Design of Reinforced Concrete Structures
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Lecture - 08
Design of Doubly Reinforced Beam Flexure - I

Good morning. Today, we shall start design of doubly reinforced beam. And we shall consider the flexure part: part 1 and we shall have followed few examples; that we shall consider it as part 2.

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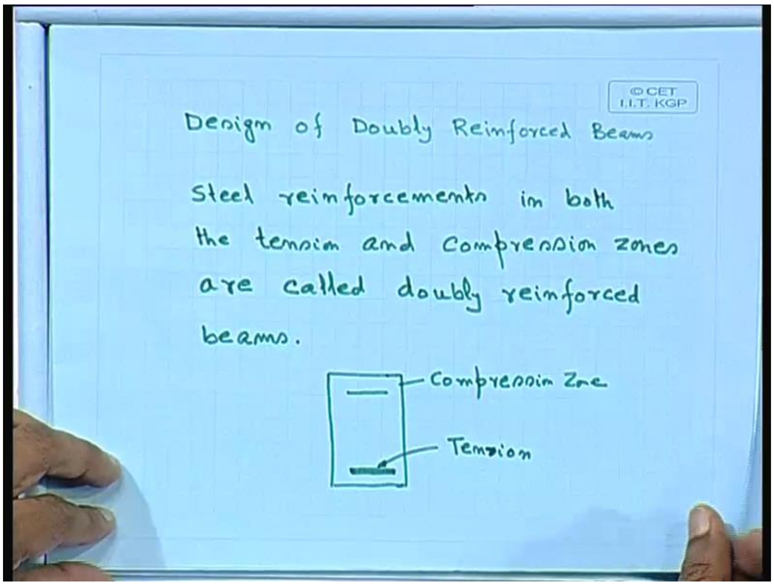

Design of doubly reinforced beam; that means: reinforcement in the tension side as well as in the compression side. Steel reinforcement in both the tension and compression zones are called doubly reinforced beams. If, you have a section which we are familiar in last few days, we are providing reinforcement in the tension zone only. If, we consider in the normal general case, the bottom portion has tension and the top portion we are providing which is nothing, but in the compression zone. We are providing steel in the tension zone as well as in the compression zone. What we can do, but where do we need it? Where do we need this reinforcement, why we have to provide that?

We need that reinforcement because, your depth is not adequate, you could not provide the depth required if you have to design it as per singly reinforced section. In that case because, it is restricted may be due architectural point of view or may be some other reason. So, we have to provide then, what we can do we provide compression zone, we provide steel in the compression zone and we do the design and that is called doubly reinforce beam. It may happen that say, due to moving load, particularly in support where you have that compression as well as tension; there is change of sign in the bending moment, there also you have to provide the reinforcement in the tension zone as well as in the compression zone.

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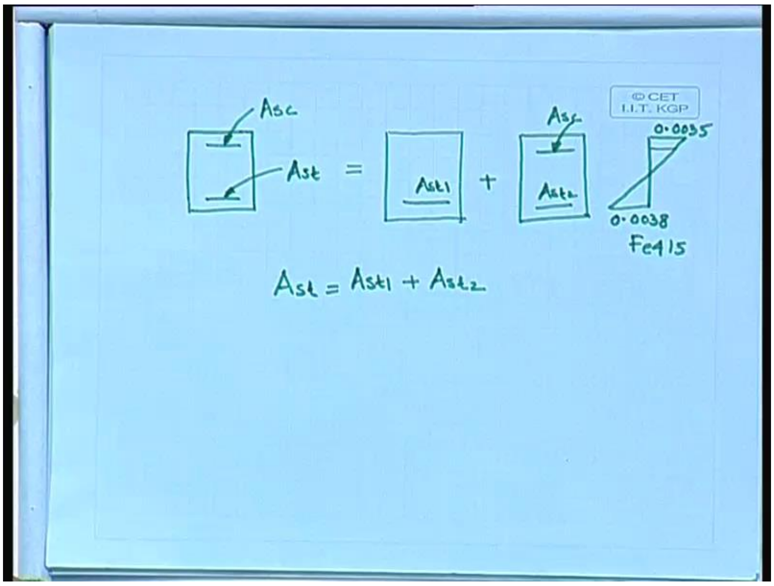

So, what is the design philosophy for doubly reinforced section? The design philosophy of doubly reinforced section; we shall provide the reinforcement in both sides. Let us say this 1 Asc: area of steel in the compression side and Ast: area of steel in the tension side. We can make in 2 parts; we can say that we are having a portion. This beam can take it a singly reinforced 1; that means, moment of resistance if we consider it as a singly reinforced section, only in the tension side.

Therefore, I can say this portion that area of steel, let us say Ast 1 area of steel Ast 1 due to singly reinforced section plus the remaining portion I can provide it assuming, that only the steel action Asc: area of steel in the compression side and Ast 2: area of steel in the tension side. So, you have steel in the tension side, we have 2 parts: Ast equal to Ast 1 considering singly reinforced section, we are assuming we are considering here we are taking it here, as a balanced section. Due to balanced section whatever steel we require that is Ast 1 plus Ast 2. Individually, this portion also will be in equilibrium as well this

1 also will be in equilibrium, Asc and Ast 2 should not be equal. It cannot be equal because, the strain in the top portion and the strain in the bottom portion, in other way the stress the stresses are different in 2 sides, stresses here and stresses in the bottom are different.

So that is why that, area of the force whatever force we shall get it the force is equal because, that 1 supposed to be in equilibrium, but not necessarily that Asc and Ast 2 will not be equal because, stresses are different in 2 different zone. I could further explain, we have if we draw the strain diagram. The strain diagram it says; some where say neutral axis.

If, we consider say Fe 415, we are assuming the steel is Fe 415. So, 0.0035 that is in the compression zone the strain, whereas, here 0.0038 seems to be considering Fe 415. We are providing the steel at this zone in the compression side, so the strain is different.

