Design of Reinforced Concrete Structures
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Lecture - 06
Limit State of Collapse Flexure

Today; we shall start, limit state method and we shall start with the topic our lecture number 6; the limit state of collapse and we shall consider flexure. So, limit state of collapse flexure that we shall consider today. So every method has certain few assumptions whatever we consider. So, that particular equation or method is based on few assumptions. So, it is true for any experimental setup also. So, whenever you're making any setup which we will consider that any real life problem there also we make certain kind of assumption and it will automatically will come, when we shall consider that problem. So, what are those assumptions?

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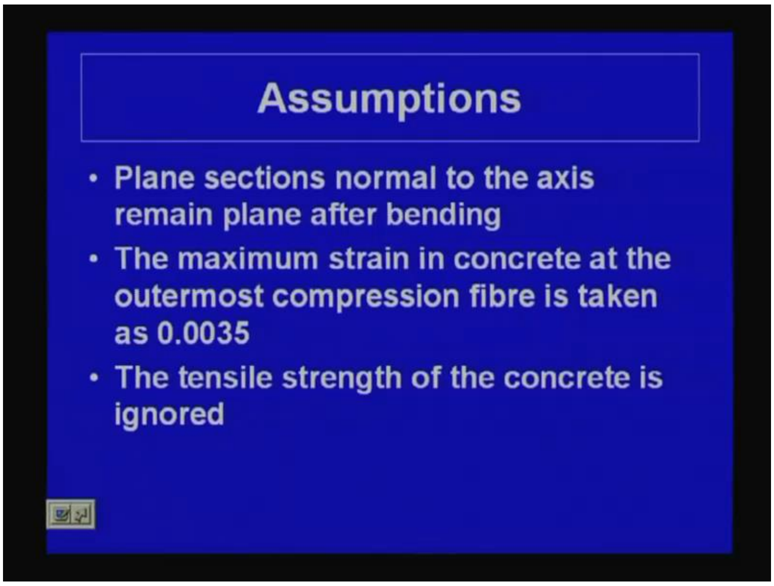

The assumptions here, we shall consider the assumptions that plane sections normal to the axis remain plane after bending the maximum strain in concrete at the outer most compression fiber is taken as 0.0035. The strain is 0.0035 that is the maximum limiting strain in concrete that we shall consider, that is 0.0035; the tensile strength of the concrete is ignored. So, that is the almost we can say primary assumption in our concrete design, the reinforced concrete design we are assuming that we shall not consider tensile strength of concrete.

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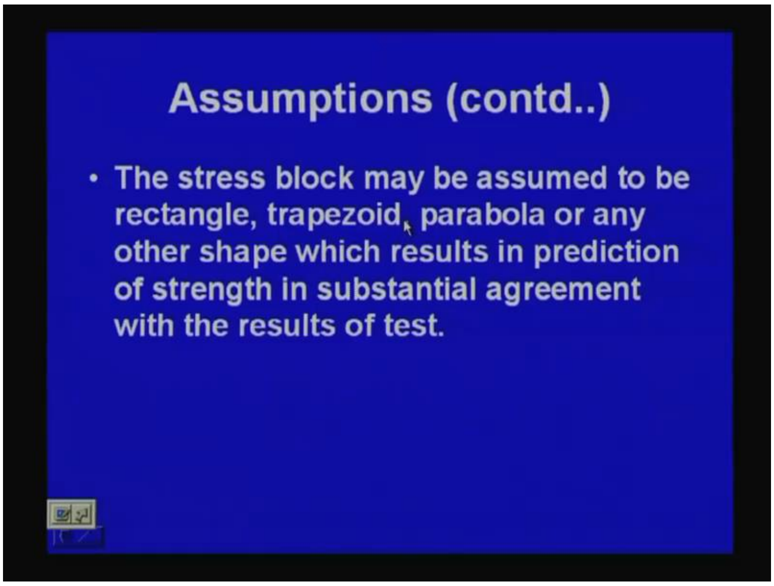

The stress block may be assumed to be rectangle; I shall come within a minute. What is stress block, the stress block may be assumed to be rectangle trapezoid parabola or any other shape, which results in predication of strength in substantial agreement with the results of test. So, even if you consider a beam and that beam, if you test it whatever the failure load you are getting and whether with your model whether getting that result than you can say that it is in a good agreement with your experimental result and the method you have proposed, the stress block that you have proposed that is valid.

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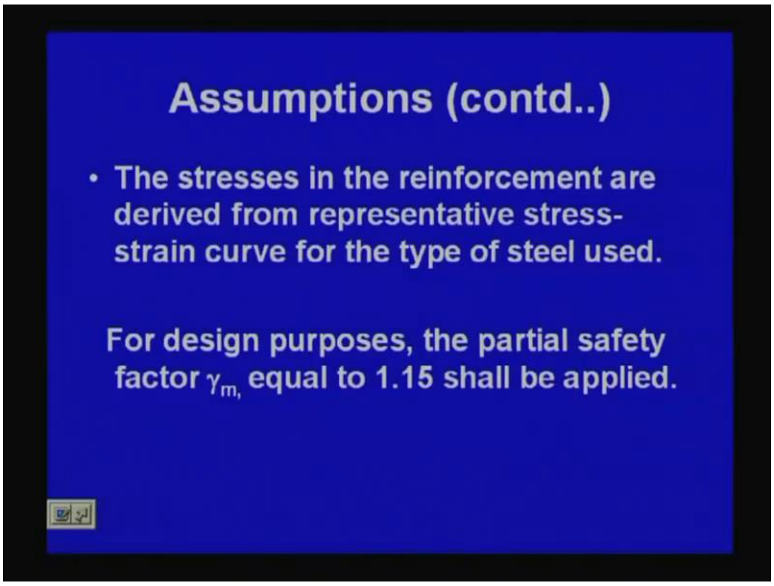

The next assumption, the stresses in the reinforcement is derived from representative stress-strain curve for the type of the steel used. So, the stress-strain curves for steel that also you have to consider. I shall show you a graph, for design purposes the partial safety factor gamma m that is for material that partial safety factor steel we consider 1.15. So, if fy is the stress yield stress. So, we have to consider fy by 1.15; that you have to consider that is the for design purposes we shall consider we never take the ultimate limit for our design calculation.

