

**INDIAN INSTITUTE
OF
TECHNOLOGY
KHARAGPUR**

**NPTEL
National Programme
on
Technology Enhanced Learning**

Probability Methods in Civil Engineering

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IIT Kharagpur**

Lecture – 11

Topic

Probability Distribution of Continuous RVs

Hello and welcome to this lecture 6 of module3 on random variable.


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Probability Methods in Civil Engineering

Module 3: Random Variables

Lecture – 6: Probability Distribution of Continuous RVs

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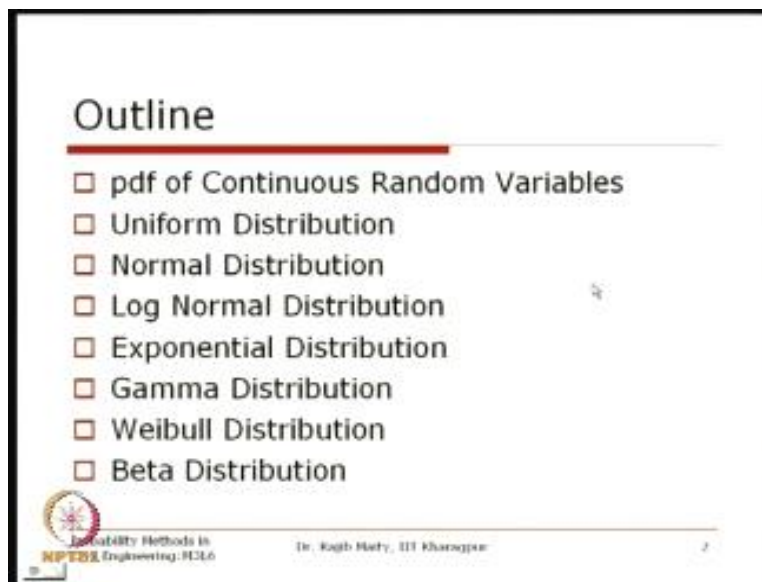
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In this lecture we will cover the probability distribution of continuous random variable. In the last lecture, we covered the discrete random variables. Similarly, for some continuous random variable, there are some standard probability distribution functions are there, which are very widely used in different problems in civil engineering. We will see those distributions maybe in today's class as well as next class we will continue the discussion through different probability distributions for continuous random variables.

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So at the starting we will quickly recapitulate the pdf that is probability density function, you know that for the discrete random variable what we refer is the probability mass function. So the probability density function and their requirement for the, to be a valid pdf for the continuous random variable we will see that very quickly as this was we discussed in the earlier lectures as well.

And after that we will go through the different distribution function. To list it we will start with the uniform distribution, then we will see the normal distribution, log normal distribution, exponential distribution, gamma distribution, weibull distribution and beta distribution. The list

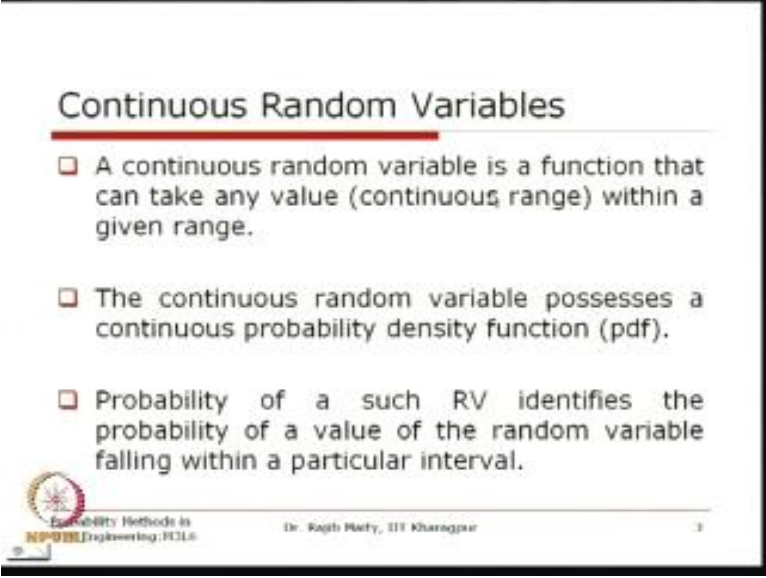
of this distribution may not be exhausted, but these are the distribution, which are generally and widely used for different problems in civil engineering. So we will cover them one after another and we will just show there for which type of problems which distributions will be most suitable and that is generally decided based on their use.

For example, when we talk about the uniform distribution this is generally, bounded from the lower side as well as upper side and in between that the density is uniform. Similarly, the normal distribution is the support of normal distribution is a spanning from the minus infinity to plus infinity. So which is the entire range of the real axis and log normal distribution, exponential distribution and gamma distribution generally, are lower bounded have been some lower bound.

And these bound is generally, at the origin that is 0 and then we will see the weibull distribution, beta distributions this beta distribution again is bounded distribution. And we will see the different possible application of this in different civil engineering problem. In this lecture and as well as, in next lecture that is in this module, what we will discuss, is their basic properties of this distribution and what are their characteristics.


Applications of this any specific distribution to some specific problem of civil engineering we will be discuss, in and the subsequent modules most probably it is in module 5. So we will start with this very brief recapitulation of probability density function for the continuous random variable.

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Continuous Random Variables

- ❑ A continuous random variable is a function that can take any value (continuous range) within a given range.
- ❑ The continuous random variable possesses a continuous probability density function (pdf).
- ❑ Probability of a such RV identifies the probability of a value of the random variable falling within a particular interval.

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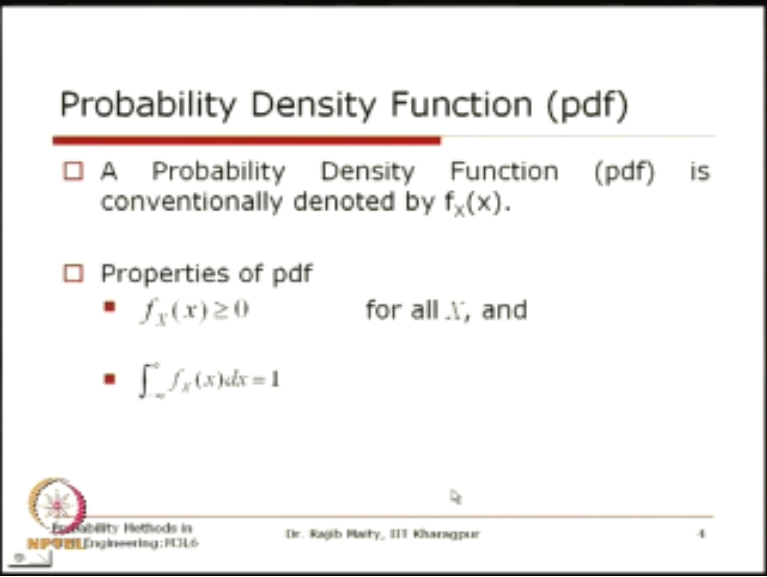
So as we discussed in the earlier classes, earlier lectures that a continuous random variable is a function that can take any value in the sense of the continuous range within the given range of that random variable. The continuous random variable possess a continuous probability density function. So this continuous probability density function we know that over the range of this possible range of this random variable over that range this function is continuous.

And obviously, some properties should be satisfied by this function to be a valid pdf. Now probability of such random variables identifies the probability of a value of the random variable falling within a particular interval. So this is basically a difference between the discrete and the continuous random variable here as we are talking the function as a density functions. So at a particular point, at a particular value of this random variable, if we see the pdf then what is giving is the density.

Now when you are talking about a small interval or small range of this one, then that area below that pdf is the probability. So that is why the probability though what it identifies is the probability of the random variable falling within the particular interval. So one small interval is


needed to define what is the probability for that random variable falling within that particular interval.

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Probability Density Function (pdf)

- A Probability Density Function (pdf) is conventionally denoted by $f_X(x)$.
- Properties of pdf
 - $f_X(x) \geq 0$ for all X , and
 - $\int_{-\infty}^{\infty} f_X(x) dx = 1$

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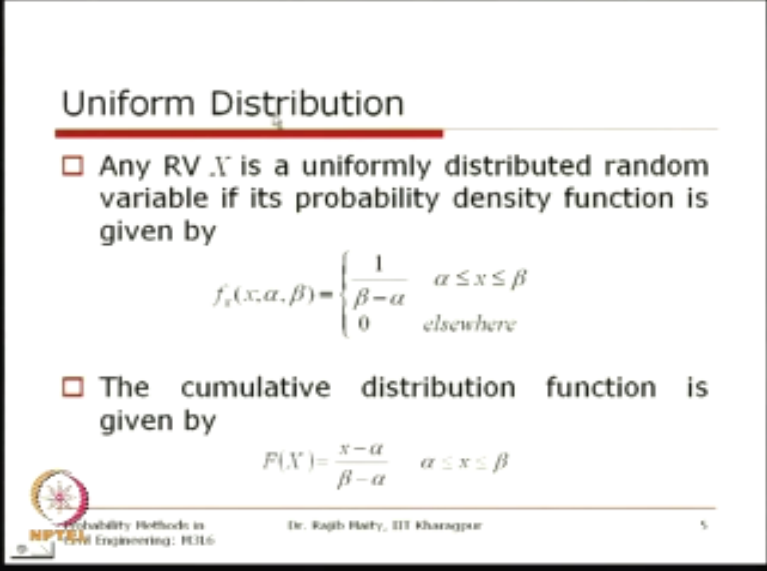
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Then these are the properties that should be followed to be a valid pdf that is a probability density function pdf is conventionally as we have seen earlier that is it is denoted by $f_X(X)$, where this is the X , which is denoting that random variable this is x , which is denoting that dummy variable are the particular value of that random variable. So there are two properties that it should follow we know that at any for all values of all feasible values of this x this value of this function should be greater than equal to 0.


And it should integrate to unity to satisfy the second axiom the total probability within the all possible that feasible over the feasible range of this random variable, which is known as the support of the random variable over this support it should be equal to equal to 1. So with this now we will see, we will go through different probability distribution function.

