Probability Methods in Civil Engineering

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Lecture No # 08

CDF and Descriptors of Random Variables

Hello there, welcome to the 3 rd lecture of our module 3 and in this lecture, we will cover mainly that,Descriptor of Random Variables and in the last class we stop somewhere at while, we have started description on the CDF, that is Cumulative Distribution Function.So what we will do we will start with that CDF, the description of the CDF and we will see some problems on it, particularly pure CDF and we will also see some example of the mixed variable, where some part is discrete and some part is continuous, we will see one such example then, we will go to some general descriptors of random variable.

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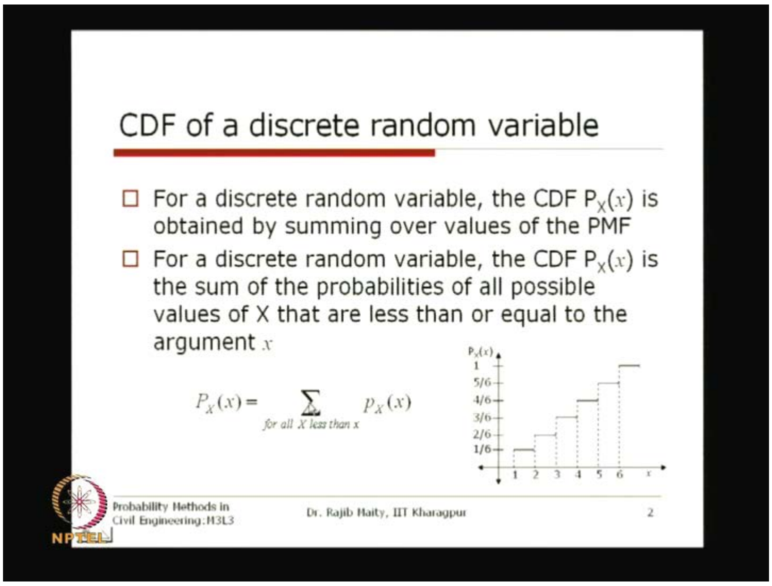

So where we stop in the last class is the description of the CDF for a discrete random variable, we know that there are two types of random variable, one is discrete and another one is continuous and for both this type of variables, we have seen what is theirdensity function, that is probability density function for continuous random variable andprobability mass function fordiscrete random variable. Now, we are starting that CDF that is Cumulative Distribution Function for a discrete random variable, so for a discrete random variable the CDF, that is P X x is obtained by summing over with the values of PMF that is Probability Mass Function.

For a discrete random variable the CDF, P X x is the sum of the probabilities of all possible values of x, that are less than or equal to the argument x. So if this notation, that is P X x stands for the CDF, the value of the CDF at x which is nothing but, the summation of the of their pmf for all possible x which is less thanthissmall x.

Now this is the, so if you take the one standard example of throwing a dice and getting that six different outcomeoutcome1,2 upto 6 and if we say that, all these are equally likely, then the pmf says that at exactly at the point forexactly for the outcome 1 the probabilities is 1 by 6 and for all such outcomes the probabilities 1 by 6, so this isthis is nowconcentrated.

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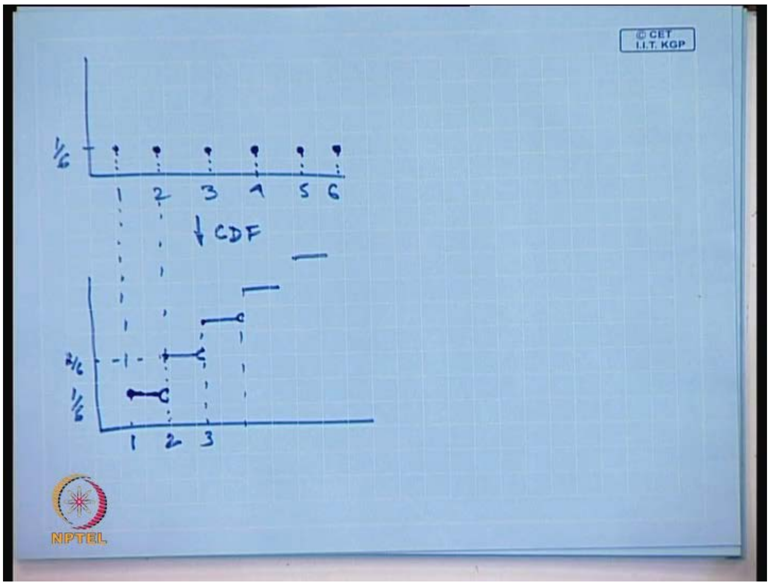

So, in the last one we have seen that its, its pmf looks like this where this 1 2 3 4 5 and 6 and all are the concentrated mass and these are all equal upto 1 by 6, basically this should be the representation of this pmf and in some book we will see there is a stemdiagram or kind of that but, this is as we are telling that, these are concentrated mass of probability,that is why the name pmf, so this dots are sufficient to declare the this is apmf but, just for the reference we can just put one dotted line like this just to indicate that this is referring to that particular outcome, if I want to know what is the CDF for this kind ofthis kind of descriptor then, you see that there is some two important thing, that we shouldwe should keep in mind the first one, that is if I just take this reference line for this one so this is this point is 1 by 6. Now, for ifwant toknow what is the value for this 2 than I know, that for two this will be exactly 2 by 6 or 1 by 3, so this probability when it is exactly equal to exactly equal to 2 then, it comes the probability comes to be 2 by 6.

Now, what about for this line in between some number now now for the for the CDF, I can say thatany number the argument can take any number between 1 and 2 than less than that value say for example, 1.75, so what is the cumulativeprobability up to 1.75 then obviously the probabilities 1 by 6, so the probability that CDF remain constant starting from this 1 and going upto 2.

Now, as soon as it touches 2 it suits to 2 by 6, so generally in the representation we should not touch theline of 2, so this is a continuous line it will simply start from 1 and go as close as it can go up to 2 but, it should not touch 2 immediately, when it touches 2, that is thecumulative probabilityat 2, that is less than equals 2that is why the less than and equal 2,that is why they here the probability distribution function value of the CDF that is sorry the cumulative distribution function value of CDF at 2 it is 2 by 6.

Similarly, so it will be 1 line and it can go as close at as it can up to 3 but, it should not touch the line 3, so here, so like this so basically, we are getting some lines continuous lines a step lines like this and it is going upto 1, so this should be the representation so this is very important that it can go as close as to thenext value but, as soon as it is touching the next value there is a sudden jump of this one so this kind of step 5 function that we can see for afor a discrete random variable, that presentation of representation of cumulative distribution function for afor a discrete random variable.

So in thisfigure you can see there are somethis steps are shown this steps are shown and there is, so thisline cannot touch this line and this dotted lines are nothing just therepresentative just to show that, this point corresponds to exactly equals to 2, so this CDF is represented only by some straight line parallel to thex axis in this six different steps and finally, which is touching to that 1, so this is the representation of the CDF for a discrete random variable.

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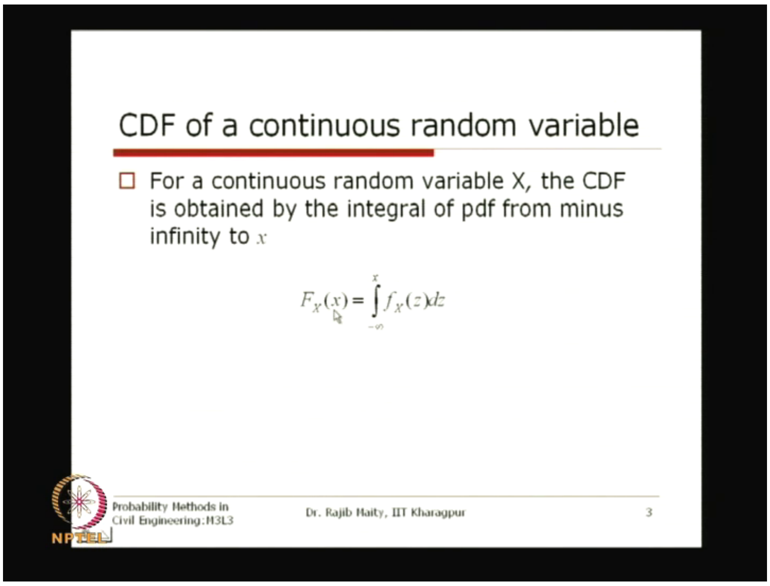

Similarly, if you see the CDF for a continuous random variable, where we know that, thisparticular function that probability density function, that we discuss in last classwhere it is having a having a continuous distribution over the entire support of the x, so if you take that, now if you want to know the value of this CDF at a particular attribute equal to x, that means this F X x that means, the total value up to that particular point we have to seen.

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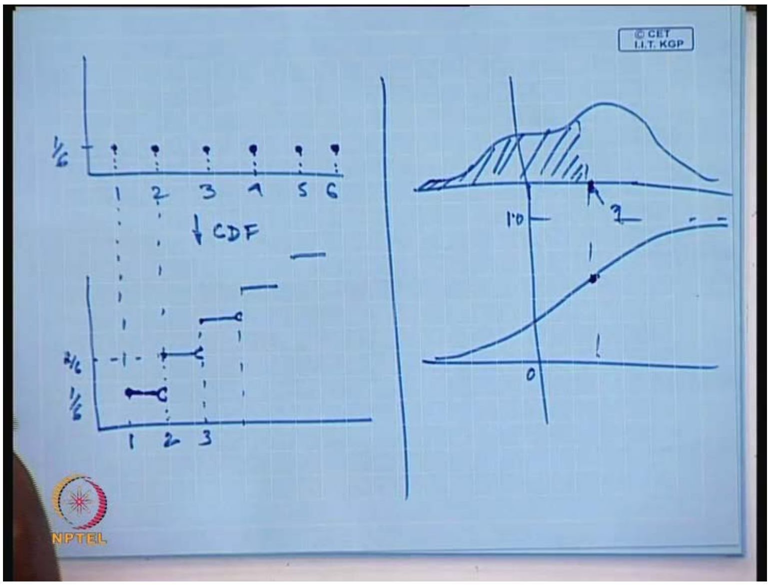

So, here if weif we see that, for a continuousrandom variable this may be the distribution looks like this,now at a particular value here if it is x, then this value if I want to draw that CDF here now, so at this value this point represents the total area from the left support upto that point x; so this value represents this one so this will bea monotonically increasing function, which will go and touch up to the maximum value that it can take is 1 and it should starts from 0.So this value is nothing but, the integration, that means the total area of this function upto that point that means the integration of that function from the left support to that particular point.

