Engineering Geology
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Lecture - 21
Stress-Strain Behavior of Soil and Rock – II

(Refer Slide Time: 00:52)

Hello everyone and welcome back. So, we are going to continue the discussion that we began last time. In the last lesson, we were discussing if you recall stress-strain behavior of soil and rock. We are going to continue that discussion, and we are going to learn more aspects of stress-strain behavior of soil and rock in this particular presentation.

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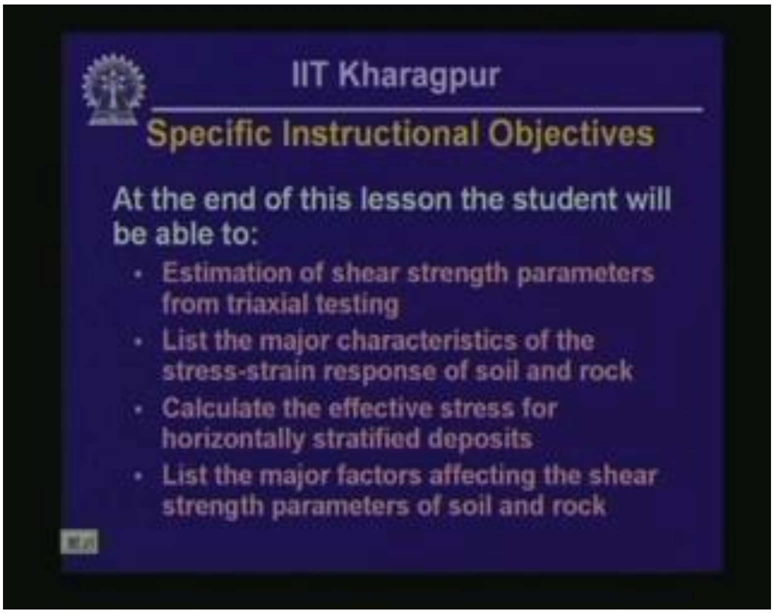

Now, the objective of this lesson is that after this at the end of this lesson, we should be able to estimate shear strength parameters from triaxial testing and then, we should be able to list the major characteristics of stress-strain response of soil and rock. We should be able to calculate effective stress for a simple geological stratigraphy. And finally, we should be able to list the major factors that affect shear strength parameters of soil and rock.

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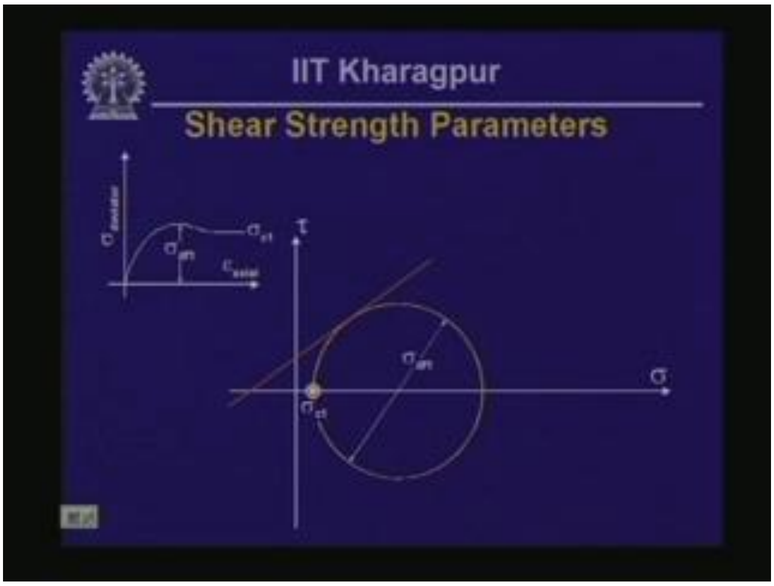

So, now we continue with the discussion that we started in the last time. If you recall, we were discussing how to analyze the data obtained from the series of triaxial tests performed on specimens of intact sample of rock or soil samples. Now, in the lesson that we had last time, we considered three different triaxial tests on three identical samples of rock or soil performed at three different shell pressures.

Now, at the end of the lesson I said what we have to accomplish for obtaining shear strength parameters is to plot more circles representing the state of stress at failure for each one of those tests. So, this is what we do in this particular slide. So, if you recall that the first test of the series of triaxial test was conducted at the smallest value of shell pressure, and the data obtained from the triaxial test, the deviatory data are presented on the plot near the top left of this particular slide, and for this data we have plotted more circle shown at the bottom right.

Now, what you should notice here is that sigma C 1 that is the shell pressure under which this particular test was conducted is the minor principle stress, and if you add deviator stress at failure for this particular test and sigma d, that is given by sigma df 1. If you add that one to sigma C 1, then you are going to get the major principle stress for this situation. So, Mohr circle for this combination of stress, this state of stress is plotted on tau sigma space shown near the bottom right.

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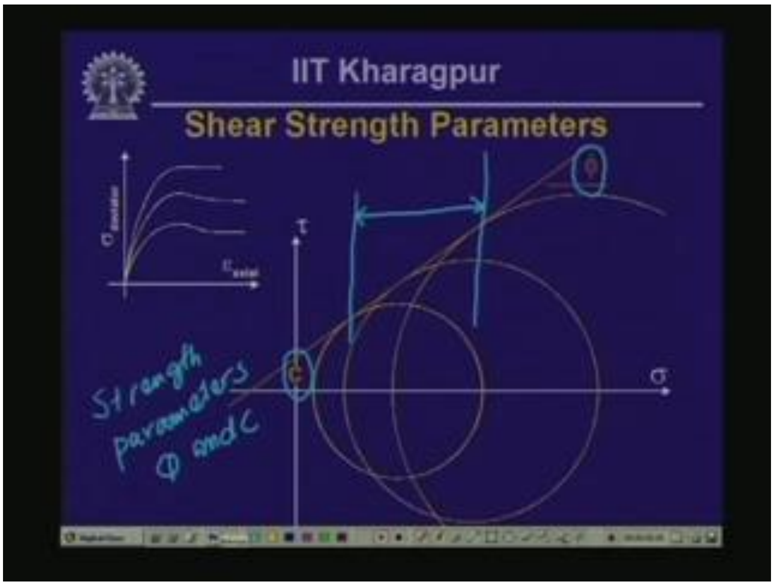

Now, what we are going to do is, we are going to plot the corresponding data from the next test which was conducted at somewhat higher level of shell pressure, and in this particular test we obtained failure deviated stress of sigma df 2. So, the failure is denoted by subscript f, and the second test is denoted by subscript 2 in this particular case. So, here as we did for the first test is that the minor principle stress is sigma C 2, slightly larger than the previous value. Then, we add to that one sigma df 2 and that gives us the major principle stress, and we compute, we plot the corresponding Mohr circle which is larger than the previous Mohr circle. Then, we do the same thing for the third test, and we get a third Mohr circle a part of which is only shown on this particular plot, and here the minor principle stress like the previous case is sigma C 3 and the major principle stress is sigma C 3 and sigma df 3.

So, all these tests you recall were conducted without considering pore water pressure, and there we have conducted in a very slow rate, so that there was no chance actually for the pore water pressure to develop. What we mean by pore water pressure, we will have to discuss as the time comes, and the other thing is if the test were to be conducted on the dry samples, you would get the same kind of stress strain plots and same sort of Mohr circles. So, repeating what we said is that Mohr circles corresponding to the three triaxial tests are shown on the tau sigma space at the right of this particular screen, and now our attempt is to find out what are the strength parameters.

