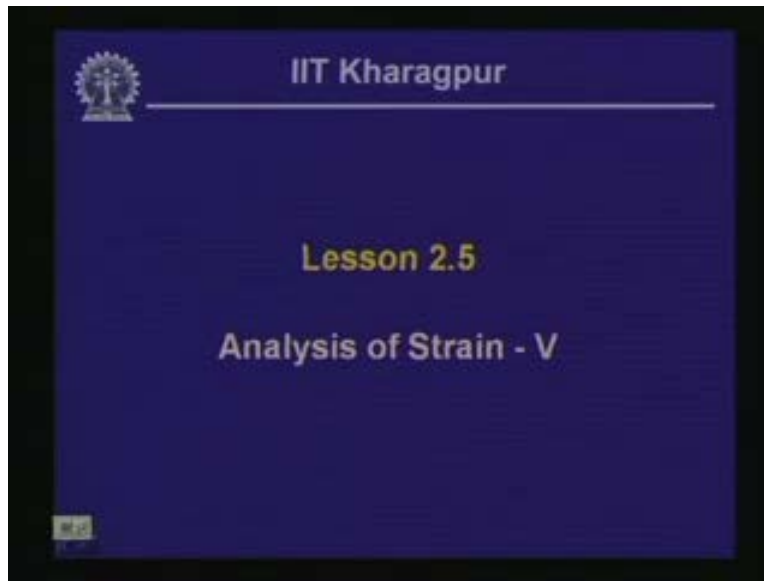



Strength of Materials
Prof S. K. Bhattacharya
Department of Civil Engineering
Indian Institute of Technology, Kharagpur
Lecture No #11
Analysis of Strain - V

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Welcome to the 5th lesson on module 2 which is on analysis of strain part 5. In the last lesson we had discussed certain aspects of strain which are caused due to the change in the temperature. Now in this particular lesson we are going to look into some more aspects of a strain which are caused due to change in temperature especially in bars or assembly which are made out of different materials. Also, we will look into certain aspects of misfit or lack of fit.

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Specific Instructional Objectives

- After completing this lesson one will be able to:
- Understand the concept of Strain due to change in temperature in compound bars.
- Understand the concept of misfit and pre-strains & pre-stress – Indeterminate system.

02:02

Once this particular lesson is completed, it is expected that one should be able to understand the concept of strain due to change in temperature in compound bars. Also, one should be able to understand the concept of misfit which we call occasionally as lack of fit and pre strains and thereby the pre stresses which are basically indeterminate systems so we will look into the aspects on these.

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Scope

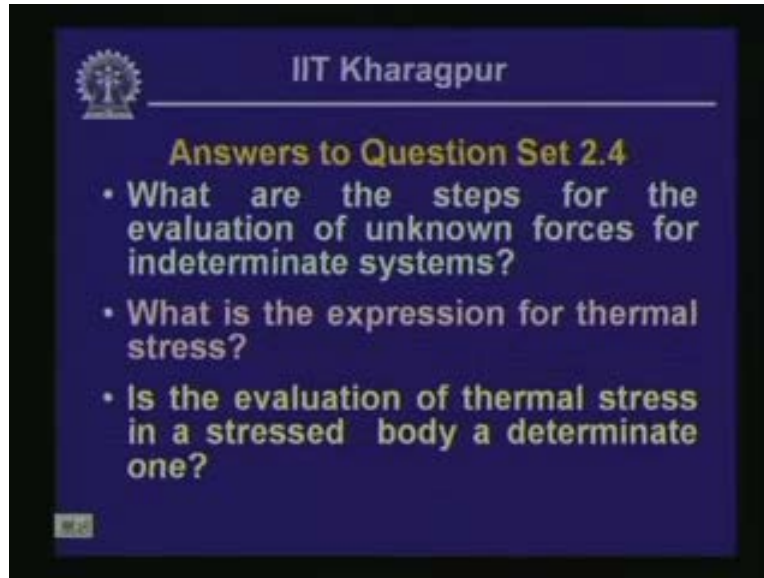
- This lesson includes:
 - Recapitulation of previous lesson.
 - Evaluation of strain in compound bars due to variation in temperature.
 - Concept of misfit, pre-strain and pre-stress.
 - Evaluation of stresses due to change in temperature in different systems and due to pre-strain.

02:11

Hence the scope of this particular lesson includes the recapitulation of previous lesson. Evaluation of strain in compound bars due to variation in temperature, then the concept of misfit

and the pre strain and thereby the prestresses. We will also look into the evaluation of stresses due to change in temperature in different systems and also due to pre strain in the systems.

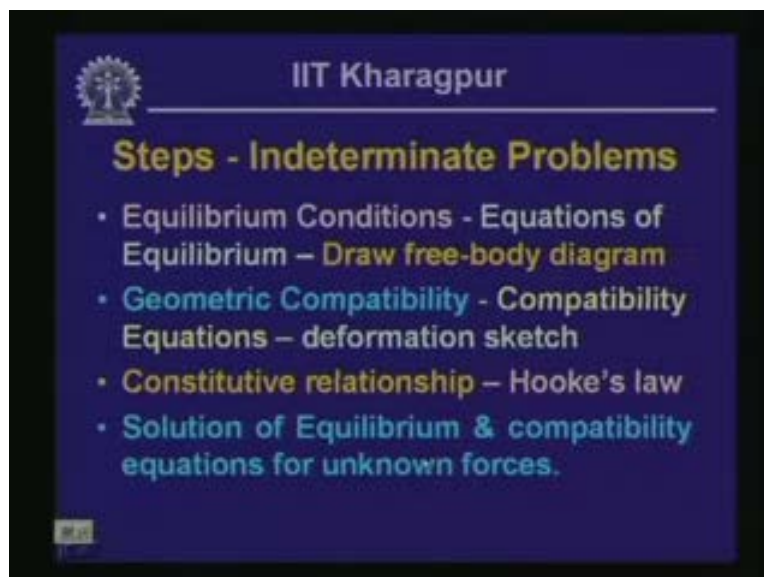
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Here are some answers for the questions asked earlier. What are the steps for the evaluation of unknown forces for indeterminate systems?

If you remember last time we differentiated between the determinate system and the indeterminate systems and we categorized the steps that are necessary for carrying out or evaluating the unknown forces for indeterminate systems. Now let us look into the steps that are necessary.