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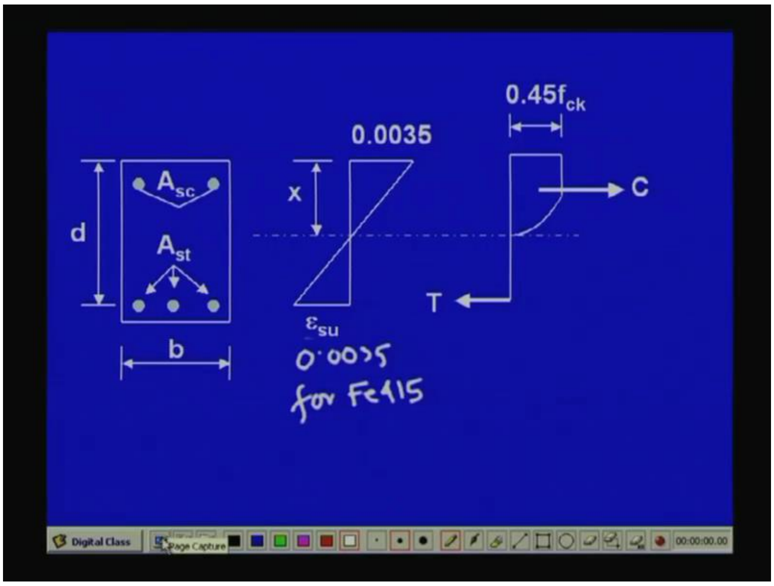

And since strain is different, I could further explain this 1 that, a doubly reinforced section; 0.0035 epsilon su I have told epsilon su which is nothing, but epsilon su which is nothing, but I can say so, 0.0035 for Fe 415. What we can do our strain diagram, 0.0035 concrete always same, but the other side; it is dependent on the your reinforcement steel reinforcement you have used. And we shall use this stress block, the stress block you are using this 1.

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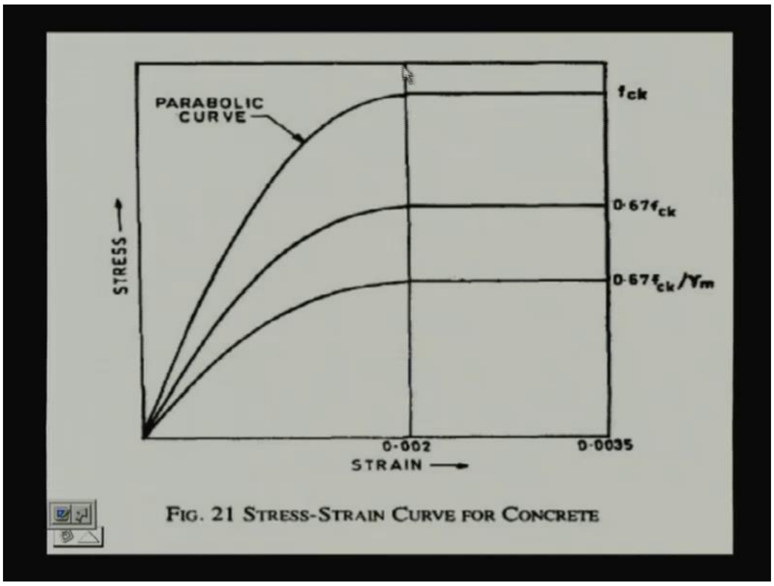

What we can do now, we are using this curve; it is dependent on the strain, upto 0.002 we shall come back upto 0.002 strain, beyond that we get constant stress in concrete, which comes as 0.45 fck. This portion less than 0.002 strains, we shall get that in a parabolic 1 we get the corresponding stress because, we have to find out the stress.

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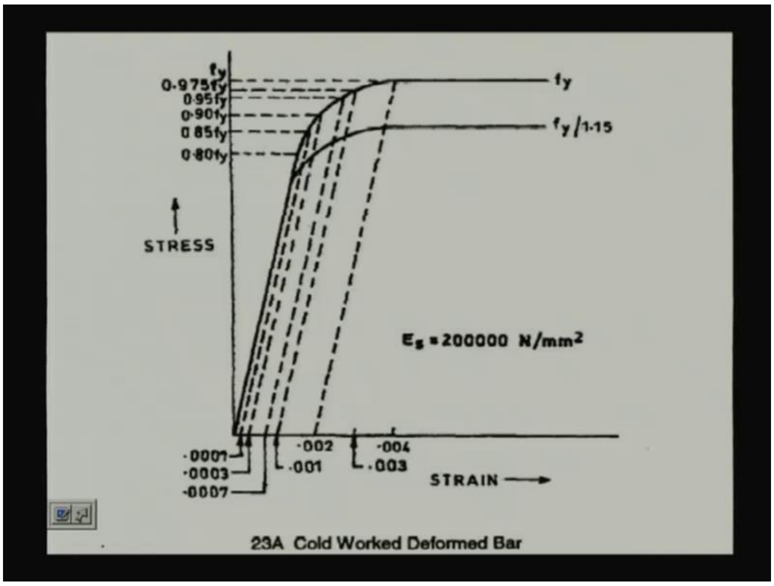

The other important part; that is the major part in our case because, since 0.0038 for Fe 415 and we are getting say 0.87 fy. 0.87 fy why because, you are using that partial safety factor for materials, that is, divided by 1.15 for steel. So, we are getting 1 by 1.15, which is nothing, but 0.87 fy, we should always remember 0.87 fy. And the corresponding strain we shall get it from this curve, for the strain computed we shall get the corresponding stress and that we shall use it to calculate our force.

