**Strength of Materials**
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**Lecture no 21**
**Lecture Title: Torsion-IV**

Welcome to the 4th lesson of the 4th module which is on torsion part 4. In the previous 3 lessons on torsion, we have seen several examples of what happens to a bar when it is subjected to a twisting moment and this twisting moment could be in a bar which is of solid circular shaft or could be in a bar which has a cross section which is tubular in form. That means there is hollowness in the shaft. We have also seen how the stresses and the deformation are generated if these bars are used for transmitting mechanical power to some other devices. Now we are going to look into some more aspects of torsion in this lesson.

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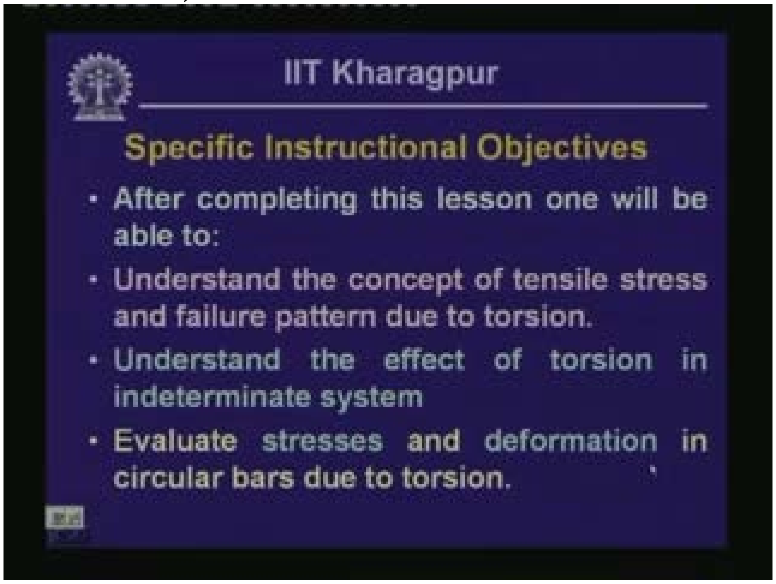


It is expected that once this particular lesson is completed one should be able to understand the concept of tensile stress and failure pattern due to torsion.
Due to the twisting moment we have seen the shear stress that gets generated and we have the maximum stress on the periphery or the surface of the bar. Then we will look into what are the values of the maximum normal stresses and what is the consequence of those normal stresses in the bar?

Secondly, one should be able to understand the effect of torsion in an indeterminate system. So long we have discussed aspects of the determinate system, that means if the twisting moment acts in a bar, then we can evaluate the resisting twisting moment from simple equilibrium equations. We are going to look into the systems where, by using equilibrium equations alone, the internal resisting twisting moment cannot be determined. So, as we have seen in the past the systems where we cannot evaluate the internal forces using equilibrium equations alone are indeterminate systems. Now for such indeterminate systems, if twisting moment acts, then what are the consequences? We will be looking into that in this particular lesson and finally we will evaluate stresses and deformation in circular bars due to torsion.

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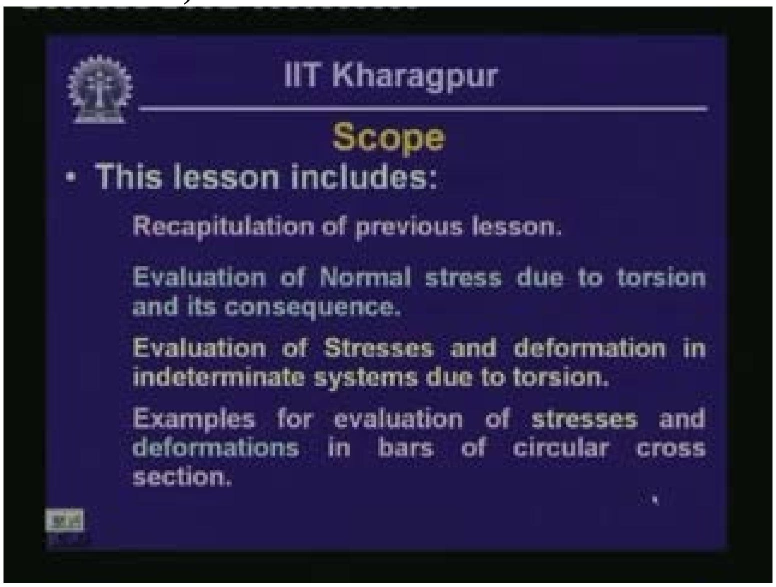


The scope of this particular lesson includes the aspects of the previous lesson as we will go through the answers of the questions. Then we will evaluate the normal stress due to torsion and its consequences and evaluation of stresses and deformation in indeterminate systems due to torsion. Then we will look into the examples for evaluation of stresses and deformations in bars of circular cross sections.

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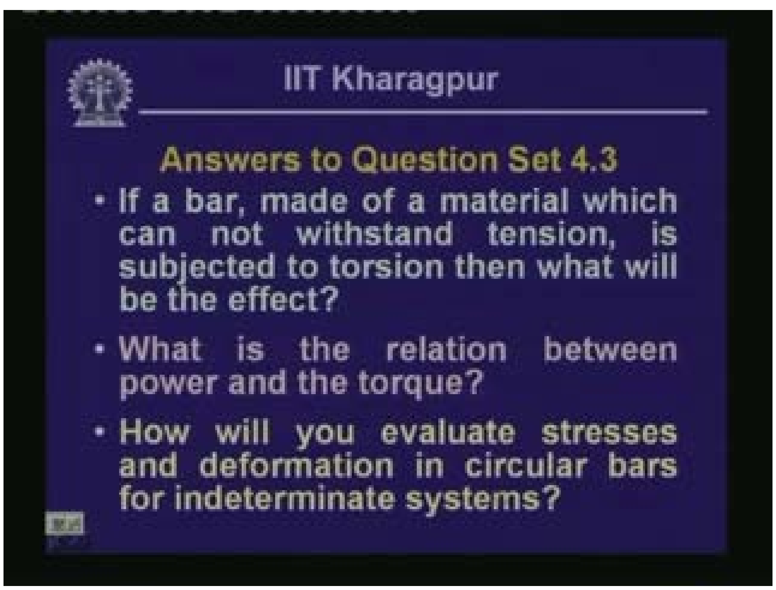


Let us look into the answers of the questions which we had posed last time. Now the first question posed was; if a bar made of a material which cannot withstand tension is subjected to torsion then what will be the effect? As we have seen that when we test a bar under axial pool, it undergoes extension and the bar which undergoes a large extension we call the ducktail material. Whereas when we pull the bar, it does not elongate much and because of the tensile stresses, it breaks or fails. We call such a kind of material as a brittle material. If we use a material which is a brittle kind of a material and if it is subjected to torsion there will be a tensile normal stress; then what is the consequence of that?

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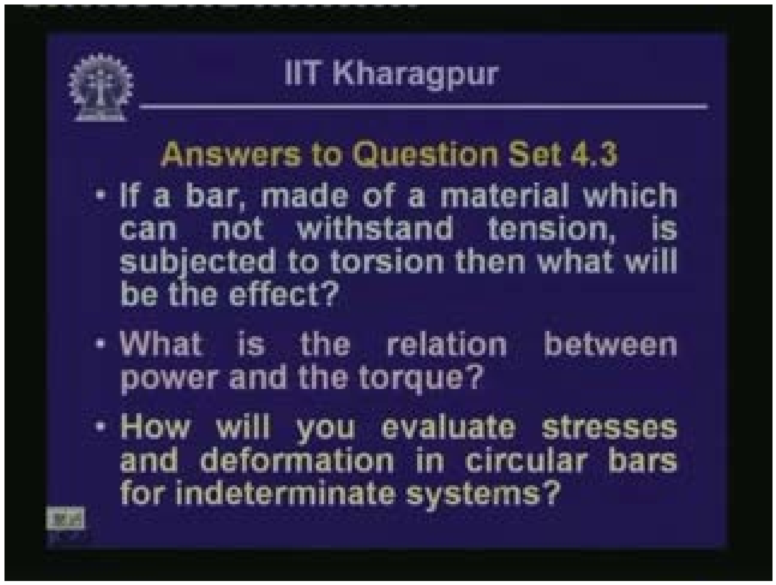


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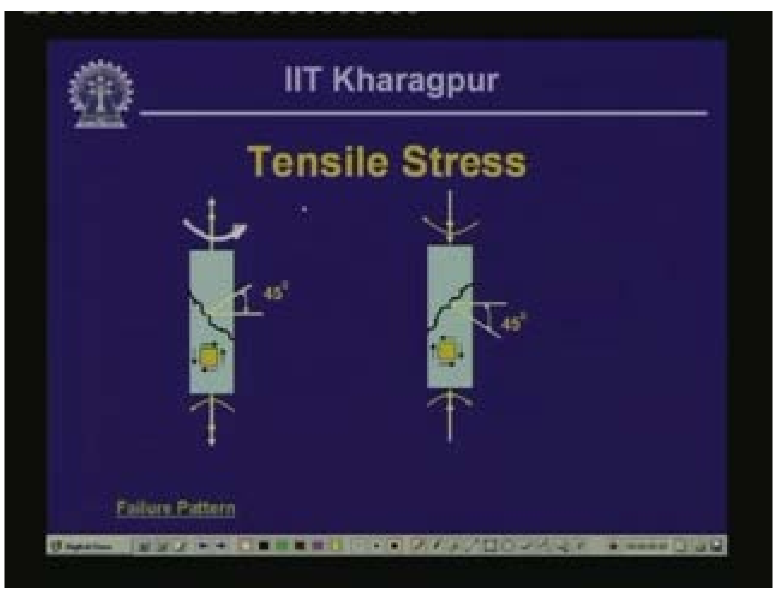


Suppose we have a bar which is subjected to a twisting moment of this form. Here this particular twisting moment is the positive moment as we had defined earlier. These are the kinds of stresses that will be generated on the surface which is the twisting moment in the form of pure shear. Let us try to find out the value of the normal stress corresponding to that.

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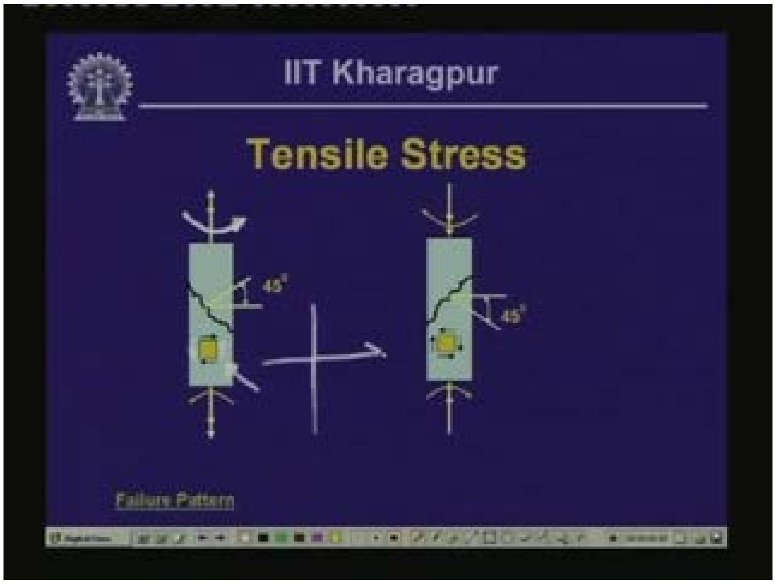


Here because of this shearing stress (Refer Slide Time: 05:06) this is a positive shear, this is Sigma positive axis and this is Tao positive axis. If we plot that since normal stress is 0, shearing stress value is over here and for the complementary part, we have normal stress 0 shearing stress over here.

