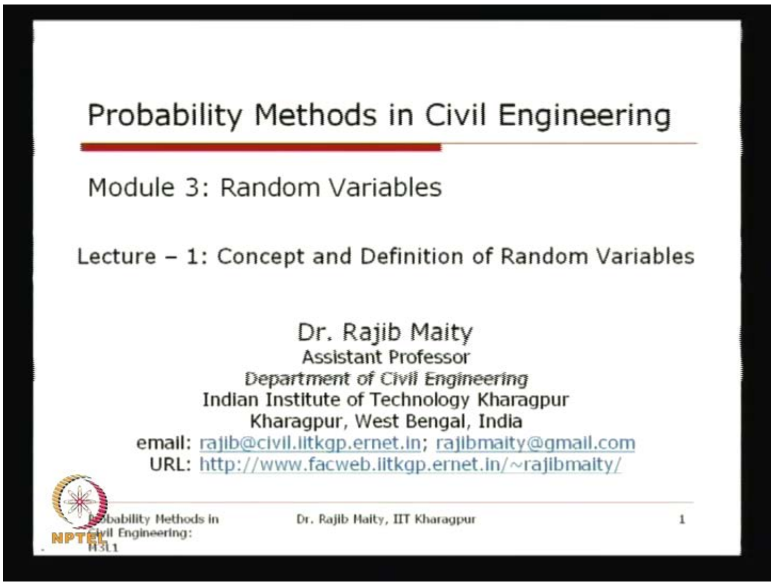
Probability Methods in Civil Engineering
Prof. RajibMaity
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

Module No.# 03
Lecture No. #06
Concept and definition of Random variables

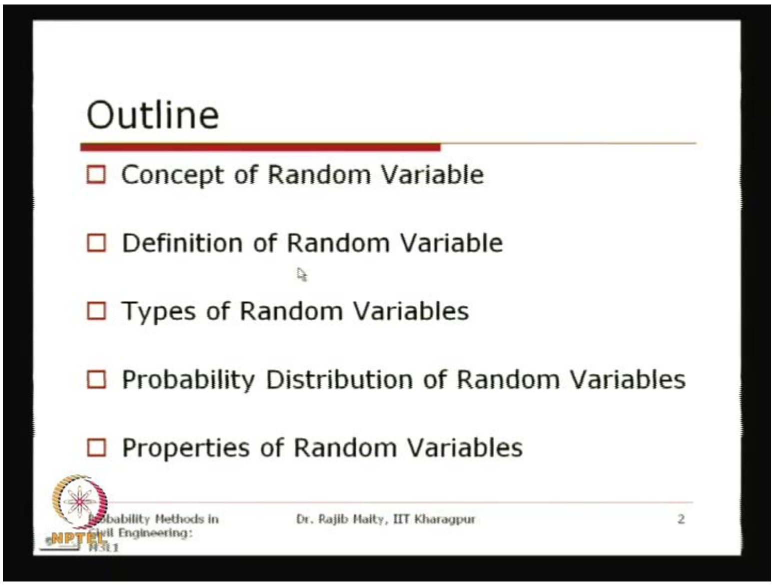
Hello there and welcome to the courseprobability methods in civil engineering.Today, we are starting a new module, module 3.

(Refer Slide Time: 00:28)



In this module, we learn aboutrandom variables and there will be couple of lectures and in this first lecture, we learn the concept of random variables and theirdefinitions.

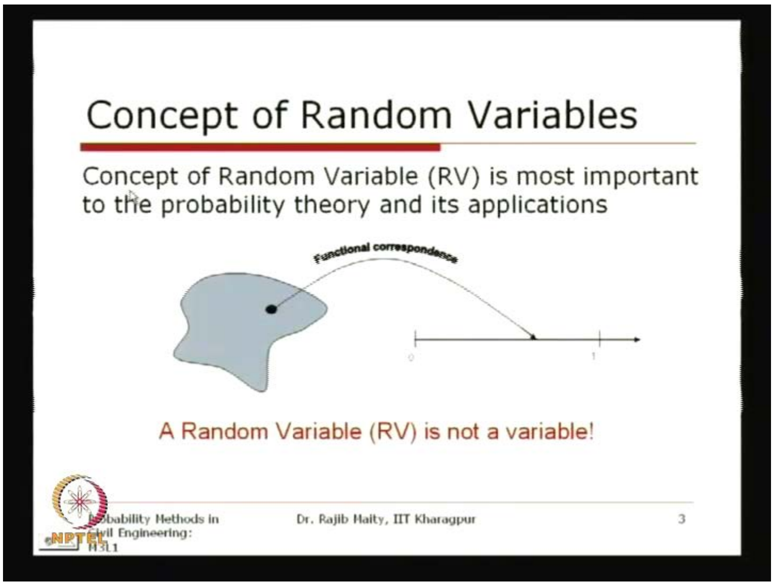
(Refer Slide Time: 00:45)



