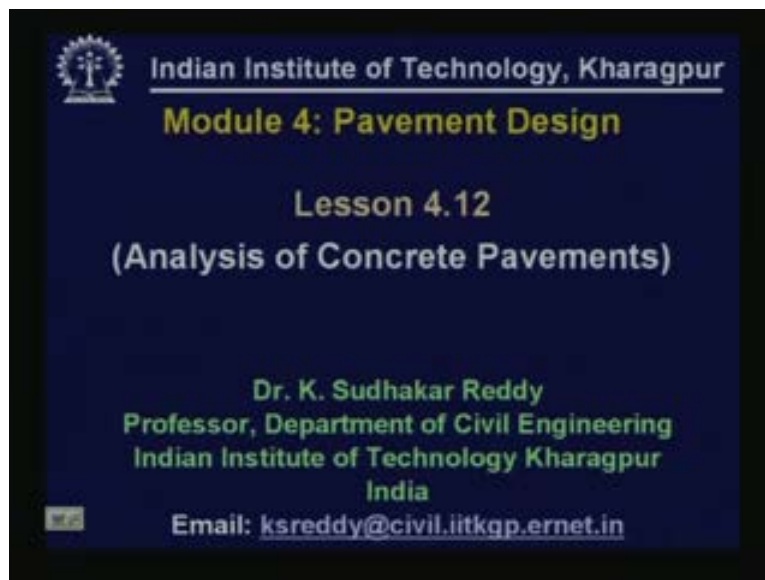


Introduction to Transportation Engineering
Prof. K. Sudhakar Reddy
Department of Civil Engineering
Indian Institute of Technology Kharagpur
Lecture - 35
Analysis of Concrete Pavements

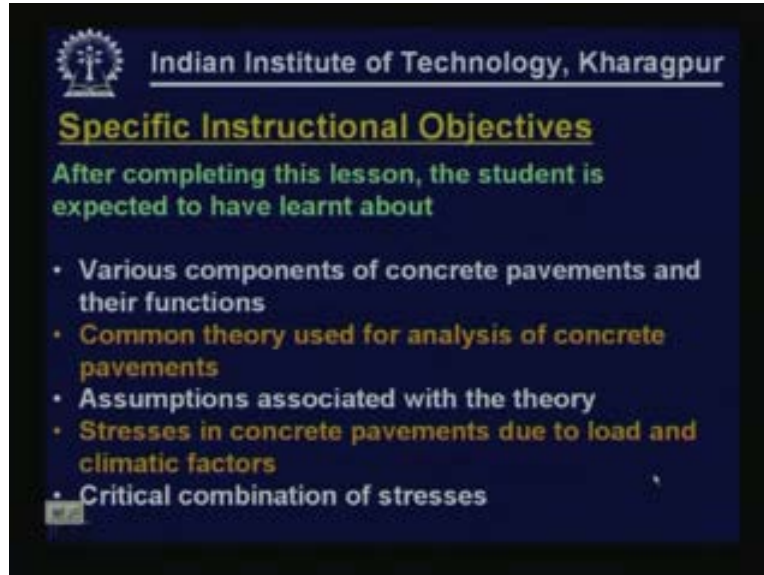
Hello viewers welcome to lesson 12 module 4.

(Refer Slide Time: 00:00:55 min)



As you know module 4 is on pavement design. This lesson will be about analysis of concrete pavements. You may recollect that in lesson eleven we have discussed about analysis of flexible pavements. Once we complete the theory that is required the models that are required to analyze both flexible type of pavement and also rigid type of pavement then we will be in a position to start discussing various design procedures that are existing.

(Refer Slide Time: 00:01:32 min)



The slide is a presentation slide from the Indian Institute of Technology, Kharagpur. It has a dark blue background with white and yellow text. At the top left is the IIT Kharagpur logo. The title 'Indian Institute of Technology, Kharagpur' is at the top right. Below the title, the section 'Specific Instructional Objectives' is written in yellow. Underneath, a statement in white says 'After completing this lesson, the student is expected to have learnt about'. A bulleted list follows, with each item starting with a white dot. The list items are: 'Various components of concrete pavements and their functions', 'Common theory used for analysis of concrete pavements', 'Assumptions associated with the theory', 'Stresses in concrete pavements due to load and climatic factors', and 'Critical combination of stresses'.

Indian Institute of Technology, Kharagpur

Specific Instructional Objectives

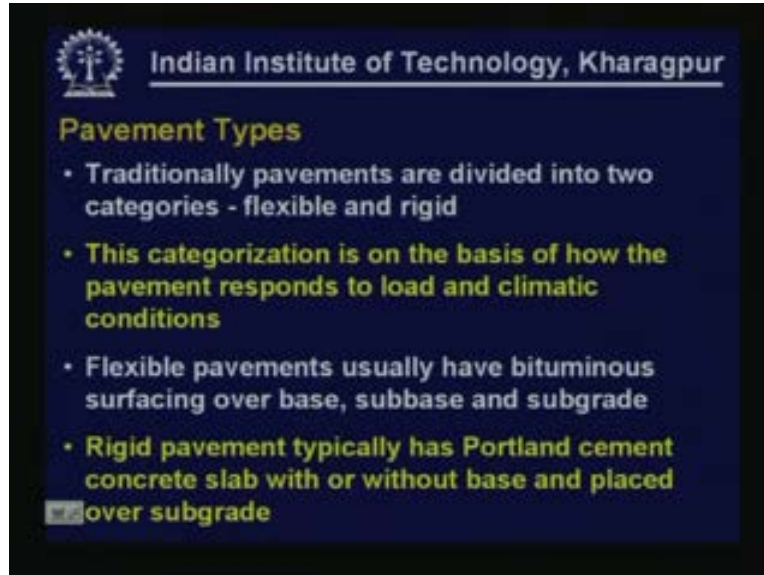
After completing this lesson, the student is expected to have learnt about

- Various components of concrete pavements and their functions
- Common theory used for analysis of concrete pavements
- Assumptions associated with the theory
- Stresses in concrete pavements due to load and climatic factors
- Critical combination of stresses

The specific objectives of this lesson will be after completing this lesson it is expected that the student would have learnt about various components of concrete pavement, unlike flexible pavement concrete pavement is significantly different in the sense that it is not usually a continuous layer it is built in terms of number of slabs connected through joints or through different mechanisms so we need to understand the difference between how flexible pavements are constructed and how concrete pavements are constructed, and what are the main components that are there in concrete pavements.

So after completing this lesson it is expected that the student would have understood what the various components are and what is the function of each component in concrete pavement. It is also expected that the student would be familiar with the common theory that is used for analysis of concrete pavements and also about the assumptions that are associated with the theory. It is expected that the student would have some understanding of how to compute stresses using this theory such as stresses associated with load and also stresses associated with climatic and other factors. And, it also expected that the student would have an understanding of how to select a critical combination of loads and climatic conditions and the associated stresses.

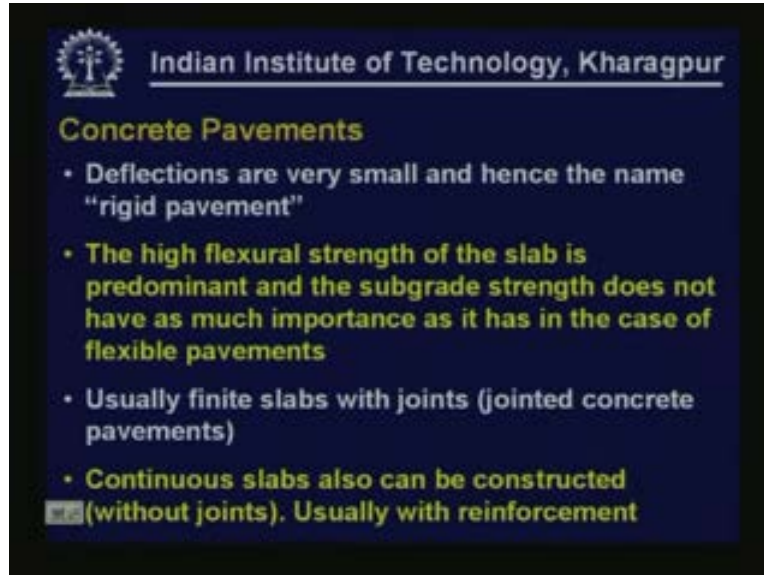
(Refer Slide Time: 00:03:16 min)



As we have already discussed in earlier lessons traditionally pavements are divided into two categories; flexible and rigid. This categorization is mainly on the basis of how the pavement materials or pavement as a whole responds to load and climatic conditions.

Flexible pavements usually have as we all know bituminous surfacing over granular base granular sub-base which in (()) (00:03:48) place or sub-grade. there will be flexible pavements without bituminous surfacing also so basically we will have sub-grade, sub-base surface it is either both or one of them and as surfacing it may be either bituminous surfacing or it can be simply granular surfacing also, it can come in other combinations also. As far as rigid pavement is concerned it typically consists of Portland cement concrete slab with or without a granular base and both these layers placed over sub-grade.

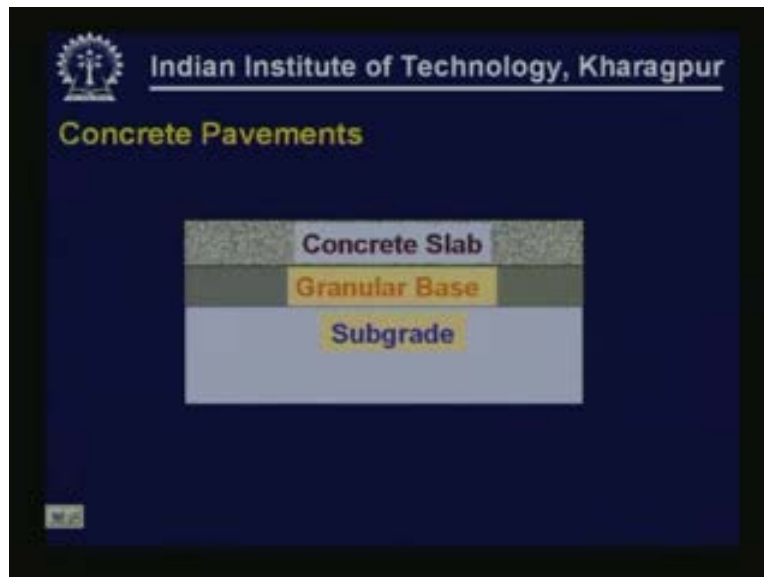
(Refer Slide Time: 00:04:27 min)



Concrete pavements are different from flexible pavement in the sense that the deflections when they are subjected to traffic loads are very small hence it is named rigid. Of course it is not truly rigid because they also deflect. the high flexural strength of the concrete slab is so predominant compared to the strength of the foundation that is mobilized so the strength of the foundation is not in fact very significant in the case of concrete pavements it is the strength of the slab or rather the stiffness of the slab that is more important whereas in the case of flexible pavements the strength of the foundation plays a major role in the performance of pavement.

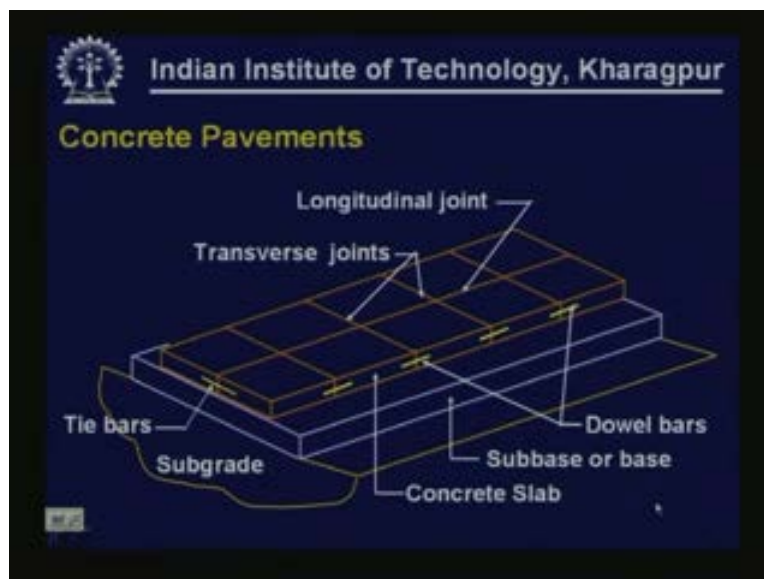
Concrete pavements are usually finite slabs with joints. You will say number of joints in a concrete slab. These are typically called as jointed concrete pavements. But you can also have continuous pavements without joints to avoid joints because joints are normally difficult to maintain and these are places of weakness, this is not very common in India. So if one wants to construct pavements without any joints it can be done but usually they are reinforced and called as continuously reinforced concrete pavements.

(Refer Slide Time: 00:05:49 min)



This is how a typical concrete pavement would look like. it will have sub-grade, sub grade though it shown to be of finite thickness it will be of infinite thickness, the top portion as we have learnt earlier will be of prepared foundation prepared to proper specification and strength over which you may have a granular base or sub-base depending on the requirement and over which we have a concrete slab.