You should also notice there is an orange line that actually is tangential to the three Mohr circles, and that is normally the case if you conduct a series of triaxial tests, then you construct Mohr circles at failure, then more often than not what happens is you could actually connect them with a common tangent as shown by the orange line plotted on the tau sigma space on this on the screen there.

Now, the characteristics of the orange line are the strength parameters that define the ultimate strength of the rock or soil samples which were tested in this series of triaxial tests. What are the characteristics? Since, it is a linear tangent in linear line; it is a straight line tangent. The common tangent to three Mohr circles is a straight line. So, the characteristics are really the angle which the line makes with the sigma axis and intercept the line has at tau equal at sigma equal to zero axis tau axis. So, these two parameters are also indicated on this on your screen there. So, the angle here is denoted by symbol phi and the intercept is denoted by using symbol c.

So, the angle phi is called angle of internal friction, and this quantity C which is measured in units of stress is called cohesion intercept. So, this C and phi essentially are shear strength parameters that allow general interest in such problems. Now, you should notice that here we are saying that Mohr that the failure envelope indicated by the orange line is a straight line, but in many cases, the envelope is slightly curved and it is actually convex. It is convex upward. So, if you have got to consider such type of material, then a linear failure envelope may not be appropriate for your problems.

Now, what we want to do in this case is to simplify here. We made a simplifying assumption that at least for the range of stresses that is considered in this particular problem, namely between here and here the failure envelope can be approximated by a straight line. That is what we said and that particular straight line is characterized by these two quantities, and they are the strength parameters. So, what we get is strength parameters are for this range of stress for the samples tested strength parameters are phi and c. This is what we obtained from the series of triaxial tests that we conducted.

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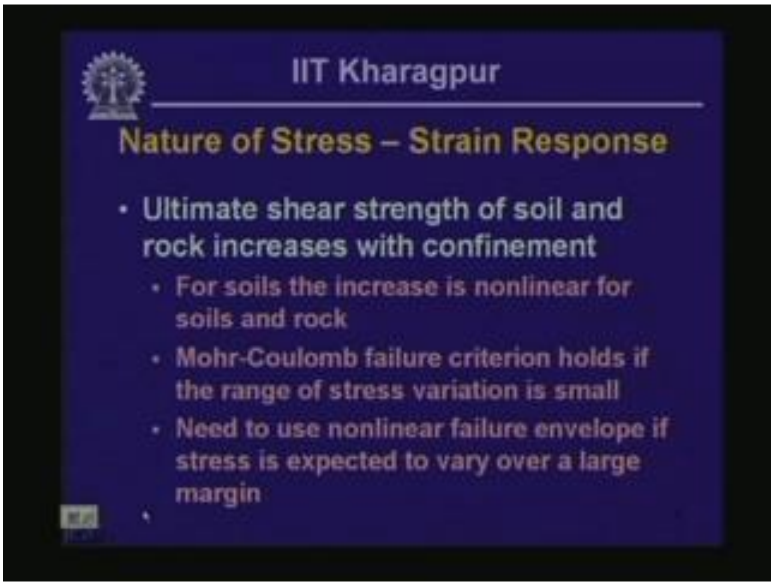

So, now what we are in a position actually to look at the major features of this stress strain response. What we have seen is that ultimate shear strength of soil and rock increases with the increase of mini normal stress or increase in confinement. That means, if you have got the same type of rock occurring at great depth underneath the ground surface as well as it has another at another level near the ground surface, you have got an identical deposit of rock. Then, the strength that the deeper element is going to exhibit the strength behavior that the deeper element is going to be able to exhibit is going to be much larger compared to the strength of the shallower element

Now, as I mentioned when I was talking about Mohr’s circles in the previous slide is that for soils and rock, the increase of ultimate strength, actually there is a mistake here. So, for soils and rock, the increase in the ultimate strength is actually non-linear. So, Mohr-Coulomb failure criterion is going to hold only if the range of stress variation for that particular problem is relatively small, and if you have to consider, if you are set with the problem which involves variation of stress, normal stress over a very large range, then you need to consider a curved non-linear failure envelope.

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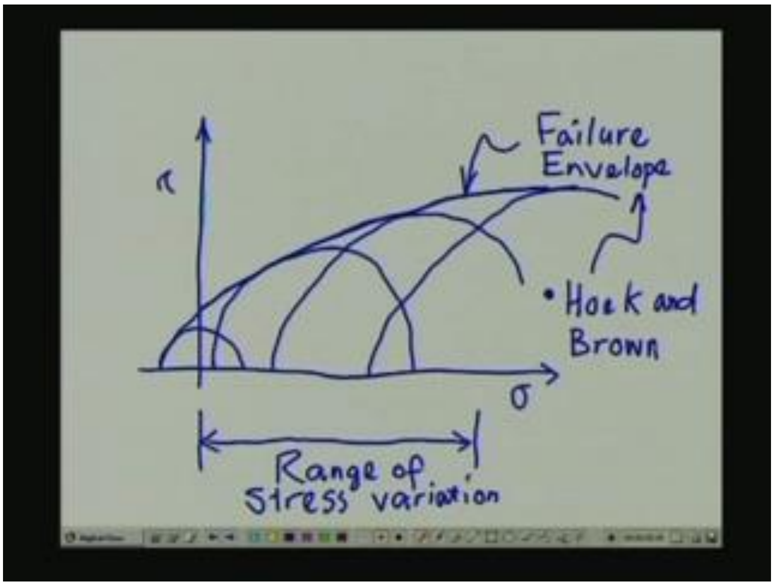

So, what I mean by that is you have got tau versus sigma plotted for a series of triaxial tests, and then you might get a failure envelope which is not linear. You can see that this particular failure envelope is a convex upward. So, Mohr circles for this particular failure envelope were possibly like this, and you might have another Mohr circle on the other side of the axis. So, in this situation if you have to consider a problem in which the range of stress could vary over a very void range, then this is the range of stress variation.

In that case, you have to consider the non-linearity in the failure envelope, and there are several types of modifications available over the linear assumption that we indicated in the slides that I discussed little while ago, and you have to consider one of those modifications, and one such modification was the failure criterion given by Hook and Brown for intact rock particularly. So, you have to consider non-linearity of the failure envelope if need arises.

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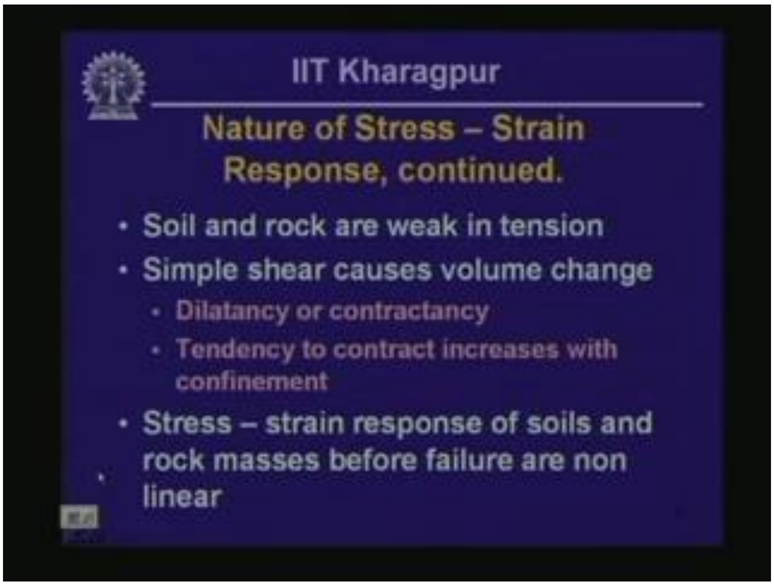

A few more features are soil and rock is weak in tension. They cannot take tension and that was clear from the failure envelope that we illustrated in the previous screen there.