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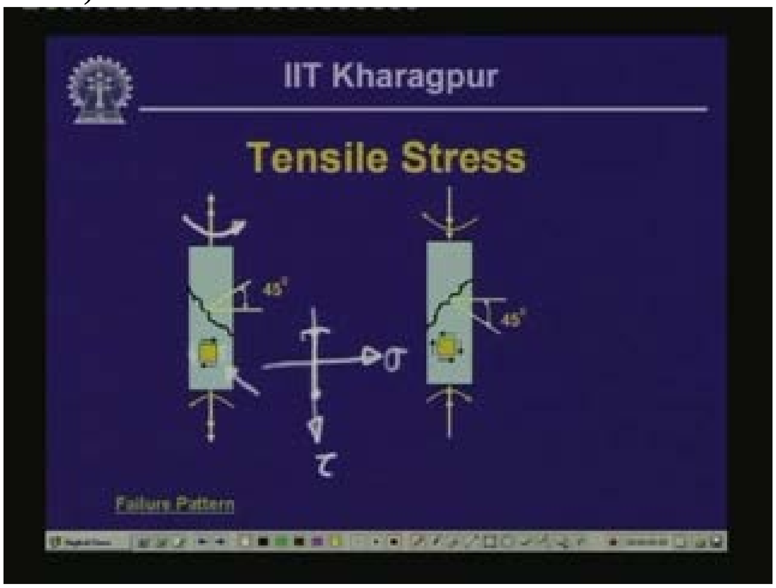


If we plot the Mohr's circle corresponding to this, then this is the value of the maximum normal stress which you call as Sigma 1 and this Sigma 1 is of magnitude equivalent to again the Tao.

This magnitude being Tao, Sigma 1 = T. So the maximum normal stress also is Tao. Now this particular point is at an angle of 90 degrees from the reference plane which is this. See the Mohr's plane. Now with reference to this it is at an angle of 45 degrees where the direction of normal stress will occur and perpendicular to this surface is the plane along which the failure is expected to occur.

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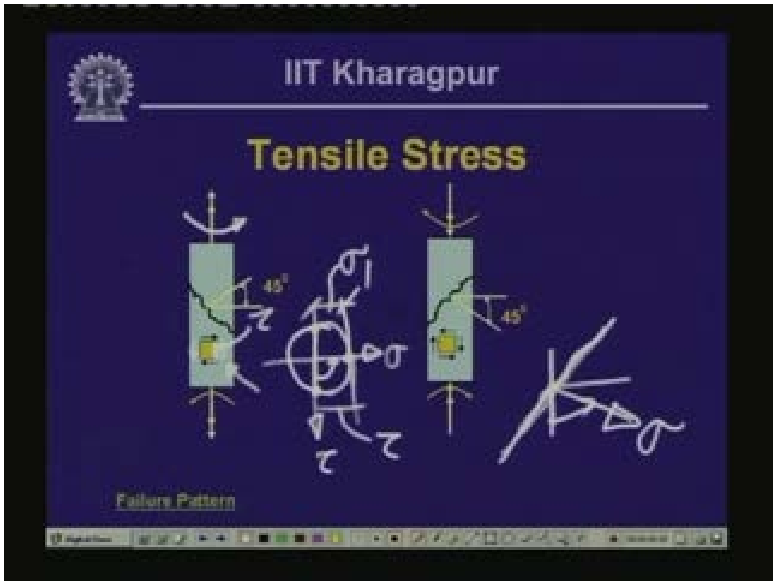


If we look into that it is the positive value which is occurring and this is the plane 45 degrees with respect to the x axis and perpendicular to this plane is the failure line and this is the plane along which it will fail. Now if we take the bar in which we have the torsion acting in this form then again corresponding to this, if we plot the Mohr's circle, we will have the Sigma axis on this side and Tao axis over here. Now this direction of shear stress being negative ,we will have the point over here, the complementary point will come over here and if we plot the Mohr's circle again, the normal stress is going to be Sigma 1here and which is equal to Tao.

This point will act in a clockwise direction with reference to the reference plane. So, this is the reference plane. Normal to this is the particular line and if we go 45 degrees in the clockwise direction then this is the direction of the normal stress Sigma 1 and perpendicular to this, is the failure line along which the failure occurs. Now this can be demonstrated using a very simple experiment.

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Let us say that we take a piece of chalk which is a kind of a brittle material and in fact if we apply a twisting moment for the first case which is in the anticlockwise direction then the failure we get is in this form as we have seen. This is the line which is inclined at an angle of 45 degrees and look into this particular line which is 45 degrees as we have seen in the first case. If we perform a similar kind of experiment though this time we apply a twisting moment in a clockwise direction as we have seen in the second case then you see the kind of failure that you have obtained which corresponds to the second case. This is a simple experiment to demonstrate that when a twisting moment acts into the bar, a failure line occurs along the perpendicular direction of the direction of the stress which is the failure plane.

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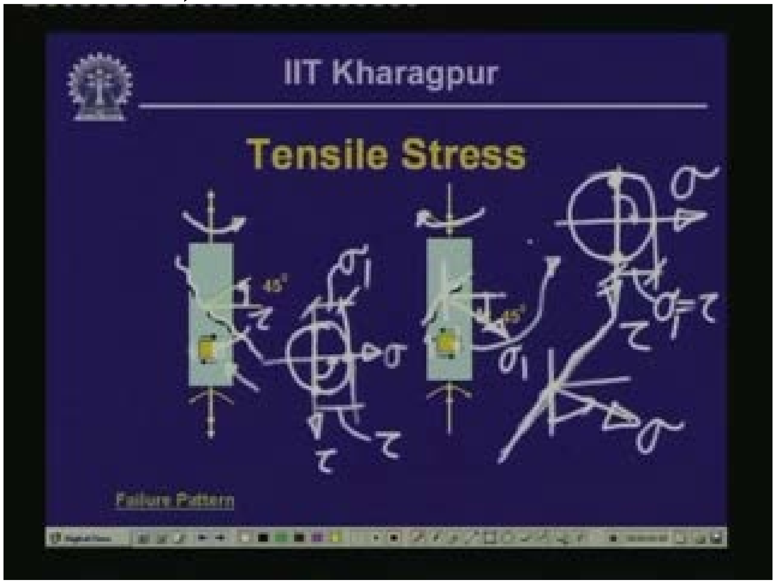

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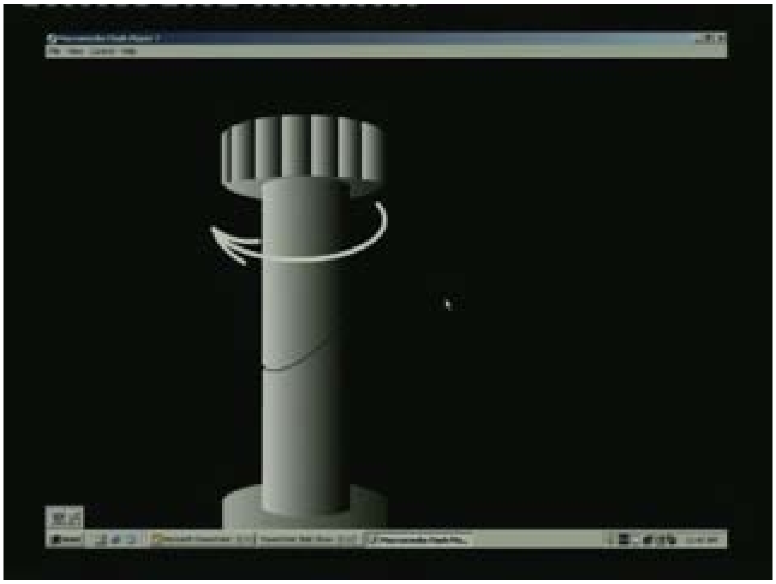

This is the failure line. This bar is over here. Let us look into the second question. What is the relation between power and the torque? Then because of the twisting moment, the tensile stress which gets generated on the surface causes failure which is perpendicular to the direction of the normal stress.

Now the second question is: what is the relation between the power and the torque when a shaft is subjected to a twisting moment?

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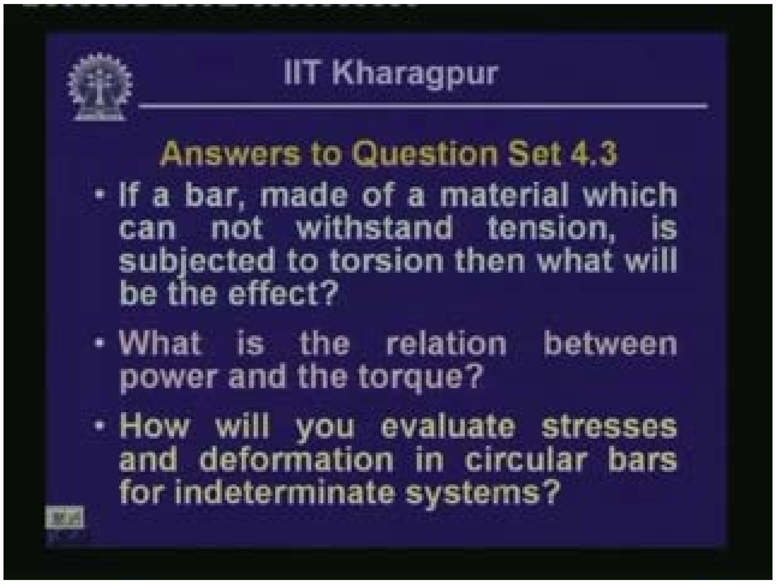

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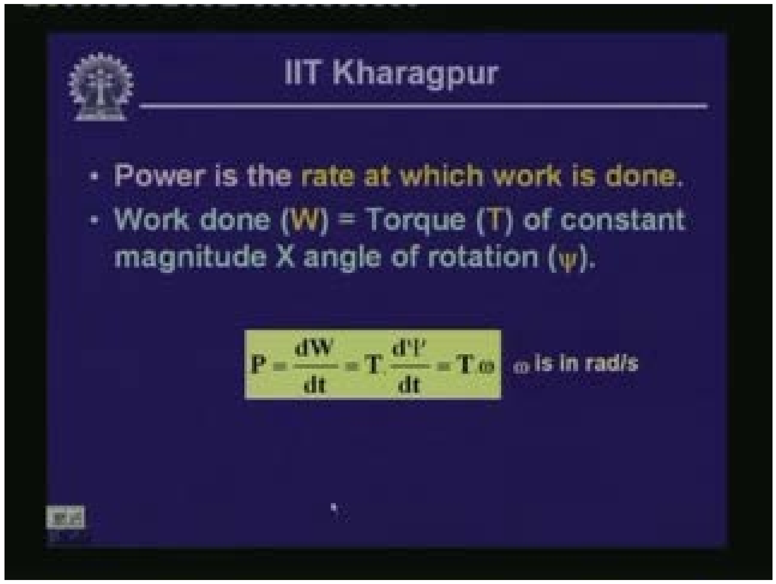


Now in the last lesson that we had looked into, a power is generally defined as the rate at which work is done. When we apply a twisting moment to a shaft, then the work done, 'W' may be defined as the torque which is of constant magnitude which is multiplied with the angle of rotation Psi. Power then can be written as equal to the time derivative of the work done, the rate at which the work is done. Now w is T psi which is ddt of T psi. Then t being constant this is T times d psi dt and d psi dt is the angular speed which you have defined as omega. So, T times omega equals the power where omega is defined in terms of radians per second.

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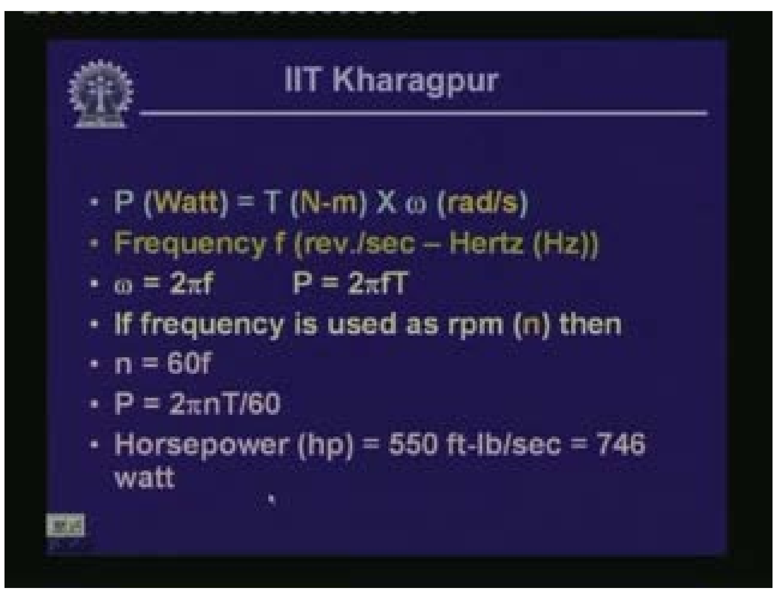

Now let us look into some aspects which we had discussed last time. We generally write power in terms of a watt which equals to the twisting moment which is in Newton meter and the omega, angular speed which is in radian per second.