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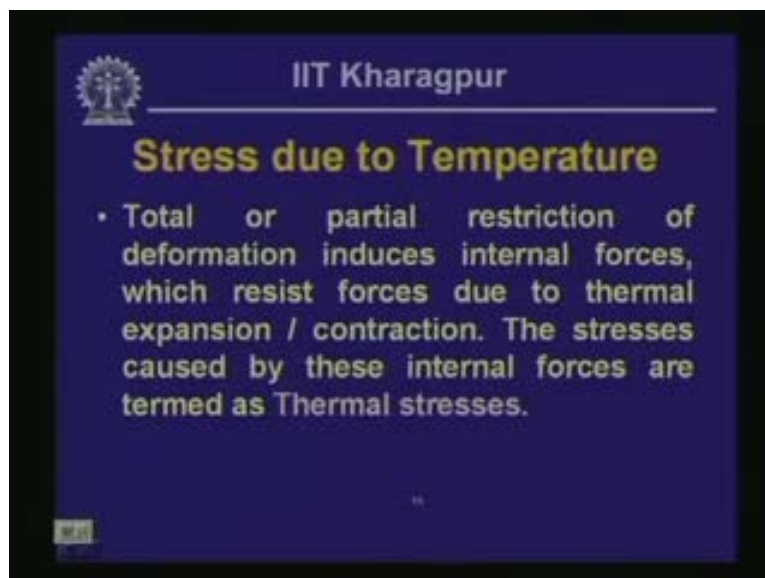
- 1) Equilibrium equation or the equilibrium conditions and these conditions or the equilibrium equations can be written down if we draw the free body diagram of the system given.

As we have seen when we had solved several problems we had taken the free body of the whole system and then from the free body diagram of the system we can write down the equations of equilibrium which states that summation of horizontal forces are 0, summation of vertical forces are 0 and the moment with respect to a point is equals to 0 in the system and that gives us the equilibrium criteria.

Secondly, the second step which is necessary for the solution now is, if the system is statically determinate then from the equations of equilibrium we can solve the system. But if the system is indeterminate then we need to solve or we need to write down the geometric compatibility which gives rise to the compatibility equation and this compatibility equation can be arrived at if we draw an exaggerated sketch of the deformation pattern. As we have seen through several examples though the deformation is small when we had drawn the exaggerated view of the deformation it becomes clearer that how the system is deforming and it becomes easier to write down the conditions thereby which we call as the compatibility condition which leads to the compatibility equation.

Once we write down the equations of equilibrium, equations of compatibility and then with the help of the constitutive relationship which gives us the relation between the strain and the stress we can write down in terms of the forces. So we have equations of equilibrium written down in terms of the forces; we have the equations of compatibility written down in terms of forces through the Hooke's law or the equations of constitutive relationship. Once we have these two sets of equations, we can solve for unknown forces so the solution of equilibrium and compatibility equations are utilized for the solution of unknown forces. These are the steps that are necessary for the solution of indeterminate problem.

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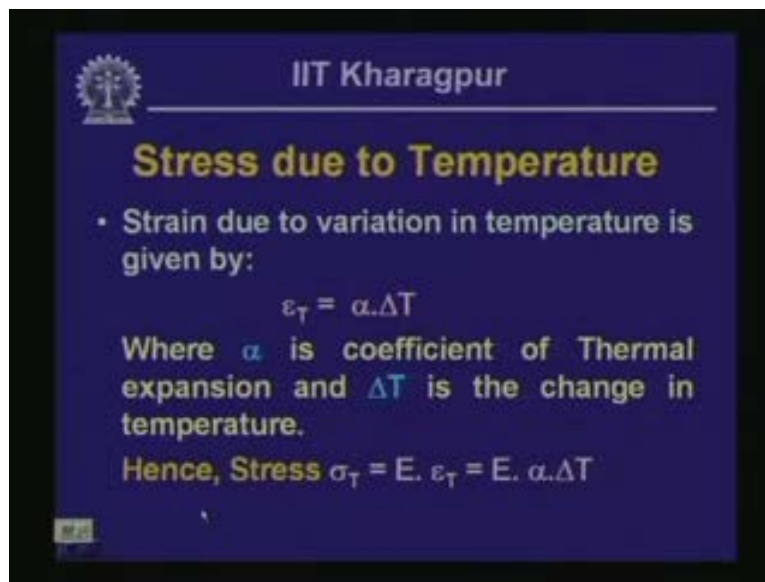


Now the second question which was posed was; what is the expression for thermal stress?

Before we evaluate the expression for thermal stress let us look into the definition of thermal stress. As we had defined last time the total or partial restriction of deformation induces internal forces which resist forces due to thermal expansion and these stresses caused by these internal forces are termed as thermal stresses.

If you remember, as I said that if you take a body and if this is allowed to undergo change in temperature, if it is not restricted or constrained, it will move or expand if the temperature rises. But then if we do not allow this bar to move freely that is if we put some restrictions, then the expansion will be restricted and thereby the internal force will be induced into the system. Now the stresses induced because of these forces caused due to the thermal expansion or due to the change in the temperature is called as the thermal stress. So, to evaluate the value of the thermal stress let us look into the expression which we had derived for the strain.

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The slide is a presentation slide from IIT Kharagpur. It has a dark blue background with white and yellow text. At the top left is the IIT Kharagpur logo. The title 'Stress due to Temperature' is in yellow. Below it, a bullet point states 'Strain due to variation in temperature is given by:' followed by the equation $\epsilon_T = \alpha \Delta T$. A line of text explains that α is the coefficient of thermal expansion and ΔT is the change in temperature. The final line states 'Hence, Stress $\sigma_T = E \cdot \epsilon_T = E \cdot \alpha \Delta T$ '.

Strain due to variation in temperature which is ΔT the final temperature minus the initial temperature is given as..... Now, if the deformation due to temperature is written as ΔL this is the deformation of the member. Now, when the body is allowed to move freely that means it is undergoing expansion due to rise in temperature and we call that expansion as ΔL . If we do not allow this expansion to happen in this member that means we are putting some restriction so thereby we are imposing some force into it which will bring back this deformation to its original position and because of that this deformation ΔL should be equal to the deformation generated because of the applied force which is equal to $\frac{P L}{A E}$ by $\Delta L = \frac{P L}{A E}$.