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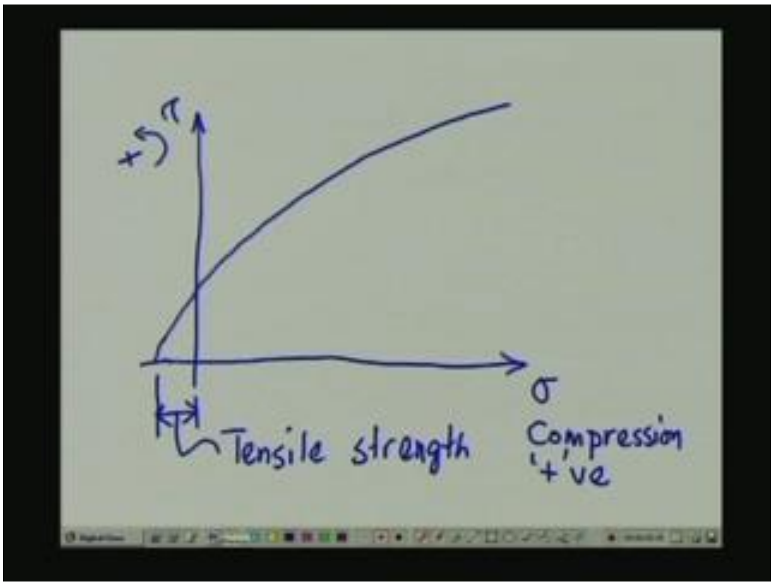

So, this we were considering a non-linear failure envelope in that case, and what you saw there is that on the tension side, you remember that we were following compression positive convention, and here we have got counter clockwise shear stress positive and for the normal stress as I indicated, compression was treated as positive. So, what becomes clear is that this value is relatively much smaller compared to the range of compressive stress that a sample cannot withstand. So, this is the tensile strength of the specimen. So, soil and rock are weak in tension. They normally tend to make a simplifying assumption that they do not have any tensile stress at all.

Second aspect is simple. Shear causes a volume change, and that was evident from the triaxial stress strain data that we schematically presented near the end of the previous lesson, and what you have here, what we are having at that stage was for application of shear stress, we were also having a change of volume, and this type of behavior of shear volume coupling is called dilatancy or contractancy.

Normally, what you expect is, when you apply shear stress then there is no volume change whereas, when you apply normal stress, then only there is a volume change, but in case of granular material, what we tend to see is the feature of dilatancy. When we apply shear stress, then the volume also tends to change. We are going to look at the details of this type of behavior in the next little bit, and also you should have noticed that shear stress and shear strain response, all deviatoric stress versus axial strain response that we illustrated when we were talking about triaxial tests in the last lesson was highly non-linear before failure, before reaching the peak value of the deviator stress, and from the discussion, it is evident actually in this particular case we are taking the peak value of the deviator stress as the condition for failure.

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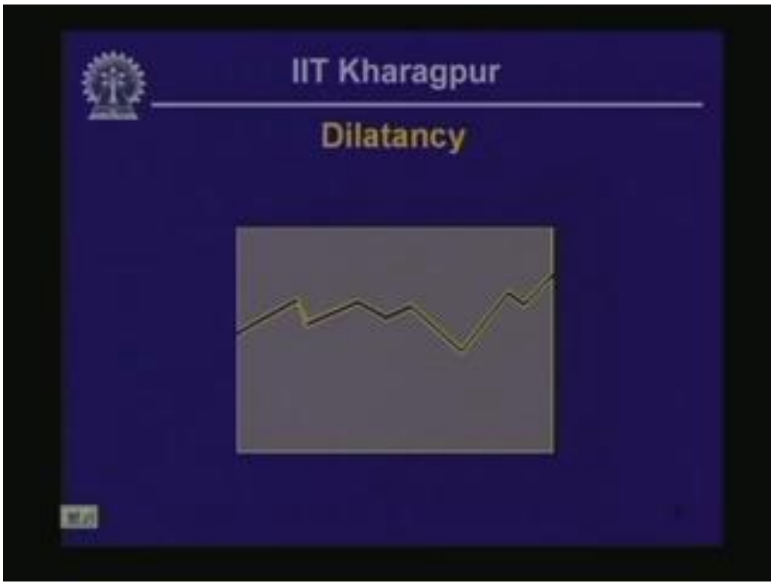

Now, we want to illustrate why there is a volume change when we apply shear stress on a soil or rock specimen. In order to illustrate that we consider a joint rock mass shown schematically on the screen there.

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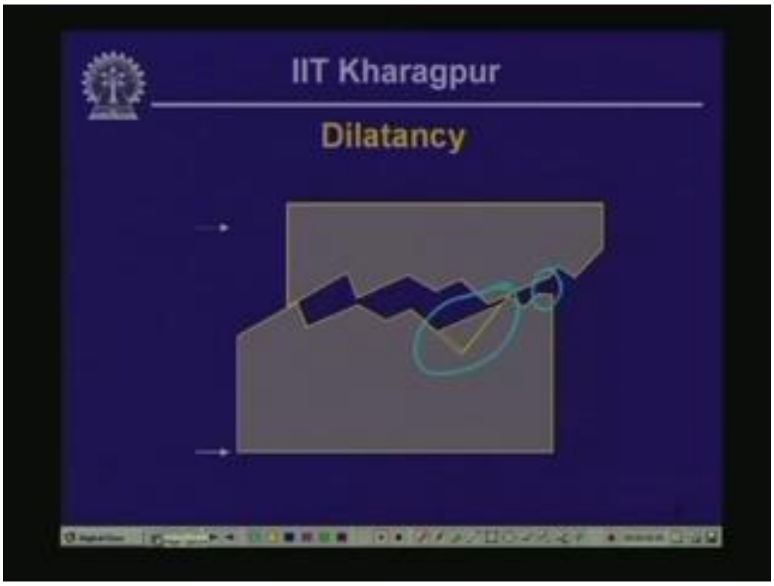

So, if we want to apply shear stress on horizontal planes at the top and at the bottom then let us try to see what will happen. It just keeps track of what is happening. You will see that in order for the sliding to take place, the rock has to be the top piece, the rock has to climb up in order to climb across the serrations of the joints, and if the normal stress applied during conducting such type of test is large enough then. In fact, the joints may shear and break off as is shown by this particular piece here and also, you would have noticed that the small piece of rock from the top right corner of the bottom piece also broke off as this schematic experiment was being conducted.

So, another thing I want you to notice here is that there are two arrows, two horizontal arrows shown here and the bottom arrow in this particular case indicates the bottom piece of rock whereas, the top arrow indicates the original position of the top piece at the beginning of this schematic experiment. So, let us do this again. Let us continue. Let us take a second look at this particular schematic experiment.

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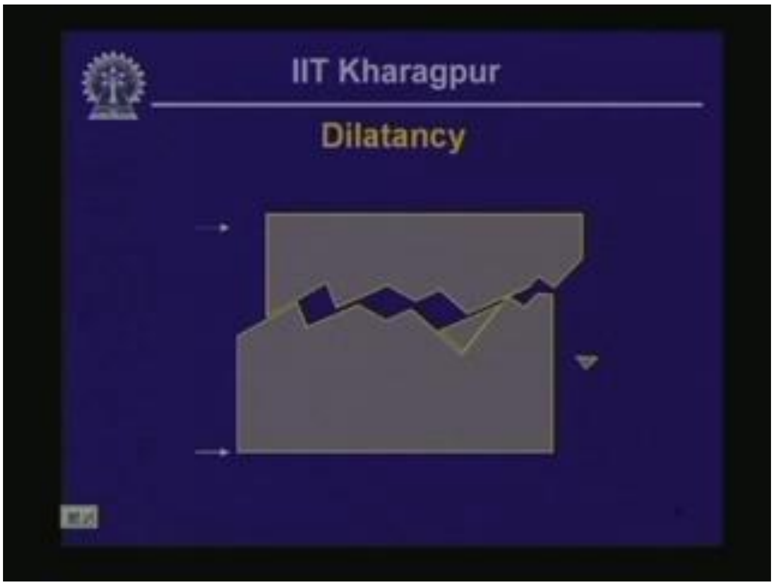

So, what happens here is that the top piece tries to climb up as it is shear to the right of this screen, and this include, this involves a change of volume and you should have also noticed that the void space in between the top space, top piece and the bottom piece increases and during this process and this type of behavior is called dilatancy.

