**INDIAN INSTITUTE
OF
TECHNOLOGY
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**NPTEL
National Programme
on
Technology Enhanced Learning**

Probability Methods in Civil Engineering

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Lecture – 15

Topic

Functions of Random Variables – Different Methods

Welcome to this lecture on this second lecture on this module 4. In this module, we are discussing on functions of random variable.

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And in today's lecture what we will do, we will just learn about this different method. So, if you recall that our last lecture in that, last lecture, we discussed about the fundamental theorem and in that fundamental theorem we have seen that, if there are different roots of a function of random variable, then how we can make a general equation to find out the probability density of that function of the random variable, from the information which is available with this original random variable, its density is known and the functional from with this new random variable is known.

So, using these two equation we have seen that, what are the, what is the fundamental relationship between the probability density function of this two random variables, and keeping that theorem in mind today what we will do is that, we will find out different methods. And these different methods are basically based on the different assumption, but so far as the fundamental concept is concerned is same what we discussed in this last class.

So, what in today's class, what we are discussing is that we will discuss about different methods to get the probability density of the functions of single random variable.

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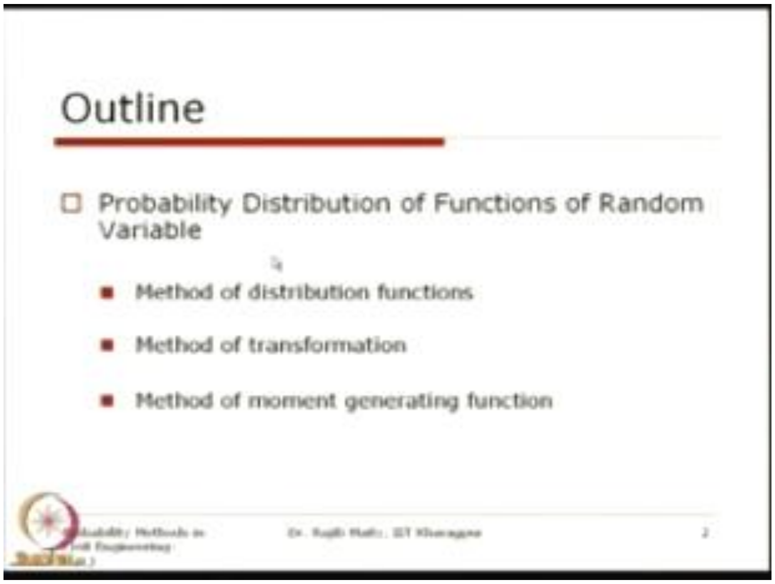

So, to get that probability distribution both the PDF and CDF there are different methods are available, and as a I told that all these methods are based on the same fundamental concept of the fundamental theorem, that we discuss in the last class. And we will use the same concept to discuss about our different methods which, these methods are based on certain assumption which are realistic, so far as application is concerned.

So, the first thing is that method of distribution function, then the method of transformation and then method of moment generating function, we will discuss. The first we will discuss that method of distribution function, so if I, before I go to that that theoretical thing.

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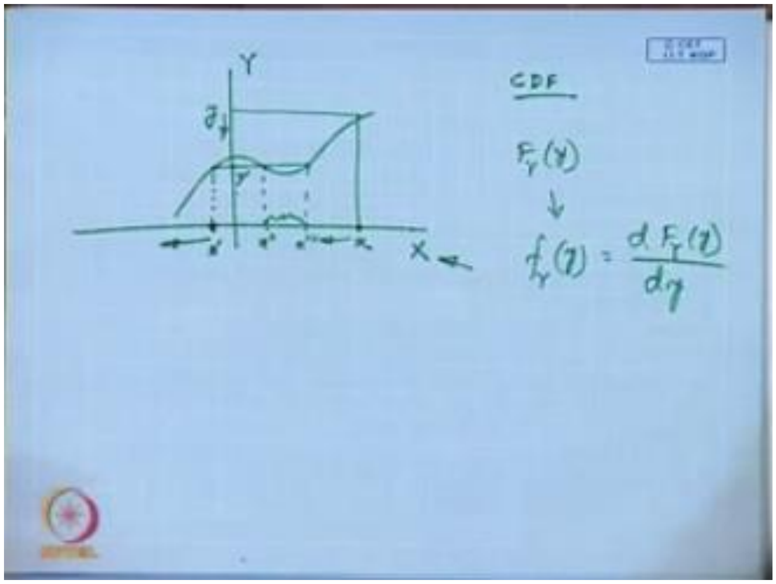

If I just draw here the concept of that, what is the fundamental thing is that just relating the fundamental theorem of our last class, that what we have to find is that if, there are two random variables one is this x and other one is your y, and there could be some relationship of this equation. And what we are supposed to do is that, we have to find out, so we know what is what is the density function is for this variable x is known.

So, what we have to find out is that, for a particular value of y we have to find out what is that, what is the domain for that particular value x, if I am first I am that considering that probability the method of distribution function, here the method of distribution function means, what we are talking about that CDF that cumulative distribution function. So, to now if this is a specific value of this random variable y is this small the lower case letter y, then what I have to find out is that, what is the corresponding domain of the random variable x.

So, if I want to know this CDF that means, this that y is less than equals to the specific value y, so that is this side. So this domain is the equivalent domain for this x is that specific value of that random variable x and its domain. So, the CDF that is probability up to that particular value y is equals to probability of the random variable x up to this specific value of this x. Now, if there are in the last class we have also seen that, if in case so here these domains are same and continuous domain, from this minus infinity to this y and for x this minus infinity to the x.

Now for some other value it may so happened that there could be some different zones okay. For example, if I just take the another value of this y, let us say that we give name as this y′, then there could be that this domain can have different domains for the random variable x. The first one this could be this x′, so which is less than equals to x′ which is corresponding to this point, plus another domain which is your, this one if I just give this names as x ″and this is x‴ then these zone as well as from minus infinity to x ′summation of this probability for the random variable x should be equal to the probability of the random variable y less than equals to y′.

So, this is the concept where what we are talking about this method of distribution function. So what we have to do is that, we have to find out what is the infers relationship and that should be express in terms of this y to get that probability, this cumulative density function for that new random variable y. And once we get, this one we will, we can get that probability density function by their differentiation. So, this is the keeping, this one is the basic concept.

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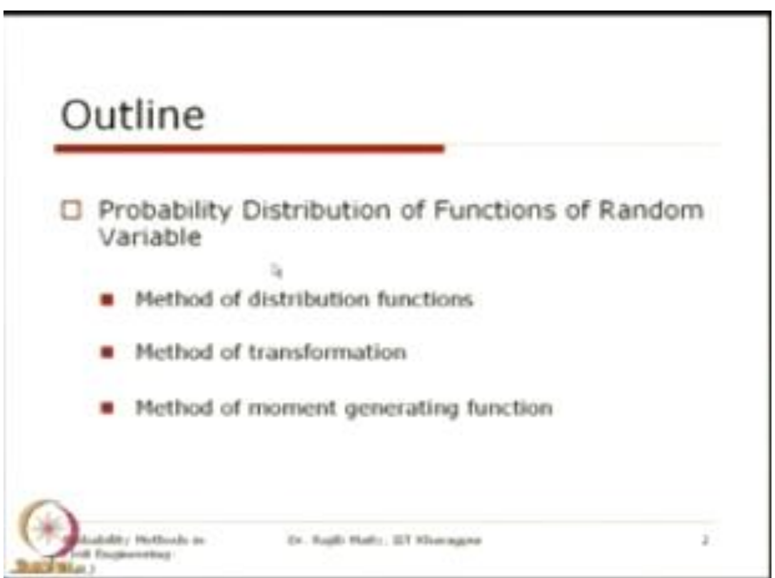

We will now, see that method of distribution function.

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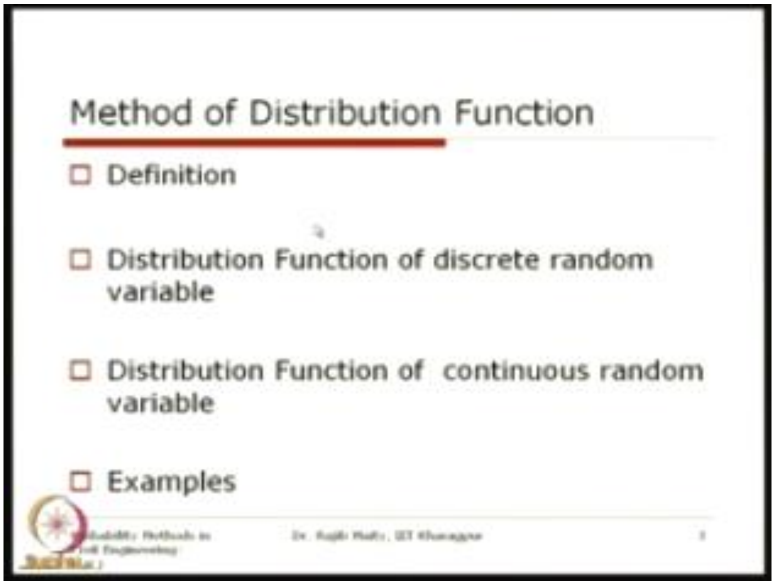

So, first what the so far as the fundamental definition is concerned that is what we are meaning is that, what how these two this two variables are related that we have discussed. Then the distribution function of the discrete random variable as well as this distribution function of the continuous random variable, there will be a little bit little difference between these two, when we are talking about that discrete as well as this continuous, so we will discuss this one and we will see it through some example.

Basically what we will be doing, we will be using the same example that we have used in the last class, and we will show that in the last class from the fundamental theorem we have solved that equation. Now, with the help of this method of distribution also we will solve the same problem to show that, how what the result comes just to differentiate what is this difference is there, if any what is the difference, so we will use same problem that we discussed.

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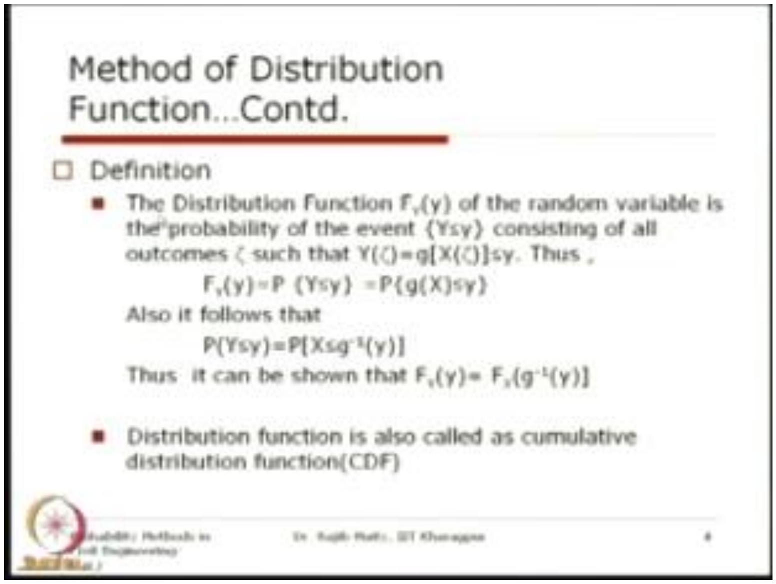

So, what we just now is discuss is that that the distribution function that is the cumulative distribution function F y, y of a random variable is the probability of the event that this y less than equals to a specific value y consisting of all outcomes ζ such that, y ζ = g(x)ζ which is ≤ y so that means here what we are trying to say this is the fundamental thing of this cumulative density at cumulative distribution function that we discuss earlier is that so if I want to know this Fy, then I have to find out the design.

