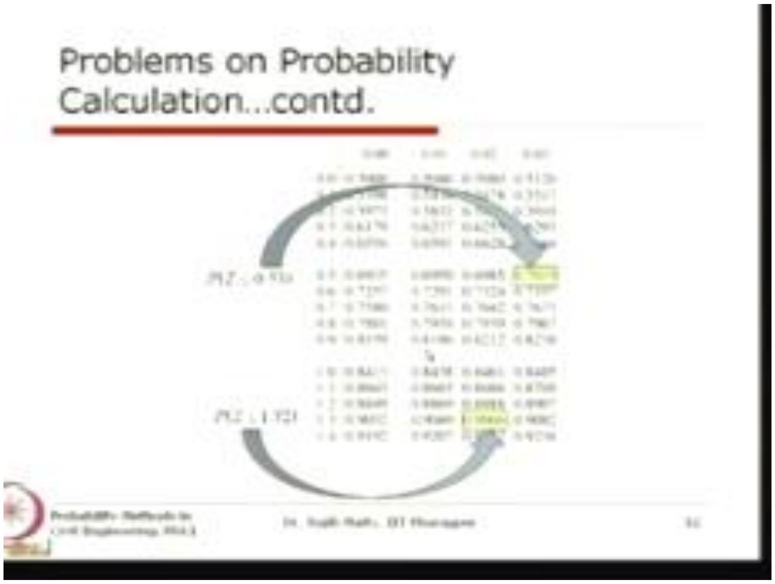

So that means, if we just go back to that original table, so, instead of looking for this z past, we will look for this, which is the, where is this probability is there. So, where is this probability there means, that for example, here the probability is 0.8186. So, if I am interested for this probability, then I should say that this is for the z value which is0.91, similarly I have to look for the probability which is = 0.25.

So, this is; obviously, as this is the symmetrical you know that, from z = to 0.5 this is for the probability is = to 0.5 the z value is 0, So, obviously, the z value will be negative in that standard normal distribution to get the probability = to 0.25, because the 0.25 is less than 0.5. Now, what we can do, if both the tables are available to us like this,

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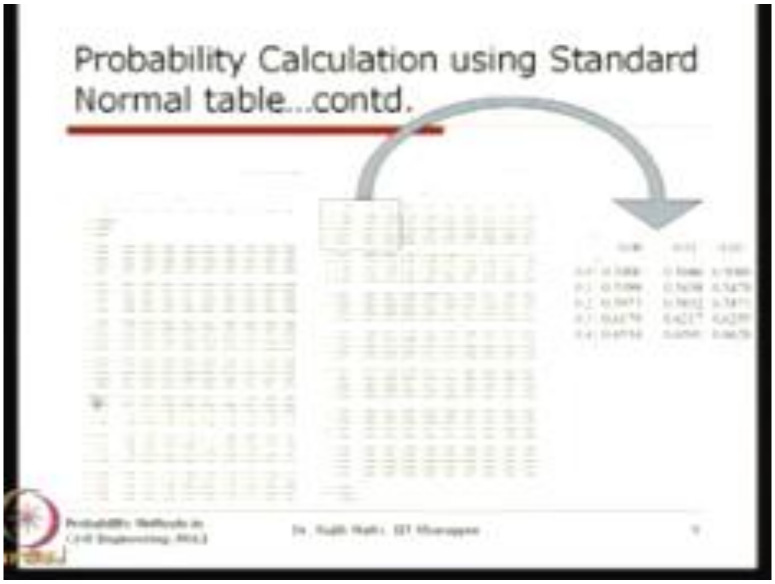

So then we will, so, from the negative side, we will just find out wherefrom this 0.25 is there and from there we will get what is this z value. If it is not available, then what should I do, how can I use that only the positive side to know what is that quintile.