Frequency which we express as one revolution per second and for one revolution that we have seen it has to undergo 360° = 2pi radian. So we can write the angular speed omega as 2 pi f.
Now suppose we substitute for omega as we have seen, p = T times omega. For omega if you write 2pi, then power = 2 pi fT where p is in watt, f is in cycles per second or revolutions per second or in hertz. Now sometimes frequency is used in terms of r p m which is revolution per minute and designated by the term n.
Then n by 60 which is revolution per minute converted in seconds = f.
So, n = 60 × f and thereby the power p =2pi star n T/60 where t is in Newton meter again and n is rpm, revolution per minute.

Sometimes we defined the power of the equipment from which the power gets transmitted through the shaft in terms of the horse power which is in fps unit. You should know the relationship between the horse power and what it corresponds to in SI units. One horse power is equal to 746 watts approximately and 550 foot pound per second. Given the relation and given the values of the power in terms of horse power or in watt, we can write down in terms of watt and then we can compute the values of the torque. So these are the relationships between the power and the torque and the corresponding units which are being used for defining the power.

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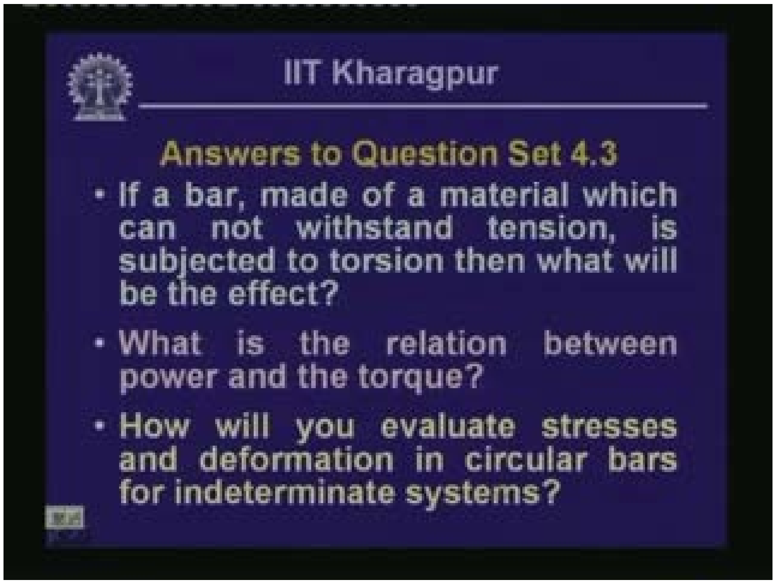


Now the last question which was posed was: how will you evaluate stresses and deformation in circular bars for indeterminate systems?
As we have discussed so long, we were looking into the systems of the bars where the bar is fixed at one end and subjected to the twisting moment or if you have a shaft which is fixed at one

end, may be subjected to non uniform torsion at different points, we could evaluate the values of the internal resisting twisting moment by employing the equations of equilibrium.

If we take the free body diagram, we can evaluate the values of the internal resisting twisting moment from it. Now if we go for a system in which the equilibrium equations alone are not adequate to evaluate this internal resisting twisting moment, then those systems are no longer a determinate system. If we come across such an indeterminate system, then what will be the ways by which we can evaluate the internal resistance twisting moment?

Now one such example is; supposing you have a bar which is fixed at both ends and then it is subjected to a twisting moment, then what will be the stresses in the bar or what will be the deformation in the bar because of this twisting moment?
If we evaluate this particular problem, we cannot solve it by using equations of equilibrium alone.
As we have seen in the past, a bar which is an indeterminate one for evaluating the axial stress and the strain would have us resort to the equations for compatibility. Here also for an indeterminate system, we will have to generate additional equations from the equations of compatibility. So once we have the equations of equilibrium and equations of compatibility, then we will be in a position to evaluate the internal resisting twisting moment from which we can compute the value of stresses and the deformation.
Let us look at how we can carry that out.

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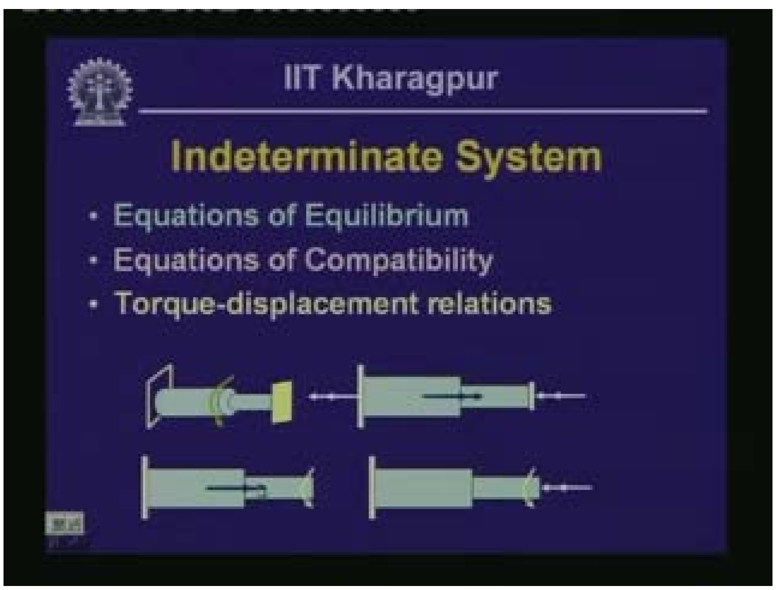


Let us look into a system where a bar or a shaft has a different diameter.
Let us say this diameter (on the left) is d1and the diameter here (on the right) is d2 and it is clamped at this particular end as well as it is clamped at this end. Now if this is clamped and is subjected to a twisting moment, (let us call this as $T_0$) then what will be the values of the twisting moment at these ends since it is clamped and also what will be the stresses and the deformations in this particular zone?

Now we can write down the reactive twisting moment like this.

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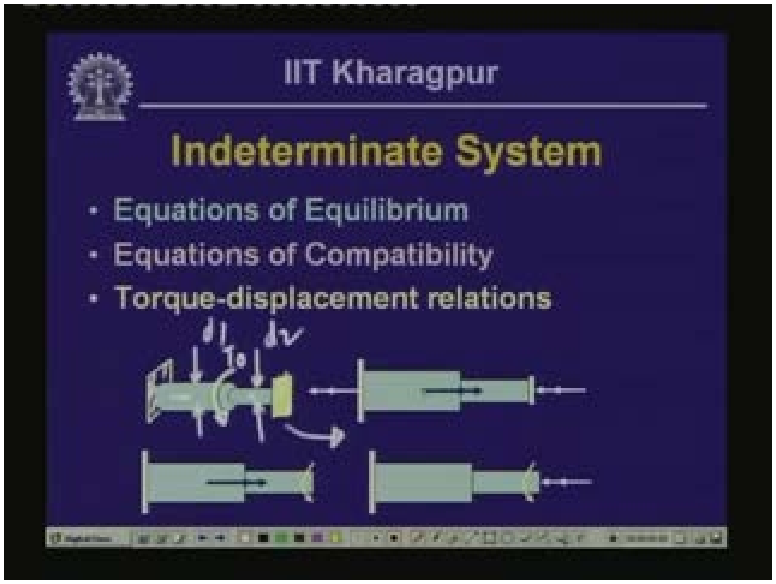


The positive twisting moment as we have defined is that when the twisting moment is in anticlockwise direction in the direction of the vector, notation or the direction of the twisting moment is towards the thumb and that is what has been shown over here. The twisting moment acts over here which is T0. If that vector direction acts in this particular shaft, then what are the reactive twisting moment at these two ends? They are in the opposite direction which is in a clockwise form and let us call this point as a and this point as b and accordingly let us call this as twisting moment at a and twisting moment at b.

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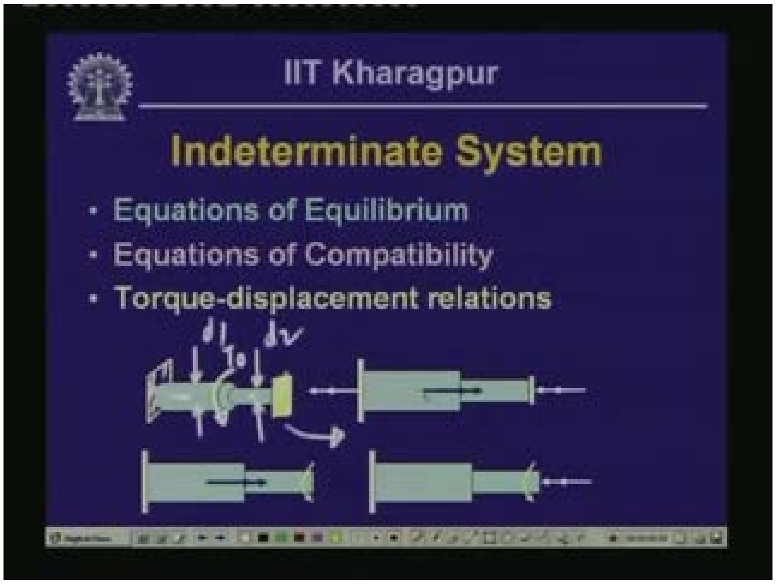

Obviously then the twisting moment $T_0$ will be distributed in this reactive twisting moment which is $T_A$ and $T_B$. So, $T_0 = T_A$ and $T_B$ which is the equation of equilibrium. The external twisting moment which acts in the shaft gets equilibrated by the support twisting moment $T_A$ and $T_B$ and so $T_0 = T_A + T_B$. This is the equation of equilibrium. We can see from this particular expression that $T_A$ and $T_B$ are the two unknown parameters and we have only one equation. From one equation, you cannot solve the two unknown parameters. You need an additional parameter to be brought in.

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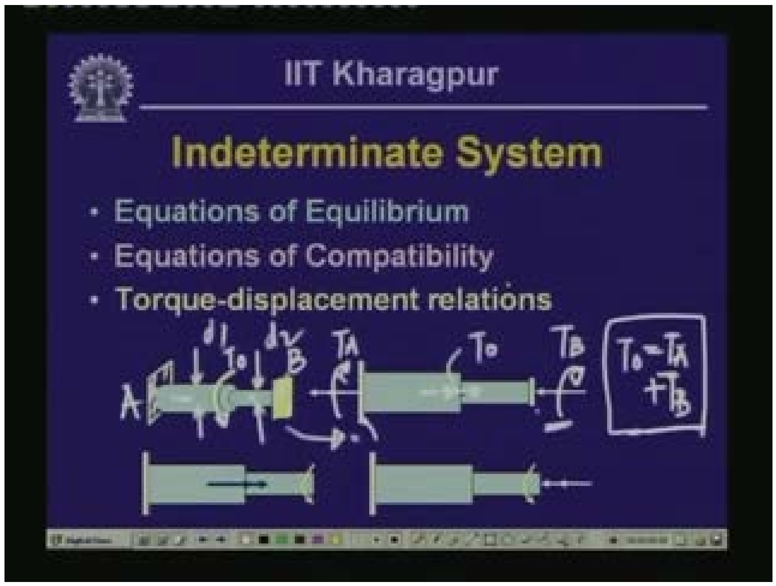


We consider a case as we have done in the past for an indeterminate system. Let us remove this particular support from this particular shaft. Now if we remove this particular support then because of the twisting moment, $T_0$ acts. Let us call $T_0$ the rotation of the shaft. Let us say that the rotation which it undergoes at this particular end (if we call this rotation as $\text{Theta}_1$ which is at end B) is caused by the twisting moment $T_0$.