I have to find out the probability that this is less ≤ y now referring to the figure that just now I have drawn is that what we are.

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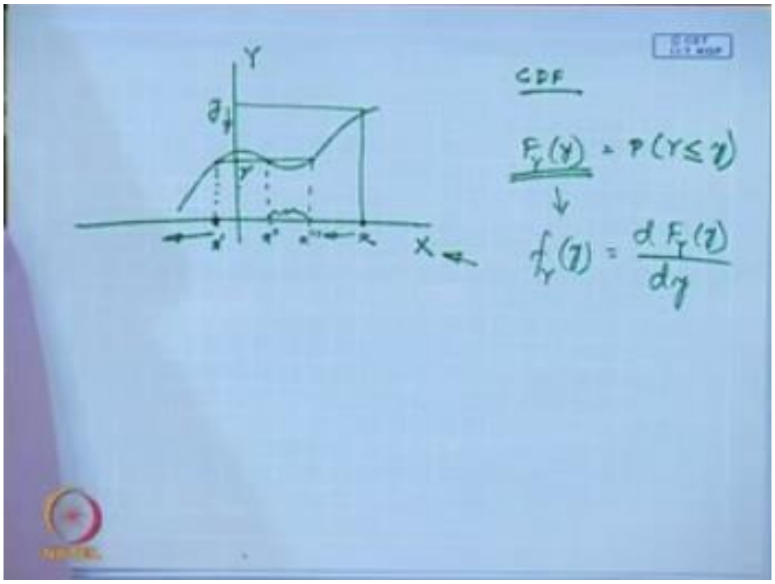

Going to get that for this $F_y(y)$ is that this $F_y(y)$ is nothing but the probability of the random variable ≤ a particular value y. So, this is from the fundamental this is from the basic definition of this cumulative distribution function that we discuss in the long back earlier classes. So, some specific value y is there I have to find out this probability what is known to us thus properties of this x is known so that is why I have to related to this random variable which is known to us, which is the x here so, thus if I now.

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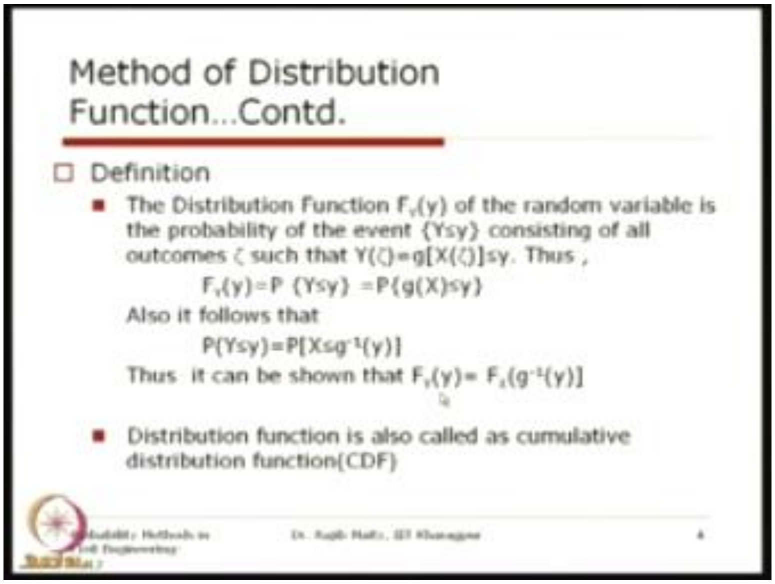

So if I now want to know this F_y which is equals to the probability of y < the specific value y which will be equal to now I am relating it to the functional form the functional dependence between x and y that is y is equals to g(x) so I am replacing that one so this g(x) should be ≤ y. So I have to find out some domain in the x which corresponds to that a for the random variable y which is < this specific value of y. So it follows that probability of y ≤ y is equals to x ≤ $g^{-1}y$ directly from this equation.

So this $g^{-1}y$ is another function which I am just taking the inverse function of this g(x) the functional form is there. So what is here is that I have to express that functional that depend that functional dependence in terms of in terms of y. So if I know this one now probability of x less than equal to a specific value, we can easily get it from that the properties of this random variable x which is we know that this cumulative distribution is F_x for that specific value $g^{-1}y$.

Now, this distribution function is you know that this is a cumulative distribution function and once I get this cumulative distribution function.

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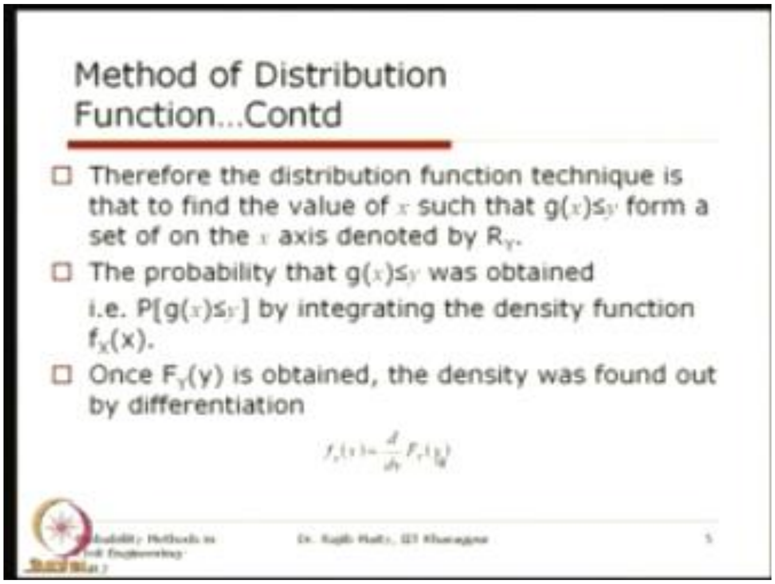

Then we can differentiate it to get that probability density function. So in brief the distribution function technique that is cumulative distribution function technique is that to find the value of x such that the g(x) is ≤ y from a set on the x axis and which is denoted by R_y. So, here if we refer again that for any specific value.

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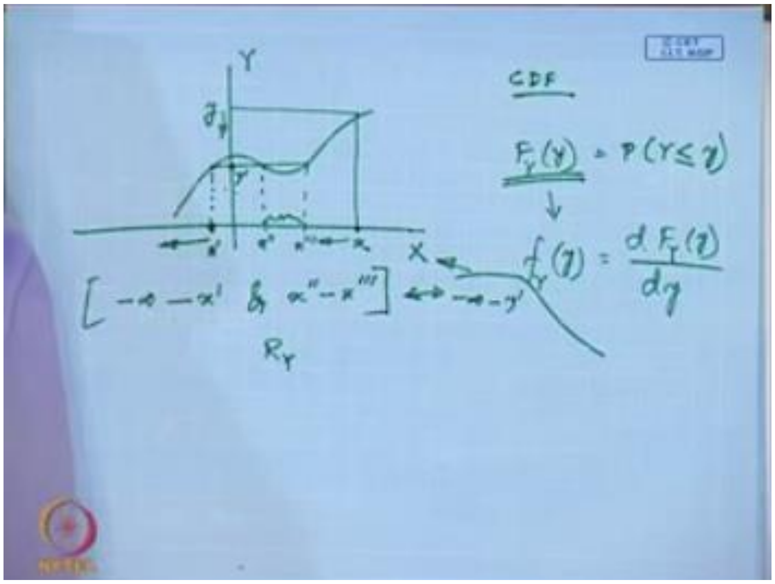

For this y, for a specific value of this random variable y, I have to first identify what is the set of x that corresponds to that specific value of y. So, this one for this here this y´ that set is nothing but, one is that from - ∞ to x´ and x ´´to x´. So this is the full set of the x, that I have which is corresponds to that - ∞ to y´ so this is this is that set which we are talking about that R_y that is the corresponding set for this specific value of the y, this is what is explain here. So, first of all this for this in this technique.

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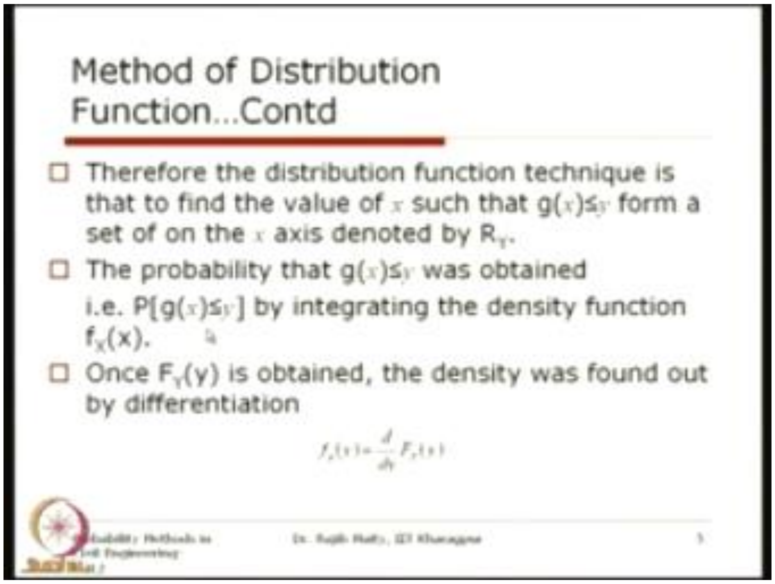

The distribution function technique we have to find the value of x such that this g(x) is < y form a set on the x axis and this is denoted by R $_{y.}$ Now, the probability that g(x) ≤ the specific value y is obtained that is the probability of g(x) ≤ y by integrating the density function of Fx, so integrating this density function of Fx. So this is this F x is your probability density function of x.

Now if I integrate it we will get the probability that cumulative distribution function that CDF of x and from that CDF of x we will get that what is that distribution of x for a specific value that this probability will be obtained which we discuss in this slide.