Now the parameter P by A the force divided by the cross sectional area we can write as stress. So stress σ is equal to $\frac{P}{A} = \frac{E \Delta L}{L}$ and this L gets cancelled out. So the where E is the modulus of elasticity of the material α is the coefficient of thermal expansion and ΔT is the change in temperature so this is the expression for the stress due to the thermal effect or thermal stress.

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Stress due to Temperature

- Strain due to variation in temperature is given by:
$$\epsilon_T = \alpha \cdot \Delta T$$
$$\delta T = \epsilon_T \cdot L = \frac{P L}{A E}$$

Where α is coefficient of Thermal expansion and ΔT is the change in temperature.

Hence, Stress $\sigma_T = E \cdot \epsilon_T = E \cdot \alpha \cdot \Delta T$

Let us look into the last question which we had posed which is the evaluation of thermal stress in a stressed body a determinate one. The variation of the temperature is basically not possible to solve using equations of equilibrium and hence they are not statically determinate.

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- Forces developed due to temperature changes can not be determined from equilibrium equations only. Hence these are statically indeterminate and equation of compatibility is necessary.

So, forces developed due to temperature changes cannot be determined from equilibrium equations only and thereby these particular kind of systems are statically indeterminate or where there is a change in the temperature in the system when we evaluate the forces because of that or when we evaluate the thermal stresses due to change in temperature, the system is

statically indeterminate. We need the help of compatibility equations so that we can solve the stresses due to temperature change.

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What will be the effect of change in temperature or what is the effect of thermal stress in compound bars?

Compound bars are the ones where we have a system made out of materials of different types thereby their thermal expansion or contraction is of different magnitudes. Now if we look into a system which is quite common like bolt which is put into a sleeve and this is the bolt head let us say that this particular part is the outer sleeve and this is the bolt which has a threaded part here and this is the washer plate which is provided at the end.

Now the whole assembly is allowed to undergo a change due to temperature; let us say that there is a rise in the temperature the whole system undergoes change in temperature and let us assume that the thermal expansion for the outer sleeve is larger than the central bolt. So if we say the coefficient of thermal expansion for the sleeve is α_s so α_s is greater than the coefficient of thermal expansion for the bolt which is α_b . When this whole system is undergoing a change in the temperature, to look into how much stresses that is getting generated in the system because of the change in the temperature.

Let us proceed step by step; first we remove the head of the bolt and allow the sleeve and the bolt to expand freely because of the change in the temperature. In the second figure as it is shown we have removed the bolt head and allowed the whole system to expand freely. As a result since the thermal expansion coefficient of the outer sleeve is larger than the bolt inside it is expected that the outer sleeve will undergo more expansion than the central bolt, since it is free to move it has expanded.

Let us say that the expansion of the sleeve from its original position is equals to δ_1 this was the original position and it has moved to this particular position, so the distance that it is

moving is δ_1 which is due to the change in the temperature given as $\alpha_s L$ the expansion of the sleeve. Consequently the bolt also will undergo expansion but since the thermal expansion coefficient is less than the outer sleeve it will have relatively lower value than δ_1 and let us say that this was the original position of the central bolt and this is the position which has been reached after the thermal expansion so let us call that as δ_2 so $\alpha_b L$.

Now, when they are not constrained or not restricted they freely expand and having expansion or the deformation is δ_1 and δ_2 but. Since the bolt head is present it is not allowing it to move freely as a result there will be forces generated and thereby stresses generated. Now the force in the central bolt and the outer sleeve will be such that they come to a common position, thereby the central bolt will have a tensile pull and the outer sleeve will have a compressive force, these two forces are equal in fact this and this in a combined form will be equals to this.

So, in effect, if I impose these two forces their effect is 0 so this is being pulled and this is being compressed. Therefore as a result they will come to a common position which will be the stress position for the whole system. It has expanded freely by δ_1 the outer sleeve now because of the compressive force that is getting applied on the outer sleeve, this will come back to the position from this position to the final position here a third figure indicates the final position of the outer sleeve and the central bolt where both are in the same line having the same deformation.

Now to bring back to this particular position from the free expansion of the outer sleeve this much is the deformation that is required to be brought back by application of this compressive force, let us call that as the sleeve force P_s . If we call this as δ_3 , this is the amount of deformation that will be generated in the sleeve because of the application of this force P_s . Now to bring to this particular position to the central bolt we need to apply a tensile pull so that that is elongated by this much of amount. This is the position after thermal expansion and this is the position where we need to bring this bolt in. Therefore this is the amount of deformation that is undergoing in the bolt which is equal to δ_4 . So these are the four deformations that at different stages these numbers are undergoing the final deformation this is the final form from its original length which is equals to δ . Thereby δ equals to δ_1 will be equals to δ_1 was the free moment of the sleeve because of the temperature and it has been brought back to the final state by application of the force P_s which is δ_3 . So δ_1 minus δ_3 is the final position δ for the outer sleeve.

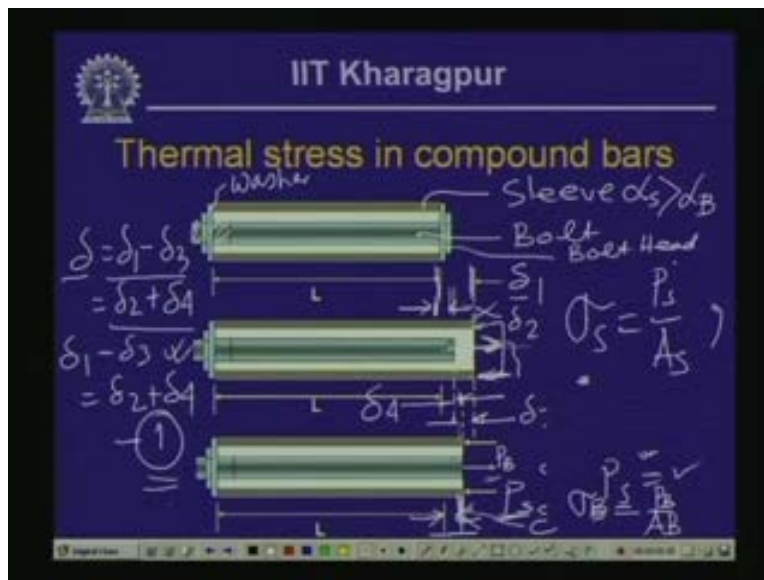
For the central bolt the free thermal expansion was δ_2 . From its initial position it has expanded to δ_2 . Now to bring this to the final position of the combined system it has to be expanded further by application of this tensile pull P_b which is δ_4 . So this is equals to δ_2 plus δ_4 so the final deformation in the whole assembly of the compound bar equals to δ . So the final deformation δ is or so we can write the compatibility equation now as.....