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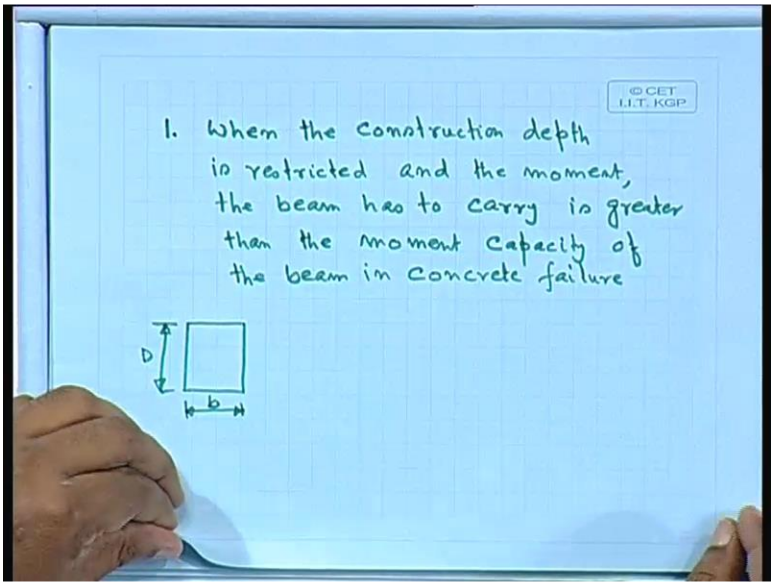

Now, let use comeback in few cases. When we have to provide the reinforcement, let use write down. When the construction depth is restricted and the moment, the beam has to carry is greater than the moment capacity of the beam in concrete failure. When construction depth is restricted and the moment, the beam has to carry is greater than the moment capacity of the beam in concrete failure.

So, moment of resistance for a section provided, if we have this section, let us say D. Due to some reason we cannot go beyond this D, 450 millimeter 500 millimeter like that. But moment computed applied that is more than the moment of resistance of this section. In that case, we have to provide the compression that your reinforcement.

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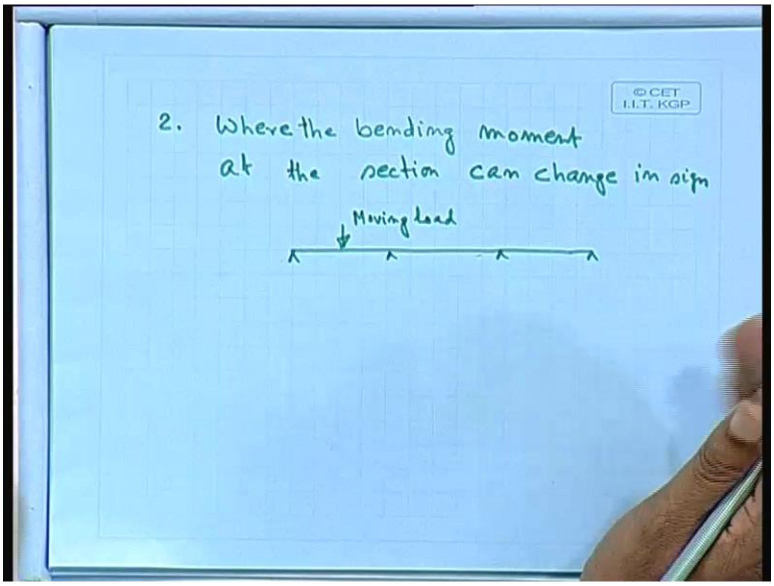

The other 1 we can take; let us say number 2, where the bending moment at the section, can change in sign. We can say that a continuous beam possibly and there is a moving load; the vehicle is moving over the bridge if it is continuous. Most of the cases our bridges are simply supported, but if the bridge is continuous, in that case in the support, the bending moment can change its sign. And in that case because, in the top say generally it is compression and bottom tension, but due to change in sign, so it can happen the other way the bottom compression top tension.

So, we can get that, we have to provide that adequate reinforcement and in that case, we have to provide that doubly we have to use it as a doubly reinforced section. Well, let use come there are different other points also.

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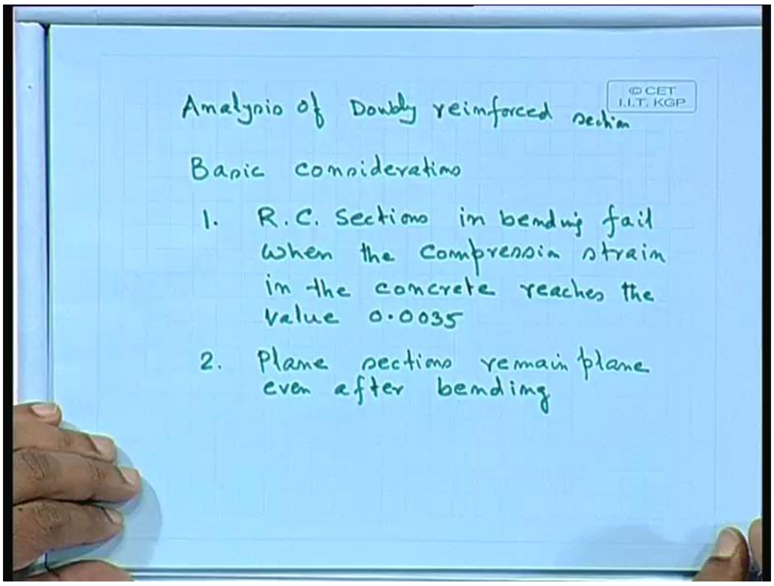

Now, let us come that your analysis of doubly reinforced section. We shall comeback for different other cases also, but before that let us take basic considerations. Number 1: RC sections in bending; RC sections in bending fail, when the compression strain in the concrete reaches the value 0.0035. We shall assume few cases 0.0035, that is, strain in concrete and here also it is valid in doubly reinforced section also.

Number 2: plane sections remain plane, even after bending. We can take 1 more consideration.