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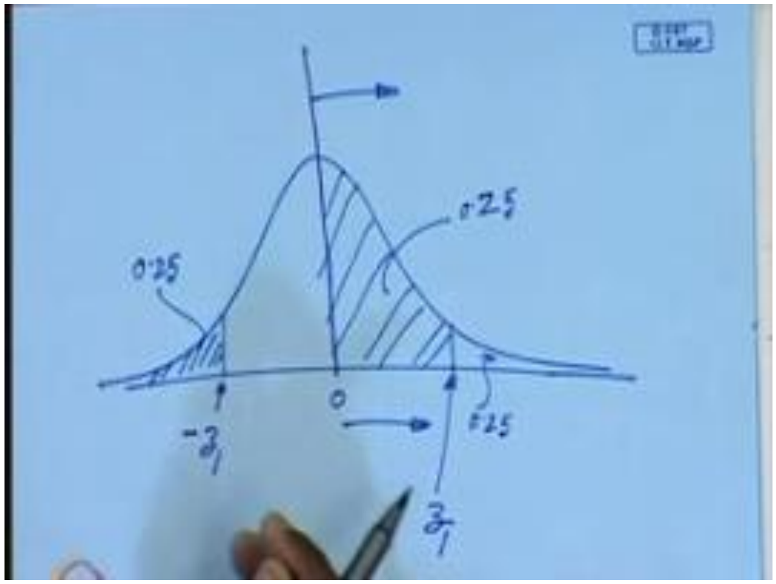

Let us explain it first graphically. That means, this is the standard normal distribution, this is your 0 and what is available to us only this side, only the positive side from 0 onwards is available to us and what we are interested to know is that, we are interested to know this value up to which the total area is = to 0.25. So, how we will get this one is that, we will find out for the value is higher so, if only the positive side is available higher, where this area is = to 0.25. Why 0.25, because I know from this value, total area below this curve towards the positive side is 0.5.

Now, if I can say that this is = to 0.25, that means, this area this, white area is also 0.25. Now, suppose that I get this value = to say z1, at z1 I have seen that this from 0 to that z1 value this probability is 0.25, that means, from z 1to + ∞ is the area is also 0.25.Now, using the symmetry of this normal curve, what we can say that, this value will be - z1. The value will be same only it will be on the other side of this 0. That means, towards the negative side of this 0. So, to make this 1 = to 0.25, this will be a - z 1.This can be done, in case from some textbook you get only the positive side values of the normal distribution.

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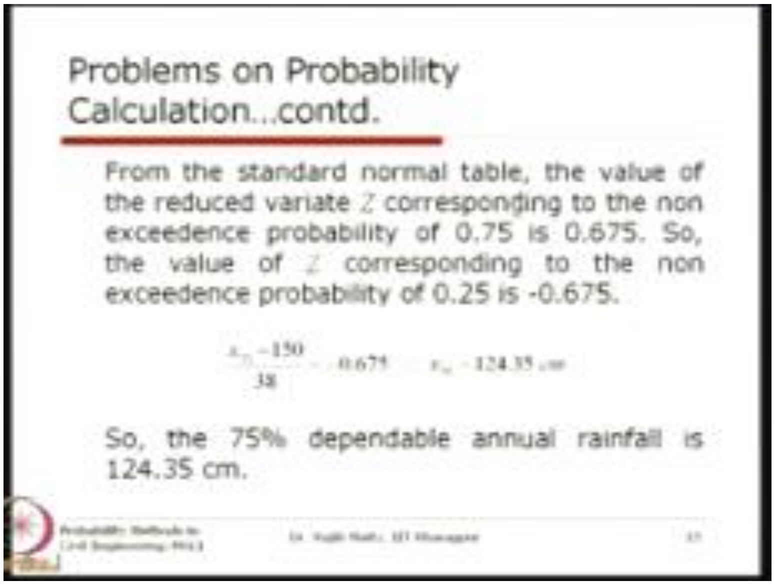

Now, here you can see that from this standard normal table, the value of the reduced variate z corresponding to the non-expedience probability of 0.75 is 0.675.So, the value of the z corresponding to the non-expedience probability of 0.25 is - 0.675. So, the value that the z 1here, that we are referring to, is your, if you see the table, you will see that it 0.675.