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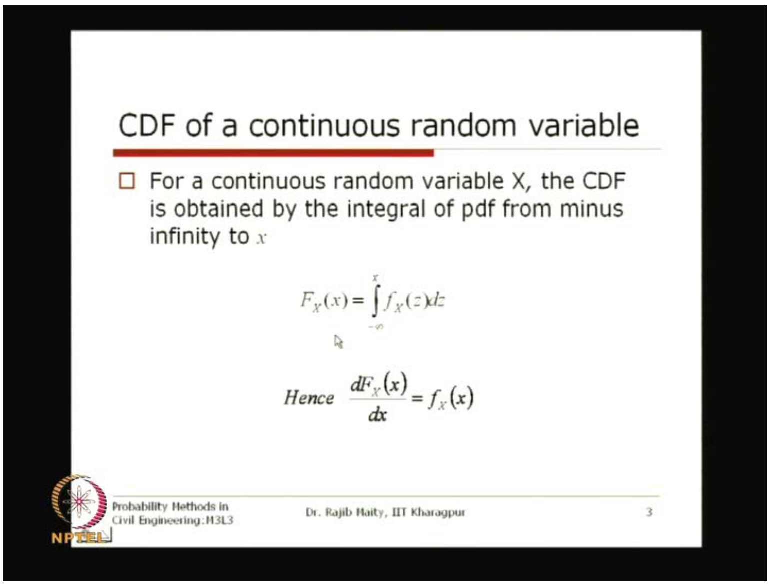

This is exactly mathematically represented here, that you can see that f x equals to from this minus infinity to that particular point integration of that, that is PDF probability density function and this will give you the CDF of a continuous random random variable.

One thing is important here, once again I am repeating the fact that this one will not give you the directprobability, so this is the representation on the probability density at a particular point but, not thenot theprobability but, this function add that point it is representing theprobability of x being less than equals to small x that particular value.

So, if from this one again, mathematically we can if we just take thedifferentiation of this CDF cumulativedistribution function, then we will end up to the probability density function.

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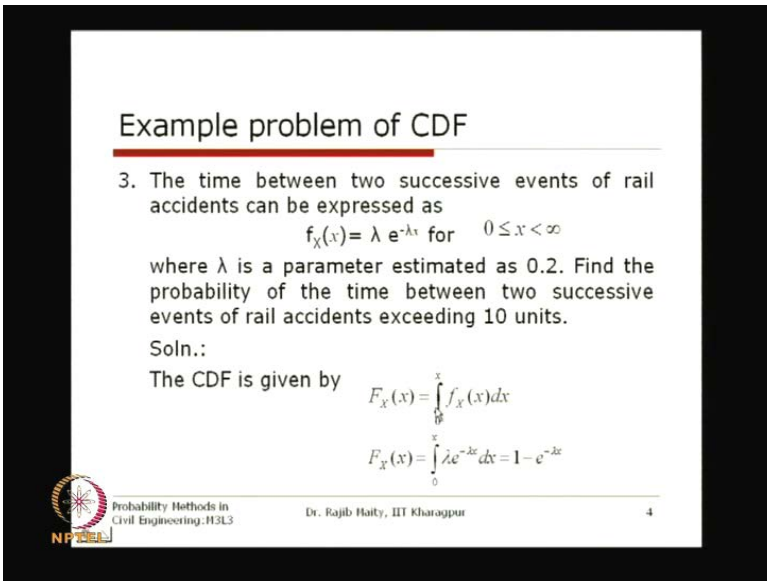

Now we will see two examples of CDF the first one isa straight forward one example of this exponentialdistribution, that time between two successive events ofrailway accidents can be expressed as thisthis probability density function, if F X x equals to lambda e power minus lambda x, which isthe support is declared as from 0 to infinity.

So, wherelambda is a is called the parameter, which is estimated to a 0.2, so these are theparameters it is estimated from we will just see and in the successive class how to estimate the parameters of the distribution, so this with this parameter lambda equals to 0.2 we have to find outwhat is the probability of the time between two successive events of rail accident exceeding 10 units; suppose thatunits suppose that units is not specified or mentioned here so such 10 units, what is the probability that two successive events of rail accidents will exceed 10 units.

So, first of all what we have to do from this probability density function, we can directly get it from this implication, now if we want to know that, what is this probability, what is the cumulative density function of this one,we will know that we have to integrate, this probability density function from this minus infinity to x, so here it is from minus infinity to 0 it isit is 0.So, from 0 to x we will integrate itso to get that so if we do this simple integration we will get this 1 minus e powerminus lambda x which is thecumulative distribution function for thisfor this probability density function, now if we put any particular value of x here, that means we are getting the getting we are directly getting the probability of the random variable x being less than equal to small x.

Now, ourquestion is find the probability of the time between two successive events of the rail accident exceeding 10 units; now it is given that exceeding 10 units that means if I put here 10, that means I will get the probability of non-exceeding 10 units; so if I want to calculate the exceeding than the totalprobability we know 1, so you have to deduct thatparticular value from the total probability to get what is theprobability of exceeding 10 units.

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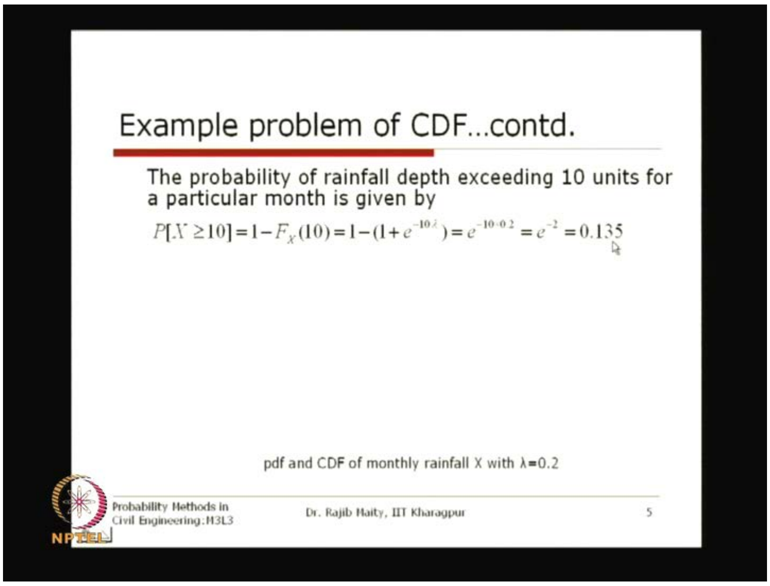

Exactly the same thing has done here, that is probability of the exceeding 10 units is equals to probability x greater than equal to 10, so x greater than equal to 10 is nothing but, 1 minus x less than equals to 10 and x probability x less than equals to 10 is nothing but, F x 10. So this F x 10 we are directly getting from this ourprobability density function, this will be minus means after taking out this bracket this will obviously become plus, sothis is eventually coming to this after putting this value of this lambda this will eventually come e power minus 10 multiplied by 0.2, that is a value of lambda.

So this will be minus if you if you are putting this parenthesis if we take out this parenthesis obviously this will be the plus, so which is shown it here so the probability is 0.1 3 5 so 13.5 percent it isprobability is that, the rail accidentsorry this will be railway accident exceeding 10 units for a particular for that case is 0.135.

Now if you want to see how it is distributed, this show where that probabilitydensity function is shown and this probabilitythat cumulative density function is shown,now if we want to see how it is varying over this x, then this is thegraph.

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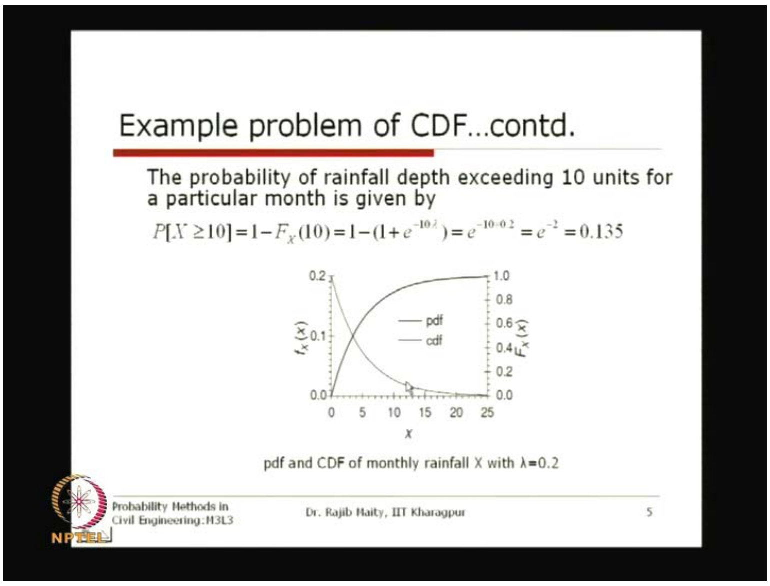

So, you see here so at x equals to 0 the value of these probability density function is 0.2 obviously which is the value of value of lambda and gradually as it is goes to infinity, you seeit is becoming it is coming down and beingasymptotic it being asymptotic to the value 0, now if I want to get that CDF that means this CDF is this one; so that CDF will be nothing but, the integration of this area, so gradually the that integration will increase and its starts from 0 and will be asymptotic to the value value 1 and it will be 1 at x equals to infinity, so this curve that you see, this is your CDF and this one this is your PDF for the exponential.

This is thatwe call an example of the exponential distribution that is showed you in the last class, so these are thethese are how this PDF and CDF for this case looks like.

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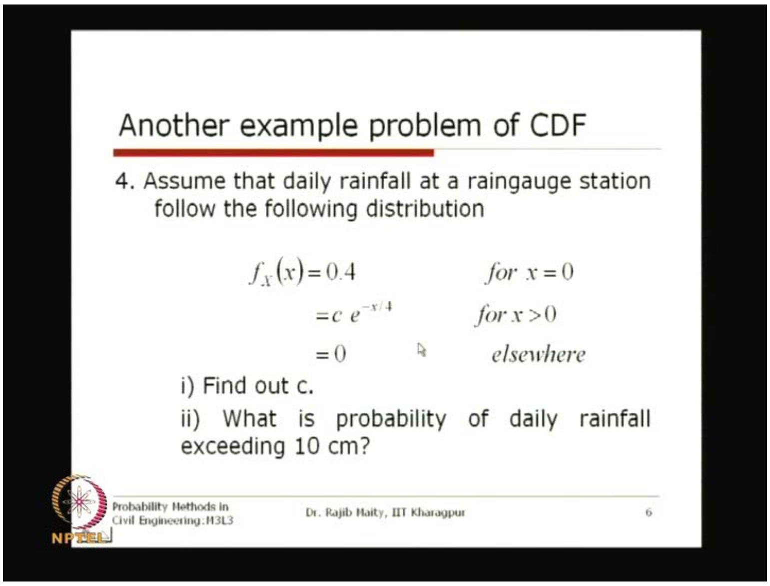

Anotherinteresting problem is taken here; this ison thus daily rainfall at a rain gauge station.Assume that, the daily rainfall at a rain gauge station follow the following distribution now the daily rainfall if you take from a from a particular station than what we see thatmost of the time there will be many 0 values and for some non-0 values it will come so, what is done iswhat is done is first of all you just collect thatreading.