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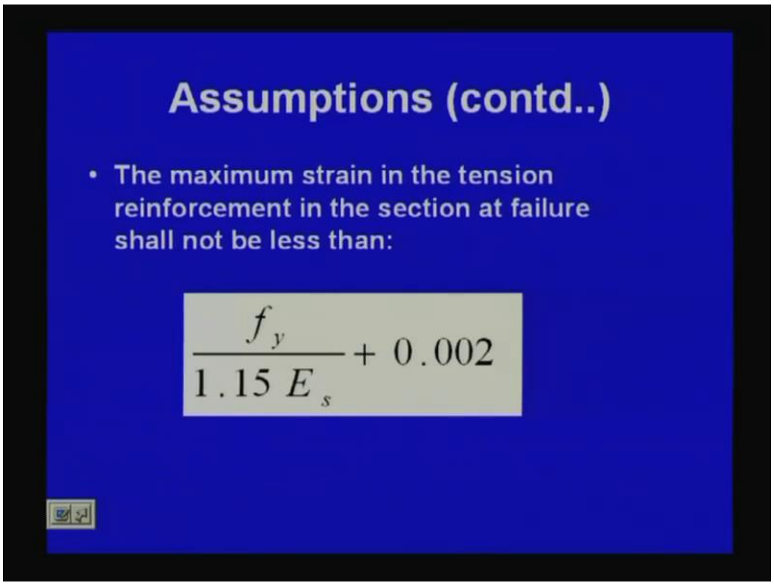

The maximum strain in the tension reinforcement in the section at failure shall not be less than fy by 1.15 Es plus 0.002. So, fy by 1.15 Es plus 0.002. This your that strain that should not be less than this value. So, for different grade of steel we shall get different limiting strength. We have 3 types of steel fe 250, fe 415 and fe 500 most of the cases we use fe 415 and mild steel we use it fe 250.

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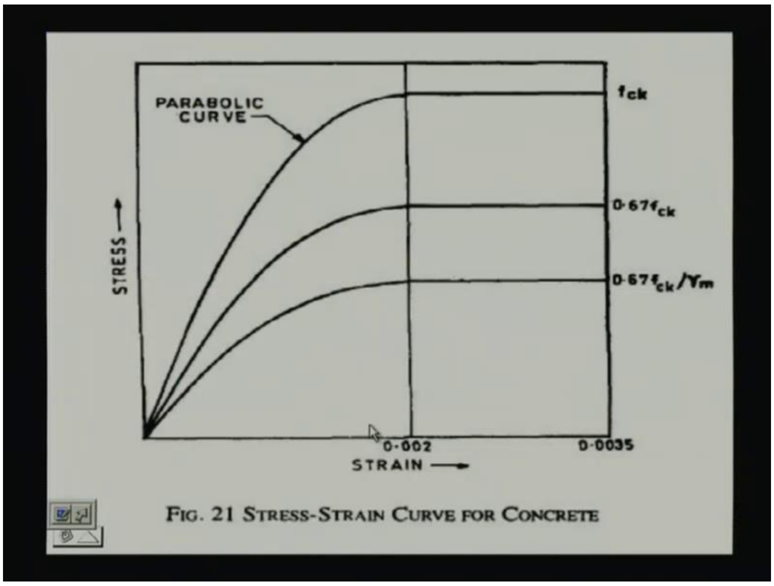

According to IS 456, this is your strain and stress stress-strain curve of plane concrete. What we consider here, please note the strain there are 2 strain 1 is 0.0035, which I have already told which is specified in the assumption and given in IS 456 2000. So, that you should remember that 0.0035 that is the limiting strain maximum strain allowed; in concrete, plane concrete and for design purposes we shall use this curve. What we consider, 1 this is the 1 that cube strength M 20 Newton per square millimeter. M 25 Newton per square millimeter. We shall take 0. 6 7; that means, two -third of that cube strength or characteristic strength we shall consider for our design purpose.

Further we shall consider; this value that is 0.67 fck divided by gamma M; that is partial safety factor material concrete, gamma M for concrete we shall consider 1.5 gamma M for steel we shall take 1.15 and, which is justified because we are considering we take steel that is manufactured in the workshop in the control environment so; obviously, we can take less, but whereas, concrete that is made in the field itself. So, we are taking little bit higher and that 1 we are considering, we are taking 1. 5. in that curve, you will get it in figure 21 for your reference in IS 456.

So, 1 more thing I would like to point out, this part is parabolic and another part straight, but in real situation, that if you do the stress-strain curve or if you do the experiment. If you apply the load, you will get certain drooping portion also; that means, it will go up this way it will go up and then, slowly it will come down like this; it will go certain level, but we are not considering for our design purposes, for our design purposes we are taking this curve and when we shall take the actual value design value we shall consider these curve, which is based on this curve we are getting from the experimental result. Experimental data, then we are taking two -third of that further we are dividing by partial safety factor for materials and then finally, we shall get the curve for our design purpose.

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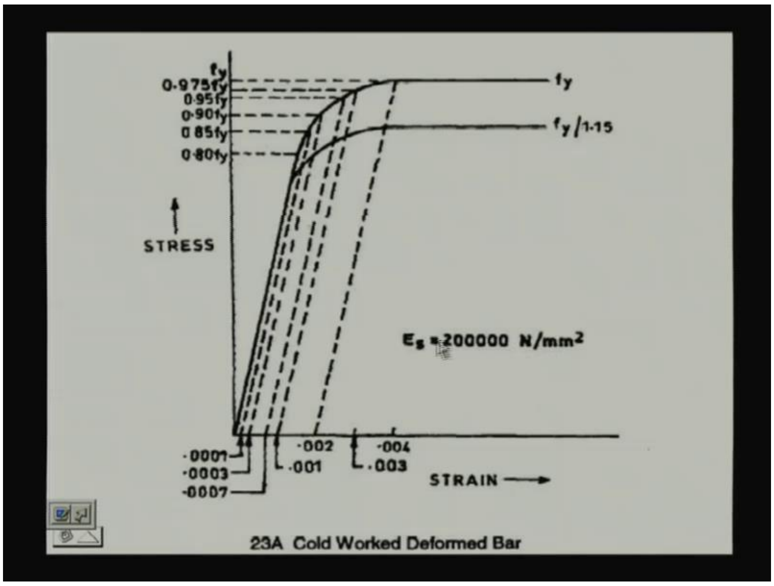

Figure 23A of IS 456 that is for cold worked deformed bar high strength and we shall consider Es modulus of elasticity for steel, we shall take this value. So, we shall take this value and we can get that different for different strain we can get the stress value. So, we shall consider for high yield strength deformed bar we shall take this stress-strain curve for steel.