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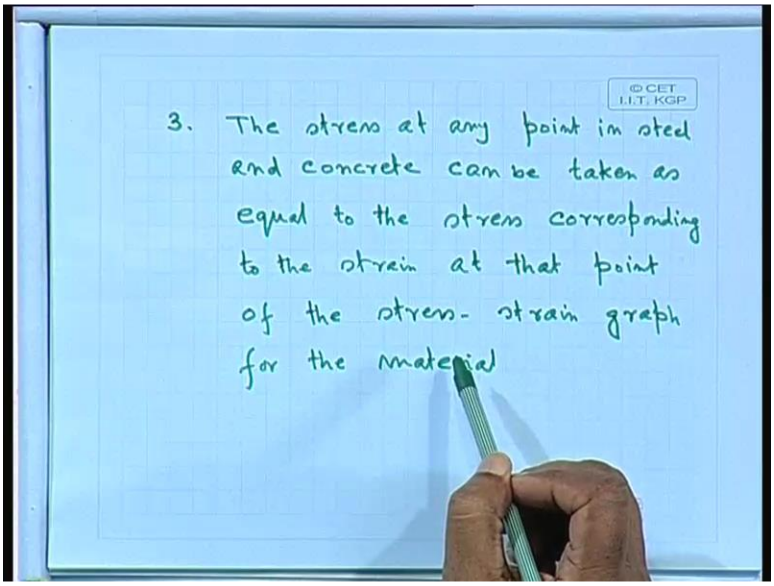

Number 3: the stress at any point in steel and concrete can be taken as equal to the stress corresponding to the strain, at that point of the stress-strain curve for the material. I repeat; the stress at any point in steel and concrete can be taken as equal to the stress corresponding to the strain, at that point of the stress-strain curve for the material. So, we shall get stress and strain, at a particular section that level we shall get the strain.

Strain is same for concrete and steel that is same, but the corresponding stress that we shall get it from the corresponding stress-strain graph. From the concrete we shall get the parabolic and strain portion and for the steel the corresponding graph which is given in IS 4.56, from there we shall take the yours the stress.

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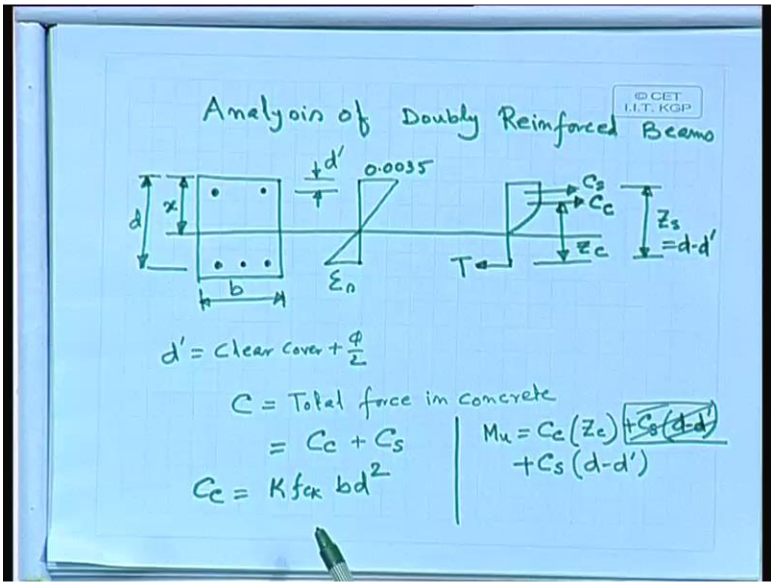

Well, now come to few procedures. What we have to do, we have a section and we have neutral axis. The effective depth d, width b, stress strain curve let us draw 0.0035. What about these portions? Let us say this is effective depth d dash equal to clear cover plus pi by 2. Our interest here this is epsilon s, our interest here we can divide it into 2 parts: 1 that I have already told, 1 that singly reinforced section plus you supplement with steel in the compression side and as well as in the tension side, we shall get the curve.

We have let us say this is T Cc due concrete only, compressive force due to concrete only and we have Cs. Therefore, C total force in concrete equals Cc plus Cs. Cc we can get say due to concrete failure, I can write down Cc equals K times fck bd square, K 0.138 fck bd square if it is Fe 415, you can write down. And the Cs it is nothing, but the stress in steel multiplied by the area of steel in the zone. And we have 2 lever arms: 1 lever arm this is 1 let us say Zc and the other lever arm let us say Zs Zc and Zs, d is the effective depth, d is never within the between the 2 reinforcement, d is always from the top fiber, d is always from the top fiber and that is why we have got another parameter say k 2 d, it means: that from the top fiber what is the position of this resultant force compressive force which this 0.42 d or 0.41 C or 0.42 d that we are getting.

So, you can write down Mu equal to Cc concrete times I can write down here say Zc the lever arm plus cs times it is always d minus d dash it is always d minus d dash. So, I can, I can write down instead of that, let us write down here plus Cs d minus d dash. I mean to say j dash equal to d minus d dash and this Cc is nothing, but kfck bd square depending on the steel used we shall get.

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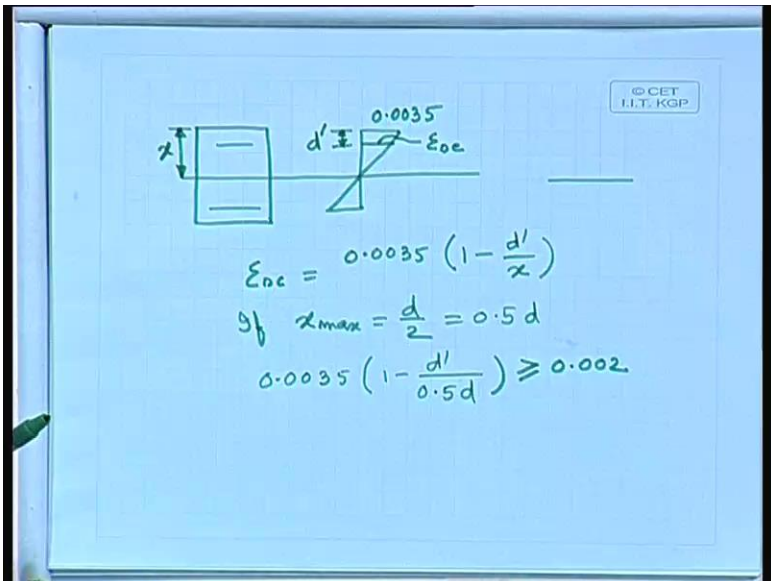

We have to find out the strain. The section we are providing here 0.0035, we have to find out epsilon sc in the strain in the compression side at the steel level, where we are providing the steel, that is, at a distance d dash from the top fiber, d dash from the top and we have x. What about epsilon sc epsilon sc? Will be equal to 0.0035 1 minus d dash by x; epsilon sc 0.0035 1 minus d dash by x. If, we assume if x maximum could we say d by 2, so we can write down d dash by 0.5 d.