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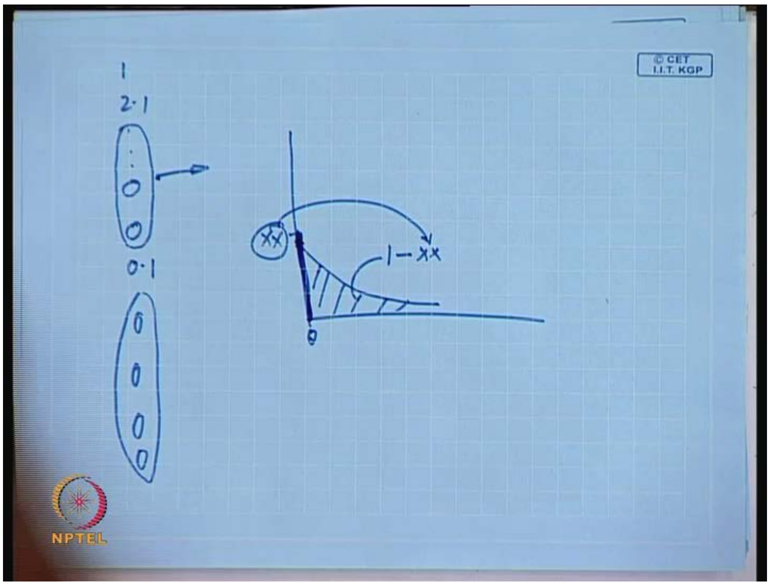

And in thatreading, we gets some non-zero values, so 1 2 point 1 like this,we will get and then we get many 0s again some value 0.1 and so, what is meant is there thisdaily rainfall value that kind of value is the series is having many 0 values, so if you want to calculate, so those probability distribution function what we generally do is that we generally exclude these 0 value first and we see what is the first the probability of getting the value 0 and then we feed the another probability distribution for this non 0 value, this is a this is one example ofone example of the mix distribution where the at 0 there issome some probability is concentrated, so if you see this one so at 0 if it is 0 at 0 some probability is concentrated here and for this non 0 values it may have some distribution.

Obviously, the more thethat the magnitude of rainfall depth obvious the density will come down, so this is one example of, so here some probability is concentrated here and it is coming up. Now, you can see that at 0 if some value is concentrated here so the total area under this graph obviously would be 1 minus what this value what is this value is concentrated as at 0.

So this example, we have just we are considered here and we have just taken one example that this f x equals to here you see that 40 percent probability is concentrated at x equals to 0,now that for x greater than 0 x greater than 0 the CDF c is c e power minus x by 4 and elsewhere it is 0,that is why minus infinity toless than 0 this value is 0; so this the complete description of the CDF and this is the example of the mix distribution, where there is a probabilitymass here concentrated at 0 and a continuous distribution for greater than 0 values.

So first, we have to find out c and then you have doanswer that what is the probability of daily rainfall exceeding 10 centimeter, so this x is having the unit of centimeterhere, so first what we have to do so find out c, so c is a constant here, so if we have to find out the proper value of c that means, the total area under this curve should be equals to 1 that thing we have do we have to satisfy.

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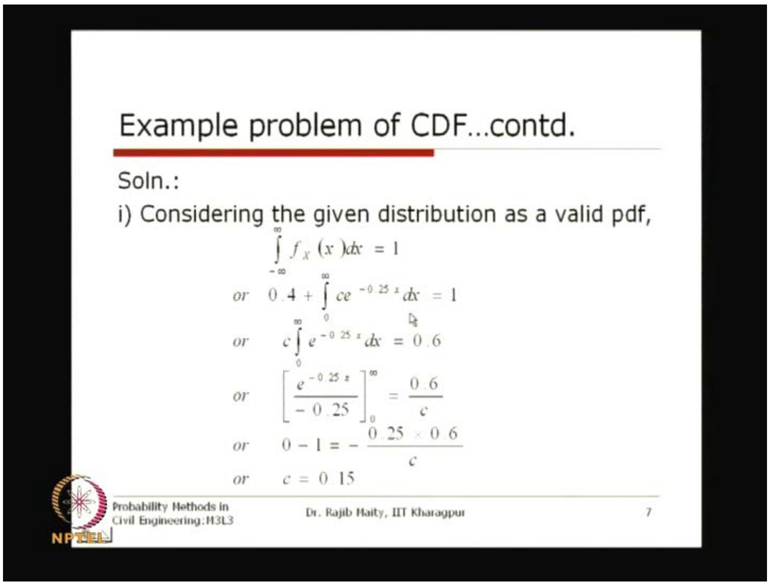

So, it is done it should from the minus infinity plus infinity, from thesecond axiom of probability we know that this totalso the condition of this PDF that this total integration should be equals to 1, now I know that, from minus infinity to less than 0 the value obviously 0, for at 0 the concentrated probability mass is 0.4 and from 0 to infinity thisthat function we know and this should be equals to 0. So if you just if you just do this integration and solve this equation for this one unknown here, so we will after doing some step you will get that c equals to 0.15, so the value of c we got so, the complete description of this probability distribution will be F X x equals to 0.4 for xequalsto 0 is equals to 0.1 5 e power minus x by 4 for x greater than 0 and 0 elsewhere.

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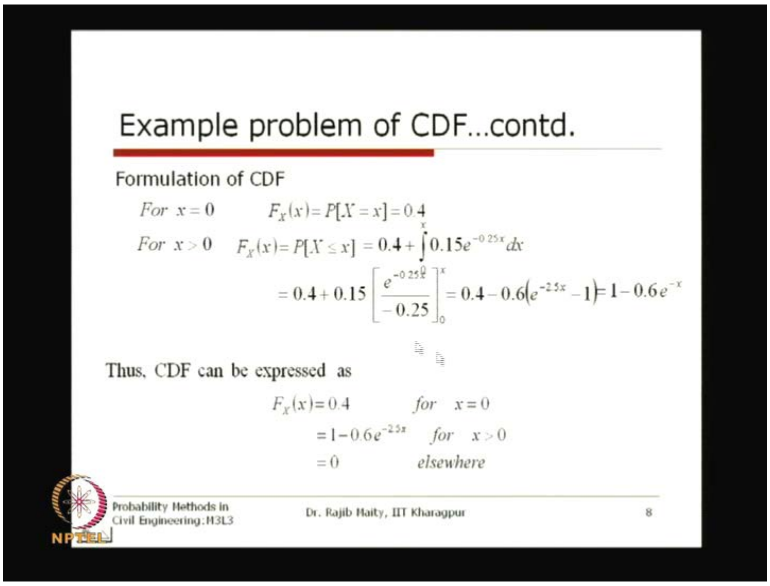

Sohere it is written that for x equals to 0 that, now if I want to get the CDF that cumulative distribution function for x equals to 0, this F X x that is x exactly equals to 0 equals to 0.4, now for this range that is x greater than 0, F X x which is nothing but, the probability x less than equals to 0 it should be equals to that concentrated mass at x equals to 0.4 plus the integration from 0, to that particular point x of this function.

So if you do this few step we will get the distribution function as that for this x greater than 0 in this zoneis 1 minus point 6 e power minus x, now the once you get this CDF then the rest of whatever theanswers we are looking for that is for example, herewe are looking the answer for this x greater than 10 centimeter,so this is the final representation of this CDF that is f x equals to 0.4 for x equals to 0 1 minus 0.6 e power 2.5 x is greater than 0 and 0 elsewhere, so this will be 0.2 5 x so instead of minus x this will be 0.2 5 x is the correction here needed.

So this, with this final form of the CDF that we got now we will put that x is xequals to 10 to get that the probability here it is asking what is probability that rainfall again here, it is shown that exceeding 10 centimeters so we will firstcalculate the probability from this CDF what is thatprobability of this daily rainfall less than equals to 10 centimeter, thanfrom total probability one if you just deduct we will get that answer.

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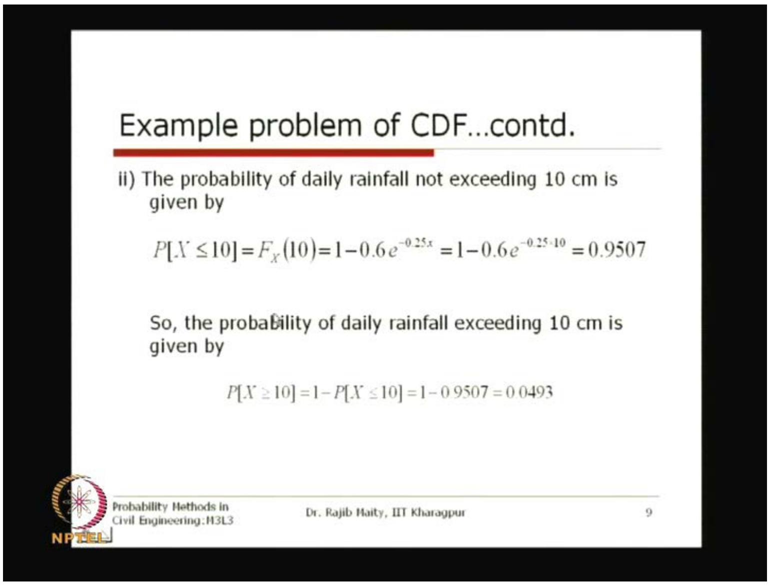

So, from this 1 if you just put thatx less than equals to 10, that is f x equals to 10 which is nothing but, 1 minus this equation 0.9507 we will get from this 1, so the probability of daily rainfall exceeding 10 centimeters, obviously will be greater than 1, so 1 minus probability x less than 10 is equals to 0.0493 is thisah probability.

So, up to this what we have seen is that, we have just seen that PMF for which is for the discrete random variable, that is probability massfunction than we have seen that probability density function, which is for the continuous random variable and last we discuss about thiscumulative distribution function for both discrete and continuous random variables.

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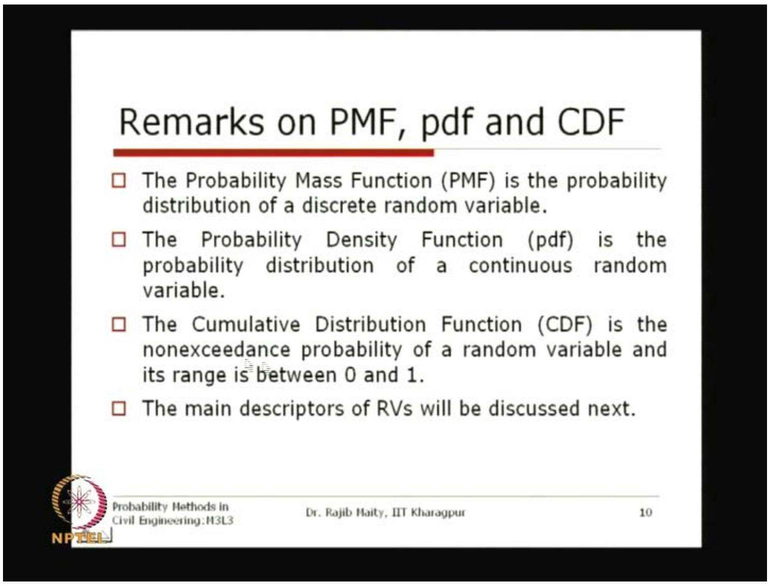

So, some notes here on this the probability mass function PMF is the probability distributionof a discrete random variable, the probability density function PDF is the probability distribution of a continuous random random variable, Cumulative distribution function is the non-exceedance probability non-exceedance probability of a random variable and its range is between 0 and 1.