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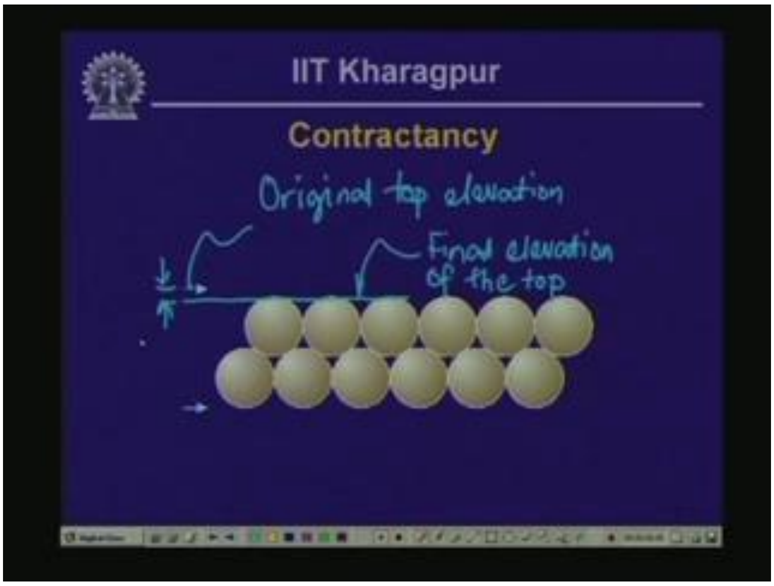

Let us consider the reverse of it and in this case, we are going to consider an assembly of spheres. We could have done the same thing using two pieces of rock which we were considering in the previous example, and going in the reverse direction in time, but in order to give you a new flavor I considered a piece of block of soil like material composed of uniform spheres.

Now, let us consider that these spheres are packed in a relatively loose configuration as shown on the screen. Now, what we are going to do in this case? We are going to try to keep the bottom of these two layers of spheres at the same location, and we are going to try to shear the top of the specimen by applying shear stress to the right on the horizontal plane to the right of the screen. So, just have a look at what happens and how the deformation evolves.

So, the spheres are going to try to climb down and occupy the void space, and in the process what is going to happen is that they are going to sink downward. They are going to sink into the origin into the space which was originally void space. So, what is happening here is that we are going to end up with an assembly of spheres which are going to be in a state of denser packing compared to the packing that we originally began with. So, if you have got a specimen which was originally loose, then if we apply shear stress, then what we are going to have. We are going to reduce the void ratio and we are going to end up with this state of packing which is a much denser assembly of individual soil or rock fragments, and that is indicated when you compare the top level of these spheres with the original elevation of the top indicated by the arrow at the top of this particular screen.

So, this is the final elevation, top final elevation of the top and the arrow noted on the screen indicates the original top elevation, original top elevation, and this much of volume change is because of the contractancy. Let us do it again. Let us have another look at it. So, this is how contractancy develop in this particular case because we began with originally loose assembly of spheres. If we were to be begun with an originally dense assembly of spheres, then we could have gotten dilatancy as we had in the case of the rock specimen, the experiment of the rock specimen that I have showed a little while ago.

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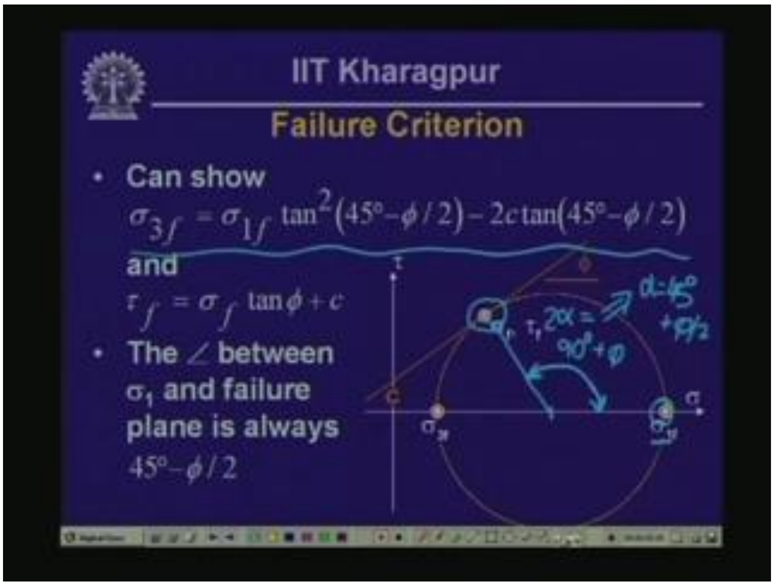

So, now we can talk about the failure criterion in a little bit more detail. If you have got, if you consider that the simple case of linear failure envelope, and such type of linear failure envelope is called Mohr-Coulomb failure envelope, then you can show that the minor principles stress at failure sigma 3 f is equal to the major principle stress at failure sigma 1 f multiplied by tan square of 45 degree minus phi by 2 minus 2 C tan of 45 degree minus phi by 2. Also, you can show that the shear stress at failure represented by the point of tangency of Mohr circle representing the state of failure, and Mohr-Coulomb failure envelope. At that point of tangency, tau f is going to be given by sigma f tan phi plus c, where phi is the friction angle as I indicated before and C is the cohesion intercept as shown on the sketch near the top bottom right of this particular slide.

Now, the angle you should also note that the angle between the major principle stress at failure, and the failure plane is always 45 degree minus phi by 2. We are going to prove the first relationship, this one here in the next little bit, but before I do that let me explain the second point. So, let us find out what is this angle.

So, this angle here is equal to 90 degree plus phi and from the discussion that we had on Mohr circles the other day, you can imagine that the angular spacing between the state of stress represented by sigma 1 f and the point of tangency is going to give by half of this included angle which is 90 degree plus 5. So, in this case alpha is equal to 90 degree plus 5, 2 alpha is equal to 90 degree plus 5 actually. So, that gives us alpha is equal to 45 degree plus phi by 2.

Now, since sigma 1 f, the plane on which the sigma 1 f is occurring at 90 degree to the direction of sigma 1 f, so what we have is the angle between sigma 1 f and the plane at which the failure is taking place is going to be given by 45 degree minus phi by 2. In other words, you recall the triaxial sample and in this case, let me open a new page here.