If we can have, let us draw the stress strain curve or I think I can show it here.

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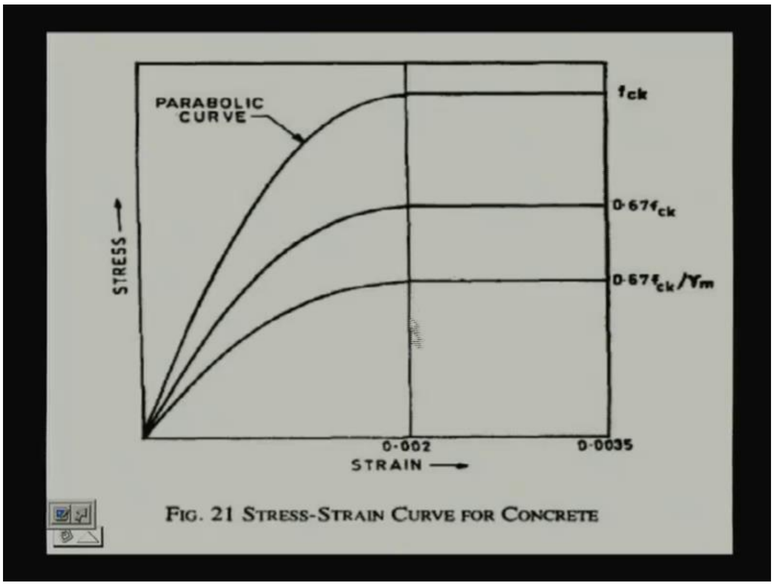

FIG. 21 STRESS-STRAIN CURVE FOR CONCRETE

Our objective here; the strain where we are providing the steel reinforcement that d dash, if it is more than 0.002 the strain greater than 0.002 what we can get, we can always get 0.45 fck, we can get 0.45 fck.

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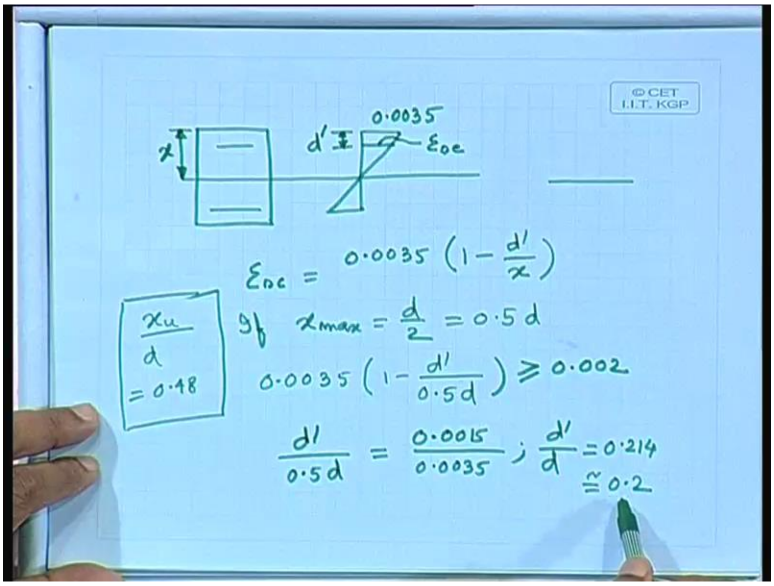

We are assuming if x max equal to d by 2 0.48 d xu by d for Fe 415, that we get 0.48. We can get it in is 456, we can also derive it also. xu by d equal to 0.48. That means, it is coming say reasonable assumption that 0.5 xu equal 0.5 d maximum. So, we can write down this expression, where it comes as d dash by 0.5 d equal to say 0.0015 by 0.0035. Therefore, I can write down d dash by d equal 0.214 approximately say 0.2. We can say d dash by d if it is 0.2 or less than that, we shall get the stress in concrete 0.45 fck, we shall get the stress that is 0.45 fck.

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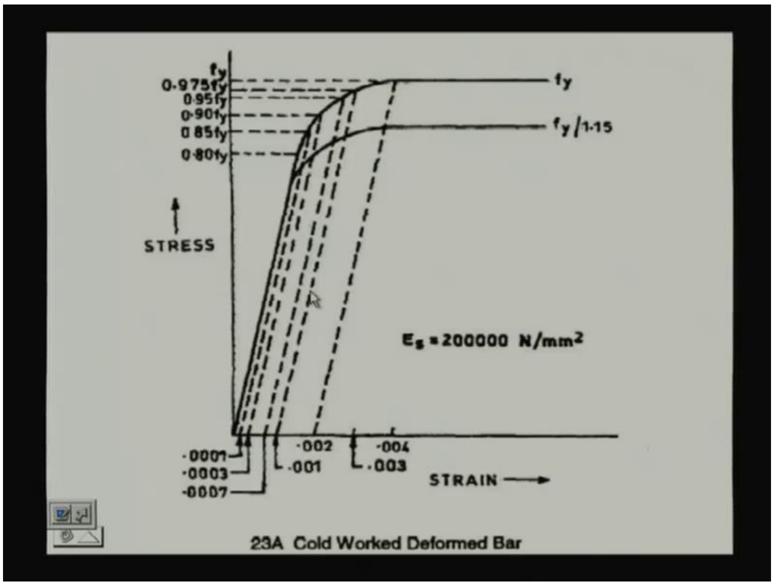

The other portion because, we are getting the strain we are getting strain at that level. So, corresponding stress we shall get it from this curve. So, the corresponding stress we shall get it from this curve. Instead of that let us tabulate it.