So now the main descriptors,now sometimes if we do not get anya particularclose form of that close form of this probability density function or probability mass function of a variable, then from the observed data there are some descriptors of this random variable that we will see, so that approximately with this descriptorsa random variable the nature of random variable can be known to us so this is our next focus to know that, what are the different main descriptors of a random variable.

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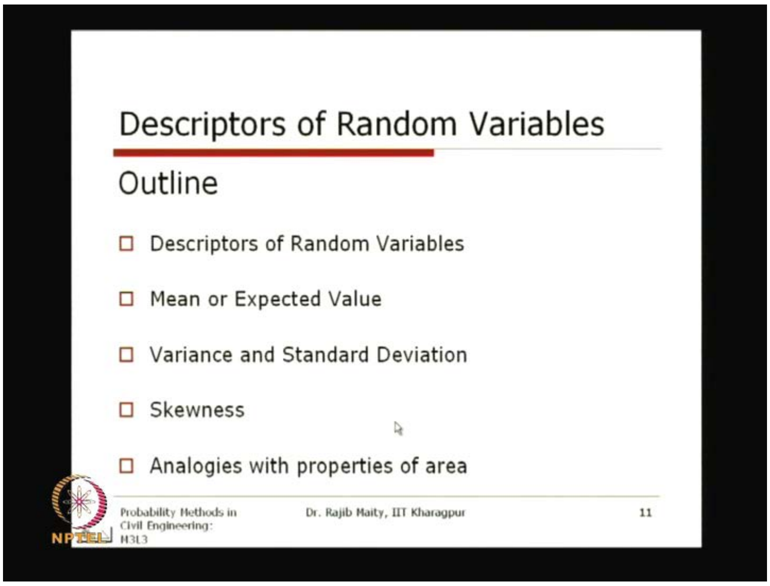

So what we will see that, what is meant by this descriptors of random variables that we will see first, then we will know what is mean or the expected value, then varianceor the variance and standard deviation., then skewness, then one where is there for that called called kurtosis, and then we will also see the analogies with the properties of the area, that is area under the PDF probability density function in terms of incase of the continuous random variable, that we will see just to rely that how this how this thing can be call as a moment that we willthat we will see.

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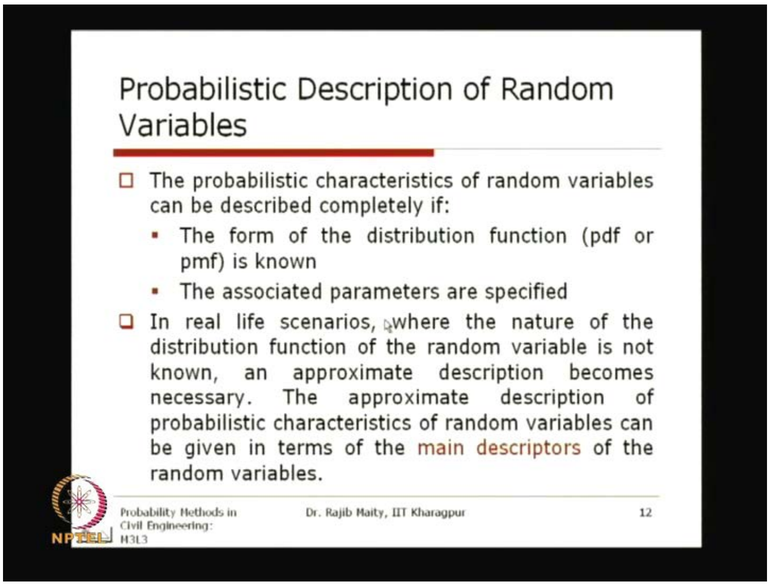

So, probabilistic description of random variable, if you want to see take somesome random observation of a random variables, some random sample data if we take take then we will we can see these thing, that is the probabilistic characteristics of a random variable can be described completely, if the form of the distribution function PDF or PMF is known obviously what we discuss so far. And the associated parameters are specified for example, that afor the exponential distribution, that lambda e power minus lambda x so that lambda is the parameter, so if you know that what is that density function PDF that is it is exponential form and if you know the value of lambda then it is completely known to us, that the total description is given to us; so this is what isits meant by this point.

Now, in the real life scenarios where the nature of the distribution function of the random variable is not known, an approximate description becomes necessary, the approximate description of the probabilistic characteristicsof the random variable can be given in terms of the main descriptors of therandom variables, so this is why so if we do not know the exactly the close form of this equation this is why this main descriptors becomes very important, just to know the nature of thedistributionof that particular random variable.

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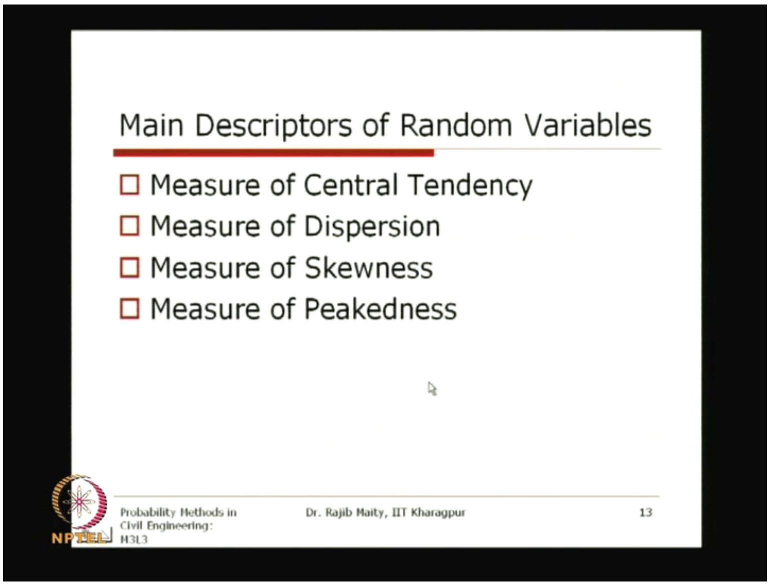

So, there arethe first four description, that we that are very important and obviouslythere are higher side but, first four is very is important and mainly is residential for that nature of this distribution the first one is the measure of central tendency, these measure of central tendency means; where the centre is, now the centre is again a subject of importance in what sense we are looking for thecentre, so a given a random variable given the distribution, where it is it iscentre is, that can be there are three different ways, that you can say that, what is the it is central tendency. Second thing is the measure of dispersion, how so about, that central point, how the distribution is disperses, how it isspread around thatparticular central value.

Then measure of skewness, whether there is any skewed, whether it is skewed, whether it is symmetric, so this is a measure of symmetry, you can say that whether it is symmetrical or has some skewnessto either to the left or right of this central point and measure of peakedness, so whether the of peak of that, distribution is very high or low like that so you will go one after another, we will start with this measure of central tendency.

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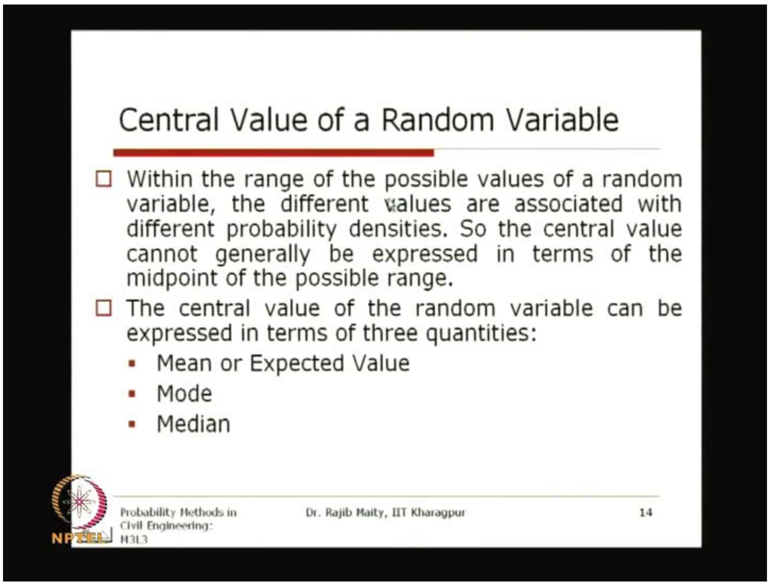

The central value of a random variable, so within the range of the possible values of a random variable the different values are associated with differentprobabilitydifferentprobability density, so the central value cannot generally be expressedin terms of the midpoint of the possible range.

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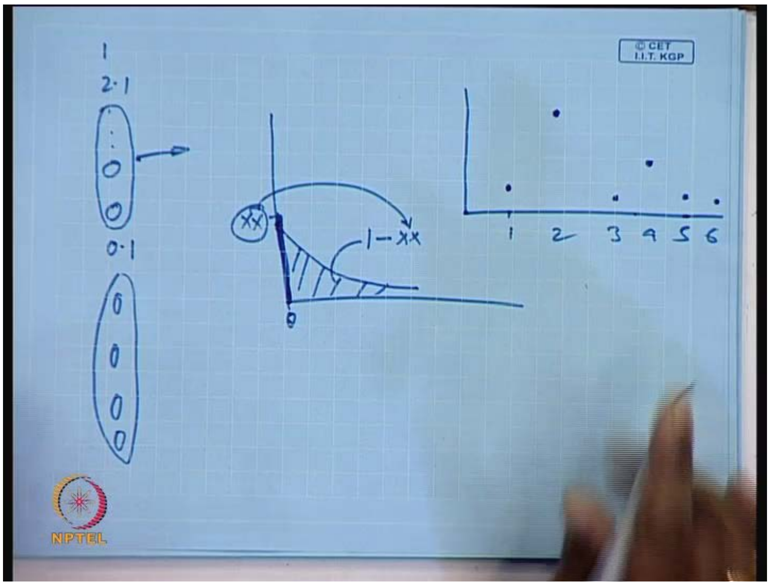

So just just for a just for a smallthing if we just say, that if we if we say thatif you just take that particular example, say if I take the example of the discrete random variable first for the dice, we have seen that 1 2 3 4 5 and 6 and if you see the all these things are equation probable, then we can say the centralist somewhere around 3.5, the outcomecentral tendency is 3.5.

Now, think in the situation, where this out comes or notequally likely, so the mass is concentrated 1 is from for here, for 2 it is here, 3 it is here, 4 it is here; so if it is like this then it may not be it may may not be that that thatmidpoint of this outcome may not be the central tendency of this 1; obviously when I am putting this dot, obviously I have keptin mind, that the summation of all theseshould be close to one those exemptions what I mean is that obviouslyafter satisfyingall the axiom, that is needed before I can declare that this is a valid PMF, so if thewhat here it is it is it isunderstoodthere is central tendency is that, the central tendency need not be always (( ))the to the midpoint of the observation that we see, it depends on this how much density is associated to each each outcome of that particular random variable, so this is what is meant for this central tendency.

And there are three different three different ways, how we can how we can say, how you can express that the first the central tendency, central value of the random variable can be expressed in terms of three quantities.

The firstis the mean or expected value, second is mode, and third one is median.

