INDIAN INSTITUTE
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Probability Methods in Civil Engineering

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Lecture – 12

Topic

Probability Distribution of Continuous RVs (Contd.)

Welcome to this today's lecture we will continue the same topic that we started in the last lecture and this lecture is on the some standard probability distribution on continuous random variable. In the last class we covered some of them and in this class also we will cover some of them. As I mentioned in last lecture that all this theory, all this description, all this properties of this standard distribution will be used, while we are taking some modeling approaches for some civil engineering problems in the subsequent modules.

And also I mentioned in the last lecture that whatever the distribution that we are discussing, those are mainly focused to the problems related to the civil engineering maybe the list may not be complete, but these are the distribution, which are used in the civil engineering. So in today's lecture when we are talking the same topic on this probability distribution of continuous random variable this is continuing from the last lecture.

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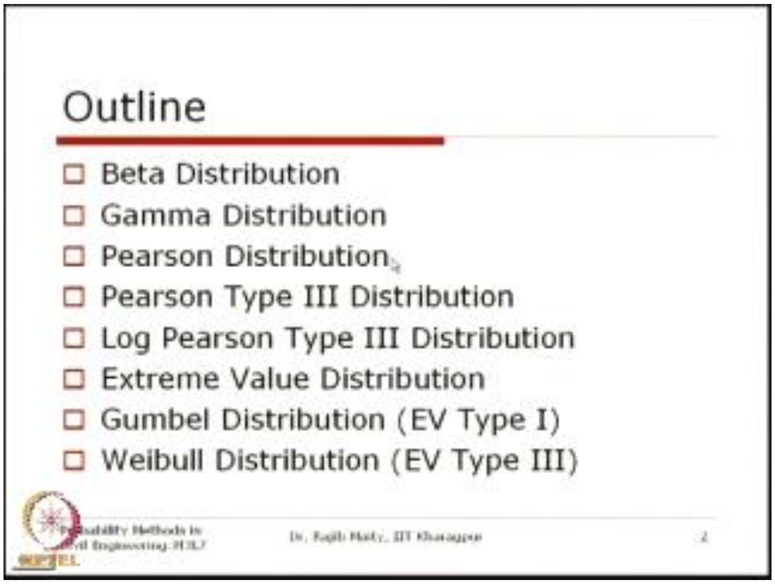

And in this lecture we will covered this distributions, the Beta distribution we will start then Gamma distribution, Pearson distribution. Basically, this Pearson distribution is a family of distribution and there are many such families, for this Pearson is the one of them. And in this family there are seven different types of distribution and in civil engineering particularly in the hydrology and water resource specialization this is important for the analysis of this hydrology, hydrological analysis that is known as this Pearson type three.

So, this Pearson type three distribution will be covered from this family and also some cases, where it is known as this log Pearson type three distribution. And after that we will discuss about the general extreme value distribution, again in this extreme value distribution family, there are three different types depending on obviously depending on their some of these parameter properties.

And most useful in the sense in this context of this civil engineering problem this Gumbel distribution, which is the extreme value type one. And the extreme value type three distributions is known as this reverse Weibull distribution and we will also cover the general Weibull distribution as well. So, maybe this line what is written that extreme value type three, this extreme value type three is related to the reverse Weibull distribution and reverse Weibull distribution and this Weibull distribution also will be covered in this lecture.

There are some more distribution before I proceed to this main part of today’s lecture which I should mention that there is some more distribution which also will be covered. For example, that chi-squared distribution, t-distribution, f-distribution and these distributions are mostly important not only in civil engineering in all different problems where we need to test some statistical data.

So, for those kinds of statistical test and the hypothesis testing, which will be mostly covered in the last module of this course. And in those applications those distributions are also important and those distributions also will be covered either in this module or in the last module. So we will see that basically today’s lecture we are starting with those distribution, which has not covered in the previous lecture. So, we will start with the beta distribution.

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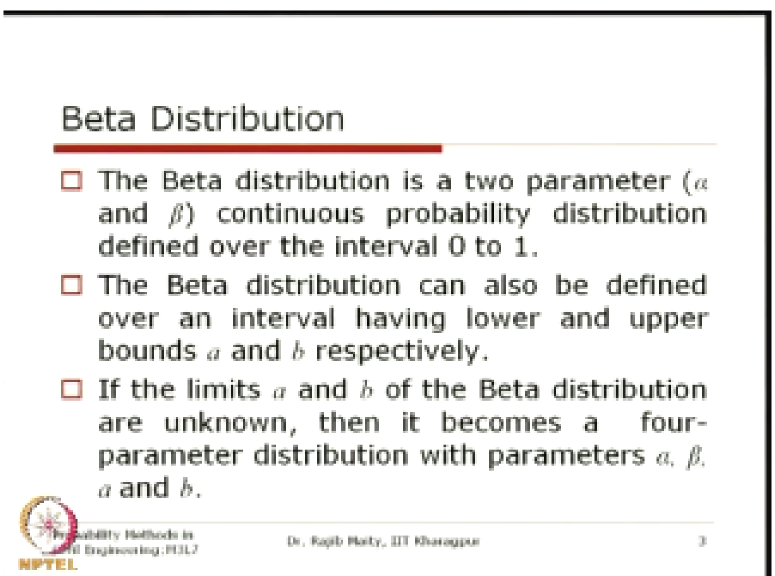

This beta distribution basically is a general case of the uniform distribution. There is a similarity between this beta distribution and the uniform distribution, in the sense that both these distribution is having their specific bound. So, both that lower bound and upper bound we have seen that for example, there are some distribution which are not bounded, bounded in that sense that the support is the entire real axis.

For example, the normal distribution, which is whose support is from the minus infinity to plus infinity. Then we have discussed in the last lecture that the exponential distribution, that exponential distribution we have seen that it can take positive value. So it is the lower bound is at 0, so that distribution here in this lecture we will see the gamma distribution, that gamma distribution is also having the lower bound at 0.

Similarly, some other distributions now the uniform distribution that we saw in the last lecture, that it is having both the bounds lower bound as well as the upper bound. Similarly, that beta distribution is also another distribution for which that lower bound as well as the upper bound exists. So this beta distribution is a two parameter which is α and β two parameter continuous distributions and the distribution is defined over the interval 0 to 1.

So, this interval when we are defining this beta distribution on the interval 0 to 1 this is known as standard beta distribution, this is similar to that, similar in the sense that we also use the same terminology for this normal distribution to this standard normal distribution in the last lecture. Similarly, here when these beta distribution is defined over the interval from 0 to 1, it is known as this beta that is known as standard beta distribution.

Generally in general case the beta distribution can also be defined over an interval having both the lower and upper bounds in the general form like a and b respectively. If the limits a and b of the beta distributions are unknown in some cases then it becomes a four parameter distribution. So this α and β are the two parameters as well as these bounds are also becomes two more parameters. So that will become a four parameter beta distribution.

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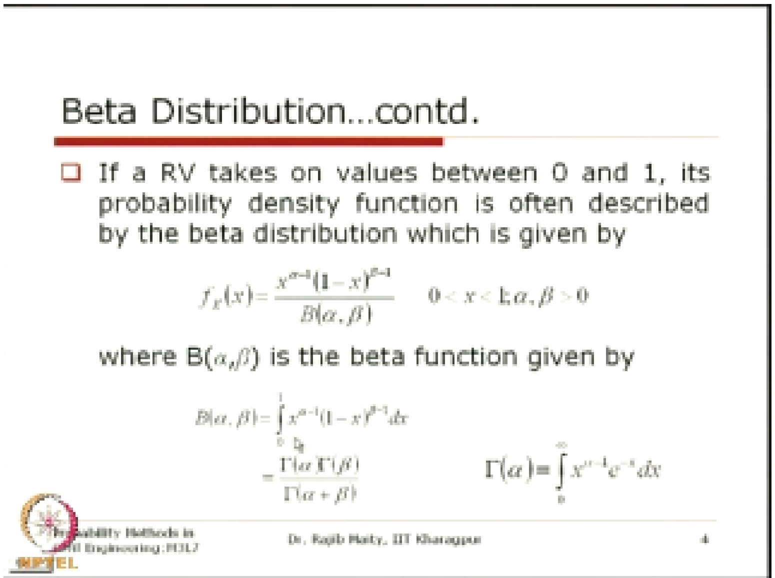

Now, this beta distribution if the random value takes on the values between this 0 and 1. Its probability density function is described by this beta distribution, which is given by this form. That is f_x (x) the density function is $x^{\alpha-1}$ x $1-x^{\beta-1}$ by a function which is known as the beta function and this beta function is having the parameters α and β. Note here, that the range that we are that the support of this random variable is from 0 to 1 and both this parameters α and β are greater than 0.

Now, this beta function that we are talking is an integral form of like this $x^{\alpha-1}$ x $1-x^{\beta-1dx}$ so, this integral form is known as this beta function with parameter this α and β. This can be shown that this can be related to the other functions like this other function which is known as γ function so this is related to this γ,α multiplied by γ,β divided by γ,α + β now this γ function again is another integral form which is shown here. This γ with a single parameter α which is a integration from 0 to ∞ ^{x} α - $1e^{-x}$ dx.

Now this γ function also will be used when we discuss this γ distribution and there are some very specific properties are there that we can utilize. For the time being as we have shown that this α is > 0 so for some specific values this γ function is known so it can be shown in such a way that this integration if we know this value from this 0 to 1 range. Then the value of this γ function can be calculated based on the simple properties and those properties we will discuss in details when we are talking about this γ distribution so with this gives in the complete distribution that is complete density function for the β over the support 0 to 1.

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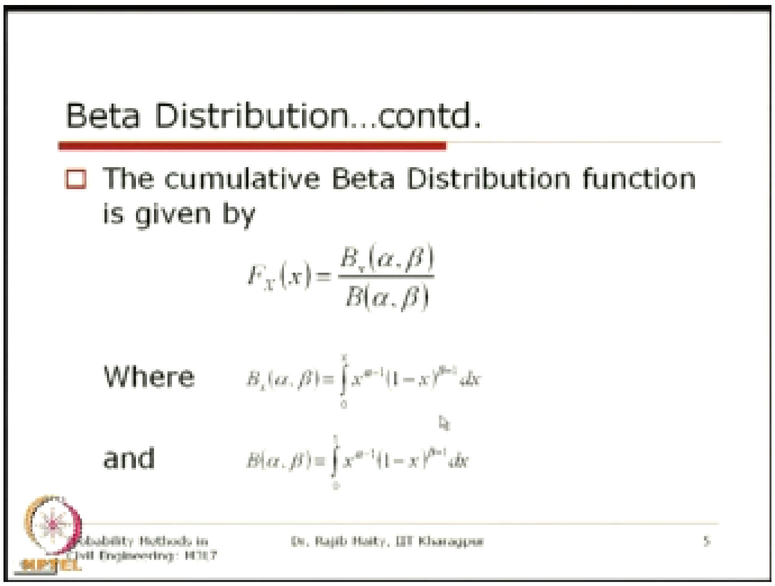

Now, if we calculate this cumulative beta distribution function which is you know from the previous lectures that this is a integration form the left support that is from the 0 in this case to some particular value x, that is the cumulative probability so these will come to this form which is this βx, α,β now this is defined by this integration form that is 0 to x x^{α}-1 1 -x^{β} - 1, this is from the same principle of this cumulative distribution and it is normalized by the same β function where this β is explained earlier that is 0 to 1 x $^{\alpha}$ -1 1 - x $^{\beta}$-1dx.Now, we will just see that how this beta distribution is related to the uniform distribution or in other words how we can say that this uniform distribution in special case of this beta distribution.

And this distribution we generally apply where both the upper limit and the lower limit are known for a random variable in that case this distribution can be used and depending on these parameters this is that α and β we will see in a minute that how just controlling this parameter, how the safe and location of the distribution can be controlled that we will see now.