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Uniform Distribution

- Any RV X is a uniformly distributed random variable if its probability density function is given by
$$f_x(x, \alpha, \beta) = \begin{cases} \frac{1}{\beta - \alpha} & \alpha \leq x \leq \beta \\ 0 & \text{elsewhere} \end{cases}$$
- The cumulative distribution function is given by
$$F(X) = \frac{x - \alpha}{\beta - \alpha} \quad \alpha \leq x \leq \beta$$

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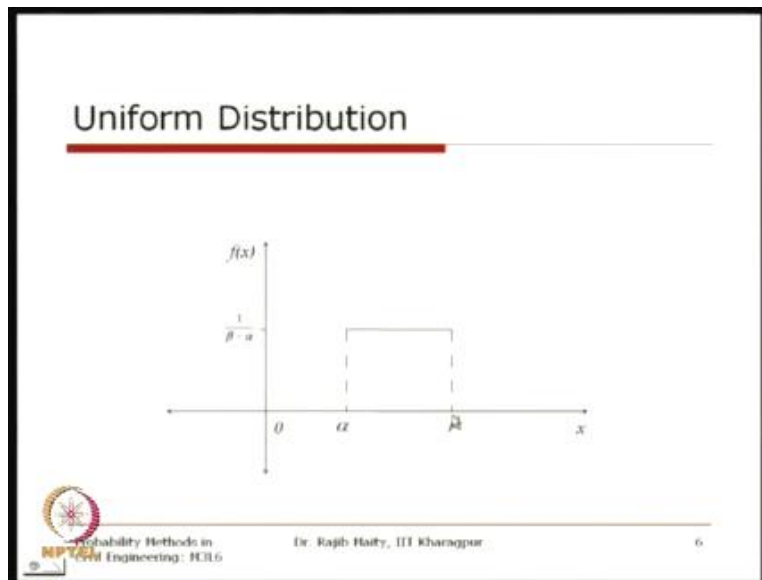
And first we will start with the uniform distribution. So any random variable X is uniformly distributed random variable, if its probability density function is given by this equation that is $F(X)$ this will be $X F(X)$ with this variable x and this $\alpha\beta$ or any such thing, which is shown here are generally, the parameter of the distribution. So here the parameters are α and β . So with this parameter this distribution is expressed by $1/\beta-\alpha$.

For the range when this x lies between α and β that is the lower limit is α and upper limit is β outside this range anywhere, the value of this function is equals to 0. Now if we see this function then before we call that this is a probability density function we have to see that whether the those two properties are followed or not.

So we see that as this β is greater than α so, this quantity is greater than 0 always and in the outside this region this is equal to 0. So the first property that is it should be greater than equal to 0 is satisfied and obviously, this $1/\beta-\alpha$ if we now integrate it over this α to β . So this range from this β to α , which is again this $\beta-\alpha$ this will be equal to 1.

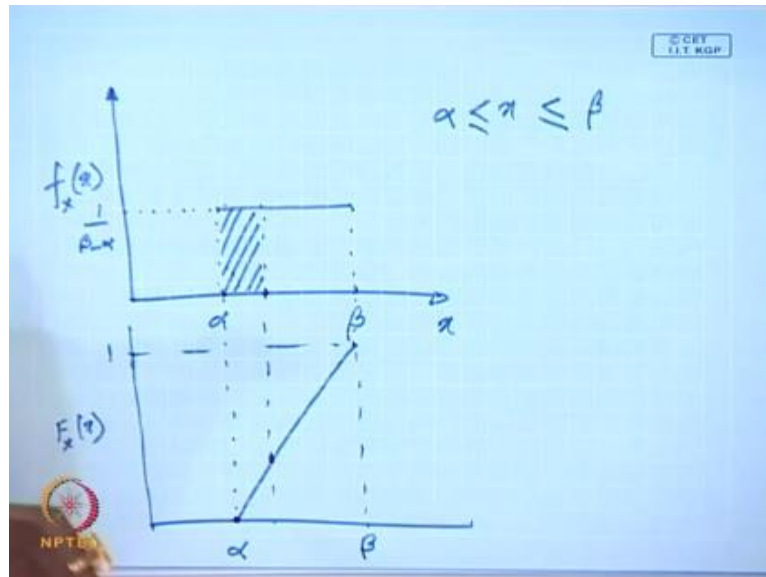
So this is the complete form of this uniformly distributed uniform distribution and we know that if we say that the cumulative distribution function will be given by this $x - \alpha / \beta - \alpha$. Here the diagram or the how this distribution looks like.

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Is shown here this is your α limit and this your β limit now if you just see it once here.

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That now this is what we are calling as this α and this what we are calling as β and now this side is your that value of that of that function $f(x)$ and this is your the x . So, here the possible range is between α and β and this is a close boundary close boundary means that \leq is shown. So this is the close boundary that means it is inclusive of these two values so add this value the probability is 1 by from this point to this point this probability if I have just shown as a indicative line is $1/\beta - \alpha$. so from this range to this range the density is uniform all over this all over this region that is why the name is uniform distribution and just for this reference we are just noting these two lines from this α to β .

Now, when we are talking about what is it is cumulative distribution that means any value with in this range up to the x we have to calculate what is the total area up to that point as we have seen in the earlier description as well from for the CDF. So at this point I have to point that what is the value for that region in this so this point when you're talking about this cumulative distribution function this point implies that the total area covered up to that point so obviously the total area covered at α is nothing 0.

So, this should corresponds to this point and when it is in this way if we just move this x up to this β obviously from this second property it should go it should touch the values of total area will be equals to 1. So this value should come as 1 and as this is uniform this should be a straight lines starting from 0 at α and 1 at β . So this will be your CDF that is cumulative distribution function for uniform distribution. So which is here shown here.


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Uniform Distribution

- Any RV X is a uniformly distributed random variable if its probability density function is given by

$$f_x(x, \alpha, \beta) = \begin{cases} \frac{1}{\beta - \alpha} & \alpha \leq x \leq \beta \\ 0 & \text{elsewhere} \end{cases}$$
- The cumulative distribution function is given by

$$F(X) = \frac{x - \alpha}{\beta - \alpha} \quad \alpha \leq x \leq \beta$$



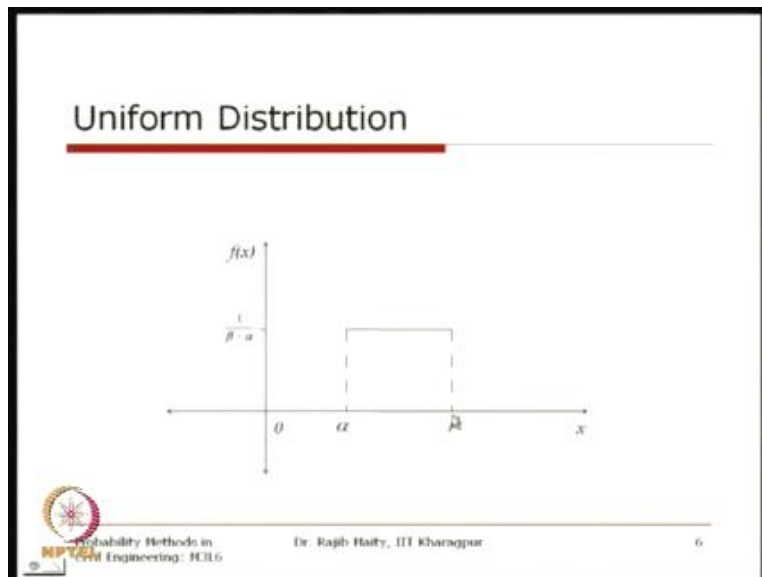
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We can see from this that cumulative distribution function that is $x - \alpha / \beta - \alpha$. Now if you put that x is equals to α here then you are getting it here to be 0 and if you put that x equals to β then you are getting this value as to be your 1 so which is shown in this diagram as well.

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So this is that density that is the density function.

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Mean, Variance and Coefficient of Skewness of Uniform Distribution

- For a uniformly distributed RV X ,
- Mean is given by $\mu = \frac{\alpha + \beta}{2}$
- Variance is given by $\sigma^2 = \frac{(\beta - \alpha)^2}{12}$
- Coefficient of Skewness is zero as the distribution is symmetric about the mean

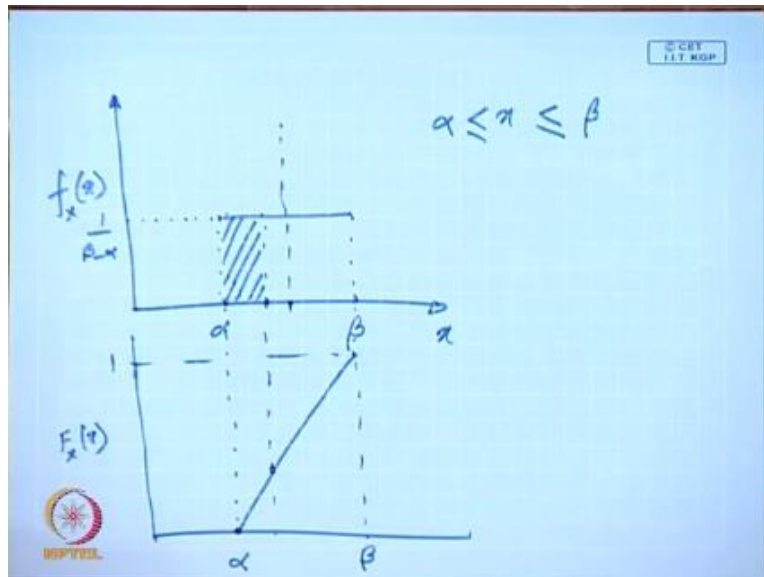
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Now the, from different initial parameters that we have discuss in the earlier lectures that is mean variance and coefficient. If we calculate these things for the uniform distribution is comes as the that is mean is given by μ equals to $\alpha + \beta / 2$ which is obvious from this diagram as well so that mean value of this one as this distribution is as these distribution is uniform so mid mean value should be.