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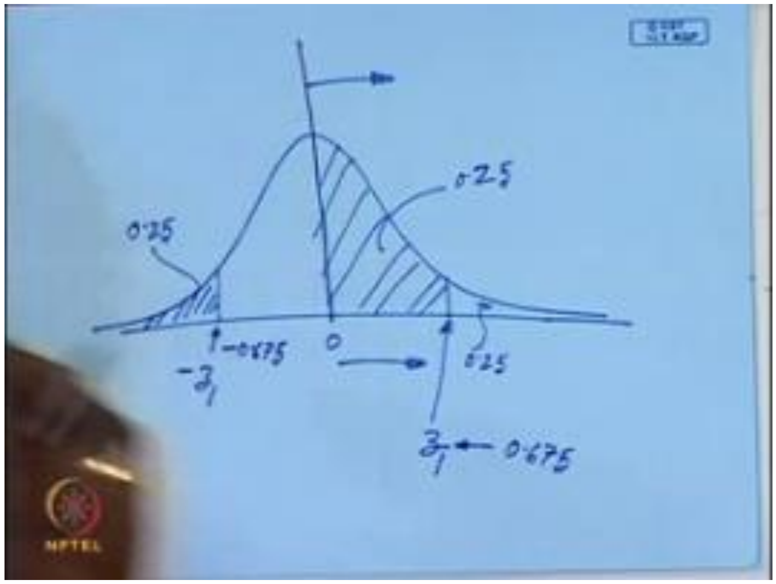

So, that means, here that we are value, ware referring to it is - 0.675.Now,

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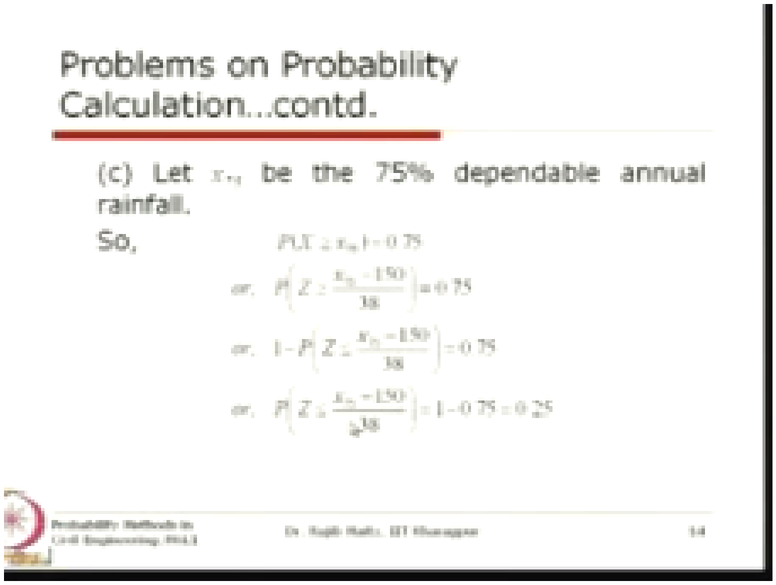

If this is the thing, what we are doing is that. So, this is your now the reduced variate z, what we have seen it earlier, that is x 75 - 150 / 38, that this probability is = 0.25, that means, this value which is nothing, but the z = -0.675.

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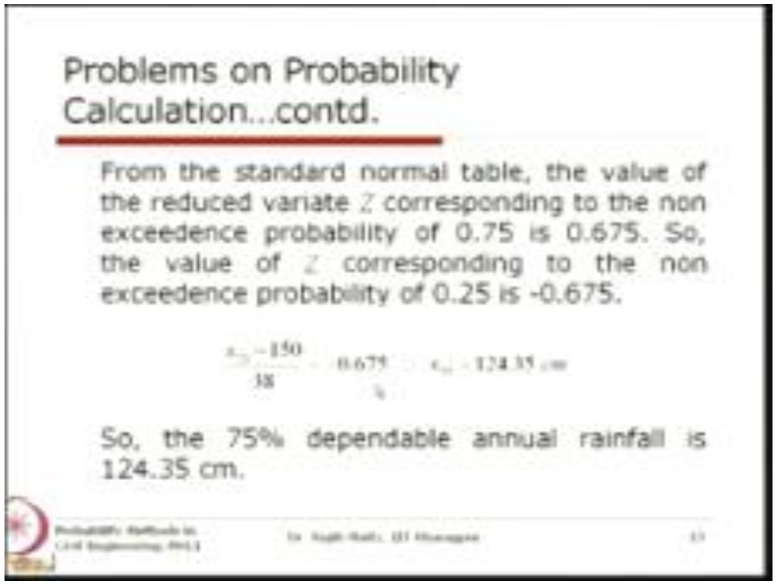

This is what is explained here.