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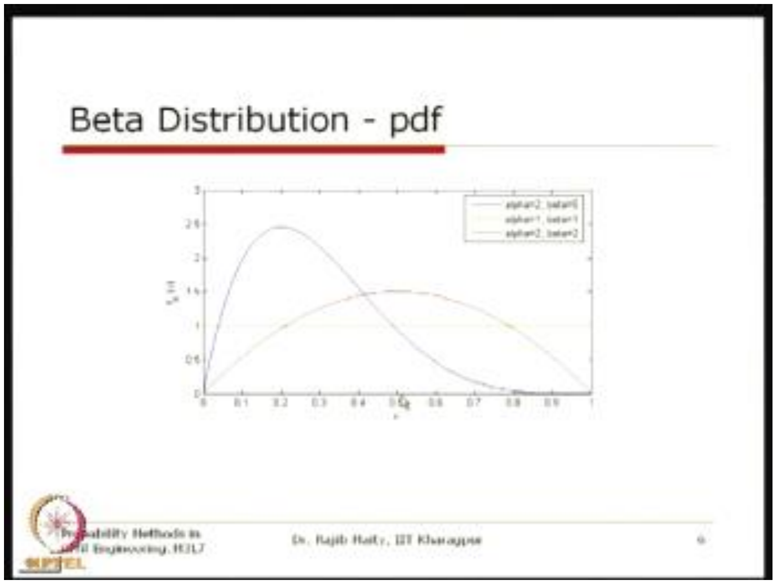

So, here you can see we have plotted three different pdf that is probability density function of the beta distribution. We are just varying the it is parameter now if you first see this blue line this is the parameter the first thing that before we come to this one the first thing that we see that you see it is support of this x is starting from 0 and going up to 1. Now this for this blue 1 if you see that this $\alpha = 2$ and $\beta = 5$ some value has been taken like this. Now if we take the value $\alpha = 1$ and $\beta = 1$ then you see that this is becoming flat so which is nothing, but the uniform distribution over the range 0 to 1.

So if for a beta distribution if we set the parameter $\alpha = 1$ and $\beta = 1$, which is equivalent to the uniform distribution over the range 0 to 1 so we can say that this uniform distribution is a special case of beta distribution now if we keep both the parameters same for example $\alpha = 2$ and $\beta=2$ then you can see that this distributions looks like this which is a symmetric distribution and having it is mean at point 5 mean mode obviously as it is symmetric so it is mean, mode and median coincides to this point 5 it is not only for α =to 2, $\beta = 2$ if you keep this parameter same for example α = 3 and β = 3 then it will remain symmetric as symmetric keeping it is mean, mode and median at point 5.

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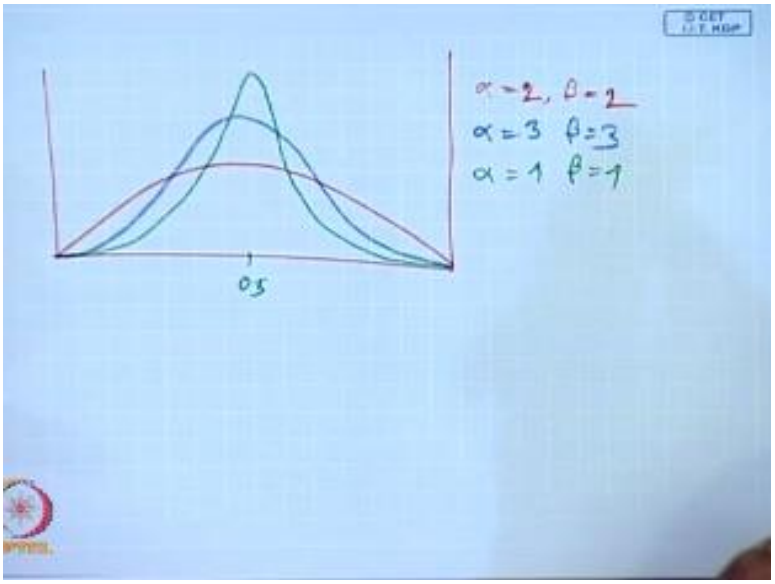

Basically it will change like this so it is peak will increase so this is what is approximated they are as this α = 1 and β equals to oh sorry α = 2 and β = 2. Now if we just increase this that α equals to say 3 and β = 3 then this will approximately look like this. It will be symmetric and similarly if we further increase these parameters 4β = 4 and this again. We will show it is peak will increase but it will remain the symmetric with respect to this mean we will be at 0.5.

Now depending on now we discuss about the if we change it instead of just making it equal if you just change it if you see it on the slide again.

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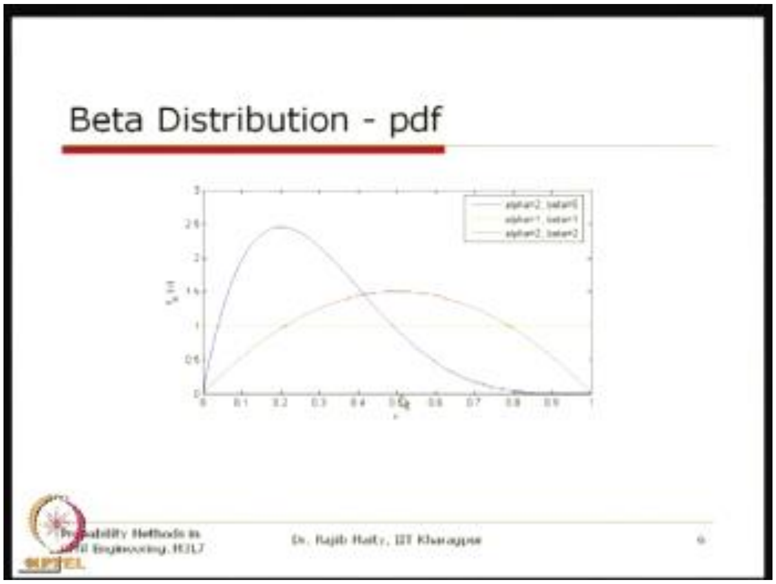

That this one that if we just change this α and β are not equal then how it comes so, here the α = 2 and β = 5 you can see here and from our previous lectures discussion on this positively skewed, negatively skewed you can see that this distribution is positively skewed where this α = 2 β = 5. In fact, for any α which is less than β the distribution will be positively skewed and it is opposite if we change this relation.

That is if we increase this α when α becomes greater than β it will become the negatively skewed. So, what I am trying to say.

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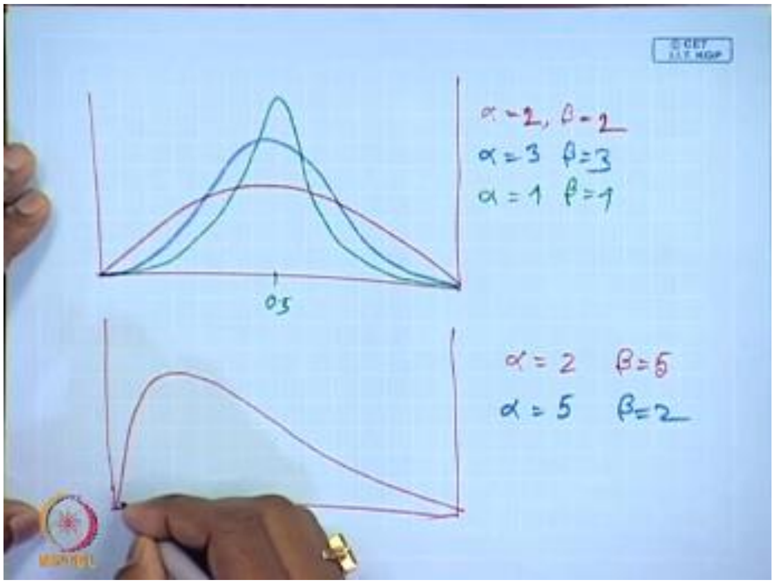

Just same plot if I just take a different frame here so what is shown there is a positively skewed like this for this α = 2 and β = 5 now, if we take this one just reverse this α = 5 and β = 2 the distribution approximately it will look like this which is a negatively skewed. So what we are trying to discuss is that this beta distribution having it is specific bound from lower bound and upper bound. Just controlling these two parameters we can give it is different shape different skewness and it can even be the uniform distributions just selecting the parameters are both are equal to 1.

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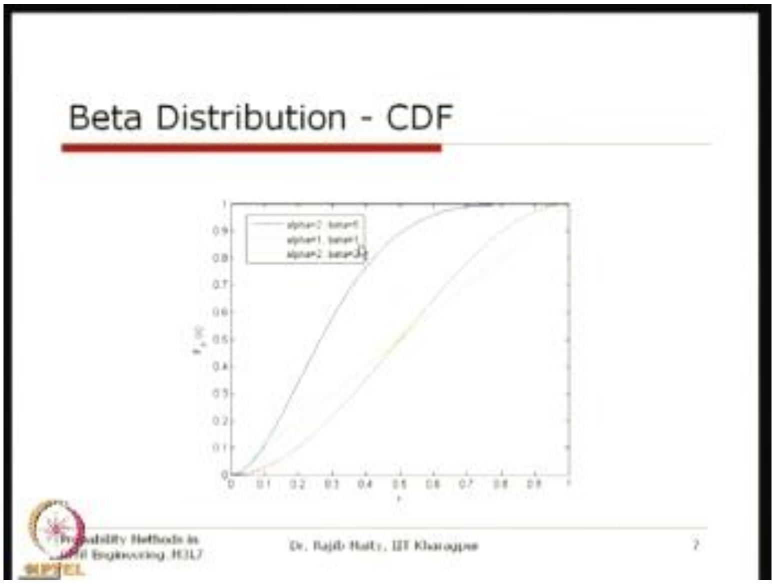

Now, we will see it is cumulative distribution for this β distribution with those same parameter here, that is first the blue one is showing you that $\alpha = 2$ and this $\beta = 5$ and green one is $\alpha = 1$ and $\beta = 1$ we have seen that in this case the distribution is uniform so this green line is a is a straight line starting from 0 and ending of that 1. So this is the CDF for the uniform distribution when $\alpha = 2$ and $\beta = 2$ we see that this is giving a nature that is cutting the point at exactly at 0.5 when this α and β are same.

Now if we increase this both these values α equals to instead of 2 and 2 if we just give that $\alpha = 3$, $\beta = 3$ or even higher, so what will happen this the slope of this one will increase. So, if I just show it here.

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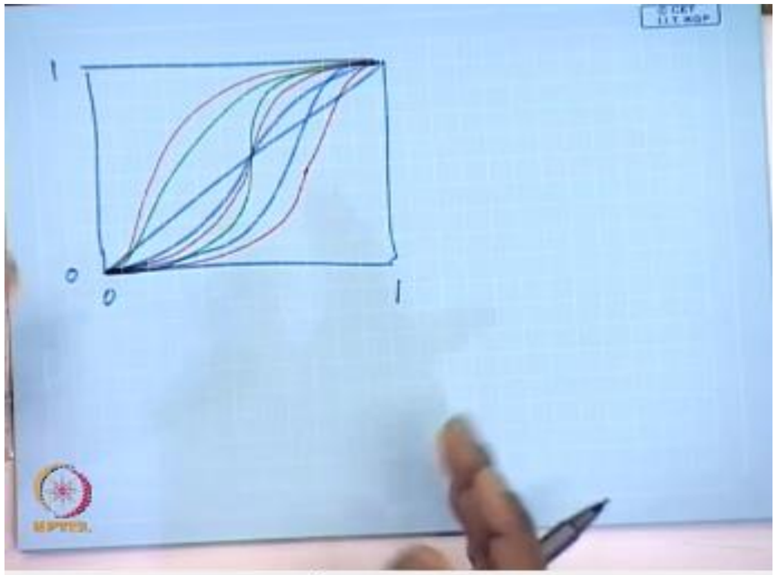

Then what will get is this one, so this for your this is for your uniform distribution this is starting from 0 to 1 and this is x axis is 0 to again, so this is your for the uniform distribution now if we increase this value from $\beta = 1$, $\alpha = 1$ this will looks like this, if you further increase this keeping this intersection point same this will have more slope here and similarly it will go on increasing with the increased value of this β, okay.

Now when we are talking about this positively skewed and negatively skewed how this distribution will look like is like this that when it is becoming positively skewed this intersection point will come towards this, so more if it is more positively skewed then this can be even like this. So it will go to this higher probability quickly which is the standard thing for this positively skewed.

Now if it is negatively skewed then what will happen this will go, so this will take the lower probability percent then it will go and meeting there, more negative it will just show like this, it will go in this fashion. So just controlling the parameter what we are trying to do is that, we can cover a wide range on this on this paper. So just depending on this how this parameters are selected. So this is a very important distribution and we will see that it is application for this different field in the civil engineering in the subsequent using these properties.