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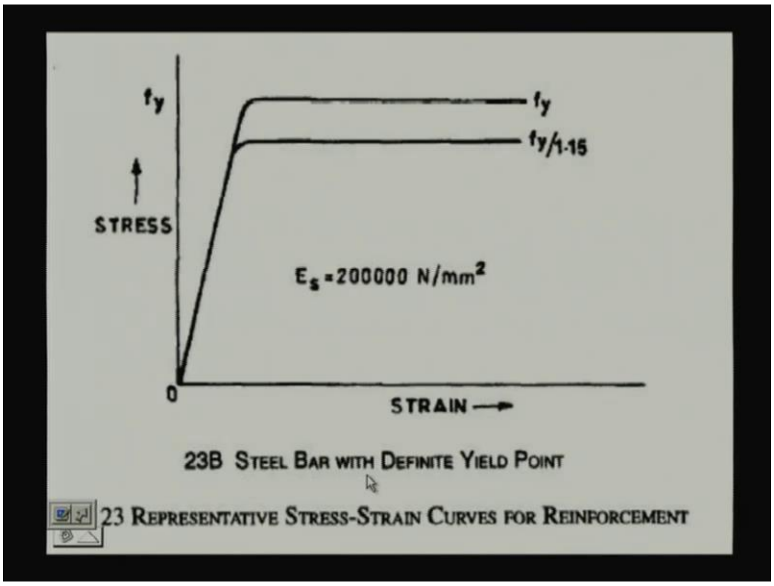

This is figure 23B steel bar, with definite yield point for mild steel we shall consider, we shall take, these curve this is the test data and this is after dividing by partial safety factor.

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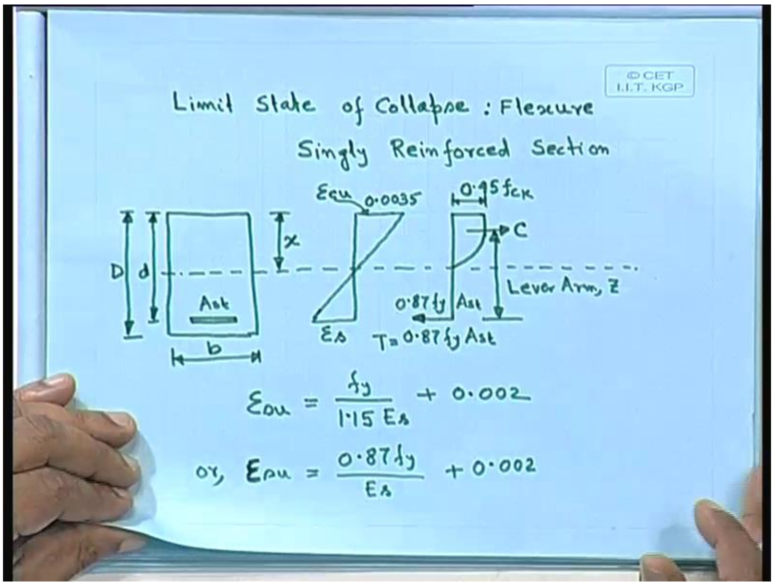

So, let us come to the limit state of collapse. We are starting with the limit state of collapse, ultimate strength and we shall consider bending flexure; further we are considering that singly reinforced section. So, let us draw a cross section of the beam and we are having area of steel d effective depth d and though we don’t need D, but even then let us write down, for the completeness this is your D. We have width of the beam b and we should have the neutral axis; let us draw it here also further. So, that we can draw the stress strain curve, this is the neutral axis position from the maximum compressive stress. We shall get the compressive stress at the top fiber that is the maximum. And from there we are measuring the depth of the neutral axis which is defined as x.

In working stress method; we have not considered we have not taken the strain criteria, but in limit state method we are specifying the strain that is the limiting value of strain, that we are specifying. For concrete the limiting strain 0.0035 that you should always remember 0.0035; we shall go up to the steel position. This is 0.0035 and this value say epsilon s. Let us say, these value epsilon cu, what about the stress block, the stress block comes the same 1 which we have shown you that curve we are having maximum 0.0035 somewhere here 0.002 because your are starting from 0. So, some where we shall get 0.002 finally, we shall get the maximum strain allowed 0.0035, which will never go beyond this 0.0035.

According to this curve that mentioned in the code, we shall consider this curve. This our stress block also this value comes 0.45; we shall consider this value as 0.45. So, we are taking 45 percent of the cube strength to be more specific I can say, I am taking 45 percent of the cube strength, for our design purpose we are taking 45 percent the cube strength. What about these steel this, this will be 0.87 fy multiplied by let us write down here, please allow me times Ast 0.87 fy. Ast here I think it is better, if I repeat once more here 0.87 fy that is the stress permitted in steel multiplied by area of steel. This 1 that your T. So, tensile force T on the steel 0.87 fy Ast we shall get the compressive force that is C.

What is the our interest here, that beam which under bending, under flexure how much maximum load it can take. What is the moment carrying capacity of that section that is our target? If we can find out that, value than we can say this section having this many number of bars and the cross sections that width and depth. We can say this is the maximum moment carrying capacity and that we have to find out. So, if we can find out that 1, then from analysis the result you have got moment different moment that you have got in different places. So, you can provide your section. That way also you can do; you can do in another way also; that means, you are taking a particular section let us see, the moment carrying capacity and let us provide. It should be always it will be more than the computed value the moment at the particular section.

This is lever arm we specify by Z so, lever arm Z; so, what is the difference here in working stress method we shall not always compare with the working stress method, but since we have done in the last class the working stress method. Let us compare for the singly reinforced section for flexure. We shall here we are considering we have started with the strain, in working stress method we have not considered strain we have not taken strain. Only difference is the stress block, only difference is the stress blocks in that case the stress block linear here, we are getting certain portion parabolic certain portion straight.

But otherwise; your formulation everything will go in the same direction here also, we shall consider the same lever arm and it should be in equilibrium position compressive force equal to tensile force and moment carrying capacity nothing, but compression compressive force times lever arm or tensile force times the lever arm. Only the difference the stress value that portion is different it will be different for this case because we are starting from the strain diagram.

Epsilon su as per the code to repeat fy that is 1 of the assumptions also or we can write down in a familiar way; which is nothing but, 0.87 fy or let us write down. So, this is our corresponding strain this 1 will be epsilon su. This 1 will be epsilon su that ultimate strain in steel. We are considering singly reinforced only reinforcement in the tension site and we know that, 0.0035 is the limiting strain in concrete in compression. We shall get, the corresponding epsilon su for different grade of steel we shall get different limiting strain.