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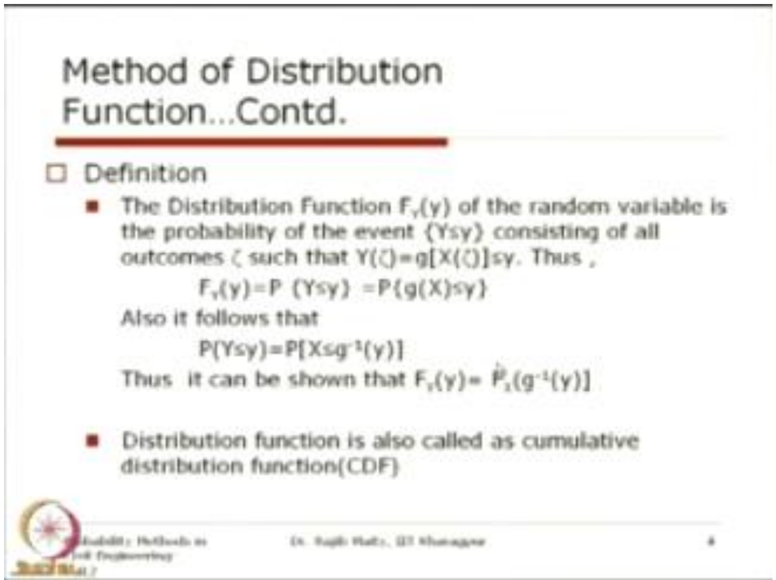

That is if I know that specific value of this y and if I know the probability that cumulative distribution function of x, then from this form we can get that probability that is probability of this specific < specific value of y is equals to from that cumulative distribution function of x.

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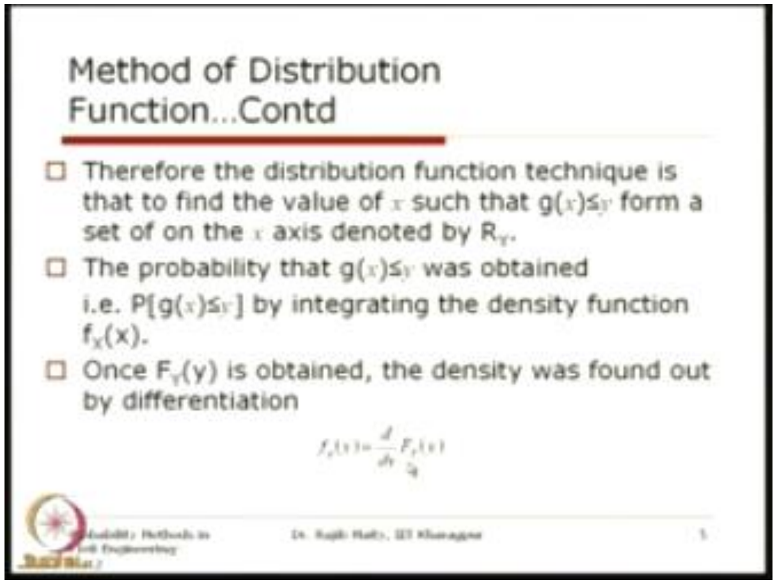

Now, once we get this $F_y(y)$ is obtained then we know that its density that is PDF, probability density function can be identified, can be found out by the differentiation with respect to y, so we know this one from this basic principle of this CDF and PDF.

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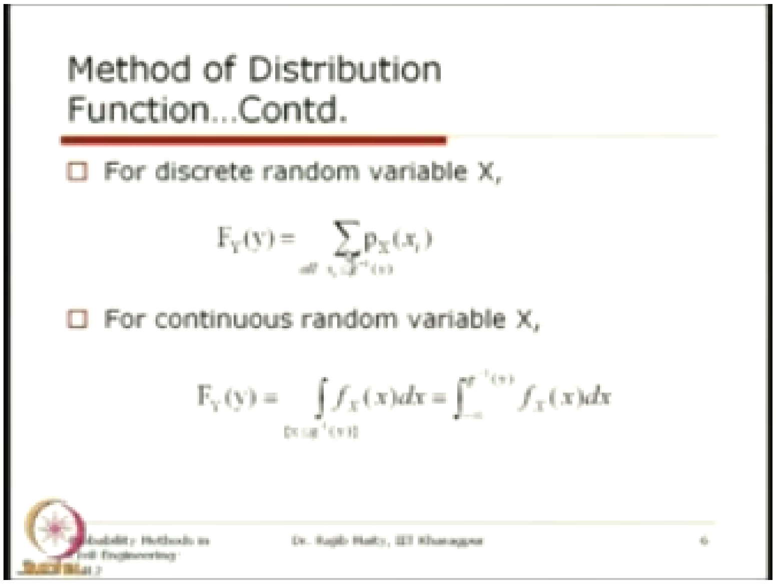

Now so we will just see that two different thing for this discrete random variable and for this continuous random variable there is a slight difference in the sense may be when we are talking about this equation form the difference may not be, so may not be may easily understood, but in the next method when we are going to talk that is method of transformation that time it will more clear.

So but here what we are trying to say just we will just differentiate from in terms of integration or in terms of sum summation what the case maybe some may be that integration is for the continuous random variable and summation for the discrete random variable because we know that for the discrete random variable the probability is are concentrated at a particular value so we have to sum them up.

So, here so that is why it says the for the discrete random variable x that F_y, when we are getting that another function is equals to the summation of the probabilities of this x_i for all such x_i which is < that $g^{-1}y$ that $g^{-1}y$ means that inverse function of that functional relationship between x and y. So, we have to first identify that which are that specific values of the x which is less than this $g^{-1}y$ and for those specific values whatever the probabilities are there for the x should be summed up sorry should be summed up and that should be equal to the probability of that specific value of this y. Now for the continuous one continuous random variable we know that we if we have this probability density function $F_x(x)$ then to find out this F_y (y) it should be integrated for the for the set for which this $x \leq g^{-1}$

So you know that so this set this from just now discuss in this pictorial form this is your basically that R(y) that set for which this condition is satisfied. So if we integrate over this region than we will get so other words are from this leftmost support that is from the minus infinity to that inverse we will get this form and this will be equal to the probability of y up to the specific value small y.

So here means even though this is shown in a very compact form it may not be depending on this how many roots are available here for example here that we have shown in this figure.

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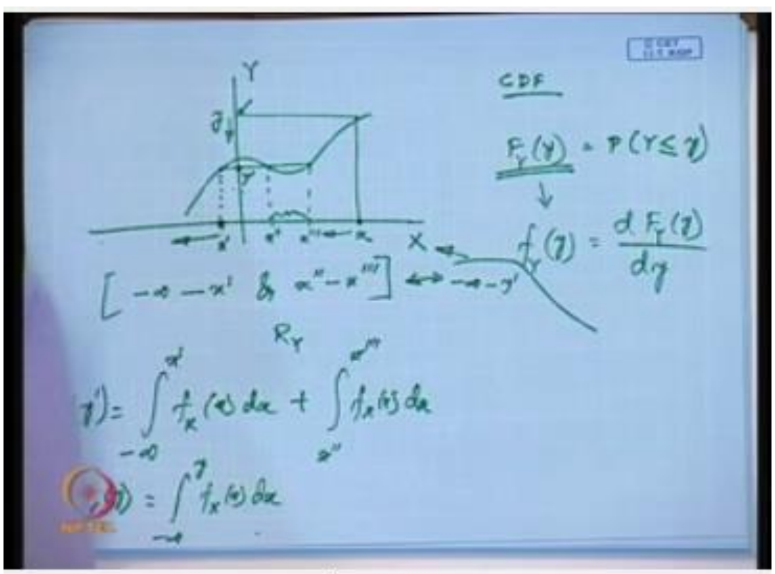

That for this one the integration must be the summation of this so minus infinity to the x prime. So the integration from this minus infinitive to x prime that F(x) of this x dx plus I have to take another integration that is from the x double prime to x triple prime that is this zone F(x) dx. So if we get this form then we then this will corresponds to that F(y) of that value F(y)’ here this F(y) prime we will get. But if I just want to take that this point that is F(y) of y this is simple you can get that from the minus infinitive to y Fx(x) dx, so this will be equal to these probabilities.

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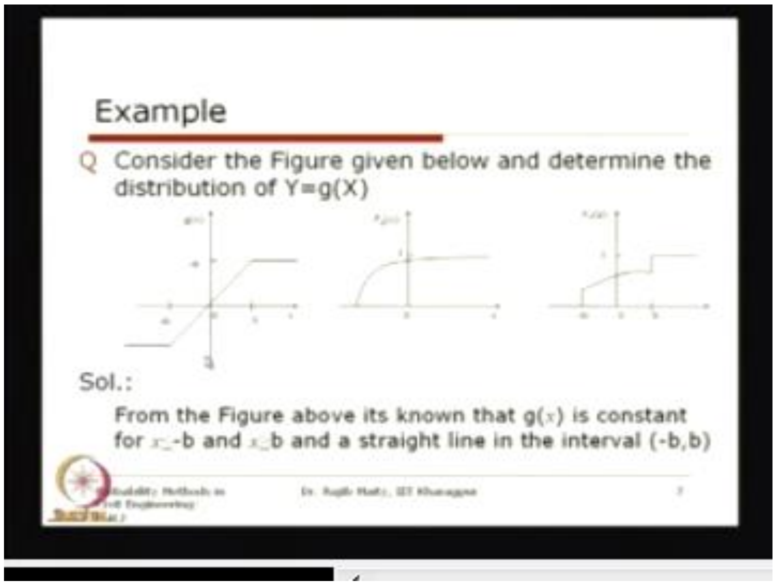

So now we will just show this is not a very not a specific to any particular civil engineering problem but this is very important to understand the concept that we here discussed so far and this is taken from the Papoulis book Papoulis and Pillai book. So here what is shown here is that a relationship between this x and y this g(x) = y you know that this is the functional form, so this is if this is the type of relationship.

Then we will see what how it happens, because whenever we are talking about we are just giving some values to this g(x) corresponding to a specific value of y then here in this form we can see we have taken this example just to demonstrate that if these things become becomes flat this functional relationship becomes flat at some point then what will happen this, at flat at some point means mathematically I can say that the rate of change of y that is g(x).

The rate of change of y with respect to rate of with respective to x it is constant. So it becomes when it becomes flat then how this see this distribution looks like that we will see, so here what we have to do the basically is that we have to have to first of all find out the three different region of this function. So in such cases, so where there you can see that there are three distant, different, dependence for over this three different zones.

There is one is that with respective to x if I see that one the first zone is that less than equals to -b, second zone is from this –b to + b and the third zone is from the from the that greater than equal to b. So for the for the first region that is when the x is less than equals to –b then the value of this g(x) or the y is –b and for this region that is when this x>= b the value of y is b this will not be –b.

So value is value of y is b and for the zone from the –b to +b it is varying linearly. Now in this case how if we know the distribution of this x what should be the distribution of g(y). So for this region it is whatever we have discussed so far is a is easy to get because I just get any specific value of this g(x) that is of this y find out what is the corresponding value of this, value for this x and find out their density function.

So that is easy but here we are more interested to know that how it is for this zone where this relationship becomes relationship is constant so this two zone we will see, so this figure what it is showing is that how this Fx(x) looks like and a function is taken, so that function the pictorial view of that function is looks like the, this obviously it starts from 0.

We know that it is CDF cumulative distribution function, and it is shown that it is assume that to 1 at infinity. Now so what we have to first as I told that.