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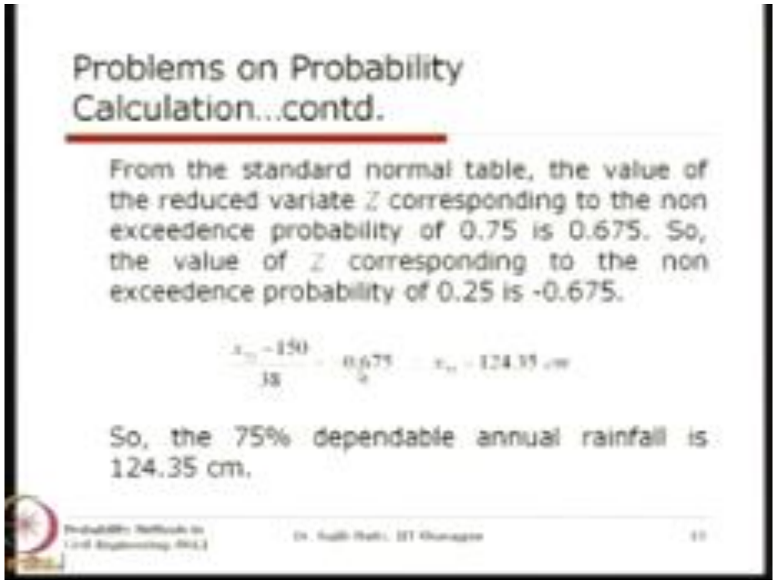

So, this is = to - 0.675 and if I doing this 1 x 75 is 124.35, that means, the 75percent dependable annual rainfall for that catchment is 124.35 centimeter.

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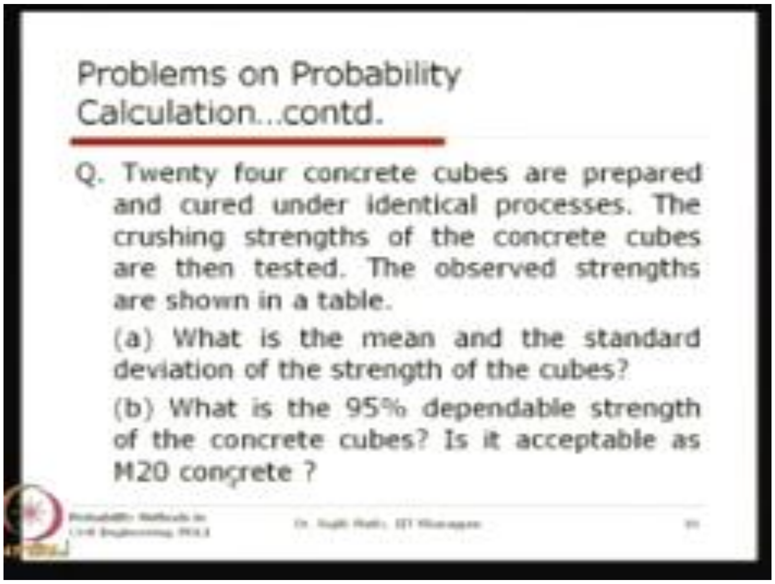

Okay we will go to the next problem, where also the similar the normal distribution is used and this problem is on the testing that strength of concrete and you know that the characteristics strength of the concrete is determined based on this 95 percent dependability.

Now, what is done you know that there are some several concrete cubes of specific dimension 150 millimeter by 150 millimeter by 150 millimeter is casted and they are cured in some identical condition and for 28 days at the end of 28 days, we generally get the value of the crushing strength in the laboratory and different crushing strength are obtained.

Now, from there, we have to see that, what is the strength we have to draw, because obviously, if we go for the different concrete cube, many concrete cube for all the values . So, as many concrete cube, those many values we will get. So, we have to conclude something probabilistically.

