Introduction to Transportation Engineering
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Lecture - 34
Analysis of Flexible Pavements

Hello viewers welcome to lesson 11 of module IV. As you know module IV is on pavement design. This lesson will be about analysis of flexible pavements. In the previous lessons of this module we have discussed about general philosophy of pavement design. We have considered various loads that come on pavements in terms of traffic loading. We also discussed about climatic and other parameters that need to be considered for design of pavements. We also discussed about characterization of different types of materials that we use in pavements, soils, bituminous materials, aggregates and so on.

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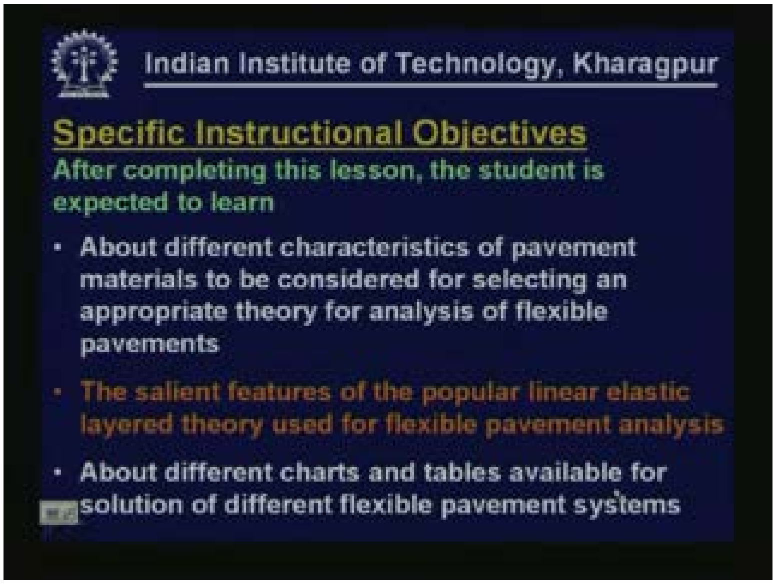

In this lesson we will be concentrating on the analysis of flexible pavements. The specific objectives of this lesson will be after completing this lesson it is expected that the student would be in a position to understand about different characteristics of pavement materials to be considered for selecting an appropriate theory for analysis of flexible pavements.

As you know we use different types of materials in pavements, each material is different in terms of its behavior so we need to be able to model all these different materials independently and together as a pavement structure so that we can select an appropriate theoretical model for analyzing these pavements so that we can conclude stresses, strains, deflections when these pavements are subjected to loads and climatic conditions.

It is also expected that the student would have learnt the salient features of some popularly used theoretical models which are used for analysis of flexible pavements. It is also expected that the student would have learnt about different charts and tables that are available for solution of different types of pavement systems such as single layer system and multilayer systems. Even though nowadays there are software available for solution of any type of pavement system but for a student to be solving classroom problems and solving problems in examination it is essential that we should learn what are the things that are available using which we can still analyze pavements and that is what we are going to be concentrating on in this lesson.

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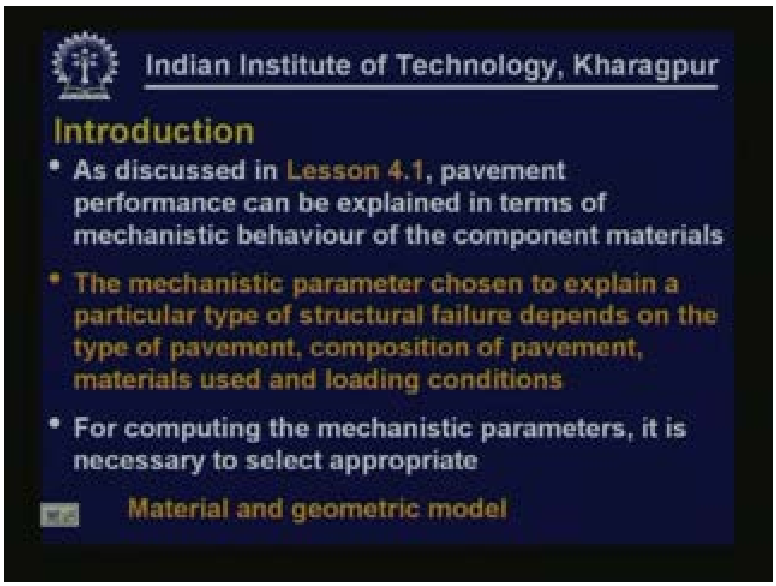

To begin with as we discussed in the first lesson of this module which was about general philosophy of pavement design pavement performance can be explained in terms of the mechanistic behavior of its component materials. Pavement is going to be comprised of different types of materials. So, if we can understand how all these materials respond to the application of load then we can explain the behavior of pavements in a rational way in a better way.

The mechanistic parameters chosen to explain a particular type of structural failure depends on pavement type, because the pavements are going to be undergoing various types of structural failures or other types of failures so we would be selecting different types of parameters to explain different types of failures. These parameters would depend upon the type of pavement, the composition of the pavement the materials that we use in constructing these pavements and loading and climatic conditions to which this particular pavement has been subjected to.

So accordingly we may have to be talking about different parameters for different types of pavements and we should be able to analyze and then find out the value of that particular parameter. Also, for computing these selected mechanistic parameters. It is essential to select appropriate material and geometric models. Like in the case of any other structure it is necessary to select a proper geometric model and also a proper model which represents the material that are used in the pavement.

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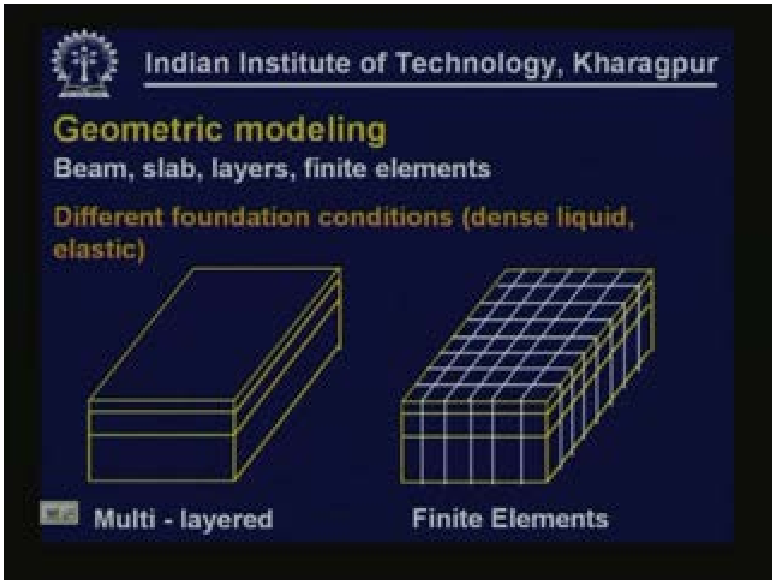

There are various models that are available to geometrically characterize any given structure. Especially when we concentrate on pavements the common models that are used are beam, slab, a system of layers or a system of finite elements. These configurations come with different foundation conditions the foundations either being considered as elastic foundations or dense liquid foundations.

For example, you see on the left hand side of this slide a multilayered system which is the most commonly adopted geometric modeling of a pavement because we understand pavements are constructed in layers, it comprises of different layers so this is a very logical way of modeling pavements. On the right hand side you see a finite element model in which the pavement is discretized into number of small elements. This would be required especially when the nonlinear behavior of different types of materials that we use in the pavement are to be properly modeled.

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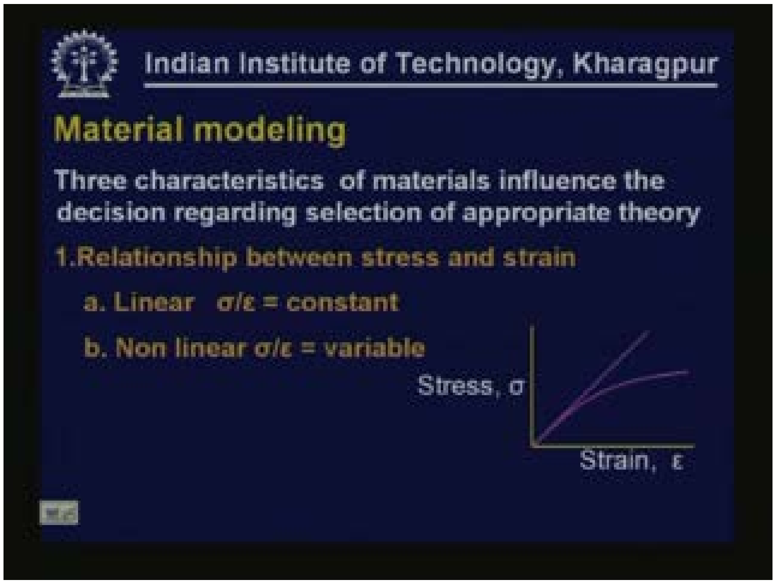

When it comes to material modeling materials are modeled on the basis of three main characteristics. These are the relationship between stress and strain as you can see in this plot if the stress versus strain relationship is linear the relationship is called as a linear relationship and the material is called as a linear material. On the other hand if the relationship is not represented by a straight line then the material is non linear. The material is also described to be having stress dependent material properties.

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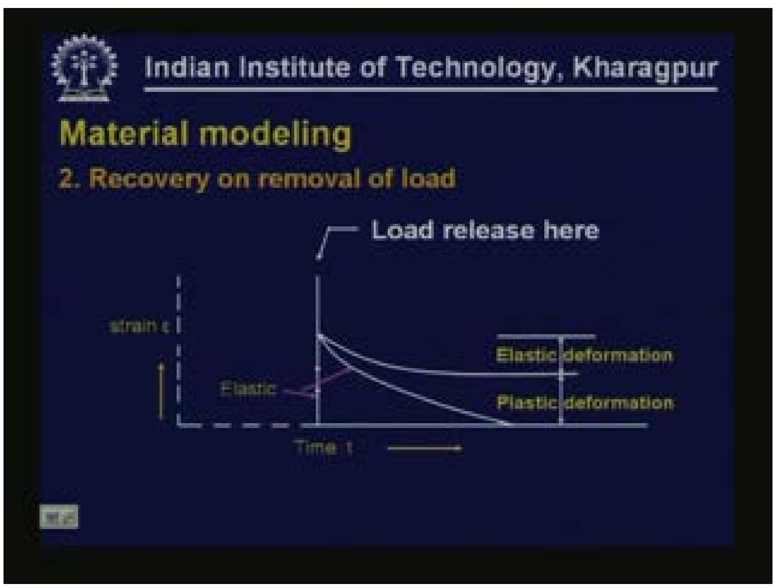

The second characteristic that we consider for selecting an appropriate material model is the response of the material on removal of the load rather the recovery of the material on removal of load.

As you can see from this diagram for example if the material is stressed the corresponding strain is epsilon. On removal the material may instantaneously recover as shown by this and it's an elastic material so it may recover completely but with time slowly. This is a material that is recovering fully because complete deformation is recovered. This is also an elastic material (Refer Slide Time: 8:29) but it has time dependent behavior whereas this material has recovered partly and part of the deformation is not recovered which is known as plastic deformation. The part of the deformation that is recovered is known as elastic deformation so this is partly elastic and partly plastic.

So we can see there are different types of materials purely elastic, viscoelastic and this can be considered to be elastoplastic, partly elastic, partly plastic and so on. So depending upon the materials that are used different materials will have different behavior. But hardly any pavement material can be considered to be having instantaneous recovery. Most of these materials are recovered to a great extent depending upon the condition to which these are subjected to but most of the recovery is over some time period.

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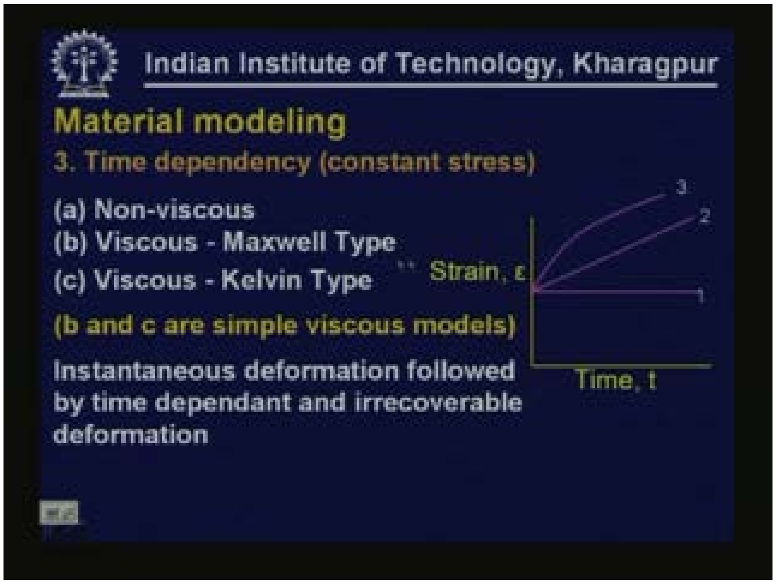

The next characteristic that we consider for selection of a material model is the time dependency. For example, if we stress a material to a constant stress magnitude sigma and epsilon is the resilient strain so under the sustained stress sigma with time if there is no change in strain this is the non-viscous material. But if strain changes for constant stress these are viscous materials. Therefore depending on the relationship between strain and time if it is a linear relationship then this is a linearly viscous material and if it is a viscous non-linear relationship between strain and time then that is a non-linearly viscous material. So you have either a non-viscous material where the strain is not a function of time or you have viscous material where the strain for a constant load constant stress is a function of time and that variation can be linear or non linear so various types of mechanical models are commonly used to describe the viscous behavior of materials. The two commonly used models are, these are the simplest models that are used to describe viscous behavior (Refer

Slide Time: 11:08) Maxwell model and the Kelvin model, b and c the second and third models are simple viscous models. There can be more complex models to properly model the complete viscous behavior of different types of materials.