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Equal to should be at the midpoint of this two ranges so that midpoint is nothing but your $\alpha + \beta / 2$ now before coming to this variance if I see the coefficient of Skewness then coefficient of Skewness if we see then we know that which is the measure of the symmetry with respect to the with respect to it is mean. So we see that this is also symmetric with respect to the mean that is why Skewness also will be 0. So this is so mean.

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Mean, Variance and Coefficient of Skewness of Uniform Distribution

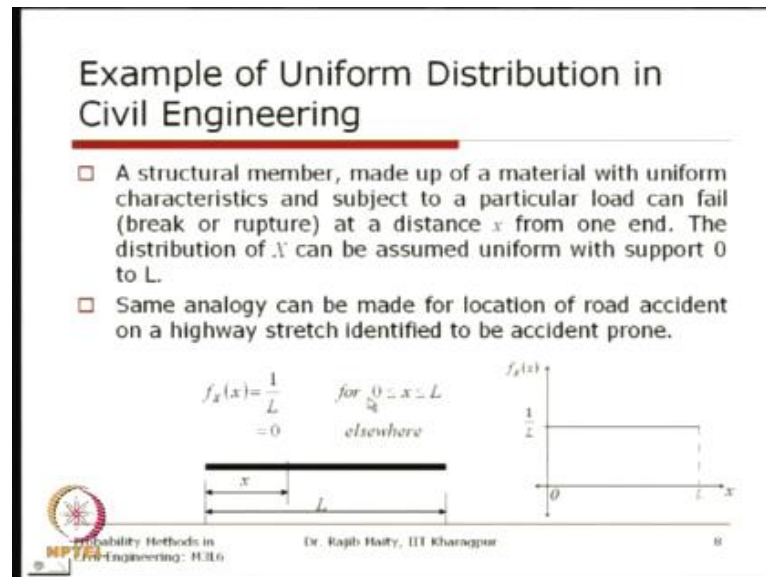
□ For a uniformly distributed RV X ,

- Mean is given by $\mu = \frac{\alpha + \beta}{2}$
- Variance is given by $\sigma^2 = \frac{(\beta - \alpha)^2}{12}$
- Coefficient of Skewness is zero as the distribution is symmetric about the mean



Is given by this average value of this lower and upper bound of that distribution and the coefficient of Skewness is 0 as the distribution is symmetric about the mean and variance also the equation that we use that it should be from the mean and if we take the square and then we multiply it with that probability density function and integrate over the entire support. Then we get that the variance should equals to $\beta - \alpha$ whole square divided by 12. So, this is your variance and obviously Skewness is 0 as discussed.

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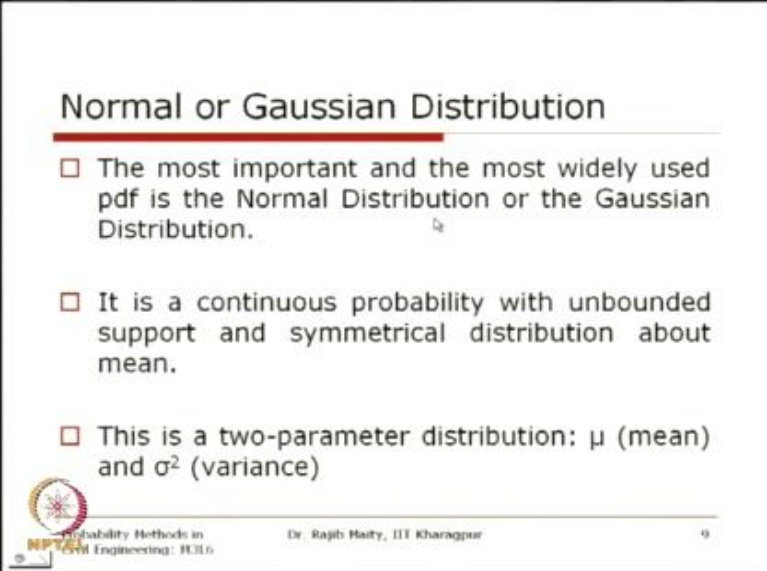
Now any specific civil engineering applications sometimes if we see then if we take the example of a structural member which is made up of the made up of a material which is a uniform characteristics all over and it is subject to a particular loading condition then that structural member can fail failing means it can break or rupture at a distance x from the 1 end. Now the distribution of that X can be assumed to be uniform with the support 0 to L now, if this 1 so this is the structural member now the location of the failure from the 1 end of that member that if it take that particular distance from 1 end is the random variable then it can we can it can happen anywhere.

So the probability can the that distribution of the probability for this random variable X can be assumed to be uniform and this is equals to $1 / L$ so and this is valid from the 0 to the entire length of this structural member. So that is why this is $1 / L$ and it is 0 elsewhere now if we draw it is pdf it looks like this that is from 0 to L it is uniformly distributed which is equals to $1/ L$. Considering the fact that the total area below this curve below this pdf should be equals to 1.

Now, this analogy this example can be analogously can be extended to the other examples like that road accident on a highway stretch identified to be the accident prone now, if the total length


of this road is equals to 0. So that location of this accident at can happen any point over this stretch. So, that is also can be followed as a uniform distribution. Now, we will show to start with we started that a uniform distribution which is easy in the sense from this mathematical concept and we will proceed now to the normal.

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Normal or Gaussian Distribution

- The most important and the most widely used pdf is the Normal Distribution or the Gaussian Distribution.
- It is a continuous probability with unbounded support and symmetrical distribution about mean.
- This is a two-parameter distribution: μ (mean) and σ^2 (variance)

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Or the Gaussian distribution and this is this normal and Gaussian distribution is one of the most important distribution in every field including civil engineering and most many applications has found to be is very useful this normal distribution is useful for many applications in different research field. So that is why this normal or the Gaussian distribution is most popular distribution among the all continuous probability distribution and this is a continuous probability distribution with unbounded support, unbounded support means mathematically it can take the values from $-\infty$ to $+\infty$ and it is symmetrical distribution about the mean. So these two properties are there and this is a two parameter distribution again.

That is the mean is μ variance is σ^2 for the uniform distribution also we have shown that there are two parameters α and β which are the basically the bound for the distribution here also there are

two distribution one is this μ and another one is the σ^2 . We will see how the distribution looks like and it is different properties in the success in this that.

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Normal or Gaussian Distribution ...contd.

□ pdf of a RV, X , having Normal or Gaussian distribution with parameters μ (mean) and σ^2 (variance), is expressed as :

$$f(x, \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad -\infty < x < \infty$$

The pdf results in a bell shaped curve symmetric about the mean.
The cumulative distribution function is given by

$$F_X(x) = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2} dt$$



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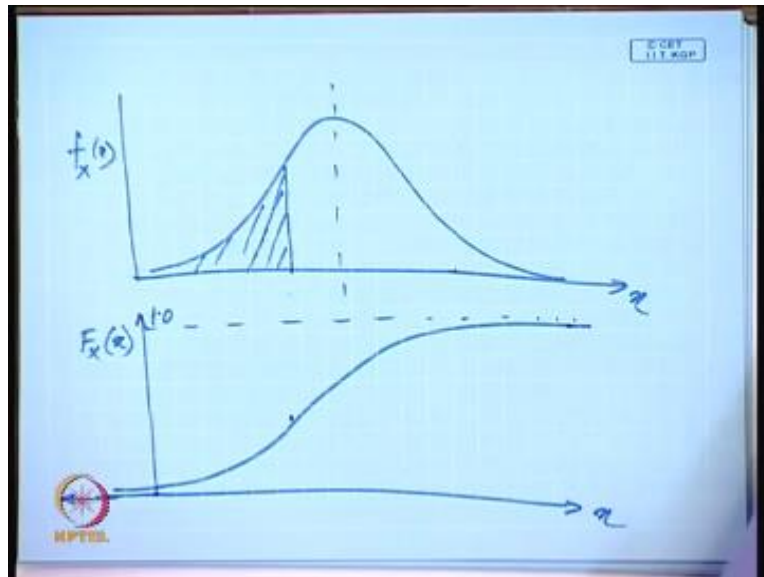
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This is the pdf for this normal distribution so pdf of the random variable X having normal or Gaussian distribution with the parameter μ and other parameter is σ^2 . It is expressed as that $F(x)$ x with this two parameter equals to $1/\sqrt{2\pi}\sigma$ remember that this σ is outside this square root if it is within obviously 1 square will come here multiplied by exponential of $-1/2 (x - \mu/\sigma^2)$ and this support for this x is from $-\infty$ to $+\infty$ as just now discuss.

So this pdf results in a bell shaped curve and which is symmetric about the mean in the earlier cases also if you see that this distribution generally looks like this.

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So this distribution generally looks like this, so this is having its mean and obviously the mean mode and median are same here and this is with respect to this, this is known as the bell shaped curve and this is symmetric with respect to this mean and if you see again if you see it is cumulative density function then obviously we can say that it starts from this $-\infty$ and go up to this $+\infty$.

So if we take that any particular point here and total area if we calculate and put it some value here then this cdf that is which is your pdf and this is your cdf and this generally goes and become again it touch this 1 $\infty, +\infty$ it is asymptotic to this line one and this is asymptotic to the line 0 at $-\infty$. So this is the how this cumulative distribution looks like and this is how the pdf looks like for one normal or Gaussian distribution.

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Normal or Gaussian Distribution
...contd.