In addition to this we need to have the equation of equilibrium for this particular system. If we look into the equation of equilibrium they are the forces which had been applied to bring back to its final form which is P_b equal to P_s so this is the equation of equilibrium 2 and this is the equation of compatibility 1. We have two equations now and this particular equation is written in

terms of forces. If we substitute the values of δ_1 , δ_2 , δ_3 and δ_4 we write them down in terms of forces and α_1 and α_s .

Finally we can write down the expression for using these two equations P_b is equal to P_s and the expression for δ s which we can convert in terms of forces. So we can evaluate values of P_b and P_s which are in terms of $\alpha \Delta T$ the length and of course the cross sectional area of the numbers and the modulus of elasticity of the material that is being used. Once we know the values of P_b and P_s we can compute the stresses in the number. We can compute the final value of δ which is or which will give the value of δ , which is the final deformation in the whole assembly. So, from the values of P_b and P_s we can calculate the values of stresses which are σ_b and σ_s ; and σ_s the stress in the sleeve.

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So in the process we can compute the values of the stresses and also we can evaluate the value of the deformation δ for such compound system.

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Now let us look into another term we call as misfit or another term which we call pre strain. Many a times we get such structural system consisting of several members now it may so happen that any of the members of the whole structural assembly are not the exact length as it should be. Either it is longer or it is shorter. Now if that happens, when we try to assemble the whole structural system using these individual members, the length which is shorter or longer causes some problem in the assembly. This is what is termed as misfit, it is not fitting appropriately in the system and that is why this particular term misfit comes in.

If the structural system is a determinate one, let us say we have one bar which is pinned at this end and this bar is connected by another bar and connected at this position. Let us assume that this AB and this is CD. Now, if this particular member of the structure CD is not exactly to the length as it should be; let us call this length as l , if this particular member is having some defect and the length is not perfectly l , either it is smaller or larger. Eventually this member is not going to fit in the whole structural system and this we generally designate as misfit.

Many a times of course this misfit is introduced in the system intentionally, many a times it is required to have a system where we like to introduce some kind of strain beforehand. When we introduce this misfit in the system, thereby we try to introduce some amount of strain before really it is loaded and thereby subjected to strains. So when we form such structural system where we introduce the strain beforehand, we call that particular system as pre strained system and thereby from the strain, the stresses can be evaluated and we call those kinds of systems as pre stressed system.

One of the examples which are very common is the spokes in a bicycle wheel where the spokes are in fact strain beforehand so that the whole wheel remains in position. If they are in loose form it will collapse. Coming back to this example when this particular system is a determinate one this misfit is not going to cause any problem to the structural system. That means if this particular member CD is longer, then what adverse can happen is the bar AB may have some

geometric imperfection in terms of rotation. It may not look straight as it should be or if the length is shorter than l , then it might so happen that the horizontal bar will no longer remain horizontal but it may rotate to certain extent but in the process what is going to happen is that the whole structural system will not be subjected to any kind of strain or stresses.

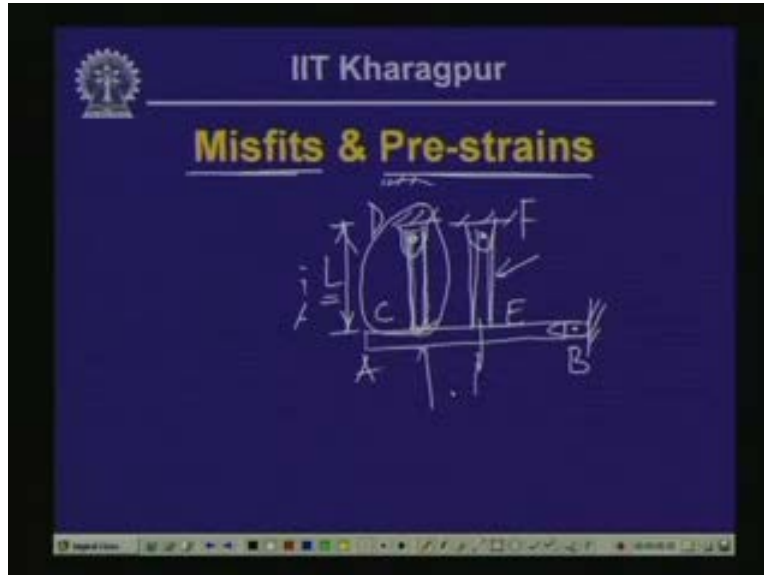
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On the contrary, if we look for structural system which is indeterminate and if we have any member which is shorter or longer than its actual length then that induces strains or the stresses in the system. So, if we consider an example where the system is an indeterminate one let us look into that; say we have a bar which is connected at this point and we have two bars connected at this point, this is a, this is b, this is c, this is d, this is e, this is f.

Now let us assume that the length of both bars CD and EF is l . Now if one of these bars CD is longer than the length l then there will be problem that we cannot fit in the member CD in the whole structural assembly as a result what is going to happen is that. If we like to fit in that longer bar in the whole system what we need to do is, we stretch this whole assembly fit in the bar CD in its position and place them in position and release this force. As a result what is going to happen is the whole of assembly the bar EF bar AB. They will be subjected to some amount of strain and there will be stresses induced in the bar. So, if the system is indeterminate one if we have length of the member not exactly as we desire then there is a possibility that the structural system will be subjected to some amount of strain and these strain or the stresses are pre introduced in the system before the system is loaded or any loads are applied on that system. This is what we call as pre-strains or pre-stressing and many a times we introduce this kind of pre straining or pre stressing in the system intentionally to avoid some problem in some situations, which we will look into at related stage.

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Then let us see the consequences if we have this kind of misfits or pre-strains.