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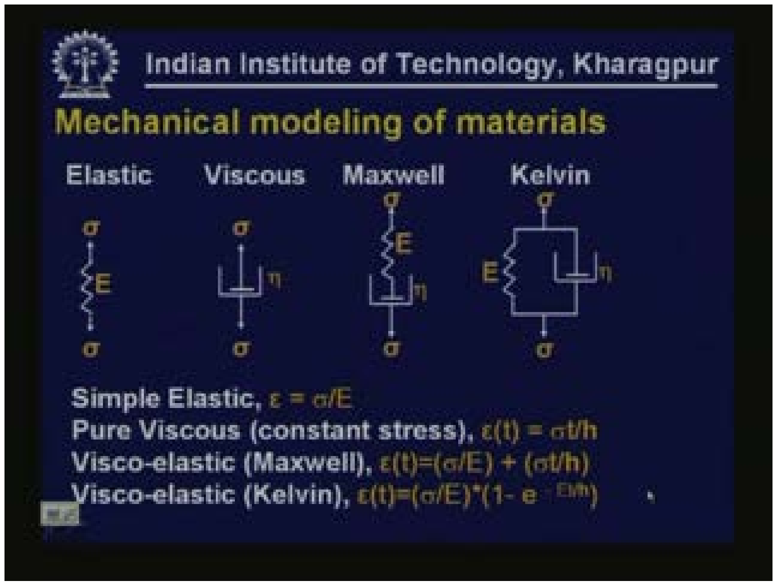

Normally the instantaneous deformation is followed by time dependent and usually irrecoverable deformation. These are some simple mechanical elements that can be used to describe the mechanical behavior of different materials. For a simple pure elastic material a spring can be used with an elastic component a constant value of e so when this element is subjected to a stress sigma the strain is simply given as sigma by E, and this we all know.

For a pure viscous material subjected to constant stress is given by this, this can be represented by a dash pot so the strain is a function of time and given by the stress applied as a function of time and this is eta, this is the viscous coefficient. Similarly, a slightly more complex behavior that can be given by Maxwell model where the strain is represented by a combination of, this model is represented by a combination of spring and a dash pot, the strain as a function of t is given by stress divided by the elastic constant e plus stress and as a function of time and then viscous coefficient.

This is a Kelvin model in which spring and the dash pot are put in parallel and when this is subjected to sigma the corresponding strain as a function of time is given as stress by elastic constant into 1 – e to the power e into time divided by eta.

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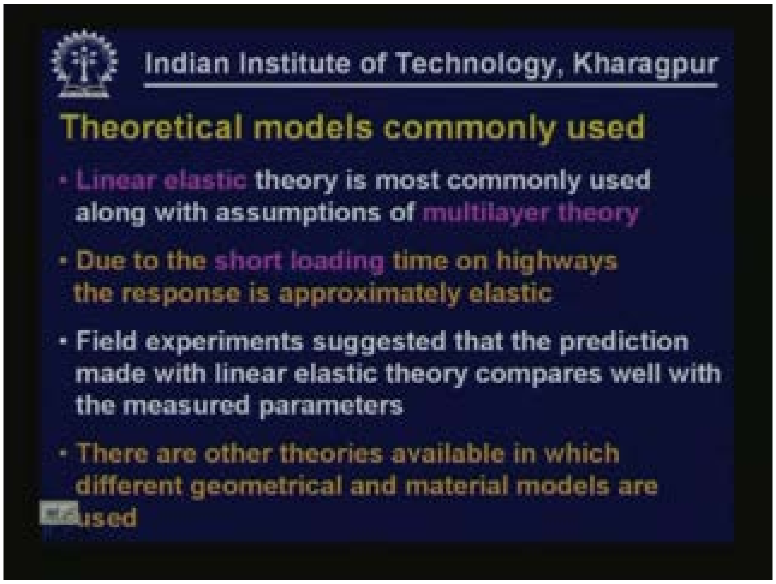

The commonly used theoretical models for analysis of flexible pavements are linear elastic theory which is the most commonly used theory and the geometric model that is used is layered theory. So linear elastic layer theory is the most commonly used model for analysis of flexible pavements.

Layered adoption of assumption we understand because pavement is a layered system. But we have to be justifying the assumption of linear behavior of pavements. This is due to the short loading time that we see in pavements because the pavements are subjected to moving loads they travel at different speeds 40, 60 even 100 km per hour. The duration of the load pulses to which the pavement is subjected to will normally be very small like ten milliseconds fifteen milliseconds twenty milliseconds so this is the order of the duration of the load pulses.

So, for such small load pulses the behavior of the pavement is generally observed to be elastic, and most of the deformation is normally recovered although there is some irrecoverable component of deformation so that's the reason why most of the agencies use linear elastic layer theory for analysis of flexible pavement. Also, various field experiments conducted throughout the world in various countries have suggested that the stresses and strains and deflections observed in field can adequately be explained by using linear elastic layer theory. That’s the reason most agencies use linear elastic layer theory. There are of course other theories available in which different geometric and then material models are used.

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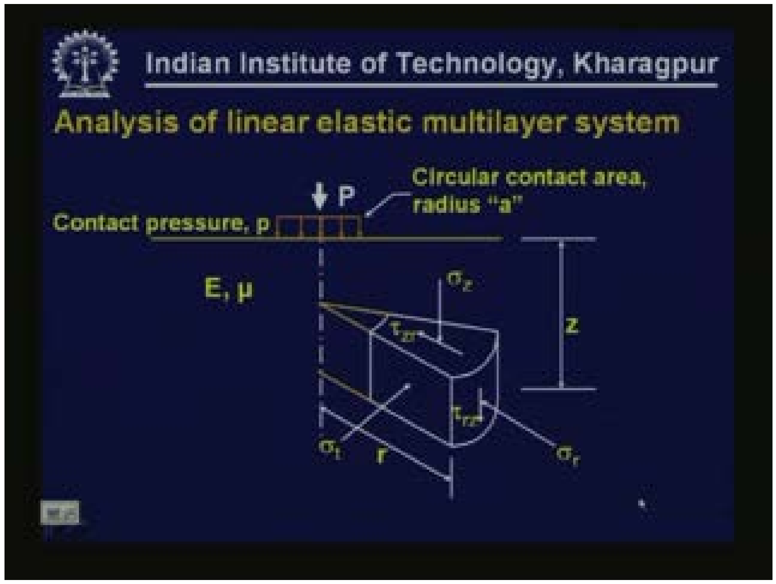

To discuss the analysis of linear elastic multilayer system, a simple system would comprise of a single layer characterized by elastic properties elastic modulus value and Poisson ration value and in a single system the pavement would be subjected to normal surface load P distributed over circular contact area with a radius of 'a' and the corresponding contact pressure can be calculated from applied load and the contact area. Say, if you consider an element at depth Z and at a radial distance 'r' there would bE3 normal stresses sigma suffix z, sigma radial along the radial direction and sigma tangent tangential along the tangential direction and also one shear stress. So normally these are the stresses that we will have to be analyzing.

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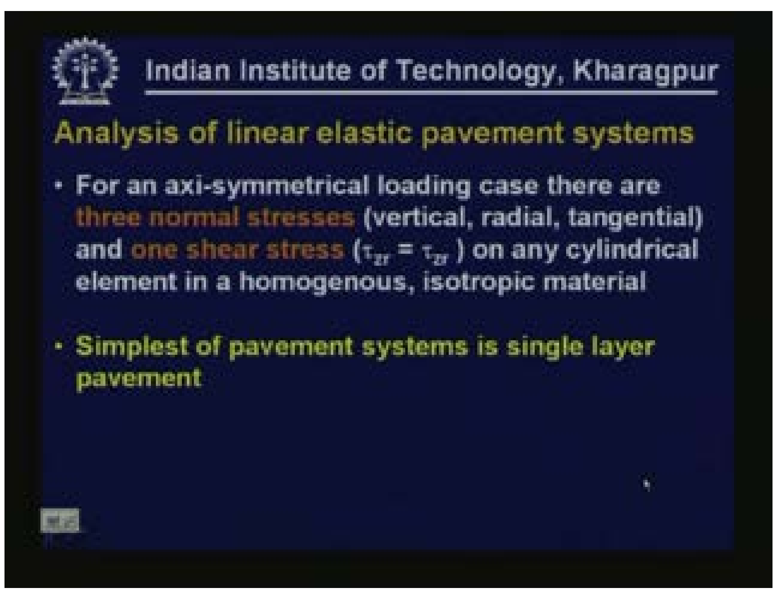

For an axi-symmetric case as we have just seen there are three normal stresses; vertical, radial and tangential and one shear stress on any cylindrical element in a homogenous isotropic material. The simplest of pavement systems is a single layer pavement system.

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Let us consider the analysis of one layer pavement system although you normally do not find any single layer pavement system because pavements normally comprise of different layers. However, if we are trying to find out what are the stresses the subgrade is subjected to it can be analyzed as a single layer system.

Also, when we have very thin layer of pavement placed over subgrade if we ignore the strength of the subgrade contribution or rather the thin pavement then the both the layers together can be approximated to be single layer and that can be analyzed as a single layer. Also, many-a-times it becomes very convenient computationally to convert a given multilayer system into an equivalent single layer system and analyze it because in some problems we have to solve the multilayer system repeatedly thousands of times. So in such a case if the multilayer system can be converted into an equivalent single layer system then lot of computation time can be saved so this is usually done. Hence it will be very convenient to learn about analysis of single layer systems.

Boussinesq as you might have learnt in solid mechanics subject Boussinesq was the first to give a solution for a point load on an elastic half-space. Elastic half-space is a single layer system.

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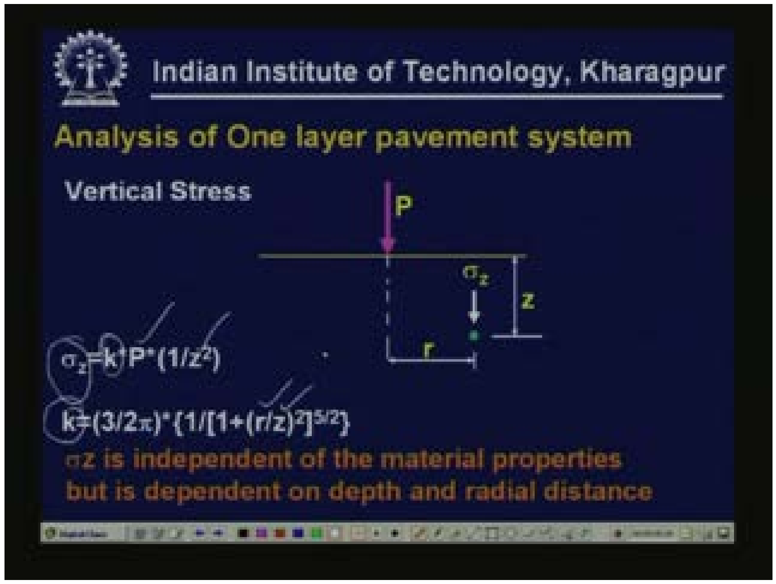

The Boussinesq analysis considers a concentrated load, P applied at this surface so for this system vertical stress at any given location or at a given radial distance from the load and at a given depth from the pavement surface can be given by this expression; vertical stress as a function of load applied and the depth at which the element is located multiplied by a constant k where k is a function of the location of the point in the pavement system.

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The Boussinesq can also solve solution of a single layer system subjected to a concentrated load can be expanded to a uniformly distributed circular load by integrating the point load or the entire area. the circular contact area can be distributed into small elements and if the load that is coming on the small element can be considered to be P concentrated load for which we have the solution then we can calculate the vertical stress at a given location with reference to this load applied at this location so this can be integrated over the entire area giving us the vertical stress due to a circular contact area and uniformly distributed load. This obviously is a more realistic loading condition.

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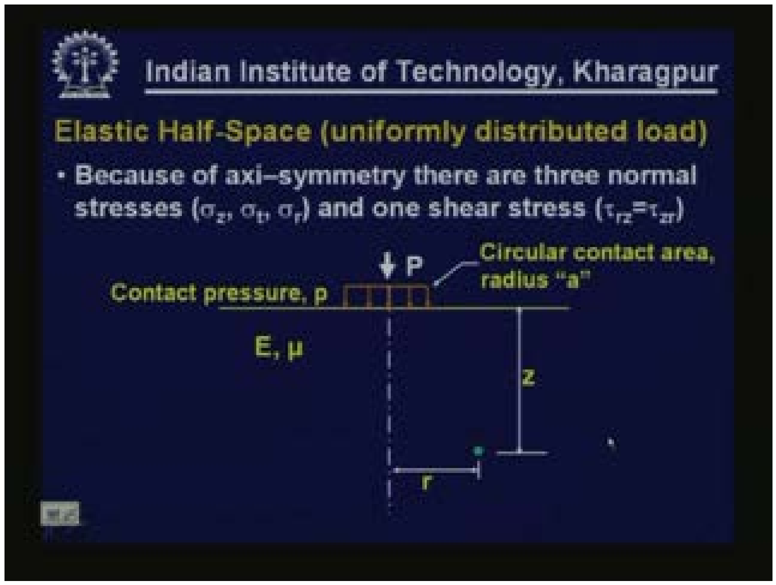

Because of axi-symmetry as I mentioned earlier in the case of an elastic half-space when it is subjected to uniformly distributed load circular contact area we have he axi-symmetry case so we normally have three normal stresses and one shear stress to be analyzed. There are number of charts that are available. As I mentioned earlier nowadays there are number of software available to easily analyze any pavement system but for those who want to do classroom problems and those who do not have access to any of the software there are number of charts and tables available for different types of pavement systems in standard text books. So what I am going to discuss here in the next few slides will be some such systems that are available for solution of single layer system, two layer system and three layer system. In this slide we are going to talk about some charts and tables that are available for solution of single layer system.