Let us remove this twisting moment. Let us apply the resistive twisting moment which is $T_B$ at this end and this $T_B$ also is going to cause a twisting moment over $\text{Theta}_2$. This particular support B is fixed in position and so, it is expected that there will not be any rotation because of the twisting moment. That net rotation at support B = 0. $\text{Theta}_1$ which we get corresponding to the twisting moment $T_0$ and $\text{Theta}_2$ which we get corresponding to the resistive twisting moment $T_B$ should be 0.

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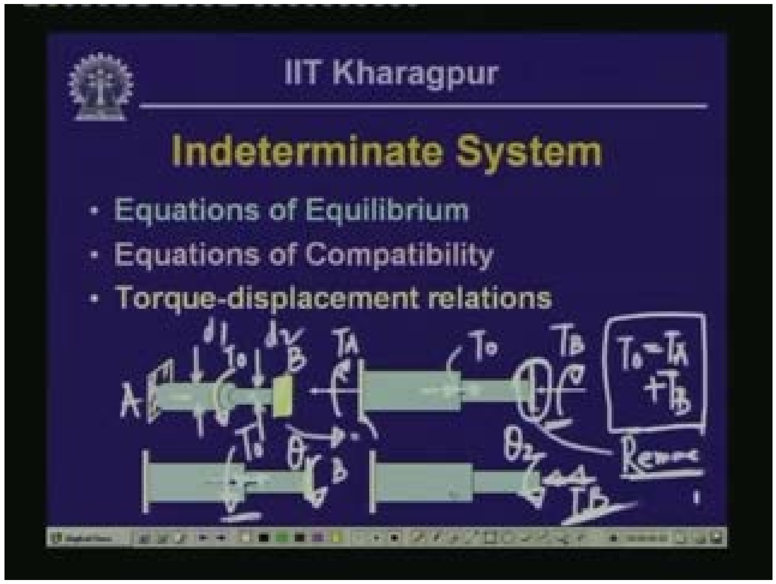


It means that if we write $Theta_1$ + $Theta_2$ = 0 and then if we write Theta in terms of a twisting moment T then we get another set of equation. You get equation 2 and this which is equation 1. We have 2 equations now and we have 2 unknown parameters $T_A$ and $T_B$. Then we can solve for $T_A$ and $T_B$. An additional equation has been generated from the deformation compatibility in this particular case and we call this as a compatibility equation.

So, for an indeterminate system in fact, we need the equations of equilibrium to be written and we need equations of compatibility to be written which is the function of the rotation Theta. Then we write the torque displacement relation so that we get this compatibility equation in terms of the twisting moment T. Then we have two equations from which we can compute the unknown values $T_A$ and $T_B$.

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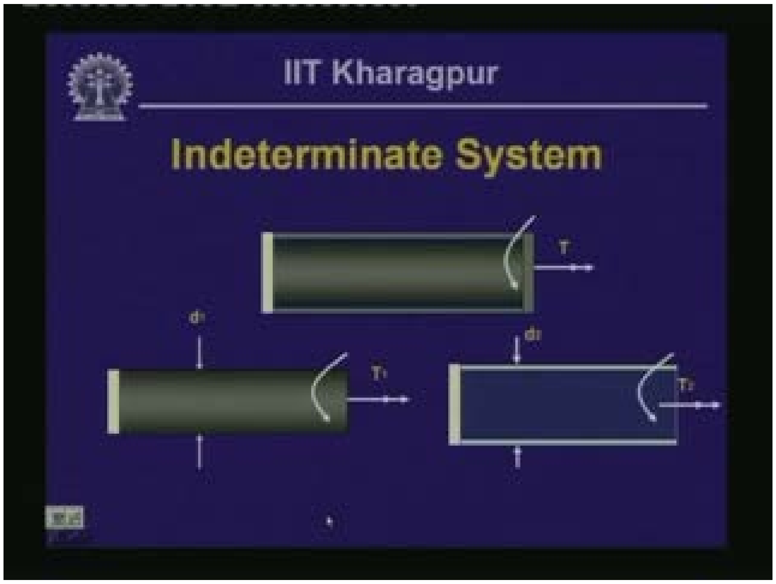


Let us look into another kind of a system in which we apply a twisting moment. Let us say that we have a solid bar, of diameter $d_1$ which is inserted within a tube. Let us call this tube as 1 and this tube as 2. This is one side of the cell and this is the other side which is the circular one. A circular tube is there within which we have a solid shaft inserted. On one end it is clamped and on the other end we have put a plate which is connected to both the tubes on the solid shaft. Now if the whole assembly is subjected to a twisting moment, then what are the consequences?

Since both the ends are fixed naturally and you have two elements; one is a tubular shaft, another one is a solid shaft and both the elements are subjected to a twisting moment in a combined form. We like to find out what the share of this twisting moment is between the two elements, the tubular shaft and the solid shaft. There are two twisting moments that will be shared by these two elements and then how do we evaluate those twisting moments?

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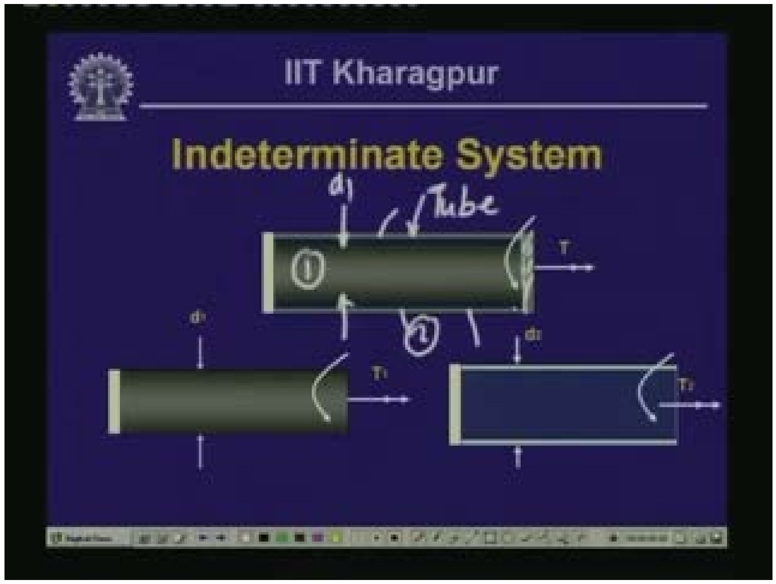


Let us suppose we take of this particular rigid plate from this and take out the shaft over here. This is the internal solid shaft which is subjected to a twisting moment the pictorial notation of which is this. Let us call that twisting moment as $T_1$.

That means the external twisting moment T which acts in the composite system is shared by the shaft and the tube. The twisting moment which acts in the central solid shaft is $T_1$. The twisting moment, which acts in the tubular form, is $T_2$.

We have $T_1$ and $T_2$ and the diameter of the internal shaft is $d_1$ and the external diameter or the outer diameter of the tubular shaft is $d_2$.

We will have to find out the values of $T_1$ and $T_2$. If we write down the equilibrium equation then the external twisting moment $T = T_1 + T_2$ because this external twisting moment T is to be shared by the solid shaft and the tubular shaft. Now again, we have only one equilibrium equation and we have two unknown parameters $T_1$ and $T_2$. From this single equation we cannot evaluate the values of $T_1$ and $T_2$.

We need an additional equation to be generated so that we can evaluate these 2 unknown values $T_1$ and $T_2$ and this additional equation can be generated if we take the compatibility into account. What is the compatibility criterion in this particular case? Here, you have the composite system where you have an external tube in which you have a solid shaft and both are enclosed within 2 fixed supports.

When this is twisted, the whole composite system undergoes rotation and also since both the tube and the solid shaft are under the constraint of these two plates, they are expected to undergo the same amount of rotation. So, the rotation in the solid shaft and the rotation in the tubular shaft should be identical and then we have the compatibility from which we can generate the additional equation.

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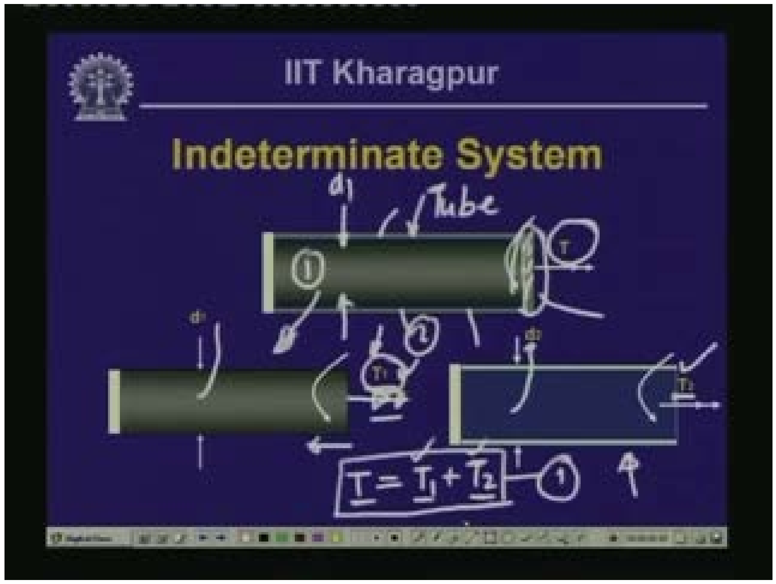


If we say that this is the rotation $Theta_1$ that this particular solid shaft undergoes because of the twisting moment $T_1$ and if we say $Theta_2$ is the twisting moment or the rotation that this tubular shaft undergoes because of the twisting moment $T_2$ then we can say that $Theta_1$ = $Theta_2$ and this is the compatibility equation. This is our equilibrium equation where you say that T = $T_1$ + $T_2$ and this is our compatibility where $Theta_1$ = $Theta_2$.

As we know that T by J = Tao by rho = G Theta/L. So, Theta in terms of t is nothing but TL/GJ. The value of Theta is in terms of T = TL/GJ. We can write that down as
$Theta_1$ = $T_1$ and L is the same because both are within the same confinement. The length of the solid shaft and the length of the tube are identical and let us call that as L. So, it is $T_1$L /G and let us call this as $J_1$ which is the polar moment of inertia of the solid shaft and likewise $Theta_2$ = $T_2$L/GJ.

If we have the same material or $J_2$ and if both the tube and the solid shaft are made out of the same material, then the shear module as value g will be identical but if this tube and the solid shaft are of different material, then you will have two different values of g and correspondingly, we will use $G_1$ and $G_2$ to make it more general. If we equate this $Theta_1$ and $Theta_2$, we can get a relationship between $T_1$ and $T_2$. Therefore the second equation will be generated from this compatibility.

Equation1 and equation 2 can give us the values of $T_1$ and $T_2$. So, these are the 2 equations T = T 1 + $T_2$ and $Theta_1$ = $Theta_2$ or $T_1LGJ_1$ = $T_2LGJ_2$ will give us the values of $T_1$ and $T_2$. That is how for an indeterminate system when they are subjected to a twisting moment, we make use of the equation of equilibrium the equation of compatibility and then the torque displacement relationship to arrive at what will be the equations from which you can evaluate the unknown twisting moment or the internal resisting moment.

Once we know this twisting moment, then we can take the free body diagram of the whole shaft at any point and then correspondingly, we can find out what are the internal resisting twisting moments and from which we can compute the value of the shear stresses which are generated from the twisting moment and the angle of a twist because of this twisting moment.