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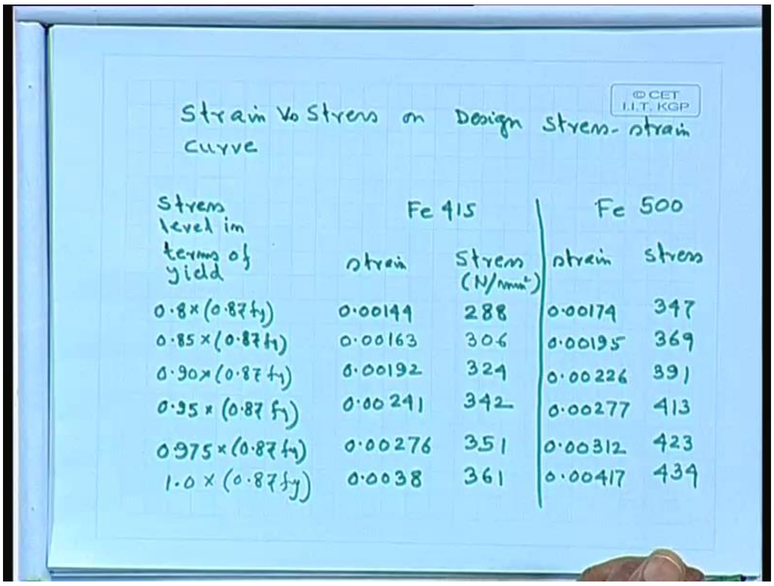

Strain Vs Stress on Design stress-strain curve

Stress level in terms of yield	Fe 415 strain	Fe 415 Stress (N/mm²)	Fe 500 strain	Fe 500 stress
0.8 × (0.87 fy)	0.00144	288	0.00174	347
0.85 × (0.87 fy)	0.00163	306	0.00195	369
0.90 × (0.87 fy)	0.00192	324	0.00226	391
0.95 × (0.87 fy)	0.00241	342	0.00277	413
0.975 × (0.87 fy)	0.00276	351	0.00312	423
1.0 × (0.87 fy)	0.0038	361	0.00417	434

Let us take Fe 415 strain and stress, we shall get at this stress level 0.8 into 0.87 fy 0.00144 the strain, corresponding stress 288 let us give the unit 0.00163 306 at 0.9

0.00192 324 0.00241 342 0.00276 35 0.0038 361. And we are getting from this curve. You can directly you can get it directly using scale directly also you can get the corresponding stress, but instead of that it is customary that we can tabulate it and from there we can just linearly we can interpolate. If the corresponding strain comes within this limit, so we can linearly interpolate and we can find out the corresponding stress. So, it is better to use this type of table.

What about fu 500? The strain let us give a margin strain and the stress. We get 0.00174 347 0.00195 369 0.00226 391 0.00277 413 0.00312 423 0.00417 434.We have for Fe 415 and also for Fe 500. We can get the strain and corresponding stress and we shall linearly interpolate to get the corresponding stress at a particular strain level. We shall come to the next level, say analysis of beam.

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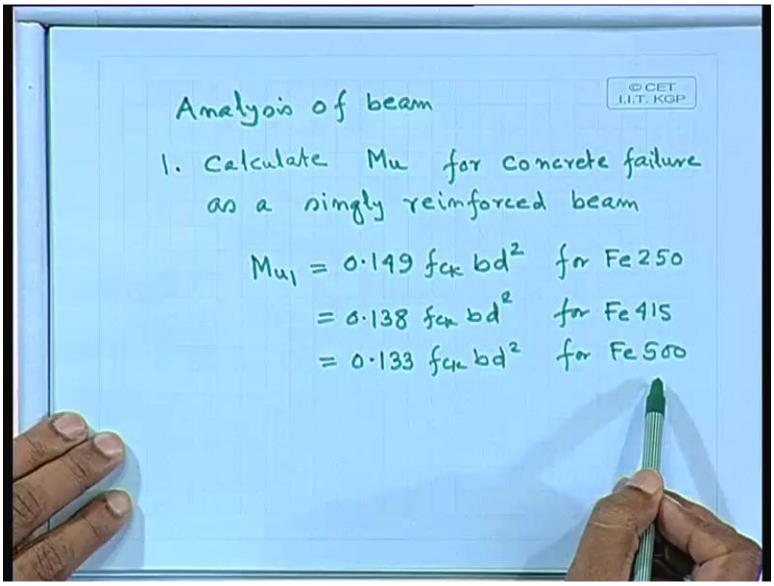

1: Let us elaborate the whole procedure. Calculate Mu for concrete failure as a singly reinforced beam. We can write down this part as Mu 1 equal to 0.149 fck bd square for Fe 250, equal to 0.138 fck bd square for Fe 415, equal to 0.133 fck bd square for Fe 500. We shall use the appropriate formula to get the moment of resistance for singly reinforced section, when the section is assumed to be failed due to concrete; concrete failure. Mu 1 for Fe 250 for Fe 415 for Fe 500.

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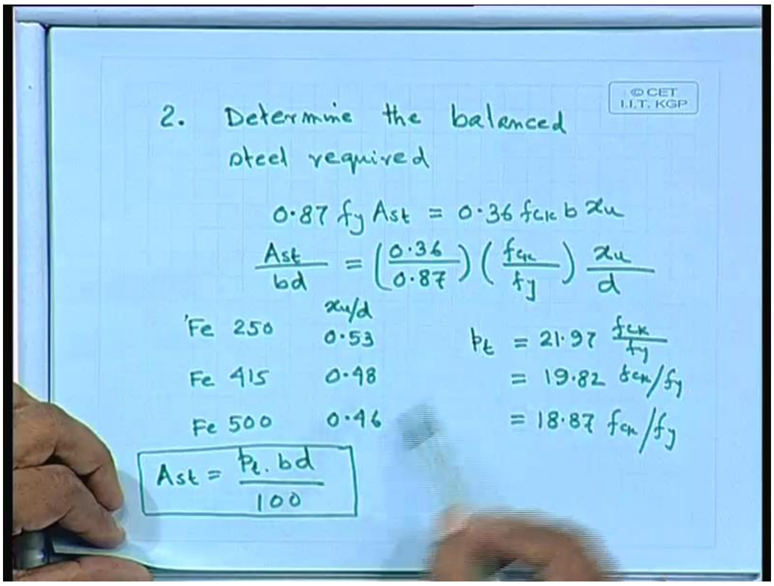

Step 2: determine the balanced steel required. We shall use the formula that 0.87 fy Ast, the steel has also reached the maximum stress equal 0.36 fck b times xu. We can write down it as Ast by bd equals 0.36 by 0.87 just rearranging times fck by fy times xu by d. Let use repeat Fe 250 Fe 415 Fe 500 xu by d 0.53 0.48 0.46 pt, what is pt? In other way, Ast? Ast equal pt times bd by 100.

