Design of Reinforced Concrete Structures

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Lecture - 5
Working Stress Method (Contd..)

In continuation of Working Stress Method, I shall continue today's lecture also on Working Stress Method. So, our today's topic, we shall continue the lecture number 5, that is Working Stress Method.

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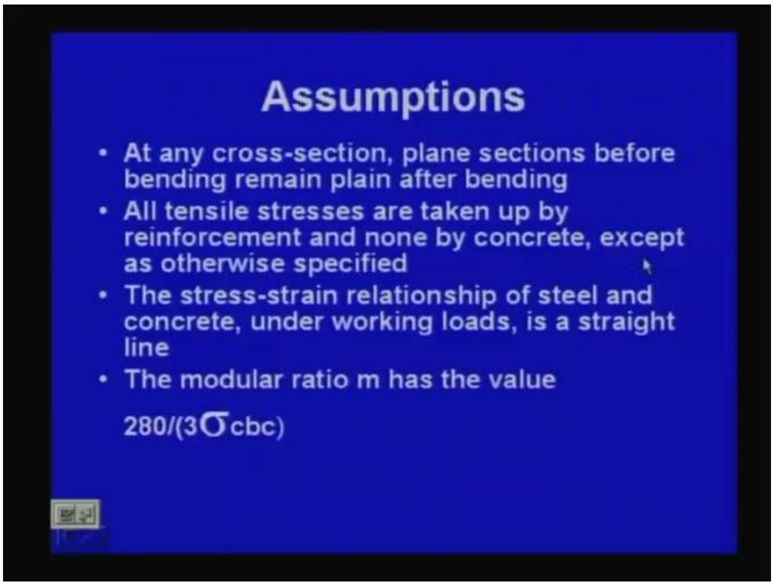

Let me just briefly tell that assumptions; yesterday I told the assumptions. So, let me just enlist it, related to that Working Stress Method. At any cross section, plane sections, before bending remain plane after bending. So, that means, actually your strain, the strain will be linear, that assumption, this assumption will be valid only when the strain is linear.

All tensile stresses are taken up by reinforcement, and none by concrete, except as otherwise specified. In the special case, because concrete can take little bit of say 10 percent, approximately you can say, the tensile case. So, in exceptional case, we can consider that concrete also will take tensile.

So, our next assumption, that in our case, we shall not consider that concrete will take tension, everything will be taken care of by steel only. The stress-strain relationship of steel and concrete under working loads is a straight line. So, this is also another assumption; and these are all assumptions I am talking from IS 456 2000. And the modular ratio m has the value 280 by 3 sigma cbc. Sigma cbc permissible bending stress in compression; sigma cbc that concrete bending stress in compression. So, these are the four assumptions and you can get it in IS 456 also.

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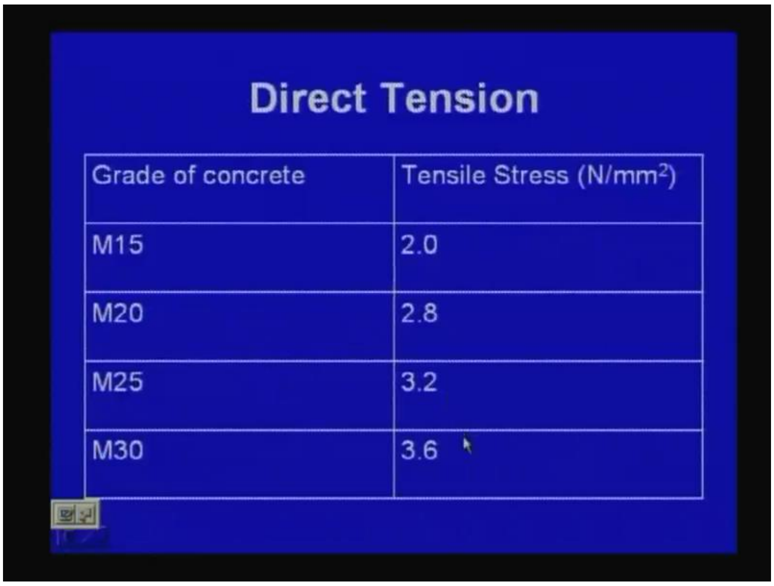

Direct Tension

Grade of concrete	Tensile Stress (N/mm^2)
M15	2.0
M20	2.8
M25	3.2
M30	3.6

I am not going to detail of these, because I have already listed this table; already I have given this table, in the last lecture. So, just to make it in a complete form that I have just written, and you will get in IS 456 also. And I have given only those four genuinely we use it; most of the cases we shall use M 20 and M 25.

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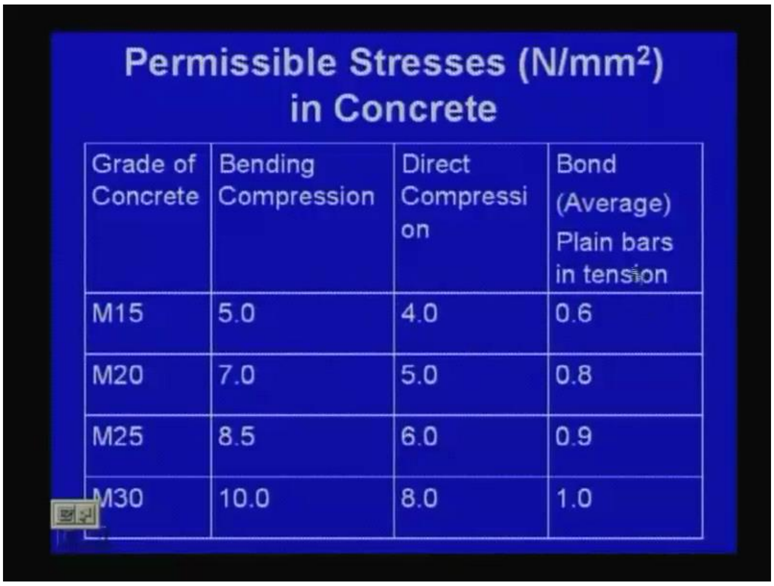
Permissible Stresses (N/mm^2) in Concrete

Grade of Concrete	Bending Compression	Direct Compressi on	Bond (Average) Plain bars in tension
M15	5.0	4.0	0.6
M20	7.0	5.0	0.8
M25	8.5	6.0	0.9
M30	10.0	8.0	1.0

Permissible stresses in concrete. And again we have given four M 15, M 20, M 25, and M 30. And bending compression and direct compression, and bond average plain bars in tension. So, whatever if it is, compression or if it is say your high instant deformed bar, then we have to make some kind of multiplication factor, and then, we can get the required value. That is the standard procedure we do in our calculation.