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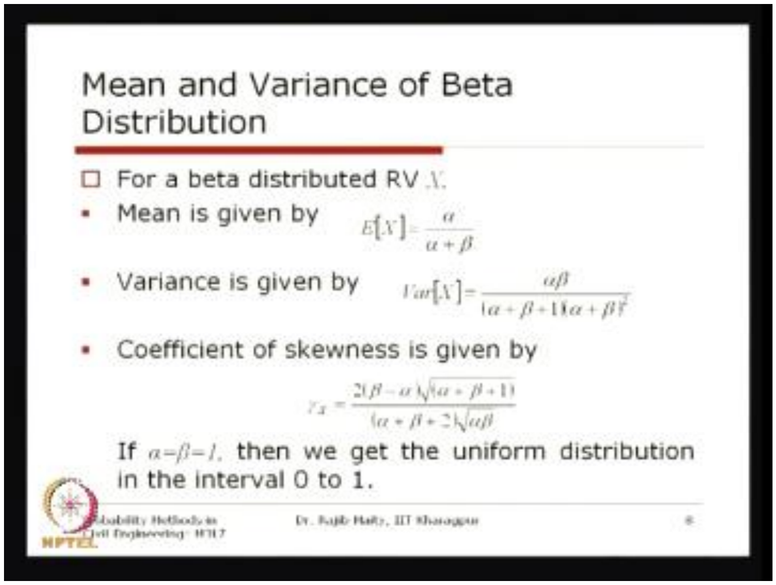

So some initial moments that we discussing now will be mathematically what we have I have just shown you graphically can be checked here through this graphical through this mathematically expression, this is your expectation expected value of this x that is the mean is $\alpha/$ $\alpha + \beta$. So depending on this parameter this is the value we will get as its mean. Similarly the variance can also be calculated we know that this is the second moment with respect to the mean.

It will come as this $\alpha\beta/ (\alpha + \beta + 1)(\alpha + \beta)^2$. Now this coefficient of skewness you can see our previous lecture how to calculate this variance of X, It is the integration from over the entire support of any distribution $(x - \mu)^2$ multiplied by this density, it will give you the variance and if you do it for this β distribution this will come to this form. Coming to this skewness again you know this is the third moment with third moment with respect to the mean.

And when you talking about the coefficient of skewness it is normalized by the cube of the standard deviation or 3/2 power of the variance. So that is normalized this is also discussed in this previous lectures. So for this beta distribution if we calculate the coefficient of skewness then it will take the form with respect to the parameters like this. Now you see here as this α and β remember that this α and β both are greater than 0. So now depending on if the α is less, β is more than this skewness is positive that is what we discuss. So and again just opposite if the α is more β is less then it will become negatively skewed.

What we showed graphically just now. And again that if α and β both are equal to 1 then we get the uniform distribution on the interval 0 to 1, okay. So this is also we have shown graphically a minute before.

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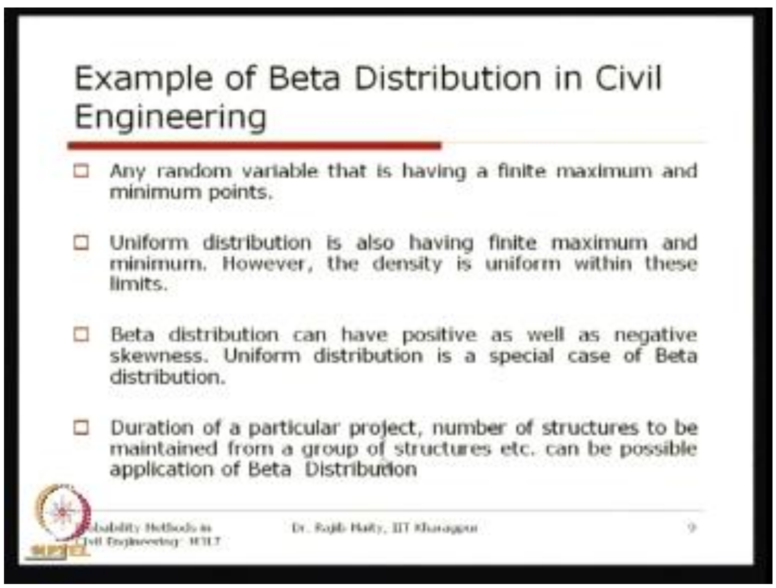

So if we just have some discussion on this β distribution that is any random variable that is having the finite maximum and minimum points then that can be modeled as a beta distribution uniform distribution is also having the finite maximum and minimum however the density is uniform within these limits. Now for the β just controlling those parameters you can have the different safe and size of the pdf, probability density function.

The β distribution can have positive as well as negative skewness uniform distribution is a special case of this of this β distribution, just we discuss. The duration of a particular project these are some exemplar in this civil engineering while we take up some project it is there is some belief that the maximum how long this project can take that generally is known from this from our earlier experience.

And you know the minimum duration is always lower bound is 0. So this total duration of a particular project is known is lower limit and as well as the higher limit from the earlier experience. So this kind of so if we now want to model this what is the total duration of a particular project can be modeled through this β distribution. Then say another example can be the number of structures to be maintained from a group of structure.

Say in a project we have constructed ten such structures in a region and we need to know that how many structures on and average needs to be repaired or actually the budget in the maintenance if we want to know then we have to know that how many a structure is supposed to need the maintenance, so we know that its lower limit obviously 0 and it is higher limits are maximum number of structure that we are having.

So having this two limits known and on the different properties of those structures we can just model this kind of problem also through this β distribution. We will see some of the application in the subsequent modules.

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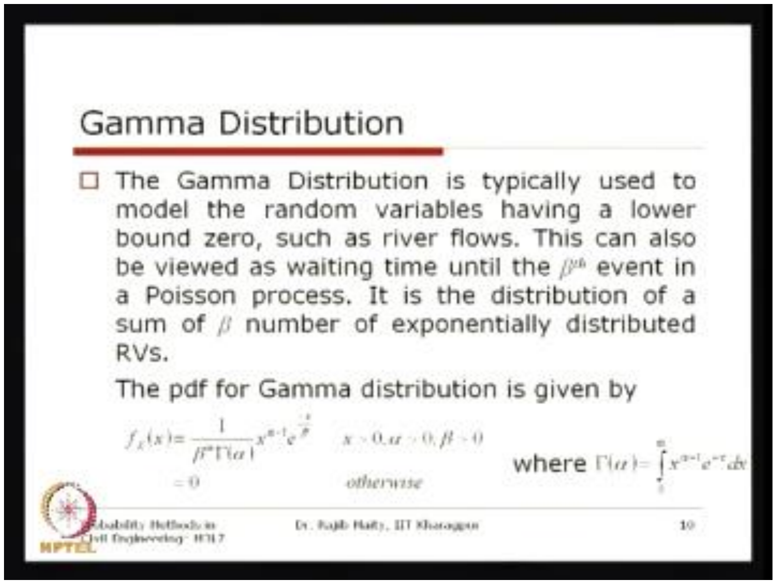

Next we will discuss another distribution which is known as gamma distribution and this gamma distribution is also very important and just while discussing it is the bounds we have seen that this the gamma distribution is also similar to our means previous lecture exponential distribution which one is also support is from starting from 0 to infinity. So it is having a lower bound 0 so this gamma distribution is also having the lower bound at 0.

So generally so the random variables that is having a lower bound at 0such as river flows this kind of cases we can use this gamma distribution to model this kind of variables and this can also be viewed as the waiting time until the β event of a Poisson process so we have discussed this Poisson process earlier and there if we just wait for the so instead of the first failure if you say that some nth of β^{th} failure then this taking that this events are mutually independent to each other what we can assume that there are.

So each distribution is exponential so we can say that this is a summation of the β number of exponential distribution and we can show that if we just sum it up then this is this is approaching to a gamma distribution. So that gamma distribution forms looks like this that f x, x = 1 by β σ γ σ $x^{\sigma - 1}$ exponential - x by β and support is as we told that is it is x is greater than 0 and both the parameters are also greater than 0 and for the otherwise it is 0 means that negative side this pdf is 0 and the γ distribution that we discuss in the while discussing the β distribution as well that γ σ with this parameter σ is defined as an integral from 0 to α $x^{\sigma - 1}$ e – x dx so now we will see there are some easy way of this there are some simple rules how to get this gamma form.

Because this gamma function is very important for this gamma distribution both as well as for this β distribution as well.

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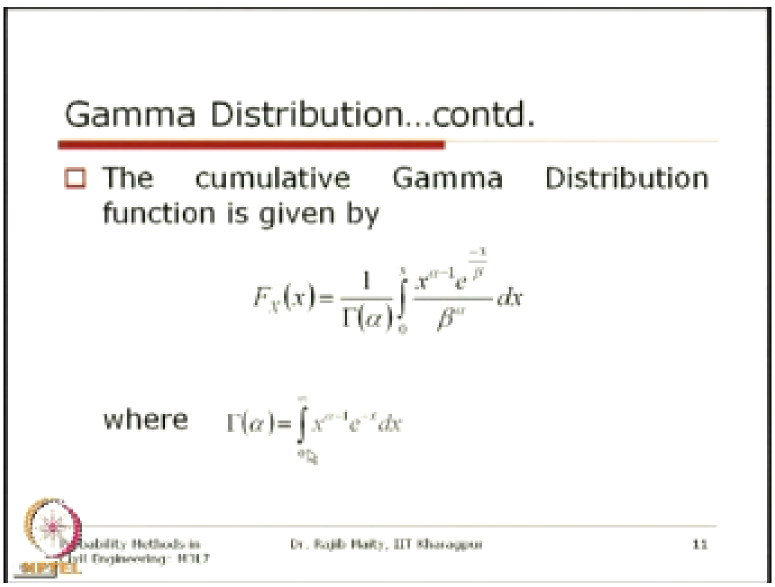

Okay so.

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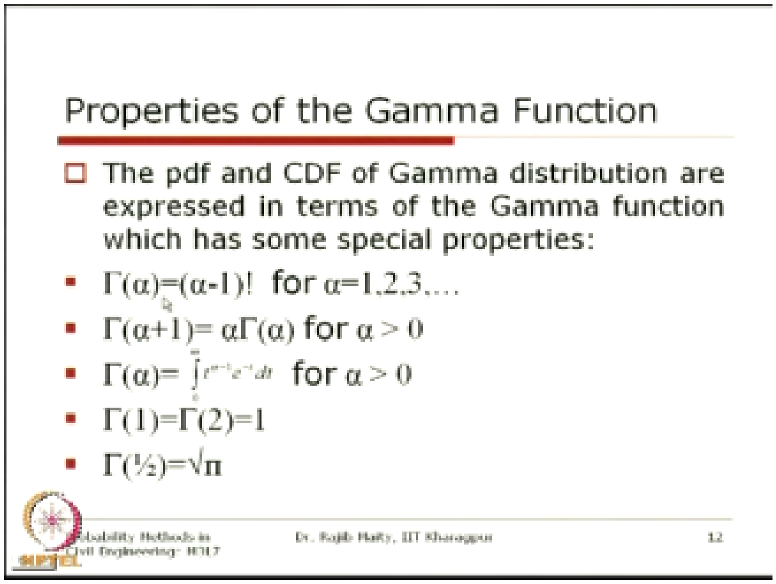

This is the form that is Γ σ = α – 1 factorial for the α = 1, 2, 3 so when this is an integer value when this α is an integer value then we can say that this value of this Γ function is = α – 1.