So, pt equal to 21.97 fck by fy equal to 19.82 fck by fy equal to 18.87 fck by fy, if we calculate we shall get this. Either you can remember this 1 or we can start from the very first principle you can calculate it. Our objectives we have to find out because, it is easier to remember which is nothing but, the steel force equal to compressive force. From there you are getting the value and we can find out Ast equal pt bd by 100, therefore we can write down pt. What I feel most of the cases at the time of design, when you are doing in design office or at your home, your books other things are available, but when you are doing in the exam, so you should at least follow certain procedure so that, you can do it without consulting any book.

So, that is why it is better to start from the first principle, though it will take time. But if your do practice if you solve so many problems, then you will find out automatically you can remember these values also.

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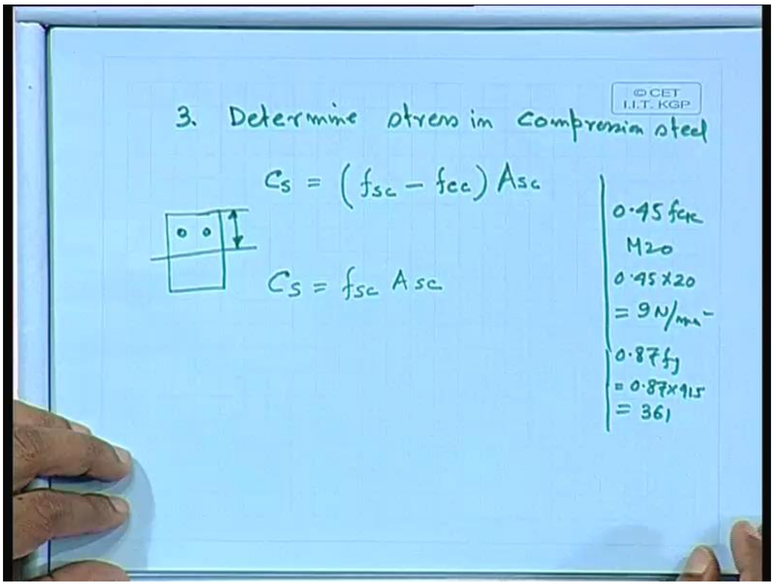

Number 3: determine stress in compression Cs equal to fsc, fsc is the steel that in the concrete in the compression side fsc minus fcc times Asc. What we mean: when you are taking this section, there are reinforcement. When you are considering the singly reinforced section, already we have taken this area. However, small it is compressive say your concrete stress, but already we have taken this area that is why, we are deducting that Asc. When you are taking the steel so we taking fsc minus fcc times Asc.

However we can neglect this fcc because it is very very small, say because you are taking 20 Newton 20 times say 0.45. So that means 9, if we find out 0.45 fck. If, we take 20 Newton per square meter M 20 grade of concrete, we are getting 0.45 times 20 which comes as 9. Whereas, 0.87 fy say for Fe 415 which comes as say almost say 361.

So, in other case we can neglect this 9 and our code also that in annex G you will find out that, it neglects and we can write down Cs equal to nothing, but fsc times Asc. So, this is your that, compressive force due to that steel in the compression site.

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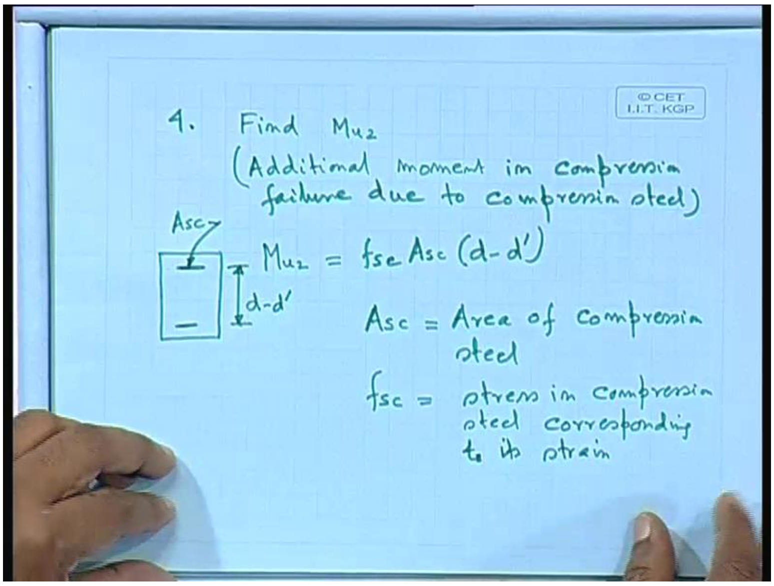

Number 4: we have 2 parts of moment of resistance; 1 is Mu 1 and another 1 Mu 2. Find Mu 2 we can say additional moment; let us say in compression failure due to compression steel. Mu 2 equal to let me draw the figure, this length is d minus d dash Asc. So, Mu 2 equal to fsc which you have got it from the strain times Asc the force times the lever arm which is always d minus d dash Mu 2 fsc ac d minus d dash. Let us write down Asc: area of area of compression steel and fsc: stress in compression steel corresponding to its strain.

So, we have 2 parts: 1 is Mu 1 and other part is Mu 2.