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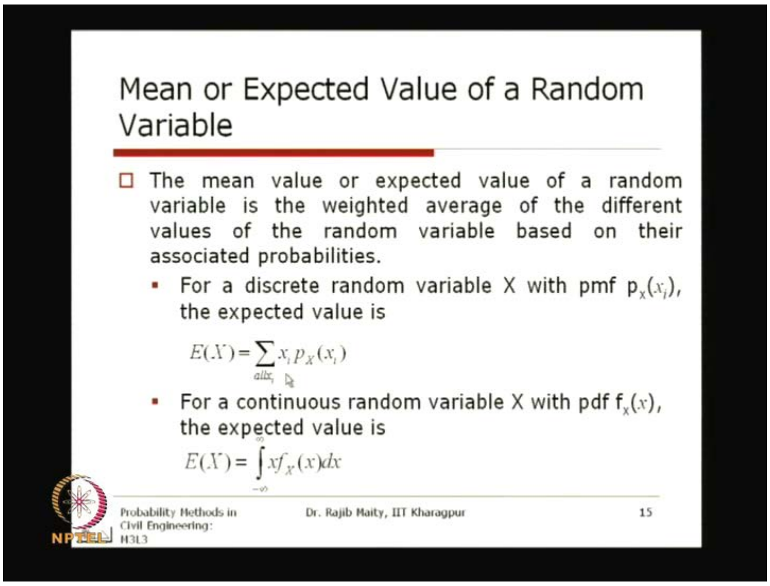

So we willwe will see the meanfirstwhich is expected value as well of the random variable, the mean value or the expected value of a random variable is the weighted average of the different values of the random variable based on their associated associated probabilities, for a discrete random variable x with PMF, P X x i the expected value isE x is equals to x i multiplied by that associated probability.So instead of just simply simply calling that, thisthe average of this, so the average of this outcome where what we are doing is that we are just taking the weighted sum, the weightedaverage the average is taken by the weight age of the associated probability so this is the expected value.

Now, instead of multiplying the same values here, that is the same values are1 1 by 6 for each and everyoutcome, I am multiplying the each out come by their by theirweighted probability, so this may be something but, the when I am putting the weight for these two, this is obviouslyvery high, so obviously, this the central tendencywill have the, will pull that central part towards this particular observation, because here the probability the concentration the mass is very high compared to the other outcome.

Now, taking this same thing, same concept for this continuous random variable, that for that continuous random variable x with PDF, F X x the expected value is that for the full range; full range of this random for the full support of the random variable x that multiplied by x gives you that particularparticularexpected value of that random variable x,we will just discussing a minute, how this is related to somesomekind of moment we will just come in that point a little later.

So, as we have seen that, forif we if when we are talking about the expected value of that particular random variable, then we are multiplying then we are multiplying that variable with that particularvalueonly with that x or here the outcome the x; now the expected value instead ofyou can, we can get the expected value of any function of the random variable, the function of the random variable will be discussed in subsequent classes but, here instead of only it is what we'retrying to say here is that, so that expected value can be obtained for any other functions of thatof that random variable as well simply by replacing this x by that function and this here x by that particular function.

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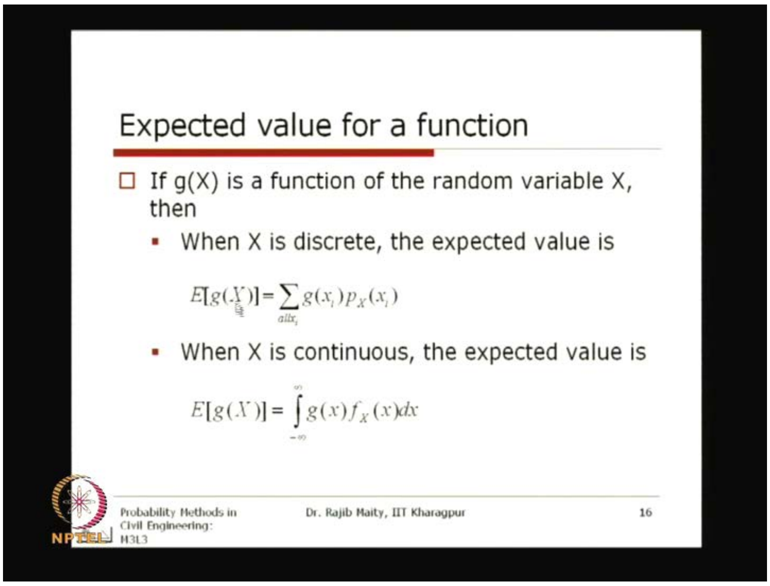

So here,that is the g x here the g x is the function the random variable xthen when x is discrete obviously we have to go for this summation multiplying that g x with that individual masses, the probability mass for that particular outcome and when X is continuous then we are taking the integration instead of multiplying only by E X by multiplying by this function that is g x into F x x d x, so we get that expected value of that random variable of the function of the random variable g x.

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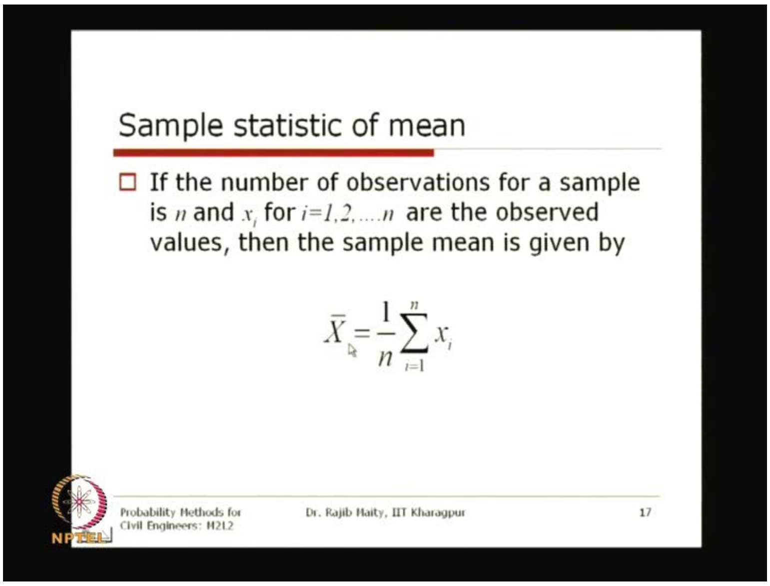

Now, there are so, if we know that probability density function then we can calculate thatthat mean, now if we have some observations then from the observations if we want to get what is that sample estimate, now there are differentdifferent criteria before I declare that, this is a this is a particular estimated of that particular variable, that will be discussed later those are known as that consistencyunbiasedness.So this thing after satisfying those things if we have the different observation for a particular random variable, then the mean of that particular random variable can be expressed as this where this n is the totalnumber of observation is taken for the random variable, so sum it up and then divided by this total number of observation, that get what we get that is the mean of that the sample mean of that observation of that sample.

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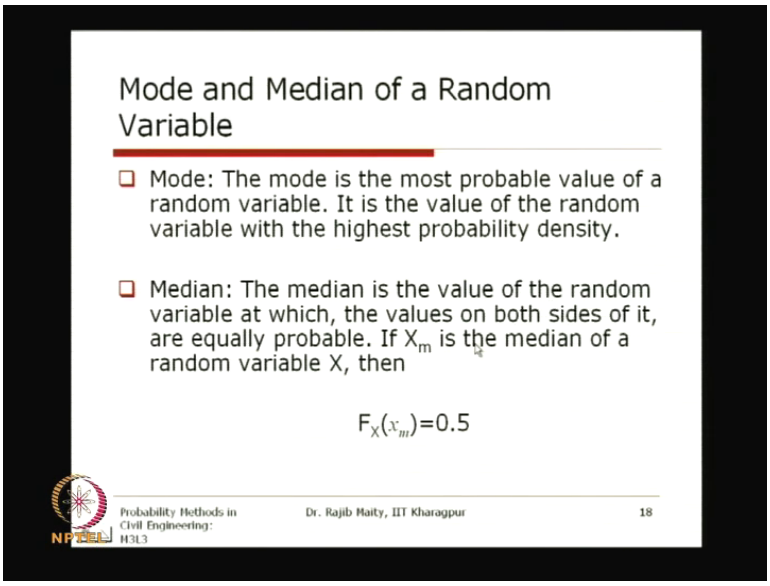

Other tools measure of central tendency is mode and median of the randomvariable, the mode is says that the mode is the most probable value of the random variable, so out of the different outcome in case of the discrete and for this over overall range where the, which one is the most most probable value for thisah this one and this is the value of the random variable with the highest probability density. Obviously when we arecalling this probability density this is for the continuous, so if you say this one show, we see that here at 2 that the probability thatmass is concentration maximum at 2, so obvious the mode here is 2.

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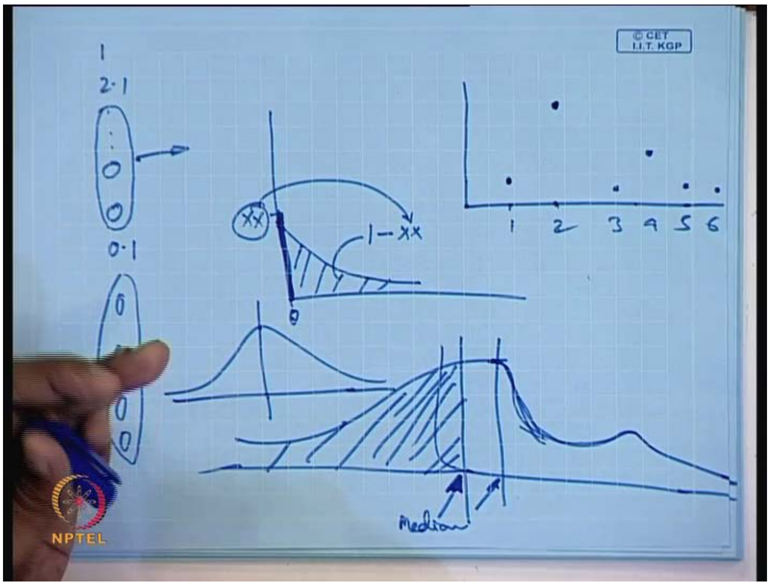

Now and and for if I just say that this is your this is your some probability distribution function then, obviously obviously at this point where you see the density is maximum this your mode, again if you see that, there are some values and there are some peak so there may be some other things so these are generally known as the bimodal or multi model, so this is having more than1 1 mode so this is the secondary mode here, where there is a secondary peak.

For example the standard normal distribution if you see this is unimodal and the mode ishere at 0 and we will see that, what this moon that, this mean mode and median is same for this kind of symmetric distribution, that is a normal distribution this is same, that we will see see later; what we are mean here is that, for the discrete random variable the where the probability mass is maximum that is yourthat is your mode and where the probability density is maximum, that is your mode for that random variable.

And then it is median, the median is the value of the random variable at which the values on both side of it are equally probable if the X m is the median of a random variable x then F X x m is equals to 0.5.

Now, this is this is one thing that, where should, whenyou will discuss this one first in case of this incase of continuous random variable, so I am just started going on integrating from its left supportif it is minus infinity, from the minus infinity I am going on integrating and I will stop at somepoint where that, where the total area covered is equals to 0.5, so that means this is the point where the probability less than that particular value is 0.5 greater than that particular value is 0.5, so this value is your median.