□ pdf of a RV, X , having Normal or Gaussian distribution with parameters μ (mean) and σ^2 (variance), is expressed as :

$$f(x, \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad -\infty < x < \infty$$

The pdf results in a bell shaped curve symmetric about the mean.
The cumulative distribution function is given by

$$F_X(x) = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

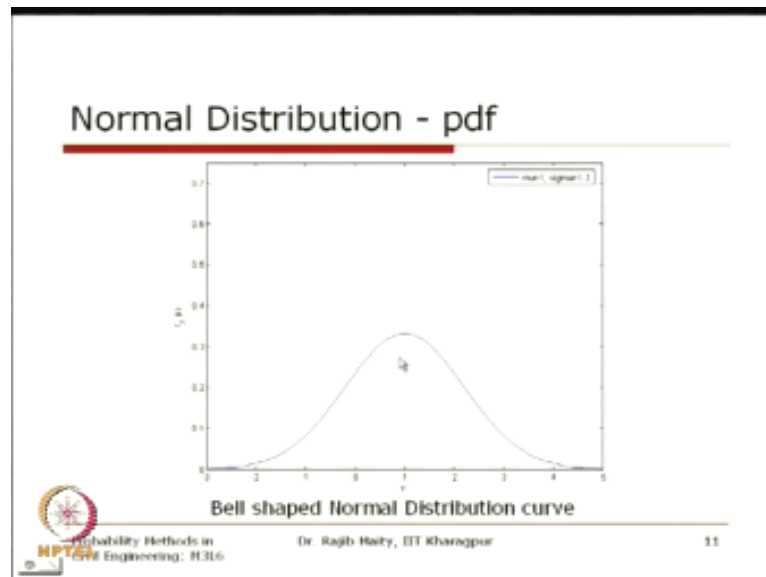
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So mathematically we know that to get that cumulative distribution function we have to integrate from this left hand support that is $-\infty$ to it can go up to x and this integration from this $-\infty$ to x of this pdf this integration of this one from this it will give you the cdf. Now this integration is difficult and we will see that how this is over come through this numerical integration and with the available chart that we will discuss in a minute.

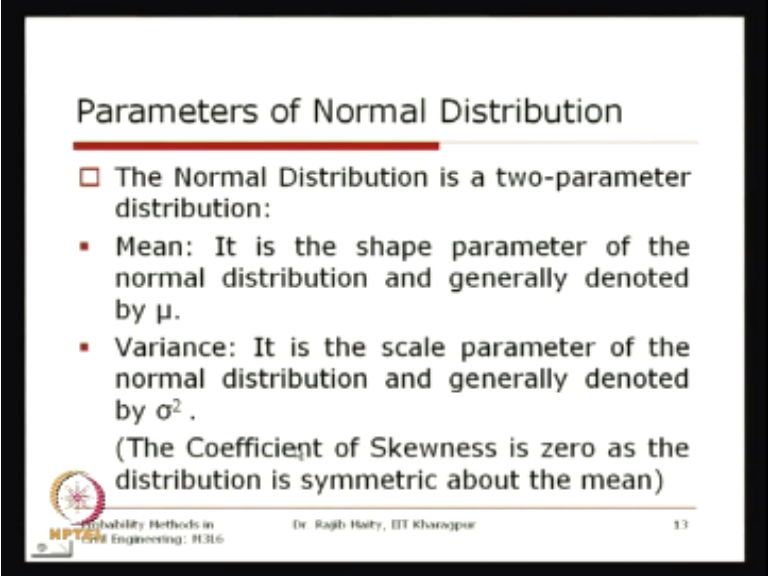
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Before that we will show this the example of this bell shaped curve that it is that we have shown is the this bell shaped curve the parameter for this one this curve is that $\mu = 1$ and $\sigma = 1.2$. So as this parameter $\mu = 1$ you can see that it is the maximum density is at this point 1. So this μ is basically the location parameter where the maximum density is located, so that is simplified by this μ and σ also it is generally showing the spread over this mean.

And we will show this one in the examples how this things can affect the shape of this shape of this curve and this is your for the same pdf with $\mu = 1$ and $\sigma = 2$ if we calculate what should be that cumulative distribution function that is cdf then it looks like this and this line you can see that this is asymptotic to 1 at $+\infty$ and this one towards the left it is the asymptotic to 0 towards $-\infty$.


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Parameters of Normal Distribution

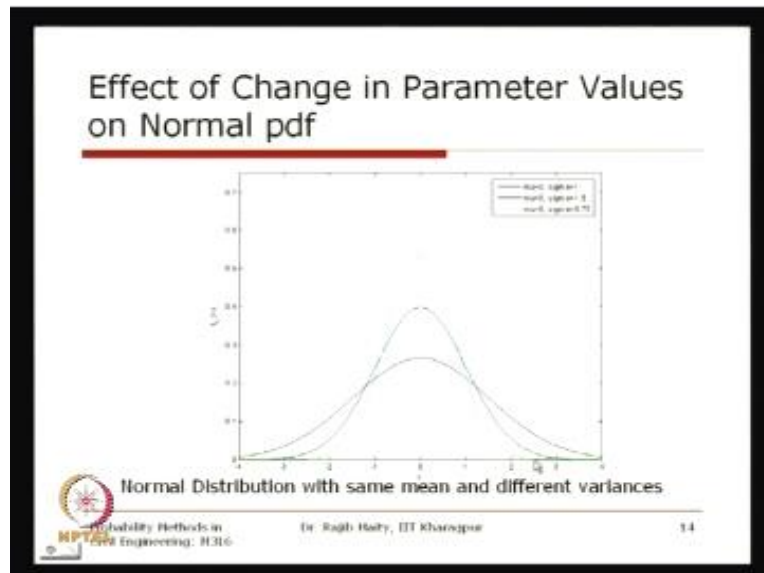
- The Normal Distribution is a two-parameter distribution:
 - Mean: It is the shape parameter of the normal distribution and generally denoted by μ .
 - Variance: It is the scale parameter of the normal distribution and generally denoted by σ^2 .

(The Coefficient of Skewness is zero as the distribution is symmetric about the mean)

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Now this parameters of this normal distribution as we discuss that this normal distribution is a two parameter distribution. So the first parameter is your mean it is the shape parameter of this normal distribution and it generally denoted by this μ and the second parameter is the variance which is the scale parameter of the normal distribution and this is generally denoted by σ^2 . Now the coefficient of Skewness is again 0 similar to the uniform distribution what we discuss earlier this is 0. Because this is this distribution is also symmetric about the mean that is why this Skewness is 0.

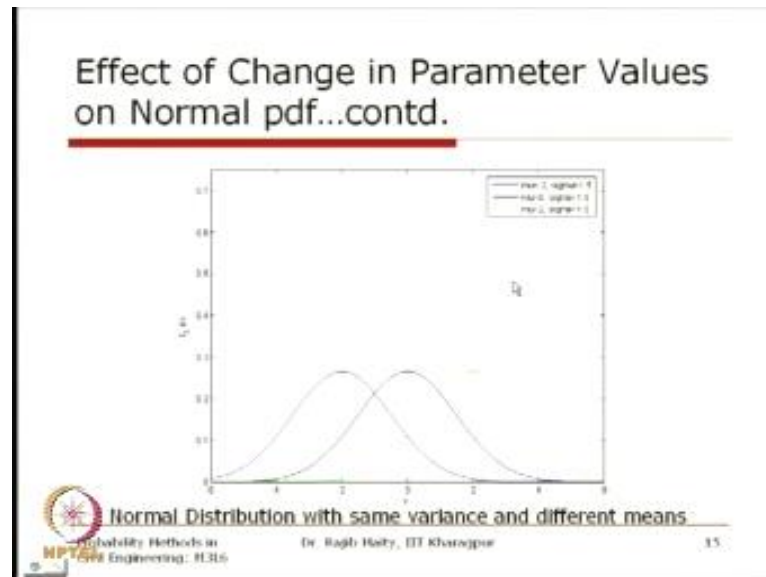
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Now if you see the effect of change of this parameter as we were talking that effect of change in this parameter value on this normal pdf then it looks like this, we have plotted here three different normal distributions the pdf for the normal distribution with different parameters. The this the blue line that is the middle one if you see this is for the this is for the similar value 1.5 for all these three distribution that is shown here having the same mean $\mu = 0$.

So that is why for all this distribution you can see that maximum density is concentrated at $x = 0$. Now this blue curve this blue one is having the sigma value equals to 1.5 whereas this green one is having the sigma value is 0.75 and this black one is having the sigma value is equals to 1. Now you see for this black one the sigma value is the is 1.5 which is the maximum which is maximum here. So this is that is why keeping the mean same for all three distribution, this is more spread the spread is more about it is mean.

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And so it is reflected from it is value of this parameter σ which is 1.5 similarly for this green one the σ value is the minimum and which is 0.75 that is why it is this spread about the mean is the minimum most among this three distribution curve so this sigma generally controls the spread about the mean which is reflected from this background now the effect of change in the parameter value the second parameter now here we are taking this is that a μ .


Now in this three plot again what we have kept same is that σ now for all this three curves $\sigma = 1.5$ now so as this σ is same then you can see the this spread about the spread about the mean the respective mean is same for all three curves now as we have change for this blue one $\mu = -2$ for the black one μ is 0 and for green one $\mu = 2$ so you can see that this is generally shifted from the one location to another location for blue one it is centered at -2 for black one it is at 0 and for green one it is at 2 so that so where this it is centered so that is controlled by this parameter μ .

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Example of Normal Distribution in Civil Engineering

- If a number of concrete cubes, prepared through identical methods and cured under identical circumstances, are tested for their crushing strength, it is observed that their crushing strength is a normally distributed RV.

The crushing strength that is available in at least 95% of the samples is called the 'characteristic strength' or the 95% dependable strength.



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Now if we take some standard example of this normal distribution in civil engineering if a number of concrete cubes so one example is that strength of this concrete if a number of concrete cube prepared through the identical methods and cured under the identical circumstances are tested for their crushing strength it is observed that their crushing strength is a normally distributed random variable now the crushing strength that is available in at least 95% of the sample is called the characteristic strength of the 95% dependable strength.

You know that example of the characteristic strength in the for the strength of the concrete is that it is that in the 95% cases we if we see that the particular strength of that cube is exceeded that is generally denoted by this crushing strength now that how we can say that this is the 95% cases it is exceeded so it is if generally found to follow a normal distribution keeping it is mean and we considered that strength where it should be exceeded at the 95% cases to designate that particular strength to be the characteristics strength of the concrete of the particular concrete.

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Example of Normal Distribution in Civil Engineering

- If a number of concrete cubes, prepared through identical methods and cured under identical circumstances, are tested for their crushing strength, it is observed that their crushing strength is a normally distributed RV.

The crushing strength that is available in at least 95% of the samples is called the 'characteristic strength' or the 95% dependable strength.