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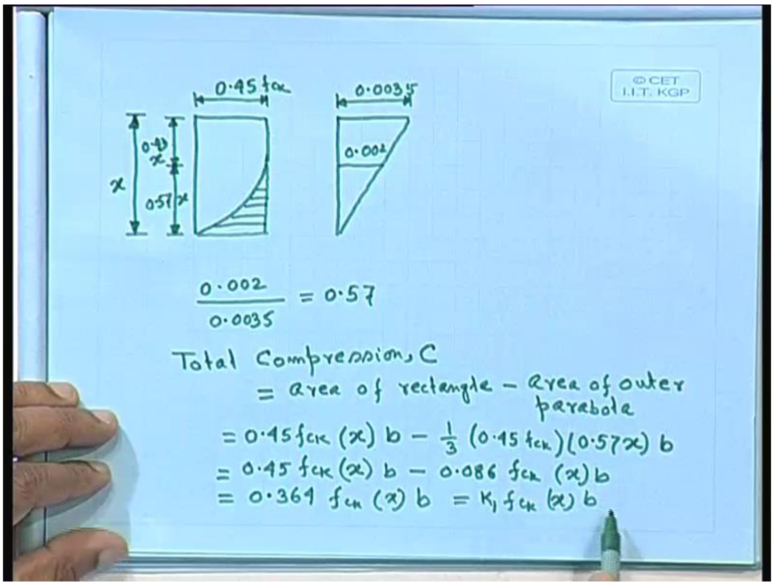

Let us draw once more, the stress block, the strain and also draw the strain here, the strain here 0.0035 this is 0.002. So, what about this value this 1 will be equal to this value let us find out 0.002 by 0.0035 and which will be 0.57. If we take depth of the neutral axis from the maximum compressive stress site x this is 0.57 x and this 0.43 x. So, 0.57 x and 0.43 x; we can write down, total compression C area of rectangle. The full 1 minus area of outer parabola, area of outer parabola means, this portion equal to 0.45 fck this 1 multiplied by the neutral axis to make it say for doubt let us give parenthesis it is not multiplication 1; x is the depth of the neutral axis multiplied by b minus one-third you have to find out this area 0.45 fck. This 1 also is 0.5 fck multiplied by 0.57 x multiplied by the area, width b.

The b is the cross sectional width, which equals 0.45 fck x times b minus 0.86 fck x b equals 0.364 fck x b. We take this 0.364 fck x b, x is the depth of the neutral axis fck the characteristic strength b width of the beam 0.364, this 0.364 we can take as 1 factor that k1.We can also write down equal to say k1 fck x b. This is 1 factor generally considered that k1,x b.

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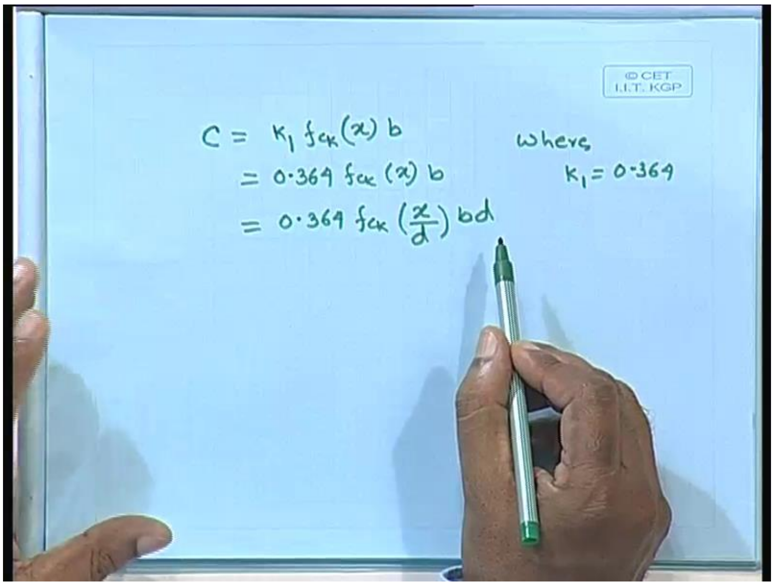

We can, further we can write down as C equal to, I have told k1 fck x b in our case 0.364 fck xb where k1 equal to 0.364. We write down the equation in a different form x by d multiplied by bd. So, 0.364 fck x by d, this is multiplied by bd, bd is the cross sectional area please note taking d as the effective depth not the overall depth. Always we shall take, bd whenever we consider that is the effective depth we shall take for area of steel what percentage that steel we shall consider Ast by bd b width and d effective depth please note we shall always take d effective depth.

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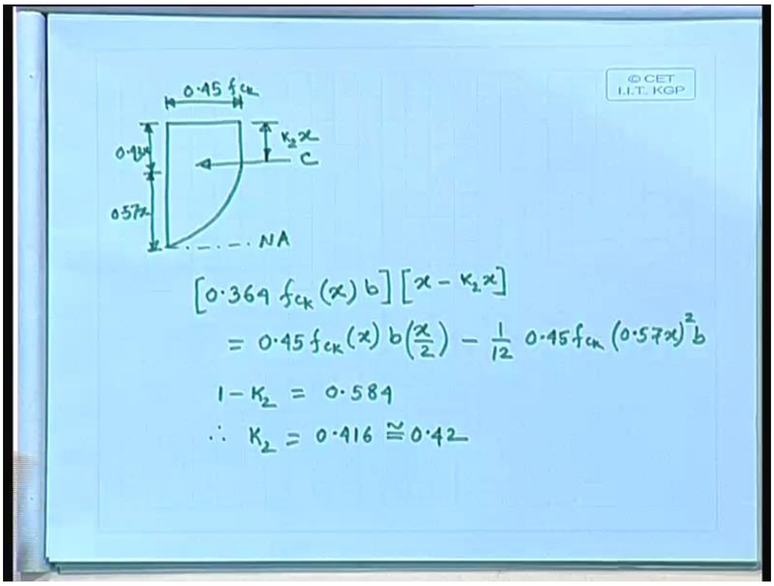

Let us write down, once more the stress block 0.43 x 0.57 x compressive force. We shall add one more factor, that is k 2 times x, x is the, this depth what is the factor k 2 that we shall find out. We shall take moment; we shall take the moment about the neutral axis. This is our neutral axis, if we take moment about the neutral axis already we have computed that c 0.364 fck x b that is the total force. Already we have computed that; 0.364 fck x b. This is 1 term, that is the total force and we are taking moment about this line x minus k 2 x. Which is equal to 0.45 fck x b times x by 2; the total rectangle 0.45 fck times x the total rectangle and cg width and x by 2.

So, we are getting that total rectangle minus one-twelfth; let us make it, x whole square b. If we take that area so, we are taking out this parabolic portion area times that cg. So, we shall get this 1, which equals 1 minus k2. If we simplify it equal to 0.584.Therefore, k2 equals 0.416 or we can say let us say 0.42. So, the compressive force will act from the top fiber having, the maximum compressive stress from there at a distance of k 2x which is nothing but, 0.42 x. So, we have got 1 factor k2 and another factor k2.