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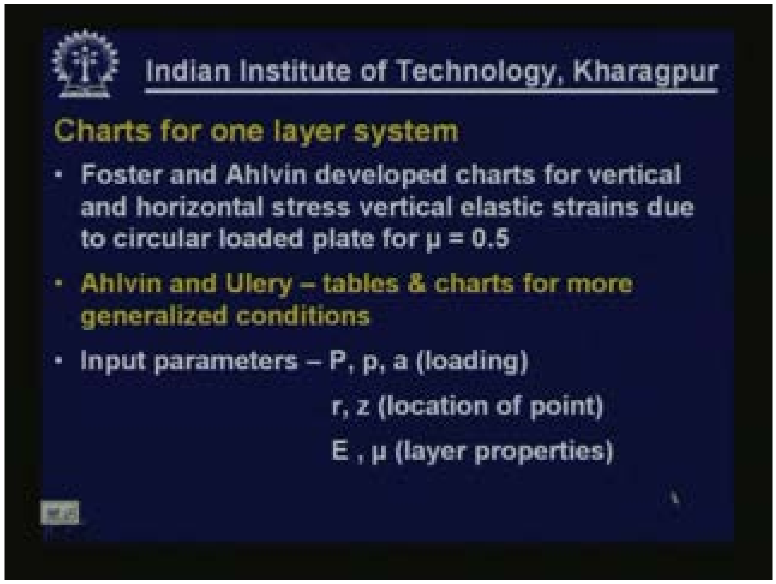

The first set of such charts were developed by Foster and Ahlvin. They have developed charts for vertical and horizontal stress and vertical elastic strains due to circular loaded plate for a Poisson ratio value of 0.5. Subsequently Ahlvin and Ulery developed more generalized solutions and presented them in the form of tables and charts. The input parameters that are to be considered for solving elastic half-space will be the load that is applied and the contact stress. Once we have these two parameters we can of course calculate the radius of load contact area assuming this to be a circle and location of the point at which we are trying to analyze stresses and strains and of course the elastic properties of the layer that is elastic modulus value and Poisson ratio value of the layer.

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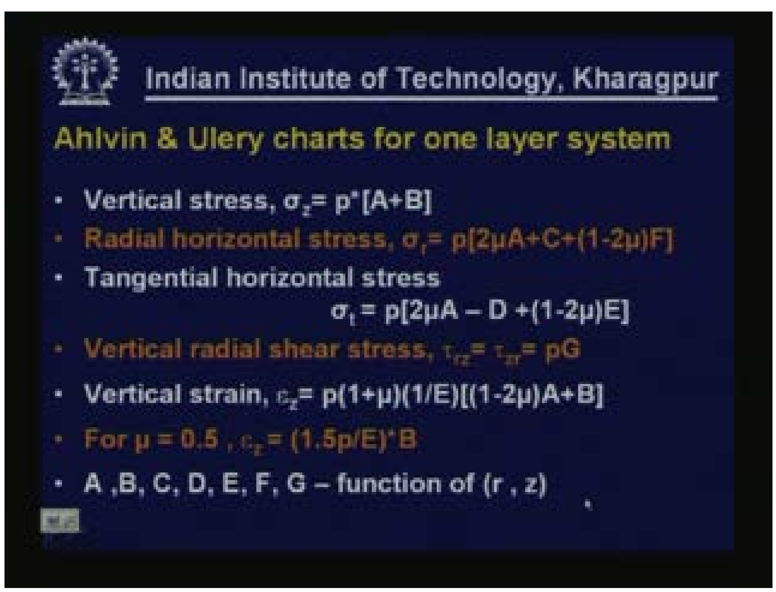

Ahlvin and Ulery have given charts for one layer system for the following parameters. We can calculate vertical stress sigma suffix z, we can calculate radial horizontal stress sigma suffix r, tangential horizontal stress sigma suffix t, vertical radial shear stress, vertical strain and other parameters. As you can see here all these parameters sigma suffix z, sigma suffix t and sigma suffix r are function of P that is applied and some coefficient A, B, C, F, D and so on G. So obviously we have to find out for a given system what are the coefficients A, B, C, D and so on to be adopted. These values are function of R and Z which represents the location of the point in the pavement system.

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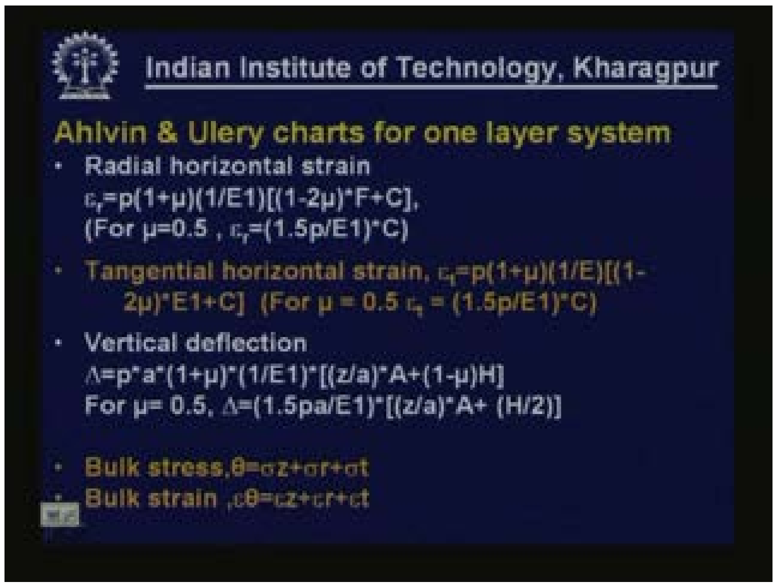

Similarly we can see expressions for other parameters like radial horizontal strain, tangential horizontal strain, vertical deflection, bulk stress, bulk strain all in terms of the input parameters ‘p’ ‘a’ elastic modulus value E1, Poisson ratio value mu and the coefficients that are to be selected from the charts and tables that are been given by Ahlvin and Ulery A, B, C, D, E, F, G, H.

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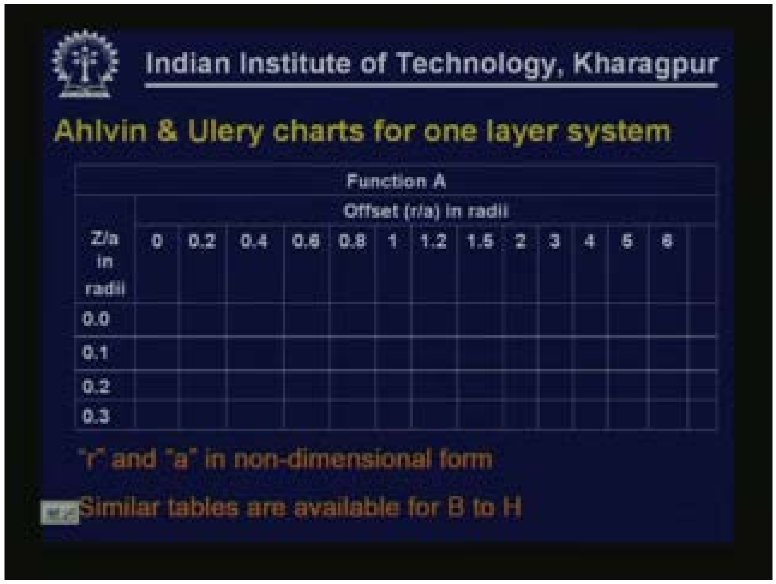

Typically the charts look like this. For example, for identifying or determining the value of function 'a' if we know the location of the point in the pavement that is radial distance and depth from surface this can be expressed in terms of radial offsets (Refer Slide Time: 25:03) given by r by a where 'a' is the radius of load contact area and z by a so the corresponding value of 'a' can be selected from this table. So we have similar tables available for all the other coefficients b to h.

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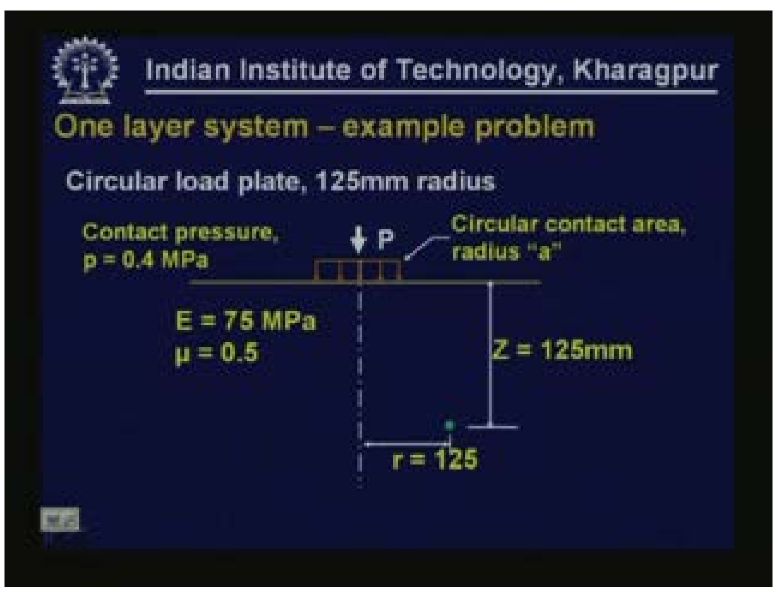

We will take up an example problem here. In this case we have considered a circular load plate of 125 mm radius. An elastic system has got a modulus value of 75 MPa and Poisson ratio value of 0.5 and we are considering a point which is at a radial distance of 125 mm and at a depth of 125 mm from the surface and we are assuming this to be a circular contact area and the contact pressure is assumed to be 0.4 MPa.

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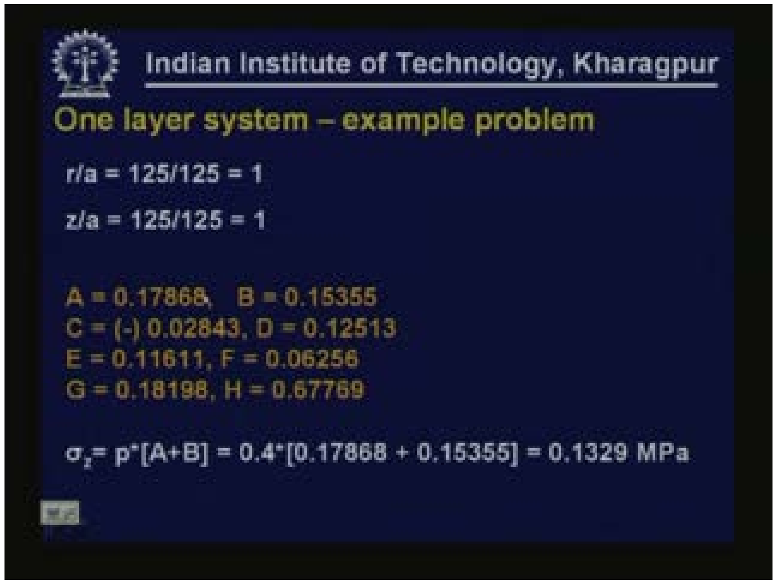

Therefore for the given problem r by a is 125 by 125, radius of load contact area is 125, radial location of the point at which we are trying to calculate stresses and strains and deflections is 125 so r by a is 125 by 125 = 1, the point is at a depth of 125 so z by a is also 1 hence for values of r by a = 1 and z by a = 1 these are the values of various coefficient that I have selected from the tables. For this sigma suffix z can be calculated as ‘p’ applied pressure was point four and coefficients of a and b so this is the vertical stress at that location.