So, we start with one value, and then, we can multiply with certain factor, and then, we can get the adequate value. That I shall tell you in due course when you shall come across those types of problems.

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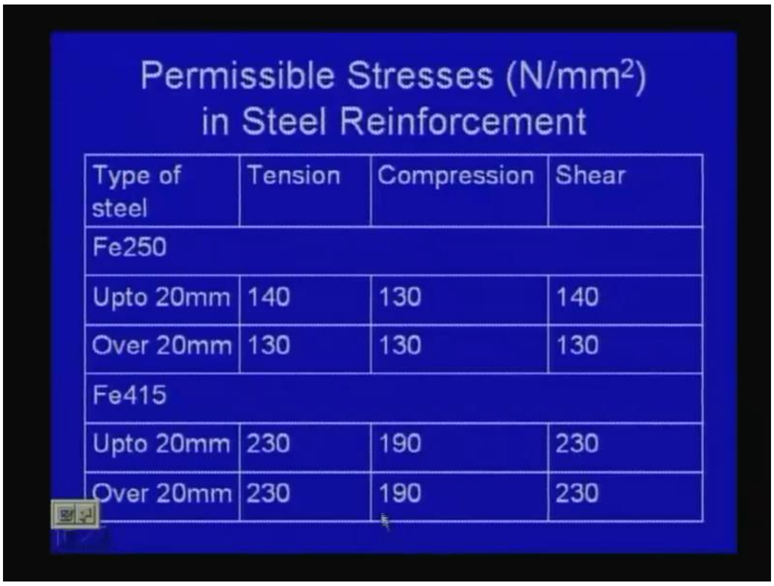

Permissible Stresses (N/mm²) in Steel Reinforcement

Type of steel	Tension	Compression	Shear
Fe250			
Upto 20mm	140	130	140
Over 20mm	130	130	130
Fe415			
Upto 20mm	230	190	230
Over 20mm	230	190	230

And permissible stresses in steel reinforcement. This one also, this table also I have given. And here you will find out that difference, particularly for Fe 250 - mild steel, that up to 20 millimeter that one kind of stress pattern and up; over 20 millimeter diameter we are having another one.

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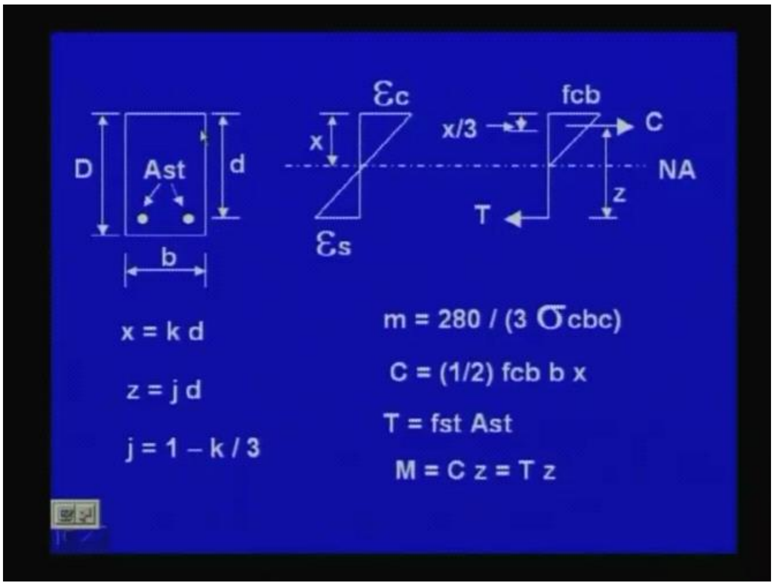

And I have already told that in the last class. And soo, this is your, that section. We are talking, say rectangular section. We can say that t section also possible, l section - inverted l section - is also possible; that we shall do it. Mainly we shall do it in the limit state of method, because that is the method, which we shall consider ,and we should shall use it. So, we are getting that one say width d; depth - overall depth - capital D; effective depth - small d; and area of steel that ast means area of steel in the tensile reinforcement. And this is the screen diagram, and this one, you see that one we are getting at this level; not at the bottom; we can get in the bottom also, but we are not interested; we have interested up to the steel bar level.

And this is your, that neutral access position from the top, that x, and which we specify x equal to kd. So, this is one factor k, which is important, and z equal to jd, which we shall get it that one, say lever arm; this is your lever arm. This is the one tense; compressive force, is the cgo of this triangle, where it is acting, so, c; and tensile, force t in the steel bar; and you will get the lever arm z, which is again specified as z equal to jd and we know that j equal to nothing but 1 minus k by 3. M equal to 280 by 3 sigma cbc, that we have to calculate the modular ratio, instead of calculating from es by ec, instead of calculating from es by ec, we shall calculate it from m equal to 280 by 3 sigma cbc.

We shall calculate from the bending stress, in compression, of that concrete, from there we shall calculate; because we do not what is the value of ec, it is difficult to get, but it easier to get sigma cbc also, because we can get that characteristic strength, then we can do the required factor, and then ,we can get the sigma cbc.

And c equal to, and c, that force in the concrete part, that in compression. So, we have already given half fcb b into x, t equal to fst ast, m could be calculated on the basis of cz - compressive force multiplied by the lever arm. We can calculate the momentum of resistance m or we can calculate on the basis of t.

And, in fact, we do it, we calculate it, that whenever a concrete section is given, that we shall solve one problem today. Whenever a concrete section is given, then we have to find out the moment of resistance, and that moment of resistance, you have to find out we have to calculate moment of resistance due to concrete failure, that if concrete fails, and we have to calculate the moment of resistance, if steel fails. And out of these two, which one will be the minimum, that we have to take, because it can fail before the other one reaches the failure. So, that is the standard procedure we have to do it, and I shall show you one problem today, for that, say how to calculate moment of resistance.