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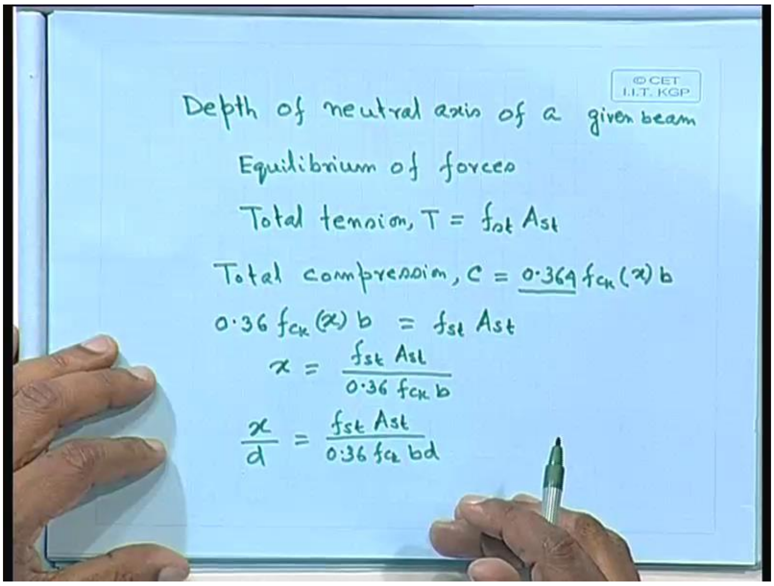

Depth of neutral axis of a we shall get it again from equilibrium of forces. Total tension T fst times Ast stress times Ast total compression C equal to total compression. We have derived here to show you total compression C equal to 0.364 fck x b. So, let me copy it here; 0.364 fck x b therefore, for equilibrium case; we also write down instead of writing 0.364 we also write down as 0.36 also. We can take it you know because that way we are estimating a little less we are taking a little less. So, that also we consider; so, let us take 0.36 fck x b; let me underline that we have noted down 0.364 and here 0.36 we are taking a little less value fst Ast x equal to fst Ast divided by 0.36 fck b.

We can also write down other way also we can write down this 2. As x by d equal to fst Ast 0.36 fcx b times d. We write the equation in this fashion that x by d, x by d we instead of directly computing that value we specify in this ratio that x by d. So, if we know the fst we can find out that x by d ratio. Now, if we take the section is under reinforced; that means, the we shall reach the yield stress in steel cost that case fst will be equal to 0.87 fy; that is the permissible stress according to IS 456.

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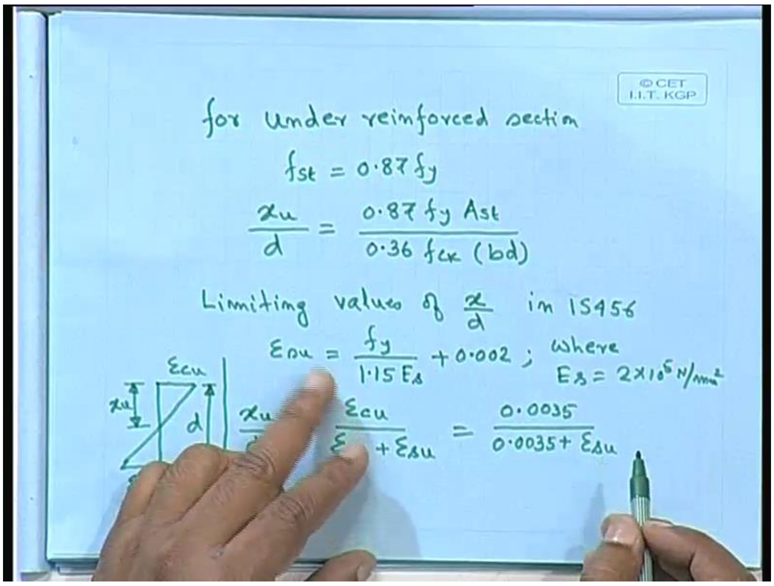

So, we can write down the equation, for under reinforced section fst equal to 0.87 fy xu by d. We are giving a special name xu that is the xu is the that name ultimate that you can say that, depth of neutral axis .You can say, su by d equal to 0.87 fy Ast by 0.36 fck b d. Let us check, limiting values of x by d specified in IS 456 epsilon su equal to fy by 1.15 Es plus 0.002 where s equal to 2 into 10 to the power of 5 Newton per square millimeter. xu by d equal to epsilon cu divided by epsilon cu plus epsilon su. Where, from I am getting from the strain diagram. This is epsilon cu and this 1 epsilon su; epsilon cu epsilon su. So, we can get let us say this one xu and total depth d.

So, we can find out from this curve from this similar triangle in civil we can find out xu by d epsilon cu by epsilon cu plus epsilon su equal to epsilon cu. We know which is nothing but, 0.0035 divided by 0.0035 plus epsilon su epsilon su is different for different grade of steel and which we shall get it which is given in our IS 456 we can get using this formula.

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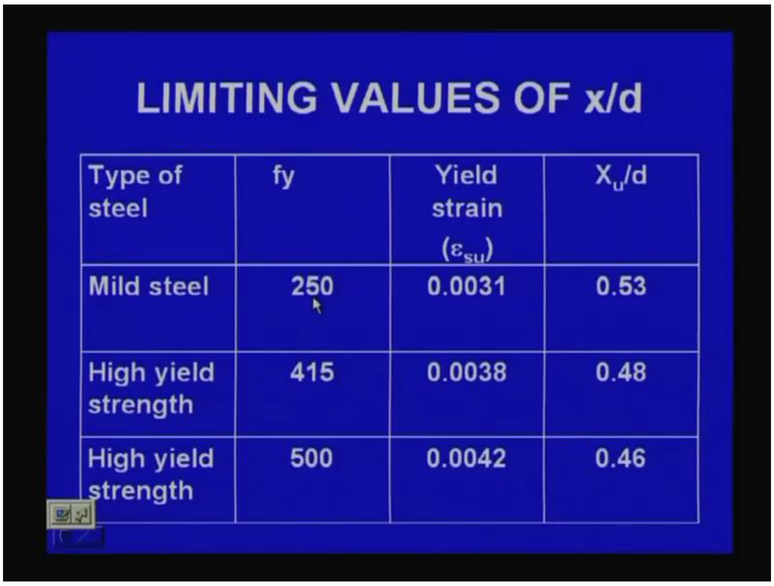

So, limiting values of x by d type of steel mild steel, high yield, strength high yield strength 250 fe 250 fe 415 Fe 500 using the formula, we can get the yield strength 0.0031 for fe 250 0.0038 for fe 415 0.0042 for fe 500 simple calculation from there we can make and we should remember at least we should be remember 0.0038. So, please note for concrete 0.0035; most of the cases we use fe 415. So, we should note the limiting strength which is nothing, but 0.0038. At least you should remember these 2 values 0.0035 for concrete and for the steel maximum used fe 415 0.0038 and we can get the limiting values xu by d 0.53 0.48 and 0.46.