So our outline for today's lecture is arranged broadly like this.First,we will discuss about concept ofrandom variable and then, we willunderstand the definition of random variable.Then, we will discussdifferent types of random variables andtheir probability distributions.

This random variables are important in the sense, that this is the, this concept is important to understand how it is used in the probability theory and this probability theory for this random variable is generally,is is generally assessedthrough the through their distributions.So,this is known as the probability distribution of random variables.After that, we willlearn different properties of random variables.

(Refer Slide Time: 01:40)



So, asjust what we are telling, that this concept of this random variable, which is abbreviated as rv, is most important to the probability theory and its application.Now, if we see that, how what actually thisrandom variable mean, if this shaded area,what you see here is the is the sample space, then this sample space consist of all feasible outcome ofoneexperiment.

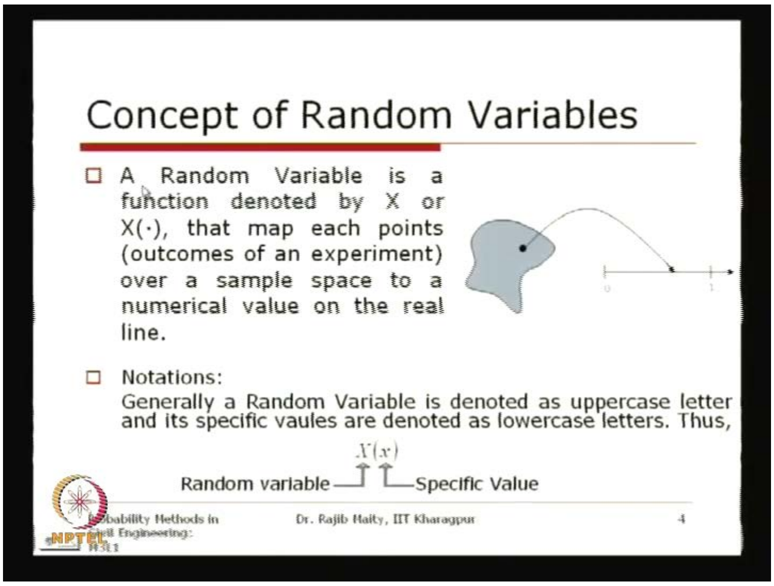
Now, this feasible outcome of this experiment can be continuous or can be discrete points fromthrough this sample space.Now, from the sample space, we have to, for the mathematical analysis, we have to map the outcome of the experiment, ofoneexperiment to theto some number, according to our convenience.

Now, to map this, each and every output ofa random experiment to this real line, is generally through a functional correspondence and this correspondence, this functional correspondence is onerandom variable.

So, what is important and what is, what we want to understand from this slide is that, this random variable,even though this variable isshown here in the in the name of the random variable, is not a variable. So, which is generally, what you want to say is a functional correspondence or is a function. That function generally map between the sample space ofoneexperiment to the to the real line.

Now, this mapping is done based on our convenience and that we will see in aminute.