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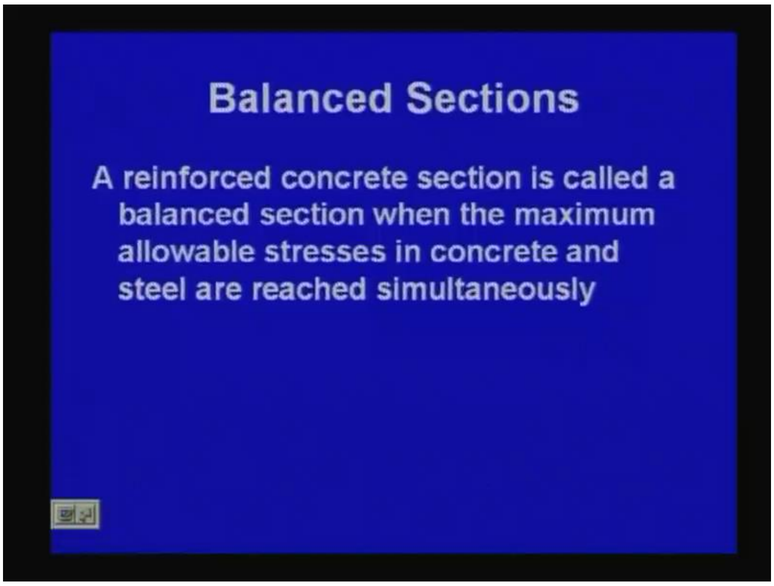

Before going to the calculations, let me tell you that what is a balanced section. Balanced sections, it means, a reinforced concrete section is called a balanced section, when the maximum allowable stresses, that means, sigma cbc in concrete, and sigma st in steel, so when the maximum allowable stresses in concrete and steel are reached simultaneously.

So; obviously, we can say that is the one optimum section, if we can reach say in steel as well as in concrete, same time, then only we can say that we have designed optimum section, and truly speaking, it is not possible. We cannot provide the section, what we can come closer. But generally, we have to do that closer, but we cannot, exactly we cannot make it. If we have to make it, then your depth of the section it will be irregular. The number will not be, say, some regular number. So, number could be something, say, fractional. So, that is not a good choice, because if we go for that one, if you would like to optimize in that aspect, the other aspect is that form work, shuttering. That one also it takes lot of that cost; that it cost much. So, that one also you have to take care and you have to use the form work repeatedly. It is not like that, for each and every beam or column, you will use different formwork; you have to do it in such a way, so that you can use your formwork. Then only you can optimize your cost also. So, that is another aspect.

So, our strength criteria definitely one aspect, but we have to consider other aspect also, other practical difficulty also. So, that is why we compromise with certain kind of strength, if we can reach within 10 percent or 15 percent, that means, it is an… this is an optimum design, that we can say.

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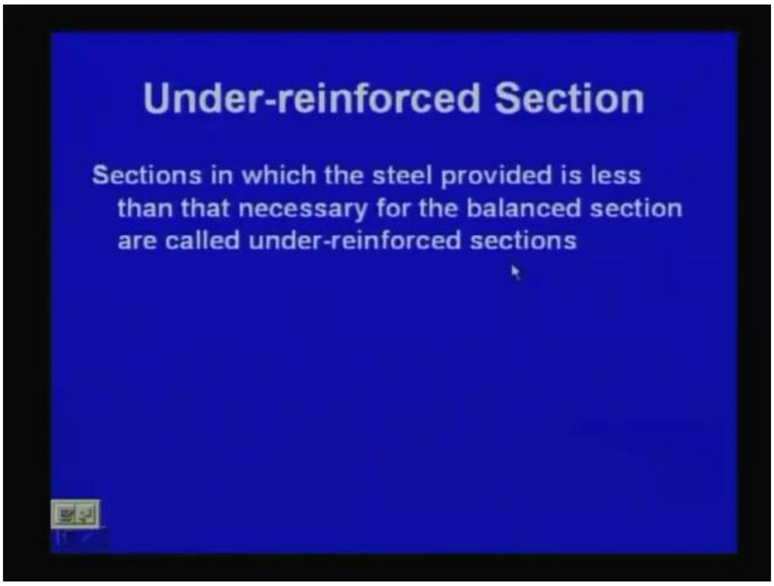

Next one we call it that under-reinforced section. So, that means, when the section is not balanced, so when the section is not balanced, we mean to say that it is also possible that related to steel I can say that, reinforcement steel, either that could be less or that could be more, because balanced is something that which is equal to the concrete strength.

Now, when we are talking, say, steel only. So, steel area could be more or could be less, because we have only two other options. So, if it is, say, more, that is we call it one part; if it is less, then we call it under-reinforced section. So, the under-reinforced section, it means, the sections in which the steel provided is less than that necessary for the balanced section are called under-reinforced section.

So, this is the section, where we are having the reinforcement less than the balanced one. It means that concrete will not fail first, steel will fail first, and that one, we require also particularly forced, that means, when you are having say steel, it will fail fast, it means that you are giving certain time for failure, because steel is ductile material, it will give some time for deformation.

So, the occupants of the particular building will get some time to get away from that; particularly for say earthquake. If you like to design earthquake resistance structure, so it should be better that we should use under-reinforcement section, and also, if we can use mild steel which is more ductile, so we shall get, you will get some time to get away. But generally, opposite it is not possible now, it is to extensively used - the mild steel, but the section, at least it should be under reinforced.

[Conversation between student and Professor – Not Audible]

Because mild steel, we shall come to that next, we shall come, we shall find out, because strength of mild steel is different than strength of, say, your high deformed that HYSD bar. So, obviously, you need more steel, and that is why we prefer that, say we go for HYSD bar.

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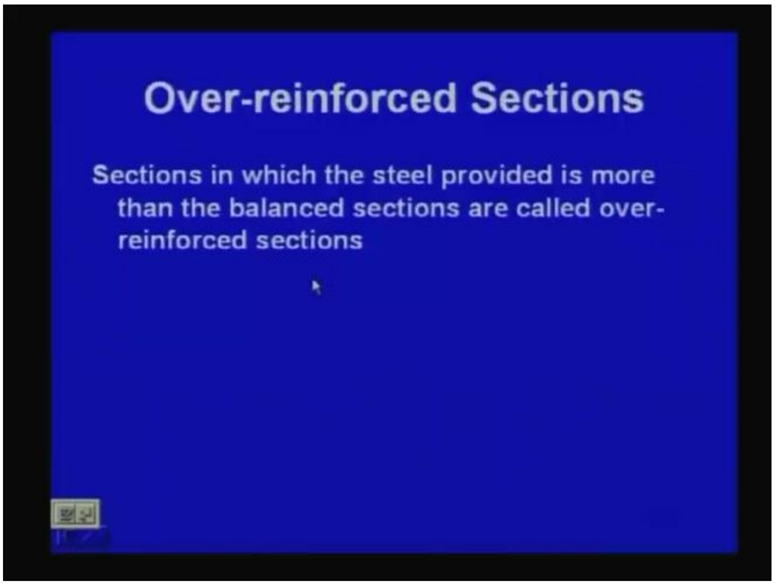

So, the other one is called over-reinforced sections. Sections in which the steel provided is more than the balanced sections, they are called over-reinforced sections. So, we shall consider this one as a over-reinforced section and under reinforced section.