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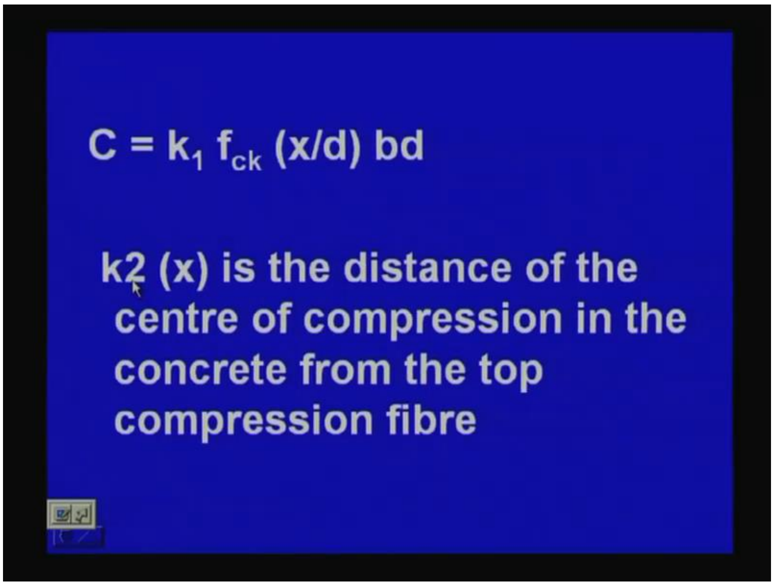

So, from that equation we can get it 0.53, 0. 480 point 46. So, again you please not this value 0.46 xu by d limiting value 0.46 point fe 415 is used. We shall consider, we have already derived this equation C equal to k1 fck x by d times bd. This 2 should be suffix. So, k2 times x is the distance of the centre of compression in the concrete from the top compression fiber that, I am repeating number of times. So, k2, x1 parameter is k1 the other parameter is k2.

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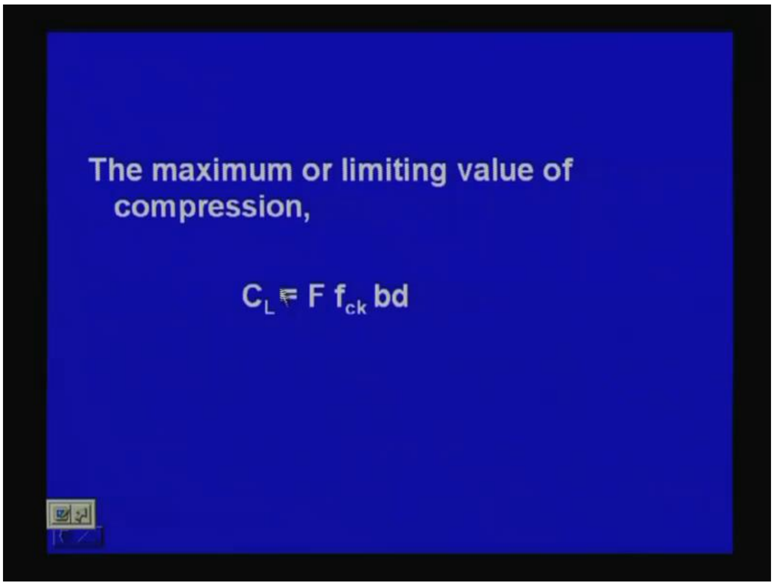

And the third-one; the maximum or limiting value of compression F times fck bd. So, 1 parameter K1 another parameter k2 and the third parameter we are taking F.

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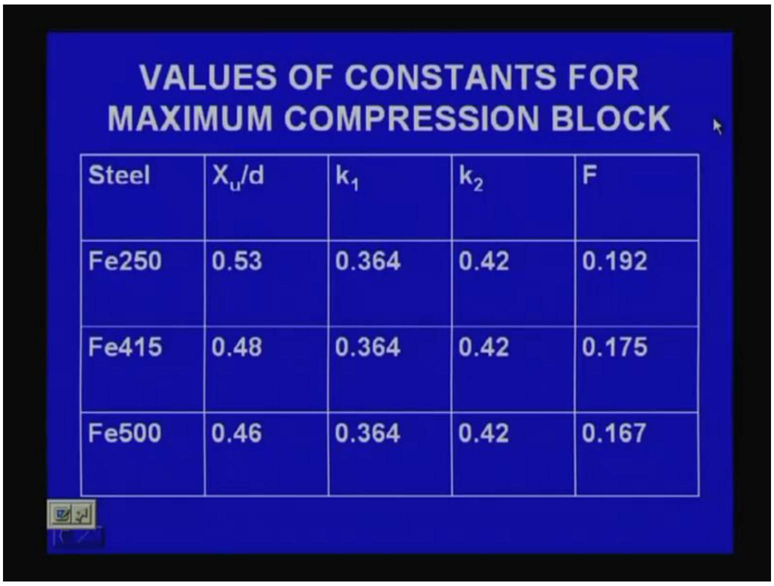

Steel	X_u/d	k_1	k_2	F
Fe250	0.53	0.364	0.42	0.192
Fe415	0.48	0.364	0.42	0.175
Fe500	0.46	0.364	0.42	0.167

So, values of constants for maximum compression block. So, we are considering that is as a block that, 1 straight portion and another 1 parabolic portion we are considering for different steel, that already we have noted down xu by d that we have noted down 0.53 0.48 0.46 k1 that is 0.364 that is constant for steel that k1 value similarly k2 also same 0.42 for all the cases and if that is 0.192 0.175 and 0.167 for different steel we are getting different limiting value. We shall get, these values and we should at least we should note down again 0.175; that at least you should note down the 0.175.