So this is how we define that characteristics strength of the concrete cube.

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Additive Properties of Normally Distributed RVs

- If a RV X is normally distributed with parameters μ and σ^2 , then another RV $Y=a+bX$ is also normally distributed with parameters^b $a+b\mu$ and $b^2\sigma^2$.
- If X_i (for $i=1,2,\dots,n$) are n independent and normally distributed RVs with parameters μ_i and σ_i^2 , then the RV $Y=a+b_1X_1+b_2X_2+\dots+b_nX_n$ is normally distributed with parameters

$$\mu_Y = a + \sum_{i=1}^n b_i \mu_i \quad \text{and} \quad \sigma_Y^2 = \sum_{i=1}^n b_i^2 \sigma_i^2$$



Now there are some nice properties of this normally distributed random variable which is known as this additive property if a random variable X is normally distributed with it is parameter μ and σ . We discuss the normal distribution having two parameters μ and σ then if we get another random variable which is related to this earlier one that is X in the through this equation that is $Y = a + bX$ that is the random variable is multiplied by b and added with a then this Y is also a normally this Y .

We will also the normally distribute however it is parameters will change like this that it is the first parameter in case of this μ it will be $a + b\mu$ and it is second parameter which is $\sigma^2 = b^2\sigma^2$ so while getting this new parameter what we are doing is that we are just putting the mean value that is that first parameter value here and getting the mean for this μ random variable and when we are talking about the variance that is the spread around the mean then the constant term that was adding that is not that is not affecting.

But what is affecting is by it is multiplying coefficient and it should be squared so that is $b^2 \times$ by this σ^2 sigma similarly if there are n numbers of such normally distributed random variable and if we can say that these are independent to each other then if you create another new random

variable which is $Y = a + b_1 \text{ multiplied by } X_1 + b_2 \text{ multiplied by } x_2 \text{ similarly up to } b_n \text{ multiplied by } X_n$ then this Y will also be a normally distributed and its parameters will be so we will just put the individual mean of these random variables.

To get the mean for this y which is so the $\mu_y = a + \text{summation of } b_i \mu_i$ so $a + b_1 \mu_1 + b_2 \mu_2$ etc up to $b_n \mu_n$ and for this σ^2 square it should be the coefficient for those random variable that square times their individual variance and their summation up should give this 1 this second property generally leading to the central limit theorem and that we will discuss in the subsequent lectures we will see that we can relax the requirement of this normal distribution for this random variable.

If these are simply if they are independent and identically distributed itself we can say that this y will have this normal distribution which is the result of the central limit theorem will be discussed later and the first property when we are talking about that is the $Y = a + b x$ this y can be treated as the function of x which is the which will be discussed in greater detail in the next module where we will discuss about that functions of random variable.

So while discussing the functions of the random variable we will know if we know the properties that parameters for one this by one random variable how to get the parameters for the or the distribution as well as parameters as well as distribution for it is function so this normal distribution is one example that we have shown here in while discussing the functions of random variable this will be discussing in a general way irrespective of this of any particular distribution of the original random variable.

(Refer Slide Time: 31:33)

Standard Normal Distribution

- The normal distribution having mean $\mu=0$ and variance $\sigma^2=1$ is called the Standard Normal Distribution.

It is given by

$$p_z(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \quad -\infty < z < \infty$$

The cumulative probability for Standard Normal Distribution is given by

$$P(Z \leq z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$$



Now another distribution which is derived from this normal distribution is known as the standard normal distribution this standard normal distribution when a normal distribution is having it is mean $\mu = 0$ and variance $\sigma^2 = 1$ then this particular normal distribution with this specific values of this parameters is known as the standard normal distribution.

(Refer Slide Time: 32:10)


Normal or Gaussian Distribution
...contd.

□ pdf of a RV, X , having Normal or Gaussian distribution with parameters μ (mean) and σ^2 (variance), is expressed as :

$$f(x, \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad -\infty < x < \infty$$

The pdf results in a bell shaped curve symmetric about the mean.
The cumulative distribution function is given by

$$F_X(x) = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

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So, in this original distributional form that is what we have shown it earlier that this distribution, if you just put $\mu=0$ and $\sigma=1$ what the distribution form we will get that will be the standard normal distribution instead of using x there we are using z as the dummy variable.

(Refer Slide Time: 32:33)

Standard Normal Distribution


□ The normal distribution having mean $\mu=0$ and variance $\sigma^2=1$ is called the Standard Normal Distribution.

It is given by

$$p_z(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \quad -\infty < z < \infty$$

The cumulative probability for Standard Normal Distribution is given by

$$P(Z \leq z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$$



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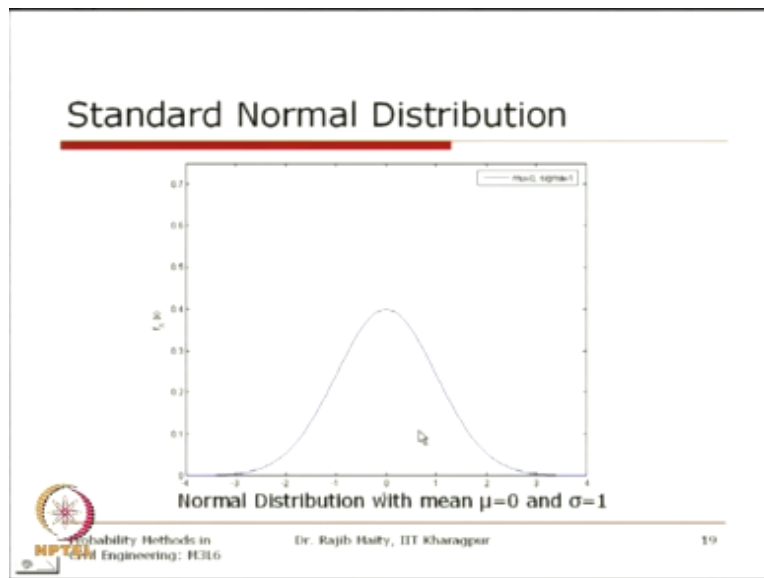
So, this is your standard normal distribution to continue with our same notation this is also as continuous instead of p this will be f and this is the Z and this is the z . So, this is specific value and this is the random variable. So, whose distribution is $1/\sqrt{2\pi} e^{-z^2/2}$ and obviously, the z is having the support from $-\infty$ to $+\infty$. So, this is also normal distribution, which is having the mean μ and variance 1. The cumulative distribution for this standard normal distribution again, can be found out from integrating it from this left support to the specific value z , which is $1/\sqrt{2\pi} e^{z^2/2}$.

So, integrating from this left support to the particular value x and this is also will be the F and this Z less than equal space particular value z , which is giving you the cumulative probability distribution. Now, we should know, what is the use of this distribution and why we need this specific distribution with this specific parameter is that this integration whenever we are talking about this integration, which is important to calculate the probability it is not this integration cannot be done in the close form.

So, we have to go for some numerical integration and the numerically integrated values are available for this standard normal distribution. Now, for any general normal distribution we can

convert it first to the standard normal distribution and get its desired probability and that we will discuss in a minute.

(Refer Slide Time: 34:24)




So, before that this is how this distribution that is pdf probability density function looks like and as you know that it is $\mu=0$ and $\sigma=1$ so, that is why it is centered at 0. Basically, its support is again that from $-\infty$ to $+\infty$ the, but one thing you can see that most of that probabilities is concentrated between -3 to +3.

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Utility of Standard Normal Distribution

- If a RV X is normally distributed with mean μ and variance σ^2 , then the probability of X being less than or equal to a x is given by
$$P(X \leq x) = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$
- As the above integration cannot be evaluated analytically, numerically computed values are tabulated (taking $\mu=0$ and $\sigma^2=1$) in the standard normal table. For normal distributions with $\mu \neq 0$ and $\sigma^2 \neq 1$, the cumulative probability is determined by converting the RV X to the reduced variate $Z = \frac{X - \mu}{\sigma}$



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Now, while the thing that we are discussing why we need this standard normal distribution is as follows, if a random variable X is normally, distributed with mean μ and the variance σ^2 then the probability of X being less than or equal to x is given by this particular distribution. Now, as the above integration cannot be evaluated analytically as we discuss just now the numerically computed values are tabulated taking this $\mu=0$ and $\sigma^2=1$ that is the standard normal distribution.

So, for the standard normal distribution these values are numerically computed and listed. Now, for all other normal distribution with any values of this parameter that this if this μ is not equal to 0 and this σ^2 is not equal to 1, then the cumulative probabilities can be determined by converting this x to it is reduce variate. So, how it is converted that is this X that particular random variable is deducted from it is mean that is the whatever the value mean is there and it is divided by σ . So, we get in new random variable which is Z . Now, we have seen just now in a minute we have seen that.

(Refer Slide Time: 36:33)

The image shows a handwritten derivation on a blue background. At the top, the equation $Y = a + bX$ is written. Below it, two arrows point from the terms $a + b\mu$ and $b^2\sigma^2$ to the a and b in the equation above. In the center, the equation $Z = \frac{X - \mu}{\sigma} = -\frac{\mu}{\sigma} + \frac{1}{\sigma}X$ is boxed. Below this, the coefficients are identified as $a = -\frac{\mu}{\sigma}$ and $b = \frac{1}{\sigma}$. Then, the mean is calculated: $a + b\mu = -\frac{\mu}{\sigma} + \frac{1}{\sigma}\mu = 0$. Finally, the variance is calculated: $b^2\sigma^2 = \frac{1}{\sigma^2} \cdot \sigma^2 = 1$. There are small logos in the bottom left and top right corners of the slide.