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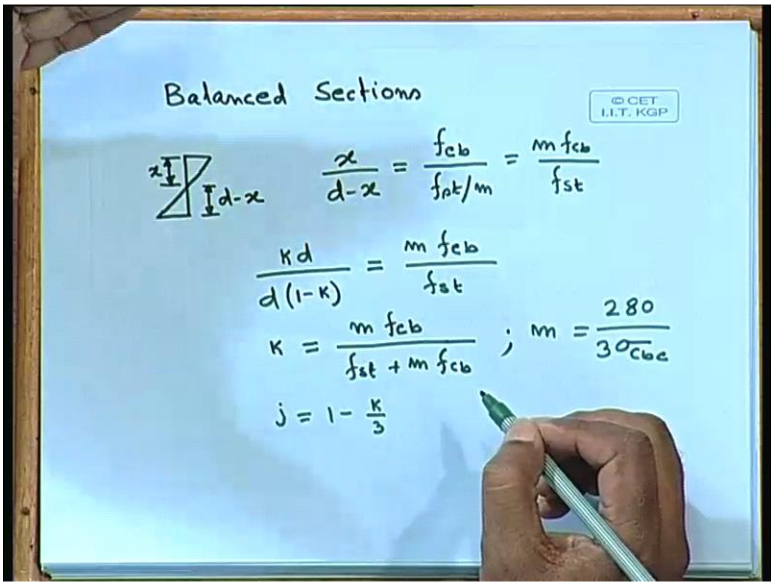

So, now, let us see the balanced sections, because we can find out the steel also. Let me just… I think, I can go back to the previous slides on that, yes. So, this is the one we are interested. So, x and this one d minus x; this is x and this is d minus x. So, now let us find out that or we can write down here also, at least for this page. We are talking this one x and this part - effective depth - d minus x. So, we can write down fcb at the top.

So, we can write down kd; x equal to kd. So, that I am writing here. And mfcb by fst. So, we can write down k equal to, let me make it you, please check it, because we are not going to detail of that calculation. And j equal to 1 minus k by 3. So, for the balanced section, we can find out ka, that k d, so that means, if we know fcb, fst, and fcb, and m; m, obviously, we can find out am equal to 280 by 3 sigma cbc. We should remember this one; and fcb, fcb means sigma cbc; and fst sigma st.

So, for a particular grade of concrete, and for a particular grade of steel, what we can do, we can find out the value of k. And if you know k, then we can find out j also, and if we can find out j, then we can find out the jd, then we can find out moment of resistance, all those things we can calculate. So, here that fcb, fst that one you are limiting permissible values, and that one we shall get it from code; our code gives that permissible stresses for a particular grade of steel, for a particular grade of concrete.

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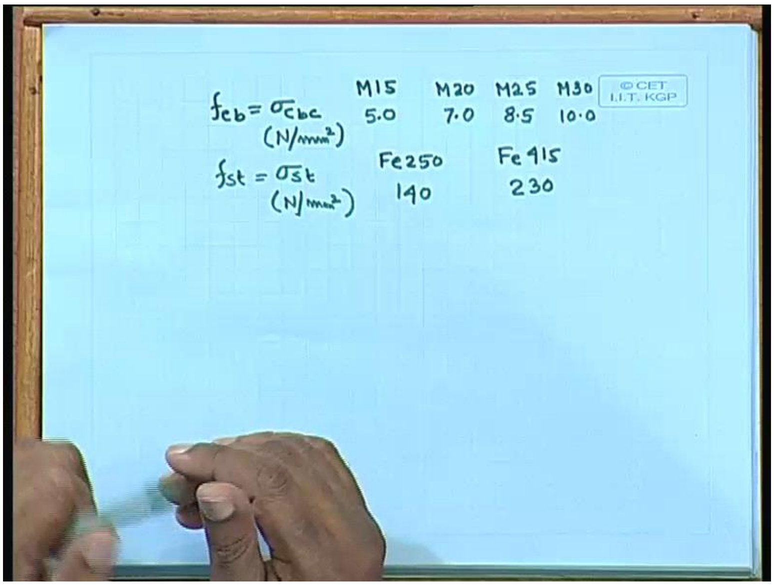

So, we can go; here let us see. So, we can go. Here let us write down fcb equal to sigma cbc, and just make it here for M 15, M 20, M 25, M 30; 5.0, 7.0, 8.5, 10.0. So, these are permissible stresses for different grades of concrete. Fst for balanced section, it should be sigma st. Let me complete this one with giving the proper unit, and this one also Newton per square millimeter. So, for Fe 250, Fe 415, we have 140, we have 230. There is one more grade that is Fe 500; that is one more grade that is Fe 500. And that also, we can use it, but I am giving only those, because you can get those things in your code.

So, in this particular class, if we cover all the each and every numerical value, then it will be difficult to cover all those things. So, we are only using those grade of concrete and steel, which we generally use. And other, say Fe 400 you can get it in IS code. So, IS 456 that also we can find out, and there you can give it in detail.

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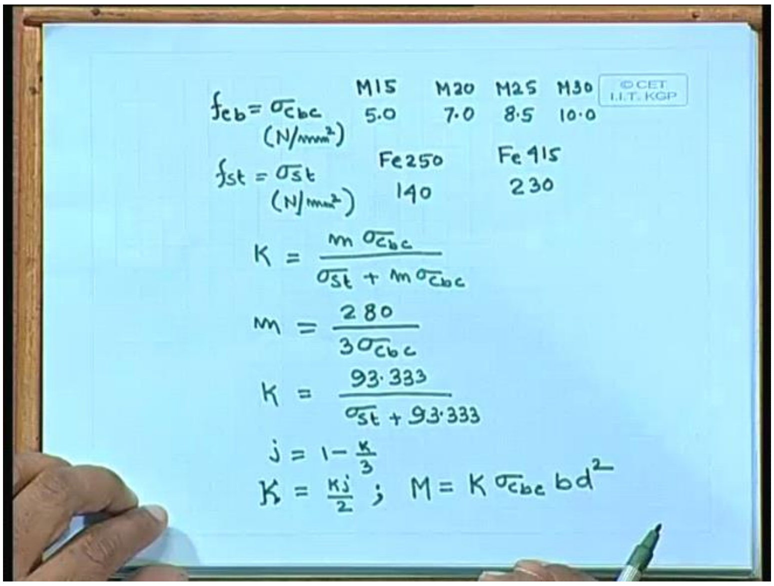

So, k equal to m sigma cbc divided by sigma st plus m sigma cbc. I repeat once more, m equal to 280 by 3 sigma cbc, and k equal to, please note, there is one more k will come, that is capital K. So, but this one for the time being we are only working on say small k - that kd. So, that one comes as… and j equal to 1 minus k by 3; and capital K equal to kj by 2; and moment of resistance m, that one will be equal to capital K sigma cbc bd square. So, m equal to capital K sigma cbc bd square.