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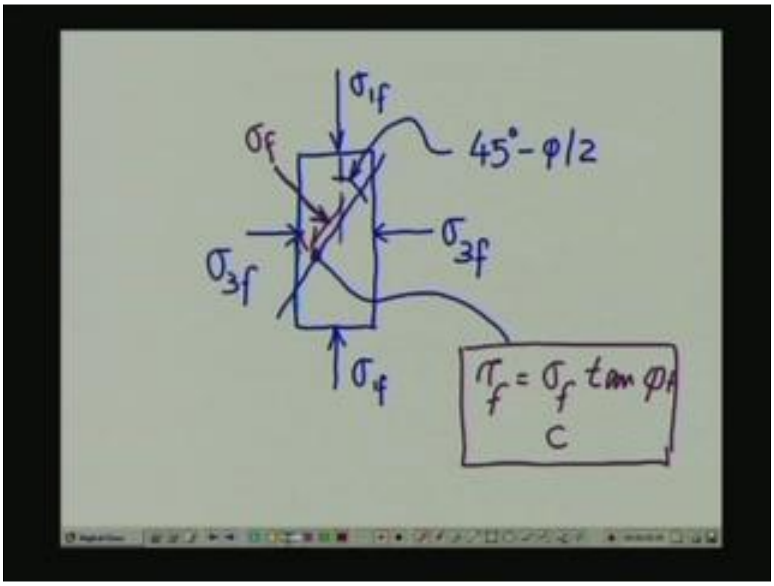

So, if you have got, if you recall that triaxial sample that we were testing sigma 1 f was in the vertical direction oriented in this manner, and sigma 3 f could have developed in this manner for the triaxial test that I illustrated in the last lesson. In this case, you are going to have failure on a plane which is inclined at 45 degree minus phi by 2 2 vertical. So, that is what I mean. So, on this particular plane, the relationship between tau f and sigma f, so if you imagine that there is a normal stress on this plane and let us call this 1 sigma f, and let us call this 1. Actually it is going to be other way round. No, that is alright.

Now, this is going to be tau f. So, then the relationship between tau f and sigma f is going to be tau f is equal to sigma f tan of phi plus c. So, then on this particular failure plain, the value of the shear stress that leads to failure reaches a critical value given by a combination of the normal stress on that particular plain, and some material properties namely the friction angle and the cohesion intercept. This is the failure criterion that is called Mohr-coulomb failure criteria.

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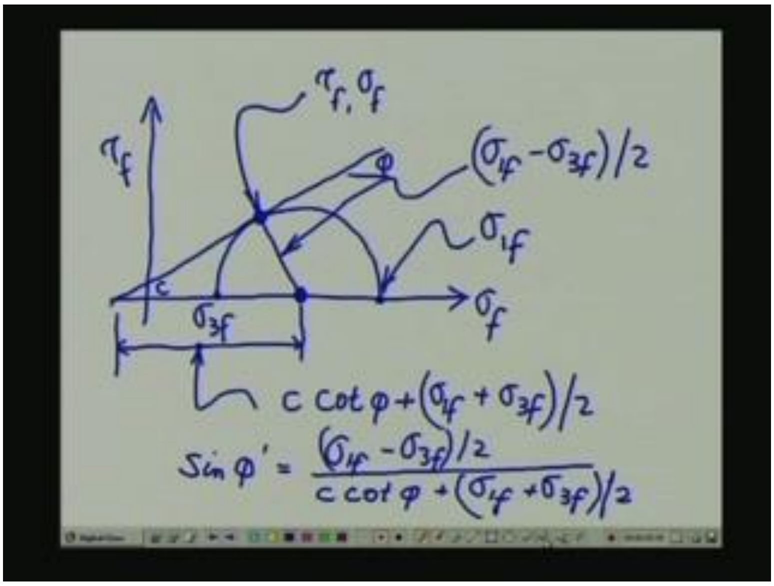

Now, let us try to prove the first expression sigma 3 f equal to sigma 1 f tan square 45 degree minus phi by 2 minus 2 C tan of 45 degree minus phi by 2. So, for doing that what we do is check the geometry. So, here what we have is sigma f versus tau f, and then let us consider Mohr circle at failure and that Mohr circle is going to be tangential to that. Mohr circle will be a Mohr-Coulomb failure envelope given by phi and a cohesion intercept c. Now, this point here as you have learnt already is given by tau f minus sigma f. So, the normal stress on that particular plane is sigma f and the shear stress is tau f.

Now, let us consider the geometry of this situation here. Since, this angle that the failure envelope makes with sigma axis is phi, then this distance here is going to be given by C cos phi plus sigma 1 f plus sigma 3 f divided by 2. Recall that this value here is sigma 1 f and this value here is sigma 3 f, and we also know that this particular distance or the radius of Mohr circle is given by sigma 1 f minus sigma 3 f over 2. Using these two results, we can write sin of phi prime is going to be given by sigma 1 f minus sigma 3 f over 2 divided by C cotangent of phi plus sigma 1 f plus sigma 3 f over 2. Recall, I mean try to remember this last relationship that we derived here, and I have to erase everything from this particular screen other than that, and actually try to remember the geometry that we were considering as well.

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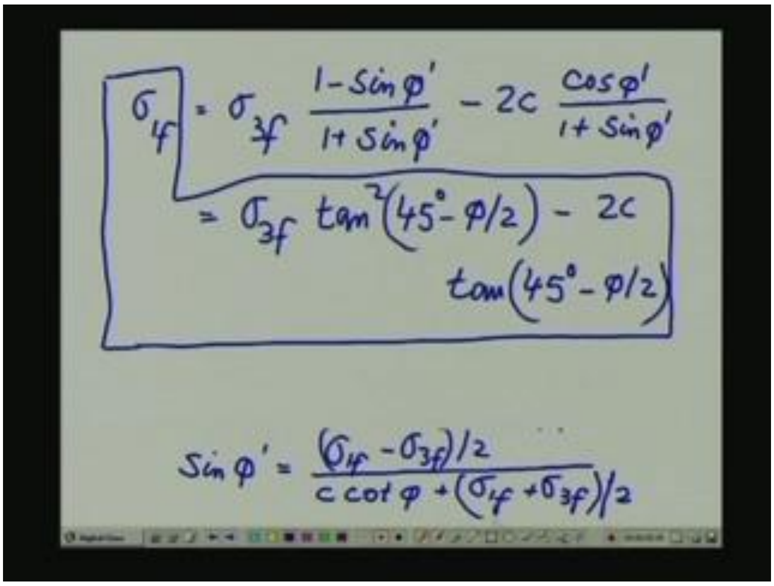

So, then what we do is we can collect tan and we can go through the geometry, go through the algebra, and we can show that sigma 1 f is equal to sigma 3 f multiplied by 1 minus sin of phi prime divided by 1 plus sin of phi prime minus 2 C cos phi prime divided by 1 plus sin of phi prime, and that in turn will give us sigma 3 f multiplied by tan square 45 degree minus phi by 2 minus 2 C tan of 45 degree minus phi by 2, and this is the relationship that we were trying to derive.

So, there is a definite relationship between sigma 1 f sigma 1 that the major principle stress at failure and the minor principle stress at failure, and the material properties namely the friction angle and the cohesion intercept. So, if you know the values of the friction angle and the cohesion intercept, and also if you know what is the shell pressure or what is the minor principle stress, you could predict at what major principle stress or what the value of the actual stress is at which the sample is going to fail. Let us take an example on this one.