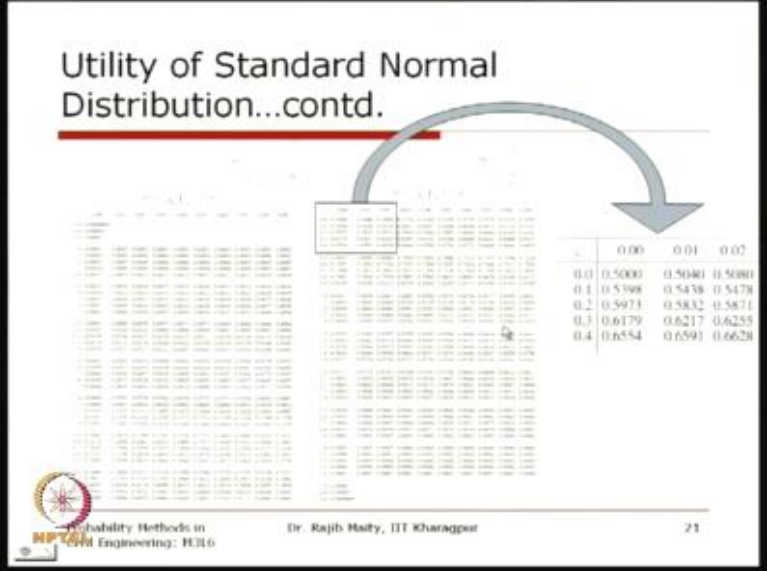
$$Y = a + bX$$
$$a + b\mu \quad b^2\sigma^2$$
$$\boxed{Z = \frac{X - \mu}{\sigma} = -\frac{\mu}{\sigma} + \frac{1}{\sigma}X}$$
$$a = -\frac{\mu}{\sigma} \quad b = \frac{1}{\sigma}$$
$$a + b\mu = -\frac{\mu}{\sigma} + \frac{1}{\sigma}\mu = 0$$
$$b^2\sigma^2 = \frac{1}{\sigma^2} \cdot \sigma^2 = 1$$

If that $Y = a + bX$ just few earlier, if this is normal distribution now we have seen that it is mean the mean of this Y is $a + b\mu$ and it is variance is b^2 sorry $b^2\sigma^2$. Now, what we are doing here the conversion is thus $z = (x - \mu)/\sigma$, if I just write in that form that is $\mu/\sigma + 1/\sigma x$ so, here we have $a = -\mu/\sigma$ and here $b = 1/\sigma$. So, that the mean for this z will be $a + b\mu$, if I take it from here $a + b\mu$ and if I put this value is equals to $-\mu/\sigma +$ this 1 by this b is $1/\sigma$ μ multiplied by μ , which is 0 and for this variance for this z is $b^2\sigma^2$, which is again this b^2 is $1/\sigma^2$ multiplied by $\sigma^2 = 1$.

So, we have converted it in such a way that is the mean for this new random variable is 0 and variance for this new random variable becomes 1. So, through this conversion of any normally, distributed random variable we can generate a another new random variable, which is again, normal distribution with mean 0 and standard deviation 1, and this 1 this conversion is irrespective of any specific value of μ and σ that original random variable is having.

(Refer Slide Time: 38:42)

Utility of Standard Normal Distribution...contd.



z	0.00	0.01	0.02
0.0	0.5000	0.5040	0.5080
0.1	0.5398	0.5438	0.5478
0.2	0.5793	0.5832	0.5871
0.3	0.6179	0.6217	0.6255
0.4	0.6554	0.6591	0.6628

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So, once we can convert this one then we can use this standard normal distribution chart, which is generally looks like this. So, this is your that pdf of this normal distribution and this cumulative values are listed as follows that is for any specific value of this z this is basically started from -5 and goes like this and coming and going up to 5 and we know that from this from this effectively from -3 to +3 itself most of the probabilities are exhausted.


So, if I just zoom it here then you can see for the value when this z value is at 0 that is adjust at thus at the mean we know that the total probability covered the property of a symmetry should be equals to 0.5, which is shown here. Now, as we are for any value suppose that if I take that point 0.21. So, this is your point 2 and this is a second decimal 0.21. So, upto point z equals to up to 0.21 the probability covered is 0.5832.

Now, we will see one example, how to calculate the probability for the for any normal any normal distribution that we will see. So this is how we have to read this standard normal distribution and these tables are generally available in any standard text book.

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Some notes on Normal Distribution

- From the standard normal distribution curve, it can be observed that 99.74% of the area under the curve falls inside the region bounded by $\pm 3\sigma$.
This is particularly important in real life scenarios where a RV may be bounded by $X=0$ but can still be considered to be normally distributed if $\mu > 3\sigma$.



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So what we have found from this distribution is that from the standard normal distribution curve it can be observed that this 99.74 percent of the area under the curve falls inside the region bounded by $\pm 3\sigma$, so for the standard normal distribution what we saw that $\sigma=1$ so from this -3 to +3. So this much this much probability which is almost closed one is already covered in that this is particularly important in the real life scenario.

Where a random variable may be bounded by $X = 0$, but can still considered to be normally, distributed if $\mu > 3\sigma$, so in the real life sometimes we can come across to the situation that those random variables are effectively a lower bounded by 0, but if we see that it is mean is away from the origin with a magnitude of 3σ . Then we can once see that whether, that can also be considered to be a normal distribution as we know that below this 0 means left side of this 0. So towards a negative value effectively the probability is 0.

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Probability Calculation

- Let us consider a RV X that is normally distributed with $\mu=4.35$ and $\sigma=0.59$. Now, the probability that the RV will take a value between 4 and 5 can be conveniently calculated from the cumulative standard normal probability table.

$$\begin{aligned} P(4 \leq X \leq 5) &= F\left(\frac{5 - 4.35}{0.59}\right) - F\left(\frac{4 - 4.35}{0.59}\right) = F(1.1) - F(-0.59) \\ &= 0.8643 - 0.2776 \\ &= 0.5867 \end{aligned}$$



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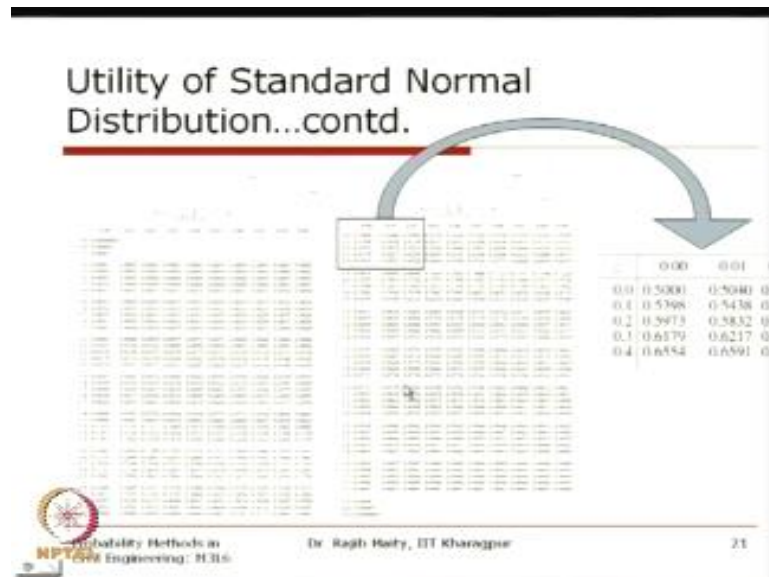
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Now if we just want to do one small exercise how to calculate the probability using this standard normal distribution let us consider a random variable X that is normally distributed and once we say that this is normal distributed we have this specified this parameter. So with $\mu = 4.35$ and $\sigma = 0.59$, so now if we look for the probability that this random variable will take a value between 4 and 5 then this can be calculated from this cumulative get standard normal probability table how.

So our intention to the probability of X line between 5 to 4 should be equals to first what we are doing is that we are we are converting this two limit that is the 5 and 4 this we are converting to it is reduced variate. So how to convert it to the reduced variate that particular value minus it is μ / σ so if we are we reduce it the upper that upper limit corresponding reduced variate lower limit corresponding reduced variate.

So this is again $4 - \mu / \sigma$ now this value ends up to the 1.1 and this is -0.59. Now this one as these are reduced variate. So this is having the $\mu = 0$ and variance is equals to 1. So this we can read from this standard normal distribution for 1.1.

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So if you refer to this chart then we can see that what this 1.1. So this cell if we can read it that is 0.4838. So what is meant is that starting from here up to 1.1 total areas covered is 0.8438.

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Probability Calculation

- Let us consider a RV X that is normally distributed with $\mu=4.35$ and $\sigma=0.59$. Now, the probability that the RV will take a value between 4 and 5 can be conveniently calculated from the cumulative standard normal probability table.

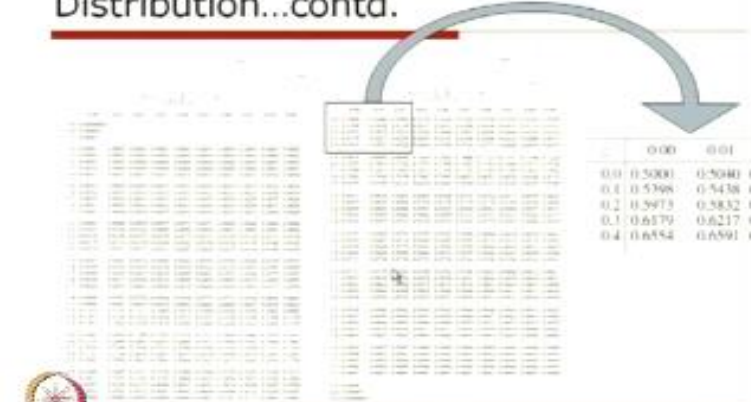
$$\begin{aligned} P(4 \leq X \leq 5) &= F\left(\frac{5 - 4.35}{0.59}\right) - F\left(\frac{4 - 4.35}{0.59}\right) = F(1.1) - F(-0.59) \\ &= 0.8643 - 0.2776 \\ &= 0.5867 \end{aligned}$$



So this is 86 4 3.

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Utility of Standard Normal Distribution...contd.