This K, that is one factor, if you remember for homogeneous rectangular section, in the last lecture we have found that is 0.167 times the stress times bd square. So, here also, this capital K also, we shall get similar type of one fraction we shall get. So, let me go little bit faster here.

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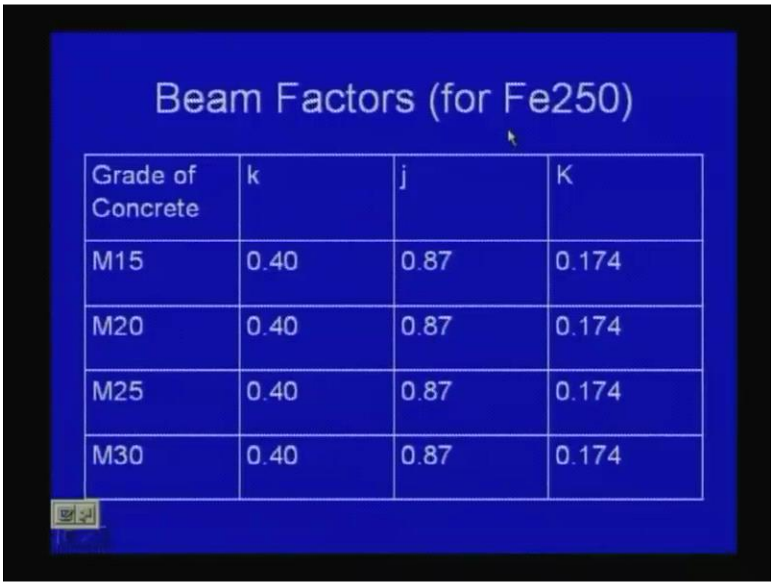

Beam Factors (for Fe250)

Grade of Concrete	k	j	K
M15	0.40	0.87	0.174
M20	0.40	0.87	0.174
M25	0.40	0.87	0.174
M30	0.40	0.87	0.174

So, we can find out that beam factors for Fe 250; we can find out beam factors for Fe 250, that k we can find out for different grades of concrete; k you can find out that all of them having same, because for Fe 250, you please note, this is your that equation. So, in this equation, you will find out it is independent of sigma cbc; it is independent of any concrete value. We are getting only sigma st. So, that is why you will get for any grade of concrete, you will get that k same, and you can calculate j, and you can calculate capital K. So, if you come back to these beam factors table, the small k, which the factor of k d, from where you can get that say 0.4 times small d - that effective depth - so, we shall get that x. J which will give me the lever arm, and capital K - that coefficient - you can get 0.174. So, all of them we are getting 0.174 and this one for mild steel.

So, this table for mild steel, for Fe 250. And I have given only M 15, M 20, M 25, M 30, just to show that for all grades of concrete we are giving the same value.

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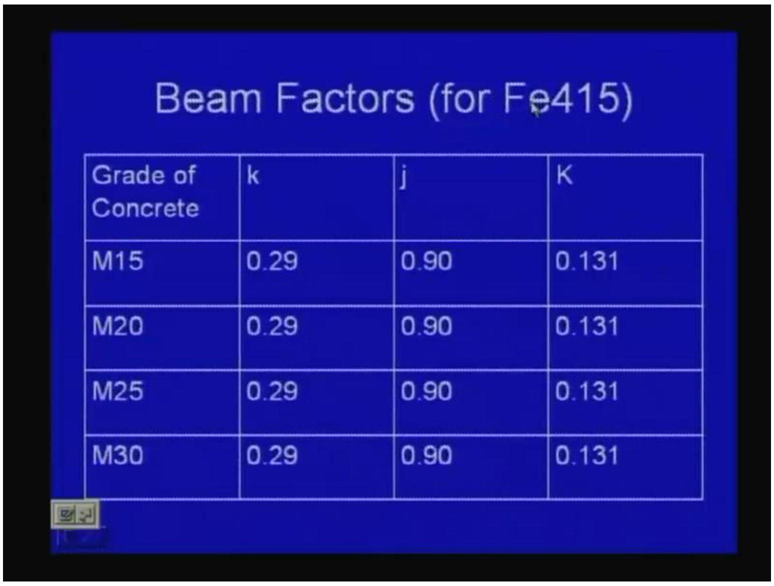

Next one, that is for Fe 415. Fe 415 we are getting that k 0.29, j 0.9, capital K 0.131. So, this value you get it, say 0.167, the one we have told that one, and where we are getting point 0.131; the other one, you are getting 0.174. So, that means, you can approximately, that is not a bad choice. One can also get some kind of result even if you get say homogeneous, but that is not… obviously, that is not a correct one. So, the correct one what we are doing now.

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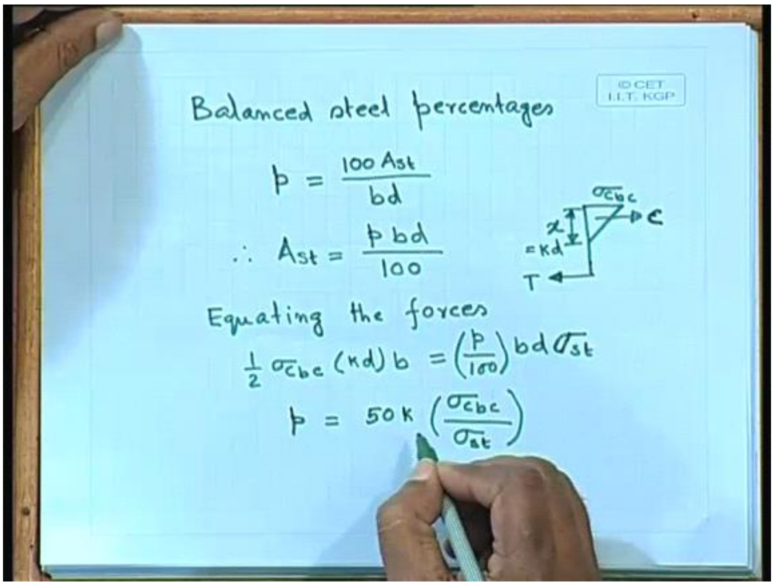

The next one, I shall tell you. So, I can go little further - the balanced steel percentages. So, if we say pt or p, whatever we consider, p equal to say 100 ast by bd. This is the percentage of steel, what we are considering, and so, ast equal to… So, we can equate forces, half sigma cbc times kd times b. This is the compressive force b times x; this the compressive force, b times x, that we are talking.