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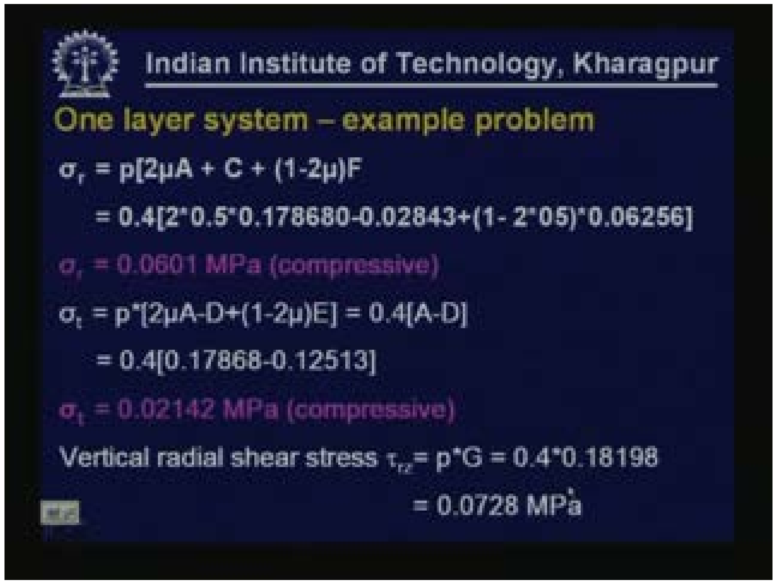

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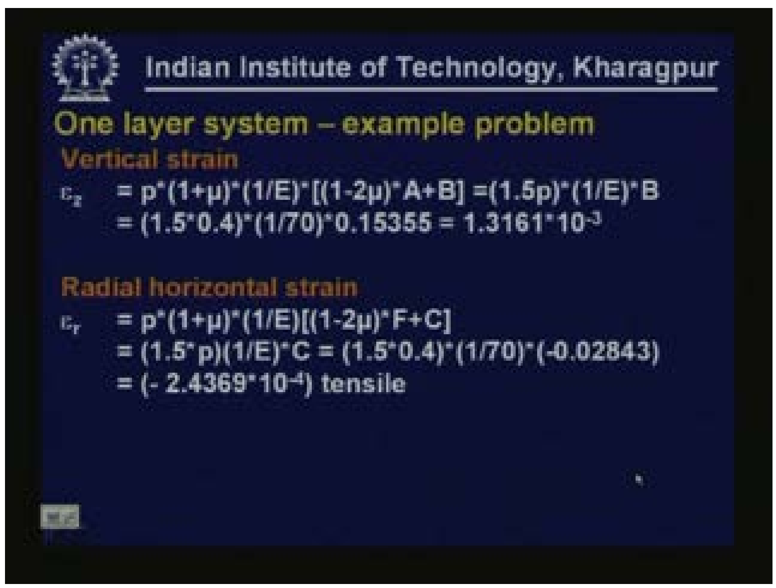

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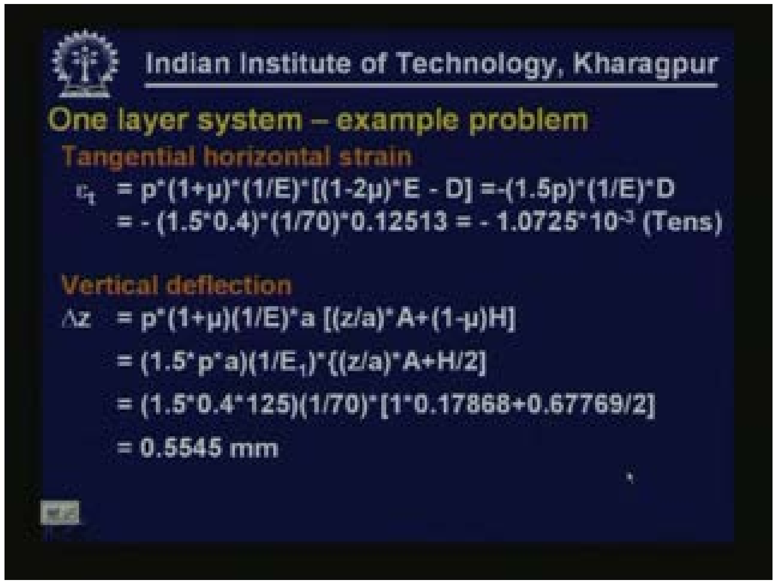

Similarly, I have selected all the other parameters C, F and other parameters and the values that I have calculated for different other parameters sigma suffix r, sigma suffix t and so on are presented here for your reference. Here we have vertical strain for the same problem and also the radial horizontal strain for the same problem. Once we are in a position to select all these parameters a to h we will be in a position to simply substitute these factors into equations and then calculate any parameter we want. Therefore for a single layer system we have charts and rather tables available using which we can calculate almost any parameter at any location in the pavement system. But as the number of layers increase and the number of variables that we have to handle increases then presenting them becomes more complex so when we move to two layer, three layer, and four layer systems we will be in a position to calculate fewer parameters.

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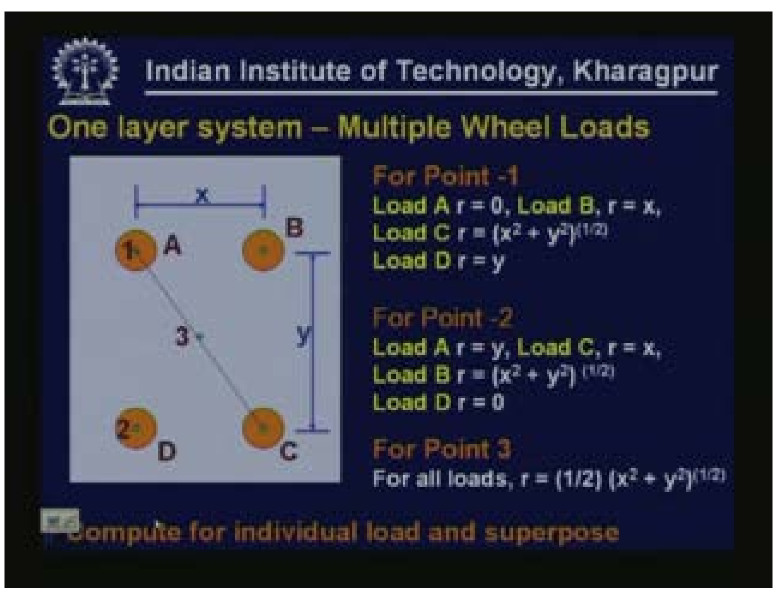

Let us consider a one layer system which has to be analyzed for multiple wheel loads. What we have seen so far is the pavement system subjected to single load but if we have a multiple wheel load, for example there are four wheel loads here so how do we analyze for stresses and strains at any given point. In this case we have four loads a b c d and we are also considering another three points 1, 2 and 3. For example if I want to analyze for point one what will be the stress and strain and deflection at that point due to all these four wheel loads.

What has to be done is we will simply do super positioning. And whatever is the stress because of load a load b load c and load d at point one if you can analyze that we will algebraically add all of them.

So, for 0.1 due to load A the radial distance is 0, for load B radial distance is x and for load D radial distance is y as you can see from this figure but for load C the radial distance is given as x square + y square and root. So once we have the radial distance and for all the four loads we may be talking about a particular depth it can be surface also so depth is constant therefore it is the same for all the four wheel loads so we know the depth we know the radial distances for all the four wheel loads, we simply find out the coefficient for all of them and possibly we are trying to calculate vertical stress, so calculate vertical stress for all the four wheel loads a b c d and add them algebraically.

Similarly, if we want to find out stresses for vertical stress for 0.2 for load A r will be y, load C r will be x, load D r will be 0, load B r will be x square plus y square under root. Similarly for point three for all loads r will be x square + y square under root 1 by 2. So what we are trying to do here is basically compute for individual wheel load and superpose.

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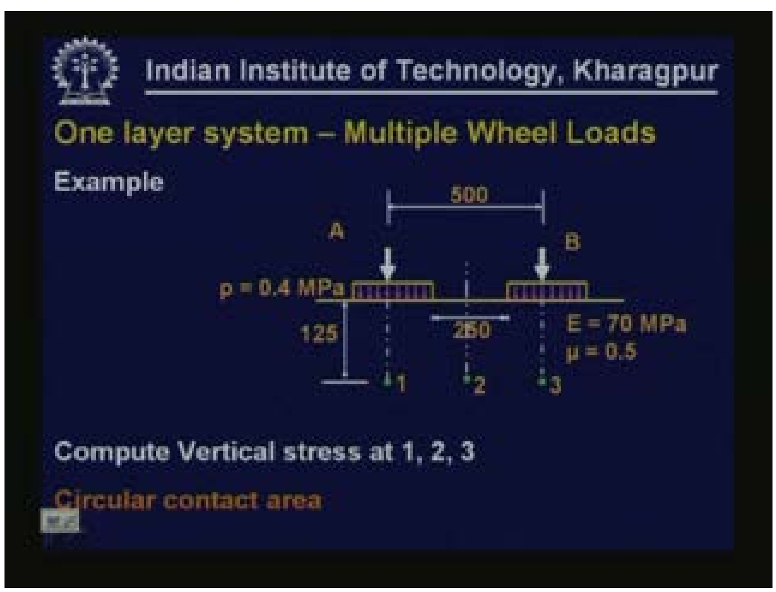

We again have a similar example here, calculation of stresses due to multiple wheel loads. Here I have considered a dual wheel system. There are two wheel loads separated by a distance of 500 mm, contact pressure is 0.4 MPa so we are trying to calculate stresses at a depth 125 mm, the distance between the two wheel loads is also given, properties of the elastic layer is given, modulus value of 70 MPa Poisson ratio value of 0.5 is there. Now we are trying to compute vertical stress at points 1, 2, 3 and we are assuming that these loads are having circular contact area.

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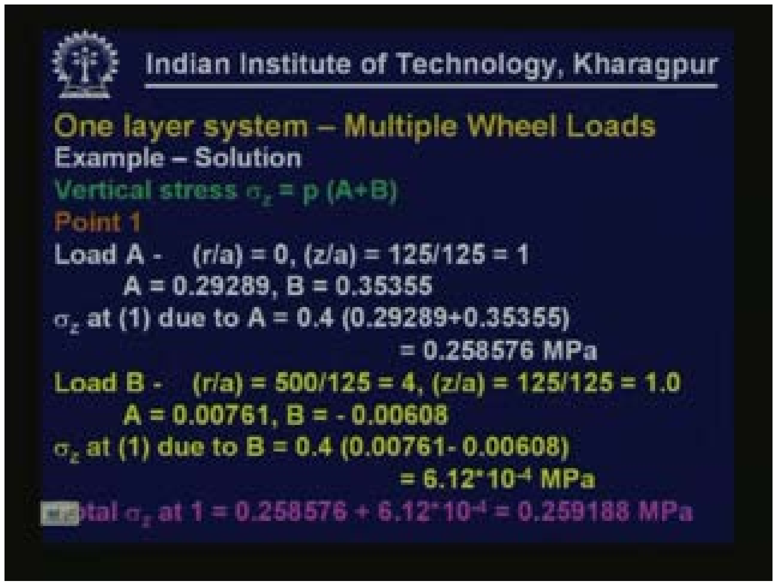

This is also very similar to what we have just discussed in the previous few slides. For point one due to load a r by a is 0, z by a is 125 by 125 = 1 these are the coefficients that are selected from the tables for r by a = 0, z by a = 1. So sigma suffix z at 1 due to a is 0.258576 MPa. Similarly for load b r by a equal to five hundred by 125, if we refer to the figure we will know the radial distance that is five hundred divided by 125 becomes 4, z by a is again 125 by 125 = 1. So the coefficients for load b; a is 0.00761, b is – 0.00608 for this the vertical stress of course this is very small 6.12 into 10 to the power of – 4 MPa.

Therefore if we are trying to find out total sigma suffix z at one this is summation of the stresses due to the two wheel loads that becomes 0.259188. Incidentally the stress due to the second load because of its radial distance has been very insignificant.

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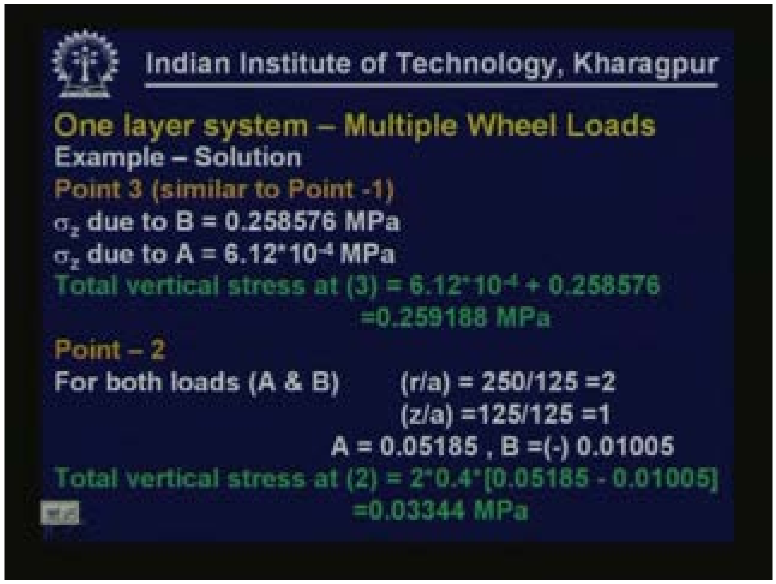

Similarly we can calculate stresses for 0.3 and 0.2 in the same manner.