Similarly, you will go on adopt this probabilities and where it will where it will touch that 0.5 for thatcorresponding CDF, thethat particular value so if we will just add this plus this so where it will touch this 0.5, that particular value will be the median for this discrete random variable.So that median means that less than that particular value and higherand about that, the higher than the particular value both are equally probable as a total, so left hand side total probability is 0.5, right hand side total probability is 0.5, that midpoint is your median.

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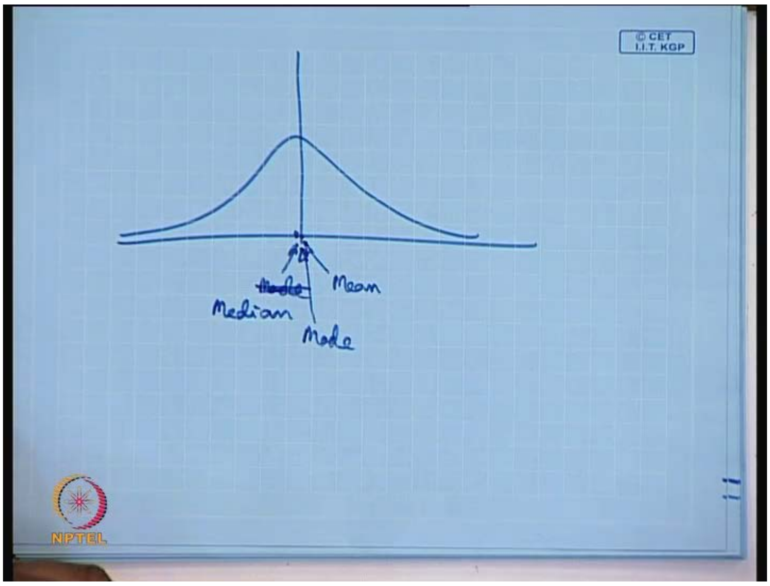

Now, what we are discussing in case of thisnormal distribution, so when we say that this is a normal distributionhaving a completely symmetric distribution and it is it is having some a mean here, so in this case you see that, exactly at that particular point where this touching the peakbeing the nature of the symmetry this is the this is covering the total area 0.5, so this is your this is your mode.Now if you take thatso if if you take that intake that integration take that mean, then you will see that exactly this point is becoming your mean and being this is the highest density point, this the same point is yours is your mode; so for the normal distribution,the mean mode and sorry median this is that where the 50 percent probability is coveredthat is called the median so for this normal distribution this mean mode and and the median are same point for this symmetricdistribution that is a normal distribution.

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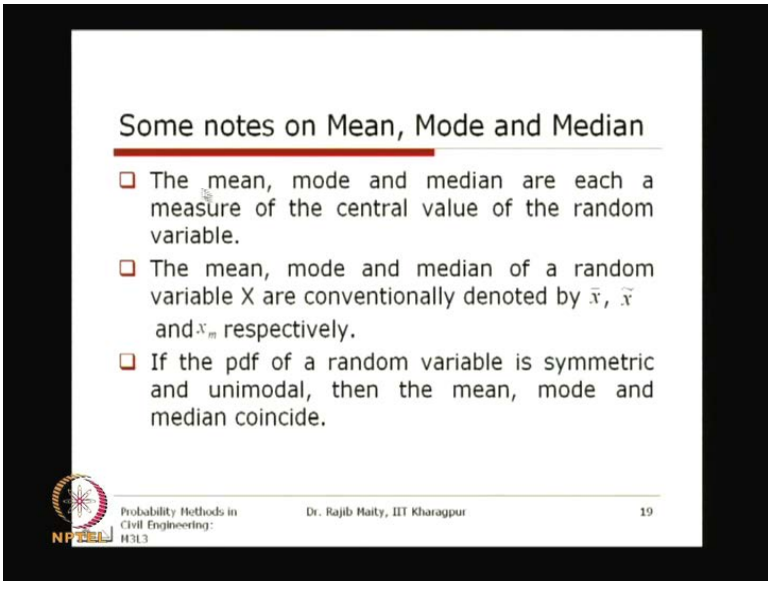

So, this ismeant that the mean mode and median are each a measure of central value of the random variable the mean mode and median of a random variable x are conventionally denoted by this x bar, which is mean this x tilde, which is mode and x m is the medianof the particular random variable x.

If the pdf of a probability density function of a random variable is symmetric and unimodal, obviously which is the normal distribution it is symmetric, it is unimodal that is having only 1 mode, then the mean mode and median coincide, just what we discuss just now.

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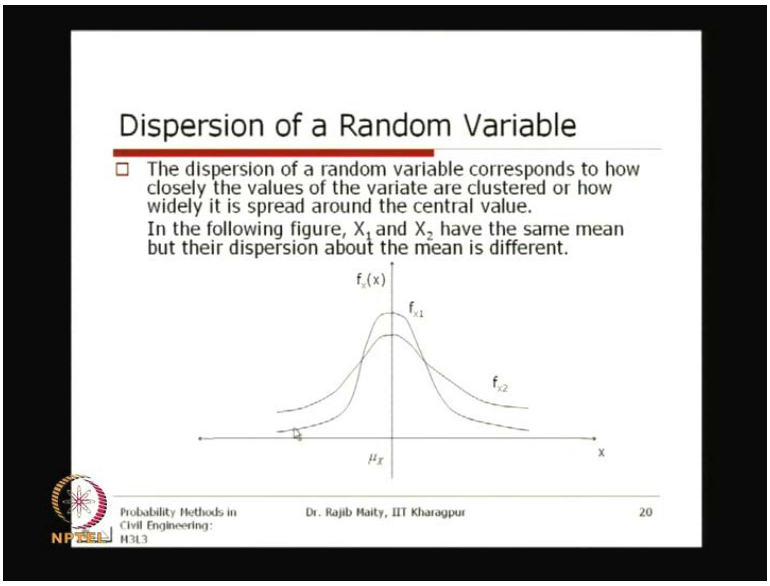

Now, the second descriptor of this random variable is the dispersion, so dispersion what is meant is a spread over this mean, that is I know once I identify where it is tending towards the central I want to know, how it is distributed above that mean so the dispersion of a random variable corresponds to how closely the values of the variate are clustered or how widely it is spread around the central value, in the following figure if you say that, this is the X 1 and X 2 have the same mean but, their dispersion about the mean is different, so this one obviouslythis one which is for this x 1, that is f x 1 this is the less dispersed compared to the other one which is that f x 2.

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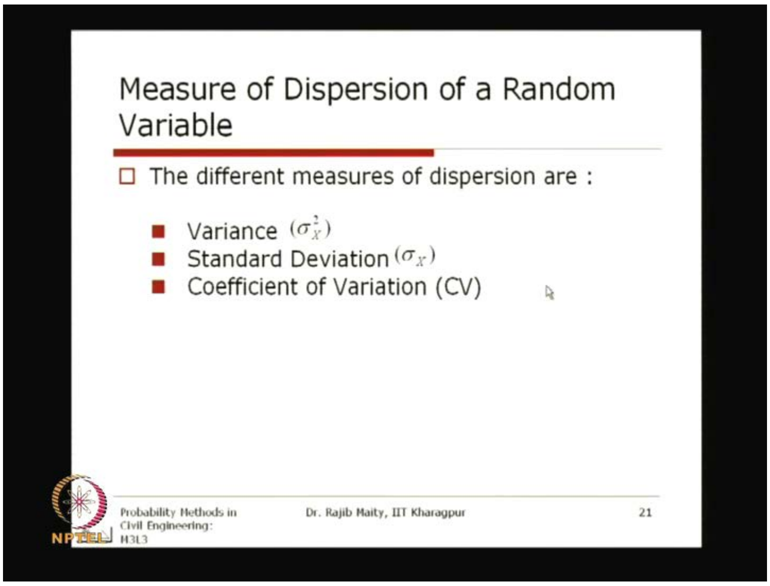

Now, how to measure this one that is the, this measure of this dispersion, there are generally we use 3 3 different measure the, first one is variance which is denoted as sigma x square, standard deviation sigma x, and coefficient of variation C V.

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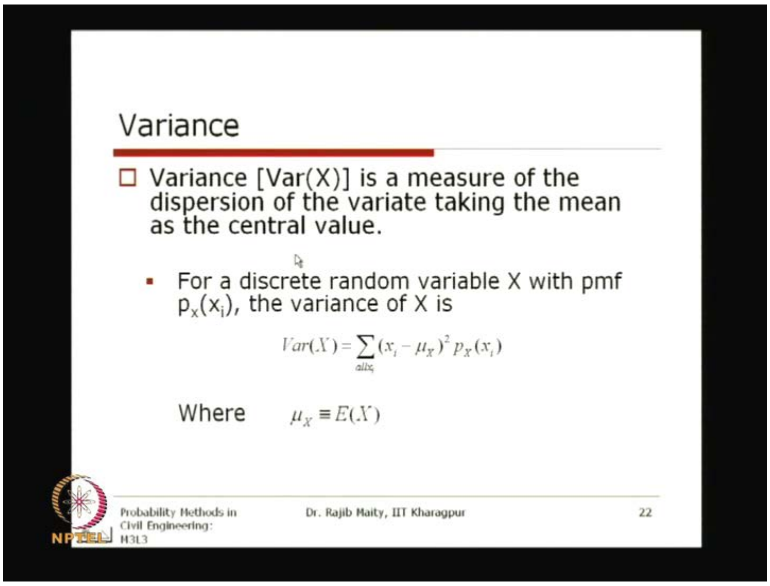

Now the variance, if we see thatvariance, which is generally also denoted that the var of x of this random variable X is a measure of the dispersion of the variate taking the mean as a central value, now when we are measuring, that spread around the mean out of this 3 central out of the 3 measures of central tendency, if we pick up the mean and thenif you calculate how it is spread around that mean, then that is known as the variance.

For a discrete random variable x with pmf, p x X i the variance of the x is this variance is equals to summation of X i minus mu x, that square multiplied by p x X i so this the measure of this variance,I will just explain how this things are meant with the context of this area with the context of this moment, how it comes; now this mu x as I told mu x is nothing but, the expected value of that particular x.

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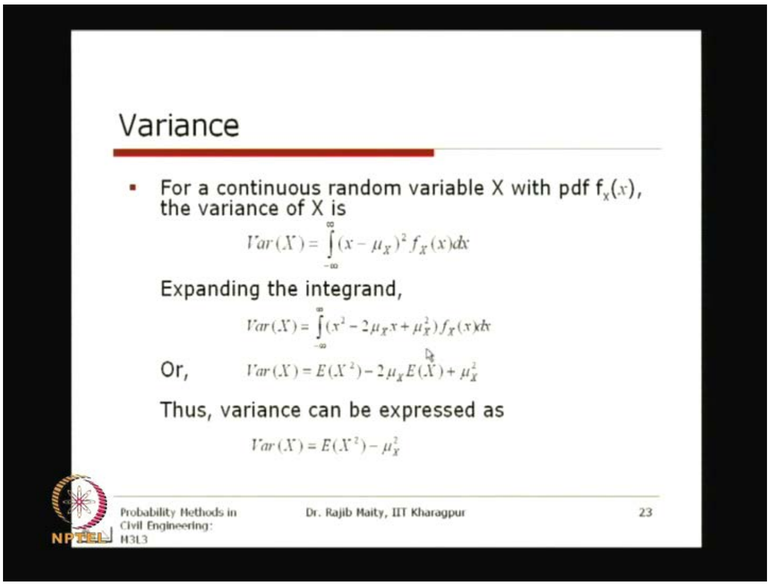

Now, now so let usfirst complete this one before I go for this pictorial representation, so this is for this is for the discrete random variable and this one is for the continuousrandom variable, now if the continuous random variable X having the pdf of this f x x than the variance of x is the we x minus mu x which is the expectedvalue that square multiplied byf x d x this is, this gives you that, that variance of X, now there are some if you just expand it a little, then we can come out to this one, just to expand this particular value andtake this one as this that is, thatit will comethat x square multiplied by f x d x.