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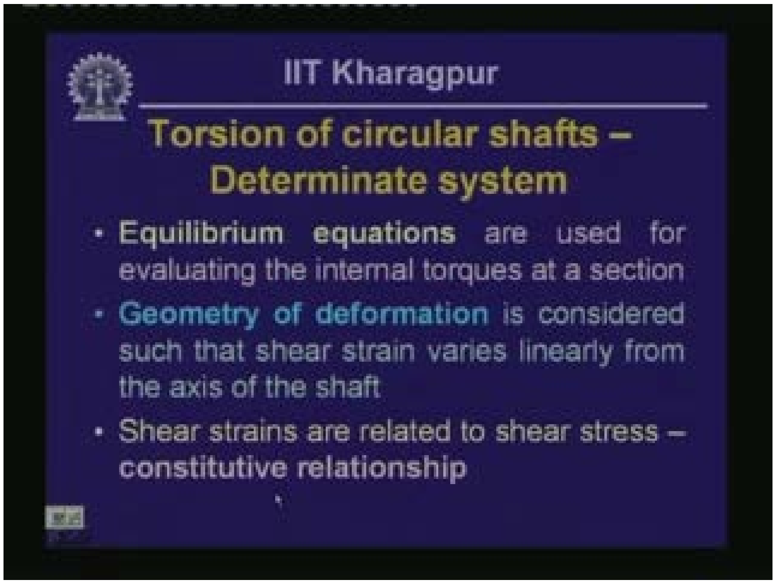


Let us look into the whole of what we have done so long for the determinate and indeterminate system. Now when we talk about the determinate system the equations which we need are the equilibrium equations and equilibrium equations alone are adequate to evaluate the internal resisting twisting moment and all we need is the torque displacement relations or torque rotation relationship from which we can compute the value of the stresses.

The geometry of deformation, as we have seen, which is in terms of the rotation varies and shear strain varies linearly from the axis of the shaft. We have seen that Gamma = d Theta/d x times rho. It varies linearly with respect to the radius. From the center as it goes, it varies linearly and shear strains are related to shear stress which is the constitutive relationship. With these 3 aspects we can solve any determinate system.

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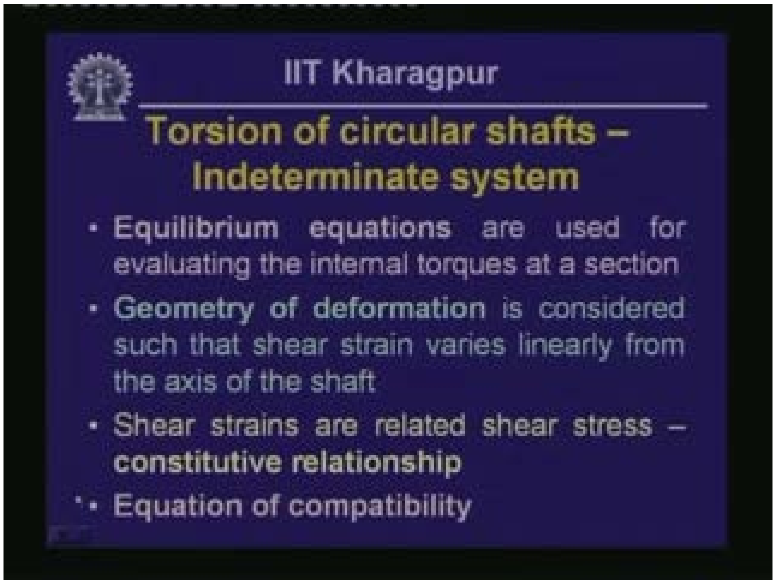


In the case of the indeterminate systems, they are also identical except that you need equations of equilibrium. You need to look into the geometry of deformation and of course the shear strains are related to the shear stress which is the constitutive relationship. Now apart from this, these alone are not adequate to give the equations for evaluating the internal resisting twisting moment. In case of the indeterminate system, we need another criterion which is the equation of compatibility and from this equation of compatibility we can generate another equation from which you can solve the internal resisting twisting moment and an internal twisting moment at any point of the shaft in terms of the support moment.

Now you are in a position to say that if a bar is supported at one end and subjected to a twisting moment, then we can compute a twisting moment accordingly at any point whether it is uniform torsion or non uniform torsion or whether the shaft is a uniform one, non uniform and the stresses corresponding to the angle of rotation.
If the bar is not supported on one point alone but is supported at two points or two sides then when it is subjected to a twisting moment, the equations of equilibrium alone are not adequate to evaluate the internal resisting twisting moment and we need the spot of equations of equilibrium to generate an additional equation from which we can compute the values of internal resisting twisting moment.

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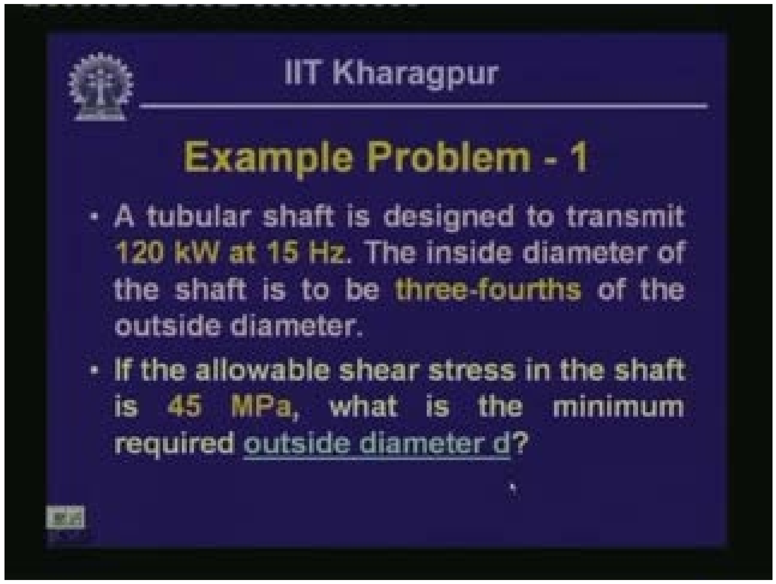


Let us look at some examples to demonstrate this. The first example is the one which I had posed last time and we discussed this aspect last time that the shafts are generally used to transmit power and in this particular case, a tubular shaft is designed to transmit 120kw power at a frequency of 15Hertz. The inside diameter of the shaft is to be three fourths of the outside diameter. If the outside diameter is d, then the inside diameter is ¾(d) which is .75 times d. If the allowable shear stress is 45 mpa then what is the minimum required diameter, d? What will be the minimum required outside diameter, d? Let us look into this particular example.

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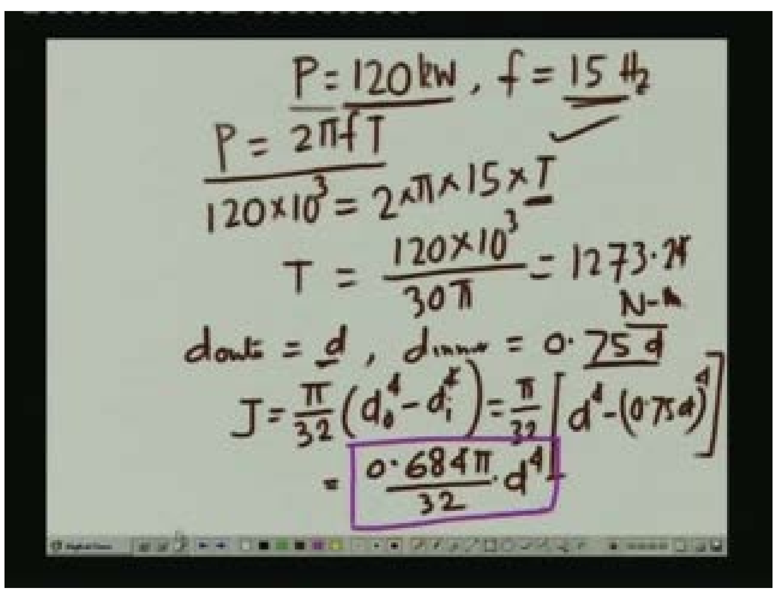

Here the value of power is given as 120kw and the frequency at which the shaft operates is 15Hertz. As we have seen p = twice pi f T. There is a relationship between the power and the torque. Power here is in watt and since it is given in kilo watt this is 120 multiplied by 10 to the power of 3 which is equal to 2 multiplied by pi multiplied by 15 multiplied by T. T is the twisting moment which is in Newton meter. So, T =120 multiplied by 10 to the power of 3 divided by 30pi and this gives us a value of 1273.24 Newton meter. This is the value of the twisting moment that has been generated because of the power driven which is 120kw at a frequency of 15Hertz.

Given that the outer diameter of the shaft d outer is equal to d and consequently d inner is equal to ¾ of the outer diameter which is 0.75 times d if these are the values of the shaft, d and 0.75 d. Then the value of the polar moment of inertia J is equal to pi by 32 $d_0$ (outer) to the power of 4 – $d_i$ (inner) to the power of 4 which is equal to pi by 32. Now d outer here is d so we have d to the power of 4 minus (0.75d) to the power of 4. This gives us a value of 0.684 pi by 32 multiplied by d to the power of 4 which is the value of J. This is the value of the polar moment of inertia J.

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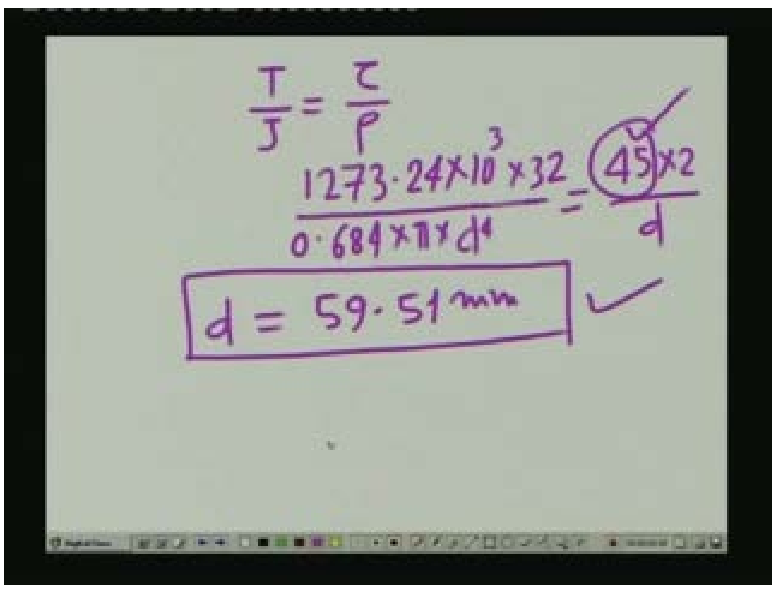


We know that the stress is limited to 45MPa. So, T by J is equal to Tao by rho. Now we will have to find out the diameter d. T = 1273.24 multiplied by 10 to the power of 3 Newton millimeter divided by 0.684 which is what we have computed times pi times d to the power of 4. So, this (the numerator) gets multiplied by 32 which is equal to Tao. It is limited to 45MPa and rho is outer diameter by 2 and 2 divided by d.

So from these if you compute the value of d you get 59.51 mm. This is the diameter of the shaft that we will have to use so that the stress is within 45 MPa.
If we have to use a tubular shaft which transmits power at 120kw and at 15 horse frequency then the external diameter of the shaft that is necessary is equal to 59.51mm if we have to restrict the value of the shearing stress within 45MPa.

If we use the diameter more than that then the stress level will be lower but if we go a diameter lower than this value, then the stress level will go higher and as a result the shaft will fail. Let us look into the second example which is of the category that we have discussed today, which is an indeterminate system.

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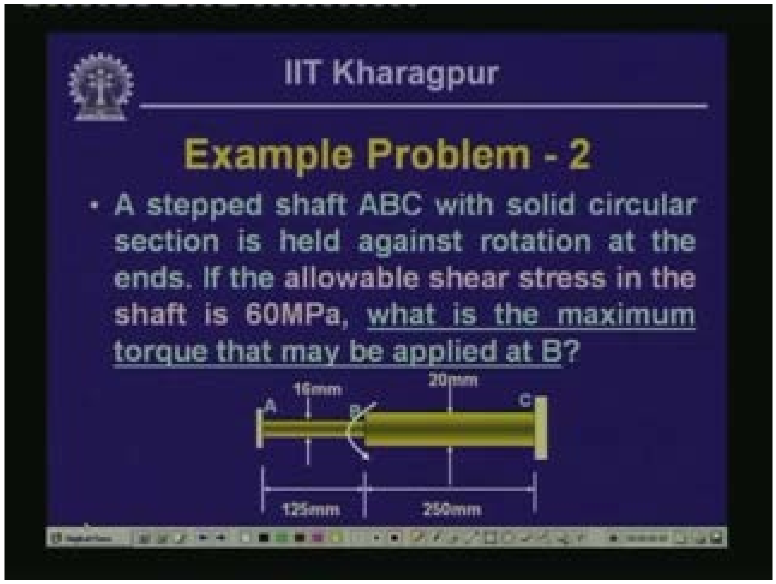


You have a shaft A B and C with solid circular section. It has 2 parts which is state 1 and the first part has the diameter of 16mm, the second one has a diameter of 20mm and it is held again in rotation at A and at C. Assuming that the allowable shear stress in the shaft is 60 MPa, Tao is limited to 60 MPa. Then, what is the maximum torque that may be applied at B?