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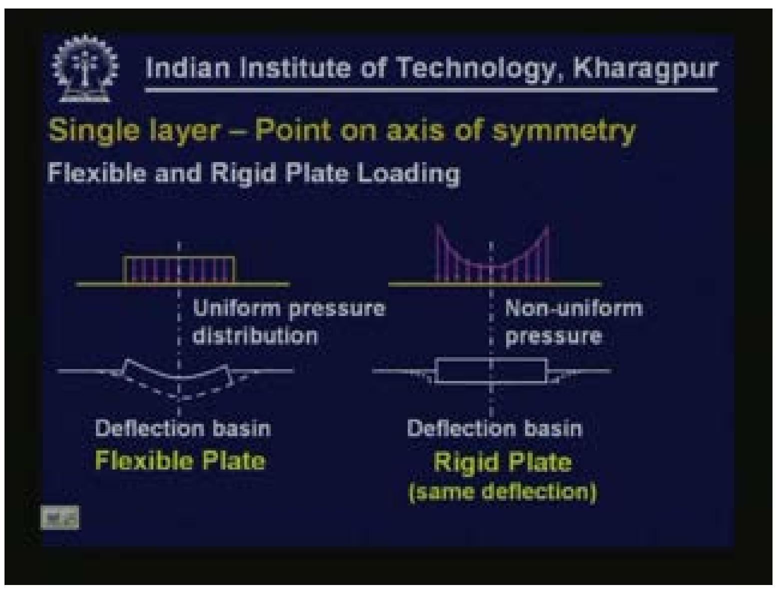

When we consider single wheel layer systems there are two different things that we have to consider. The loading can be considered to be either as a flexible plate or as a rigid plate depending upon how the load is applied to the pavement surface. For example, if you consider load transmitted to the pavement through a pneumatic tyre it is not a very rigid plate so it can be considered to be a flexible plate.

On the other hand if you have a very stiff plate that is plate over plate which is stiffened together if we have a system of very rigid stiff plate then we consider that to be a rigid plate. A rigid plate basically does not deflect but a flexible plate deflects.

As you can see from this diagram on the left hand side we have the system of flexible plate loading on the right hand side we have rigid plate loading. So on the left hand side the system gives you uniform pressure distribution and on the right hand side you have non-uniform pressure distribution. So the analysis for these two systems will be slightly different.

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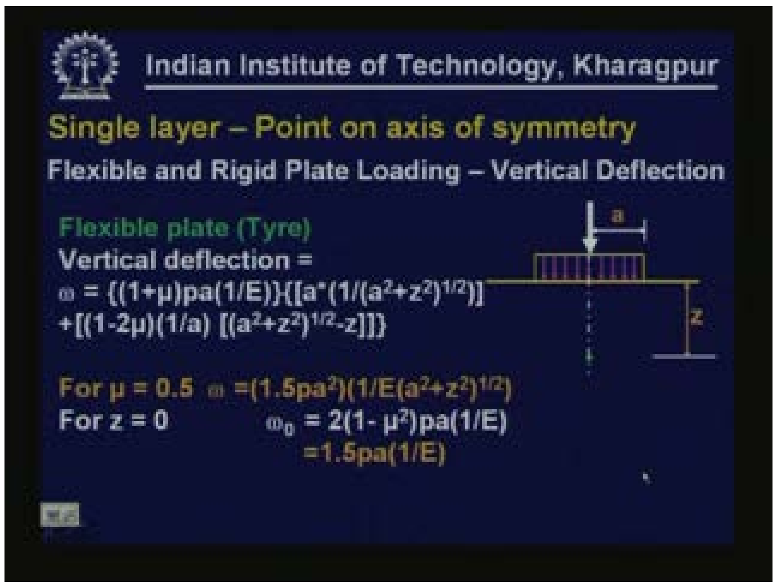

For example, if there is a flexible plate the vertical deflection W is given as a function of pressure that is applied, radius of load contact area and of course the modulus value of the pavement, radius of load contact area at the depth at which we are trying to find out stress, and Poisson ratio value. For a Poisson ratio value of 0.5 this simplifies to 1.5 pa square into 1 by e a square + z square under root. For point on the surface the surface deflection is given as 1.5 pa by e. This is a very convenient expression for us to use, we will see how it can be used. The surface deflection for a flexible plate is given as 1.5 pa by e where ‘e’ is the modulus value of the elastic half-space, ‘p’ is the contact pressure, ‘a’ is the radius of load contact area.

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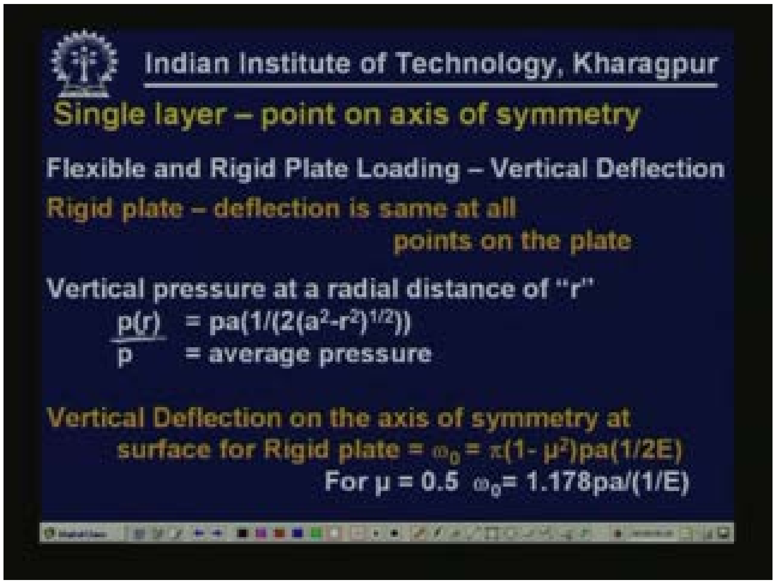

Similarly for rigid plate, and by this expression we will know how the pressure is going to be distributed, and how it is going to vary radially but we normally use the average pressure. So the vertical deflection along the axis of symmetry at the surface for a rigid plate is given as pi into 1 – mu square pa by 1 by 2e where 'e' is the modulus value of the elastic half-space. If we assume the Poisson ratio value to be 0.5 so the surface deflection simplifies to 1.178 pa by so normally we approximate this to 1.18 pa by e. So this again another very convenient expression, we will see how we can make use of these two expressions, one for rigid plate and another one for flexible plate.

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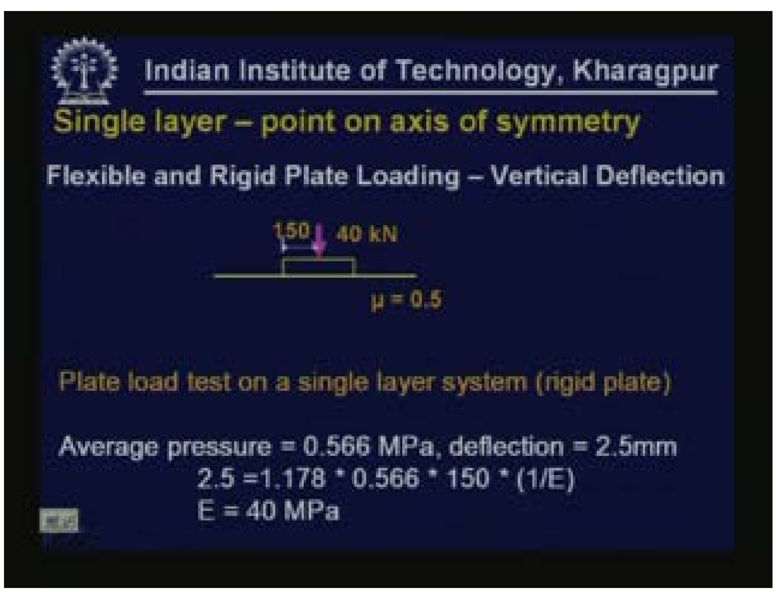

Let us consider an example. Let us see here we have a problem which has been subjected to a plate load test. The load applied is about 40 kilo Newton and the radius of load contact area is 150 mm and the Poisson ratio value is assumed to be 0.5 as you can see from this diagram. So we have conducted a plate load test on this single layer system, and as this is being considered as a plate load test so this can be considered as a rigid plate.

We can observe the load that is applied and we also know the contact area so I calculated the average contact pressure as 0.566 MPa 40 kilo Newton by area of the plate. For this load the observed deflection is about 2.5 mm at this location. This is the surface deflection (Refer Slide Time: 37:20) along the axis of symmetry of the load. Substituting this in the expression for surface deflection for rigid plate 2.5 mm this is the surface deflection that is measured is given as 1.178 we can approximate this to 1.18 also into p into a divided by e, e is what we do not know but we have all the other inputs available. From this we can estimate what is the elastic modulus value of the layer which works out to 40 MPa.

This is a very convenient system though not very rational because this is a very static load, the loading time is very large but still we can get some idea about the elastic modulus value of a single layer system provided we have a single layer system we can conduct a plate load test on that and using the expression for surface deflection we can work out the modulus value of the elastic layer system.

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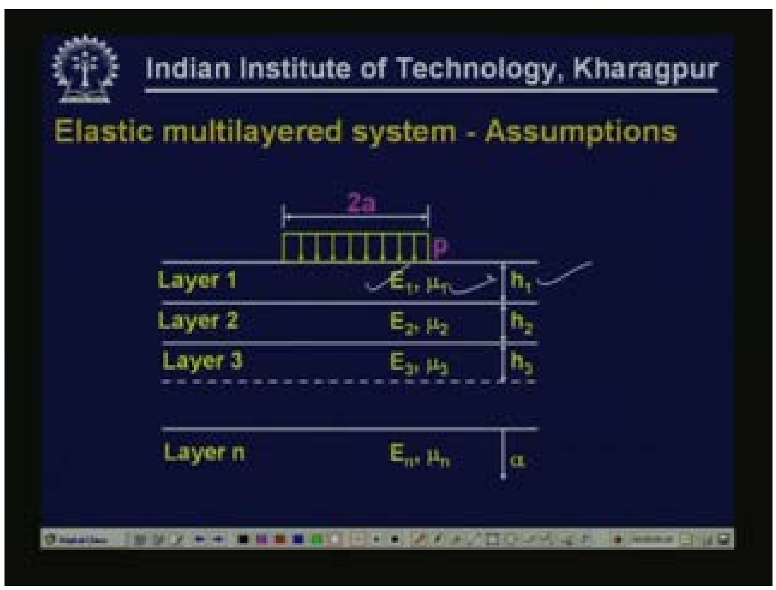

The next system would be an elastic multilayered system. It comprises obviously of a number of layers one to n layers. Each layer is characterized by the thicknesses and elastic modulus value and Poisson ratio value. And typically what we consider is a circular contact area load, uniformly distributed pressure and applied at the surface.

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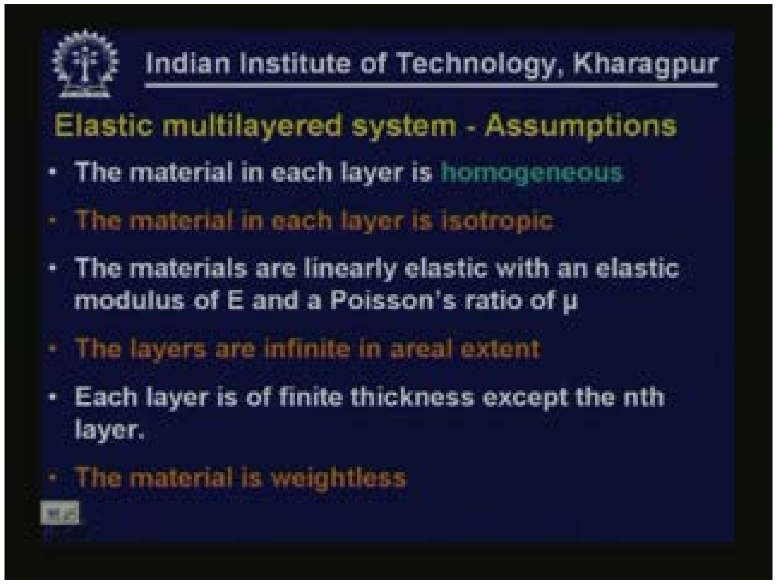

Let us see the assumption that we make in the case of multilayered system. In this we assume that the material in each layer is homogeneous. We also assume that the material is isotropic. We assume that the materials are linearly elastic with an elastic modulus value of e and a Poisson ratio value mu. We assume that the layers are infinite in aerial extent horizontally we assume them to be infinite in extent .we also assume each layer is of finite thickness except the last layer bottom most layer which is infinite in vertical extent also in one direction and we also assume the material to be weightless.

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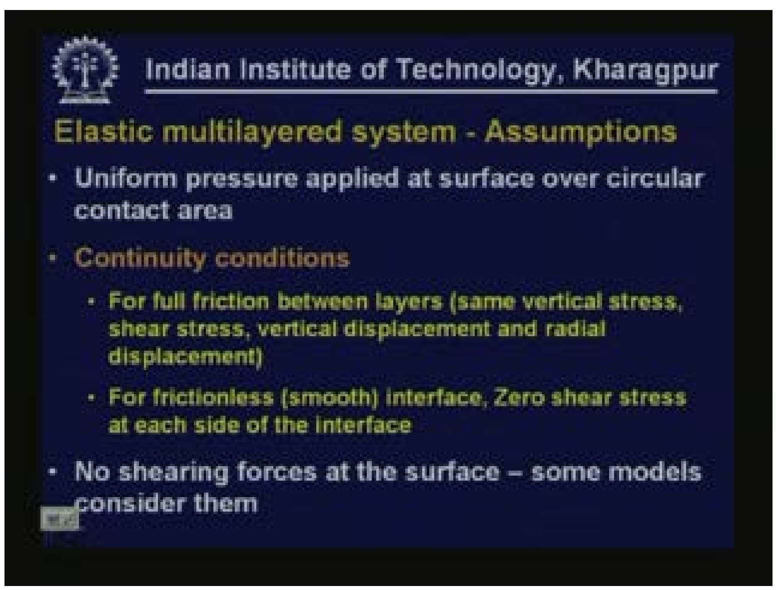

Other assumptions we make about the loading conditions and boundary conditions are uniform pressure is applied at the surface over circular contact area. Although we can still solve the pavements for other types of surface loading conditions the continuity conditions are between different layers, full friction is assumed between layers that means there is same vertical stress, same shear stress, same vertical displacement and same radial displacement at the interface we assume full friction and full contact also. We can also analyze the pavements for frictionless interface also assuming that the interface between different layers is smooth that leads to zero shear stress at each side of the interface. We normally do not consider shearing forces at surface although some models are available which are capable of analyzing pavements for surface shear forces.