Now, youjust recallthatthat expected value ofthe function, now here the function is x square, so function is x square multiplied by f x d x, which is nothing but, the expected value of xx square, this is the function of the random variable, which is the x square minus 2 mu x; now this constant when we are taking this one, so this is already known this is constant so the 2 mu x can be taken out, 2 mu x multiplied by x f x d x, x f x d x is nothing but, that expected value e x plus this again the constant, so thus plus mu x square. Now after just doing this, so then again this e x is the mu x, so minus 2 mu x square plus mu x, so this is e x square minus mu x square so variance can also be represented like this.

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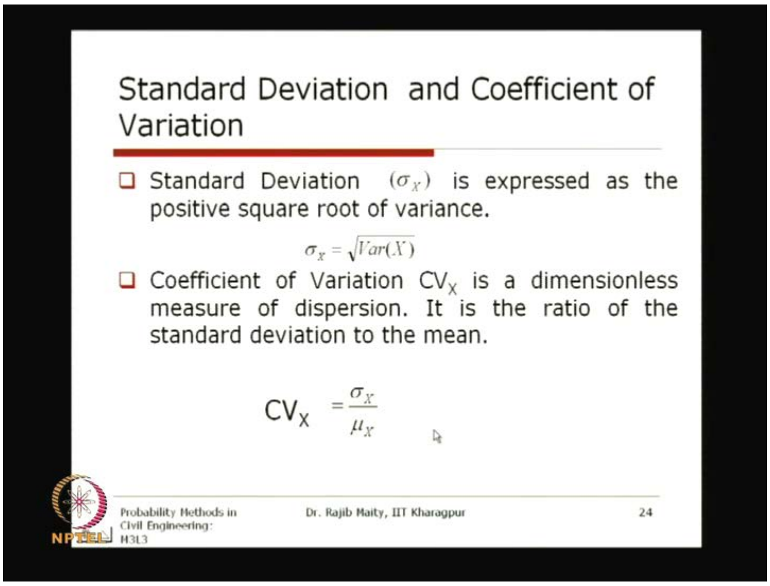

So, this standard deviation standard deviation is the another measure ofthis of thismeasure ofdispersion where this standard deviation sigma X is expressed as the positive square root of thevariance f x which is the square root of this variance of X.

And the coefficient of variation C V x is a dimensionless measure of dispersion it is a rescue of the standard deviation to the mean, so this C V x is equals to sigma x by mu x. Now from this for all other higher thing that is the first the how it is becoming the higher thing that, I will just explain in theexplain it here that is what you are doing when we arewhen we are measuring this that spread that is a that is that isthe spread around this mean what we are trying to take is that, first what we aredoing we're taking this is your mean and we are taking that X minus mu x.

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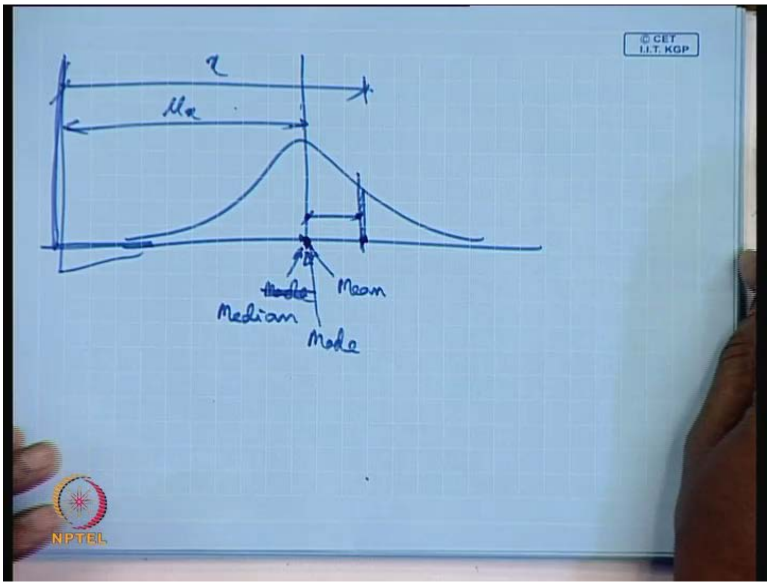

That is a particular value x if I take thatminus mu x suppose that this is somewhere where where is thewhere is the origin is here, so I am taking the value xhere, this is your x this is your mu x that is I know that is your mean, so I am taking the x minus mu x that is nothing but, from the distance from the mean to that particular value.Now this one I am just multiplyingitwith that particular density at thatpoint that is the d x and multiplying that one with the distance to this one, this is a kind of the second moment from starting from this mean of the particular distribution.

So,we will see in ain a moment,how it can be represent, how it is represented as the second moment about this mean and we can go for this, so the mean is nothing but, the first moment withrespect to the origin and variance is the second moment with respect to the mean; now like that I can go to the third moment with respect to mean fourth moment with respect to the mean fifth, sixth in this way we can go and each and every momentwill give some property of the distribution.For example, the second moment that is this one with respect to this mean is giving you the measure of the dispersion, how it is dispersed around the around the mean.

So we will come that 1 how it is represented incase of the mu particularly. So before that we will see, so here that that this distance square multiplied by thisdensity that is f x multiplied by thisd x the small inferential smallarea.

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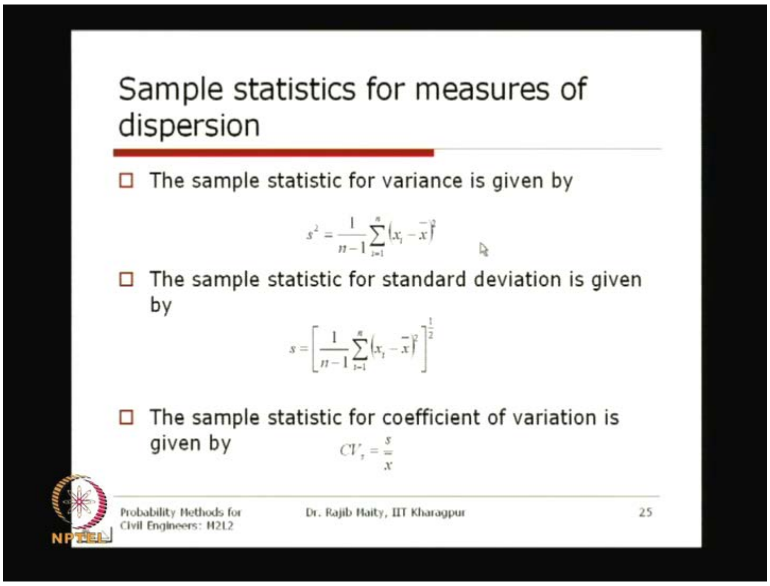

Now, if I want to know the sample statistics of this measure of dispersion the sample statistics of the variance is given by so this sample statistics means again for the random variables some observations are taken x i for n if the number of samples are n, then x i minus x bar which is themeanof that particularever that mean ofthat sample sample mean that minus this one square summing up them and divided by n minus 1 this n minus 1 minus 1 is due to the to make this estimate as unbiased.

The sample statistics for the standard deviation as we told that is the positive square root of this variance, so this full quantity power half that is square root of the this one which gives the standard standard deviation, for this standard deviation we go for this first root but, for the higher moments when you go for this measure ofthat skewness and all we generally do not take thatthe third root of that one to this one, this is this is here for the for the first one that is for themeasure of variancewe take the root just to see that if you see this expression, then you will see that unit of the standard deviation isequal to the unit of the random variable itself.

So, this is athis is sometime has greater help so that is why we take that 1 square root and we declared that 1 as a standarddeviation, which we do not do for thehigherordermomentsand the sample estimate of thiscoefficient of variation is the sample estimate of the s and this x bar is that sample mean ofthis to get that coefficient of variation.

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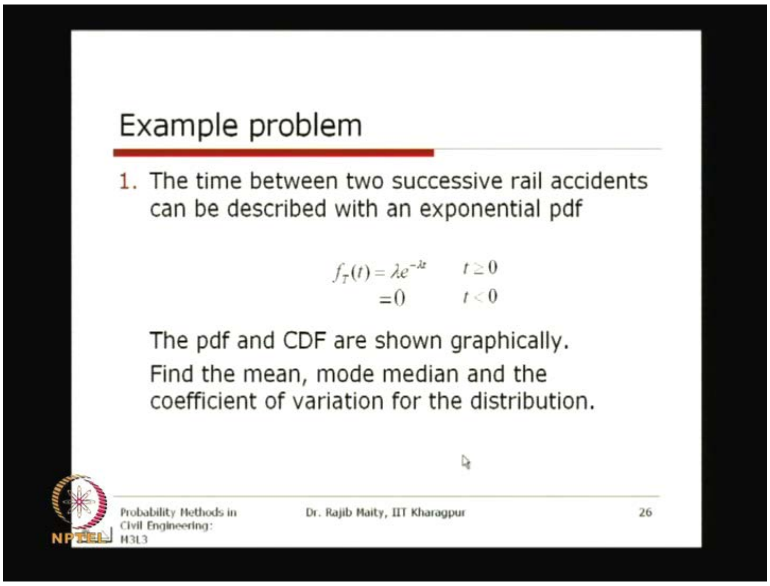

So, one example problem we will take for thisthing, that is whatever we got the mean and this standard deviation and variance, so the time between two the same example that we discuss in this in the context of this CDF the time between two successive rail accidents can be described with an exponential pdf that is f T t equals to lambda e power minus lambda T t greater than or equal to 0 and 0 for this otherareasthat is less than 0.The pdf and CDF are we have seen itearlier that is how it looks like pdf and CDF in such cases, so we have to find out the mean, mode, median, coefficient of variations, so whatever the statistics we have seenjust so far we will just see, how it is for this exponential distribution, how it can be calculated for this kind of distribution.