(Refer Slide Time: 00:06:18 min)

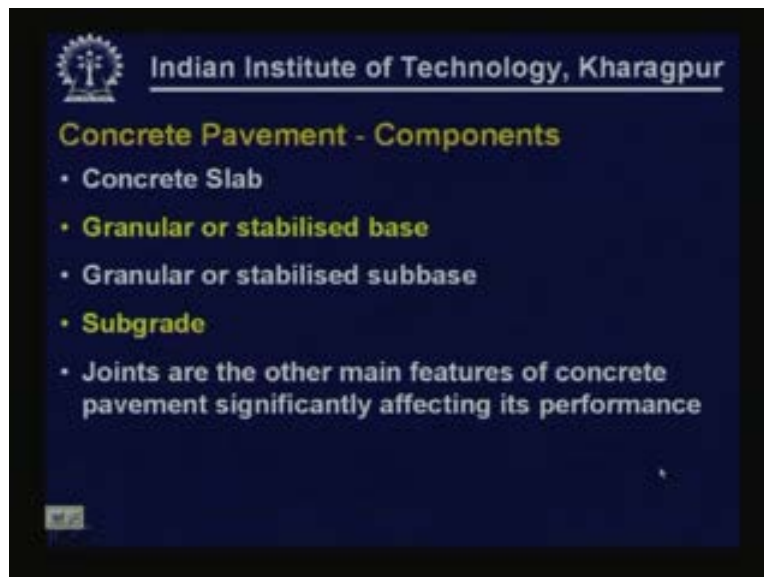


This is the general arrangement of a concrete pavement. right at the bottom you will have the sub-grade above which there will be a sub-base or base that is this layer (Refer Slide Time: 6:37) over which will be placed number of panels of concrete slabs.

And how many panels or what size will depend upon what is the width of the pavement that is being constructed, one lane, two lane or four lanes etc so accordingly you may see a number of lanes therefore the size of the slab also will be dependent on the width of the pavement that is being constructed.

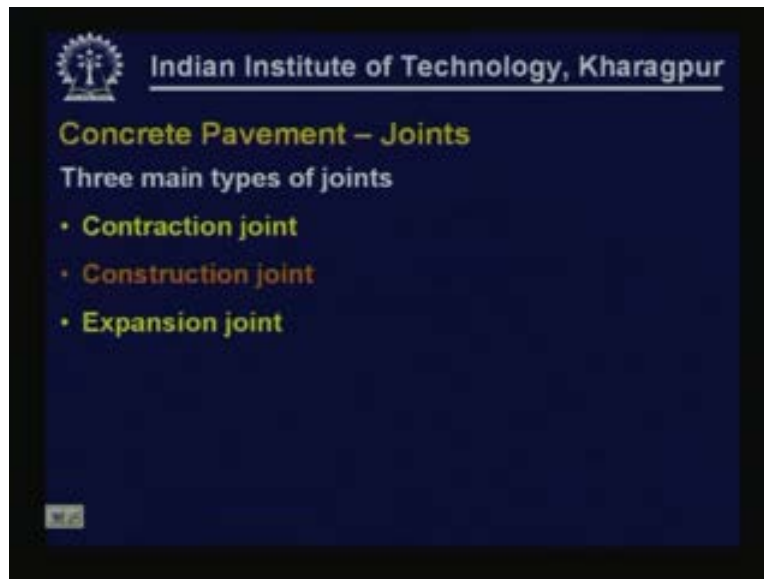
The other component that we see here are dowel bars. In fact these are bars that connect adjacent slabs. We also see another bar here which is known as tie bar they have different functions which we will be discussing in the subsequent slides. The longitudinal joint separates the panels that are placed in different lanes and there will be transverse joints at selected spacing. The spacing at which the transverse joints are to be provided and the need for a longitudinal joint will be a function of various conditions. These things we will be discussing subsequently. So in general a concrete pavement will have sub-grade, granular base, concrete slab, it will have longitudinal joints, it will also have transverse joints and it may have dowel bars across transverse joints, it may also have tie bars across longitudinal joints and at other joints also.

(Refer Slide Time: 00:08:35 min)



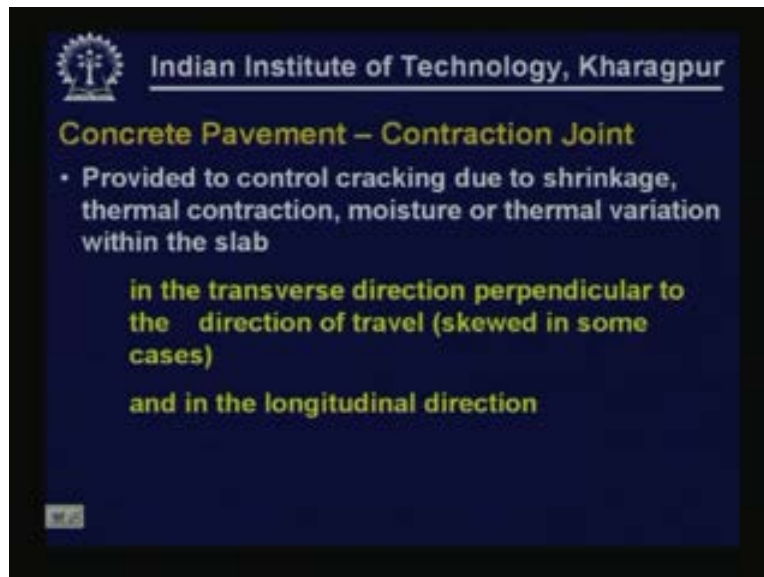
So as we have just seen these are the components that a concrete pavement will have, concrete slab, granular or stabilized base, their base can be either a granular base or it can be stabilized base, it can be dry lean concrete base, it can also have a granular sub-base, it can be stabilized sub-base also and it will of course have a sub-grade also. Joints are the other main features of concrete pavement which significantly affect its performance.

(Refer Slide Time: 00:09:07 min)



There are various types of joints. In fact there are three main types of joints contraction joint, construction joint and expansion joint.

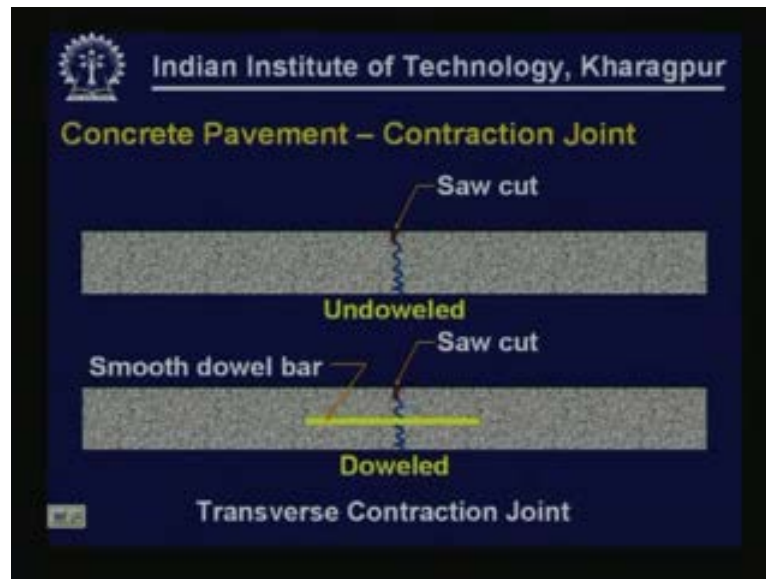
(Refer Slide Time: 00:09:22 min)



Contraction joints are provided to control cracking in the slab due to shrinkage, thermal contraction, moisture or thermal variation within the slab. These are basically there to relieve the stresses that are produced because of variation temperature, variation moisture basically variation volume either of the slab or with reference to slab, variation volume of the foundation these are usually provided in the transverse direction and at predetermined spacing these would be perpendicular to the direction of travel. In some cases these are provided with some skew not

exactly perpendicular to the direction of travel, this is to avoid the critical combination of both wheel load stresses and other stresses being very critical at these junctions. They can also be provided in the longitudinal joint depending on the requirement of load transfer across and also the dimension of the slab in the lateral direction.

(Refer Slide Time: 00:10:36 min)



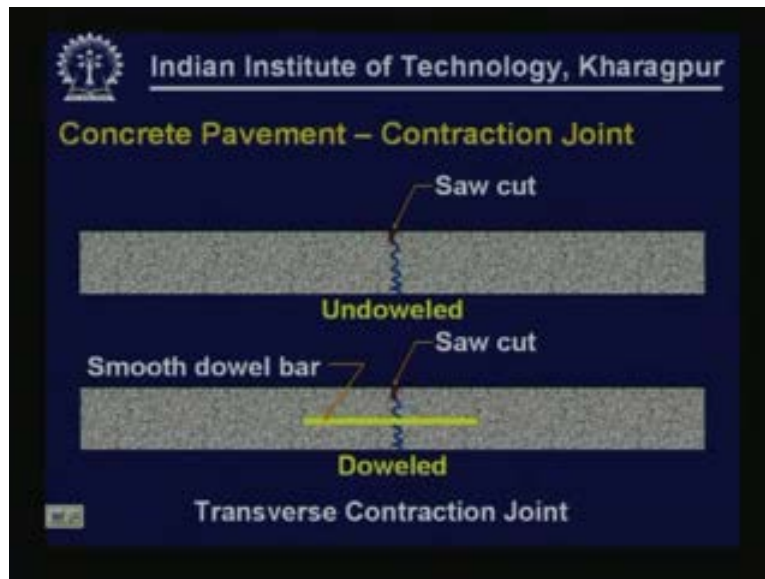
This is a typical contraction joint. The figure that you see in the upper portion of the sketch is a contraction joint without dowel bar. So this is titled as undoweled contraction joint. These are formed by making a cut in the pavement. you can see there is a cut made in this portion, so this is cut by saw and after a certain time after which the concrete is placed once it is cut we make a joint of weakness in the pavement so automatically after sometime the crack will be initiated and will progress towards the bottom so you will see a full depth crack there. But what is interesting to note is usually if the two parts of the slabs are not separated from each other there will be granular interlocking along the joint.

So it is expected that the load placed on one portion of the slab will be transferred to the other portion through this granular interlocking. So, as long as there is sufficient granular interlocking available there the crack does not become separated, the interlock does not deteriorate that means the material is not lost then we can expect some amount of load transfer from one slab to the other one thereby the stresses are in general reduced. If these two slabs are entirely separate there is no connection between these two. Obviously any load placed on one slab will produce more stresses compared to the condition where the load is transferred to the adjacent slab, may not be fully but some part of the load will be shared by the adjacent slab.

So in the first mechanism where there is only a saw that is cut and then we have a crack that is developing in this concrete pavement which is undoweled contraction joint we are depending on load transfer on the granular interlocking mechanism at this joint. The figure at the bottom shows a doweled contraction joint. during construction dowel bars will be placed inserted into the concrete, these are the dowel bars, dowel bars will be inserted into the concrete then the concrete

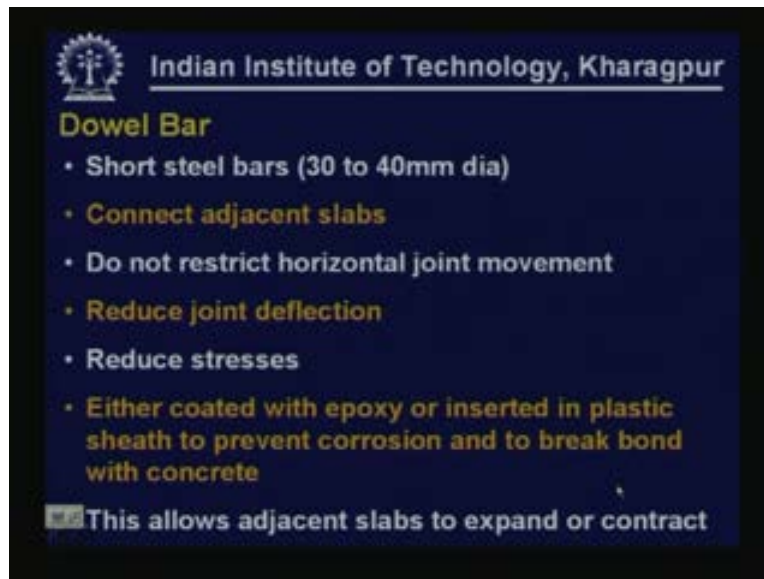
will be completed, after that at this location there will be the saw cut that is made, once that saw cut is made it will be followed subsequently by the development of this crack so in this case there will be granular interlocking also, besides this there will also be thick dowel bars that connect both the slabs.