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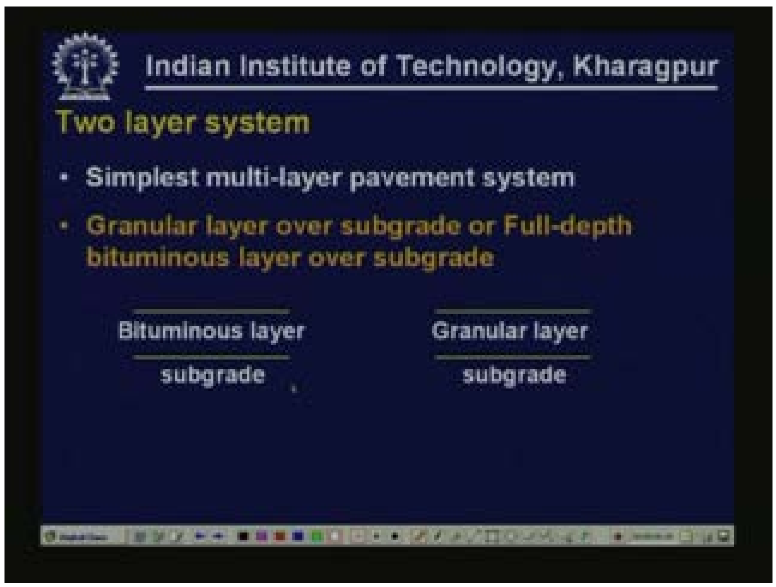

The simplest of multilayer systems is a two layer system. two layer system could simply be a granular placed over a subgrade or a bituminous layer placed directly over subgrade which is called as a full depth asphalt concrete pavement or bituminous concrete pavement which is commonly used in United States also known as the full depth pavement. Therefore it can be either a granular pavement placed over subgrade or a full depth asphalt pavement placed over a subgrade so these two examples are shown in the slide. So this is the full depth bituminous pavement and this is a full depth granular pavement.

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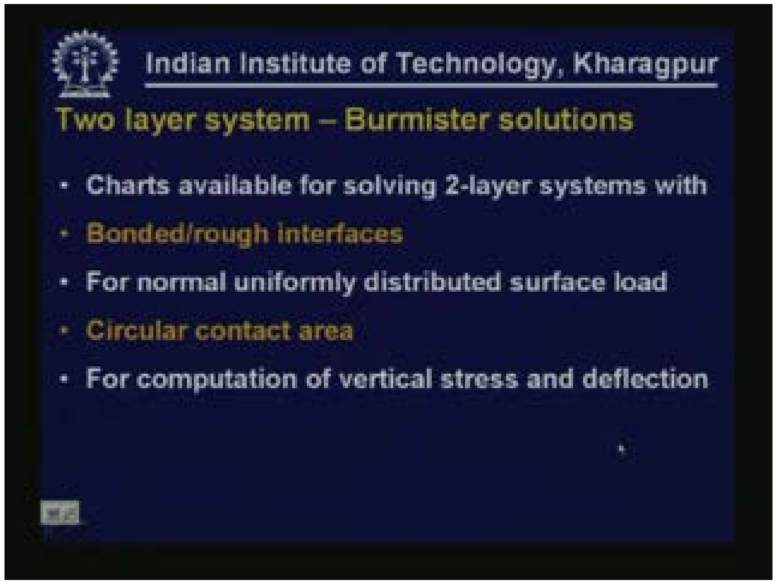

For this also Burmister has given solutions for solving two layer systems on the basis of linear elastic layer theory. Number of charts are available for solving two layer systems with both rough and tough uses. And for normal uniformly distributed surface load this is the most common assumption that we make about surface loading and for circular contact area typically charts are available for computing vertical stress and deflection.

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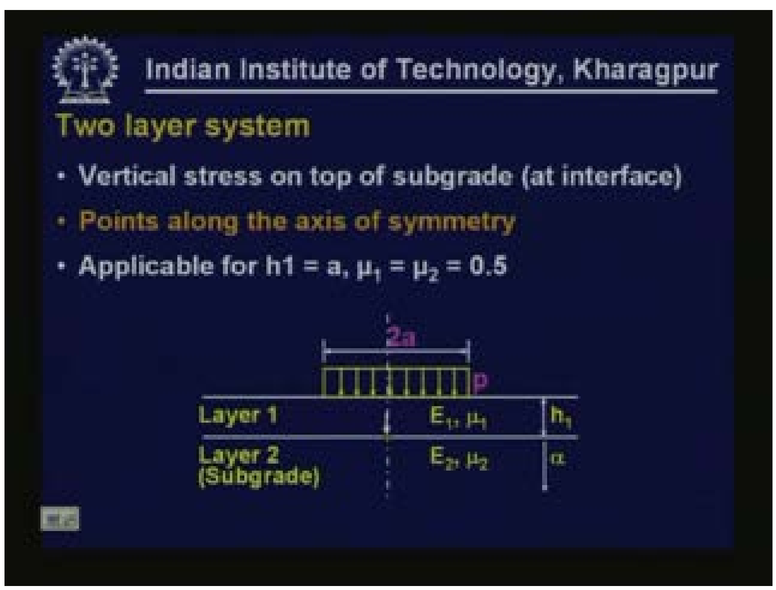

The vertical stress on top of subgrade that is at the interface between the pavement length and the subgrade can be calculated with reference to this diagram. The points along the axis of symmetry of load can be considered. So a two layer system is represented by two layers the pavement and the subgrade represented by thickness of the pavement, the bottom one has got infinite thickness and modulus value of the two layers and Poisson ratio value of the two layers and the inputs in terms of loading are contact pressure and radius of load contact area and this is the point (Refer Slide Time: 42:52) at which we can analyze the pavement for vertical stress.

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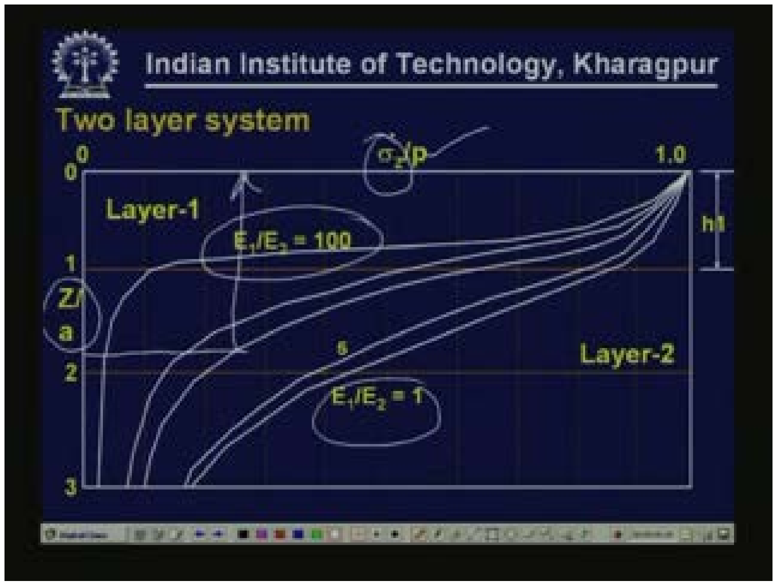

This is a typical chart that is available for finding out vertical stress. this is the vertical stress that we are trying to find out at a given depth but given in terms of non-dimensional form where p is the contact pressure that is applied at the surface and similarly the depth at which we are trying to find out the vertical stress is also expressed in a non-dimensional form where the depth is divided by the radius of load contact area. This depth is nothing but thickness of the pavement because we are trying to find out interface vertical stress.

You have number of charts available for different modular ratios E1 by E2 where E1 is the modulus value of the pavement and E2 is the modulus value of the subgrade. So, for different modular ratio values we have different charts available so for a given z by a value and for a given E1 by E2 value we can find out what is the corresponding sigma suffix z by p and by knowing ‘p’ we can calculate sigma suffix z value.

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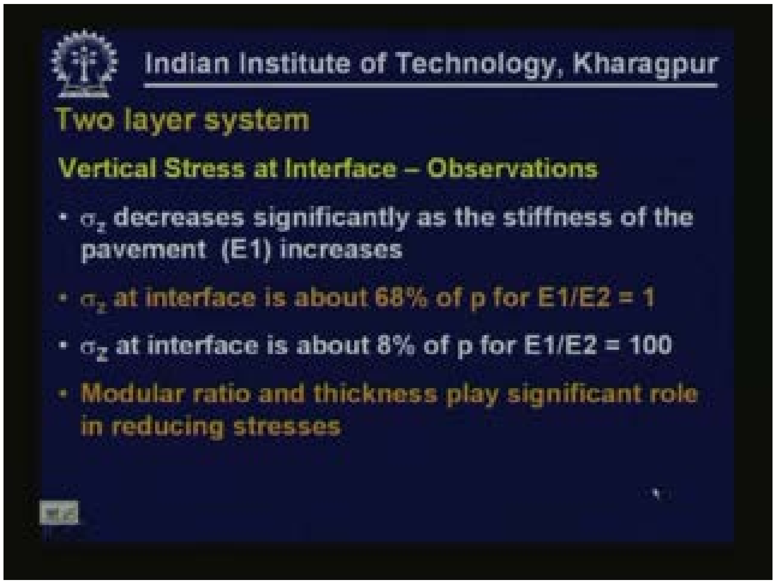

From this chart you could have observed that sigma suffix z decreases significantly as the stiffness of the pavement increases, that is E1 increases as a modular ratio value increases. Sigma suffix z at the interface is about 68% of p when we have considered E1 by E2 = 1. E1 by E2 = 1 represents a single layer system because both layers have got the same properties so it is a single layer system. So when we have a single layer system at that particular depth the stress is almost about 68% of what is applied at the surface.

But on the other hand when the modular ratio increases to about 100 that means the elastic modulus value of pavement is about hundred times the modulus value of subgrade then this vertical stress at the same depth gets reduced to about 8% of what we applied at the surface as contact pressure. So, modular ratio and thickness of pavement play a very significant role in reducing the stresses on the subgrade.

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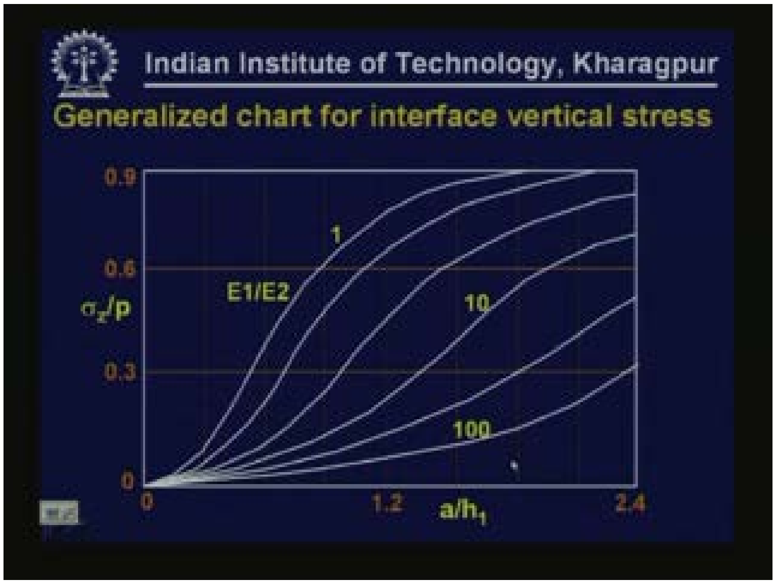

A more generalized chart for determining the interface vertical stress is given in this chart where a by h1 where 'a' is the radius of load contact area, h1 is the thickness of the pavement that is first layer so if you know the thickness of the pavement layer and also the ratio of E1 by E2 you can find out sigma suffix z.

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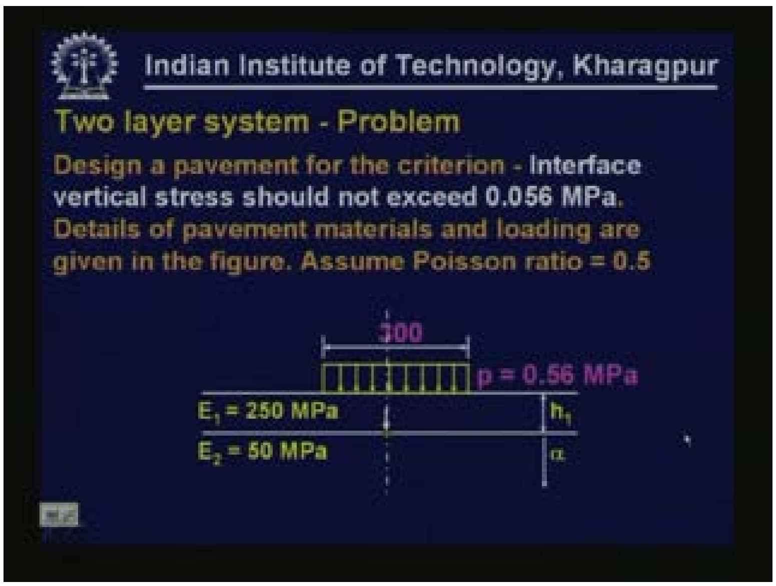

Let us take up a problem for a two layer system. This is the design of pavement for the following criteria. The criterion is the interface vertical stress should not exceed 0.056 MPa. so we have to select thickness h1 in such a way that what is the material to be used is also given E1 has to be 250 MPa this is the material we are going to use, E2 is also known that is

50 MPa and this is the vertical stress at this location we are going to be concerned about and radius of load contact area is given, contact pressure is given, all the inputs are available so we only have to find out what is the thickness that we have to place here. It is also given that Poisson ratio can be assumed to be 0.5 to simplify the calculations.