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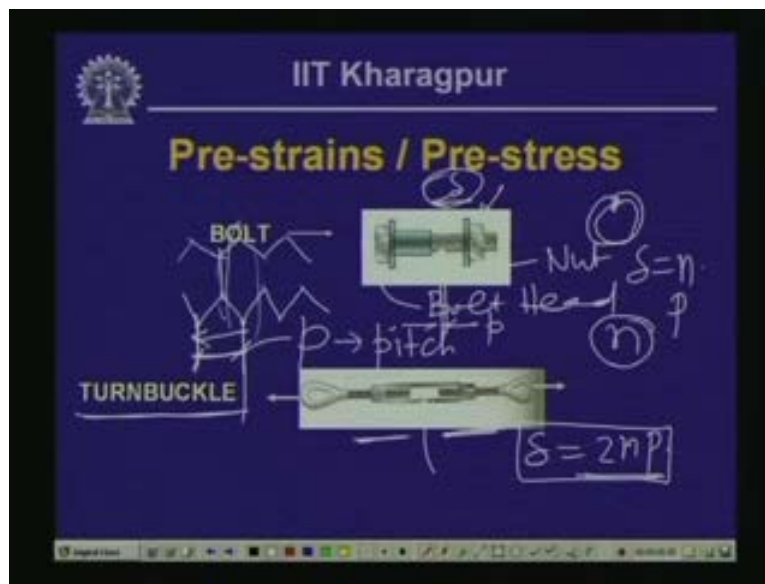


The simplest way to introduce these pre strains or the pre stressing in the system is the use of the bolts. Many times we encounter this kind of a bolting system or the nut bolt system which is shown here. This is the bolt having the nut here and this is the bolt head now when this nut is rotated if we start from one point and rotated to one complete round the nut moves forward and this moment is over a length in the threaded part of it which we commonly term as pitch. So, if we look into that threaded part which we have in this form when the nut is rotated in and around the threaded part the nut moves ahead and when it moves from one point to the other by giving a

complete 360 degrees rotation it moves ahead over these two peaks which we designate as the value p which you call as pitch. If there are n number of turns if we turned the nut by n then the total moment of the nut if you call that as δ , it is going to be equals to n times the pitch p .

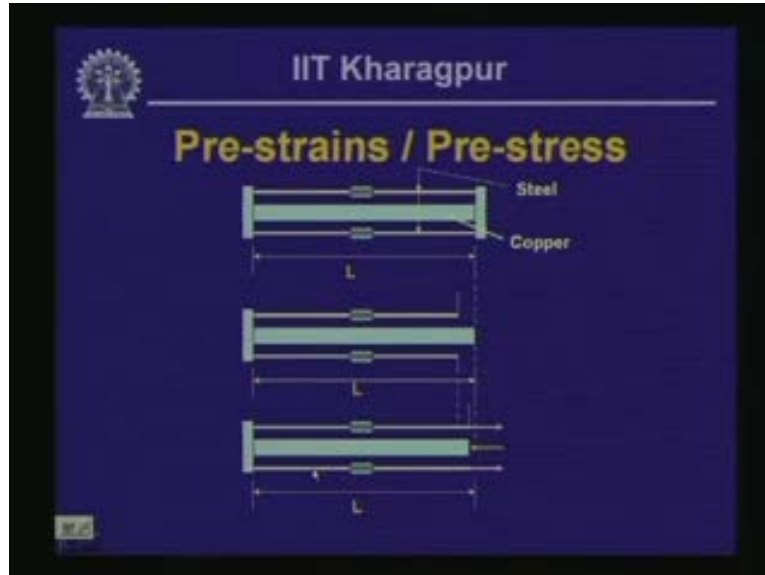
Another system which many a times are used is known as the turnbuckle. In the turnbuckle what will happen is that the threaded part is introduced on either side and this particular system which is a turnbuckle which, once it is rotated both the threaded part moves. Once the central part of the turnbuckle is rotated, since we have threads on either side both sides move forward or backwards by one pitch. So if we have one complete turn of the turnbuckle then we have two moments which is equals to $2p$. If we rotate the turnbuckle n times then the total moment which we expect of the threaded part is equals to $2n(p)$. In case of turnbuckle the deformation δ of the central part or the threaded part we can call it as is equals to $2np$.

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Many a times some deformation is introduced in the system by using the turnbuckle and we introduce some amount of pre straining or pre stressing in the member. If we have this kind of pre straining in the system then the evaluation of the effect of such pre straining or the evaluation of the stresses are exactly same as that of the effect due to temperature change. So the way we have analyzed a system for thermal stresses due to change in temperature, we carry out exactly similar steps to evaluate the effect of such pre straining system. Or in other words, we write down the equations of compatibility and equation of equilibrium and then we relate the strain to their stresses through constitutive relationship and we evaluate the unknown forces and thereby the stresses that is introduced in the assembly.

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Now let us look into that, if we introduce this kind of pre straining then how do we get the stresses in the system?

Let us say we have an assembly where we have a central shaft which is made out of copper and we have two steel wares on two sides, where two turnbuckles are introduced. This is the turnbuckle and this is also the turnbuckle. In the first instant as we have done in the case of thermal analysis, in the case of the evaluation of the stresses because of the change in the temperature we go exactly in the similar steps to evaluate the stresses due to such pre-straining.

We removed this head which is the rigid head or rigid plate and we introduced n number of turns in the turnbuckle. Both the turnbuckle is turned in times so that there is shortening of the bar. That means the threads move inside, so since the pitch as we have defined as p because of the turning of one revolution of the turnbuckle. Both the threads are moving by p square so n number of turns is there so total deformation is equals to $2np$. So let us call that this deformation which we introduced is equal to δ_1 which is equals to $2np$.

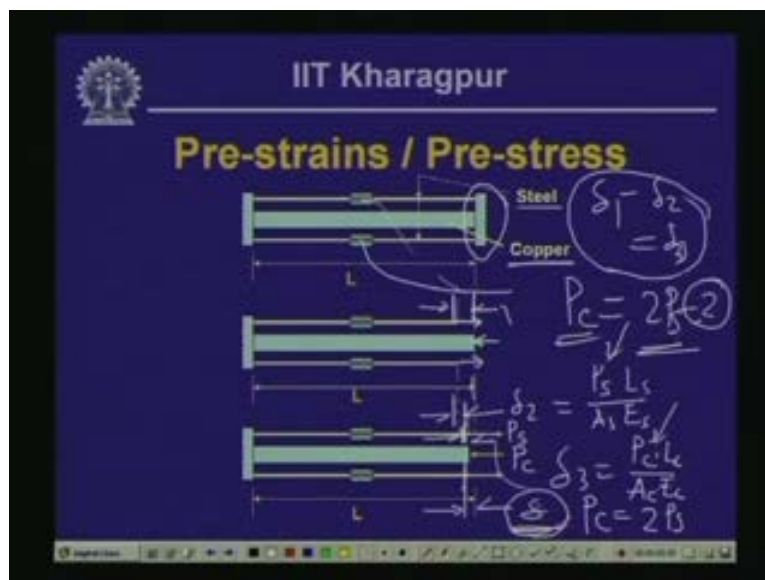
Now this is the original position of the central copper bar and if we like to look for a system wherein we will pull this sleeve or the steel ware to a position and compress this central copper to a position where they will come to a common position, then the steel ware will be extended because of the pull. And because of the compressive force the copper bar will be compressed so the final deformation should be the same in both because of the tensile pull that we are going to apply on the steel ware let us say from this position to this position when it comes if you call that as δ_2 this is equals to the force in the steel bar as $P_s(l)$ by A_e or A steel E steel.