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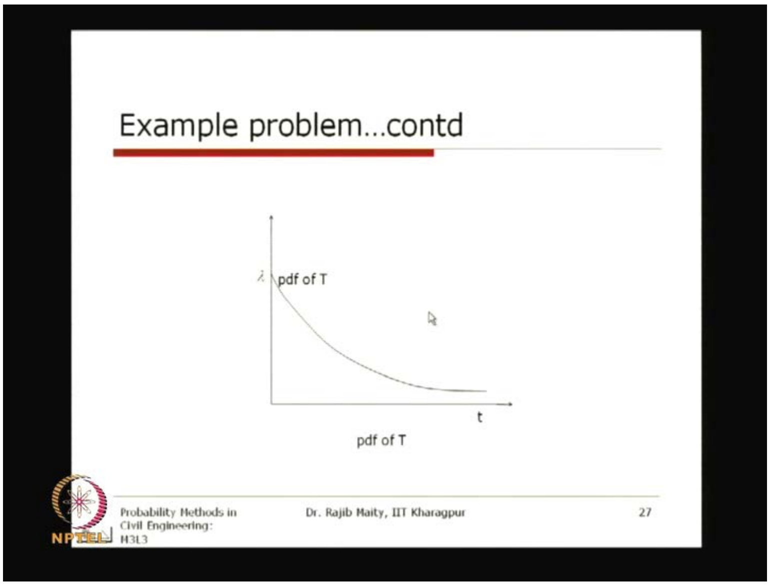

So, this is the pdf of this T which we have also seen that, this value is lambda and it isgradually coming down and getting asymptotically to this 0 at infinity, so this is the pdf for the random variable T.

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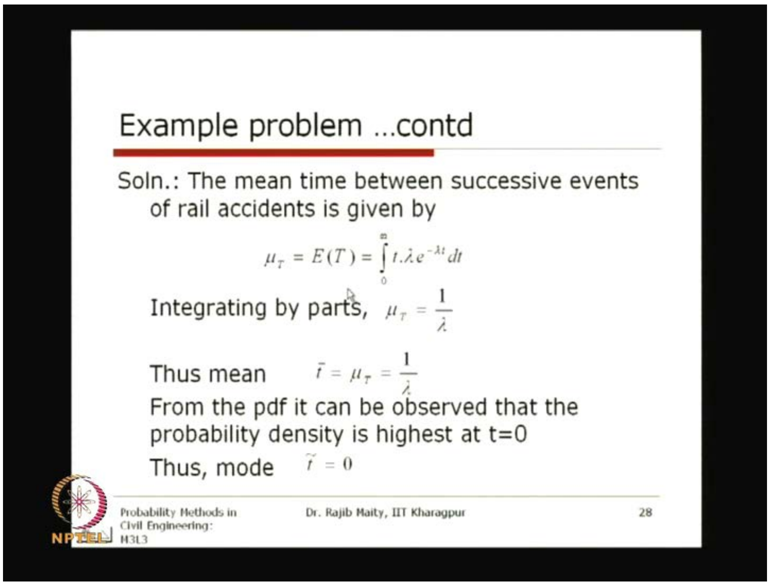

Now, the mean of this mean time between the successive events of this rail accident, if this is the distribution then, we know that mu t which is the expected value of this t is the over the entire range of this pdf the t multiplied by lambda e power minus lambda t d t, if we just do this smallthisintegration by parts, then we will see that this mu T is equals to1 by lambda. Now, here one thing you can see that so this is that this is that lambda that, we aregetting here and even we have seen that the sample estimates.

So, if youtake some sample of this particular event then if we calculate theirmean the sample mean and if you take the inverse of that sample mean then we will get the measure of this lambda, so this is how means we method of this is called the method of moment to get that estimate of this parameters, just few slide before we are discussed, we are taking that how to estimate the parameters it is one of the method, that even how to get that one but, that is not the context what we are discussing here, now we are just getting that for this distribution what is the mean, so the mean are the expected value of the random random variable is 1 by lambda.

So, this mean tbar mu t is 1 bylambda, from the pdf it can be observed thatthe probability density is highest at t equals to 0, so the density values if we see if we just look this density value so we see that at t equals to 0 itself thus value the magnitude of this pdf is maximum which is lambda; so the from the definition of the mode we can say that a mode that is t tildeis equals to 0, so mode t at 0, mean is at 1 by lambda.Now we will see where themedian is, so median means we have to calculate that, that we have to integrate this one and get some value where it is covering the 50 percent of the total area, total area is 1 we know, so 0.5 we have to integrate it from the 0 to some value x, where it will be equal to 0.5 to get the median.

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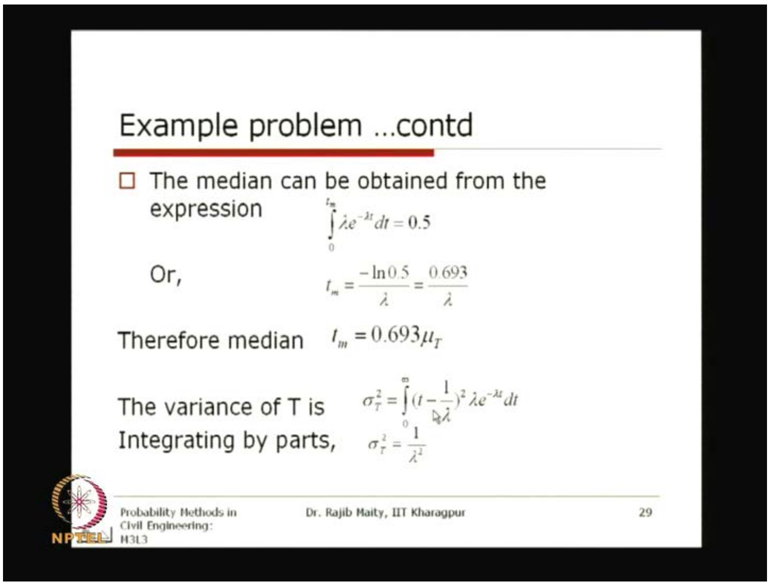

So, exactly the same thing is done, so for the 0 to t m that is the value of this median of this value should be equals to 0.5, so if you do this integration we will get that the mean becomes the t m equals to thisand t m equals to 0.693 mu t, that is 1 bylambda the variance of this t; now if you want to get thatvariance of that one, soagain we will use that same expression that is a that is random variablet minus 1 by lambda 1 by lambda is nothing but, we know that this mean that is expected value of this t so if you take the t minus 1 by lambda square then multiplied by this probability density d t then we do this integration by parts and we see that this sigma t square which is variance is equals to 1 by lambda square.

So we have seen that mean is 1 by1 by lambdafor this exponentialdistribution and the variance is 1 by lambda square.

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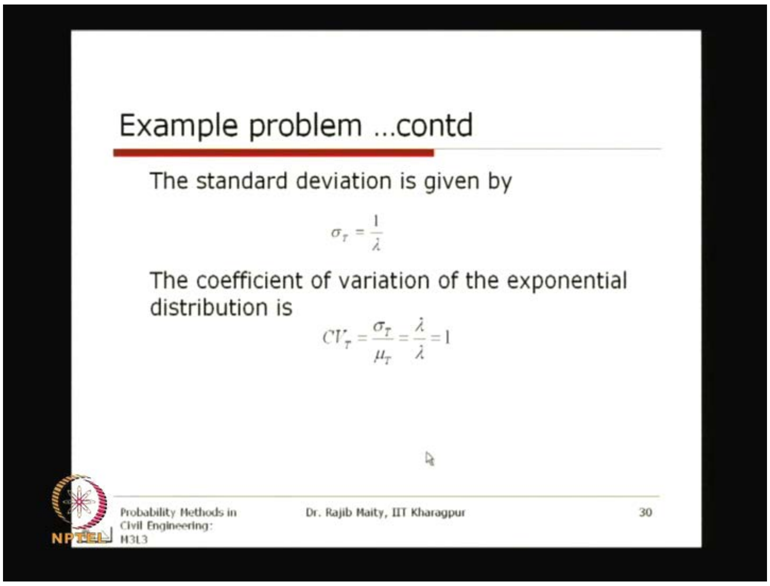

Now, the standard deviation again we know the standard deviation should be positive square root of the variance, so standard deviation is again equals to 1 by lambda, so standard deviation the magnitude of theare same the mean and the standard deviation which is again that 1 by lambda.

So, the coefficient of variation if you wantto calculate, coefficient of variation of the exponential distribution is equals to sigma T by mu T, we know that so this is 1 by lambda by 1 by lambda which is equals to 1, so coefficient of variation is equals to 1.

So there, are two othermeasure ofdispersion and one is that measure of skewness and other one is measure ofmeasure of peakedness and we will see this one later, these are the these are the higher moments what we have seen today is that is that how the first first moment about that about the origin and in thenext class as well.

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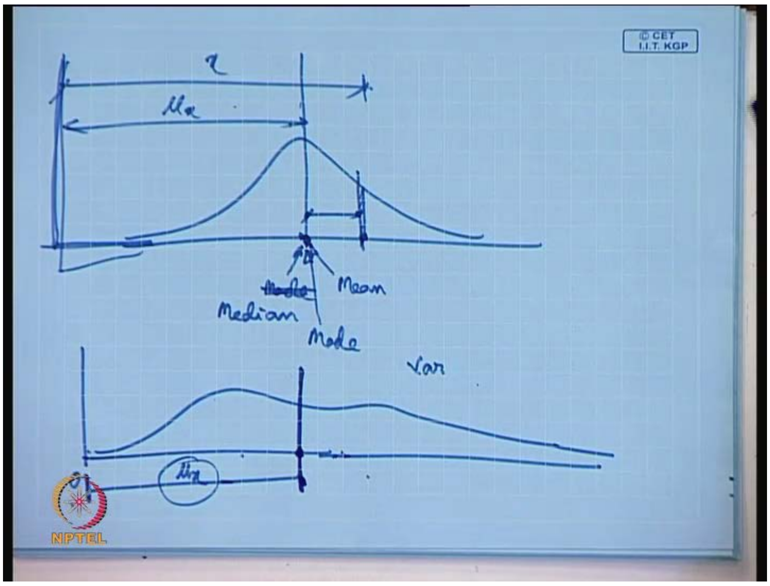

What we will see that we will we will see that, how it is becoming that mean from this so one particular 0 distribution if you take and how this moments are actually coming from this one with respect to the origin, so this is basically your mu x, how it is means the moment with respect to the origin and the oncewe get the moment with respect to the origin that means we are getting the value of mean and all other higher moments we calculate with respect to that mean.

The first that we calculate is thatwith respect to the meanthat is a second moment with respectto the mean what we got that is called the variance, one open question that I can put before I conclude today's classis that, as I told that second moment with respect to the mean is variance, what is the first moment with respectto mean, so the first moment with respect to the mean that means I have to take that particular value minus that mean minus that mean multiplied that density and that we will get so interestingly or this is mathematically very easy to state, that the first moment with respect to the mean is always 0, because you are basically whatever the positive side of that mean and whatever the negative side of that mean both are cancelled out to results in that the first moment with respect to the mean is equals to 0.

So, we start with this second moment, the second moment is the measure of variance third moment skewness, fourth moment peakedness like that,fifth moment, sixth moment, are there basically upto the fourth moment is sufficient to describe a particular random variable. Today's class we discuss up to the variance, next class we will start with the description of the skewness and the kurtosisand we will also see more details how this can be related to as a as a moment; so we will that, will be the starting off again for the first momentand that moment generating function for that for a random variable.So,thank you for today's class.