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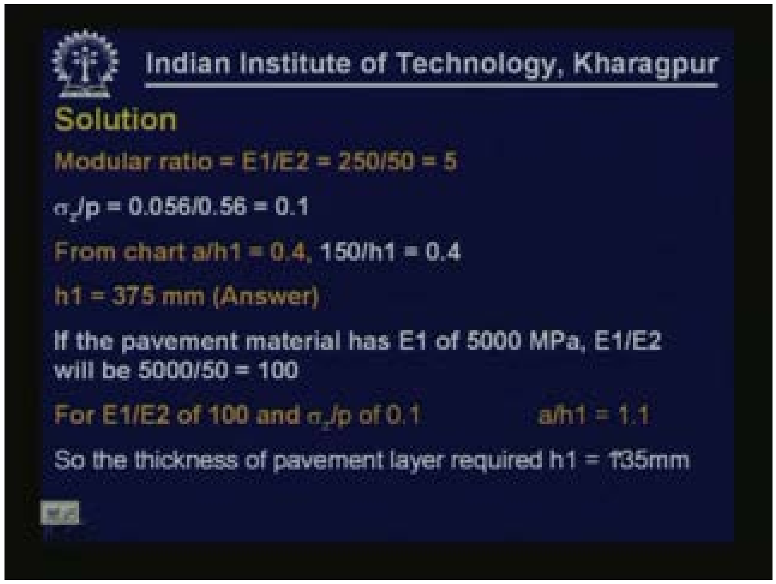

So the modular ratio E1 by E2 is 250 by 50 = 5. For this sigma suffix z by p is 0.056 by 0.56, the allowable stress is 0.056 and applied pressure is 0.56 so the ratio of sigma suffix z by p works out to be 0.1. From the chart entering with sigma suffix z by p and for E1 by E2 ratio of 5 we can get the ratio of a by h1 which was on the x axis as 0.4 that would give us a thickness of 375 mm.

If the same pavement were to be having E1 value of 5000 MPa instead of 250 MPa E1 by E2 would be100. So for E1 by E2 of 100 sigma suffix z by p of 0.1 as you have seen here a by h1 will be 1.1 so the required thickness will be 135 mm. So that will be the effect of using improved material stronger materials.

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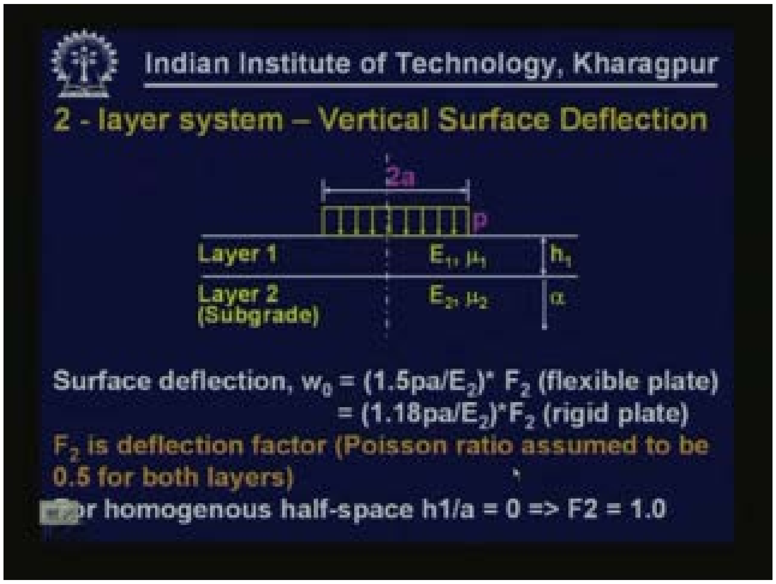

There are also charts available for determining the vertical surface deflection in a two layer pavement system. The expression is given here. This is the expression (Refer Slide Time: 48:23) for surface deflection. If it is a flexible plate 1.5 pa by E2 into F_2. This is similar to the expression that we used in the case of single layer system. But here we are using an F_2 factor. For rigid plate 1.18 I have approximated this to 1.18 pa by E2 into F_2 for a rigid plate. F_2 is the deflection factor, here we are assuming Poisson ratio to be 0.5 for both pavement layers. For homogenous half-space h1 by a = 0 gives us an F_2 value of 1. We will see how we can make use of this system.

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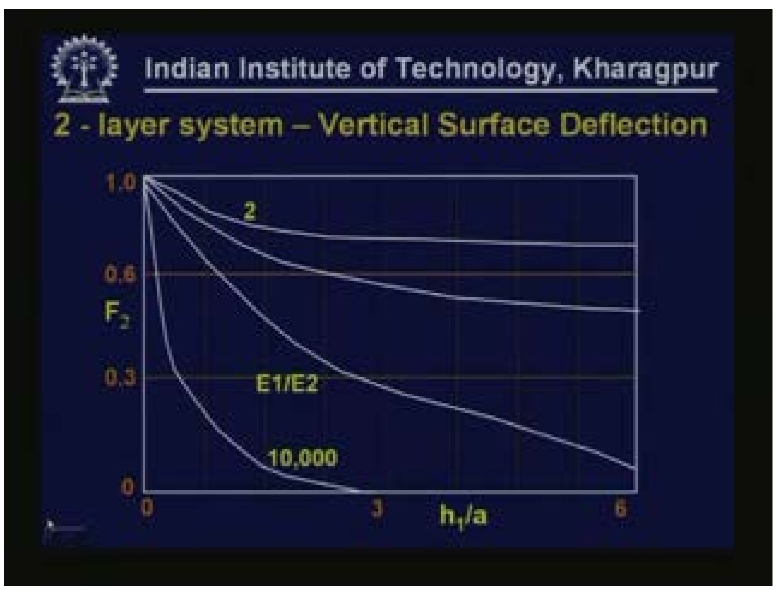

This is the typical vertical surface deflection chart for a two layer pavement system. Here if you know the thickness of the layer h1 expressed as h1 by a, if we know the modular ratio then we can find out the deflection factor F_2 and substituting this in the equation we can calculate the surface deflection either for rigid plate or for a flexible plate.

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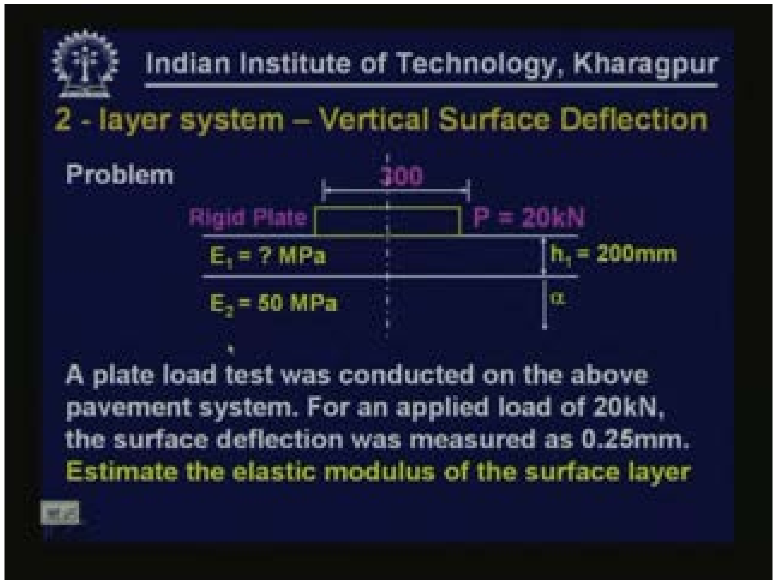

Let us take up a problem and make use of this information for calculating surface vertical deflection. here again we have a similar system load plate 300 mm diameter load applied is 20 kilo Newton, this is a rigid plate, E2 is 50 MPa, and thickness of the pavement is 200 mm therefore we are trying to find out value of E1. We have conducted a plate load test on this pavement, this is the rigid plate. So, for the applied plate load of 20 kilo Newton the surface deflection measured was 0.25 mm. So we are trying to estimate the elastic modulus value of the surface layer.

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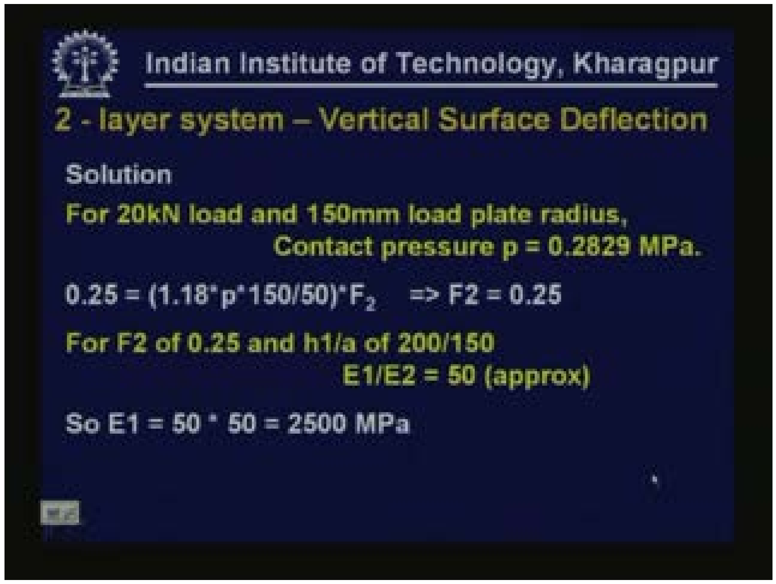

So, for 20 kilo Newton load and 150 mm load plate radius contact pressure is 0.2829 MPa so measured surface deflection is 0.25 equating that to 1.18 this is a rigid plate into p into 150 divided by p divided by 50 that is E2 into F_2 so this gives us an F_2 value of 0.25. For an F_2 value of 0.25 and h1 by a of 200 by 150 where h1 is the thickness of the pavement layer that is already given the E1 by E2 works out to be about approximately 50 from the previous chart. So E1 by E2 = 50 therefore E1 = 50 into E2 that is also 50. So E1 works out to be 2500 MPa.

Thus in the case of two layer system also if we conduct a plate load test and measure the surface deflection and if we know the other properties such as the property of the subgrade we can easily find out what is the modulus value of the pavement material.

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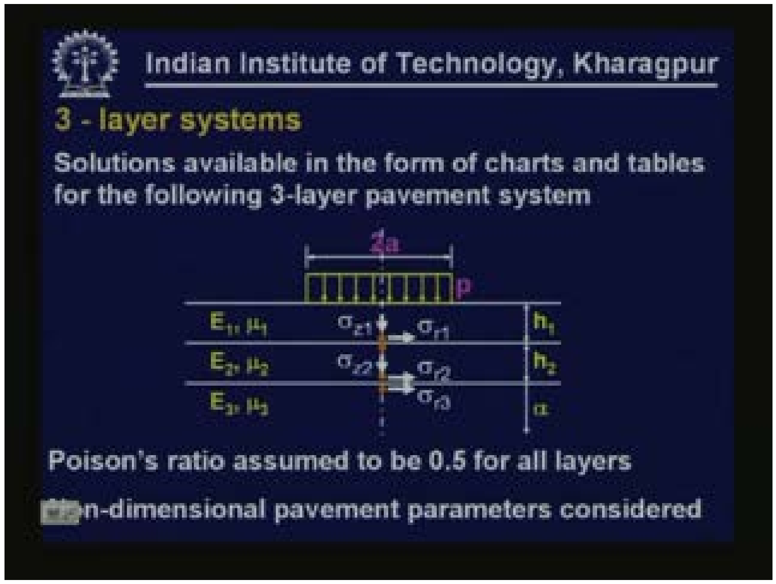

Coming to a three layer system, this being more complex there are limited parameters that can be determined using various charts and tables that are available. Here the Poisson ratio value is assumed to be 0.5 for all the layers and basically these are the parameters we can determine. Stress is at this location, this is at the interface in the first layer (Refer Slide Time: 51:51), stress is at the second interface in both layers, this system is characterized by thicknesses h1 and h2 and the elastic properties for all the three layers and the circular contact area applied at the surface.