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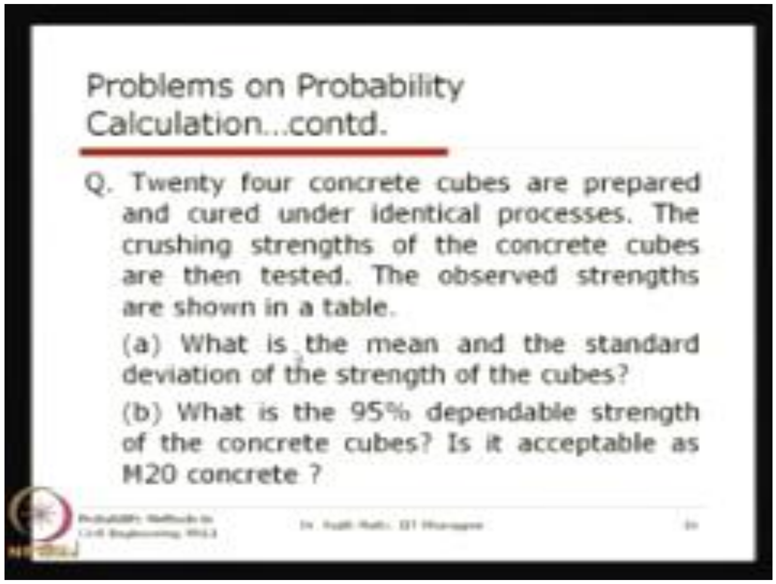

That type of problem is taken as the second problem here that 24 concrete cubes are prepared and cured under the identical process the crushing strengths of the concrete cube are then tested, the observed strength are shown in a table. The question is, what the mean is and standard deviation of the strength of the cube and what is the 95 percent dependable strength of the concrete cubes. Is it acceptable as M20 grade concrete.

Now, you have to know that, this M 20 great concrete means, it is characteristics strength is 20 Newton per millimeter square.

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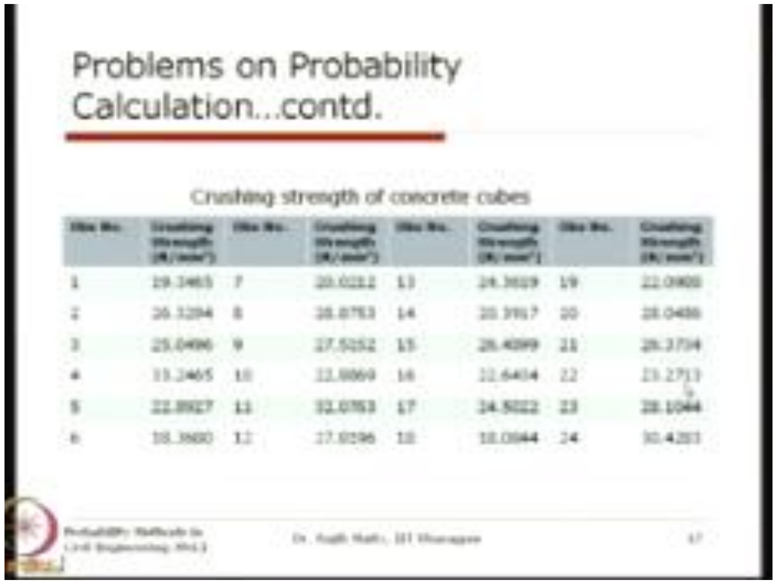

So, these 24 cubes are tested for their crushing strength and these values are shown here. So, what ware assuming that this follows a normal distribution.

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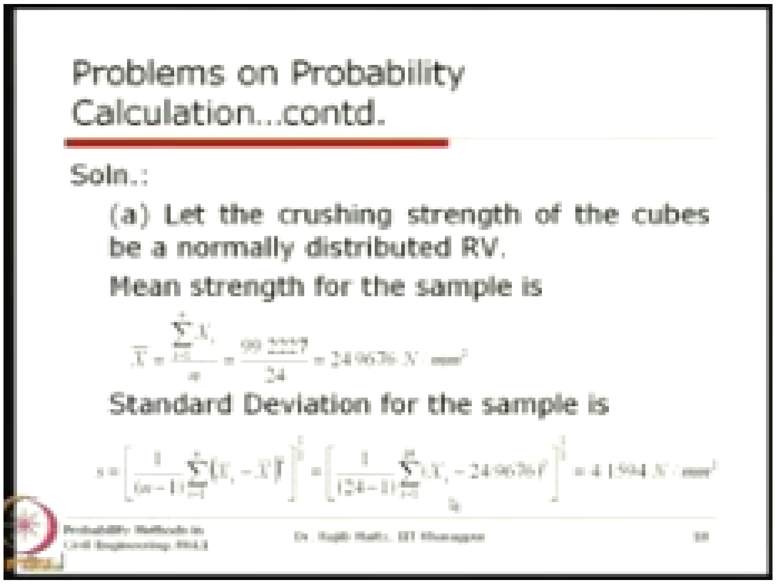