So, this is the one we are talking, this force - c, and this is sigma cbc; this one x; width of the beam, that is b. So, I can get capital C half sigma cbc kdb; x is nothing but kd; that we can get it, which should be equal to p by 100 - we are talking percentage of steel - p by 100 bd times sigma st. So, this is the area of the steel times sigma st will give me the tensile one. The other one, this one; this is tensile force T.

So, from here what we can get, p will be equal to 50 K times sigma cbc by sigma st. If just multiply, just rearrange it;so p percentage of steel will be equal to 50 times k sigma cbc by sigma st. I know that k. So, I can calculate that balanced percentage of steel. So, I can get the balanced percentage, because this one require some times that you can approximately you can find out also considering the sigma cbc by sigma cbc and sigma st, you find out k, and then, you can, immediately you can find out that p also; that you can find out.

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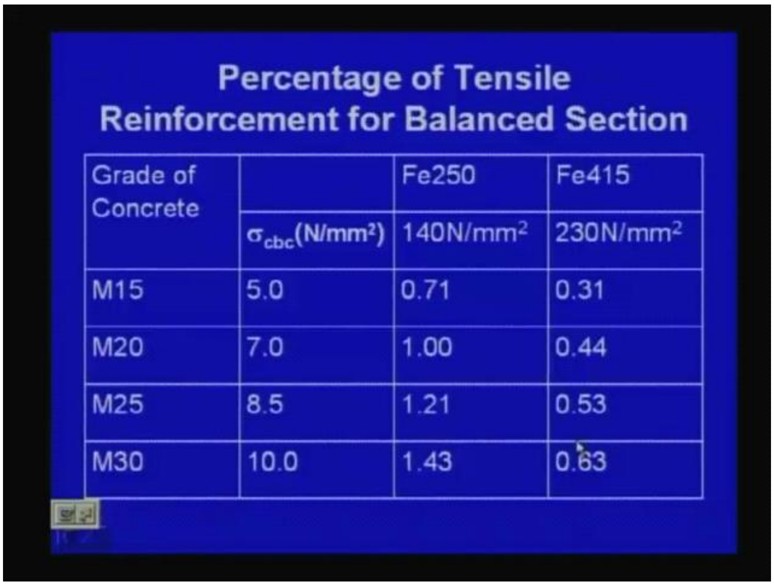

Percentage of Tensile Reinforcement for Balanced Section

Grade of Concrete		Fe250	Fe415
	σ_{cbc}(N/mm²)	140N/mm²	230N/mm²
M15	5.0	0.71	0.31
M20	7.0	1.00	0.44
M25	8.5	1.21	0.53
M30	10.0	1.43	0.63

So, percentage of tensile reinforcement for balanced section, and we can get the grade of concrete, again we have taken four grades only, and sigma cbc, just to give the value, and this is you that Fe 250, 140, 230, we have considered. So, this is your that balanced percentage of steel. So, for M 15, that means, sigma cbc and sigma st, so, .071; we can get for corresponding 0.31; here it is 1, 0.44, 1.21 percentage, where it is 0.53 percentage, almost say even less than 50 percent you can say; the tor steel you need less then 50 percent. So, you can find out that you require less than 50 percent; obviously, that we can… that is why we go for that tor steel. And we can find out that, say even it is less than 0.60; it is 1.43. So, this is your that percentage of tensile reinforcement for balanced section.

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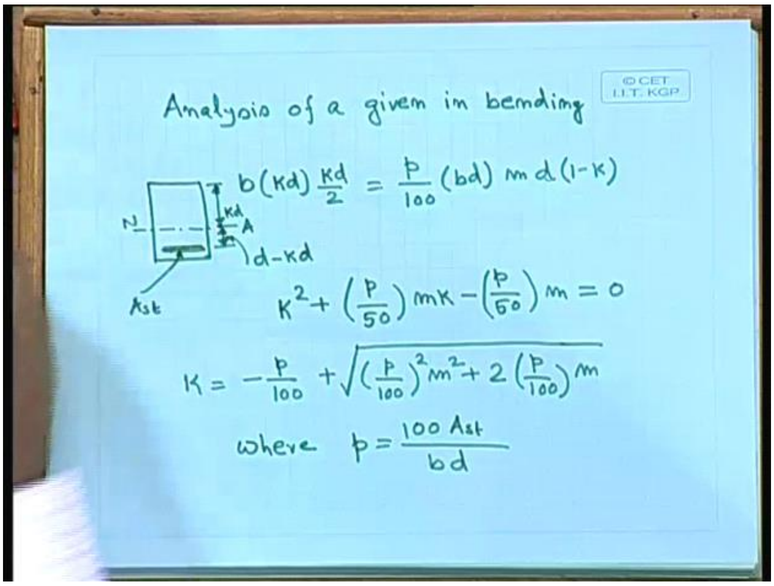

I shall come to the next point. We can find out, that analysis - one we call it design; another one we call it analysis. Analysis means that when the section is given, reinforcement, and the depth, width, everything given, then we call it analysis. Then we are analyzing that how much load it can take; what is moment of resistance.

And design means, that your moment is given, and you have to provide the suitable section; that is called design.

So, here we are doing the analysis. So, analysis of a given section, and we are talking bending only. So, we can do it. We can take the moment about the neutral axis. So, this one we are taking. So, what we are doing? We are considering, say, neutral axis somewhere here.

This part is Kd; area of steel somewhere here; area of steel somewhere here; and that distance d minus Kd. So, this area of steel. So, we can get the area b times Kd times Kd by 2. I am taking the area with respect to their moment of this area, with the respect to these neutral axis, and moment of these area, because we are considering that equivalent section; that is why you have multiplied; this is the area of steel p by - p is the percentage of steel - p by 100 times bd, because we are considering that one - always percentage of steel with respect to the width and times the effective depth.