So this is the deformation that has been occurring in the steel bar because of the application of the force P_s . The compressive force that is being applied into the copper bar if we call that as P_c then the deformation which is from this position to this position you call as δ_3 . The final form which is from original length to this particular length equals to δ and this δ is nothing but equals to δ_1 minus δ_2 so we can write the condition that δ_1 minus δ_2 is

equal to δ_3 . So this is the compatibility criteria which is occurring because of the application of the pre straining in the system also the equilibrium equations if we write that we have two steel wares which had been pulled by a force P_s and we have the central copper bar which is been compressed using the force p_c .

Therefore in an equilibrium situation p_c should be equals to $2p_s$. We have the equation of compatibility which is δ_1 minus δ_2 is equal to δ_3 which can be written in terms of P_s , P_c and we have equilibrium equation which is p_c is equal to $2p_s$ which we can call it as equation of equilibrium and these two equations in combination will give us the values of P_c and P_s . Once we know the value of p_c and P_s we can compute the stresses in the copper bar and the steel ware and thereby we can compute the final deformation that will be occurring in the whole assembly.

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Similar to the one as we have done in case of evaluation of thermal stresses we can evaluate the stress and the deformation in an assembly where we introduce pre-straining or the pre-stressing in the system.


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Example Problem - 1

- A rigid block - mass M - supported by three rods. The ends of the rods were level before the block was attached. Determine largest allowable value of M .
 $A_s = 1200 \text{ mm}^2$; $A_{cu} = 900 \text{ mm}^2$; $E_s = 200 \text{ GPa}$; $E_c = 120 \text{ GPa}$; $\sigma_{cu} = 70 \text{ MPa}$; $\sigma_s = 140 \text{ MPa}$.

determine the lengths of the two copper rods so that the stresses in all three reach their allowable limits simultaneously.



Here is a problem to be solved. We have a rigid block of mass m which is supported by three rods: two copper rods and one steel rod, one steel central steel rod and two copper rods here and here. Of course we had evaluated the stresses, it was asked to evaluate the largest value of m which we evaluated from the criteria that the deformation of the copper bar is equals to the deformation of the steel bar or δ_c is equal to δ_s that is the final deformation which will happen in the system. The criteria of equation of equilibrium are that the mass is being supported by three bars. So we had written down the equation of equilibrium and the equation of compatibility and the compatibility criteria comes from the δ_c .

The deformation in the copper bar is equals to the deformation in the steel bar and finally they will be in the same length, it will be a level surface. We had solved that we had evaluated the value of m now subsequently it is told that we need to determine the lengths of the two copper rods. We will have to find out the length of the two copper rods if it is different from one sixty so that the stresses in all three reach their allowable limits simultaneously.

Now the point to be noted is that in the previous stage we had evaluated the value of m from the criteria that the deformation in the member is same. Thereby since they are of different lengths, different strains and correspondingly different stresses we would like to find out what should be the length of the copper rod, so that both steel and the copper rod reach to the limiting value of their stresses. So the allowable stresses for these two members are given for copper and steel and they are 70 MPa and 140 MPa .

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Example Problem - 1

- A rigid block - mass M - supported by three rods. The ends of the rods were level before the block was attached. Determine largest allowable value of M .

$A_s = 1200 \text{ mm}^2$; $A_{cu} = 900 \text{ mm}^2$; $E_s = 200 \text{ GPa}$; $E_c = 120 \text{ GPa}$; $\sigma_{cu} = 70 \text{ MPa}$; $\sigma_s = 140 \text{ MPa}$.

$\delta_c = \delta_s$

determine the lengths of the two copper rods so that the stresses in all three reach their allowable limits simultaneously.



Now if we have to utilize the full stress of these two elements then what will be the length l of this particular?

Now let us look into that, if we go into the full limiting stress then what will be that value of the forces that they can undergo. Load that can be carried by the steel bar is equal to the sigma allowable multiplied by the cross sectional area, now since the limiting stress is 140 Mpa for steel bar and the cross sectional area is 1200 mm square. Then the maximum load that can be carried by the steel bar equals to 168 kN.

Similarly, the maximum load that can be carried by the copper bar is equal to the allowable stress in the copper multiplied by the cross sectional area of the copper rod which is equal to 70 Mpa into 900 is the cross sectional area so this is equal to 63 kN. These are the values that the rods three rods the copper rod and the steel rod that they can carry. If we go up to the limiting value of those two rods, then we can compute still considering that the deformation of these members will be the same from those particular criteria. We can find what length we need to achieve the full utilization of the two bars. So, from the compatibility criteria that the deformation in the copper rods will be equal to the deformation in the steel rod thereby it gives us that P_1 by A_c , a is the cross sectional area of the copper rod; e is the modulus of elasticity so this is equal to.....

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Handwritten calculations on a whiteboard:

$$P_s = \sigma_{su} \times A$$
$$= 140 \times 1200 \text{ mm}^2$$
$$= \underline{168 \text{ kN}}$$
$$P_c = \sigma_{cu} \times A_{cu}$$
$$= 70 \times 900 \text{ mm}^2$$
$$= \underline{63 \text{ kN}}$$
$$\underline{\delta_c} = \underline{\delta_s} \quad \frac{P_c L_c}{A_c E_c} = \frac{P_s L_s}{A_s E_s}$$

.....and this is equal to, if we compute this comes as this is equals to..... so this gives us the value of L_c the length of the copper rod should be equals to 288 mm.