(Refer Slide Time: 13:30)



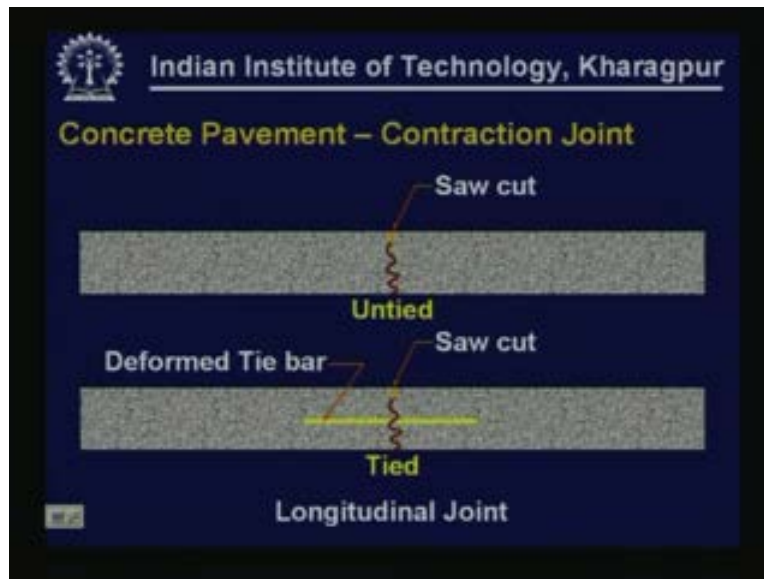
So load transfer will be through this granular interlocking and also through the dowel bar system that is placed. There will be a number of dowel bars placed across the width of the slab so all these dowel bars acting as one system will be transferring load from this slab to the adjacent slab. But load will be transmitted from this slab to the adjacent slab through the granular interlocking mechanism also.

(Refer Slide Time: 00:14:03 min)



The dowel bars are basically short steel bars, the diameter will be typically 30 to 40 mm and the length also will be about 400 to 500 mm. They connect adjacent slabs however they do not restrict the horizontal joint movement. If the two slabs have to be separated from each other because of contraction that is permitted because the dowel bar is not going to be bonded to the slab on both sides, at least on one side it is going to be smooth so that separation is going to be permitted. They are meant to reduce joint deflection and as a result reduce the overall stresses in the slab. These are either coated by epoxy or inserted in plastic sheath to prevent corrosion and also to break bond between the bar and the concrete. This breaking of bond allows adjacent slabs to either expand or contract independent of the movement of the slab in which the dowel bars are inserted.

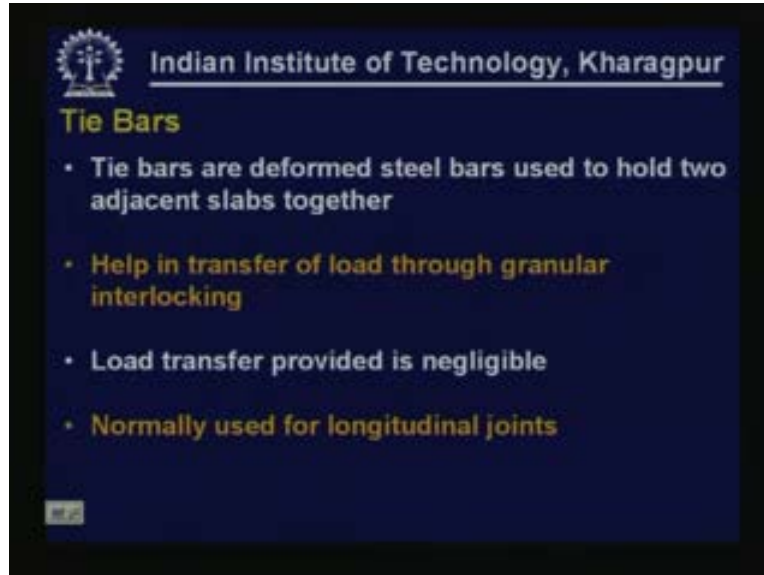
(Refer Slide Time: 00:15:20 min)



There is another type of contraction joint, the one that is shown above is already discussed in the previous slide which is very similar. It is untied contraction joint or it is undoweled contraction joint. Basically there is nothing but only saw cut is made which is followed by development of a crack, it is similar to what we discuss in the previous slide. But the one that is shown at the bottom of this slide is a saw cut followed by a crack but here what we see are tie bars inserted into the concrete while it is being placed and these are deformed tie bars whereas dowel bars are smooth, these are deformed tie bars.

We can easily understand the function of a tie bar, these are not very thick. The diameter will usually be small. The main function of tie bar is to keep these two slabs together and thereby hoping that load transfer is going to be possible through granular interlocking mechanism. We are only trying to see that the two slabs do not get separated at these joint cracks and thereby ensuring that the granular interlocking is still going to be available and as a result we expect to have some load transfer mechanism available there. And by inserting these tie bars we are trying to see that these two slabs do not get separated and there is no gap there.

(Refer Slide Time: 00:16:59 min)



As I just indicated tie bars are deformed steel bars and these are placed to hold the two adjacent slabs together. They indirectly help in the transfer of load through granular interlocking. Load transfer is provided by the tie bars themselves there are also steel bars placed across the joint but the load transfer provided through the tie bars is usually negligible and not generally taken into account. These are normally used for longitudinal joints.

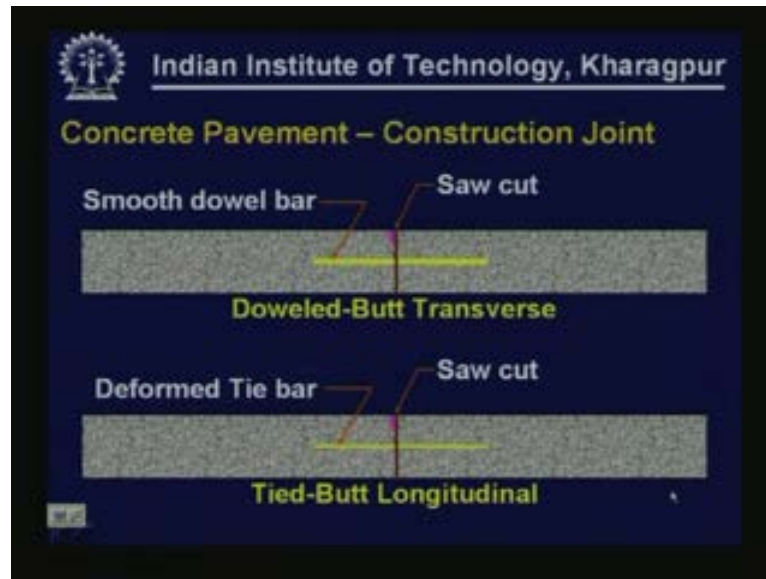
(Refer Slide Time: 00:17:38 min)



There is another type of joint that is necessary because at the end of construction one cannot construct a hundred kilometer length of slab continuously it has to be stopped and started again so you have construction joints when you stop work and start again.

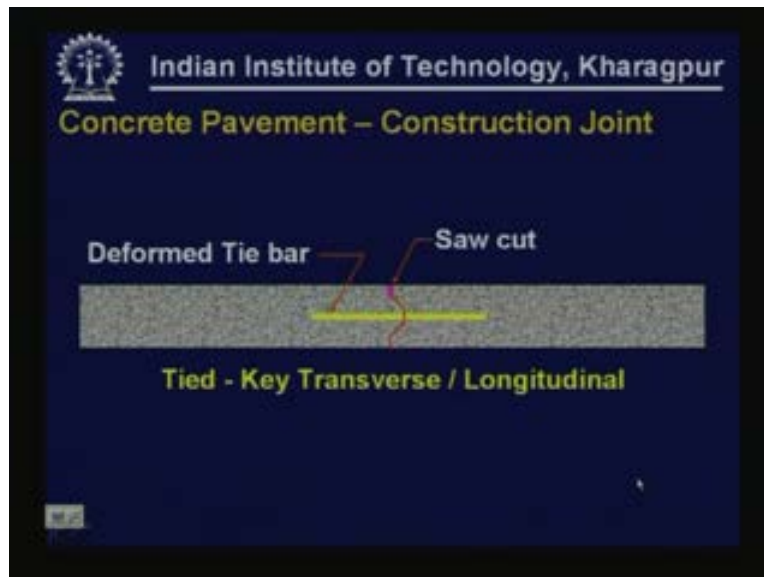
Thus construction joints are provided to separate successive construction activities. Normally what is done is that they are planned to coincide with other joints like transverse contraction joints.

(Refer Slide Time: 00:18:15 min)



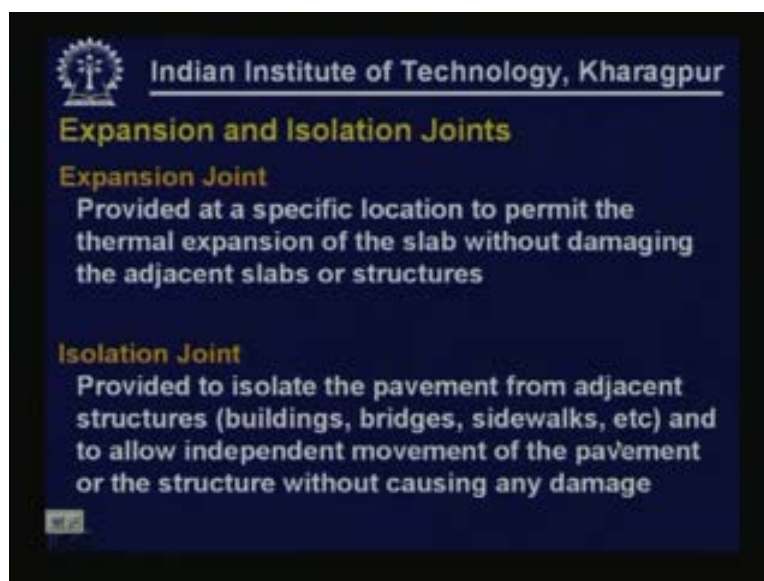
You can see two typical construction joint types that are normally provided butt joint with dowel bars. So you have this butt joint provided here and then you have a saw cut made and you also have smooth dowel bars inserted at the time of placing the concrete. This is where we want the load transfer to be through dowel bars but where load transfer across the joint is not a major problem especially for pavements where there is not much of a heavy traffic for low volume roads you may not provide dowel bars so we can just keep them together by using tie bars so this also is a butt joint. Again this can be provided along longitudinal joints. You can also have construction joint in the transverse direction and also in the longitudinal direction. When you have transverse direction normally we try to provide dowel bars to connect this to the next slab. In the longitudinal direction load transfer is not such an important consideration so we normally provide tie bars to connect these two these two slabs in the longitudinal direction.

(Refer Slide Time: 00:19:40 min)



We can have another type of construction joint that is instead of making a butt joint we can make a key joint. As you can see here there is a key arrangement here and these are also complemented by insertion of tie bars if it is a longitudinal joint you can also have it for a transverse joint then you can have dowel bars also placed across these two slabs. But since you are providing a key normally you may not require dowel bars there because through this key load transfer can be ensured.

(Refer Slide Time: 00:20:21 min)

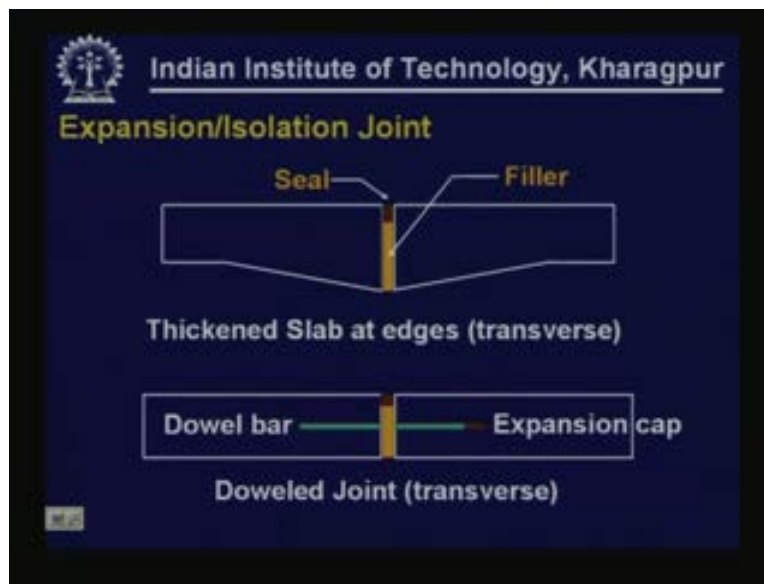


We also have expansion and isolation joints in general they are similar in their function. Expansion joint is provided at specific location to permit thermal expansion of slab without

damaging the adjacent slabs or structures. Obviously when temperature increases informally with reference to the temperature at which the concrete is placed then the concrete is going to expand the concrete should be permitted to expand otherwise there will be significant amount of compressive stresses that are going to be generating in the concrete slab. To avoid that normally expansion joints are going to be provided at specified spacing.

Isolation joint is slightly different in the sense that these are provided to isolate the pavement from adjacent structures such as buildings, bridges, sidewalks, etc because the concrete slab may be passing through various structures so it should not be pushing against those bridges or culverts, abetments so there is going to be joint that is going to be provided between the structure and the concrete slab. So this is to allow independent movement of the pavement or structure without causing any damage.