Please note - width times effective depth; not the overall depth, because there is one more term - that is capital D - which is called overall depth; there is one more term capital D, which is called overall depth, but we are not considering overall depth. So, whenever we shall consider the percentage of steel of the particular section, then we shall take it b times d. So, b times d multiplied by p by 100 will give me the area of steel, multiplied by m is giving me the corresponding, say, equivalent concrete section; that is the… for any composite, say, wood or, say, I think you have done in first year class, wood and, say, steel bar. So, that either you have to make the steel bar, you have to make it say the equivalent, say, wood area, or wood you have to consider it as steel. So, wood will become thinner, but if you make, say, steel convert it to wood, then it will be bigger.

So, here we are considering the same way, and multiplied by d times 1 minus k, that is the distance and we shall get this equation. From there, we can find out K square, and in the last class also we have done it, and K equal to - last class we have told that is a reinforcement index, and here we are talking, say, percentage of steel, and where p equal to… So, we can write down p 100 Ast by bd. And from there, we can find out the corresponding K. That K we can calculate, the periodicity given, then you can calculate that K.

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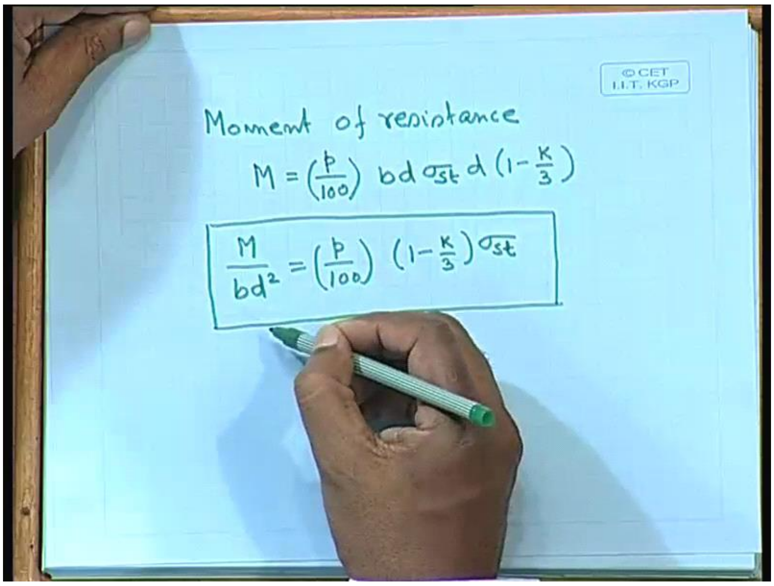

So, we can just repeat, moment of resistance; let me write down, moment of resistance, M equal to p by 100 bd sigma st d 1 minus k by 3. Also, you can write down in this way, m by bd square equal to p by 100 multiplied by 1 minus k by 3 sigma st.

So, generally in sp 16, for in any design aid or you can make your own also, that we can use this equation, which is independent of b and d. We can get a car or make a table, knowing that giving, say p, for a particular grade of steel, for a particular grade of steel we can find out, and for the different percentage of steel, we can find out that corresponding m by bd square. And, then also, we can do it; that means, we can make those things ready, and we can immediately, it will be faster in your design process, because every time you need not either calculator or computer, you need not do it. So, what you can do it in this way, you can make that cards or tabular form, and immediately you can choose, that your section.

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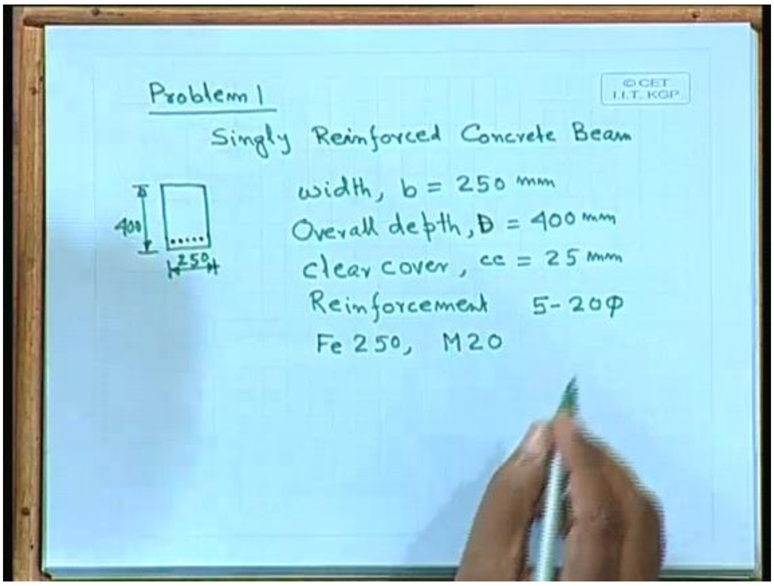

Now, let us do, say, one problem. I hope we shall be able to finish it. Let us describe the problem first. Singly reinforced, please note, I have already mentioned one term - singly reinforced; that means, single; so, that means, there is a term, should be something else, generally we have double. So, that we shall tell afterwards.

So, we are having one beam, width b 250 millimeter, overall depth 400 millimeter. Let us put is as capital D; this is capital D; clear cover we shall consider, say cc 25 millimeter; reinforcement, 525; we designate 5, we specify 5, that means, we are considering mild steel; that is one mode actually there, that I shall tell you in due course; and Fe 250 and M 20 grade of concrete.

So, this the description of the problem. This is 250; overall depth, 400; there are 5 bars; there are 5 bars, 525. So, this is your that problem; we have to solve it. So, sigma Fe 250; height 20 millimeter, sigma st, it is up to 20 millimeter. So, you have to use 140 Newton per square millimeter; not 130 Newton per square millimeter; M 20, sigma cbc.

So, always you should write down first, the values whatever you require, and m equal to… So, we know m, sigma st, sigma cbc.

So, shall I continue with the next step? I think I can change it.