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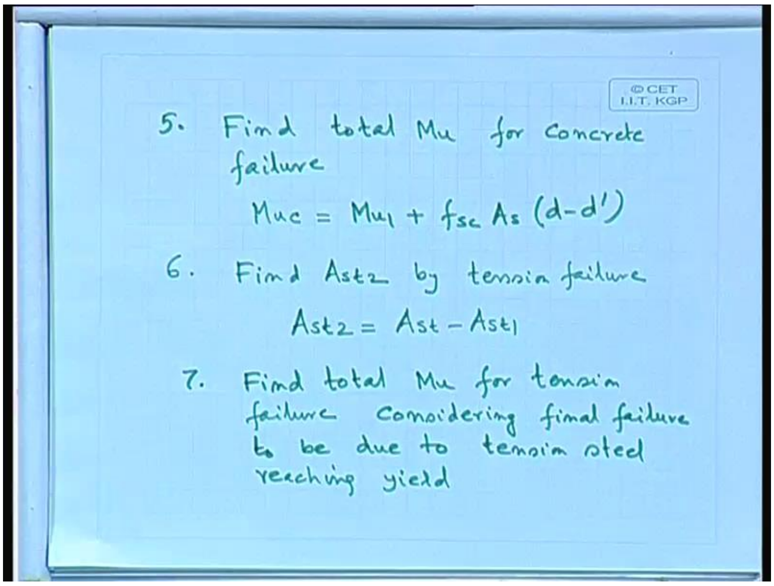

Number 5: find total Mu for concrete failure. We can say you can designate it as Muc equal to Mu 1 plus fsc Asc times d minus d dash. Number 6: find Ast 2 by tension failure. We are analyzing the beam that where Ast is given. So, I can write down as Ast 2 will be equal to Ast minus Ast 1. Ast 1 because, we are having 2 parts; area of steel is Ast the total tension reinforcement in the tension side. Due to the singly reinforced section the corresponding balanced steel we shall get it that is say Ast 1 and remaining portion Ast minus Ast 1 which is Ast 2 that, we shall get it to provide that you are in equilibrium position for the compression reinforcement and that 1 we call it as a stress steel beam theory.

So, we are having 2 parts; that is why, we are providing as if we are providing steel at the top, we providing steel at the bottom and then we are increasing the moment of resistance of the beam. Number 7: find total Mu for tension failure considering final failure to be due to tension steel reaching yield. We are taking this case; we have already done in the concrete side concrete failure. Now we are talking the other side the tension failure. Here we are assuming that we shall reach the yield stress of steel that is 0.87 fy.

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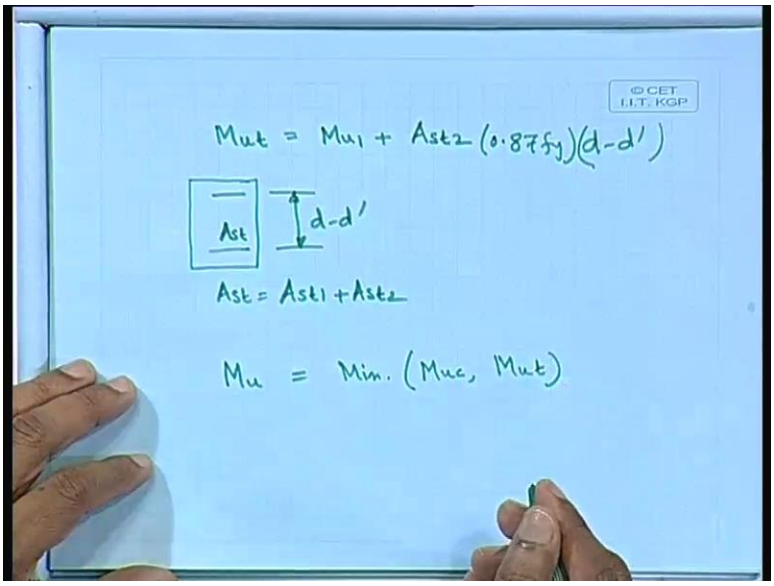

Then what we can write down; Mut we have already done Muc so, Mut will be equal to Mu 1 plus Ast 2, Ast 2 is the remaining portion times 0.87 fy d minus d dash. This mu 1 plus Ast 2 times 0.87 fy d minus d dash. We are taking the same principle Ast Ast equal to Ast 1 plus Ast 2 and the difference are different, so d minus d dash. So, we know Muc and we know Mut, out of them which 1 is the minimum 1 that 1 will be your moment of resistance of that doubly reinforced section. And this is called analysis of doubly reinforced section.

The section where all the detailing is given that your depth, width, reinforcement in the top, then in the compression side in general case, reinforcing the bottom I mean to say in the tension side tension zone, concrete grade steel grade and we have to find out the moment of resistance. So, if we summarize we shall get, we are making it in 2 parts; first part as if that we do not have the compressive reinforcement. So, if we take out the compressive reinforcement, we can consider it as a balanced section.

So, what is the moment of resistance due to concrete failure; that we can find out which is nothing, but Mu equal to K fck bd square K in particular case for Fe 415 that is 0.138. So, Mu equal to point 1 8 Fck bd square that just we can remember. The corresponding area of steel Ast 1, we can get it from the balanced section because, steel also simultaneously reaching the yield stress, so Ast 1 we can find out. The total area of steel

Ast is provided minus Ast 1 that 1 we are getting, that it will be in consideration with say doubly reinforced section.

So, Asc at the top in the compression side and corresponding Ast 2 the balance of Ast minus Ast 1, from there we shall find out the corresponding moment of resistance. So, we shall we are doing in 1 side we are doing it due to concrete failure, the other side we are doing it due to steel when it reaching the yield stress. We shall find out the 2 moment of resistance and we shall which one is the lowest; obviously, we have to take that. So, Mu moment of resistance equal to minimum of say, I can say Muc and Mut. This is called analysis.

We shall continue with that design of design of doubly reinforced beam, when moment that Mu is given that applied moment is given and width of the beam given depth also, given say we have to find out the reinforcement of that thing, that is called design. So, let us stop in this one. So, we shall continue again in the next class, today itself.