(Refer Slide Time: 00:21:44 min)



These are two typical types of joints that are provided. Expansion joint by definition has to have a separation, there has to be sufficient gap provided for the slab to expand and then close the cap so there has to be sufficient width of gap to be provided obviously one has to maintain this cap by sealing it to prevent the foundation material from coming through the joint and also from water going into the pavement and weakening the foundation. So this is usually sealed with some suitable filler material.

If there is no load transfer mechanism between these two slabs that means we are not providing any dowel bars. Then to account for the excessive stresses that are going to be close to this joint one can either thicken the slab near the edges or alternatively we can provide dowel bars here. As you can see here we can provide an expansion cap here to permit the lateral movement of the slab when it is trying to expand.

(Refer Slide Time: 00:22:56 min)



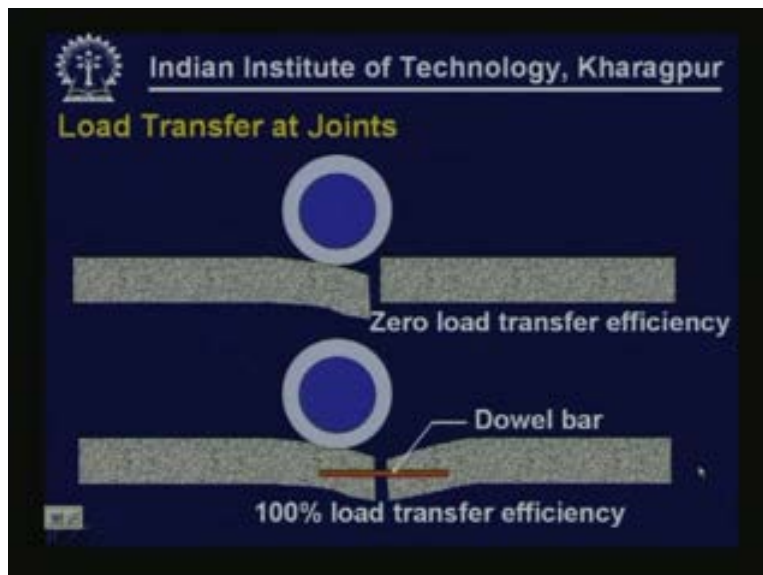
Indian Institute of Technology, Kharagpur

Load Transfer at Joints

- It is important that load applied on a slab is shared by adjacent slabs also for better performance of the pavement
- If load transfer across slabs is poor, distresses such as faulting, pumping and corner breaks occur
- Load transfer is attained through different mechanisms
- The ability of the pavement to transfer load at joints is referred to as "Load Transfer efficiency"

We are usually concerned about the load transfer mechanism available at joints and in most of the current design procedures the efficiency with which load can be transferred from one slab to another one is usually taken into account. So it is very important that load applied on one slab is shared by the adjacent slab for better performance of the pavement to reduce the overall deflections and to reduce the stresses in the slab and also on the foundation. If load transfer across slabs is poor distresses such as faulting, pumping and corner breaks and various other problems occur. Load transfer can be attained through different mechanisms some of them we already discussed like granular interlocking through dowel bars, the ability of the pavement to transfer load at joints is referred to as load transfer efficiency.

(Refer Slide Time: 00:24:01 min)



Indian Institute of Technology, Kharagpur

Load Transfer at Joints

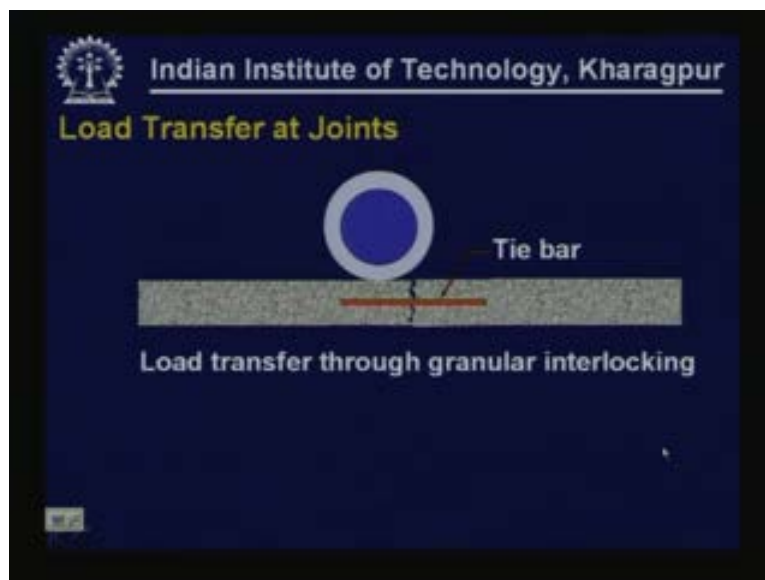
Zero load transfer efficiency

Dowel bar

100% load transfer efficiency

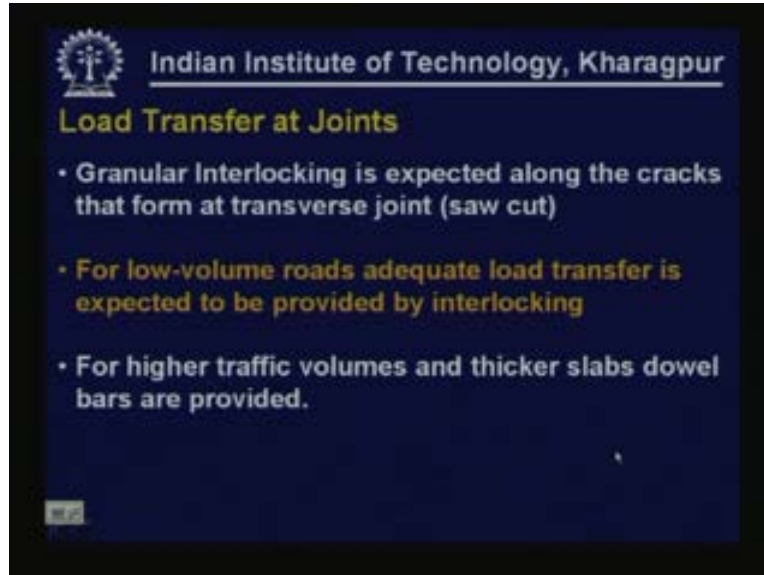
The load transfer efficiency can be explained with these two diagrams. In the diagram that is shown above the load that is placed on the left hand side there is no connection between these two slabs. Obviously the slab on the right hand side is not participating in sharing the load placed on the left hand side. So this is a situation where there is zero load transfer efficiency. But for the same situation same slab assume that we have connected these two slabs by dowel bars, a system of dowel bar it not just one single dowel bar but it is a system of dowel bars placed at a specified spacing. So what is actually depicted here is the deflection on the left hand side is almost equal to the deflection on the right hand side. This means that there is almost hundred percent load transfer efficiency. That means both the slabs are sharing the load in equal amount. That means the joint is able to transfer; it's almost like a monolithic construction as if the slab was just extending at that joint.

(Refer Slide Time: 00:25:15 min)



We can also expect load transfer through granular interlocking also if we just provide only tie bars. So this is a situation where we have crack developing through the slab and we have provided tie bars to ensure that the slabs do not get separated, this is one type of load transfer mechanism. Another mechanism is we can also have cracking, granular interlocking, we can also have dowel bars so both granular interlocking and dowel bars together provide this load transfer mechanism.

(Refer Slide Time: 00:26:01 min)

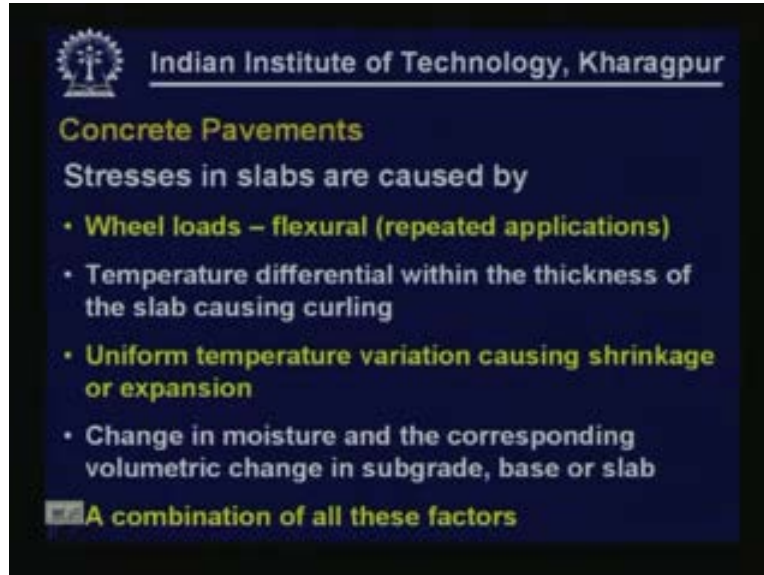


As I just discussed the load transfer at joints can be through granular interlocking. It is expected along the cracks that form at the transverse joint which is developed below the saw cut. For low volume roads adequate load transfer can be expected to be provided by interlocking. Usually we would not provide dowel bars in the case of very low volume roads. So we assume that the granular interlocking mechanism that is going to be developing is capable of providing sufficient load transfer efficiency to keep the stresses within permissible limit so normally low volume loads would not provide dowel bars. For higher traffic volumes and thick slabs obviously dowel bars are going to be necessary.

Coming to the analysis of stresses in concrete, slabs stresses in concrete slabs are usually produced due to various reasons. The number one reason is stresses that are produced because of application of wheel loads. Obviously these are going to be flexural stresses because they make the slab to deflect and bend.

What is to be noted is it is not just one single application that we are concerned about. Of course when the pavement is subjected to one very heavy load then that itself may be sufficient to cause failure of the slab. But we are also concerned that especially in the case of highway pavements there are going to be large number of repetitions of loads to be applied. Especially if you are considering a period of ten to fifteen years the number of loads that are going to be applied in terms of commercial vehicles or in terms of axles can be several millions. So we have to understand that millions of loads are going to be applied if we are talking about highway pavements or for any other pavement also there are going to be repetition of loads of different magnitudes. Hence all of them are going to be producing some amount of smaller or larger damage.

(Refer Slide Time: 00:28:11 min)



There are going to be stresses produced because of temperature differential within the thickness of the slab. Because if there is temperature differential within the slab that means top temperature is more than bottom temperature or bottom temperature being more than top temperature so slab either curls up or curls down. This also can produce stresses in the slab. We will discuss how to calculate that, what type of stresses is produced and so on.

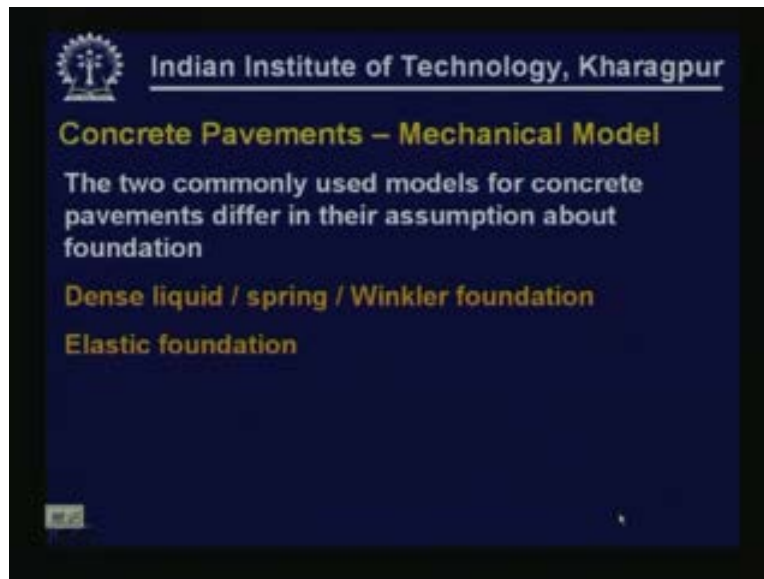
Another reason for stresses that is being produced in concrete slabs are if the temperature increases uniformly for example when the slab was placed there is certain temperature but afterwards the temperature is either increasing or decreasing, we are not talking about daily variation of temperature where the daytime being hotter at top and then cooler being at bottom and night time being cooler at top and then hotter at bottom we are not talking about that. But from season to season that is during one season the temperature is going to be let us say ten degree centigrade and at another season the temperature could be 50, 60 degree centigrade so accordingly there is going to be uniform expansion or contraction. If this uniform expansion or contraction is restrained then there are going to be stresses that are going to be produced. But if it is not restrained there will not be any stresses.

Therefore depending on the degree of restrain that is going to be provided to the expansion or contraction the corresponding stresses are going to be generated. so there are going to be stresses because of wheel loads that are applied and there are going to be stresses because of variation in temperature within the thickness of the slab because it's going to either curl up or curl down or there is going to be stresses produced if restraint is provided to uniform expansion or contraction of the slab.

There can also be stresses if there is change in moisture and corresponding volume change either in sub-grade or in base or in slab. If any one of these components undergo volumetric change and if that is restrained by the other component then there are going to be stresses generated in both the components. For example, if slab is trying to undergo volumetric change then foundation is

restraining that or if foundation is trying to undergo volumetric change because of variation in moisture content then slab is trying to prevent that so normally there are going to be stresses generated in both the components. But what we have to consider is a critical combination of all these stresses produced by various factors.