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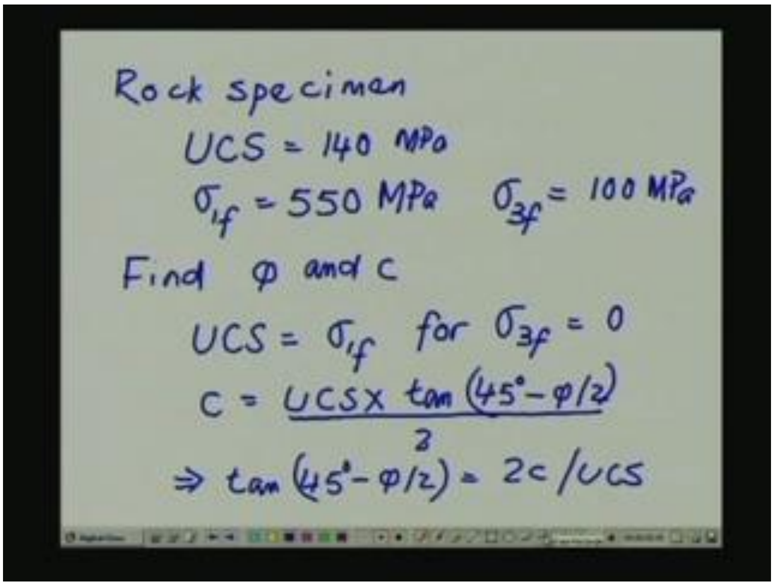

Let us consider a rock sample, rock specimen for which the UCS is equal to 140 MPa and sigma 1 at failure is 550 MPa in one triaxial test and sigma 3 at failure in the same triaxial test is 100 MPa. Now, our problem is to find phi and C for this particular rock specimen. So, how do we proceed?

We first take a look at the equation that I derived and noting that UCS is essentially equal to sigma 1 f for a case, where sigma 3 f or the minor principle stress at failure is equal to 0. So, if you note that, then what you could write is C is equal to UCS (Unconfined Compressive Strength) multiplied by tan of 45 degree minus phi by 2 divided by 2, and from that what we get is tan of 45 degree minus phi by 2 is equal to 2 C divided by UCS. Remember this relationship and I am going to erase everything on this tablet. Then, we make use of the triaxial test data, and let us see what we can come up with.

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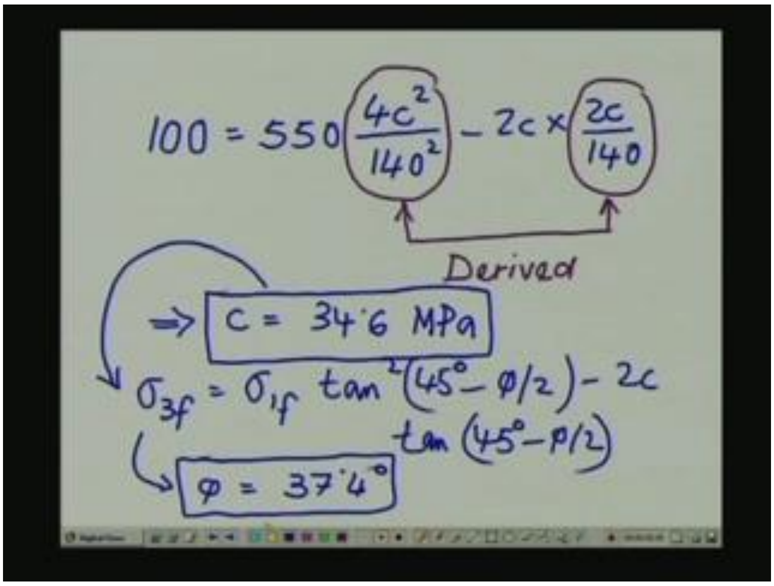

Again we make use of the equation that we derived substituting for sigma 3 f and sigma 1 f. The minor and major principle stress is that failure what we see is this. So, these two trans, this one here and this one was as we derived just a few minutes back when I immediately before I erase the tablet if you recall. From this what we get is C. When we complete the calculations, then C can be shown to be equal to 34.6 MPa, and when you back substitute this value of C in the original equation, namely sigma 3 f equal to sigma 1 f multiplied by tan square 45 degree minus phi by 2 minus 2 C tan of 45 degree minus phi by 2.

The equation that we derived earlier if you substitute that value here, then what you get is phi is equal to 37.4 degrees. So, these are the results and then, your C is 34.5-34.6 degrees and phi in this particular case is equal to 37.4. So, here we made use of one unconfined compression test and one triaxial test for getting the strength parameters of interest.

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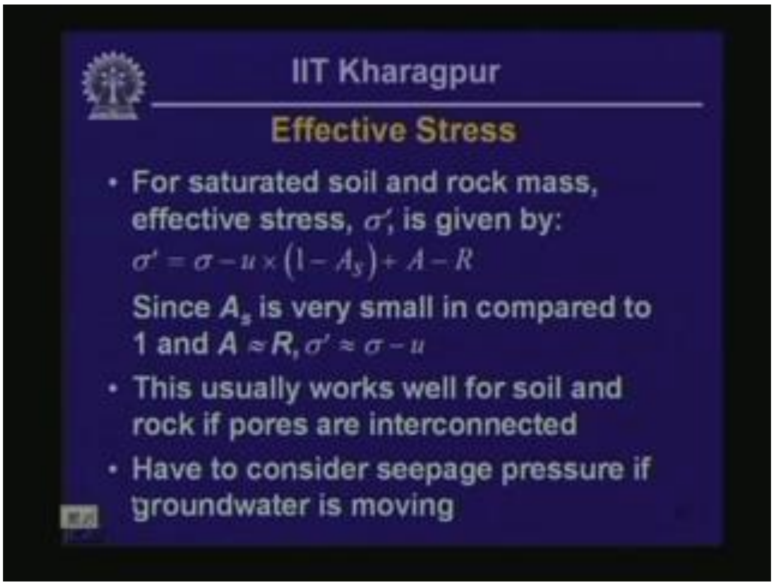

Now, we introduce the concept of effective stress for saturated soil and rock mass. Effective stress is given by this expression here, sigma prime equal to sigma minus u multiplied y 1 minus A S plus A minus R. Now, what are these trans is going to be let me explain that before I proceed any further.

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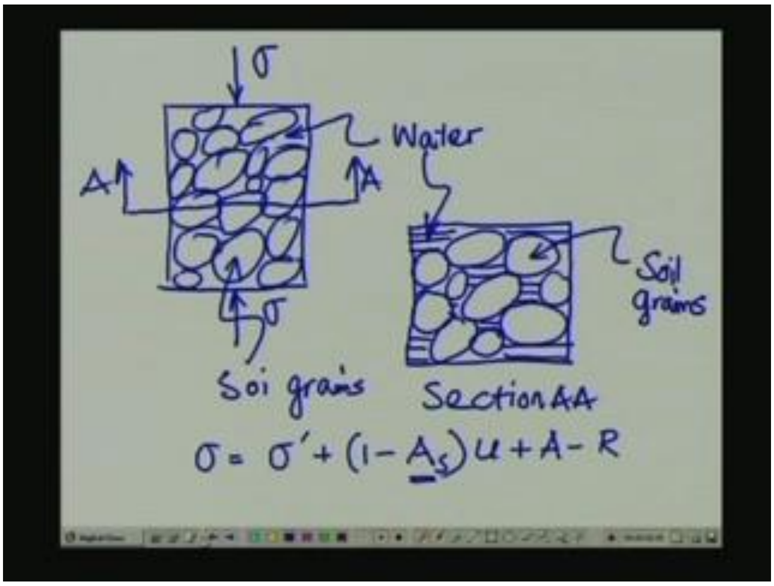

Let us say you have got an element of soil which is composed on several different grains, and what we have is a saturated sample and the pore space is full of water in this case. So, then we take a cut across the sample, and if you look at the plane view of this particular cut, then what you might see is that the portion is going to be showing soil grains whereas, in other areas what you are going to have is water. So, these are soil grains. So, let us give a name to the section. So, this is section AA.

So, these are soil grains, and in between is water. These are also soil grains. So, now what we have is that if you consider that this particular element is subjected to a compressive stress in that direction, then a part of the compressive stress is going to be transferred through intergranular contact between individual soil grains, and part of it is going to be taken by pore water. Since, the intergranular contact is very small, is of a very small proportion in comparison with the pore space with the area under water, then what we could write is that. Actually let me write that one.