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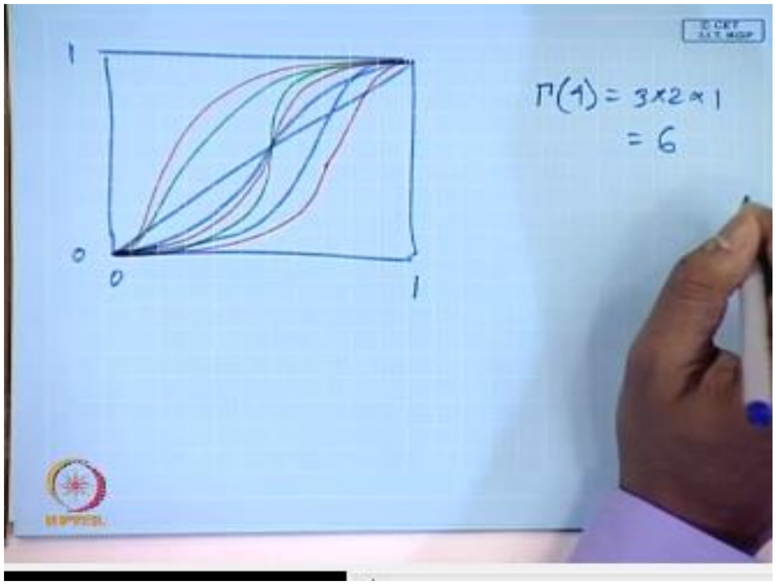

So just a simple example can be like this that Γ 4 = 3 multiplied by 2 multiplied by 1 which is equals to 6 so the second rule that tells that this Γ α + 1. This Γ α + 1 = α Γ α this is for any value not only for integers for any positive value this relation holds good so if I just want to.

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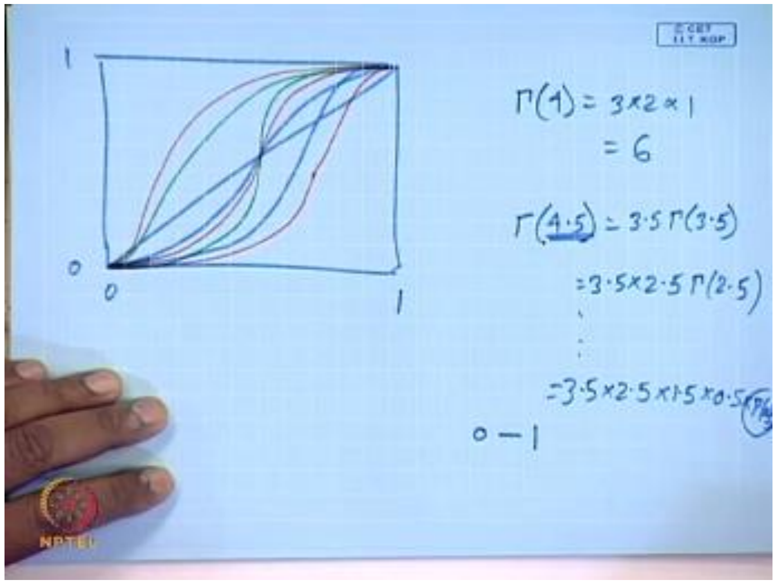

Know that this Γ 4 say 4.5 then what you can write this one is that 3.5 Γ 3.5 similarly this Γ 3.5 can be return that3.5 multiplied by 2.5 then Γ 2.5, in this way you 0.5 multiplied by Γ 0.5.Now you cannot go further because that is the negative side so ultimately for any value any positive number you can take it reduced to some Γ function which is between 0 to 1 so if these values are known to us than any Γ value can be calculated so generally in the text books these values are known this 0 to 1 Γ values known from there we can calculate this 1 one special thing that is there that is that Γ half this Γ .5 √ л so this Γ .5.

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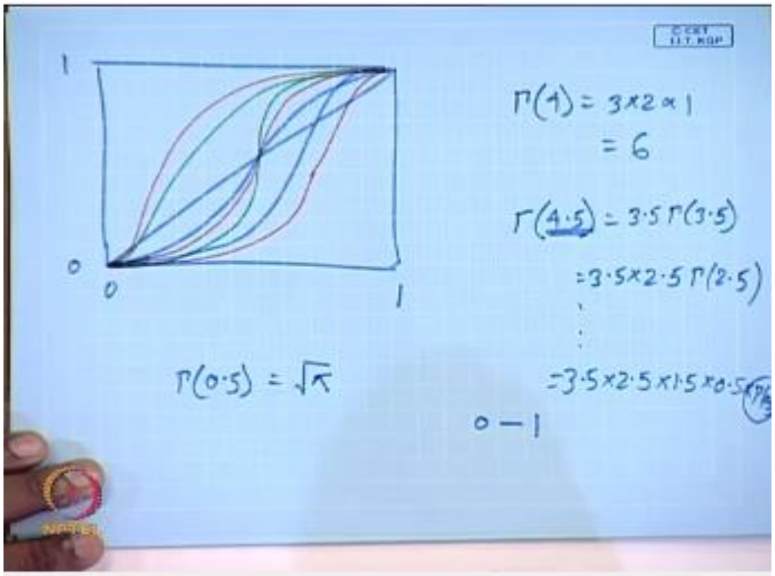

s = √ л this is known and other values can be can be obtained through this numerical integration this can be this is not a close form integration this can be obtained from the numerical integration from 0 to infinity t power α - 1 exponential - t d t and this is generally available from some standard probability table as well so you can calculate the any value that we need for this Γ function and this Γ function is useful for both for this Γ distribution as well as for the β distribution.

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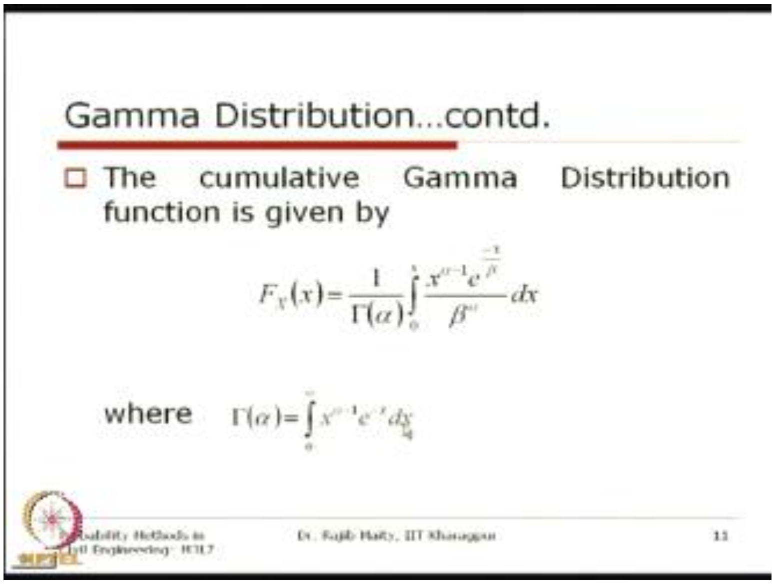

Now we will see the cumulative distribution of this Γ distribution function which is you now again given by this is the normalizing factor is taken out from 0 to x that is the left support to a specific value x of this integration for again Γ function = 0 to infinity x power α - 1 e^{-x} dx.

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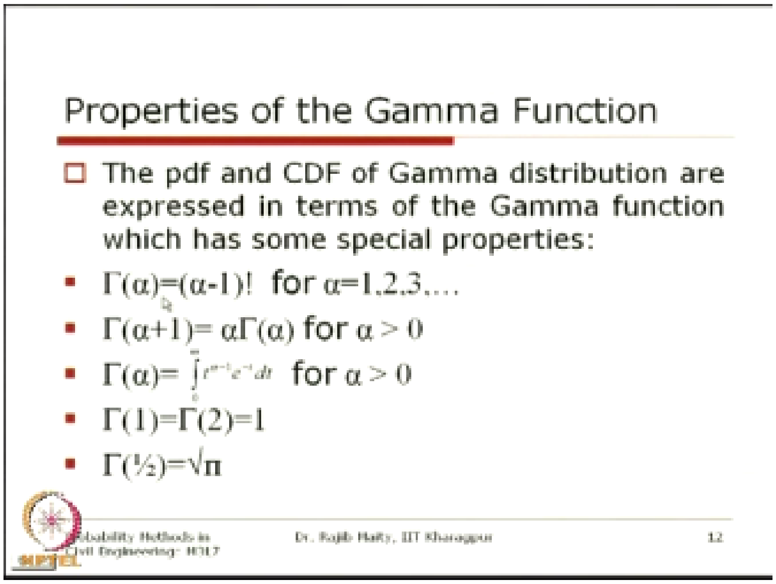

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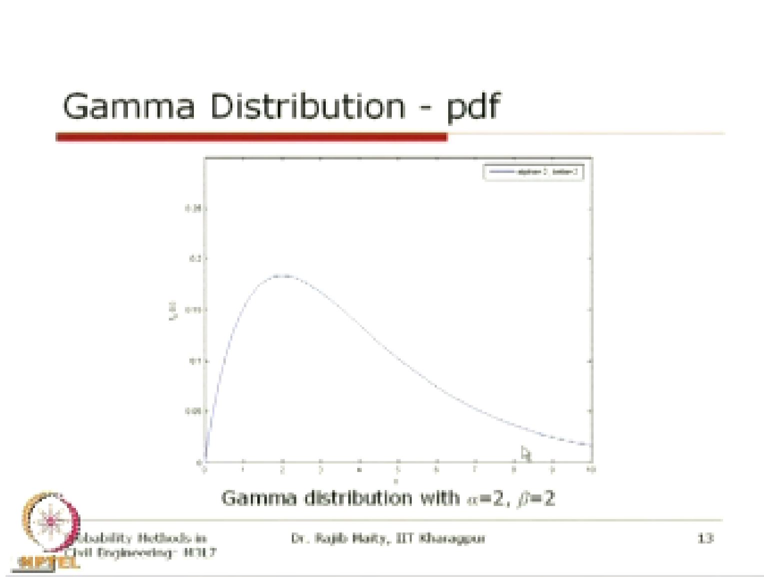

Now if you see 1 pdf of this Γ distribution you can see that it is ranging from 0 to infinity for a specific parameter set here α = 2 and β = 2 you can see this is a positively skewed distribution and basically this Γ distribution is always positively skewed and it is lower bound is 0 so these α β parameters also shown here α = 2 beta equals to 2 the distributions looks like this just for any standard example how this density function for this Γ is look like.

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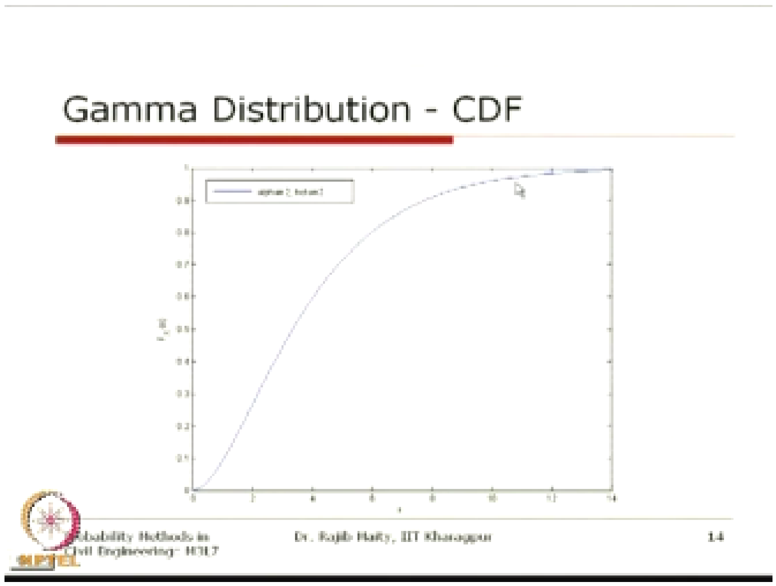

For the same parameter set α = 2 and β = 2 this is the CDF cumulative distribution function for Γ distribution looks like this so it is starts from 0 obviously going to the total probability1 and it is from support is 0 to infinity so this one is asymptotic to 1 at infinity.