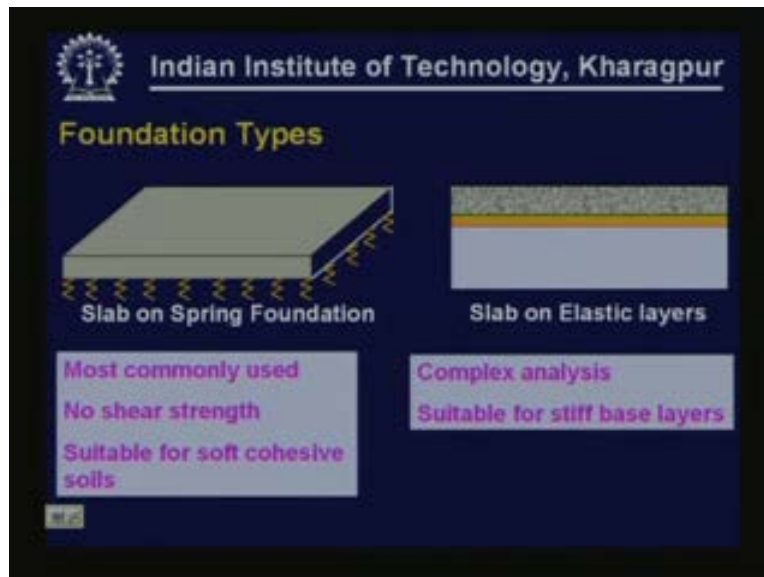
(Refer Slide Time: 00:30:55 min)



For analyzing concrete pavements we should be able to analyze these pavements and then find out if a load of certain magnitude or certain configuration is placed on the slab of certain type or strength what will be the wheel load stress. Also, we should be able to calculate things like if there is so much of variation of temperature within the slab what will be the corresponding stress, although if there is uniform variation of temperature what is going to be stressed and so on. So we have to have mechanical model or theoretical model using which we will be able to analyze the pavement.

There are two commonly used models for concrete pavements. These two differ in terms of their assumption about foundation. What I am talking about here are simple theoretical models that are available. There are lots of variations that are currently practice to these simple models more complex models are available. But I am going to only discuss about those simple models that are in practice. So as I said the main variation is in terms of the foundation and assumption made about the foundation. We either assume the foundation to be as dense liquid or spring or winkler foundation they all mean the same thing. That means the foundation consists of number of springs. Alternatively we can assume it to be elastic foundation.

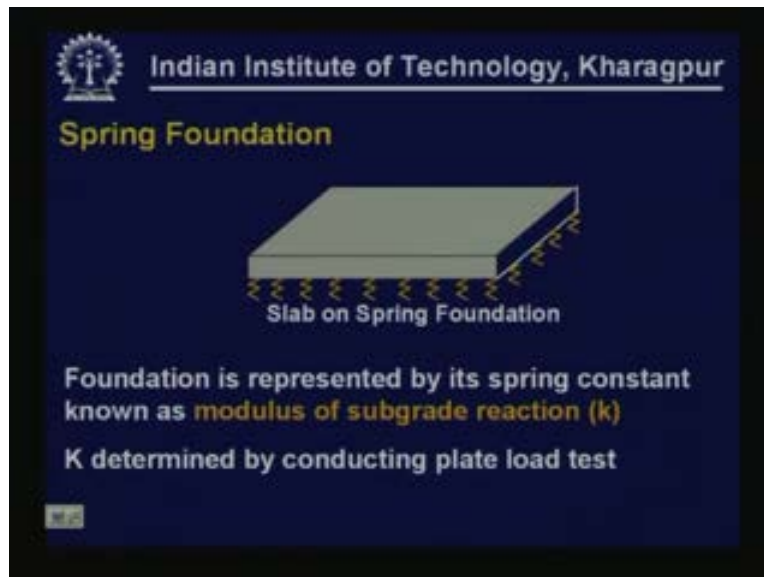
(Refer Slide Time: 00:32:22 min)



This slide here shows the two types of foundation. On the left hand side you have slab placed on a spring foundation or winkler foundation or dense liquid foundation. This is the most commonly used foundation type for analysis of concrete pavements. It assumes no shear strength for the foundation and these are usually suitable for soft cohesive soils.

On the right hand side the foundation that is considered is a combination of elastic layers. So here is a slab placed on number of elastic layers usually it is sub-grade and base or only the sub-grade. This analysis is slightly more complex compared to the analysis that we make for slab over spring foundation but this is usually suitable for concrete pavements with stiff base or stiff foundation.

(Refer Slide Time: 00:33:22 min)



As I said spring foundation is the most common assumption that we make about foundation of concrete pavements. in this the foundation is represented by its spring constant known as modulus of sub-grade reaction k . k is normally determined if this is being determined from a field test it is determined by conducting a plate load test on the foundation.

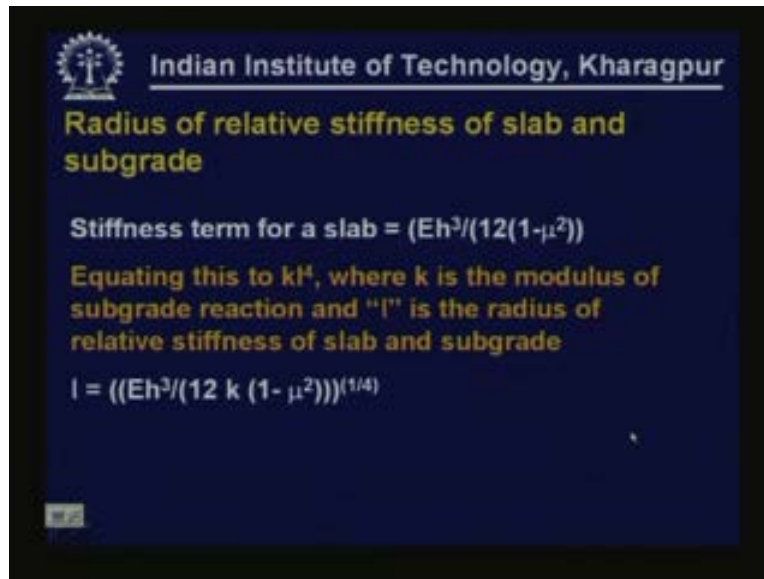
(Refer Slide Time: 00:33:51 min)



There is a parameter that we normally use to represent the relative stiffness of the slab and the sub-grade. The slab will have one stiffness and the foundation will have different stiffness. To have an index which represents the relative stiffness of slab with reference to the sub-grade we use a term called as radius of relative stiffness. The assumption using which the radius of relative

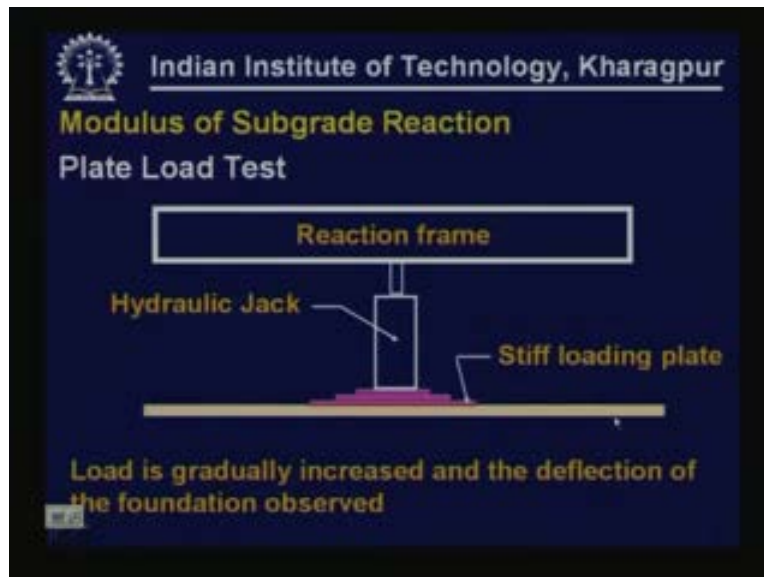
stiffness is adopted is when the slab deflects the pressure that is produced at different locations is proportionate to the deflection at that location. So the reactive pressure is proportional to the deflection so k is the proportionality constant which we call as modulus of sub-grade reaction.

(Refer Slide Time: 00:34:48 min)



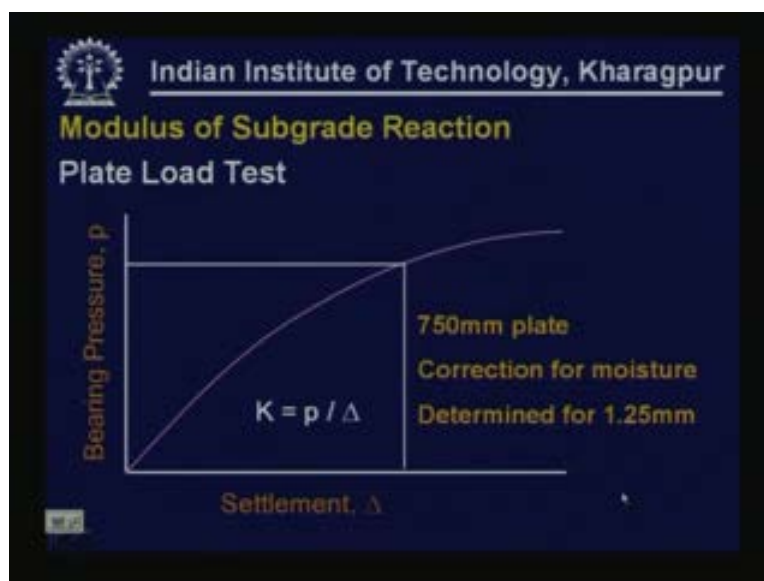
So the radius of relative stiffness of slab and sub-grade is given by comparing the stiffness term of the slab and a similar term that we use for the foundation. The stiffness term for the slab can be given as Eh^3 where e is the modulus value of concrete, h is the thickness of the slab divided by 12 into $1 - \mu^2$ where μ is the Poisson ratio value of the concrete. Equating this to k where k is the modulus of sub-grade reaction and l to the power of 4 where l is the term that was just coined radius of relative stiffness and equating these two l can be expressed in terms of Eh , k and μ by the expression given here. So this is l radius of relative stiffness of slab and sub-grade given as a function of modulus value of concrete, thickness of concrete, modulus of sub-grade reaction of foundation and then Poisson ratio value of concrete.

(Refer Slide Time: 00:35:59 min)



Determination of modulus of sub-grade reaction can be done by conducting a plate load test. We have briefly discussed about plate load test when we were discussing about material characterization. In this we apply load in an incremental manner. We use a stiff plate a series of plates used to stiffen the loading plate, this is the rigid plate (Refer Slide Time: 36:31) so we apply incremental load and measure the surface deflection of the pavement and then plot the load versus deflection diagram.

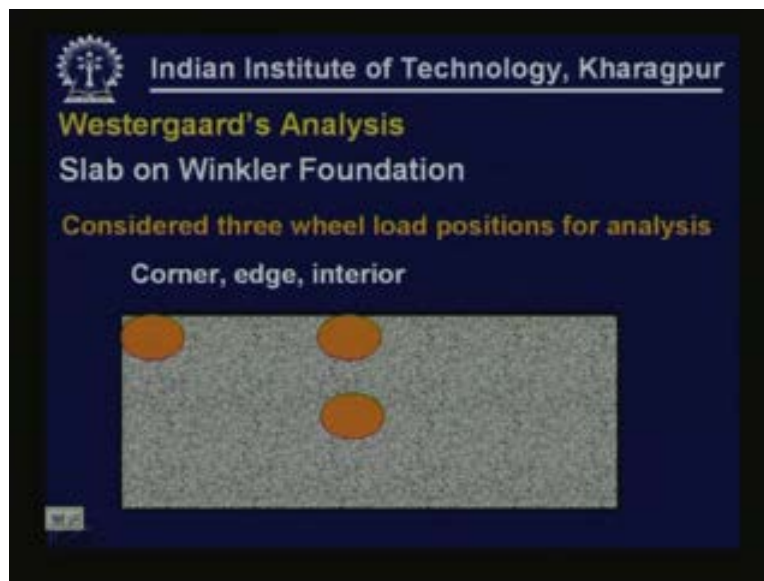
(Refer Slide Time: 00:36:42 min)



The slope of this diagram at a specified deflection is taken as modulus of sub-grade reaction. normally we use 750 mm dia plate to conduct plate load test and we also have to normally make

corrections for moisture, we normally use the K value corresponding to weakest condition in designing pavements but if you conduct this plate load test at a different point of time where the foundation is {non} (00:37:14) in its weakest condition then we normally have to make some correction corresponding to the weakest condition. The K value is determined corresponding to 1.25 mm.