First we are calculating their mean and their standard deviation. So, first the mean is that 24.96, you know that will ∑ divided by 24 is giving that 24.9676 Newton per millimeter square. And second one is that the standard deviation, the sample estimate you know, that1 by n - 1 x i - x bar square from i = to 1 to n, here n = to 24.So, using this one we get the standard deviation is = to 4.1594. So, we got this two values.

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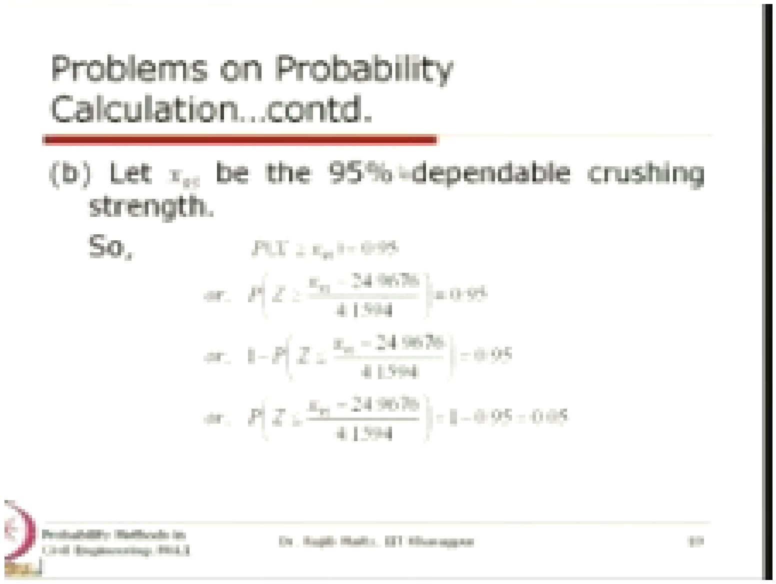

Now, the 96 percent dependable crushing strength is similar to the same that in the earlier problem, we called that 75 percent dependable rainfall. So, that means, the probability of x greater than = to x 95, it is denoted as x 95, following the same principle in the earlier problem. This - that mean divided by the standard deviation we are getting their reduce variate. We are getting it the less than equal to sign, that is 1 - probability of this and ware getting that this = to 0.05. Now, we have to find out the probability = to 0.05 for which reduce variate.

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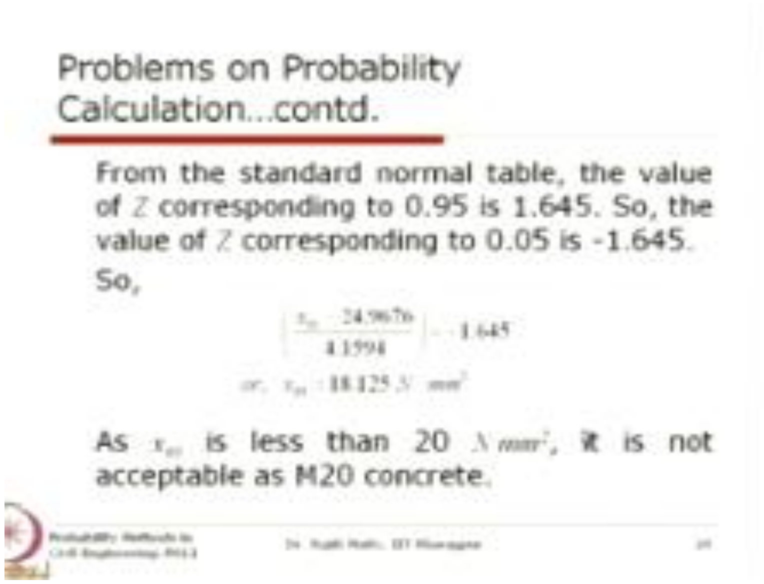

So, here you can see that, from the standard normal table, the value of the z corresponding to0.95 is 1.645. So, the value of z corresponding to 0.05 is - 1.645. So, this x 95 point that - 24.9676 divided by their standard deviation. This is the reduced variate, which should be = to - 1.645. So, x 95is 18.125 from this experiment that we got .So, the x 95 that is 95 percent dependable that strength is less than 20 Newton per millimeter square which is the requirement for the M20 grade concrete.