The diagram illustrates the utility of the standard normal distribution table. A large table is shown with a curved arrow pointing from a specific cell to a detailed view of that cell's data. The detailed view shows the following data:

	0.00	0.01	0
0.0	0.5000	0.5040	0.5
0.1	0.5398	0.5438	0.5
0.2	0.5987	0.5832	0.5
0.3	0.6179	0.6217	0.6
0.4	0.6554	0.6591	0.6

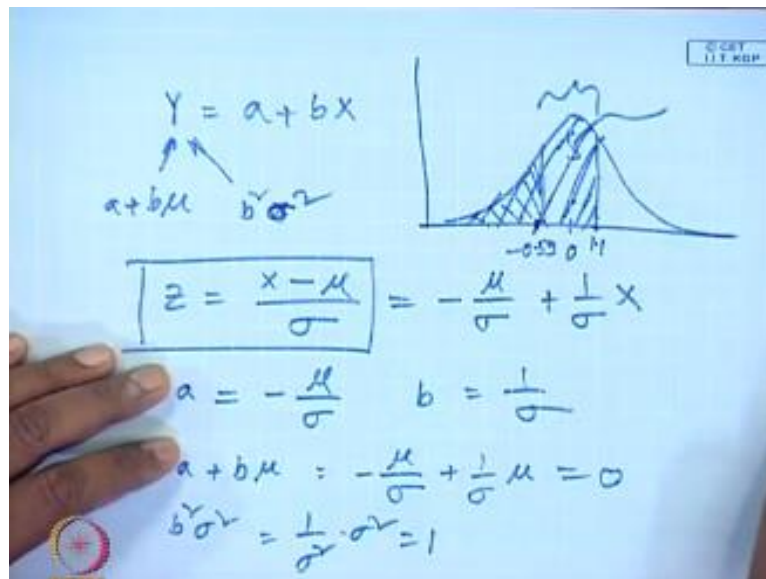
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Maybe this will be 8 4. So, this might be a mistake that is this might be that 8 4. So, this is how we read this probability for a particular value of the standard normal distribution. And similarly we can read this value for the other limit and we can deduct this probability to get this value. So what is actually here is done graphically is like this.

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So first of all this is reduced a variate so if this is your 0 then the reduced variate is looks like this now from the 1.1 what is the value we get from this table is the total area from this from the left support to that 1.1. So this total area that is the total probability we will get from that standard normal distribution table and for the left one is the minus 0.59 may be somewhere here minus 0.59.

So what you are doing that up to this much what is this area that we are deducting. So that we will get what is there in this area only so what is the total area between this two limit we will get. So the total area up to this one minus total area up to this point to calculate the probability between this to limits.

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Probability Calculation

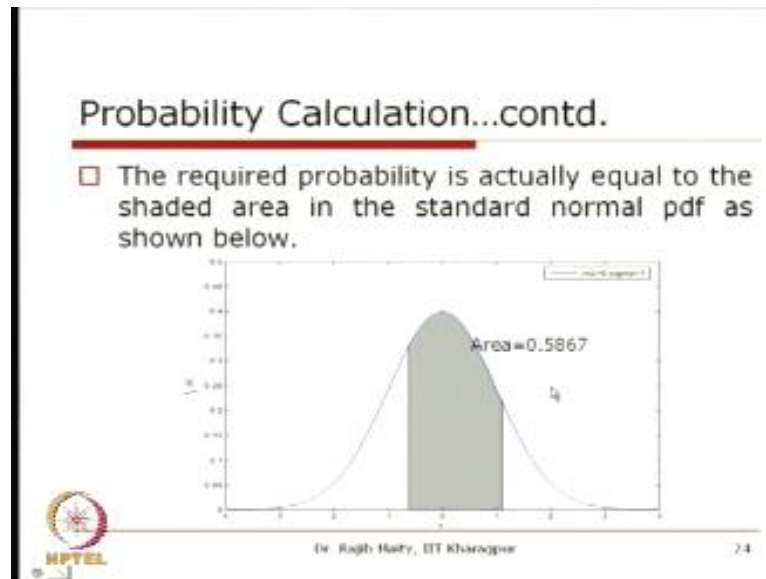
- Let us consider a RV X that is normally distributed with $\mu=4.35$ and $\sigma=0.59$. Now, the probability that the RV will take a value between 4 and 5 can be conveniently calculated from the cumulative standard normal probability table.

$$\begin{aligned} P(4 \leq X \leq 5) &= F\left(\frac{5 - 4.35}{0.59}\right) - F\left(\frac{4 - 4.35}{0.59}\right) = F(1.1) - F(-0.59) \\ &= 0.8643 - 0.2776 \\ &= 0.5867 \end{aligned}$$



So this is exactly is done here first of all we have converted to the reduced variate, which is 1.1 another1 is this point -0.59 this 5 corresponds to 1.1 and this 4 corresponds to 0.59 and this two values are taken from this standard normal distribution table.

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So here this graphical repartition is shown here again that is 1.1 this area. So up to this that value which is indicating is the total area from this minus infinity to this point. So that is why we have do to get only this much area we have to deduct the area, which is which is up to that point of - 0.59. So, this area should be deducted to get the area in between these two limits.

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Log Normal Distribution or Galton Distribution

- Any RV X is a Log Normal random variable if its probability density function is given by
$$f(x; \mu, \sigma^2) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2} \left(\frac{\ln x - \mu}{\sigma}\right)^2} \quad -\infty < x < \infty$$
- The cumulative distribution function is given by
$$F(x; \mu, \sigma^2) = \Phi\left(\frac{\ln x - \mu}{\sigma}\right)$$
where Φ = standard normal CDF
- If the RV X is log normally distributed, then the RV $Y = \log(X)$ is normally distributed.

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Next distribution that we are going to discuss is the log normal distribution this also known as Galton distribution any random variable X is a log normal random variable, if its probability density function is given by as follows. Again it is having two parameter one is μ and this σ^2 is $1 / x \sigma^2 \sqrt{2\pi}$ exponential $-1/2$ in bracket $\log \text{ natural } X - \mu / \sigma^2$. And it is limit for the x is from so this limit is this is a mistake this limit from the 0 to infinity.

So why this is important and what is the difference between this normal distribution is that for the normal distribution need varies from minus infinity to plus infinity, but this log normal distribution support is from 0 to infinity. Basically what we are doing again is that we are just taking that one random variable we are taking it is log and it is related through this equation that is Y is equals to $\log x$.

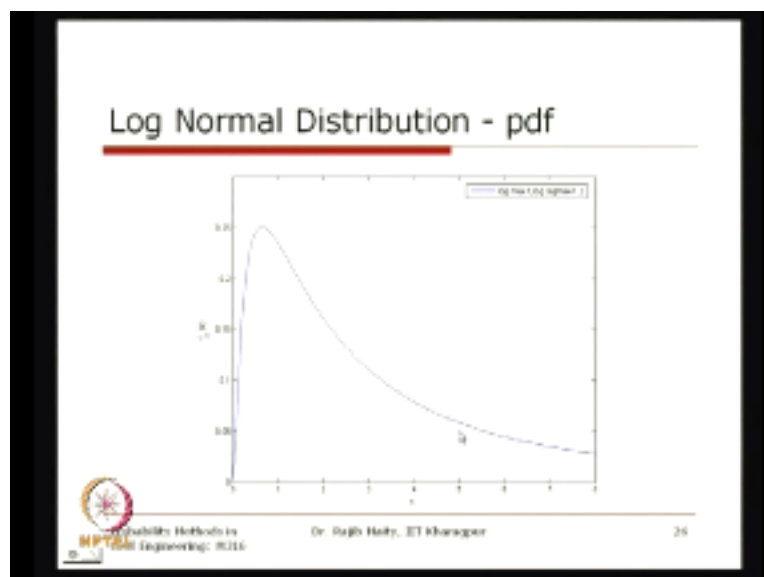
And now what is shown here is that if this random variable X is log normally, distributed then that Y , if I take that $\log X$ is the normally distributed. So, if we come across with any distribution, which is log normally, distributed us, we can take it is log and convert it to the normal distribution and again, we can use the same procedure, which is followed for the normal

distribution. Now so as it is related to this as the lognormal distribution is related to the normal distribution through this functional transformation this will again, be clear in the next module where we are discussing about the functional random variable.

So, if this distribution is known or if this normal distribution is known, what is the distribution for this x and through that one it can be usually, shown that this distribution of this x is following is having the form like this, this will be 0 this is not $-\infty$ this is 0. And again, that cumulative distribution function is given by this one where this is your that that cumulative distribution of this normal distribution and $\ln x$ minus μ by sigma. Basically, if you see this μ and sigma that is the parameter of this log normal distribution this μ is the mean of the variable X after taking it is log.

So, after taking the log of X if you calculate the mean then that is equals to it is parameter μ and similarly, if you take the variance this is this sigma square.

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Ah this is how a log normal distribution with this log μ is equals to 1 and log sigma equals to 0.12 looks like. So, this is lower bounded by 0, and goes up to plus infinity that, which is the

support for this log normal distribution. And similarly, this is here cumulative distribution function for the log normal distribution.

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Mean, Variance, Coefficient of Variation and Skewness of Log Normal Distribution

- Mean is given by:


$$\mu_X = E(X) = e^{\mu + \frac{\sigma^2}{2}}$$
- Variance is given by:

$$\sigma_X^2 = Var(X) = \mu_X^2 (e^{\sigma^2} - 1)$$
- Coefficient of Variation is given by:

$$C_{v,X} = \sqrt{e^{\sigma^2} - 1}$$
- Coefficient of Skewness is given by:

$$\gamma_X = 3C_{v,X} + C_{v,X}^3$$

The distribution is positively skewed and with decrease of the coefficient of variation, the skewness also decreases.

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Now, this mean of this log normal distribution is expectation of this X, which can be shown that $e^{\mu + \frac{\sigma^2}{2}}$ by sigma square that is this is the mean for that variable X. And this μ this is the mean after taking the log of this of the distribution of x, if you calculate the mean this is that mean, which was discussed. Again, the variance of this distribution that is variance of x is equals to $\mu_X^2 (e^{\sigma^2} - 1)$ now this subscript is shown as this is for this μ for this x this one that square multiplied by exponential sigma square minus 1.

Coefficient of variance is again, given by the C_v of this x that is the random variable X is equals to square root of $e^{\sigma^2} - 1$. And coefficient of Skewers is given by for this γ_X is equals to $3C_{v,X} + C_{v,X}^3$. The distribution is positively skewed and with decrease of the coefficient of variation the Skewers also decreases which can be reflected from this equation.