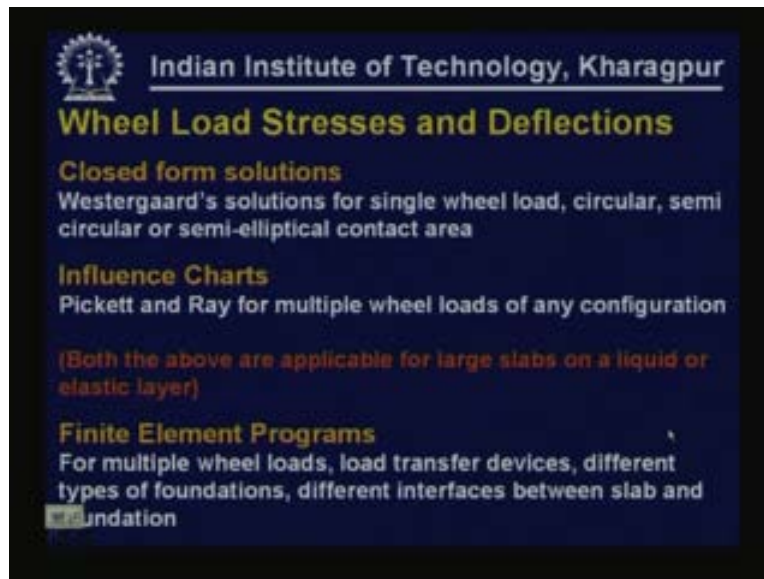
(Refer Slide Time: 00:37:26 min)



When we analyze pavements basically most of the agencies use Westergaard's analysis either in its original form or in the modified form because Westergaard has given several expressions for computing stresses in different critical regions of slab. Some agencies have directly adopted those expressions but many agencies over time have gradually modified the Westergaard's expression so that the computed values correlate with the values that have been measured during field experiments or laboratory experiments.

Westergaard's analysis assumes that slab is placed on a Winkler foundation. We have just described about Winkler foundation which comprises of springs. Westergaard's analysis typically gives expressions for computing stresses in the slab in three different regions. These are considered to be critical regions. One is corner, edge region and interior region. As you can see here (Refer Slide Time: 38:37) corner, interior and edge region of the slab. Of course if we have suitable programs that are available you can calculate stresses and strains, deflections at any point in the slab but typically solutions are available for corner, edge and interior regions of the slab.

(Refer Slide Time: 00:39:00 min)



Closed form solutions are available given by Westergaard's for single wheel load, circular contact area or semi-circular contact area or semi-elliptical contact area but circular contact area is the most commonly used shape of load contact.

Number of influence charts is also available Pickett and Ray providing solutions for multiple wheel loads of different configurations. as we know the load contact area is not in general circular in shape so using influence charts we can determine the stresses due to multiple wheel loads and wheel loads having different contact areas.

Both the types of solutions either Westergaard's analysis or influence charts developed by Pickett and Ray for multiple wheel loads are normally applicable for large slabs placed on liquid foundation or spring foundation or Winkler foundation or foundation being in elastic layer. There are a number of finite element programs that are also available at present which are capable of solving the pavement systems for multiple wheel loads for different types of load transfer mechanisms.

The two types of solutions that are available in the form of closed form solution and in the form of influence charts are not normally capable of handling different types of load transfer mechanism and also finite element programs can handle different types of foundations and different interfaces between slab and foundation, rough interface or smooth interface so all these complex situations cannot be solved by standard closed form solutions so nowadays most of these agencies normally use finite element programs to solve these complex situations where you have different types of interfaces, different types of foundations are to be modeled and different load transfer mechanism is there so to be able to solve all these problems normally nowadays finite element solutions are being adopted.

(Refer Slide Time: 00:41:16 min)

Indian Institute of Technology, Kharagpur

Wheel Load Stresses

Westergaard (1926) developed equations for solution of load stresses at three critical regions of the slab – **interior, corner and edge**

Interior – Load in the interior and away from all the edges

Edge – Load applied on the edge away from the corners

Corner – Load located on the bisector of the corner angle

But we will confine our discussions to Westergaard's solutions of wheel load stresses. Obviously all the assumptions that go with Westergaard's solutions will have to be taken together. Westergaard in 1926 developed equations for solution of load stresses as I mentioned at three different critical regions of the slab interior, corner and edge. In an interior region the load is placed in the interior and away from all the edges. That's the definition of what an interior region is. The load has to be sufficiently away from all the edges whereas edge loading is a situation where the load is placed along the edge away from the corners. On the other hand corner loading condition is defined by load being located on the bisector of the corner angle.

(Refer Slide Time: 00:42:15 min)

Indian Institute of Technology, Kharagpur

Wheel Load Stresses

The diagram illustrates a rectangular slab divided into four quadrants by a horizontal and vertical line. Three green circles represent load positions: one on the top edge (labeled 'Edge'), one at the top-right corner (labeled 'Corner'), and one in the top-left quadrant (labeled 'Interior').

Again this diagram depicts the three critical regions edge, corner and the interior region.

(Refer Slide Time: 00:42:28 min)

Indian Institute of Technology, Kharagpur

Wheel Load Stresses

Westergaard solutions for a Poisson ratio of 0.15 for concrete

Interior loading (tensile stress at the slab bottom)

$$\sigma_i \text{ (psi)} = (0.3162P/h^2) \left[4 \log_{10} (l / b) + 1.069 \right]$$

Edge loading (tensile stress at the slab bottom)

$$\sigma_e \text{ (psi)} = (0.572P/h^2) \left[4 \log_{10} (l / b) + 0.359 \right]$$

Corner loading (tensile stress at slab top)

$$\sigma_c \text{ (psi)} = (3P/h^2) \left[1 - ((a (2)^{1/2}) / l)^{0.6} \right]$$

These are the expressions given by Westergaard's for a Poisson ratio value of 0.15 assume for concrete for load being placed in different regions interior loading, edge loading and corner loading.

For example, in interior loading case obviously there will be tensile stresses at the slab bottom and then compressive stress at the top so it is sigma interior in terms of psi. So we have to understand that these equations are in a psi system, sigma in terms of psi is given as a function of single wheel load that is applied p. so this is the wheel load that is applied (Refer Slide Time: 43:20) this is the single wheel load p thickness of the slab h, l is the radius of relative stiffness **as we have discussed a few slides back** and b is the parameter which I will just discuss. So this is a function of load that is applied and thickness of the slab and l and another parameter b. l we know is a function of e, h, k and Poisson ratio. Similarly, in edge loading the expression that is given is also in terms of wheel load, thickness l and b. We also have the expression for corner loading here.

(Refer Slide Time: 00:44:08 min)

Indian Institute of Technology, Kharagpur

Wheel Load Stresses

Where,

P = wheel load, lbs

h = slab thickness, inches

a = radius of wheel contact area (circular contact)

b = radius of resisting section, inches

$= (1.6a^2 + h^2)^{1/2} - 0.675 (h)$ for $a < 1.724 h$

$= a$ when $a \geq 1.724 h$

l = radius of relative stiffness, inches

And 'p' as I have indicated is the wheel load in pounds, 'h' is the slab thickness in inches, 'a' is the radius of wheel contact area assuming circular contact area. If we know the wheel load and if we also know the contact pressure we can calculate the radius of load contact area assuming circular contact area. Then 'b' is termed as radius of resisting section again in inches given as 1.6 times a square + h square, h is the thickness of the slab under root – 0.675 into thickness of slab for the condition where 'a' is less than 1.724 times thickness of slab. If 'a' is greater than or equal to 1.724 times thickness of slab then b is taken to be equal to 'a', l of course as we already defined is the radius of relative stiffness in inches.

(Refer Slide Time: 00:45:23 min)

Indian Institute of Technology, Kharagpur

Deflection due to Wheel Load

Westergaard's equations

Corner

$$\Delta_c = \frac{P/(k l^2)}{(1.1 - 0.88 ((a^2 2^{1/2}) / l))}$$

Interior

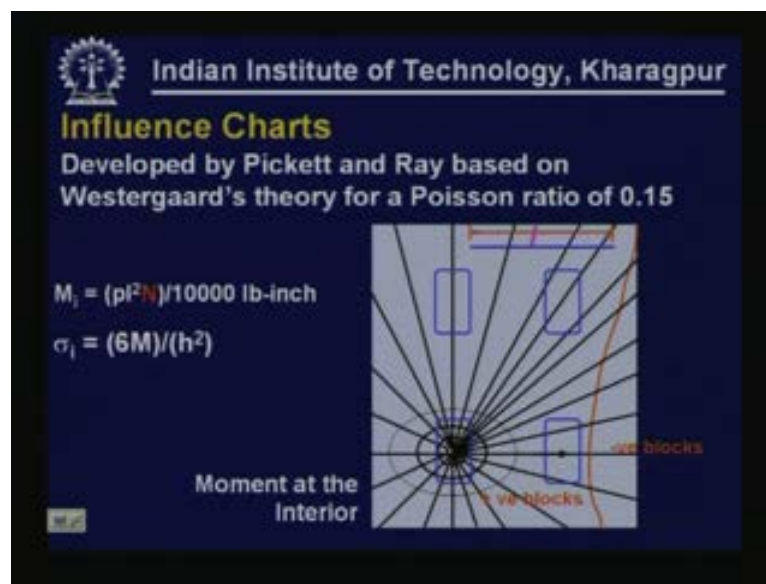
$$\Delta_i = \frac{P/(8 k l^2)}{(1 + (1/2\pi) (\ln(a/2l) - 0.673) (a/l)^2)}$$

Edge

$$\Delta_e = \frac{0.431 * P/(k l^2)}{(1 - 0.82 (a/l))}$$

We also have expressions for computing deflection due to wheel loads. Many-a-times we are also concerned about the magnitude of the deflection because this is very critical at locations where we have joints. If the slab deflects too much the foundation below is going to be damaged and the slab is going to lose support at the joints so lot of large deflections at the joints are going to be damaging the foundation because that foundation is gradually be eroded the material is going to be lost so that will lead to loss of support at these joints so we are concerned about large deflections occurring in the slab also. These are the three expressions that we have for corner loading, interior loading and edge loading and the corresponding deflections. As in the case of stresses here also these three deflections are function of load that is applied 'k', 'l' is radius of relative stiffness and 'a' is radius of load contact area.

(Refer Slide Time: 00:46:35 min)

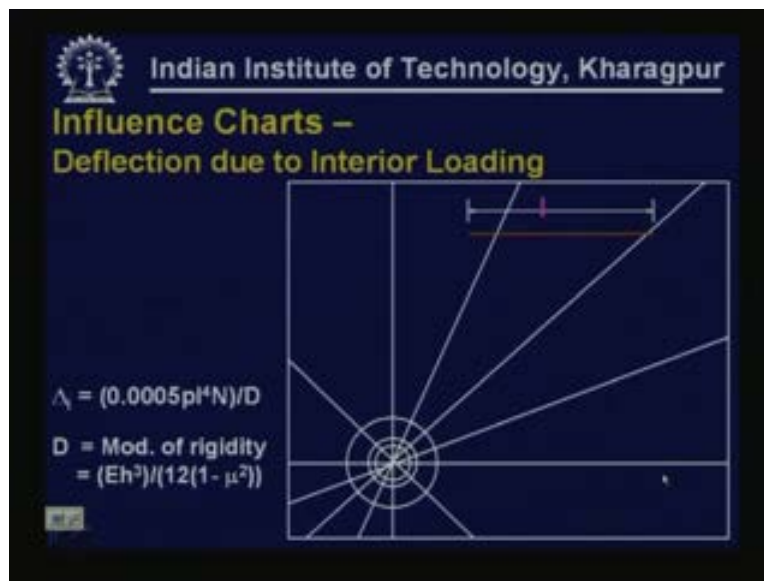


There are number of influence charts that are available developed by Pickett and Ray based on Westergaard's theory. This is for a Poisson ratio value of 0.15 this is an influence chart developed for computing the movement in the interior region. This diagram is a typical representation of the influence chart so these four blocks that I have represented here are the wheel loads. Here I am depicting the wheel contact areas to be rectangular in shape or I can draw any other diagram but these wheel load imprints have to be drawn to a proper scale. The scale to be used is represented by this line that is drawn here. So, on these influence diagrams you will see a line so you can measure that length which corresponds to l which is radius of relative stiffness.