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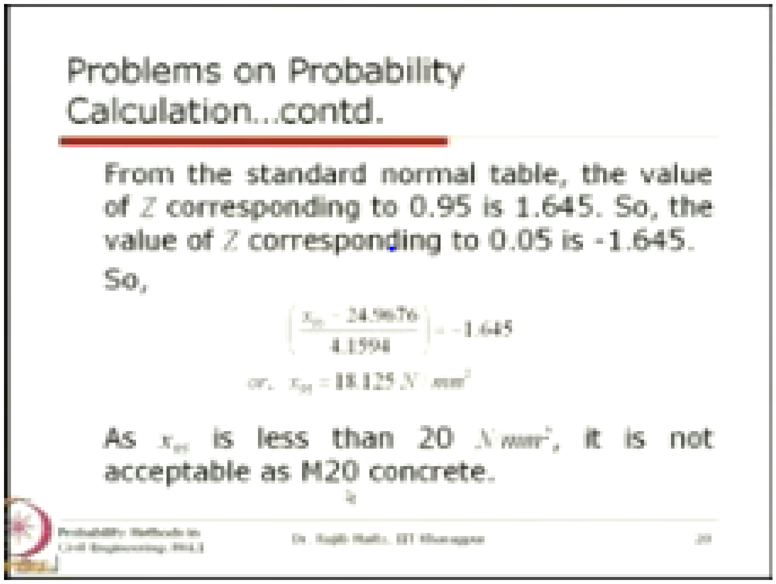

So, we can conclude that it is not acceptable as M 20 concrete.

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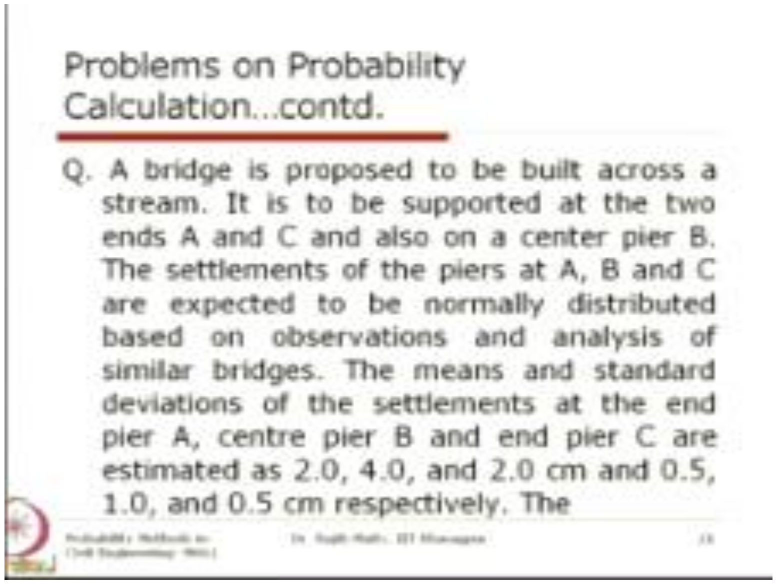

So, we can similarly we can go for the other problems, for the other civil engineering aspects, for example, we have we can, there is another problem on this settlement of this bridge p r, which is also important so far as the practical application is concerned.

So, we will take up that problem in the next class and that using. So, there also we will be using the same concept and the same standard normal distribution table to get some answers based on the probability models. And, gradually also that, what we will take up that problem also. So, from that data, we have to conclude that what distribution it is following through this probability plotting, that we will take up and then also we will take up the central limit theorem that we started with in this class. So, we will continue again in this same topic in the next lecture. Thank you.

Probability Methods in Civil Engineering

End of Lecture 31

Next: "Probability Models using Log

Normal and Exponential

Distribution" in Lecture 32

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