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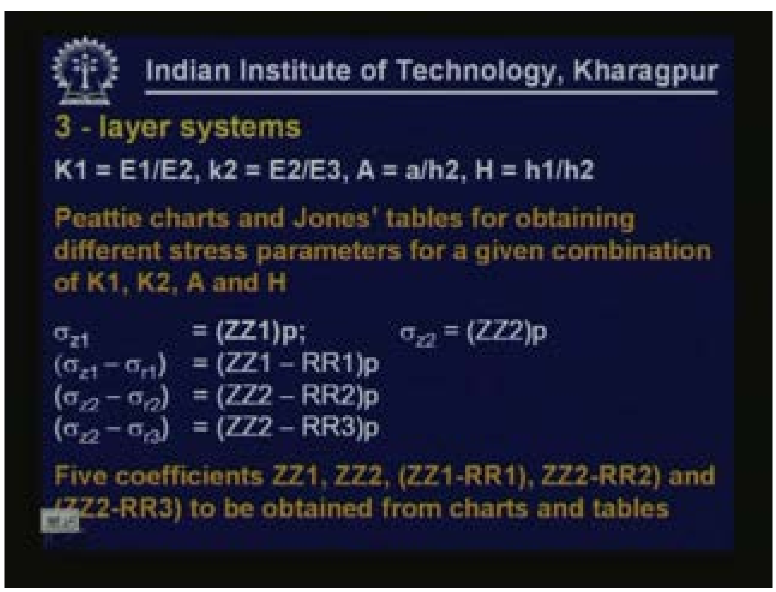

So we consider non-dimensional pavement parameters to characterize this pavement that is k1 expressed as E1 by E2, k2 expressed as E2 by E3, ‘a’ load contact area divided by second layer thickness, ‘h’ thickness of first layer divided by thickness of second layer. Peattie has given charts and Jones has given tables for calculating all these parameters. We can see the expressions; sigma suffix z1 is given as a function of p multiplied by a parameter ZZ1and ZZ1 is what we have to find out from the charts. Similarly sigma suffix z2 at the second interface vertical stress is given by ZZ2 into p. Similarly sigma suffix z1 – sigma R1 is given as a function of another parameter ZZ1 – RR1 and so on. Basically we have to find out these parameters for a given pavement configuration from the charts and tables.

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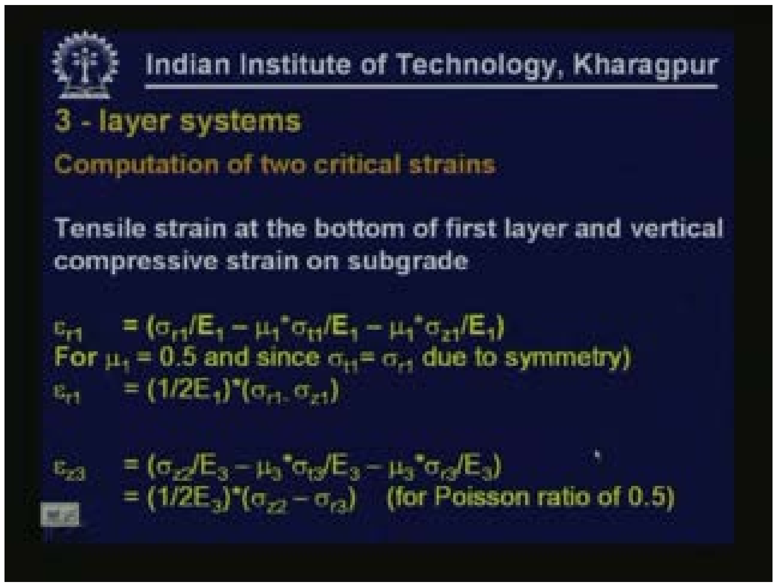

So once we have all those parameters from charts and tables we can calculate any parameter at these particular identified locations because we are going to have all the three normal stresses sigma1 sigma2 and sigma3 that is the radial stress tangential stress and vertical stress so we will be in a position to calculate the strains as we can see from these expressions.

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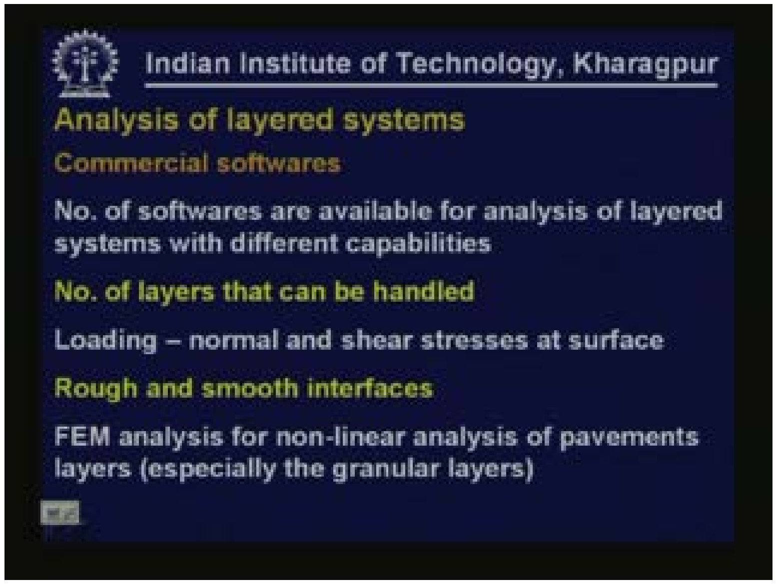

There are various software available for analysis of multiple layered systems, a number of commercial software are available. These software can normally handle various layers, different number of layers can be handled. The loading also can be normal stresses, and shear stresses at the surface can be considered. These software can normally handle rough and smooth interfaces and finite element based software are also available which are capable of handling non-linear behavior of different pavement materials especially the behavior of granular layers.

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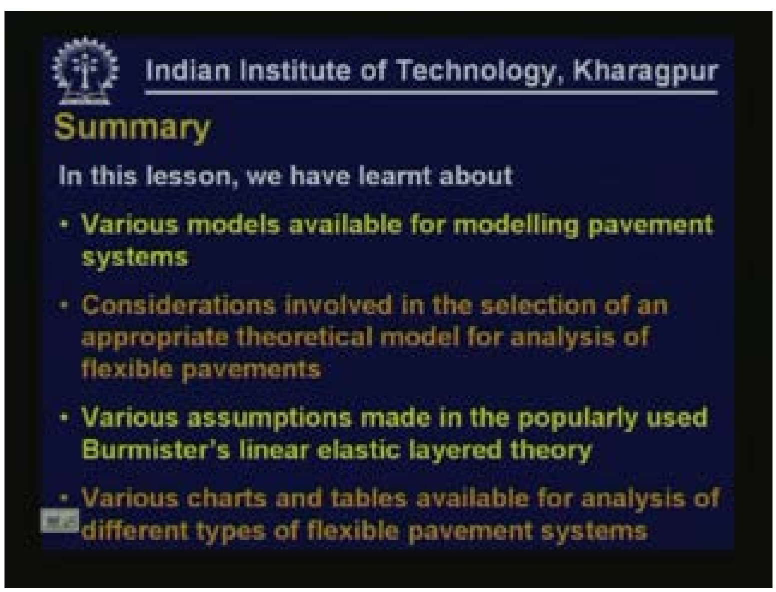

To summarize; in this lesson we have learnt about various models available for modeling pavement systems. We also discussed about the considerations involved in the selection of an appropriate theoretical model for analysis of flexible pavements. We also discussed about various assumptions made in the popularly used Burmister's linear elastic layered theory. We also considered various charts and tables that are available for analysis of different types of pavement systems.

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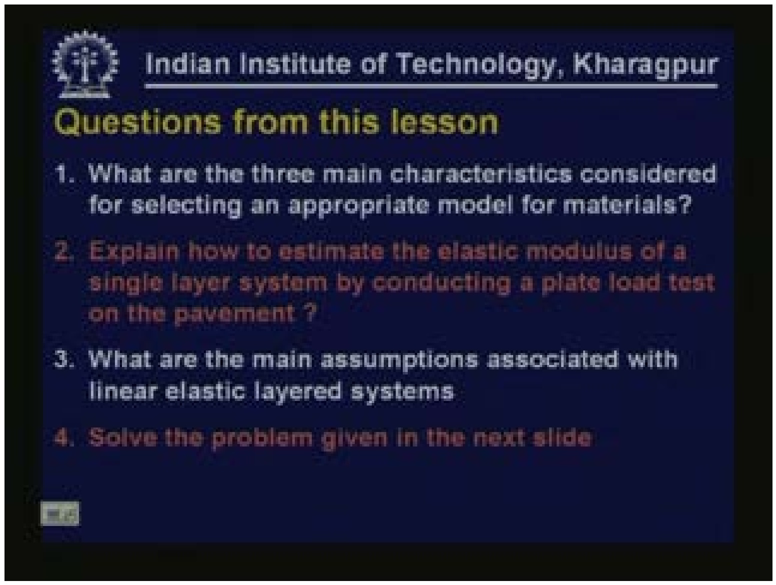

Let us take a few questions from this lesson which will be answered in the next lesson.

1) What are the three main characteristics considered for selecting an appropriate model for materials?
2) Explain how to estimate the elastic modulus value of a single layer system by conducting a plate load test on the pavement.
3) What are the main assumptions associated with linear elastic layered system and solve the problem given in the next slide.

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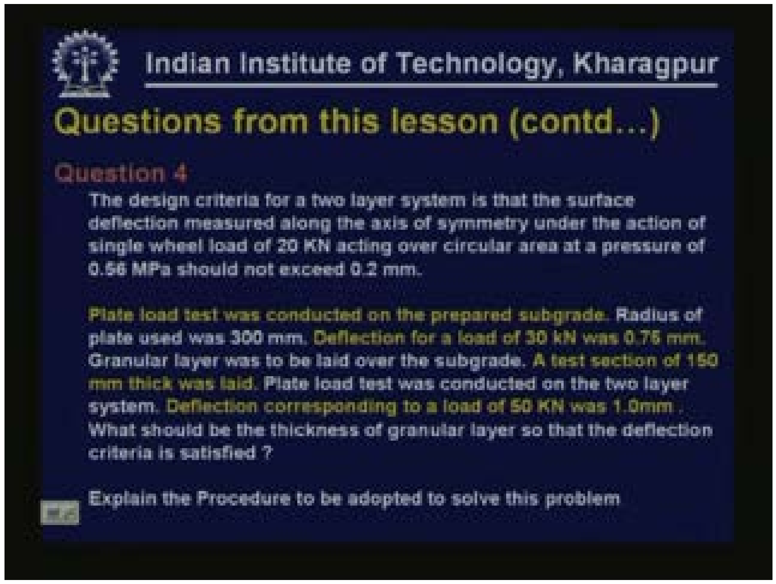

I am not going to read out the problem you can just read or we will discuss this in detail when we discuss the solution of this problem in the next class.

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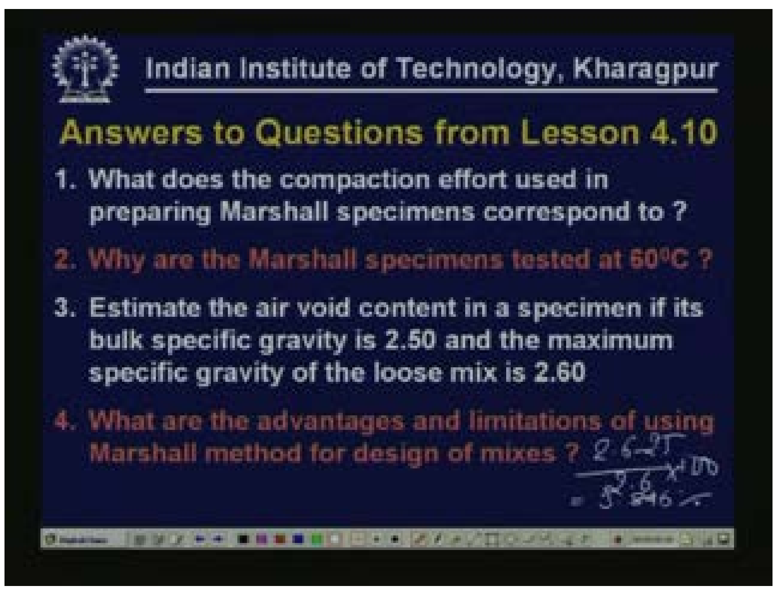

Let us take up the answers to the questions that we asked in the previous lesson that was lesson 4.10.

1) What does the compaction effort used in preparing Marshall Specimens correspond to?

Normally we prepare Marshall Specimens subjected to (( )) (00:55:41) on each phase especially for heavy traffic volumes. The compaction effort corresponds to the density that is attained after several years of traffic.

2) Why is the Marshall Specimen tested at 60 degree centigrade?

It is typically assumed that 60 degrees corresponds to the maximum pavement temperature that pavements are normally subjected to especially in the United States because this is the specification that has evolved in the United States.

3) Let us estimate the air void content in a specimen if its bulk specific gravity is 2.50 and the maximum specific gravity of loose mix is 2.60. We will quickly do this calculation. It is simply 2.6 – 2.5 by 2.6 into 100 so the value works out to be about 3.846%. Theoretical density minus bulk density divided by theoretical density is expressed as percentage.

4) What are the advantages and limitations of using Marshall Method for design of mixes?

Marshall Method of mix design is very simple, less costly, can be used by most of the agencies that's the main advantage and the specifications correlated to Marshall Method have been correlated to lot of field performance. But on the other hand the parameters that we measure that is stability flow are not really correlated to actual field performance that it does not explain fatigue performance of bituminous mixes, it does not explain the rutting behavior of bituminous mixes and these are not the parameters that are not fundamental in nature, we are not measuring the elastic modulus value and we are also not directly explaining fatigue and rutting behavior in terms of its fundamental properties that's the main problem with Marshall Method of mix design, thank you.