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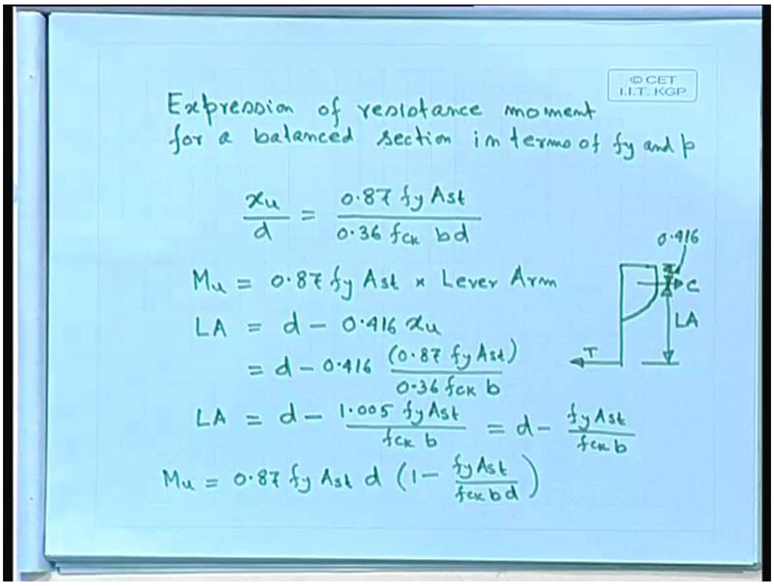

Let us take; we are deriving those things for our limit state method, our objective here. Let us derive that how far we can go and what are the things we require for design purpose. When we are considering say, singly reinforced section and for limit state of collapse for flexure. Expression of resistance moment for a balanced section in terms of fy that is steel and percentage of steel, the steel stress and percentage of steel xu by d equal to 0.87 fy Ast divided by 0.36 fck bd that we have already got it. Mu that limiting moment, that resisting moment, moment of resistance 0.87 fy Ast. This is the tensile force multiplied by the lever arm.

So, lever arm here, equal to d minus we can write down 0.42 or 0.416 also. So, the exact value also we can write down. Let me repeat once more here and some where here that compressive force C, this 1 we are talking 0.416 or 0.42 also we can consider. The lever arm, this is your lever arm lever arm equal to d minus 0.416 0.87 fy Ast divided by 0.36 fck times b; we are getting this xu we are putting it here and we can find out the lever arm. We can write down it as d minus 1.005 times fy Ast divided by fcx b; we can take this one instead of going to that position 1.005 we can simply take it as just 1.

In other way, we can write down this one as d minus fy Ast fcx times b therefore, Mu, will be equal to this is your lever arm therefore, Mu equal to we can write down 0.87 fy Ast times d; we are multiplying the 1 minus fy Ast divided by fck times bd 0.87 fy Ast d times 1 minus fy Ast by fck bd. Mu that is the moment of resistance the resisting moment that is section having area of steel Ast. We can get from this formula we can get that moment of resistance.

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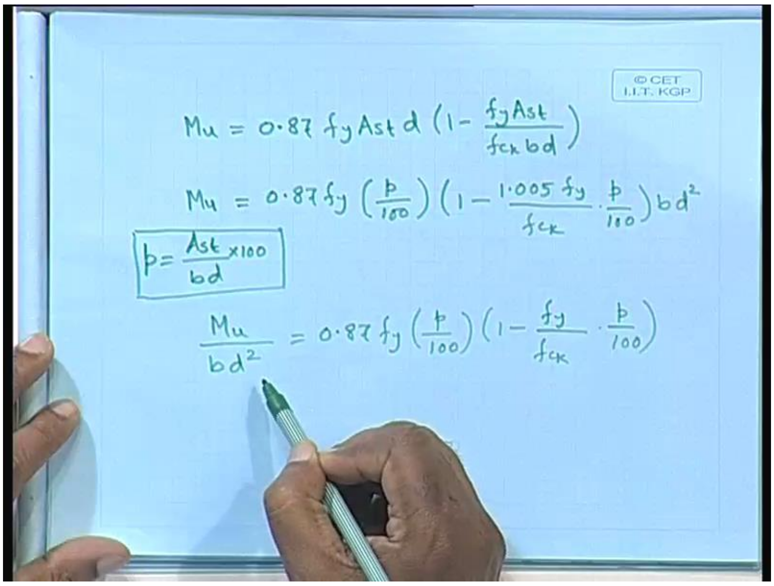

Therefore we can write down it as Mu 0.87 fy let me copy it once more, Mu equal to 0.87 fy p that percentage of steel. I can write down it as Ast by bd into 100; this is our percentage of steel. So, we can write down Mu equal to 0.87 fy Ast we can put it from here. So, we can write down it as p by 100 1 minus 1.005 fy by fck p by 100 bd square. We generally write down in this fashion, Mu by bd square equal to 0.87 fy p by 100 multiplied by 1 minus here, again I can omit this 1.005 ignore that, fy by fck p by 100.

So, for a particular grade of steel and percentage of steel given, we can find out that Mu by bd square. Where we do not need the section size, we can get 1 value; that means, 1 can plot it also for different T also we can plot it also. So, we can get Mu by bd square we can get it. For different percentage of steel, we can get the corresponding Mu by bd square. So, we can plot it also and from here, if we know this value then multiple by bd square, we can get the corresponding moment of resistance that we can find out.

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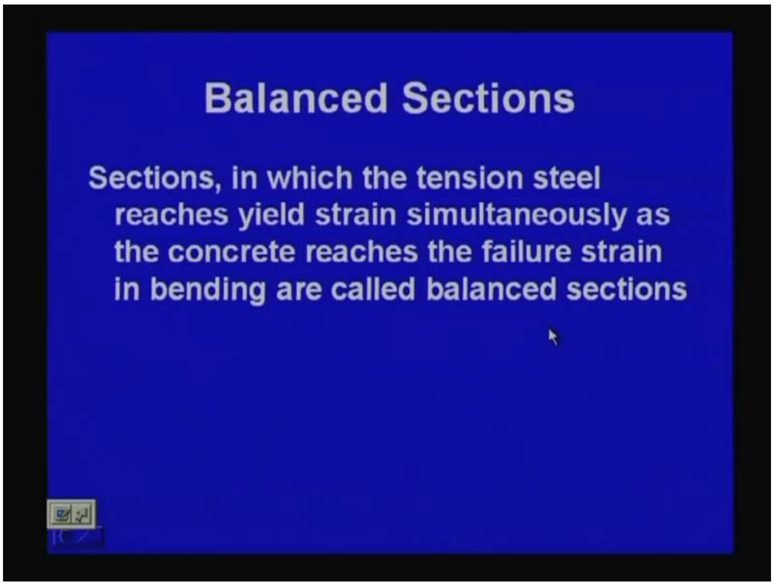

One thing here; I shall tell you, we can also find out for other cases also just to repeat the 1 which we have specified in your say mentioned in the working stress method. That is your say balanced sections. So, sections in which the tension steel reaches yield strength simultaneously; as the concrete reaches the failure strain in bending are called balanced section. Because we are considering beam and that is mainly for bending the dominating one. So, we shall consider that section as balanced section when concrete and steel both of them simultaneously reaching the limiting strength specified limiting strength then we shall take it as balanced section.