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Handwritten calculation on a whiteboard:

$$\frac{63 \times 10^3 \times L_c}{900 \times 120 \times 10^3} = \frac{168 \times 10^3 \times 240}{1200 \times 200 \times 10^3}$$
$$L_c = 288 \text{ mm}$$

So the length of the copper rod if it is changed from earlier 160 mm to 280 mm then either both the rods or all three rods the two copper rods and the steel rod will reach their limiting values or their limiting stresses 140 Mpa and 70 Mpa simultaneously and for that we will have to make the length of the copper rods as 288 mm.


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Example Problem - 2

- All members of the steel truss shown in figure have the same cross sectional area. If the truss is stress-free at 10°C , determine the stresses in the members at 90°C . For steel $\alpha = 11.7 \times 10^{-6} / ^{\circ}\text{C}$ and $E = 200 \text{ GPa}$.



Here is the second example in fact this is the example which we had set last time this is a truss which is undergoing change because of the change in the temperature. All members of the steel truss have the same cross sectional area and the truss is stress free at ten degree centigrade now we will have to determine the stresses in the members at 90 degree C the coefficient of thermal expansion for steel is 11.7×10^{-6} by degree C and the value of E is 200 Gigapascal. So we need to evaluate the stresses in the member when the temperature rises to 90 degrees. Now here this system is not subjected to any forces as such but is undergoing change in temperature and obviously the supports are unyielding or do not have any effect because of the change in the temperature.

Now, as we have done before we need to compute the equations of equilibrium, we need to write down the equation of compatibility and then we write down the constitutive relationship between the stress and the strain so that we can evaluate the unknown forces as the stresses induced because of the change in temperature is indeterminate.

If we look into that because of the change in the temperature, the members will undergo elongation because the temperature is rising from 10 to 90 degree C. Thereby as we know that $\Delta l = \alpha \Delta T (l)$. Since this is the length of this inclined member let us call that A B C and D length of member AB and BD. They are actually 3m so the length of member BC which is $3 \sin 30$ degrees length, BC is equal to $AB \sin 30$. So this is equal to 1.5m, this is the length of BC.

Now if we take the free body diagram at this particular point ad joint p then we have three force components that are acting force in BC, force in member BD and force in member BA. So this is FBA, this is FBC and this is force BD. Now if we take the summation of horizontal forces equals to 0 then it will give us FBA is equal to FBD and summation of vertical forces with 0 will give us that FBC plus FBA cos of this angle which is 60 degree FBA cos of 60 plus FBD cos 60 is equal to FBC.

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$\delta = \alpha \Delta T \cdot L$
 $L_{BC} = L_B \cos 30^{\circ} = 1.5 \text{ m}$

So, the equilibrium equation will give us F_{BA} is equal to F_{BD} and F_{BC} plus $(F_{BA} \cos 60)$ plus $(F_{BD} \cos 60)$ is equal to 0. So F_{BC} since BA and BD they are same so this is $2F_{BA}$ and $\cos 60$ as up so F_{BC} plus F_{BA} is equal to 0. Thereby F_{BC} is equal to minus F_{BA} .

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$$F_{BA} = F_{BD}$$
$$F_{BC} + F_{BA} \cos 60^{\circ} + F_{BD} \cos 60^{\circ} = 0$$
$$F_{BC} + F_{BA} = 0$$
$$F_{BC} = -F_{BA}$$

Here because of the change in the temperature the central member is undergoing extension or there is a tensile force in the bar F_{BC} then F_{BA} and F_{BD} will be subjected to compressive forces. So the whole structural system having three bars will undergo expansion because of the temperature which is equal to $\alpha \Delta T (l)$ is the deformation because of temperature and because of the forces that is getting generated as they are constrained they are not free to move.

They will have tensile and compressive forces for which there will be deformation and if we equate that we will get the equation of compatibility. So, equation of compatibility in that sense is equal to, then if we say the expansion of the central bar as δ_{BC} plus the force in the bar BC (FBC) by a of the cross sectional area of the bar BC (A) which is for the bar BC. And since a and e are same we are writing as a and e and that is the deformation of the bar which should be equal to the deformation of the bar. Now for bar BD or we have the deformation equals to δ_{BD} or δ_{BA} which is again due to temperature which is $\alpha(\Delta T) L$ and for the force for the compressive force. We will have FBD or $F_{BD}(L_{BD})$ by $A E$.

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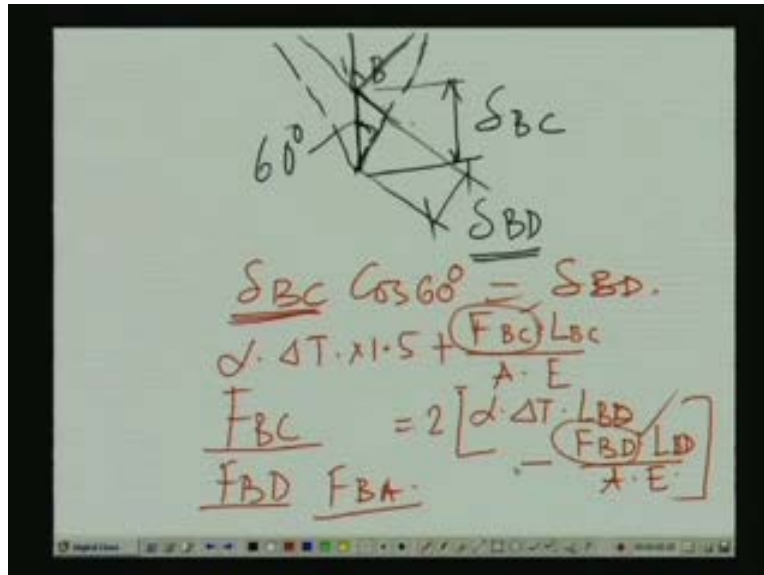
The image shows a whiteboard with handwritten equations in red ink. The top equation is $\delta_{BC} + \frac{F_{BC} L_{BC}}{A E} \rightarrow BC$. The middle equation is $\delta_{BD}, \delta_{BA} \rightarrow \alpha \cdot \Delta T \cdot L$. The bottom equation is $\frac{F_{BD} \cdot L_{BD}}{A \cdot E}$.