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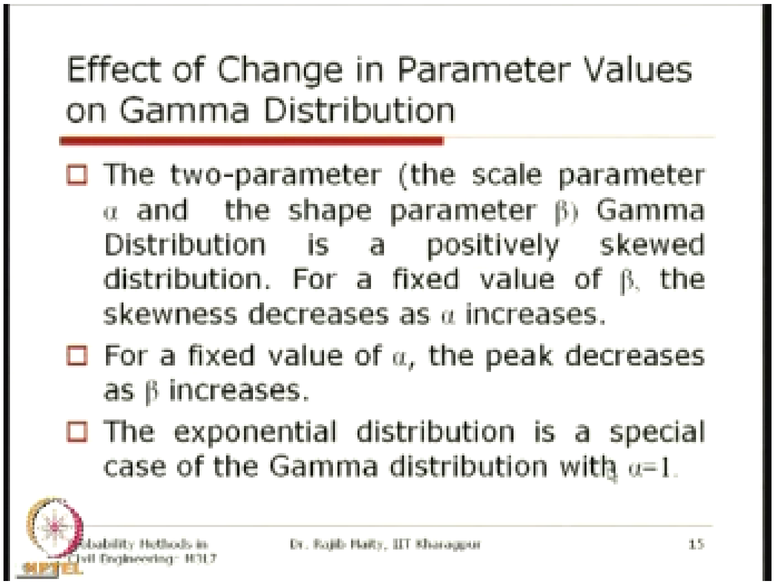

Now we will discuss about the effect of change in the parameter values on Γ distribution the two parameter this two parameters are one is the scale parameter α and another one is the shape parameter β this Γ distribution is a positively skewed distribution which we show in the pdf plot of the pdf for a particular set of parameter so this is always positively skewed now for a fixed value of β if we freeze this β and if we increase this α then this skewness increases sorry this is skewness decreases.

So for a fixed value of beta the skewness decreases as α increases for a fixed value of α the peak decreases as β increases so this thing we will just show in pictorially as well how it is changing and how these two parameter can control to basically to capture a wide range of distribution that we will see and one more thing one more very special thing here is that this exponential distribution that we discuss in the last class is a special case of this gamma distribution when we put this α = 1 basically we get this distribution as these gamma distribution will be equal to this exponential distribution.

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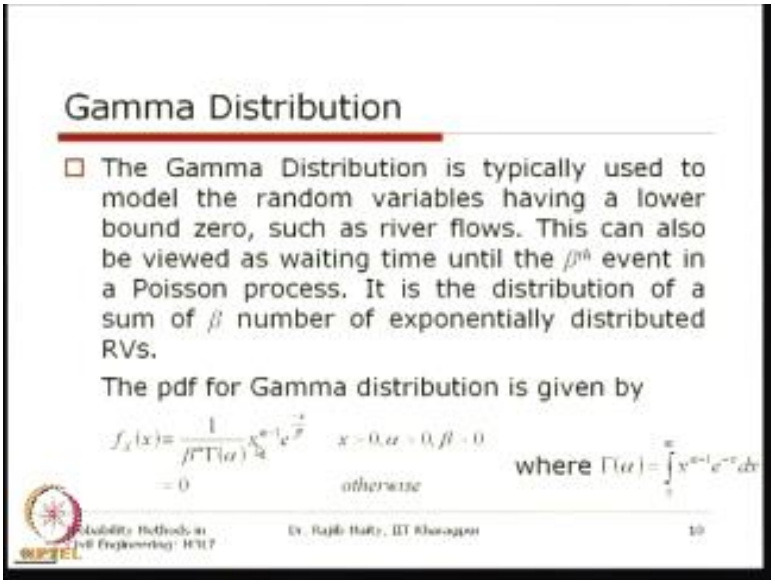

This we can check once here that this is a form. So, if we just put that $\alpha=1$so, 1 by so, this will become $1/\beta$ and we know the $\gamma 1=1$ so $1/\beta$ and this x^{1-1} so this will vanish. So, this will be1 so $e^{-x/\alpha}$ so what we have seen in the exponential distribution might be we discuss that $\lambda e^{\lambda x}$ so if we just put the $\lambda=1/\beta$ here. So, we are getting the same form of the distribution of this, of the exponential distribution from this gamma distribution.

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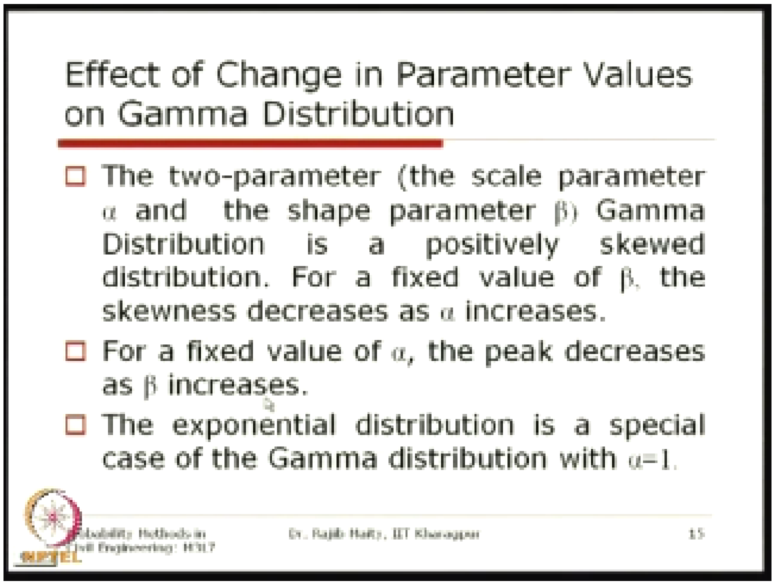

Now, we will see the how this pictorially we can see that this effect of this λ and β. What we have discussed that for a fixed value of β the skewness decreases as α increases and for a fixed value of α the peak decreases as β increases.

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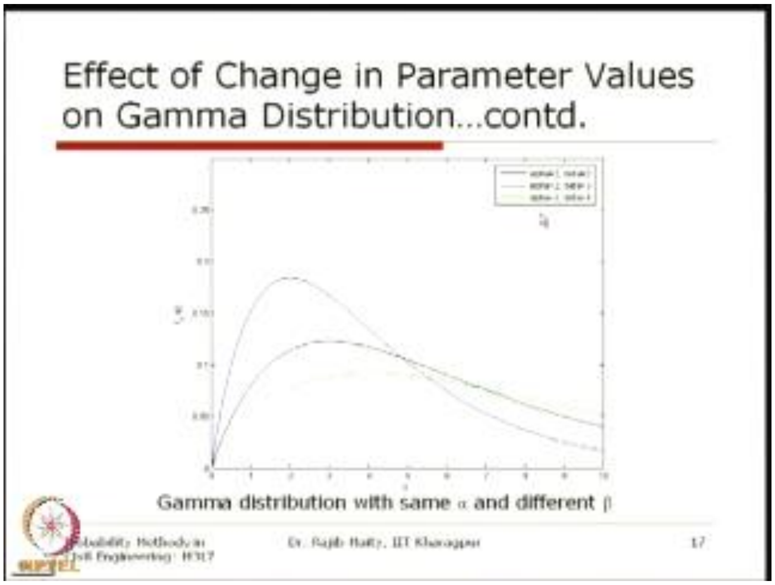

So, this is the first example here, this β all for all this three distributions, all are gamma distributions with the different values of α and β and for these three distributions this parameter β is same, which is equals to 1. So, β=1 for all these three blue, black and green color pdfs. Now, for this blue on this α value is 2, then for the black it is α is 4, for this green α alpha is equals to 8. So, you can see that how the skewness so, this is you can see that all these are positively skewed.

Now this skewness is decreasing as the, α is increasing. Similarly, if we keep that α to be same that is here again three are three different plots are shown here. Where the for all these distributions parameter α is equals to2, for this blue, black and green this α parameter is 2 and we are changing the β. So, for this blue one β is 2, for this black one β is 3 and for the green one β is 4.So, what we have seen that when we are increasing this β then the peak of this distribution is decreasing we just we discussed.

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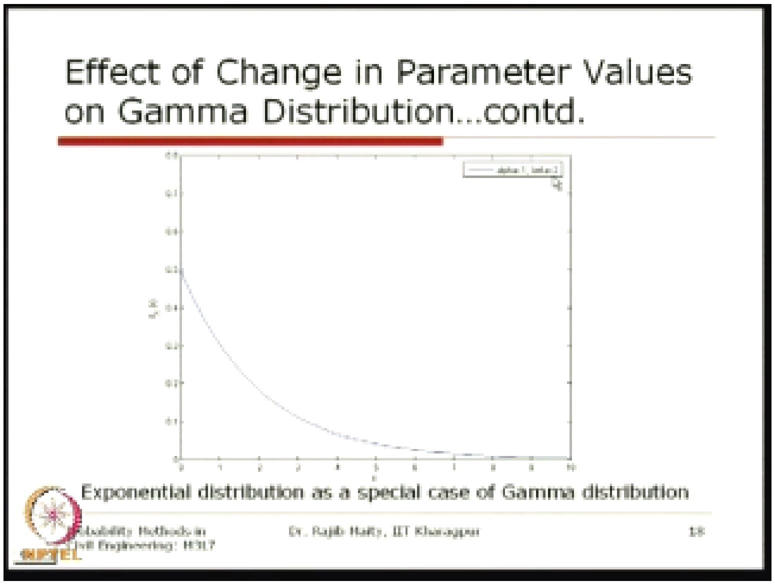

Now, if we just put this α=1 and this β=2, if we showed that if α=1irrespective of whatever the value of this β, then it is basically becoming an exponential distribution.

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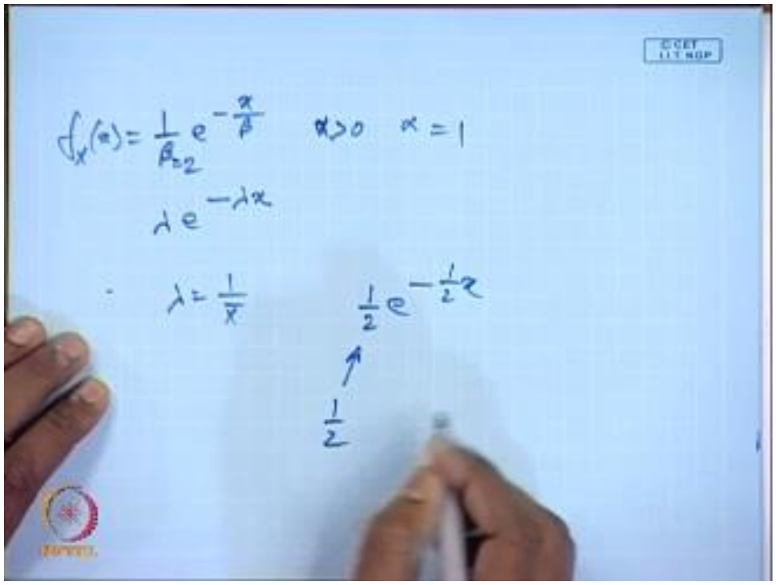

So, now we can see here that this $1/\beta$ so it is becoming $1/\beta\ e^{-x/\beta}$ that f_x from this γ if you put that $\alpha=1$. So, this support are same x is greater than 0. Now you see that when we discuss this exponential distribution might be we have used the parameter, $\alpha\ e^{-}$ sorry $\lambda e^{-\lambda x}$ and we are shown that this, when we are relating to a suspected value or this mean or that we have seen that this λ is related to this $1/\bar{x}$ so here that so this β is a here is shown that if it is that 2.

So, here the example that we have shown this β what we are using as 2 so it is mean is becoming that so it is $1/2\ e^{-1/2x}$ so it is mean value we know that not the mean 1 at the x=0 the values is 1/2.

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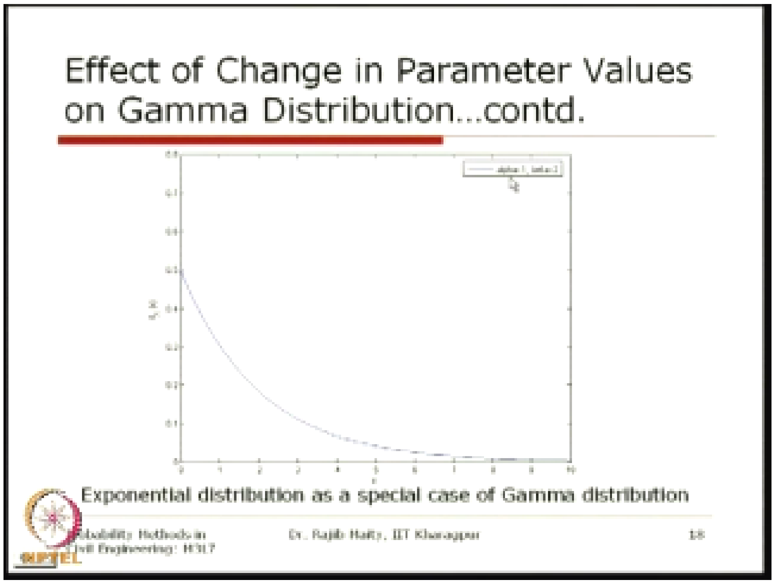

So, here also in this plot you can see that x=0 the value of this so, it is starting from exactly from 0.5. So, if we change this β accordingly maybe the starting point of this density will change, but this shape will be same as that exponential distribution as long as the α=1.