The torque T can be applied at B so that the shear stress does not go beyond 60 MPa. Here since these two ends are clamped, the resisting twisting moment will be $T_A$ and $T_C$. Obviously, from the equilibrium equation we know that $T = T_A + T_C$. $T_A$ and $T_C$ being two unknown parameters from single equation we cannot evaluate that so, it is a problem of again the indeterminate system.

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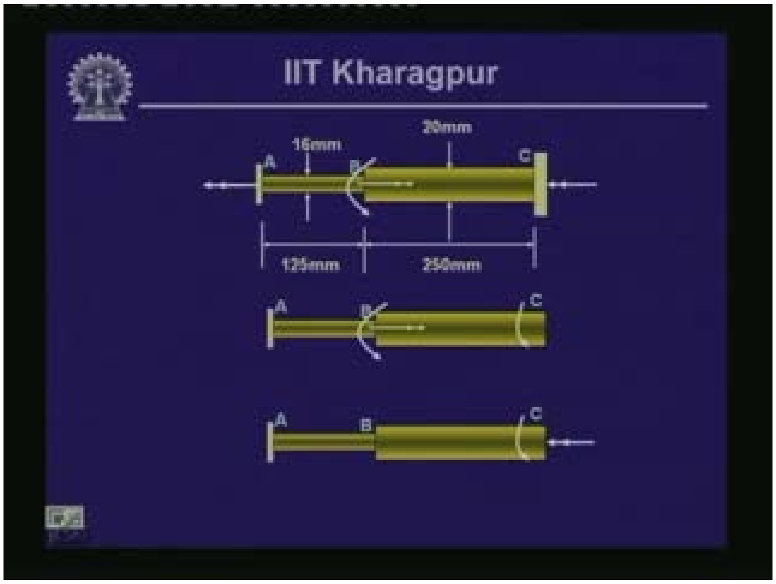


Let us look into this free body diagram. Take the free body of the shaft, the twisting moment that we will have to evaluate is T which acts at B and the resistive twisting moment at end A and C let us say, $T_A$ and $T_{C.}$ Then our equilibrium equation gives us $T = T_A + T_C$. Note here that this twisting moment which acts here is a positive twisting moment; this is the pictorial notation of the twisting moment and the support twisting moment which we have taken is in the opposite direction $T_A$ and $T_C$ and this is the equilibrium equation $T = T_A + T_C$. So, this is equation 1.

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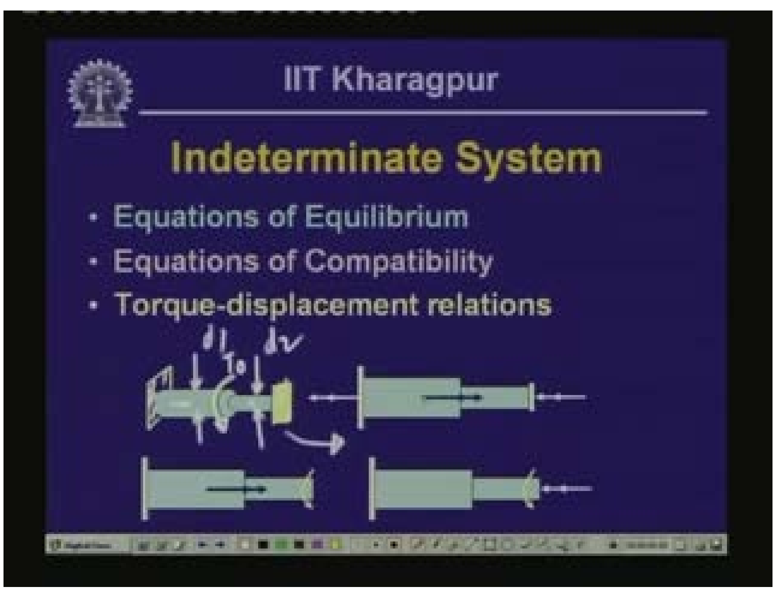



There is another equation we can generate from the compatibility criteria.

This is subjected to a twisting moment here at T and note that this acts at this point B. (Refer Slide Time: 37:19) Now once we remove this particular support, it becomes a determinate system. If we cut across here and take the right hand free body and here there is no twisting moment from part B to C, it is free of twisting moment. But between A and B we have the twisting moment, T.

Let us try to find out the rotation here. Let us call this as $Theta_1$ which will be occurring because of the twisting moment T at C and in this particular free end at C because of the resistive twisting moment $T_C$, there will be another rotation and let us call this rotation as $Theta_2$. If we combine $Theta_1$ and $Theta_2$ together since this end is fixed, it cannot undergo any rotation. So, $Theta_1$ + $Theta_2$ should be equal to 0.

This is the equation of compatibility and then if we write $Theta_1$ and $Theta_2$, from this particular free body diagram, $Theta_1$ is going to be equal to T, when it is subjected to the action of T over the length AB and so, we get T.LAB by G.JAB. This is the polar moment of inertia of the part AB. Now there are 2 different diameters. JAB and JBC are different.

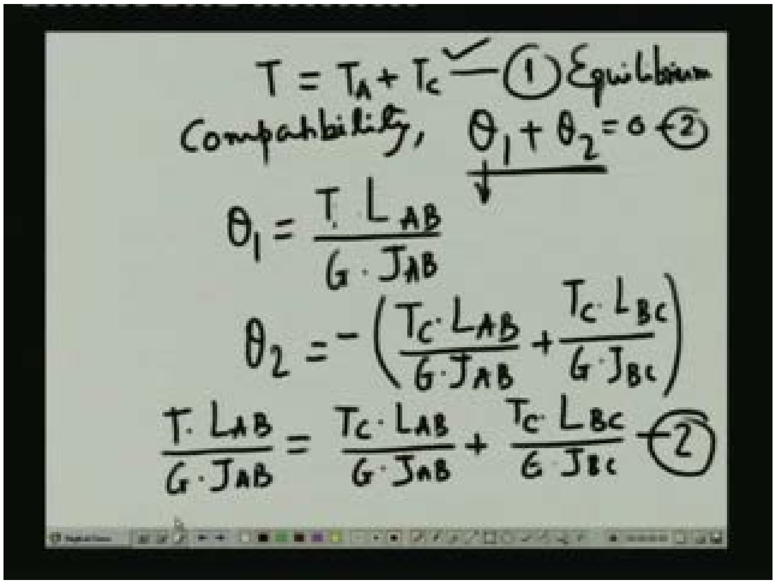
When this particular shaft is subjected to a twisting moment at C undergoing a rotation of $Theta_2$, the value of $Theta_2$ will be equal to the twisting moment $T_C$.

For part AB, it will be length AB by G times JAB. Also for the part BC, there will be a rotation. So, it will be plus $T_C$ multiplied by length BC divided by G.JBC and note here that the twisting moment T, acts in the positive direction.

As we have defined earlier that when it moves in an anticlockwise direction the direction of the twisting moment is in the positive x direction. So, the rotation correspondingly also is positive and $T_C$ acts in the opposite direction which is a negative. So, $Theta_2$ as a whole is a negative. $Theta_1$ + $Theta_2$ = 0 and we get

$Theta_1$ = $Theta_2$. From this we can find out the relationship between $T_C$ and T and we already have a relationship between T, $T_A$ and $T_C$. From these we can evaluate $T_A$ and $T_C$. Now let us compute these values.

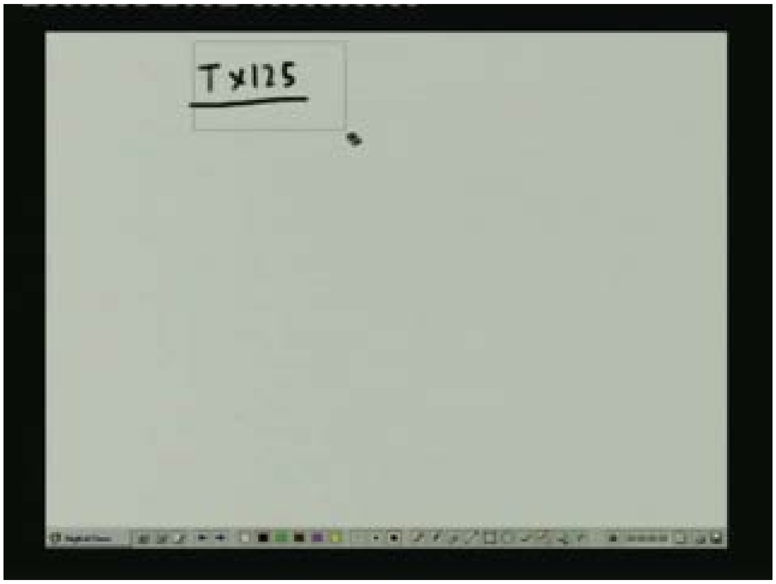
(Refer Slide Time: 40:20 - 42:57)




Now we have seen, $T = T_A + T_C$. This is our equation of equilibrium.
Equation of compatibility gives Theta$_1$ + Theta$_2$ = 0. This is the second equation.
We will have to apply the torque rotation relationship where Theta$_1$ is equal to T times length AB by G.JAB. We have seen just now that the rotation Theta$_1$ occurs because of the twisting moment T which is over length AB times G.JAB and Theta$_2$ correspondingly is equal to $T_C$ times LAB divided by G.JAB plus $T_C$ $L_{BC}$ by G.JBC.

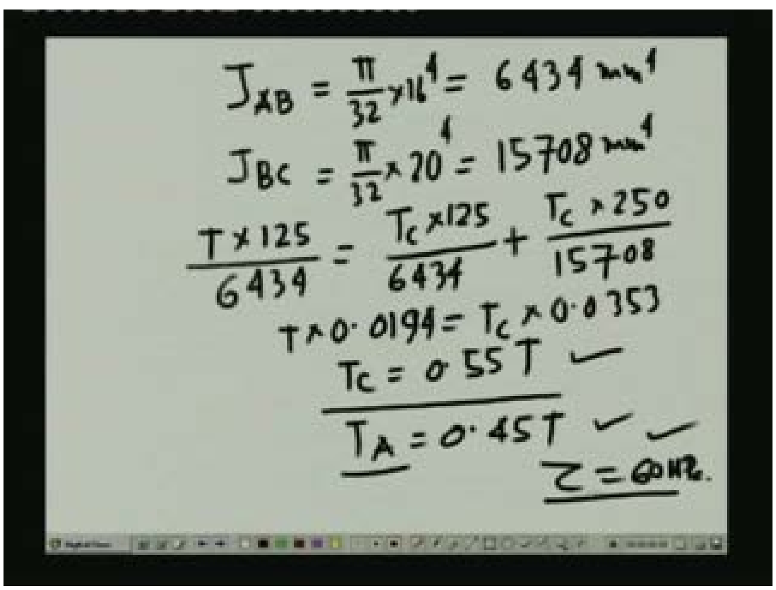
This is the total twisting moment which is negative according to the sign convention which we have used. Now if we write down this Theta$_1$ + Theta$_2$ = 0 as our equation of compatibility then we have T.LAB by G.JAB is equal to $T_C$.LAB by G.JAB plus $T_C$.LBC by G.JBC. This is the second equation after writing the relationship between the angle of twist and the angle of twisting moment and we have equation of equilibrium which is T is equal to $T_A + T_C$.

(Refer Slide Time: 42:59 - 43:11)




So if we now substitute the numerical values then we have that T length AB is 125 and G gets canceled from either side. Now before we go into this let us compute the value of the polar moment of inertia J for the 2 point.