So, for the given slab if we know the properties of e, h, Poisson ratio and k value we can calculate the value of 'l' in inches so that is the length that has been drawn. So this line here corresponds to the 'l' value that is calculated for the given slab. Therefore once we have identified what is the scale to which the imprints are to be drawn that means the length of this line should correspond to value of 'l'. Then to the same scale we can draw the imprints. What we have to do is count the number of blocks that are there within this tire imprints and count the blocks that are there within all the wheel loads that we place on the influence chart that is the

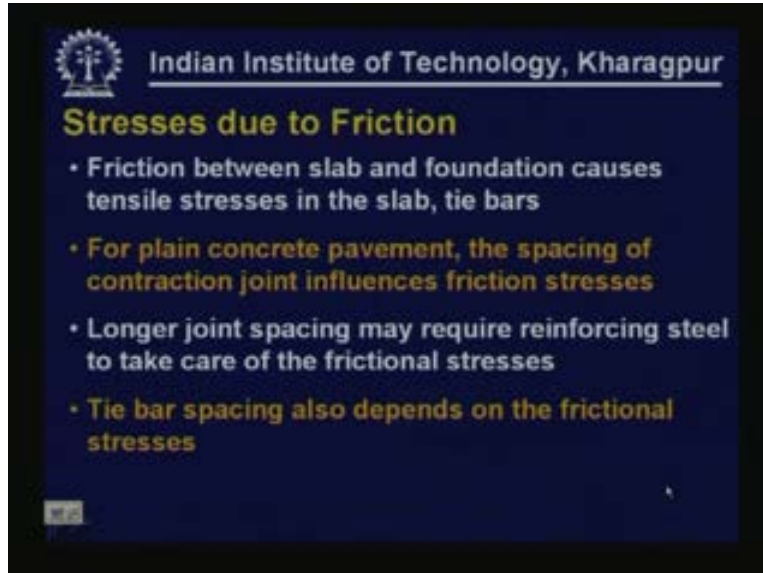
number that we have in this expression where the movement in the interior region is a function of, this is the contact pressure p , ' l ' is the radius of relative stiffness, ' n ' is the number of blocks occupied by all the tire imprints then this is the movement that we get. Once we get the movement we can compute the flexural stress using this expression. So depending on the position of the wheel loads the number of blocks that these occupy will also be varying.

(Refer Slide Time: 00:48:51 min)



We have a similar influence chart for computing deflection in the interior loading. Again here also we have to place the tire imprints on the diagram, draw them to the same scale given by the represented by the line that is there at the top, count the number of blocks ' n ' and then contact pressure radius of relative stiffness and a parameter ' d ' which is known as modulus of rigidity computed as Eh^3 by 12 into $1 - \mu^2$. So if you know the properties of the materials to be used we can compute the modulus of rigidity and we should also of course know the thickness of the slab. Similarly there are other types of influence charts available to compute other parameters. We can compute stresses or deflections in different locations for multiple wheel loads and also for wheel loads of different contact areas.

(Refer Slide Time: 00:49:47 min)



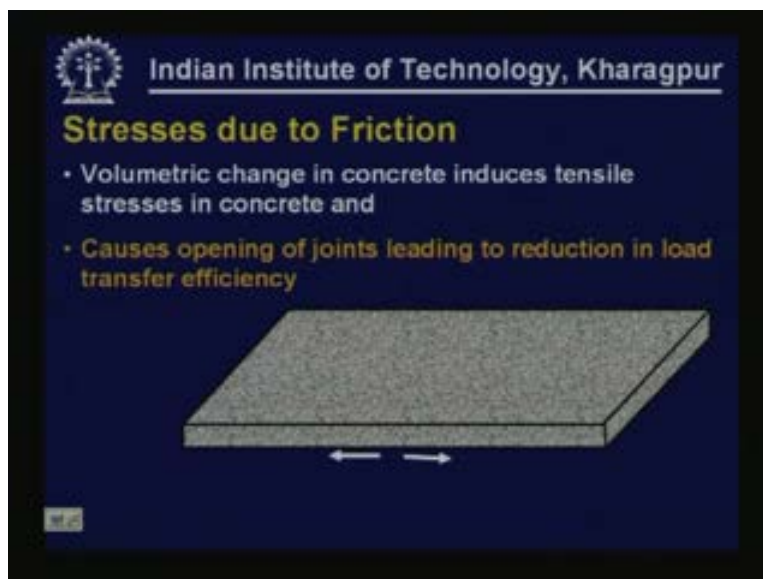
Indian Institute of Technology, Kharagpur

Stresses due to Friction

- Friction between slab and foundation causes tensile stresses in the slab, tie bars
- For plain concrete pavement, the spacing of contraction joint influences friction stresses
- Longer joint spacing may require reinforcing steel to take care of the frictional stresses
- Tie bar spacing also depends on the frictional stresses

There are also going to be stresses due to friction. This we have discussed earlier. Friction will be mobilized when the slab is trying to move. So friction between slab and foundation causes tensile stresses in the slab that is when it is trying to contract. For plain cement concrete this spacing of contraction joint influences the friction stresses. Longer joint spacing may be required for reinforcing steel to take care of the frictional stresses. Tie bar spacing also depends on the frictional stresses.


(Refer Slide Time: 00:50:27 min)



Indian Institute of Technology, Kharagpur

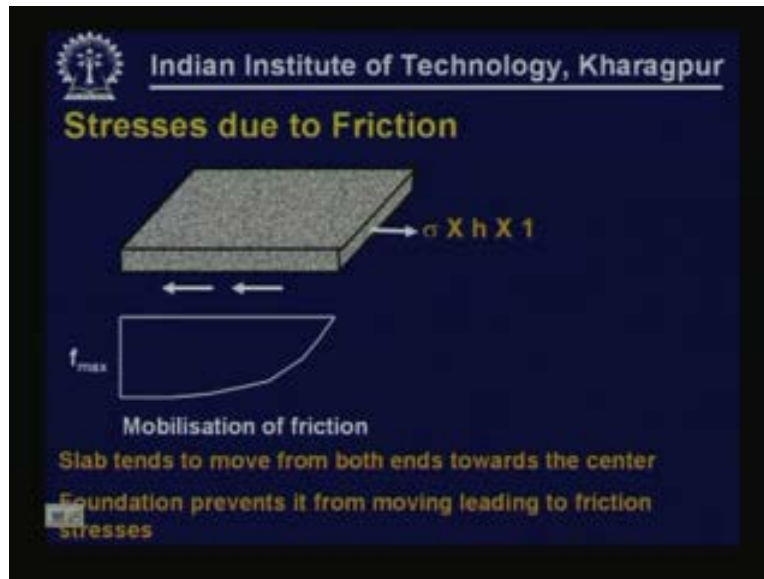
Stresses due to Friction

- Volumetric change in concrete induces tensile stresses in concrete and
- Causes opening of joints leading to reduction in load transfer efficiency



Let us understand why there will be stresses in the slab because of friction. Volumetric change in concrete induces tensile stresses in concrete. We are referring to a case where it is trying to contract. This would also cause opening of joints leading to reduction in load transfer efficiency.

(Refer Slide Time: 00:50:47 min)



When a slab tries to contract the outermost part moves the maximum, the central part of the slab moves the least so the friction that is mobilized towards the end of the slab is going to be more, and towards the center of the slab almost zero friction is mobilized so this is how the friction that is mobilized from the end to the center of the slab is going to vary. But what we normally do is we try to get an average value of friction that is going to be mobilized and that is the value we are going to use in the analysis of these slabs.

These slabs tend to move from both ends towards the center. Foundation prevents it from moving leading to frictional stresses. The restraint provided by the foundation to the movement of the slab is the reason why stresses are going to be caused. The main reason is that either the slab is trying to move or the foundation is trying to move so one is restraining the other one. Normally we are concerned about the contraction of the slab because of reduction in temperature that being restrained by the foundation. So the friction is mobilized there and the corresponding stresses are going to be generated in the slab.

(Refer Slide Time: 00:52:03 min)

Indian Institute of Technology, Kharagpur

Stresses due to Friction

Friction mobilized depends on the relative movement

Zero at center and maximum at some distance from center

Average friction (f_a) is considered

$$w \times 1 \times (L/2) \times h \times f_a = \sigma \times h \times 1$$
$$\sigma = w (L/2) f_a$$

w = unit weight of concrete, h = slab thickness

Frictional stress in concrete is dependent on the spacing of joint

So this can easily be calculated. The friction mobilized depends on the relative movement zero at center and maximum at some distance from center, average friction has to be assumed f_a . So assuming that if we have w as the unit weight of concrete, l by 2 is the portion of slab that we are considering, h is the thickness, 1 is the width and f_a is the average coefficient of friction then this is the stress that is generated and this is the thickness and this is the cross section area so the corresponding stress can be computed using this expression.

(Refer Slide Time: 00:52:45 min)

Indian Institute of Technology, Kharagpur

Stresses in Steel due to Friction

Longitudinal and transverse reinforcement and tie bars across longitudinal joints are to be designed to handle stresses due to friction

$$A_s = (whBf_a)/(f_s)$$

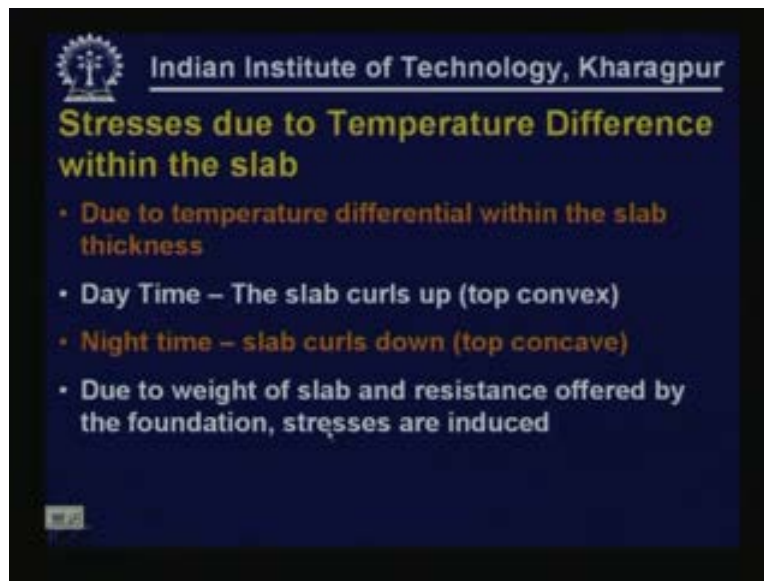
(per unit width of slab)

B

We can provide longitudinal or transverse reinforcement and tie bars across longitudinal joints to handle these stresses. The area of reinforcement to be provided is given by this expression where

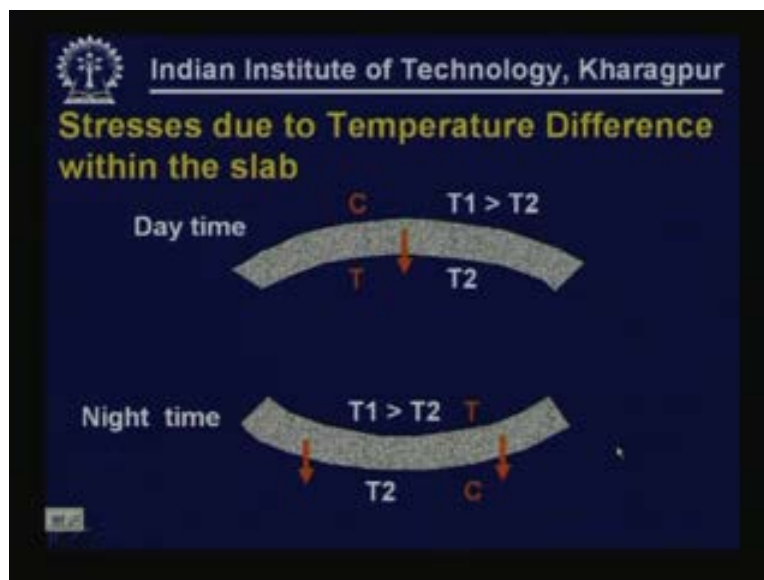
'b' is the width of the panel and f_s is the allowable stress.

(Refer Slide Time: 00:53:11 min)



As I indicated earlier there are going to be stresses within the slab due to temperature difference also. Due to temperature differential within the slab during day time the slabs tries to curl up and the top will be convex and during night time the slab curls down and the top will be concave so due to weight of the slab and the resistance offered by the foundation the stresses are going to be induced.

(Refer Slide Time: 00:53:39 min)



The slab will not be fully permitted to either curl up or curl down, there is restraint provided by the self weight and also by the foundation so there are going to be stresses getting developed. during day time the self weight is going to pull it down so there will be tension at the bottom and compression at the top because there is going to be more temperature at top compared to the temperature at bottom whereas on the other hand during night time the temperature at the top fiber of the slab is going to be more compared to the bottom fiber so the slab is going to be pulling this down so there is going to be compression at bottom and then tension at top.