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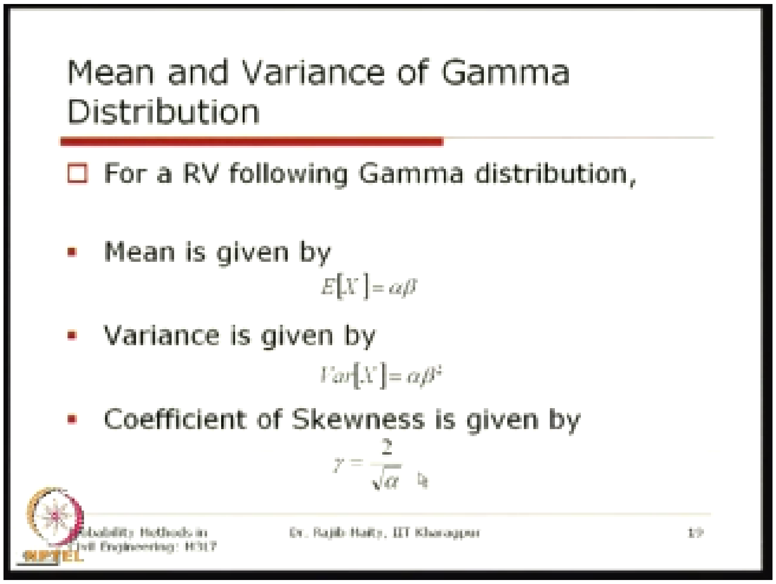

For this gamma distribution these are some initial moments that is the expected value of this α is of this $X=\alpha$ multiplied by β. The variance is given by $\alpha\beta^2$ and the coefficient of skewness is given by $2/\sqrt{\alpha}$. So, here you can see thus coefficient of skewness is depending only on one parameter, which is α and as this α is greater than 0 and this skewness always we are getting the positive one and this one is if the α increases well this skewness decreases.

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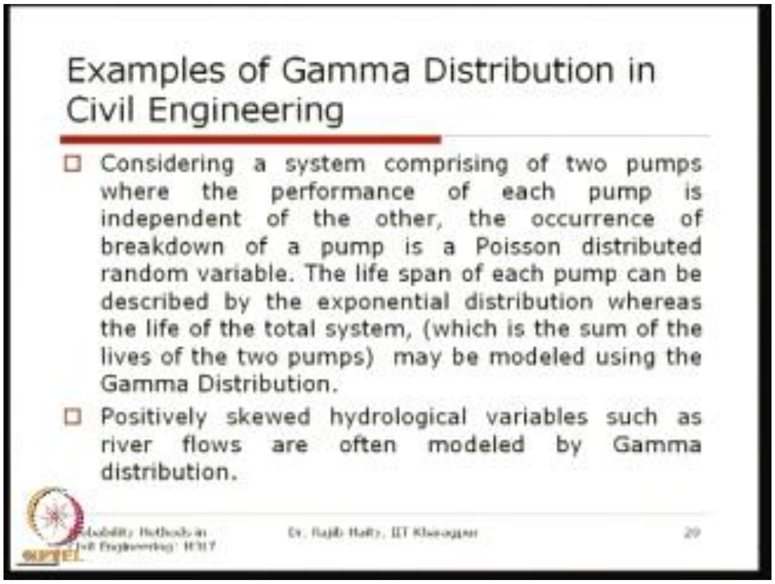

Now, some example of this gamma distribution where this possible application of this gamma distribution to the civil engineering problems suppose that there are so, basically before I come to this specific example, that when we started this gamma distribution, we told that anything that is having some distribution like that exponential distribution or in the discrete case we have discussed the Poisson distribution.

So, now, if we just add up of those distributions and the resulting distribution will basically come to a gamma distribution. Now, in different application if we can see that okay, this is the $\sum$ of this kind of the waiting time or the first failure or the second failure a system involves in different subsystem, which is following the exponential distribution can be add up to the, can be add up that is add up in the sense that these are mutually, this events are mutually independent. Then what we can then in those every case you can think of this gamma distribution.

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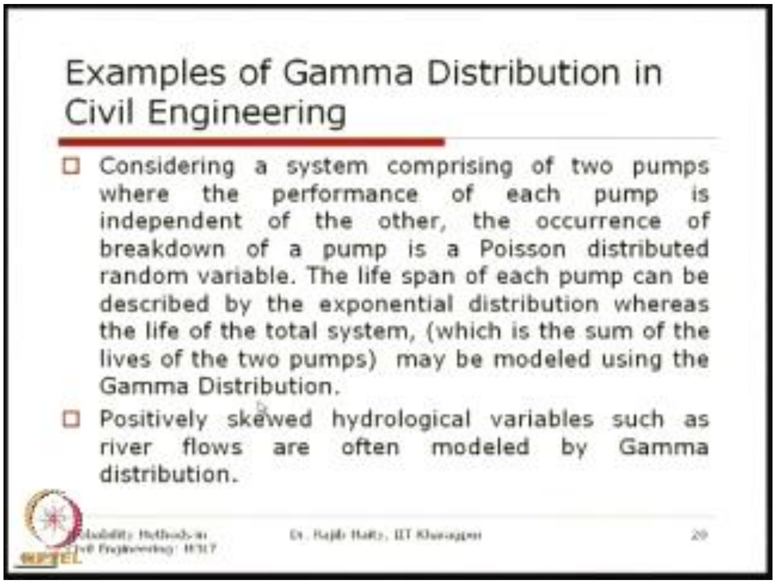

Here one such example is given as is that example of the two pump system. So, in a system two different pumps are used. So, not only two can be more than two as well, it will be just summation. So, now so these pumps are operating independently to each other. So, this independent now understand thus why we are using this independent is that we have to add up this one .Now the occurrence of the breakdown of one pump is a Poisson distributed random variable.

Now the life span of each pump can be described by in terms of this exponential distribution whereas, the life of the total system. Now when we are coming to this total system, it should be the sum of those many pumps. So ,it may be 2, 3 or whatever so which the sum is of in this case is the two pumps, so maybe modeled using through this gamma distribution. Okay, this is one example and in a particularly in the hydrological application this river-flows generally has been seen that can be very useful for this gamma distribution. In the sense that these gamma distribution just by controlling this parameter a wide shape can be captured shape.

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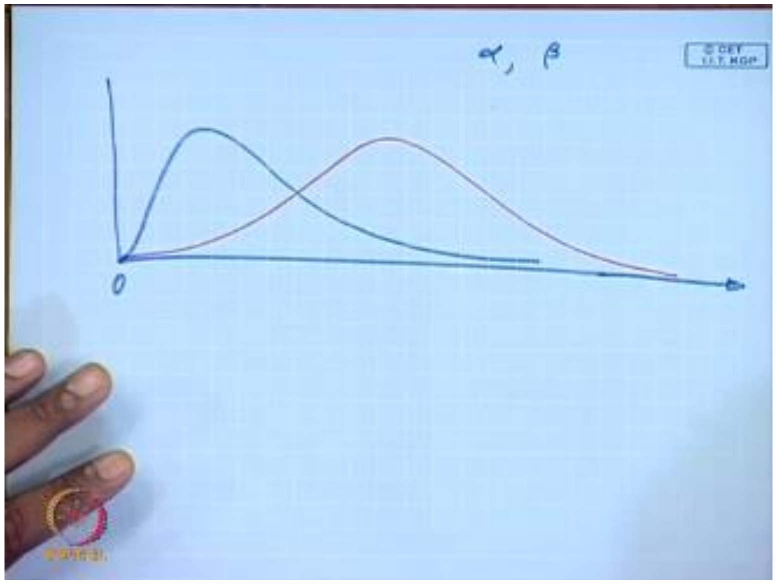

So, and one thing is that for this river flow w know that these are minimum flow is 0, this is always bounded by 0 here. Now the depending on this how the pattern how the characteristics of that flow it can be a kind of a skewness can be so very that peak can be of very low flow the density is the most respected can be low here and just by adjusting this parameter the same gamma distribution can take that any form like this. Sometime what happens for these very large rivers this is, this flow is basically a symmetrical shape and having a kind of normal distribution.

But obviously again there are also the lower boundary is 0.So just we adjusting these two parameters are α, β, of this γ distribution you can control this set starting from this exponential to this positively skewed even to the symmetric. So that is why this we found that the application of this river flow modeling is very suitable for this to use this γ distribution.

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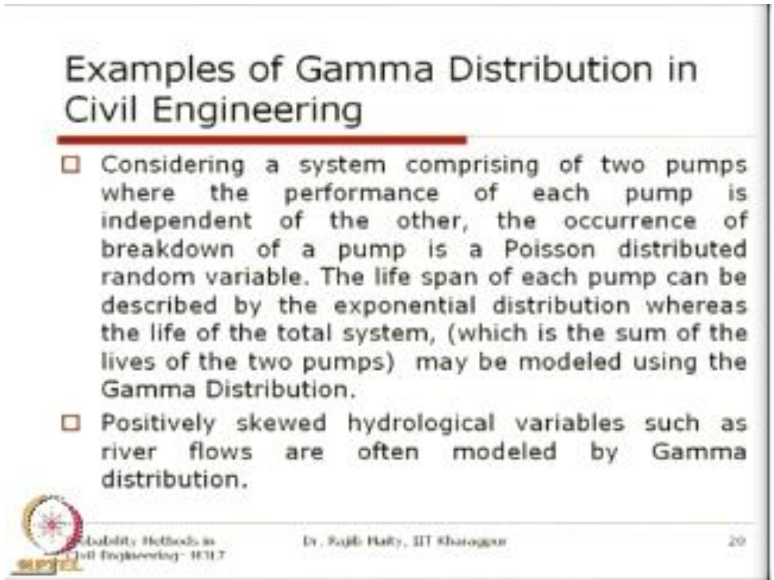

So that is why there is another application of this γ distribution to this civil engineering problem.

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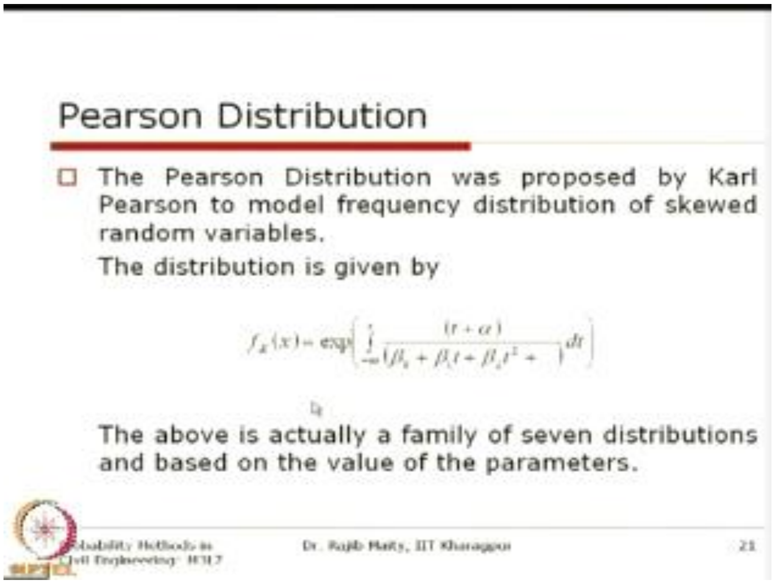

Next a class of distribution which is known as Pearson distribution the Pearson distribution was proposed by Karl Pearson to the model frequency distribution of skewed random variable. The general form of this distribution is given by this expression, which is f_Xx exponential minus infinity to x. This form is t+ α, β, naught + β 1 t + β 2 t^2, this infinite series and takes this form.

Now this form just controlling this it is parameter there are seven different types of distribution can be obtained from this Pearson distribution. Now in our civil engineering particularly we have seen this application in this hydrology and as well as, other fields in civil engineering, which is important is the Pearson type three distributions.