(Refer Slide Time: 43:14 - 45:34)




Now J for AB which is of diameter 16 is equal to pi by 32 times 16 to the power of 4 which is equal to 6434mm to the power of 4 and JBC is equal to pi by 32 times 20 to the power of 4 which is equal to 15708mm to the power of 4. If we substitute T times length AB we get 125 by J (ab) which is 6434; this is equal to $T_C$ and length AB which is 125 by 6434 plus $T_C$ multiplied by 250 divided by 15708 and we get T times 0.194 = $T_C$ times 0.0353 and this gives a value of $T_C$ as 0.55T.

Once we know this from equation 1, we get $T_A$ = 0.45T. Now we know the values of $T_A$ and $T_C$ in terms of T. Now if we have to restrict stress to a value, we need to evaluate the values of the twisting moment. What will be the value of the twisting moment T if we restrict the stress Tao to 60 MPa? The stress is to be limited to 60 MPa and correspondingly, we will have to find out the value of T.

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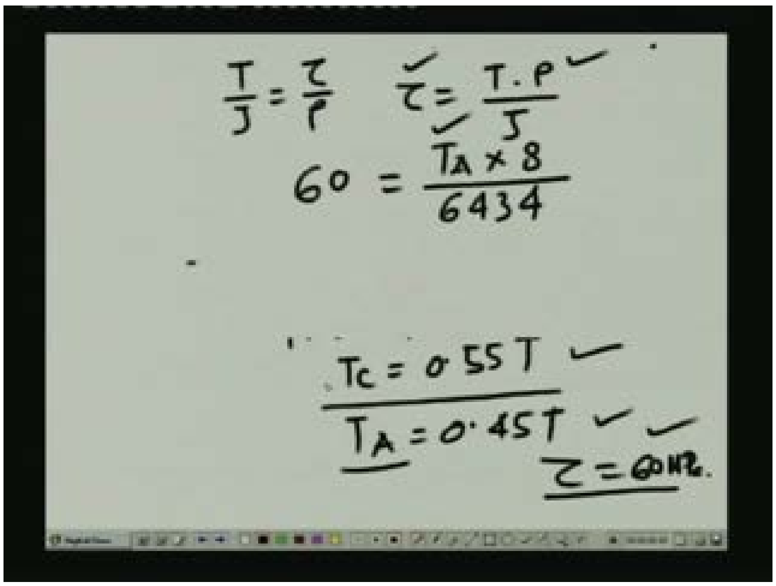


Now T by J is equal to T by rho. T is equal to T rho by J and for the AB part let us say it is restricted to 60 = $T_A$ multiplied by 8 which is the rho because 16 is the diameter divided by JAB which is 6434. Since we are using the twisting moment $T_A$ over the part AB then with the twisting moment acting at $T_A$, if we take a section in between AB, then the resistive moment also will be $T_A$. Over the part AB we compute the stress which is equal to T times rho by J. T rho is 8 and J is 6434. From this we get $T_A$ as equal to 0.45 times T. So, if we substitute that, this gives us the value of 107.23 MPa. This is the value of the twisting moment from in the part AB.

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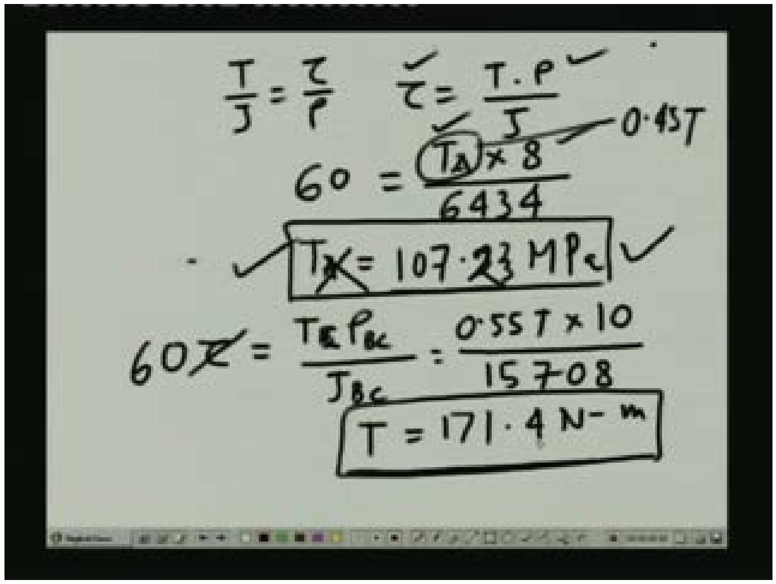
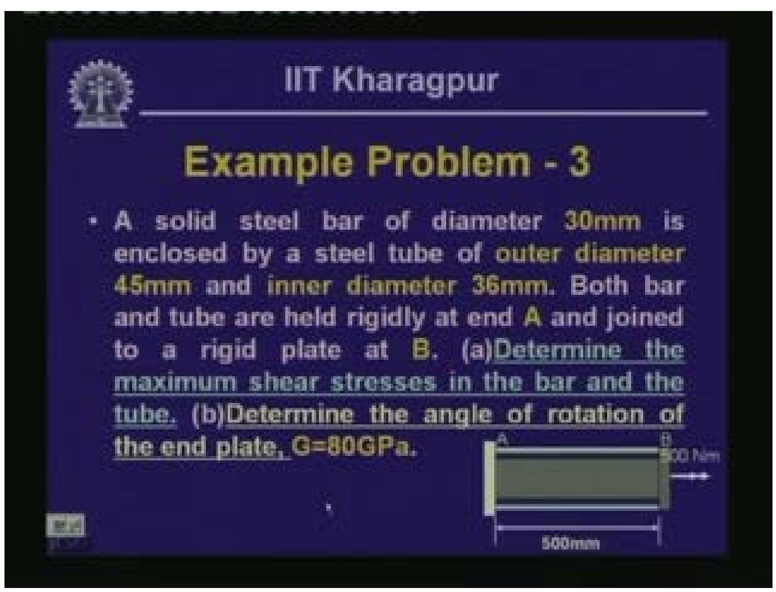



Let us see the value of the twisting moment from the part BC. From the part BC, we can say that Tao is a shear stress which is limited to 60 MPa which is equal to $T_C$ $rho_{BC}$ by $J_{BC}$. $T_C$ is equal to 0.55 times T, $rho_{BC}$ is equal to 10 and $J_{BC}$ is equal to 15708 and we are limiting Tao to 60 MPa. From the above if we compute T we get 107.23 MPa. From this if we compute T we get 171.4 Newton meter. When 171.4 Newton meter acts at B then the stress in the part AB, will go beyond 60MPa and the shaft will fail. So, we will have to restrict the value of the twisting moment at B as 107 Newton meter so that the stress level everywhere is within 60 MPa.

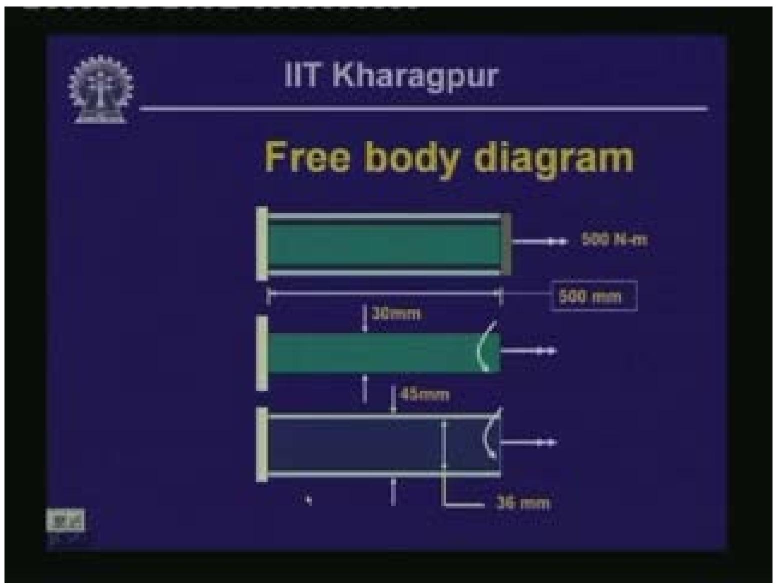
(Refer Slide Time: 48:24 - 49:31)



We have another problem which is of a similar type (of a determinate and indeterminate system). We have a solid steel bar of diameter 30 mm. It is enclosed by a steel tube of outer diameter 45mm and inner diameter 36mm. We have a solid shaft of 30mm diameter which is inserted within a tube. This is the tube of outer diameter 45mm and inner diameter 36mm. Now both the bar and the tube are held rigidly at end A and B. This is the end A and this is the end B. They are held rigidly by the plate and joined as a rigid plate at B.

Now we will have to determine the maximum shear stresses in the bar and the tube and because of this, we will have to find out how much sheer stresses are generated in the bar and the tube and also we will have to determine the angle of rotation of the end plate. The value of g is given as 80MPa. Now let us look into the free body diagram of this particular system.

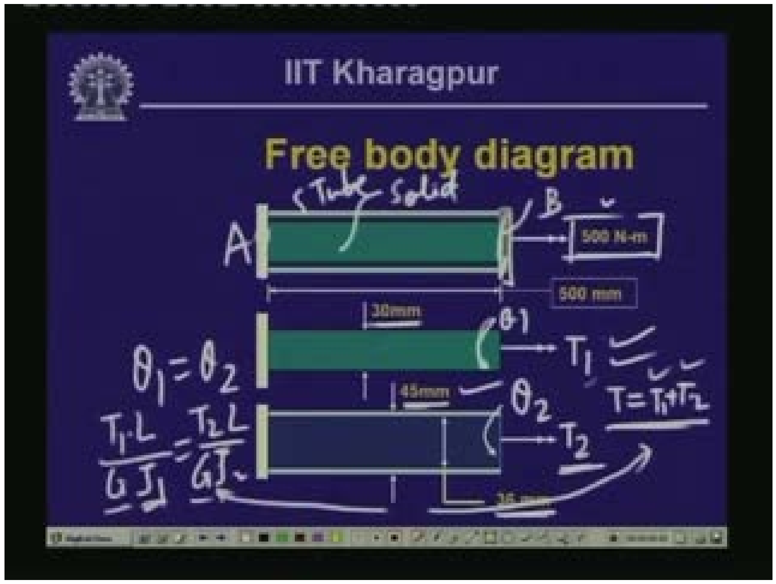
(Refer Slide Time: 49:32 - 50:51)




This is the solid bar or the solid shaft and this is the tube and this solid shaft is inserted in the tube held at this end A and we have also provided a rigid plate at B. This is held between the two ends and it is subjected to a twisting moment. The whole composite system is subjected to a twisting moment, which is of magnitude 500Newton meter. The diameter of the solid shaft is 30mm, the external diameter of the tubular bar is 45 and the internal diameter is 36mm.

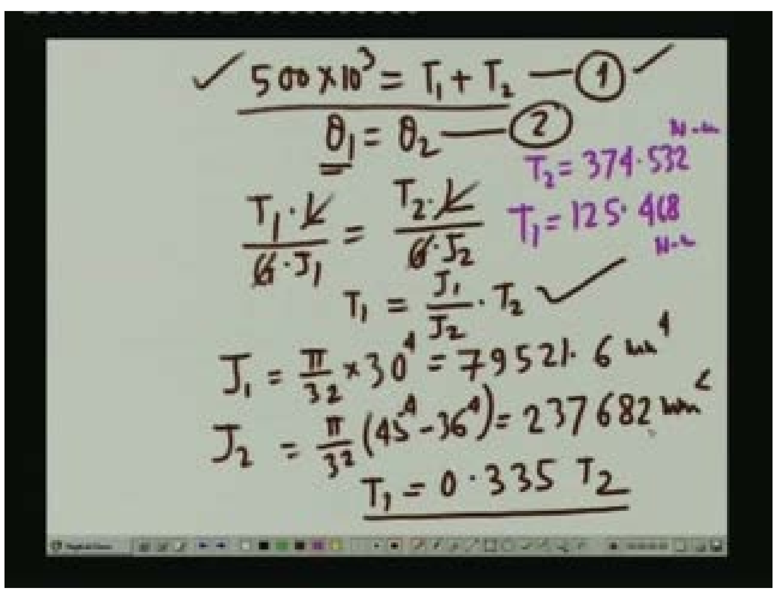
Let us say that this (tube in the middle) is subjected to a twisting moment $T_1$ and this (tube at the bottom) is subjected to a twisting moment $T_2$. The equilibrium equation will tell us that T which is equal to 500Newtonmeter is equal to $T_1$ plus $T_2$. This is the equation of equilibrium and also because of this twisting moment $T_1$, this will have a rotation $\text{Theta}_1$ and because of this twisting moment $T_2$, it will have a rotation $\text{Theta}_2$. Since the whole assembly is enclosed between two plates the rotation $\text{Theta}_1$ and $\text{Theta}_2$ should be identical because you cannot have dissimilar rotations for two different parts when they are attached between two plates. So, $\text{Theta}_1$ should be equal to $\text{Theta}_2$ and that is the compatibility.