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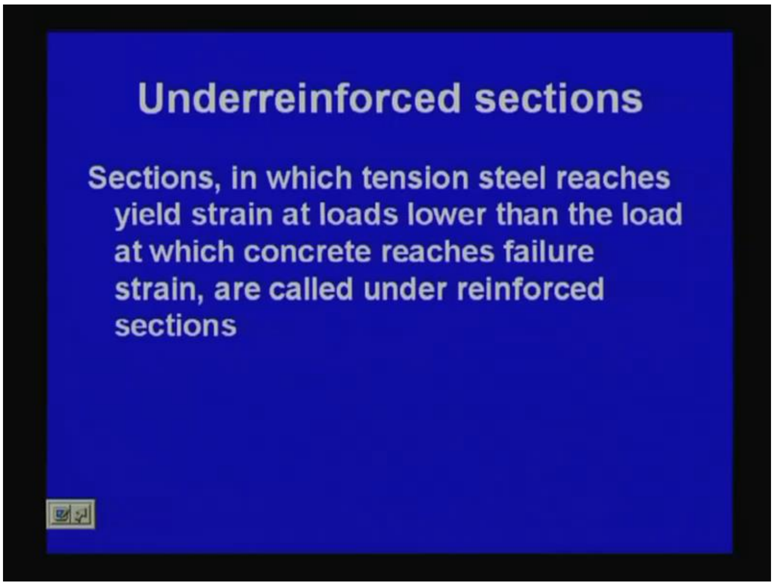

Similarly, we have under reinforced section and which is preferable most of the we always prefer this 1. So, sections in which tension steel reaches yield strain at loads lower than the load at which concrete reaches failure strength are called under reinforced sections. So, in this case the load applied; we can apply the load we shall stop, when we shall stop .When we shall measure the strain, we shall find the strain in the steel that is achieved say 0.0038 for Fe 415. We can measure the strength and we can find out 0.0035. And then we can stop there we shall not consider that concrete may be the concrete steel we have not reached that value. And that one we shall take it as a under reinforced section.

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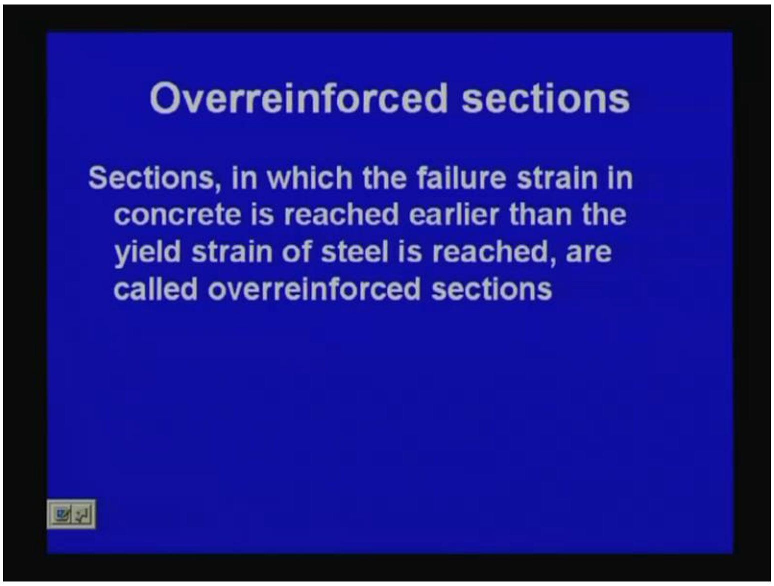

Similarly, we have the other side also. So, sections in which the failure strain in concrete is reached earlier than, the yield strain of steel is reached are called over reinforced section. So, we are having balanced section when we use the strain compared to working stress method we consider that one from the stress point of view, but here, we are considering from the strain point of view. Because we are checking the strain and then from the stress-stain curve we are getting the corresponding stress.

So, this is the basic difference in the working stress method and limit stress method and we are having balanced section, when we have reached simultaneously and; obviously, there are 2 options only either steel reached first or concrete reached first. If steel reached first then under reinforced if concrete reached first then, we shall consider a over reinforcement. And over reinforced section is not preferable because if it is under reinforced section then, it will give it will deform actually it will have some time to deform. It will yield for some time. So, occupants will get some time to that secure that particular says your place.

So, we can say that under reinforced section is preferable. We can also get that one from the limiting xu by d. The xu by d which is given from there also we can find out; the limiting value in no circumstances we should not, that go beyond that xu by d value. It should be always within that limit that xu by d from there also we can check that, whether it is over reinforced or under reinforced whether we are getting that yielding in steel or not that also we can find out.

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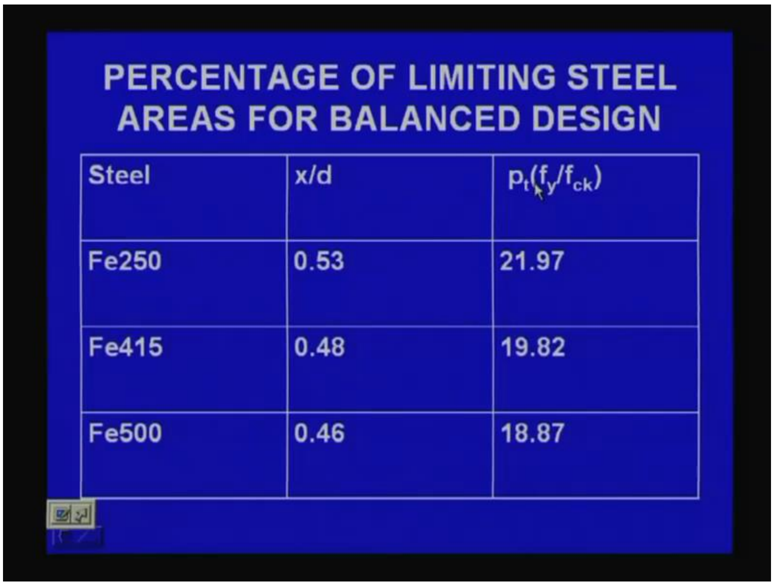
PERCENTAGE OF LIMITING STEEL AREAS FOR BALANCED DESIGN

Steel	x/d	$p_t(f_y/f_{ck})$
Fe250	0.53	21.97
Fe415	0.48	19.82
Fe500	0.46	18.87

Just to conclude this session; we shall say that percentage of limiting steel areas for balanced design. We can find out, the derivation other things I shall give in later stage. We can find out for different steel the xu by d or x by d; whatever, you say and the percentage of steel instead of percentage of steel we have retained this one pt fy by fck. So, that is equal to 21.97. So, from there we can find out, the corresponding value of pt. So, similarly for fe 415 we can also find out similarly for other one also we can find out. So, we shall conclude this particular class here and we shall continue.