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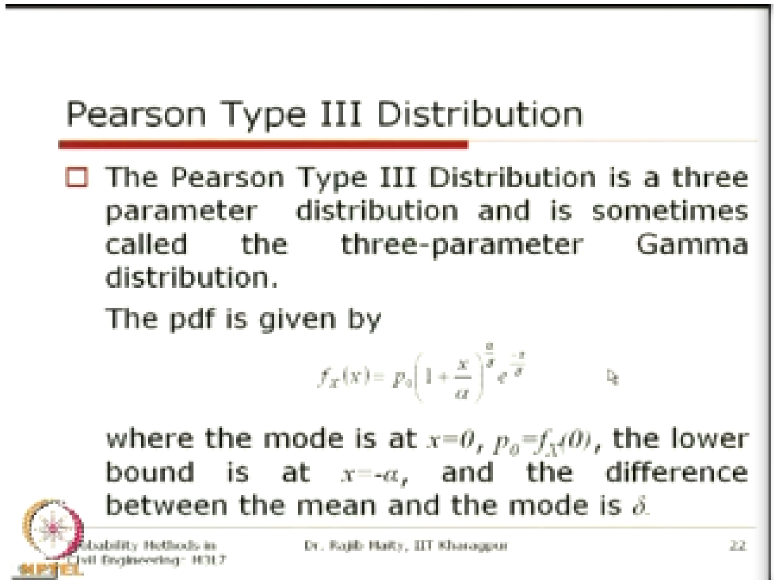

So this Pearson type three distributions are takes the form like this, which is a three parameter distribution. So this Pearson type three distribution is a three parameter distribution and sometimes it is also called as the three parameter γ distribution as well. The pdf of is of this distribution is given by f_xx p naught 1 + x / α / β exponential x / σ. Now this mode where this mode of this distribution is at x = 0 and this p naught is the value at x value of this density at x = 0.

The lower bound is at x = - α and the difference between the mean and mode is σ. Now if we just discuss it is relation to this γ distribution then we can see that mess… Particularly the question that why we call it three parameter γ distribution. We have just now we have seen that this γ distribution is a two parameter distribution here this σ is making that making the difference between this two cases that we are discussing is that difference between the mean and the mode.

Now this difference between the mean and mode, you know that this σ distribution is positively skewed distribution. So this mean and mode are not same so, but for the γ distribution this mean and mode, the difference between this mean and mode are not are not specified. Now for this

Pearson type three distributions this mean and mode is having is this σ, which is a parameter of this distribution.

Now in the γ distribution again in the γ distribution we have discussed that the lower bound is at 0. Now here we do not want keep this lower bound at a specific point. So here this lower bound is also control another parameter, which is - α. So it is more general with respect to this γ distribution which is but this it is application to this civil engineering problem is mostly to the frequency distribution.

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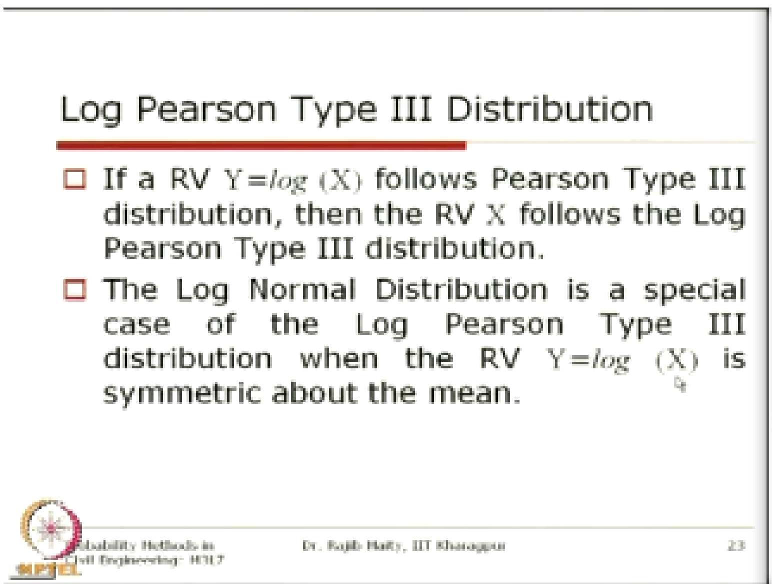

There is another one is known as this called Log Pearson type three distributions for example we have seen that normal distribution or Gaussian distribution. Then in relation to that we have also seen the log normal distribution. So for this very high value if we take the log then we have seen it is coming to this normal distribution. So that in that case we have used that and we have discussed that log normal distribution.

Similarly for this Pearson distribution type three, when we have seen that some extreme value. If we just take the log then it is it is fitting to this Pearson distribution then that original random variable is to known as that it is supporting the distribution of this log Pearson type Log Pearson type three distributions. So that is why means we can make a just correspondence between this normal abnormal and Pearson, log Pearson type distribution.

So if a random variable Y is equals to log X follow the Pearson type three distributions. Remember that what we are talking about here is the Y, if this Y is related to some other random variable through this log. If this Y is Y follows that Pearson type three distribution, then this random variable x follows the log normal type three distributions. The log normal distribution is a special case of the log Pearson type three distributions that just now we told that just the depending on how we are controlling the parameter of this Pearson distribution.

When this random variable Y log X is the symmetric about the mean. So this is also controlled this is also this can also be controlled. So we know that for this log normal distribution after taking the log it is becomes symmetric. So when this one this Y after taking this log of this X is symmetric about the mean then that special that is a special case of the this log Pearson type three distribution which is a log normal distribution.

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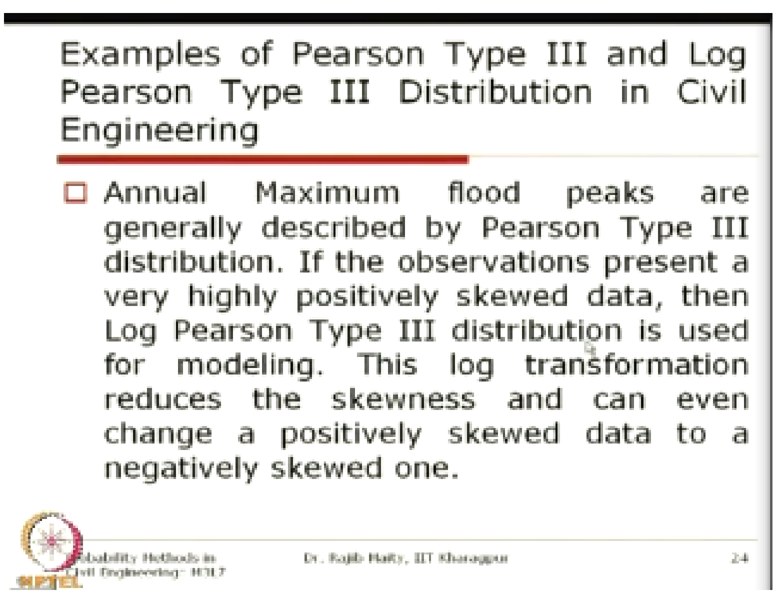

Now examples of Pearson type three and log Pearson type three distributions in civil engineering, sometimes this annual maximum flood peaks are generally described by Pearson type three distributions. If the observations present very highly positive skewed data then the log Pearson type three distributions is used for modeling. These log transformations reduce the skewness and can even change a positively skewed data to a negatively skewed one.

So these things that are when we are talking about this maximum flood this annual maximum flood. So, in a year we have from the data that we are having we are just picking up the maximum values and when we are modeling that only those peaks only those the maximum flows that is available to us. Then it is observed that this Pearson type three distribution is very important for it is particularly for it is frequency analysis that is it is return period.

Which is very important for example to design a reserve dam or that kind of thing, so just I want to know that what should be the possible return period of the occurrence of this extreme flood. So to when we are modeling those extreme floods then for a year. So, that the annual maximum I have to pick up and model it through this probability theory and it is very widely for in this cases, this use of this Pearson type three distribution and log Pearson type three distribution for its frequency analysis is very wide and we will see this application to specific to this problem maybe in the subsequent module.

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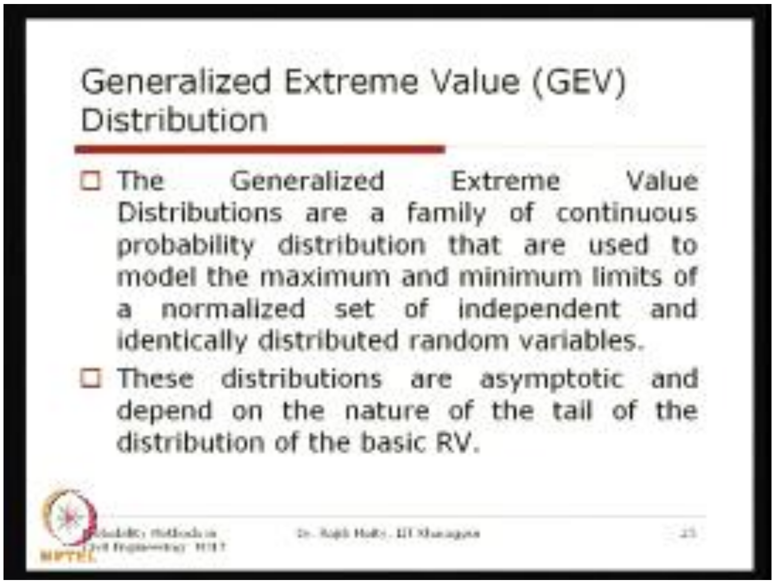

Then another family of this distribution again likes that Pearson distribution. We discussed just now is a generalized extreme value distribution. So, this general extreme value, when we are talking about in context of this civil engineering, we are referring to both this maximum and minimum values. So, not only for... So, if we take that river flows it is not only for the maximum flow as well as we are talking about that minimum flows as well. So, now, we will see that using this extreme value distribution how this three can be modeled?

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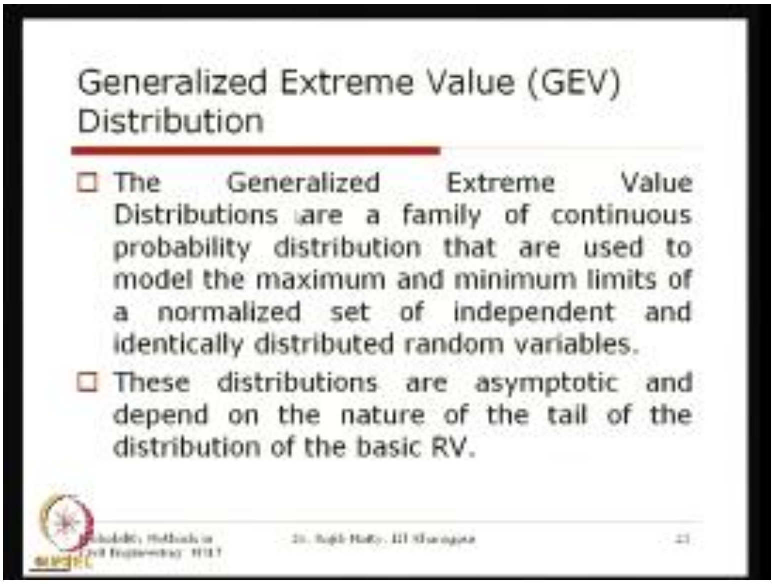

So, this generalized extreme value distribution are a family of continuous probability distribution, that are used to model the maximum and minimum limits of a normalized set of independent and identically distributed random variables. These distributions are asymptotic and depend on the nature of the tail of the distribution of the basic random variable. Now, this when we are talking about this tail, maybe this is the word that first time we are using in this course. We are, what we are talking about the tail is the towards the extreme end of a particular distribution.