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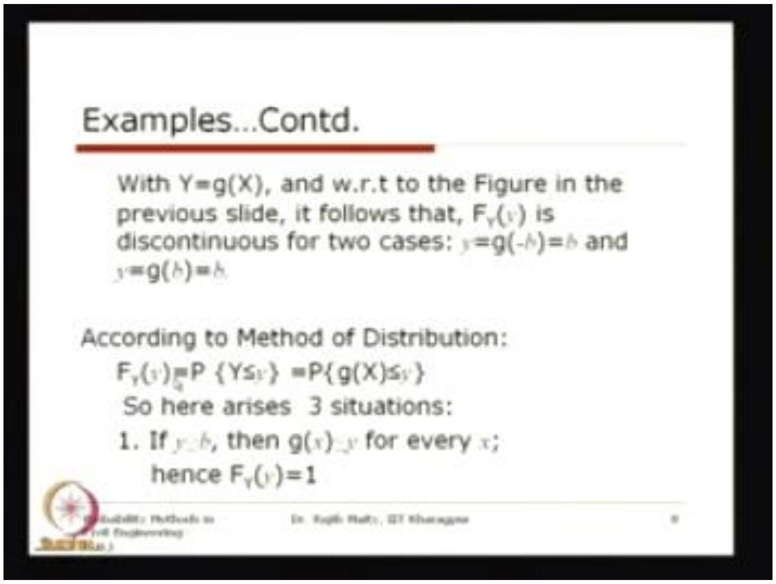

We have to find out that three different regions one is that if this y = g(x) that with respect to this figure that we have shown just now it follows that this Fy(y) is discontinuous for two cases, one is that for this at this –b < -b and at another point which is equals to b so at this point this function is discontinuous because to you know that mathematically two straight lines is meeting that point.

So according to the method of distribution the first if I this is a general form as I just explaining for the continuous random variable that Fy(y) is equals to the probability of y less than equals to a specific value of y so and this y is replaced by its functional form g(xy) now keeping this one as this as this basic equation now there are three situations can happen. The first is that y> b if y>b.

Then for the g(x)< = y that is specific value of the random variable y, for every x F(y) y is equals to 1, because that is the maximum value that it can takes, so at for this greater than equal to b this Fy is 1. Similarly if I just take that y.

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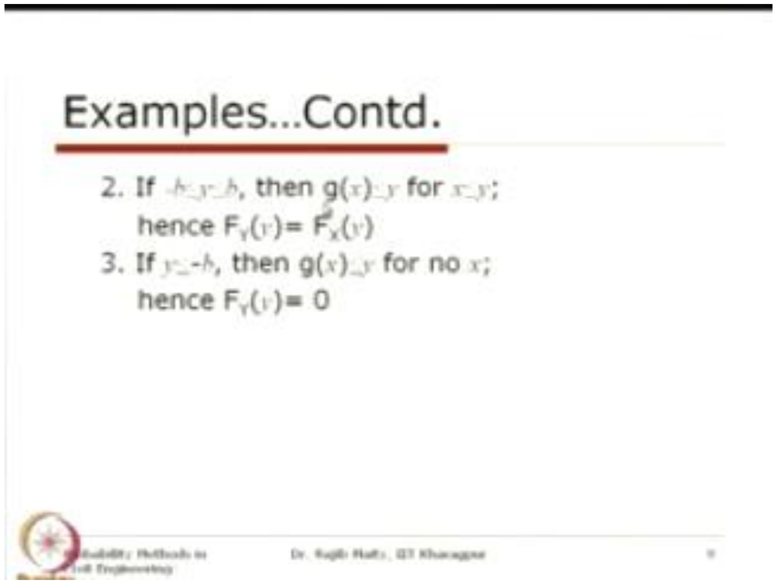

Less than = b then for this g x less than = y for no x so that is why this cumulative distribution function the value of the cumulative distribution function for this zone that is y is less than b = 0 now for this b that is from the - b + b if I come to this region then by the nature of this.

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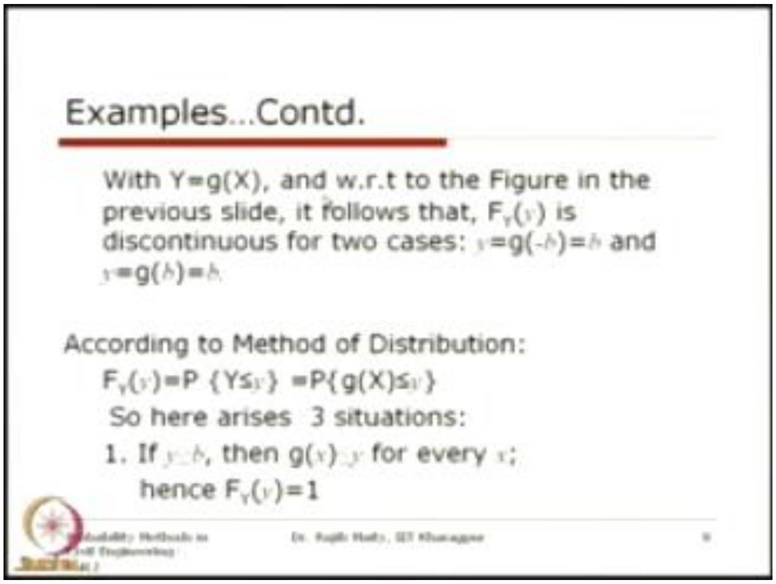

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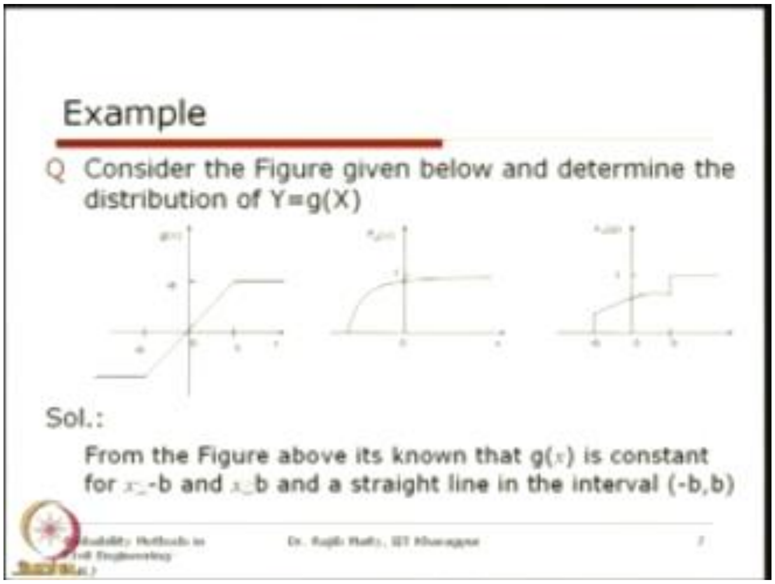

Functional dependence you can see this is just one specific value you will get for the x for a specific value of y so any value you take for this g x that is for this y from this - b to this + b then it will corresponds to a specific value of x so less than equals to that value of y will be equal to the less than equals to x of that corresponding value on this x axis means that for the random variable x so for this - b + b that distribution of this y will be same as this value for this fx and for this when it is becoming greater than b it should be =1.

And when it is becoming less than - b it should be = 0 so what now if you see it here pictorially then from this from the - ∞ - b the value of this fy is 0 then at this value that at - b this fy is becoming is equal to the value of that fx at -b and it is going on following the same curve up to the point b and at point b this is equals to value 1 now if you recall our description of the CDF means few more the earlier classes we discuss so this straight line basically is an indicative line if I just draw it draw it here then you recall that the staircase function that we that we drawn while describing that.

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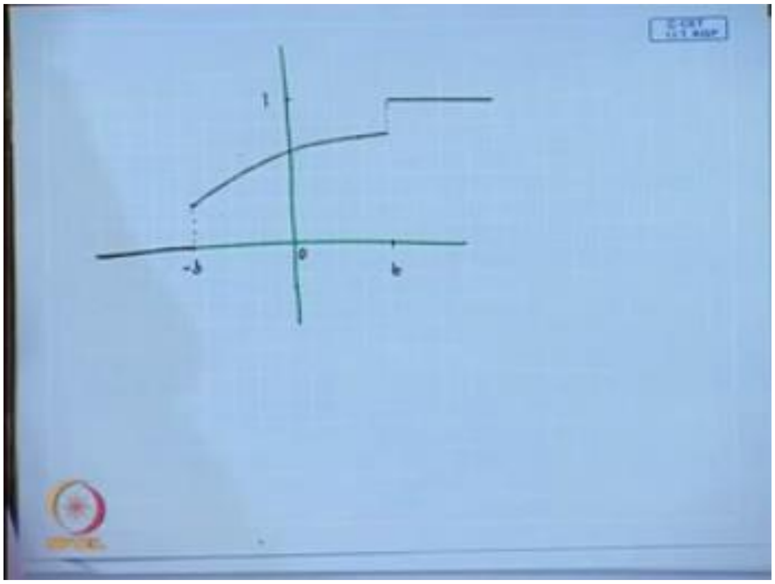

That CDF for this discrete random variable then it looks like this so what happens now I am using another thing so if this point is your –b this point is your + b then the CDF comes up to here then this is just an indicative line because at b this is equal to that particular value of this y so it follows from here and go up goes up to this + b and then this becomes = 1 this is 0 now the way the function is defined here I will just come back to this figure once again refer to this slide now that way this function is defined here.

That is the gx is constant for x less than equals to be and x greater than = b so the constant form is from this equals to be so when we are taking this form equal to that the similar form of this fx for this region from this - b to - b that means is b that constant value is included for the value up to – b so what I can show here is that.

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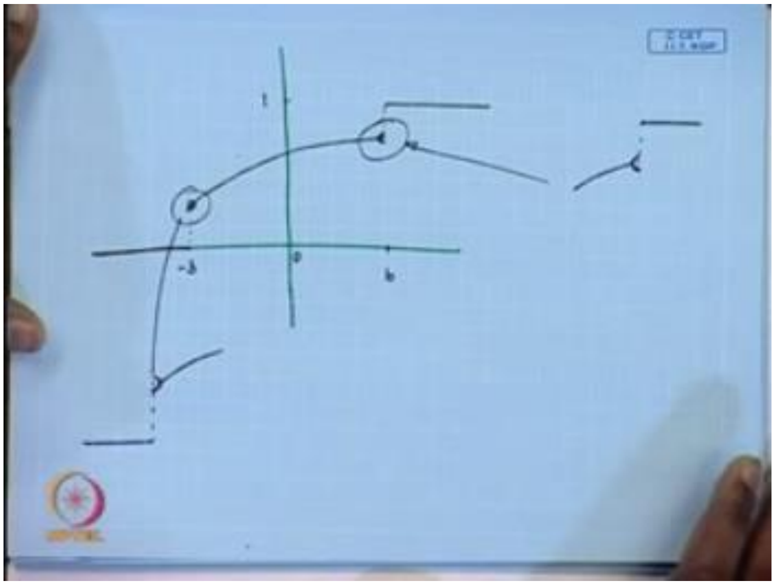

So at - b this value is 0 and at + b the value is 1 so this line which is the similar form of this fx cannot take this value so I am just if I just zoom this part is it looks like this so it can go as close as + b but it cannot take the value +b just recall our description of the CDF for thus for the throwing a dice in few previous class we discuss it can go and we cannot touch that b because at b the value is 1 similarly for this - b there will be another it cannot touch this – b so it comes like this because at - b the value is 0 so this one I think a making this.