So, let us say sigma is going a part of it is going to be borne by intergranular contact stresses, and part of it is going to be borne by the pore water pressure and this quantity A subscript s is the amount of area at which the individual soil grains are in mutual contact, and u is the pressure that is borne by pore water or that is called the pore water pressure, and on top of it, you could have some electrostatic forces of attraction and electrostatic forces of repulsion. That is particularly true in case of soils composed of charged particles such as clay soils. So, that is the expression there.

Since, A subscript s is usually very small, and that is particularly true if you have got very strong soil grains, incompressible soil grains in comparison with one is very small in comparison with one and forces of attraction and repulsion. They balance out what we could write is sigma prime is equal to sigma minus u, or the effective stress is equal to the total stress minus d pore water pressure. This simple expression works very well for solid, for soil and rock if the pore space is interconnected to saturated soil on rock, and if the pore water or the ground water within the mass of soil or rock is not in relative motion, in comparison with the solid skeleton, so this is called (( )). So, in simple cases what we are going to do? We are going to conceal, we are going to get the total stress, and we are going to simply subtract from the total stress the pore water pressure and then, we are going to get the effective stress.

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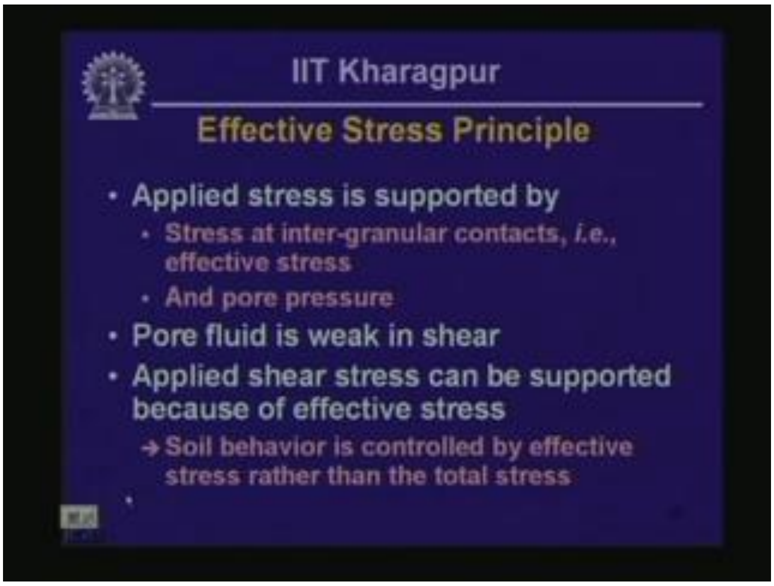

Now, we look at the effective stress principle applied on a skeleton of soil or rock, saturated skeleton of soil and rock is supported by stress at intergranular contacts, and pore pressure as we have seen already. Now, since pore fluid is weak in shear if we applied shear stress to the specimen, it can only be supported because of effective stress. So, soil behavior in shear stress, shear strain response is controlled by effective stress rather than the total stress, and this is the effective stress principle introduced by Karl Terzaghi in early 1900s which led to the development of the science of soil mechanics and geotechnical engineering.

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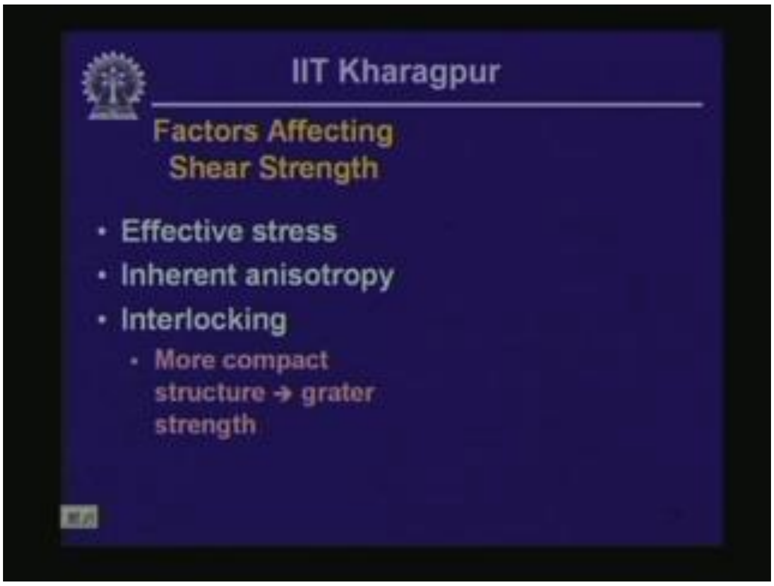

It is very important principle actually seminal. In fact, that leads to the entire development of this field of learning, then factors affecting shear strength. What are the factors that affect shear strength parameters? Number one is effective stress. So, higher the effective stress as we have noted that the shear express, the assembly can support is also going to increase.

Second thing is inherent anisotropy and that is because presence of joint sets or presents of lamination within the mass or rock or soil. That is because of the deposition on environment or because of mineralogical characteristics of the solid. Then, the third thing that affects shear strength is interlocking. As we have seen that more compact structure greater will be the interlock. There will be dilatancy and what we are going to get is a larger strength in comparison with comparatively looser assembly.

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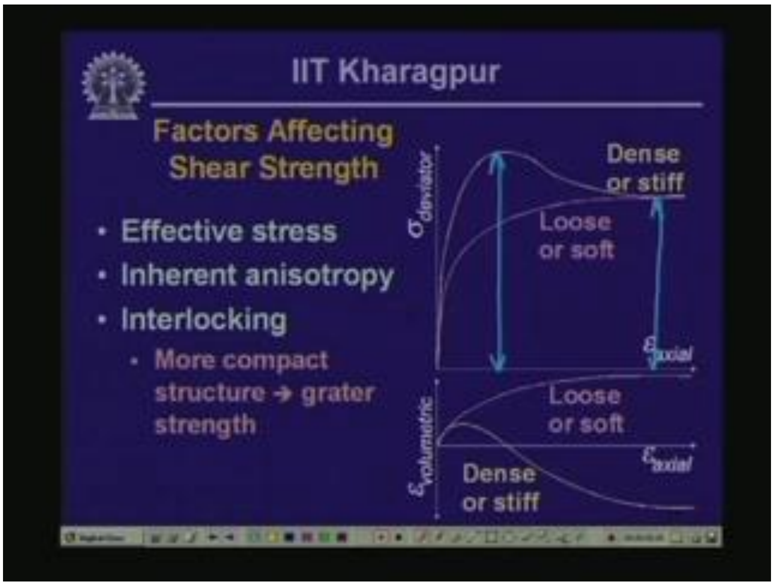

That is illustrated when we consider typical deviatoric stress versus epsilon axial plot, and volumetric strength versus epsilon axial plot obtained from triaxial tests for a loose or soft deposit, and dense or stiff deposit as we see on the plot schematic plot on the right of this particular slide. Here, as it is evident in case of dense sample, the strength is much larger in comparison with the strength of loose sample, and that is because of the dilatancy really reaches illustrated with volumetric strain versus axial strength plot near the bottom of these plots.

As we have noticed earlier, loose or soft deposits will contract on the shear and dense on stiff deposits tend to first contract when this strain, axial strain is relatively very small, and finally the sample starts to dilate as it is shown on the plot there.