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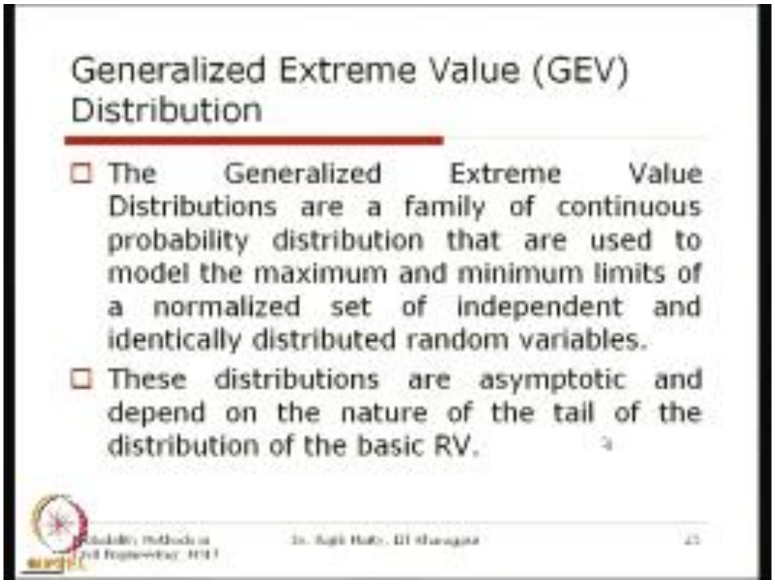

So, we just for example,

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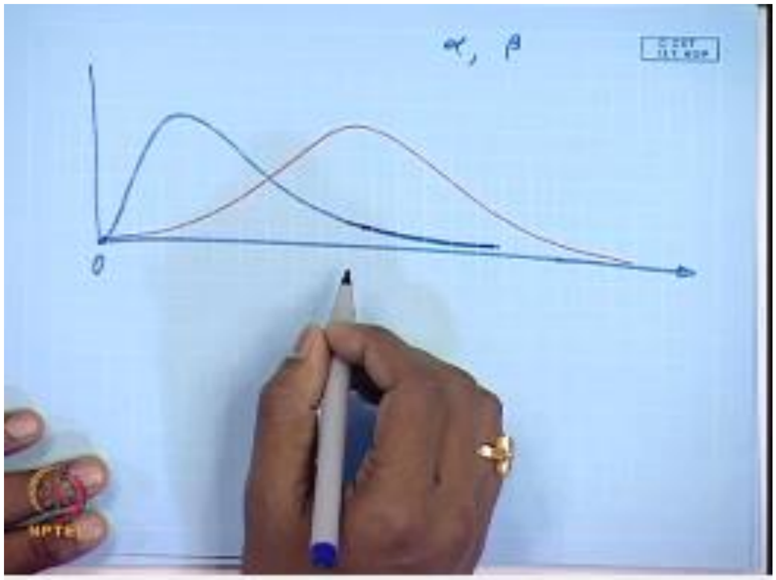

γ The just now what we discuss about this γ distribution, what we are talking about tail is that this one. Where this particularly, this kind of area, where this in the respect of this blue, generally the available data is also less and we need some special care at this zone. So, this is known as the tail of any distribution.

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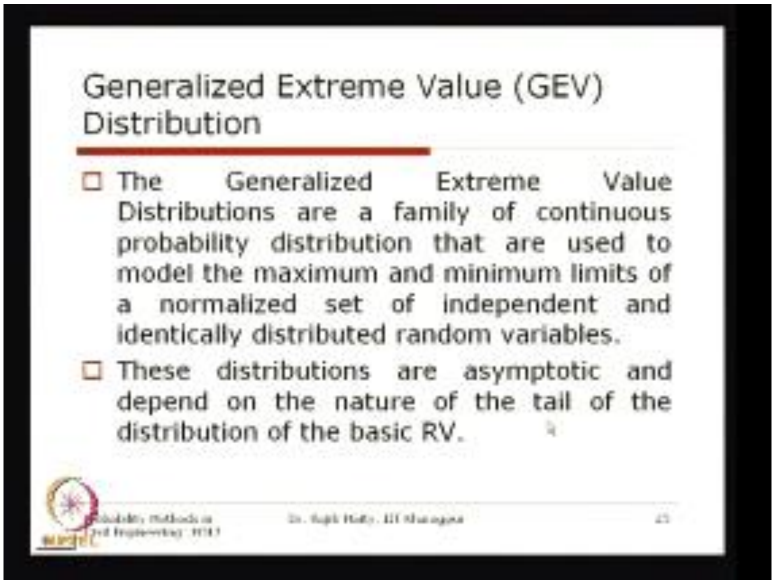

So, when we are talking about this extreme value distribution. So, these distributions are asymptotic depending on the nature of the tail of the distribution of the basic random variable.

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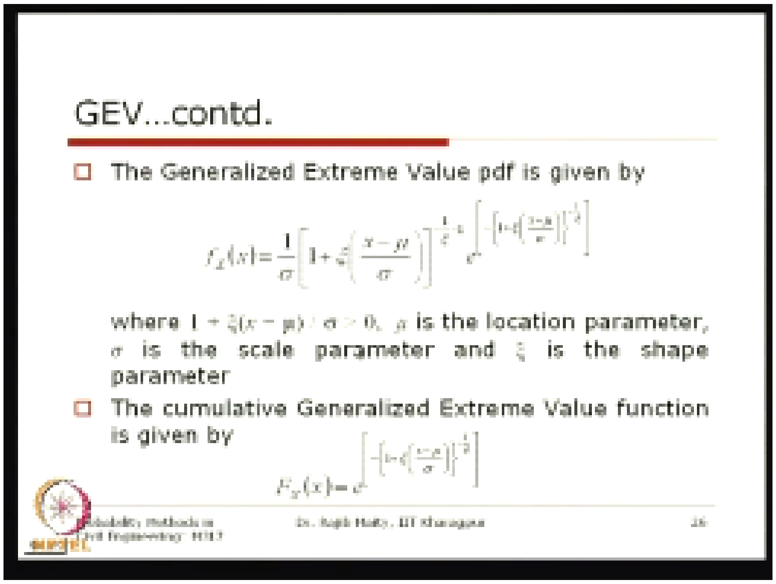

Now, this is the generalized extreme value pdf this looks like this, which is a looks a mathematically complex form. But if we just take those parameters in a group wise then it is not that difficult to tackle here. What we are talking about that here this chi x minus μ by sigma this group of parameter here there is a chi is 1 parameter then this μ hence sigma, this combination it should be greater than 0. So, that is basically the support of this distribution and this μ is the location parameter and this sigma is a scale parameter and chi is the shape parameter of this distribution.

Now, as we are having this parameter just by controlling them satisfying this condition of course then we can arrive at different distributions. So, the cumulative distribution of this one, of this general form first of all we are talking about is, we will come to this one this exponential to this group power 1 by chi.

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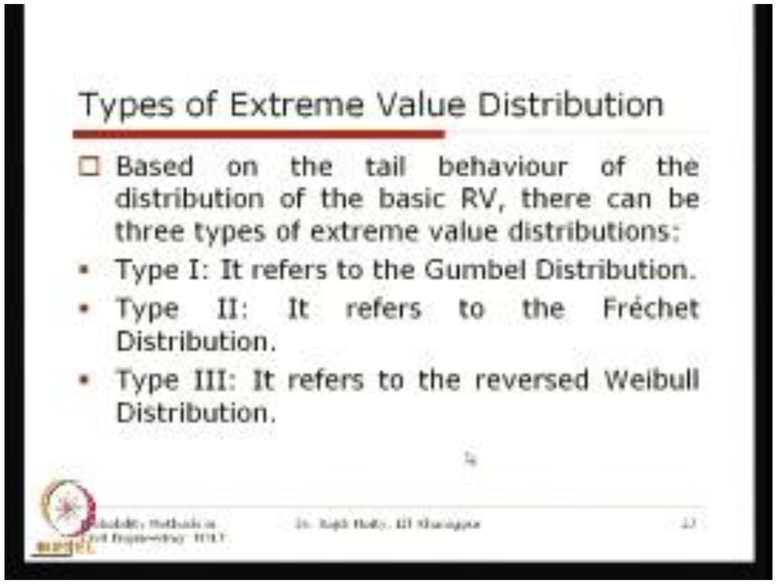

Now, this based on that tail behavior of the distribution of the basic random variable there can be three types of the extreme value distribution. Type one it refers to the Gumbel distribution we will see a in minute, the type 2 refers to the freshet distribution, type three it is refer to the reversed Weibull distribution. We will discuss the Weibull distribution and we will see that how it a related to this extreme value distributions related to the reversed Weibull distribution. Maybe this distribution, this Weibull distribution will be covered in the next lecture.

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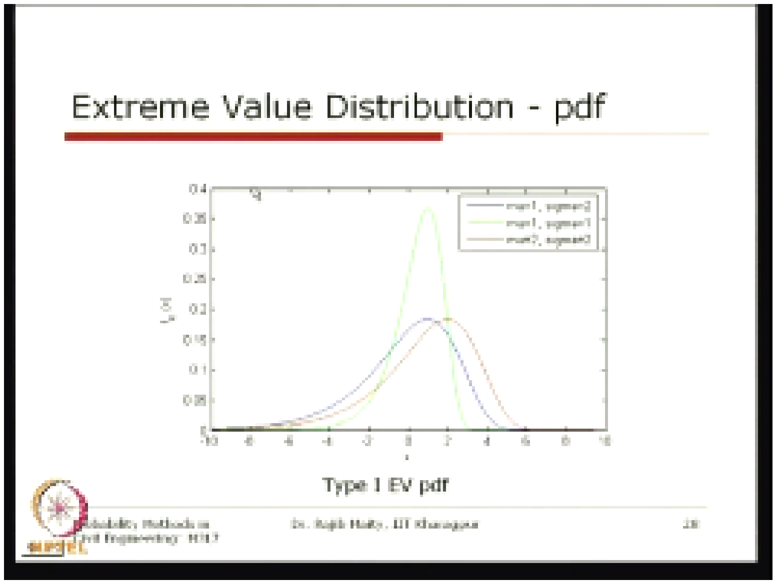

So, before that we will just see it is effect of these different parameters of this, how this pdf looks like. So, here this μ the blue one is for the μ = 1 sigma equals to 2 this green one is the μ = 1sigma equals to 1 and this red one μ =2 and sigma equals to 2.There CDF looks like this with same set up parameter this blue, this green and this red. So, how this is means it, it basically it depends on the how this one means, how distributed on this entire range of this random variable, this should be F. So, one correction here this should be F, because you know that by this time, that CDF the notation that we are following is this is the Fx. The other distribution is the Weibull distribution maybe we will discuss this distribution in the in the next lecture.

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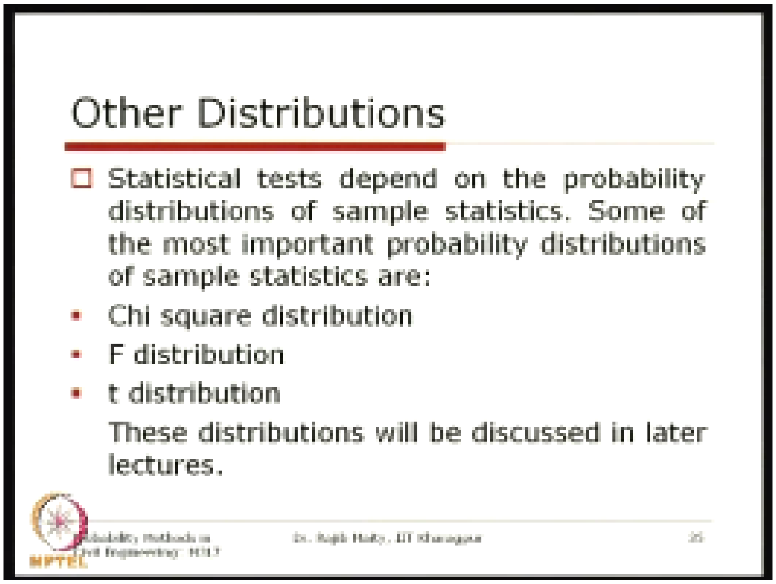

There are other distributions as well, which are the chi square distribution. While this at the starting of this lecture. We have just shown that this chi square distribution, F distribution and t distribution. These are the statistical these are very useful for the statistical test and also the Weibull distribution that we are discussing; maybe it is pending for this class maybe we will discuss in the next lecture. So, we will continue one more lecture in this module to complete that whatever the standard probability densities and their different properties and their possible application to this different civil engineering problem. We will see and these properties will be used to model the different problems in civil engineering in the subsequent module so far. Thank you.

Probability Methods in Civil Engineering

End of Lecture 12

Next: "Probability Distribution of Continuous RVs (Condt.)" In Lec 13

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