Thing clear so what we are trying to drive is a what we are trying to convey here is that when this function the rate of change of.

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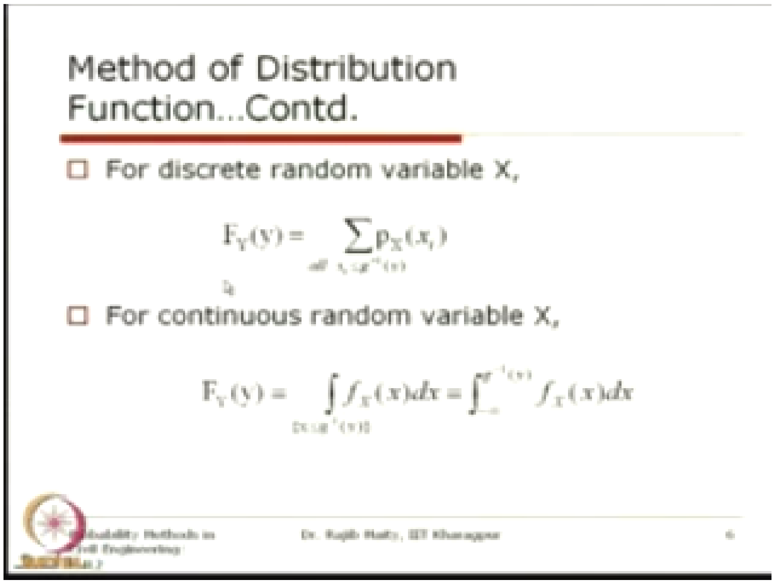

That y.

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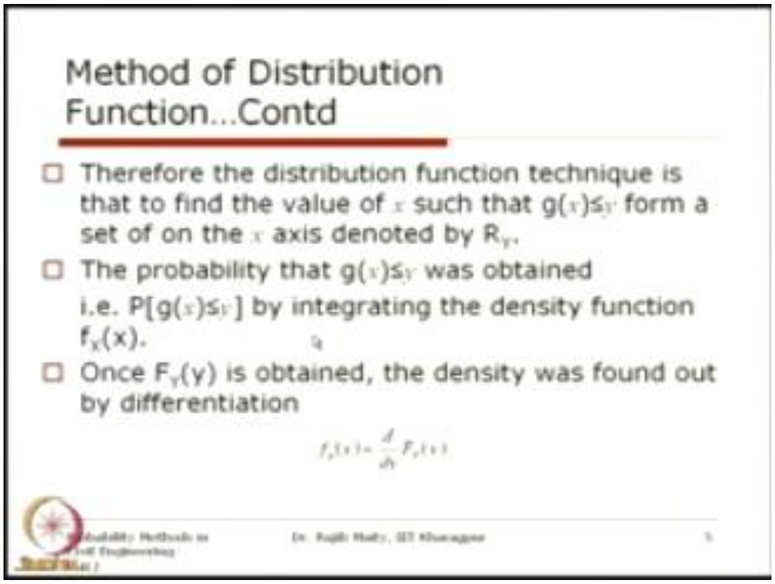

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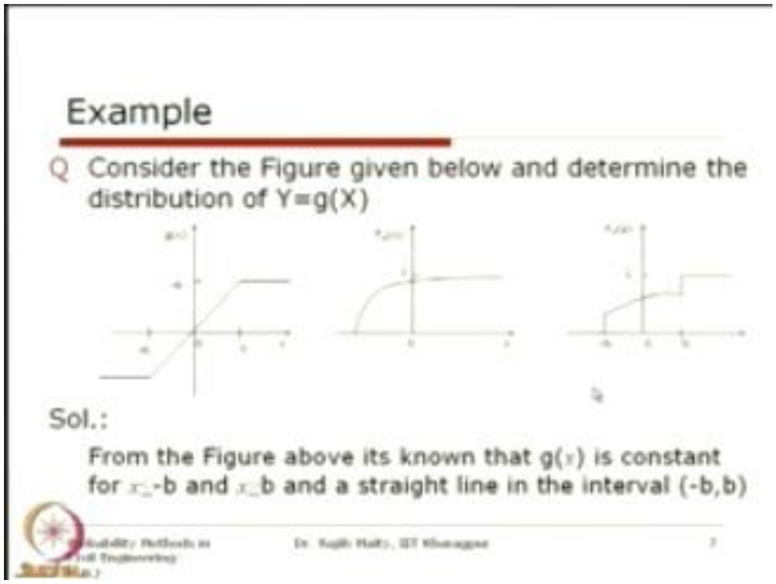

Rate of change of y become constant with respect to x at that point we can observe the discontinuity in that cumulative distribution function of y which is shown in this in this specific example.

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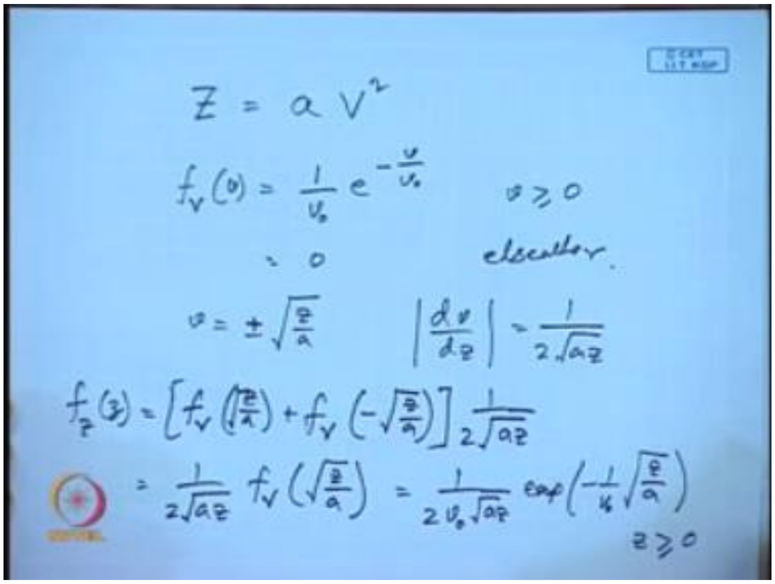

To illustrate the theory that we explain now we will take a similar problem similar means that we took in this in the last class also and we will see that how we can solve that particular thing in this context here we will take that problem there are two variables one is that this z which is basically that your wave height when there is a wind is blowing on the free water surface that that wave height depends on various factors and also it is related to the square of the wind velocity.

And so this two are related like this and obviously there will be some constant will be here that we can see so that this constant is some a now here the question is that this V if you know that distribution of this V whether we can get the distribution of this Z now this wind velocity generally what we see that this is having a distribution of distribution of this wind wave velocity is an exponential distribution say that this is looks like you know the exponential distribution like this and here the support of this one is greater than equal to 0.

In the last class the problem that we took this one is having a support over this - ∞ to the + ∞ they are we are talking about some stain energy and force which was a standard normal distribution so that that is having the entire support of this real axis - ∞ + ∞ but here it is the wind velocity so this can take only positive values with respect to as when we are talking about that wave height so it towards the towards the obstacle where we are measuring the wave height and this is the distribution.

And it is 0 elsewhere so if we know now so here also we can get that similar to this earlier the last class problem that is this g = + - √z by a we will get and obviously this -, thus -√ the negative which is not is not a valid case. And that modulus of this dv/ dz will be equals to 1/2√az. Now this f_z of this one the distribution of this new random variable z is equals to that, this distribution of this known variable at those roots that is z/a, if we want to write the complete expression ,if v-√za it is 1/2√az which is the value of this1.

Now, this is the basically this is not valid, so we have to take only this value and what we can write that 1/2√az f_v√z/a and this we can express as 1/2 v_0√az exponential (-1/v_0√z/a) where again that z should be greater than equal to 0 which is the support of this distribution. So, here, we have seen that even though they are having other roots we should not consider those root, which is the outside of the support of this random variable v, so the distribution of this z is 1/2 v_0√az exponential (-1/v_0√z/a) at the support is that z is greater than equal to 0.

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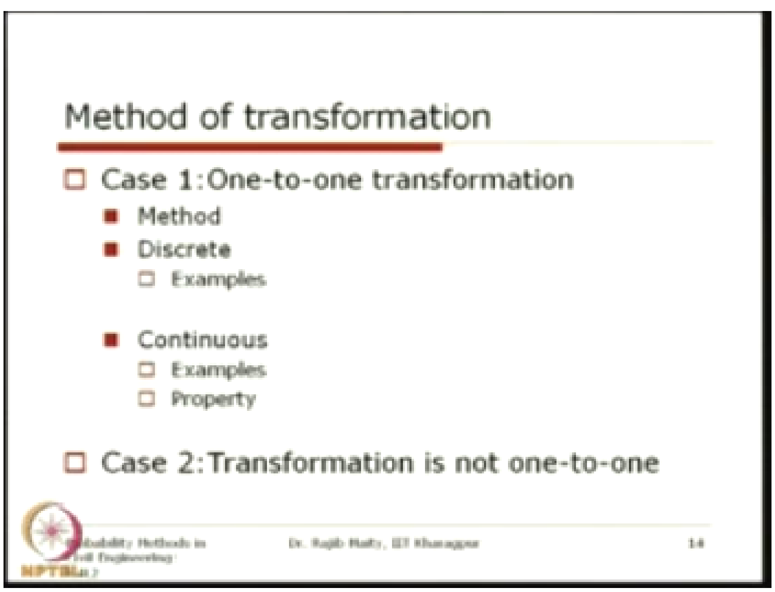

Discuss about the another method, that is call method of transformation and this method of transformation is popular in the sense that in most of the engineering application problem and most of the practical application problems as well, we have seen that whenever some function of a random variable is taken in many cases the correspondence is always one-to-one. So, what is mean by this one-to-one correspondence means that for a specific value of the original random variable, here we are using the notation as the x, so there will be only one value of the value of this depended random variable that is here we are using y.

So, for a specific value of x there is one and only one value of y and vice versa, so this kind of transformation is called that one-to-one transformation. So, if this is the one-to-one transformation then using that fundamental theorem that what we discuss in this last class, that can be reduced to a very, very effective equation and that equation can be directly used to find out, what is the probability density function directly by using the that equation.

So, that is what our second method we are going to talk that method of transformation, and obviously, we have to remember that method of transformation means, what we are referring here now is the one-to-one transformation.