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Log Normal Distribution - Sample Statistics for $Y = \log(X)$

- For a log normally distributed RV X , the sample statistics for $Y = \log(X)$ can be obtained as follows:
- The sample mean for $Y = \log(X)$ is given by:

$$\bar{Y} = \frac{1}{2} \ln \left(\frac{\bar{X}^2}{1 + C_{v,X}^2} \right)$$
- The sample variance for $Y = \log(X)$ is given by:

$$S_Y^2 = \ln(1 + C_{v,X}^2)$$

where $C_{v,X} = \frac{S_X}{\bar{X}}$



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Now, for a log normally, distributed random variable X the sample statistics for Y equals to $\log X$ that is we have some observation some sample data we have taken it is \log , if we take that \log then how we can get the mean of that converted random variable. This is obtained through this equation that is \bar{Y} after taking the \log it is mean is equals to $\frac{1}{2} \ln \bar{X}^2$ divided by $1 + C_{v,X}^2$.

So, this C_v is the coefficient of variance for that for that observation x and this \bar{x} is the mean for that particular observation of the observed value, which is square. The sample variance for the Y equals to $\log X$ again this 1 is equals to s_y^2 equals to $\ln(1 + C_{v,X}^2)$. So, this is the coefficient of variation with square plus 1 take the \log get that sample variance of y . This coefficient of variation as we know is the ratio of this standard deviation of the X divided by mean of that X .

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Example of Log Normal Distribution in Civil Engineering

- In Civil Engineering, distribution of annual river flow data may follow log normal distribution.
- The streamflow values are always greater than zero and the probability density for extremely low streamflows (observed only during droughts) is quite less.
- The probability densities are high for moderate values of streamflows (observed through the greater part of the year) and decreases progressively for increasing streamflows.
- The probability for extremely high streamflow (observed only during severe floods) is very less.



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In civil engineering the distribution of annual river flow data may follow the log normal distribution the stream flow values generally, are greater than 0 that is it is lower bound is 0 and the probability density for extremely low stream flows are quite less. This is; obviously, for some kind of big rivers and if you see that it is generally having some contribution from this ground flow or the snow-fed rivers like that.

So, generally, in some flow is maintained throughout the year. So, the it is so, that probability is low for the extremely low flows then the probability densities are increases with increase increasing amount of this annual flow for the moderate values of the stream flow and again, it decreases progressively for the increasing stream flow. So, the probability for the extremely high stream flow again, is very less.

So, what we can see is that it is starts from 0 take the peak and again, it is coming down. So, far as the density of the probability is concern over the range of this annual river flow. So t ,this can follow we cannot confirm it and we cannot say it as a general case that all the annual rivers are always follow the log normal distribution hat even though we cannot say that, but there is a possibility that this may follow a log normal distribution.

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Exponential Distribution

- ❑ It is the probability distribution that describes the time between events in an experiment where outcomes occur continuously and independently at a constant average rate.
- ❑ It is generally used as a decay function in engineering problems.



Next, we will discuss, that exponential distribution it is the probability distribution that describe the time between events in an experiment where the outcomes occur continuously and independently at a constant average rate. It is generally, used as a decay function in the engineering problems.

(Refer Slide Time: 54:40)

Exponential Distribution...contd.

- A random variable X is an exponential RV with parameter λ , if its probability density function is given by

$$f_x(x, \lambda) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

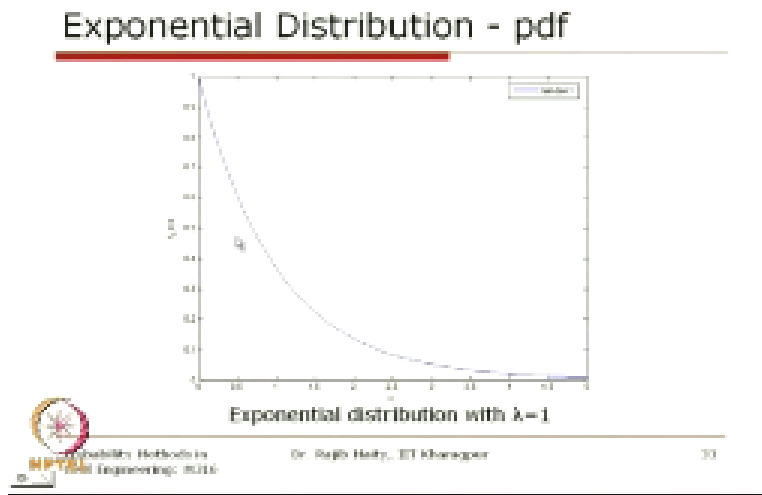
The cumulative distribution function is given by

$$F_x(x, \lambda) = \begin{cases} 1 - e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$



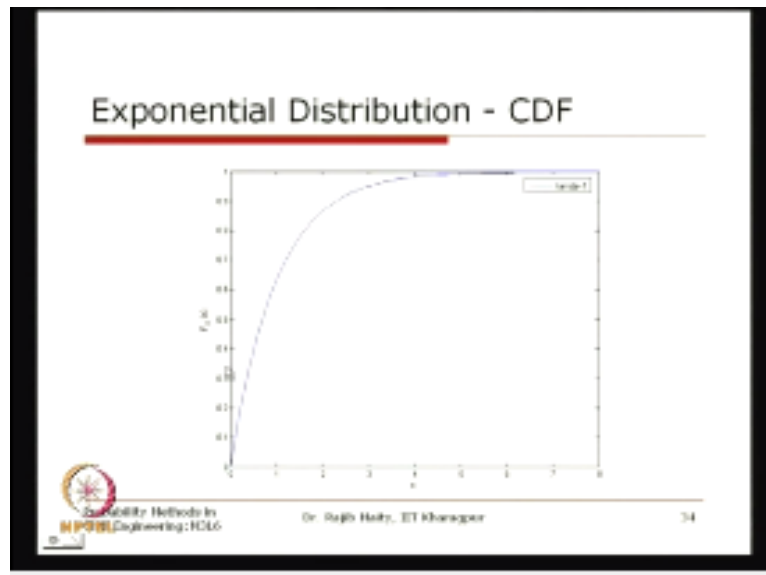
So, this distribution is a mathematically very effective to show and in the previous lectures also we have taken this example, to show it is different properties of the continuous random variable. And we know that its probability density function looks like the $\lambda e^{-\lambda x}$ for this x greater than equal to 0 and elsewhere it is 0. And here it is a single parameter distribution and the parameter is λ and if we take we in the earlier lectures also we have seen that this cumulative distribution function is can be shown as the $1 - e^{-\lambda x}$.

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And it is distribution that is pdf that is probability density function with the particular value of λ equals to 1 taken here is looks like this.

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And it is cumulative distribution again, looks like this, which is starting from 0 and going asymptotic to 1 at x equals to ∞ .

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Mean and Variance of Exponential Distribution

□ For an exponentially distributed RV X ,

■ Mean is given by

$$E[X] = \frac{1}{\lambda}$$

■ Variance is given by

$$\text{Var}[X] = \frac{1}{\lambda^2}$$

■ Skewness is 2



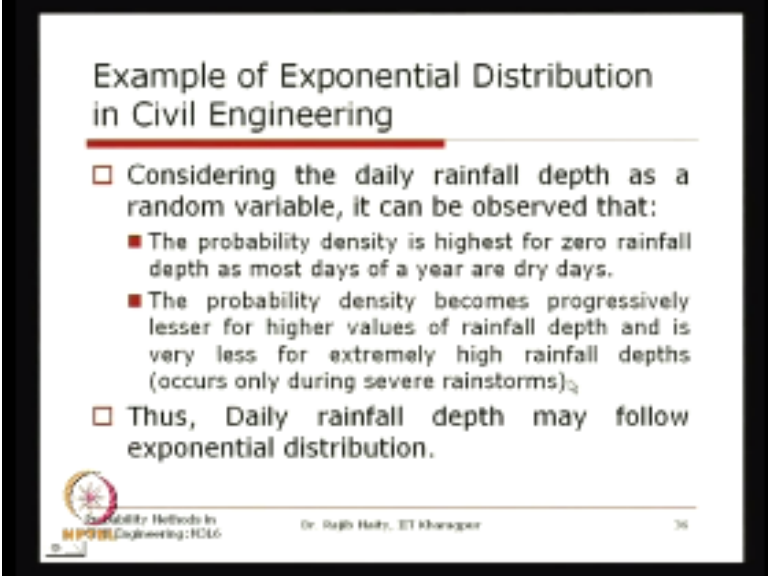
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And this things also we calculate earlier it is mean is given by expectation of X is given by $1/\lambda$ and variance is $1/\lambda^2$. It is Skewness can be shown is equals to 2 and coefficient of variance also is shown it earlier and which is equals to 1 in earlier lecture we covered this distribution as example.

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Example of Exponential Distribution in Civil Engineering

- Considering the daily rainfall depth as a random variable, it can be observed that:
 - The probability density is highest for zero rainfall depth as most days of a year are dry days.
 - The probability density becomes progressively lesser for higher values of rainfall depth and is very less for extremely high rainfall depths (occurs only during severe rainstorms).
- Thus, Daily rainfall depth may follow exponential distribution.

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So, if we take that daily rainfall depth and the probability density is highest for this 0 rainfall we know that most of the days, if in case, the most of the days are dry days then it is the maximum probability is concentrated as zero.

The probability density becomes progressively lesser for the higher values of the rainfall depth and it is very less for the extremely high rainfall depths. Thus this may follow again, it should now may should be follow for daily rainfall depth may follow and exponential distribution and with this we stop with this exponential distribution here.

So, in this lecture we cover that uniform distribution, normal distribution, log normal distribution and exponential distribution there are some more distributions are also important. And that we will cover in the next class, and then we will go through the next module and in successive modules application of this kind of distribution indifferent civil engineering problem for different modules will be explained later. Thank you.

Probability Methods in Civil Engineering

End of Lecture 11

**Next: “Probability Distribution of Continuous
RVs (Contd.)” In Lec 12**

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