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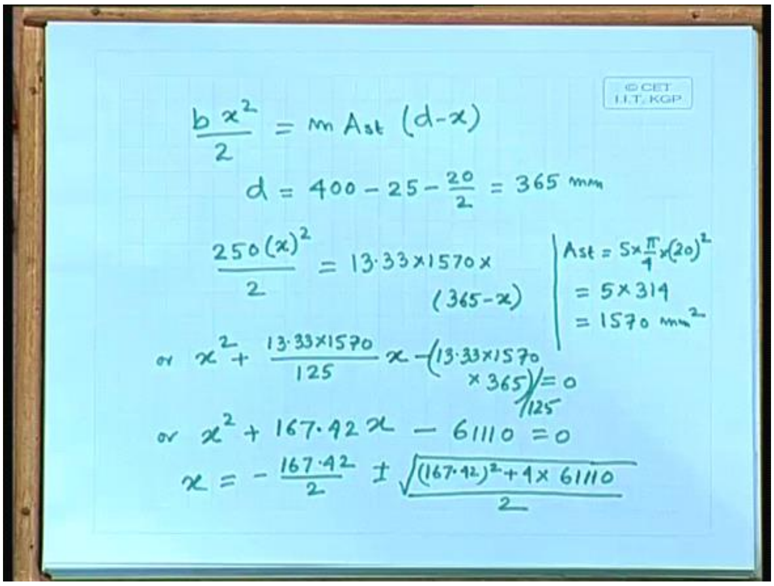

bx square by 2; d equal to 400; minus 25 is the clear cover; 20 by 2, which comes as 365 millimeter. So, 250; m is 13.33; area of steel let us calculate here, 5 pi by 4 20 square equal to… So, 5, I think. So, 1570 square millimeter. Or x square plus 1570 divided by 125 times x minus. So, I think I have to calculate it. Divided by 125. Or x square plus, which is coming as 167.42 x minus 13.33 into 1570 into 365 divided by 125, which is coming as 61110, say that is equal… So, x equal to plus minus divided by 2.

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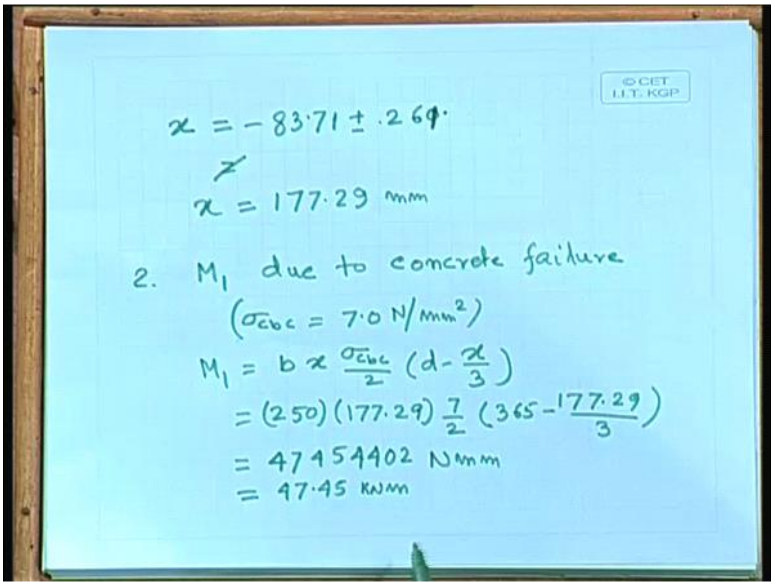

So, which comes as …. Let us little bit elaborate and 167.4226. So, 260 or let us say, 261. So, x will become 261 minus 83.71. So, 177.29 millimeter. So, we can write down M 1, number 2, due to concrete failure sigma cbc equal to 7.0 Newton per square millimeter.

M 1 equal to b x sigma cbc by 2 d minus x by 3 equal to 250 177.29 times 7 by 2 365 minus 177.29 by 3 equal to 5 minus 177.29 by 3. So, 305.9 into 7 by 2 into 250 into 177.29.

So, which comes as 47454402 Newton millimeter which comes as 47.45 kilo Newton meter. We get 47.45 kilo Newton meter; that is due concrete failure. So, we have to get the other one.

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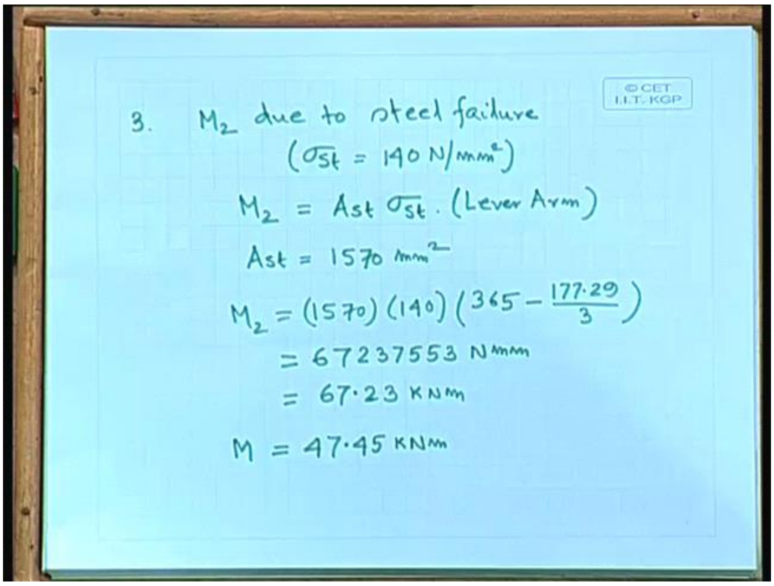

So, we can get it the other one. So, M 2, due to steel failure. Here sigma st equal to 140 Newton per square millimeter. M 2 equal to Ast multiplied by sigma st multiplied by lever arm. Ast equal to 1570 millimeter. M 2 equal to 1570 multiplied by 140 divided by 365 minus - what was the lever arm? That 177.29 by 3, which comes as… So, we are getting 67237553 Newton millimeter equal to 67.23 kilo Newton meter.

We have got M 1, due to concrete 47.45, and here we are getting due to steel 67.23; that means, moment of resistance M will be equal to 47.45 kilo Newton meter. It will be governed by that, your say concrete; it is over-reinforced; that means, it is over-reinforced section.

So, if we go; that means, concrete, that is not a desired, desired choice; that means, you can reduce your enforcement, can reduce your enforcement, and you can find out your, that your optimum section.

[Conversation between student and Professor – Not Audible]

No, no. We generally we avoid that; that over reinforced, because that one also not economic also; the other way, because if the concrete fails first, that means, that is not economic also; I do no whether I have… I have I have nothing.

So, we shall conclude with the Working Stress Method, just to give an idea, and we shall compare with the Limit State Method also, but mainly now on we shall work on Limit State Method. So, in the class, we shall start Limit State Method.

Thank you.