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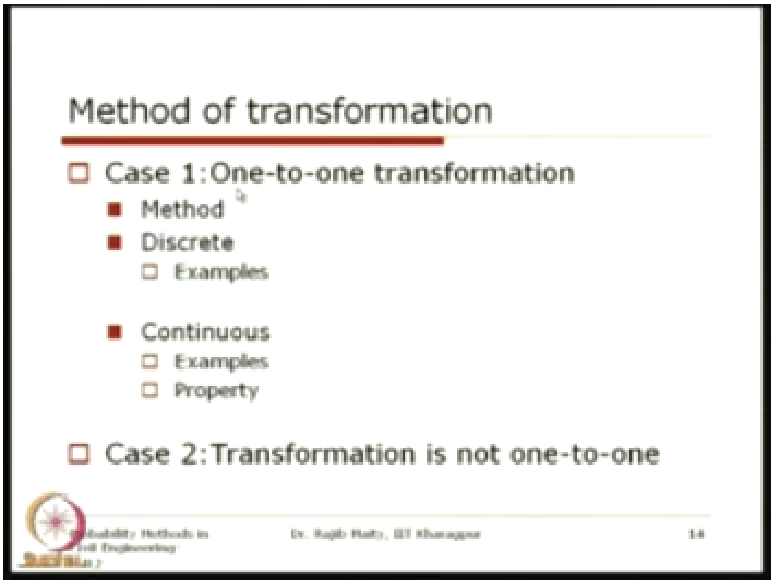

So, here that this case one is the one-to-one transformation and we will just show one example of this discrete and another example on this continuous, and that could be another case that the transformation is not one-to-one, so we are talking about this when the transformation is one-to-one.

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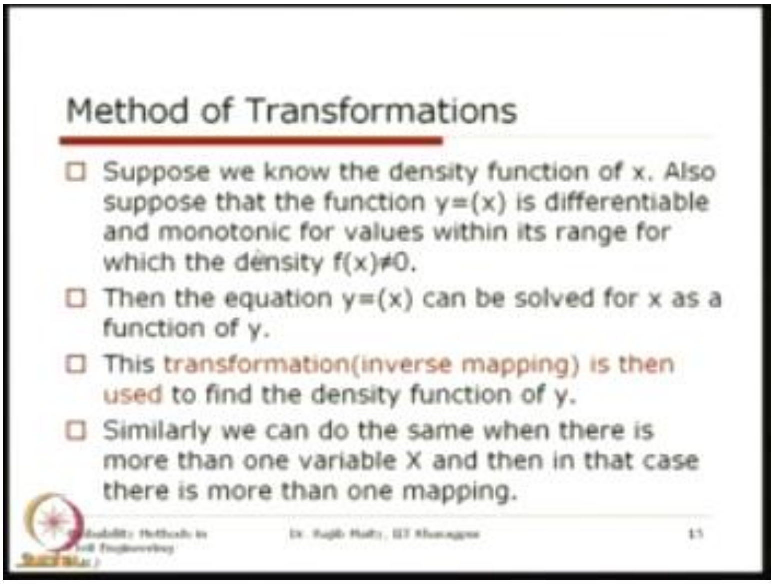

So, suppose that we know the density function of x also, suppose that the function that y is equals to g(x) sorry there will be one g that is the functional form that y equals to g(x) is differentiable, and monotonic for the values within its range for which the density f_x is not equal to 0, so this is basically what is what we are referring is that to the one-to-one. So, we will just discuss about this monotonic, what is this monotonic is talking about is that.

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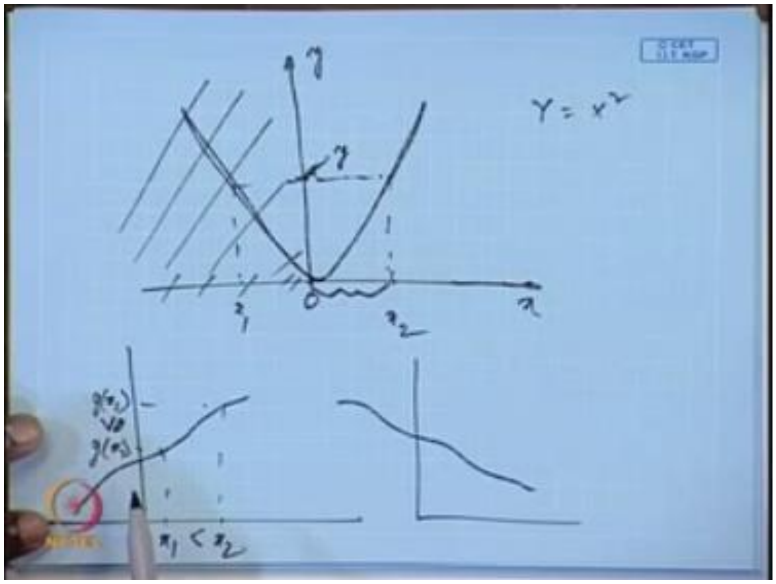

If the function either is increasing, either if it is increase, so it should increase over the range of this f_x or if it is decreasing. Now, this monotonic means it can never, if it is the increasing function in this it can never decrease, if the x_1 so fine, so if this is the value x_1 and if this is the value x_2, and if this x_1 is less than x_2 then this value that is, this is your that $g(x_1)$ and if this is your $g(x_2)$ then $g(x_1)$ always should be less than less than equals to $g(x_2)$, if I just put this sign that less than equals to $g(x_2)$, that means I am including this sign the equal to that means it can even be flat that is called the monotonically increasing.

But if I exclude this equal to sign that is known as the strictly monotonically increasing. So, strictly monotonically increasing means it can even not be equal, it will always increase for the for some higher values of the x otherwise if the equal to it is include that is known as that monotonic.

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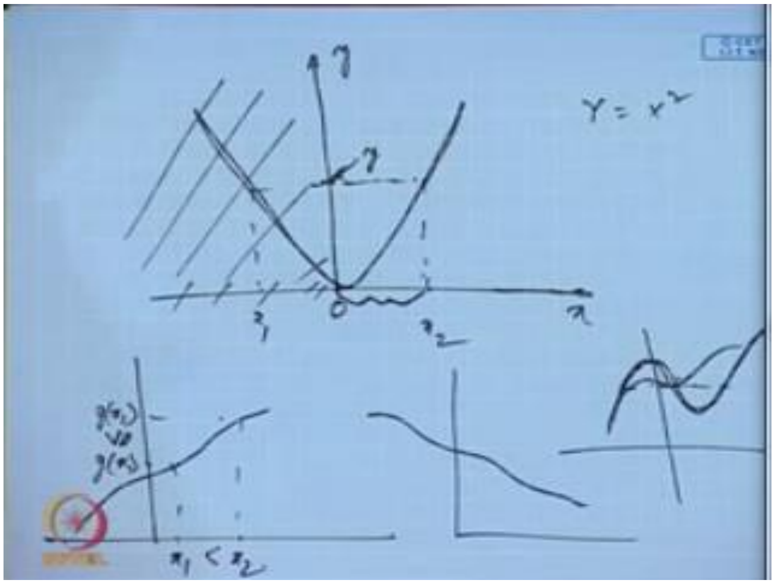

So, this function is monotonic that means, that this transformation is always one-to-one now we recall that, we for the general discussion when we are talking about this kind of equation, so there is possible the possibility of having more than one roots is only is can happen, only in case of when this function can increase and then again decrease and then again can increase, so in this kind of situation only there it is possible to have more than one root, so what we discuss in this fundamental theorem. So, here what we are talking about it can never increase that it is monotonic, that means there is always one-to- one relationship of that from this x to that g x.

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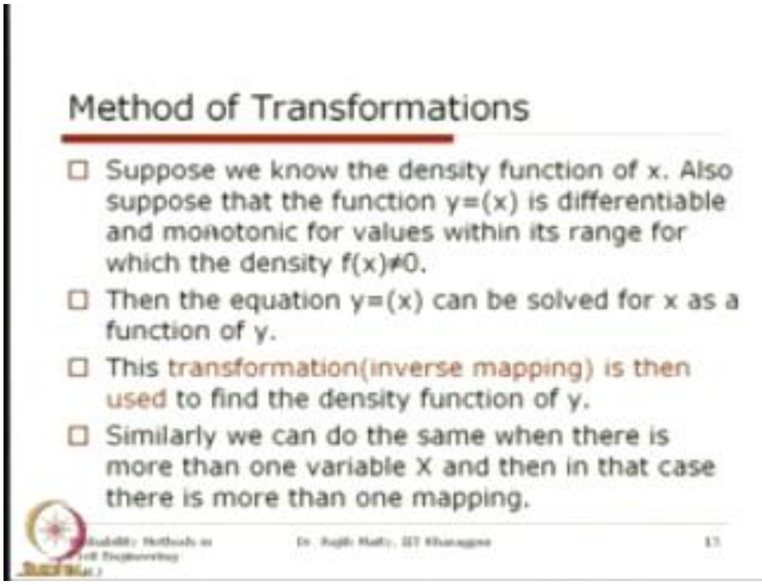

This is what is what is explained here. So, once again I read that suppose we know the density function of x also suppose that the function y equals to g x is differentiable and monotonic for the values within its range for which the density f x is not equal to 0. So, monotonic we have explained y the requirement of this monotonic, and what is monotone monotonic and there is another requirement is that differentiable.

So we will just see if it is not differentiable at a particular point, what will happen and it will be clear, when we are referring to the fundamental theorem. When the equation that is g equals to sorry the y = g x can be solved for x as a function of y, this transformation that is the inverse mapping is then used to find the density function of this y. Similarly, we can do the same when there is more than one variable x and then that case there is more than one mapping.

So this one basically we will be discuss when we are when will be discussing that multiple random variable, here we are discussing in this lecture the single random variable. So, we will deal only from one random variable to another random variable that is from x to y.

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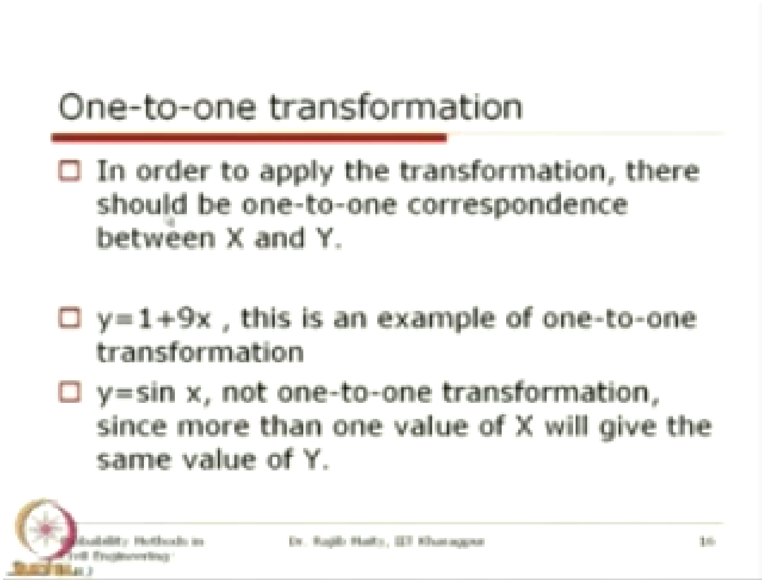

So, in order to apply the transformation there should be the one-to-one correspondence between the X and Y that is what we discuss, here is one example, that $y = 1 + 9_{X}$. So, if I take this example, then you can see that for a specific value of Y only one value of Y is there and vice versa, so this is an example of the one-to-one transformation. Now y = sine X is not a one-to-one transformation, since more than one value of X will give the same value of same values of Y. So you can see that, so if this is that.