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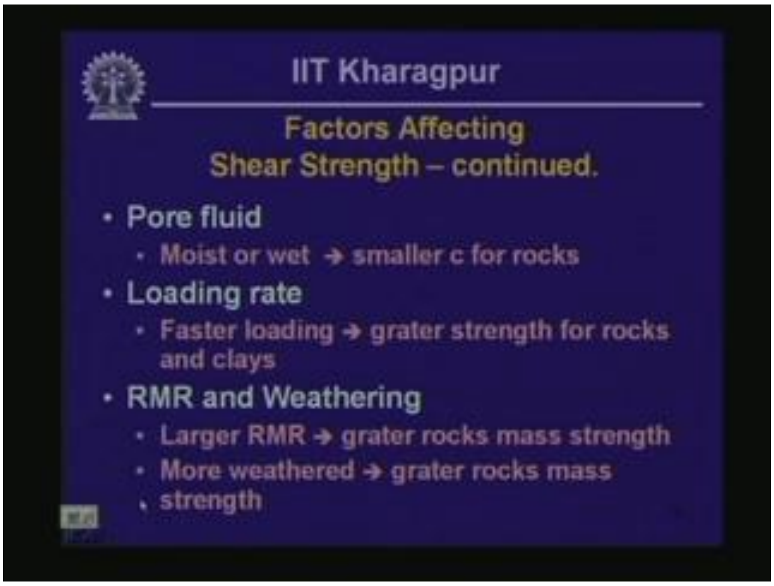

Other factors that affect shear strength characteristics present of pore fluid. Some of the rocks exhibit smaller strength under moist or wet conditions in comparison with a dry condition. Then, loading rate, if you have faster loading rate, generally for rocks and clays shear strength becomes greater. Then, if you have got whether rock or soil sample, then you are going to expect a smaller strength. If you have got a larger RMR rock mass rating, then you would expect a greater strength of the rock mass.

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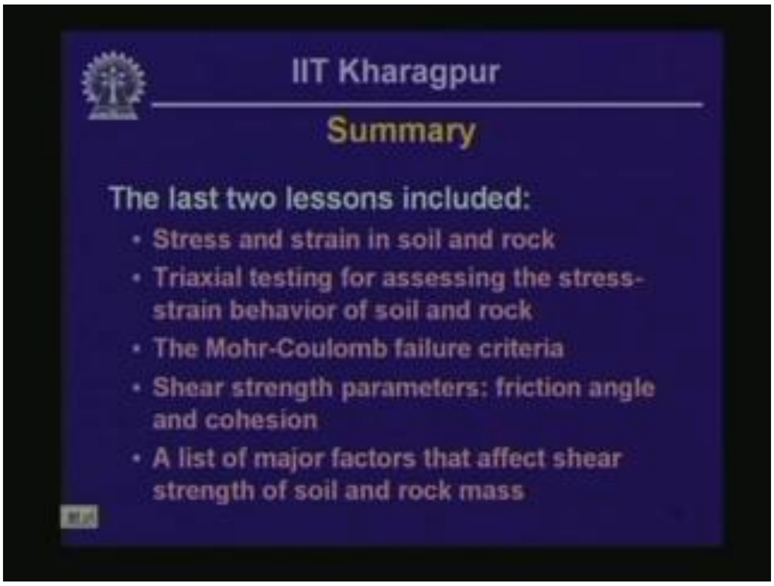

Summarize this particular lesson as well as the last one. We looked at the characteristics of stress strain behavior of soil and rock, looked at triaxial testing and how we can access the shear, how we can access shear stress strain behavior of soil and rock, then how we can obtain Mohr-Coulomb failure parameters associated with Mohr-Coulomb failure criteria from conducting triaxial tests, and we looked at a list of major factors that affect shear strength of soil and rock mass.

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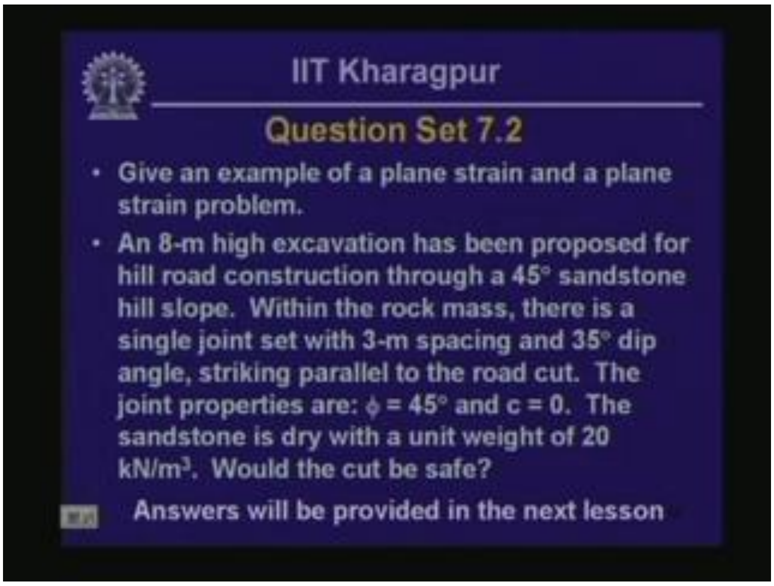

Finally, we want to wrap up this particular lesson with the question set. Try to answer these questions at your leisure. The first question that I ask is given example of a plane strain and plane. Actually the first one should be plane stress. There is a mistake there. So, give an example of a plane stress and a plane strain problem, and then second one is a really numerical problem. It involves an 8 meter high excavation for a hill road construction through 45 degree sandstone hill slope. Within this rock mass, there is a single joint with 3 meter spacing and 35 degree dip angle striking parallel to the road cut. The joint properties are given there. Phi is equal to 45 degree and C is equal to 0, the sand stone is dry with a unit weight of 20. Would the cut be safe?

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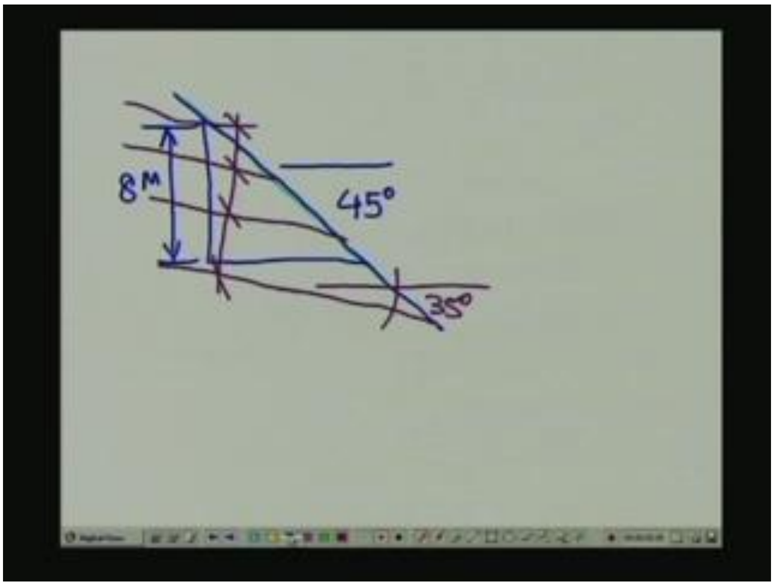

So, you have to actually look at the cross-section in this particular case. So, what I say here is really you have got a hill slope which is inclined at 45 degree to horizontal, and you want to cut an 8 meter height slope. So, this height here is 8 meters, and there is a joint set within this particular rock mass dipping at 35 degree angle. So, this angle here is 35 degree and the spacing of these joints is also given. So, you are supposed to tell me whether this particular cut is going to remain stable during construction or during the operation of the highway. That is the problem there. So, with that I am going to stop this lesson, and you try to answer these questions at your leisure. I will give you my answers when we meet for the next lesson.

Thank you very much.