(Refer Slide Time: 00:54:12 min)

Indian Institute of Technology, Kharagpur

Curling Stresses in a Finite Slab

Interior

$$\sigma_x = \frac{C_x E \alpha \Delta t}{2(1 - \mu^2)} + \frac{(C_y \mu E \alpha \Delta t)}{2(1 - \mu^2)}$$

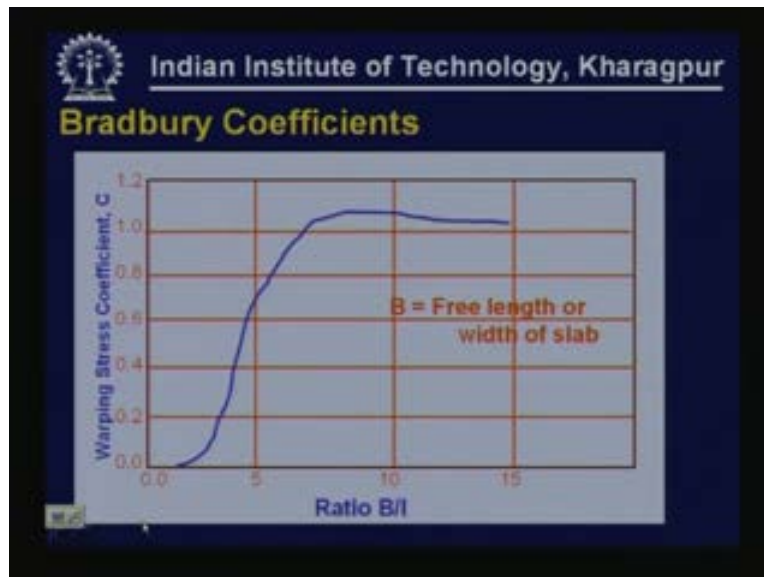
$$= \frac{(E \alpha \Delta t)}{2(1 - \mu^2)} (C_x + \mu C_y)$$

$$\sigma_y = \frac{(E \alpha \Delta t)}{2(1 - \mu^2)} (C_y + \mu C_x)$$

μ: Coefficient of thermal expansion of concrete

So these curling stresses can be computed using these expressions sigma suffix x as a function of modulus value of concrete, alpha is a coefficient of thermal expansion delta t is the temperature differential that is there from top to bottom. There are coefficients like C_x and C_y that we have to determine because the slab is finite. So, depending on the dimensions of the slab L_x and L_y we have to determine two coefficients C_x and C_y then we can determine the stress and sigma suffix x and then sigma suffix y. The other information we need is modulus value of concrete, alpha as a coefficient of thermal expansion of concrete and the temperature differential. We need to find out what is sigma suffix x and sigma suffix y. Once we get those two parameters we can compute the stresses arising out of curling.

(Refer Slide Time: 00:54:57 min)



Bradbury has given coefficients to find out what is value of sigma suffix x and sigma suffix y depending on the dimension of the slab with reference to the radius of relative stiffness. If we know the dimension of the slab for example $l \times y \times l$ then we can find out what is the corresponding coefficient that is C_x .

(Refer Slide Time: 00:55:19 min)

Indian Institute of Technology, Kharagpur

Curling Stresses

Edge Stress = $(CE\alpha\Delta t)/2$

Corner Stress – Negligible

Critical Combination of load and curling Stresses

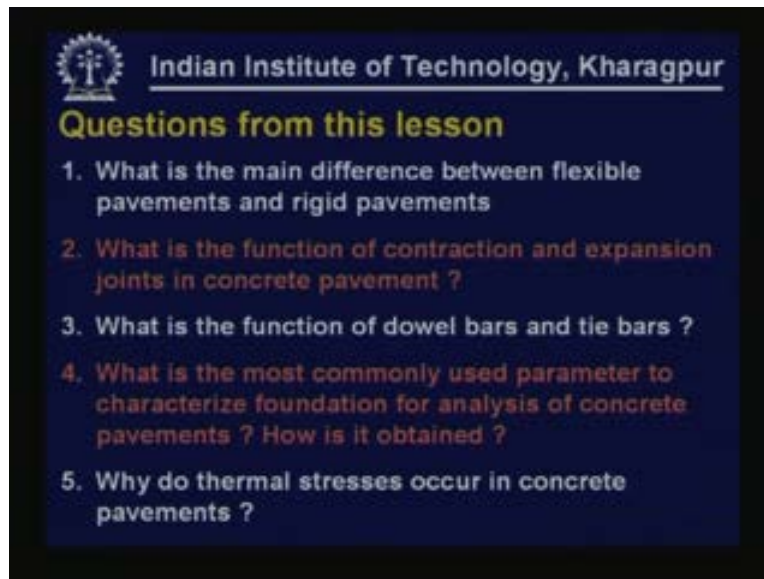
Night time – Curling Stress (tension at top) compensates load stress (compression at top)

Noon Time – Curling stress will be additive to load stress. Hence Critical

The curling stress in the edge region can be estimated by the expression coefficient multiplied by elastic modulus value and coefficient of thermal expansion temperature gradient by 2. In the corner region the curling stress is normally negligible because the restraint provided by self weight is almost negligible there. The critical combination of load and curling stresses that is to be

considered is in the case of night time the curling stress compensates the load stress because curling stress tension will be there at top and load stress compression will be there at top. On the other hand at noon time the curling stress will be additive to load stress hence this is a critical combination.

(Refer Slide Time: 00:55:58 min)



Let us take a few questions from this lesson, answers to this will be provided in the next class.

What is the main difference between flexible pavements and rigid pavements?

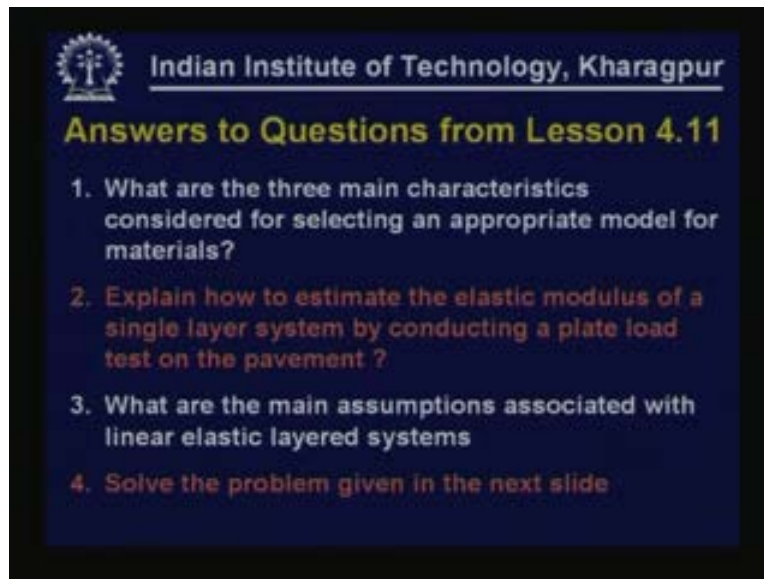
What is the function of contraction and expansion joints in concrete pavement?

What is the function of dowel bars and tie bars?

What is the most commonly used parameter to characterize foundation for analysis of concrete pavements how is it obtained?

Why do thermal stress occur in concrete pavements?

(Refer Slide Time: 00:56:24 min)



Now we will have answers for the questions that we asked in lesson 4.11.

What are the three main characteristics considered for selecting an appropriate model for materials?

The three main characteristics are; the relationship between stress and strain whether it is linear or nonlinear, the response of material after applied load is released whether it is instantaneously elastic, complete recovery, complete recovery over time or partly not recovered and some is recovered so accordingly terms such as elastic plastic, viscoelastic and various combinations or terms can be used to describe the material. The third characteristic is the time dependent behavior of the material. Accordingly it is either viscous or non-viscous. And within viscous also it can be linear, nonlinear. Depending on the relationship between time and the deformation it can be linear or nonlinear.

Explain how to estimate the elastic modulus value of a single layer system by conducting a plate load test on the pavement?

This we have already discussed earlier. If you conduct a plate load test measure the surface deflection know the value of contact pressure 'p', radius of load contact area 'a' is known, 'p' is known, 'a' is known so we know the expression for a rigid plate surface deflection. So surface deflection is equal to $1.18pa$ by e where e is the modulus of sub-grade 'p' is known, 'a' is known deflection is measured and we can estimate the modulus value.

What are the main assumptions associated with linear elastic layer system. Linear elastic layer system we assume it is comprised of number of layers. Each layer is characterized by its thickness, elastic modulus value, Poisson ratio value, all the n minus one layers are infinite in horizontal extent and the last layer is infinite in both the directions. Solve the problem given in the next slide.

(Refer Slide Time: 00:58:13 min)

Indian Institute of Technology, Kharagpur

Answers to Questions from Lesson 4.11

Question 4

The design criterion for a two layer system is that the surface deflection measured along the axis of symmetry under the action of single wheel load of 20 kN acting over circular area at a pressure of 0.56 MPa should not exceed 0.2 mm.

Plate load test was conducted on the prepared subgrade. Radius of plate used was 300 mm. Deflection for a load of 30 kN was 0.75 mm. Granular layer was to be laid over the subgrade. A test section of 150 mm thick was laid. Plate load test was conducted on the two layer system. Deflection corresponding to a load of 50 kN was 1.0mm. What should be the thickness of granular layer so that the deflection criteria is satisfied ?

Explain the Procedure to be adopted to solve this problem

I am not reading out the problem completely to save time but I will discuss the solution.

(Refer Slide Time: 00:58:13 min)

Indian Institute of Technology, Kharagpur

Answer to Question 4 from Lesson 4.11

The Problem:

Determine the thickness of the granular base so that the surface deflection under the action of 20kN (0.56MPa) < 0.2mm.

Given :

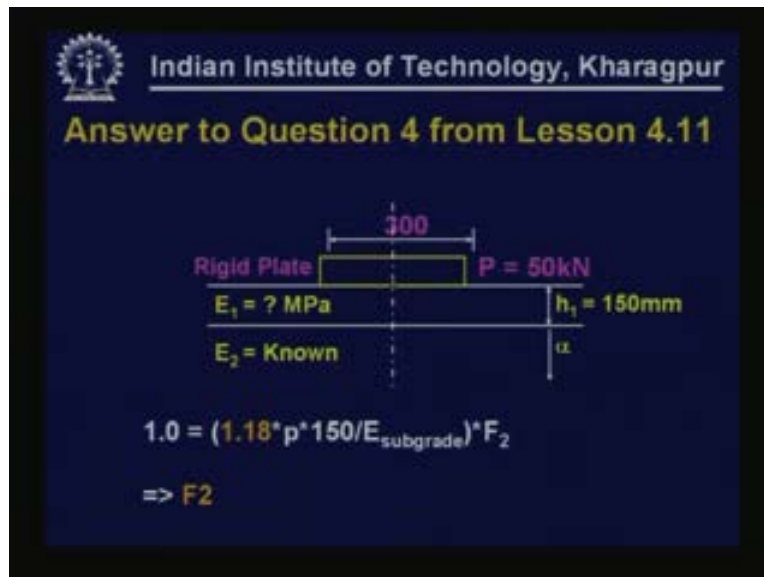
Diagram showing a plate load test on a subgrade. A plate of radius 150 mm is subjected to a load $P = 30 \text{ kN}$. The surface deflection is 0.75 mm. The subgrade modulus is $\mu = 0.5$.

Surface deflection = 0.75mm

$0.75 = 1.18 \text{ pa}/(1/E) \longrightarrow E \text{ Subgrade}$

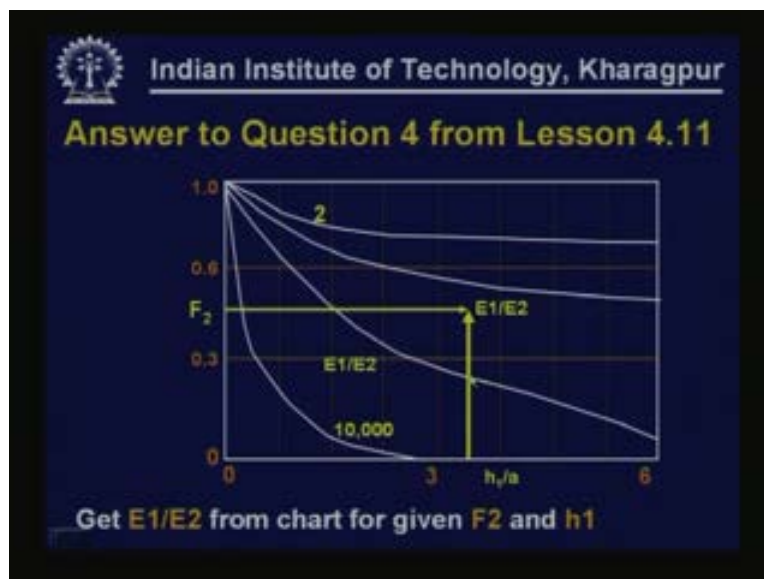
What is given is we have to find out the thickness of the granular base so that the surface deflection under the action of 20 kilo Newton load will be less than 0.2 mm. Given is plate load test is conducted on sub-grade as we had just discussed. We can find out the modulus value of the sub-grade because deflection is known, 'p' is known, 'a' is known so we can find out the sub-grade modulus value.

(Refer Slide Time: 00:58:36 min)



Then granular thickness of known thickness is built over this, plate load test is conducted on a two layer system. so for a rigid plate this is the expression for surface deflection 'p' is known, 'a' is known, surface deflection is also known, E sub-grade is also known so now we can find out F_2 from this plot.

(Refer Slide Time: 00:58:54 min)



So once we know F_2 from F_2 and from known thickness we can obtain the ratio of E_1 by E_2 . Next we have to find out what is the thickness of the layer to be provided. We know the sub-grade modulus value, the allowable deflection is 0.2 this is a flexible plate so 1.5 pa by $E_{\text{sub-grade}}$ into F_2 so E_2 has to be obtained $E_{\text{sub-grade}}$ is known, 'p' is known, 'a' is known, surface deflection

is known so once we obtain E_2 we can get H_1 by a from known F_2 and E_1 by E_2 then we can find out H_1 , thank you.