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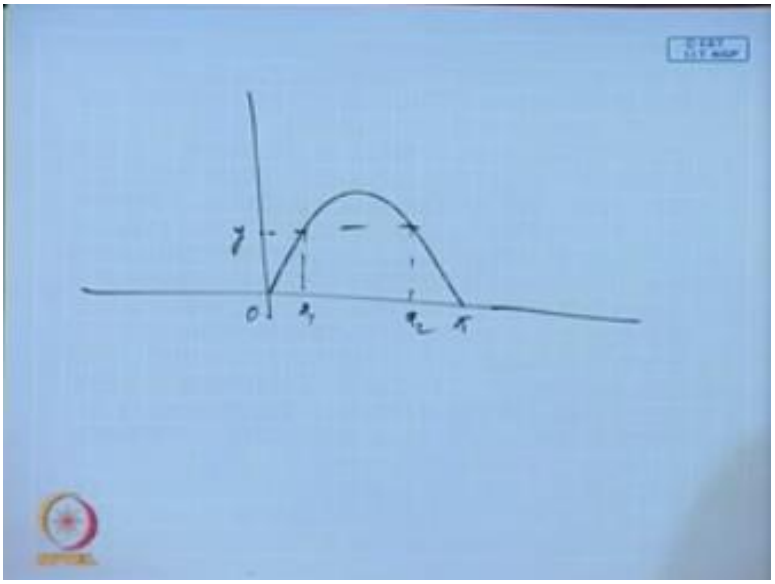

If so if the function is valid for this 0 to π then you know that for a specific for this same value of this y there are two values, this is x_1 and this is x_2. So, two values of this x are possible, so that is why this not a one-to-one transformation.

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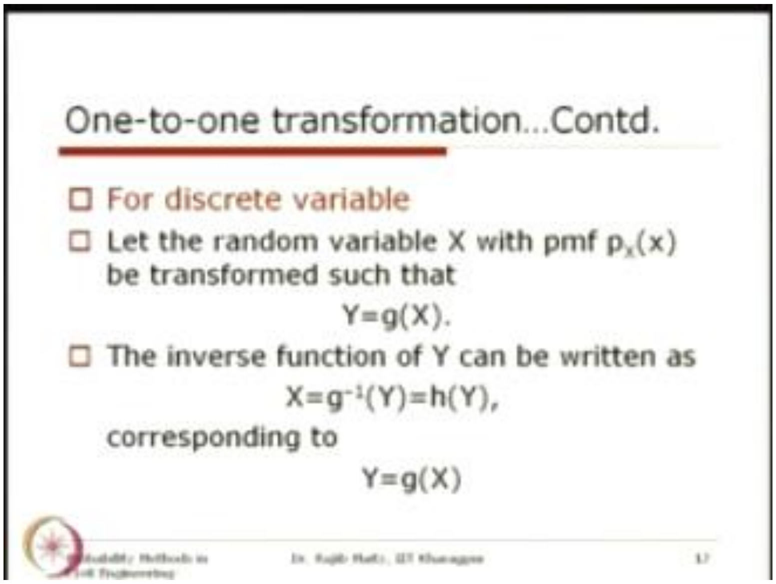

Now for the discrete variable let the random variable x with the Pmf P x of x be transformed such that, that y = g x, and the inverse function of y can be written as this x =o g inverse y, if I just give that new function of the h_y. So, h_y equals to this g inverse y corresponding to y = g x.

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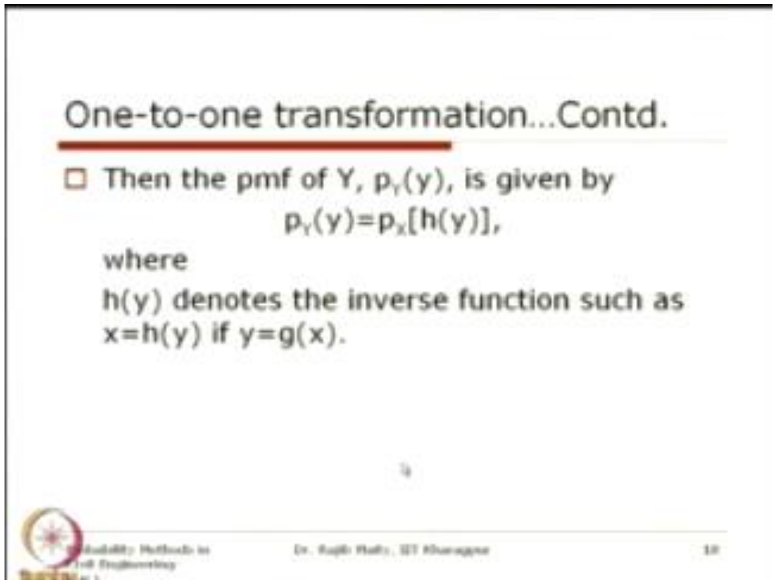

Then this pmf for this y that is P_y y is given by this P $_y$ y is = P_x h y, where this h y denotes the inverse function such as that, x equals to h_y if y = g $_x$ that is the this h_y is nothing but, that g inverse of y.

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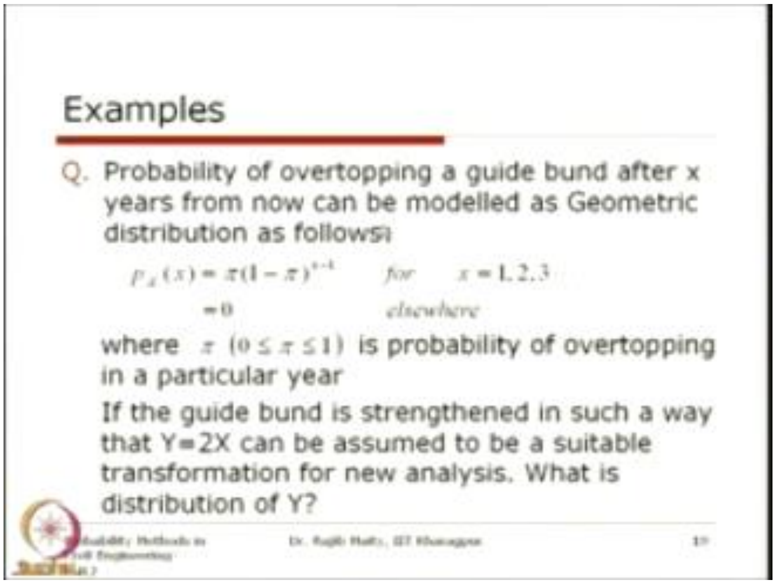

So, with this yeah one example we can talk about for this discrete random variable just we will see that one-to-one transformation obviously we are talking again here the one-to-one transformation, that is if the transformation have always only one specific value. So, this one this probability of over topping a guide bund after x years from now can be modeled as a geometric distribution.

Now we discuss in previous lectures that what is geometric distribution and this in this geometric distribution the random variable that x, that we are talking about this is a either the probability of success or maybe the success and failure you know that we generally attribute is arbitrarily. So, here the overtopping of the guide bund is one event, so that event is that event is has taken place or not that is if I say the success, means we should not means socially we should not say that the overtopping a guide bund is a is a success.

But it is just mathematically whether the event has taken place or not that is, that the guide bund is overtopped or not.

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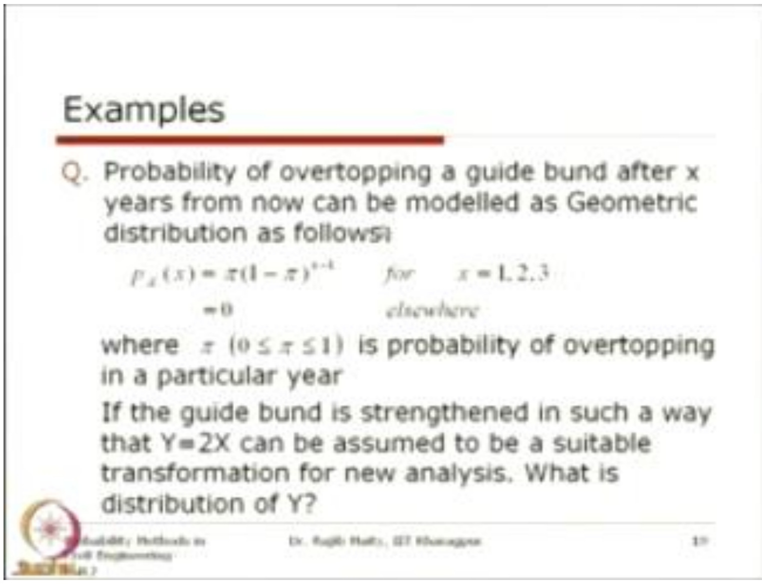

So that one that can happen in after x years, that means so from this point onwards what is the probability that the event will occur after one year, what is the probability that event will occur after two years similarly, what is the probability that it will occur after any years. So, this random variable is modeled as this geometric distribution that we discuss earlier. So, here is this geometric distribution form that is $P_x(x) = \pi\ 1- \pi^{x-1}$ for this x is the discrete value here 1, 2, 3. So whatever the year that you will take.

And this π you know is the probability of success, in the sense that π between 0 and one and the probability of the overtopping in a particular year, now the situation is this that to make the guide bund more effective, it is strengthened in such a way that if I transform that, if I want know that want do some new analysis, then the transformation $y = 2_x$ can be suitable transformation. Now, if this is assess then if I want to get some probability for this, get want to know the nature of this y then, obviously I have to refer to the probability distribution for this new random variable y.

So I have to know what the distribution of this y is so following the same principle that is first of all we have to find out what is the inverse function, because you know this I have to explain in terms of this original random variable. So this is your $x = y / 2$, because $y = 2_x$ just we discussed, thus the PDF for the y that is probability of y is equals to that function express in terms of that inverse function of that y that is g inverse y which is y / 2.

So, this one we have just put, and we have to we have to show that what is the possible value that y can take. So, this is valid for that y = to this is a mistake that should not be x now this is y, the y is = to 2, 4, 6 this onwards, because you know that y is = to twice, of the x now if we just compare that earlier distributional form this present distribution form that, if earlier it was P x x = to that is, so this is π this π multiplied / 1 - π power x -1.And this new 1 is the P y y = to π 1- π power y / 2 - 1. So, this probability of failures under the old scheme x after 1year, 2 years, 3 years, and 4 years for example, is equivalent to the probability of failure after 2 years, 4 years, 6 years and 8 years respectively and in this way it can go.

So, even though this is a very straight forward application that what we have shown here, for this is for this discrete random variable, but if we show but the concept is here is also same. So , what when we are talking about the one-to-one transformation we are finding out the respective domain for the new random variable and finding out their probabilities.