Now, if we look into the deformation configuration of joint b and if we exaggerate that let us say this is the original position of b this undergoes deformation and comes to this particular point. This was the original direction of the members and this is the deformed direction of the members which are BD and BA. Now this is the deformation that has been caused by member BC which we have called δ_{BC} and if we drop a perpendicular here, this is the length of the inclined member, basically it should move in an arc which is in a circular form but since the deformation is small we are taking this as a straight one so if we drop a perpendicular here, this is the deformation which is undergoing in the inclined member one. And if we call this deformation as δ_{BD} , this angle is 60 degrees in its original form. Since the deformation is small we still call this angle as 60 degrees hence the $\delta_{BC} \cos 60$ is equal to δ_{BD} .

So we can write $\delta_{BC} \cos 60$ degree is equal to δ_{BD} and the deformation which you have obtained in the member is $\alpha(\Delta T) L$ which is 1.5 m for this plus the force in the member BC times length of the member BC by AE is equal to $\cos 60$ is $\frac{1}{2}$ so twice of this so two times then we have $\alpha(\Delta T) L$ of the member BC minus the force which is length BD so force into BD times length into BD by AE. From this since we know the relationship between FBC and FBD so the only unknown is the force FBC and all other parameters are known so we can compute the value of FBC from which we can calculate the stress. Once we know the value

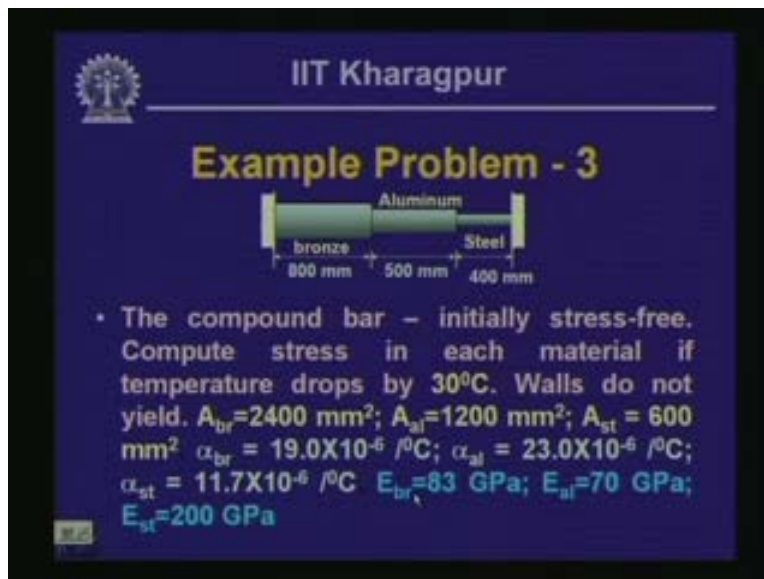
of FBC so forces in the members FBC FBD and FBA are known and force divided by the cross sectional area.

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Now since the cross sectional area is the same that will give us the stresses in the member.

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We have another example which is, we have a compound bar wherein we have three materials bronze aluminium and steel of length 800 mm, 500 mm and 400 mm and they are combined together and put within the two fixed walls. Initially they are stress free now we will have to find out the stress in each material on each part of this bar of the compound bar which is

made out of bronze aluminium and steel when the temperature drops by 30 degree C. So initially from some temperature it drops by 30 degree C considering that these walls do not deform.


We will have to find out how much stress this bar undergoes because of the drop in the temperature. Now because of the drop in temperature the way we have analyzed before if we compute the stresses that means if we remove one of these rigid walls then this bar will be free to expand or contract because of the change in the temperature. Since here temperature is reducing, is dropping down so this bar will undergo contraction, let us say this is the position where this bar comes from its original form. So this is the deformation delta now what we need to do is that to bring this bar in position we will have to apply a tensile pull in this which you call as p. This deformation when we calculate because of the change in the temperature as we know the deformation is alpha times delta t times l and over the length whole length l we have three bars 800, 500 and 400. So we can compute the values individually and calculate delta, after computing delta you can find out the value of p and equate this to the total strain is equals to the strain due to p.

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The slide features the IIT Kharagpur logo and title at the top. Below it, the text 'Example Problem - 3' is written in yellow. A diagram shows a compound bar with three segments: bronze (800 mm), aluminum (500 mm), and steel (400 mm). Handwritten notes include 'd. ΔT · L' and a stress-strain diagram with 'σ' and 'P'. The problem text is as follows:

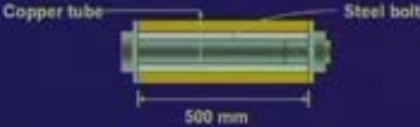
• The compound bar - initially stress-free. Compute stress in each material if temperature drops by 30°C. Walls do not yield. $A_{br}=2400 \text{ mm}^2$; $A_{al}=1200 \text{ mm}^2$; $A_{st} = 600 \text{ mm}^2$ $\alpha_{br} = 19.0 \times 10^{-6} / ^\circ\text{C}$; $\alpha_{al} = 23.0 \times 10^{-6} / ^\circ\text{C}$; $\alpha_{st} = 11.7 \times 10^{-6} / ^\circ\text{C}$ $E_{br}=83 \text{ GPa}$; $E_{al}=70 \text{ GPa}$; $E_{st}=200 \text{ GPa}$

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Example Problem - 4




Copper tube Steel bolt

500 mm

- What stresses will be produced in the steel bolt and copper tube? Quarter turn of bolt is applied. Pitch = 3mm; $A_s = 600 \text{ mm}^2$; $A_{cu} = 1200 \text{ mm}^2$. $E_s = 200 \text{ GPa}$; $E_{cu} = 80 \text{ GPa}$

We have another problem, what stresses will be produced in the steel bolt and copper tube if we allow a quarter turn of the bolt pitch of the bolt is given as 3 mm and area of cross section is 600 mm square and with this parameter you need to compute the value of this stress.

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Summary

This lesson included:

- Concept of Strains in compound bars due to change in temperature.
- Concept of misfit, pre-strain and pre-stress.
- Examples to demonstrate the evaluation of stresses due to change in temperature and pre-strain.

In summary we discussed the concept of strain in compound bars due to change in temperature, we have seen the concept of misfit pre strain and pre stresses and some examples to demonstrate the evaluation of stresses due to change in temperature and pre-straining.

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Question Set 2.5

- What is meant by misfit and what are its consequences?
- What is the principle of a double acting turn-buckle?
- What is the pitch of a bolt and how is it related to the displacement of the nut?
- Answers will be provided in the next lesson

Here are some questions;

What is meant by misfit and what are its consequences?

What is the principle of double acting turnbuckle and what is the pitch of a bolt and how is it related to the displacement of the nut?