(Refer Slide Time: 51:11 - 51:37)




$Theta_1$ is equal to $Theta_2$ and we can write $Theta_1$ in terms of $T_1$ as $T_1L$ by $GJ_1$ and this is equal to $T_2L$ by $GJ_2$. This material being the same the value will be the same and $J_1$ $J_2$ are the polar moment of inertia of the shaft and the tube. From these two expressions, we can evaluate the values of $T_1$ and $T_2$ and thereby the stresses.

(Refer Slide Time: 51:38 - 53:51)




The equilibrium equation $500(10^3)$ Newton millimeter is equal to $T_1 + T_2$ which is equation 1 and $Theta_1$ is equal to $Theta_1$ is the second equation. From the first equation, $Theta_1$ we can write that $T_1$ times L by G times $J_1$ is equal to $T_2$ times L by G $J_2$. Here since G is the same GJ gets cancelled, L is also is the same and it gets cancelled. So, $T_1$ is equal to $J_1$ times $J_2$ times $T_2$.

Now $J_1$ for the solid shaft is equal to pi by 32 multiplied by the outer diameter which is 30 to the power of 4 and this gives us a value of 79521.6 mm to the power of 4. $J_2$ is equal to pi by 32 times 45 to the power of 4 as the outer diameter minus 36 to the power of 4 as the inner diameter and this gives us the value of 237682 mm to the power of 4. This is the value of $J_1$ and $J_2$ and consequently if we substitute here we get $T_1 = 0.335\ T_2$. We have $T_1 + T_2$ as equal to 500 multiplied by 10 to the power of 3. Here the value of $T_1$ is equal to 0.335 $T_2$. If we substitute over the equation one, we get the value of $T_2$ as equal to 374.532 Newton meter and we get $T_1$ as 125.468 Newton meter.

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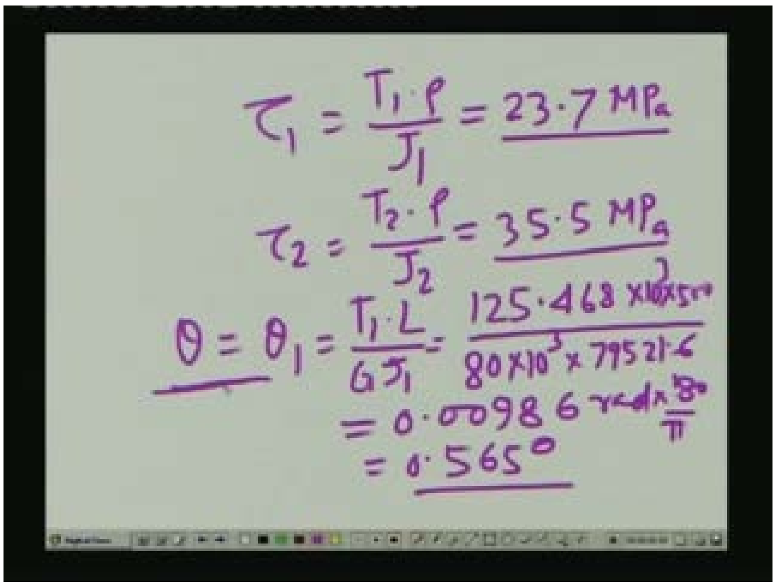
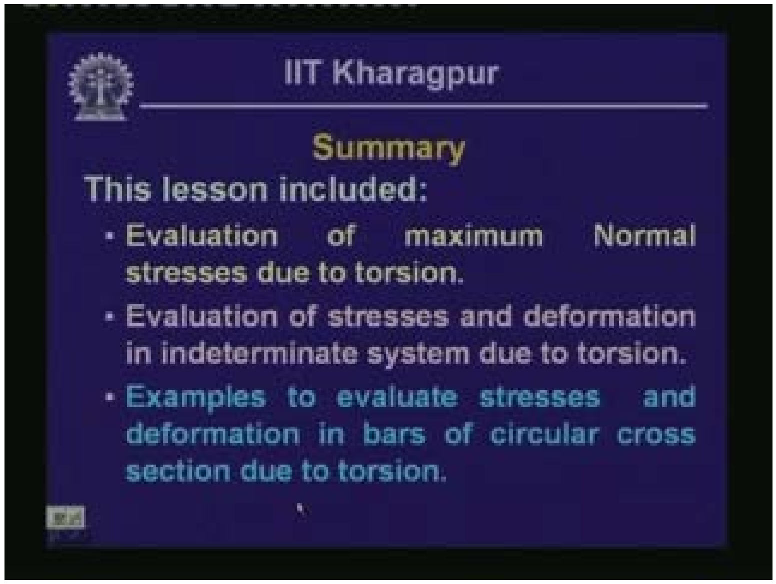



These are the values of $T_1$ and $T_2$ and correspondingly then the shear stress $Tao_1$ is equal to $T_1$ rho by $J_1$ and if we substitute the values, we will get 23.7MPa and $Tao_2$ again as $T_2$ Row by $J_2$ and if we substitute the values we will get 35.5MPa.
These are the values of the shearing stresses for the two elements.

We can compute for $Theta_1$ which is $T_1L$ by $GJ_1$. Let us substitute 125.468 multiplied by 10 to the power of 3 multiplied by L which is 500/G, which is 80 multiplied by 10 to the power of 3 times 79521.6.
If we substitute that, we get a value 0.00986 radian. If we multiply 180 by pi we get 0.5650. So, this is the amount of rotation that you have. Now if we compute $Theta_1$ we can automatically compute $Theta_2$ because $Theta_1$ is equal to $Theta_2$ and thus the rotation for the whole assembly.

Now if we want to compute the stiffness of the whole composite system as we know that T is equal to GL by J times Theta, GJ by L, the rotation for unit length is our stiffness. So, GJ by L can be computed as T by Theta. We know Theta and T and we know how much the torsional stiffness of the whole composite system is.

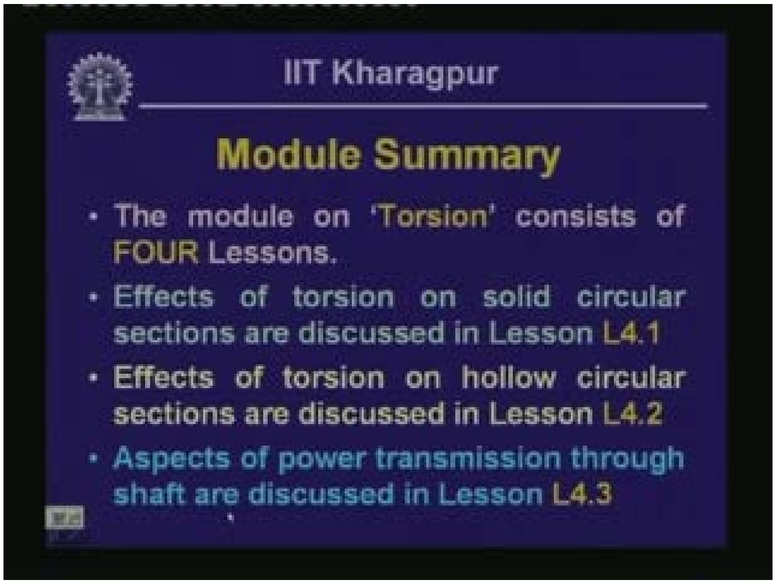
(Refer Slide Time: 55:40 - 56:25)




Let us summarize this particular lesson. We have evaluated the maximum normal stress due to torsion in the shaft and we have seen how the failure line occurs because of this maximum normal stress. If we compute the direction of the normal stress, the perpendicular direction to the normal stress direction is the one which is the failure line and it is 45 degrees, which you can see from the Mohr’s circle given.

Then we have seen the evaluation of stresses and deformation in indeterminate system due to torsion and then we have seen some examples to evaluate stresses and deformation in bars of circular cross section due to torsion. In this particular lesson, we have dealt with the indeterminate system and we have seen how the internal resisting twisting moment can be evaluated in case of an indeterminate system. In the case of a determinate system, we had computed directly from the equations of equilibrium. For an indeterminate system, apart from the equations of equilibrium, you need the equation of compatibility as well and then you can evaluate the value from the internal resisting twisting moment and you can compute the value of the stresses.

(Refer Slide Time: 56:59 - 58:01)




This is the last lesson on the module on torsion. We had 4 lessons in this particular module on torsion. In the first lesson, we had discussed the effect of torsion on solid circular sections and then we had seen how to compute the stresses and the angle of rotation because of the twisting moment when it is acting in a solid circular shaft. Consequently in lesson 2, we had looked into how we can compute the values of the stresses and the angle of rotation if the twisting moment acts in a hollow shaft that is if the shaft instead of the solid circular one is a hollow one. We have seen that in lesson 4.2.

In the third lesson which is lesson 4.3, we had seen the aspects of the power transmission through the shafts and we have seen the relationship between power and torque from which the stresses and the angle of rotation in the shaft that is utilized from one device to the other could be computed.

Lastly, in the fourth lesson which is this particular lesson we have discussed the evaluation of internal resisting twisting moment for an indeterminate system and consequently, the stresses and the deformation that occur because of the twisting moment.

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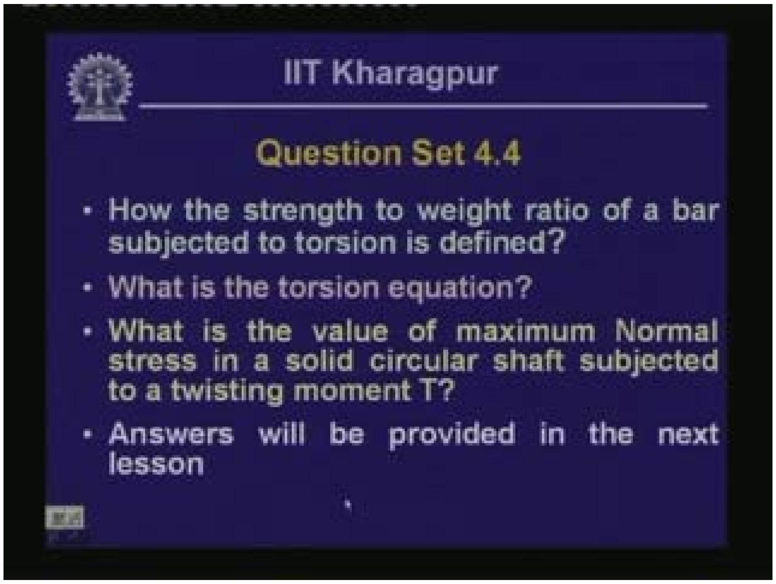


We have some questions set for you that are based on the discussions, which you have on this module. The strength to weight ratio is what you have evaluated. In fact, if you go through the lessons you will know how the strength to weight ratio of a bar subjected to torsion is defined. What is the torsion equation?

What is the value of maximum normal stress in a solid circular shaft which is subjected to a twisting moment T? Now this is the maximum normal stress which is subjected to a twisting moment that we have discussed and so you should be in a position to answer these questions.

We will give the answers for these questions in the next lesson.