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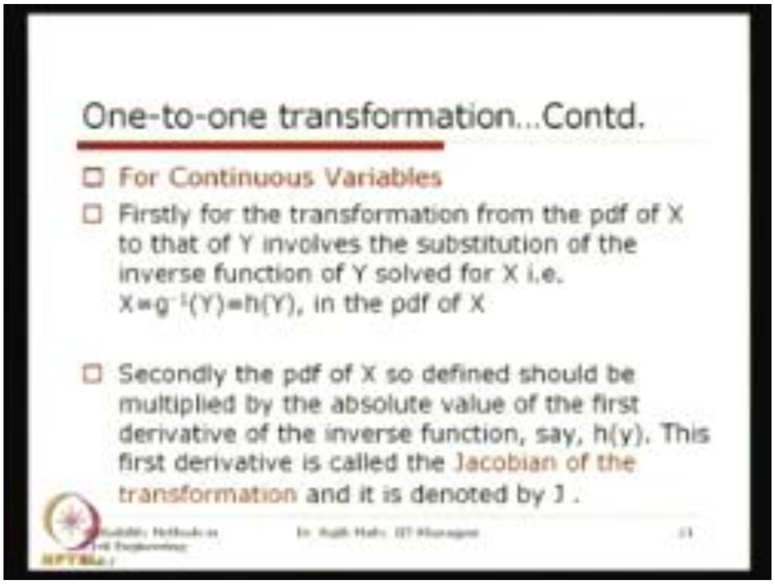

Now, we will discuss about for the for the continuous random variable, now this continuous random variable if I just want to explain first mechanically then the firstly the transformation from the PDF of x to that of y involves the substitution of the inverse function of y solved for x, that is x = to g inverse y. So, this we have to substitute in terms of the x g inverse y what exactly we did for this, just now that for the discrete random variable example.

So, secondly the PDF of x, so defined should be multiplied / the absolute value of the first derivative of the inverse function, say that hy. So, this first derivative is called the Jacobin of the transformation and it is denoted by j. So, the absolute value of the Jacobin should be multiplied with that new with that pdf of this y express in terms of this, we have shown as the y is that $x=g^{-1}y$ that is to be multiplied.

So, mechanically what this two steps is that first of all you express the same the PDF, in terms of this the new random variable and multiply it by a Jacobin, Jacobeans nothing but, the first derivative of that, of the first derivative of the inverse function of h y that is g inverse y. Now, the something if I want to represent with reference to the fundamental theorem that we discussed in the last class, that we will see then it will be more clear to us.

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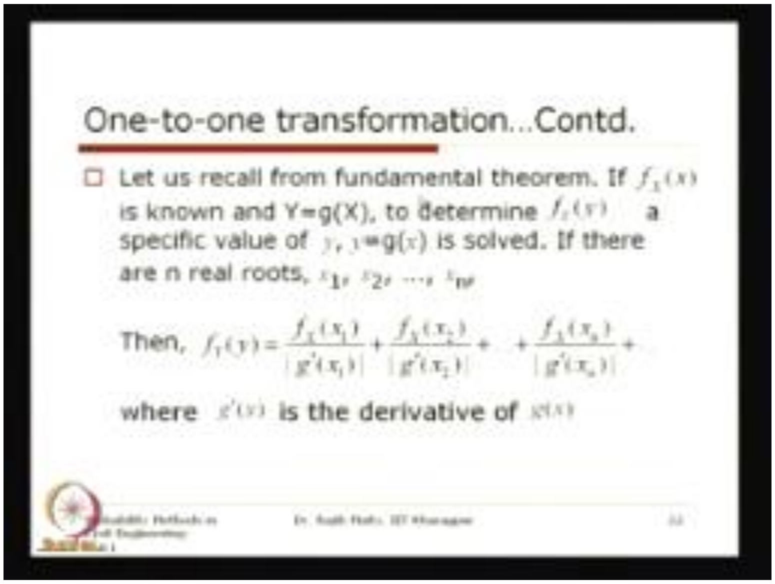

Just recall, that the fundamental theorem that we discuss in the last class, that fx(X) is known and y = to g x to determine the f y, a specific value of y = to g(x) is solved, now if there are n roots that is for this equation if there n roots x 1, x 2, x n then we have seen and we have proven that this equation holds, and we have also refer to this populism and pillar where this proof is available as well.

So, now this g prime x is also we have defined that this is the derivative, as a first derivative of this is g(x). Now, here what we are doing of, what we are saying is that this is the one-to-one transformation, that means; this root will always be one root, so when we are taking a specific value of y then there will be one and only one value of this x, because we have started with the assumption that this is the one-to-one transformation, so there will be only one. So, straight forward what we can say is that, we can just reduce this other components this is the first this one is the valid thing to what is explained here.

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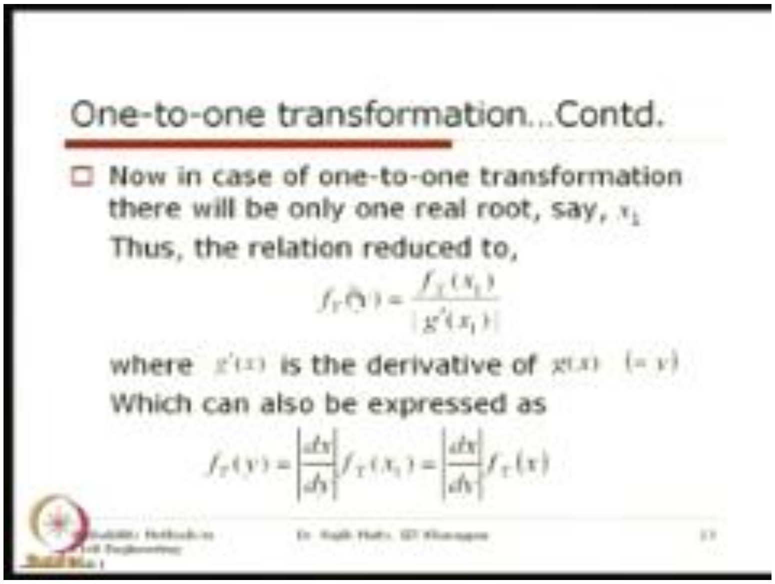

So this fy(Y) is = to this f xx 1 divided / g prime x 1, now this g x we know that this is a derivate g x is = to your y. So, now if I just this can also be expressed in terms of this fyy is = to dx dy, y is your g x that is just I am multiplying that is why I am just taking it inverse, so multiplied / fx x1 as there is only one root just to make it that, so there is no x 1,x 2, x 3, x 4, so just were replacing it / inters of this x.

Now, what we have to do is that, because this is the fy, so you have to explain this full function should be with respect to the y, so this should be express in terms this y; now what is this, this is the PDF of this x which is express in terms of this y multiplied / this factor, which is that first derivative of the inverse function, because this we have taken that inverse the dx / dy this is nothing but, the Jacobian this is what we have discuss in this first slide of this continuous random variable.

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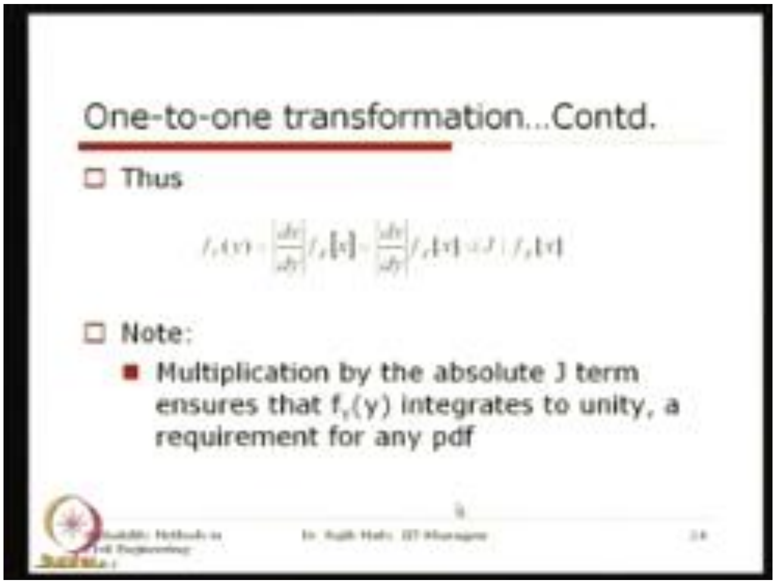

So, this fy(Y) = to this absolute value of this Jacobian multiplied / this one which is expressed in terms of this y, so this multiplication it can be proven that this multiplication with this Jacobin is nothing to ensure that this integrates to unity, which is the requirement of this PDF you know.

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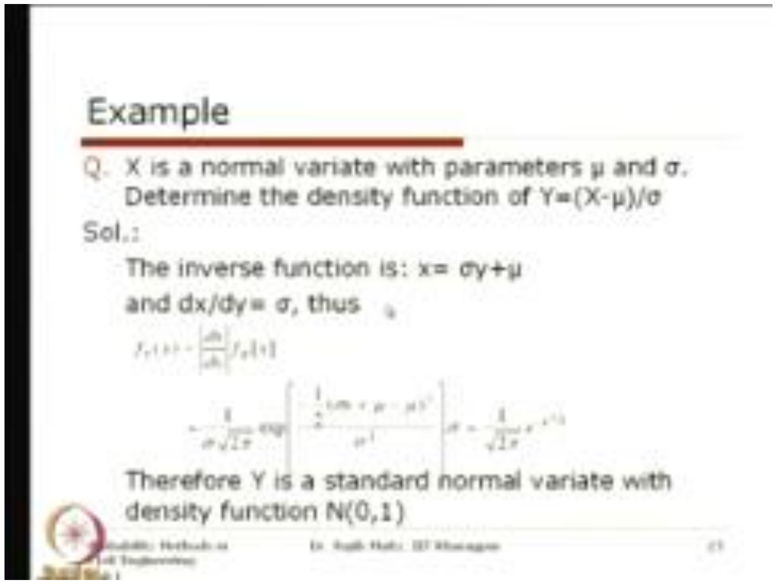

So, we can take some example that is first and foremost that example, that we can take is that this for, this from the normal distribution to this standard normal distribution; we know that this is known to us earlier we explain that is for this standard normal distribution, if you just take this one this relationship that is x - μ/ σ where μ is the mean and σ is the standard deviation.

So, this distribution of y will become a standard normal distribution with mean 0 and standard deviation 1.Now, so this is one, one-to-one transformation and if this is the one-to-one transformation, how it will happen that whether using the same equation that you have used it can be shown very easily, that how it is it is transforming to this standard normal distribution.

Similarly, for this log normal distribution we know that y is = to log x if that if that that y is = to your that normal distribution then x is the log normal distribution. So, we know that distributional form is normal then from that equation y = to log x, this is also one-to-one transformation. So, whether using this one, whether you get that distribution of this log normal distribution that we have already discuss, now we will see that whether from the normal distribution, whether that that log normal distribution can be achieved or not.

Similarly, for the others distributions also, we will see a through different examples of this civil engineering problem in the next class, thank you.

Probability Methods in Civil Engineering

End of Lecture 15

Next: "Functions of Random Variables –
Different methods (Condt.)" In Lec 16

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