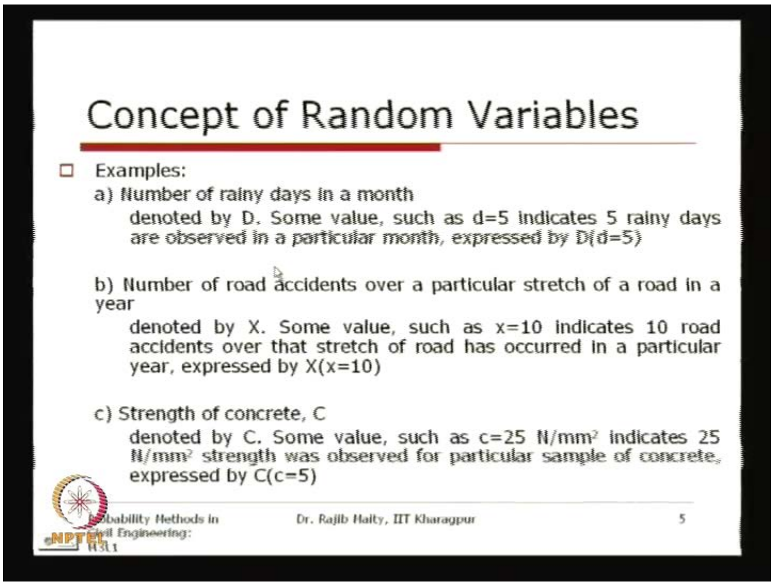
(Refer Slide Time: 03:36)



Thus, if we say the definition,if you say, then we will say that a random variable isa function denoted by x or x some specific number,it can be denoted by anyanyletter.Generally, these things are denoted by this small lower case of this of this random variable.These random variables are generally expressed interms of the uppercase letters.So, this random variablex is a function that map each points, that is a outcome ofan experimentover a sample space to the,to a numerical value on the real line.What just we have shownin thisfigure, in the last slide.

Now, before we discuss some of the, some of the further concept of this random variable, so there are different notations are being used to denote this random variable and the specific value.For this lecture or you will see in some of the classical text book, that this notation will be followed for this for thiscourse.Generally,this random variable are denoted by the uppercase letters and its specific valuesare denoted as lowercase letters.Thus, in this notation, this uppercase letter is indicating the random variable,which is denoted by this xand any specificvalue to this random variable is denoted by this small x, which is denoted, which is shown here.So, this is the specific value of the random variable x.

(Refer Slide Time: 05:20)

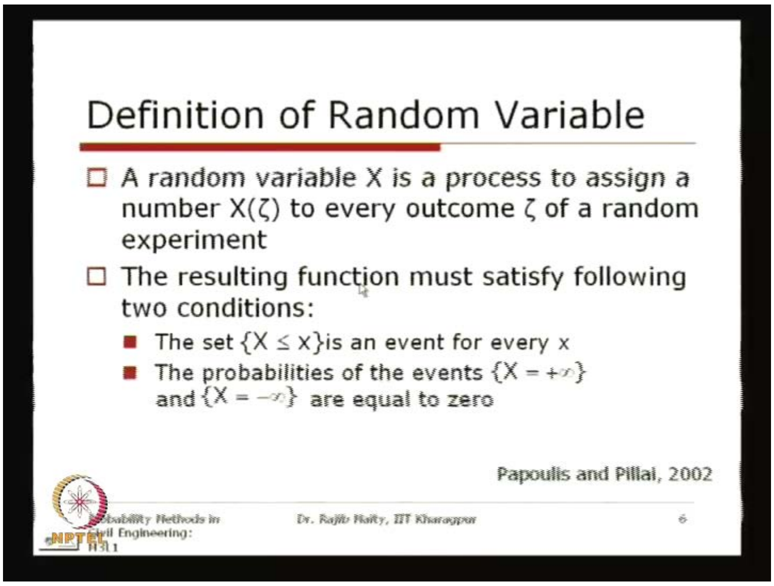


Now,we will seesome some example of this randomvariable which we just discussed.Suppose that,number of rainy days in a month;so, if wecan,you can say that this is a random variable and we do notknow.So, this value that number of rainy days in a month can varyand if we denote this random variable as letter capital d andsome value say d equals to 5.So, this indicates that 5 rainy days are observed in a particular month.Now, which is, which can expressed by d in parenthesis d equals to 5.

Second example, say the number of road accident over a particular stretch of the road in a year. Suppose, this is denoted by x and some specific values, such as say x equals to 10, indicates that there are 10 road accidents over that stretch of the road has occurred in a particular year.Now, this value 10 or this value 5 can change in the successivesuccessive time step or in the success; for in this case,in the successiveyear, that can change.So, this number of road accidents is my is my random variable, which fora particular year it takes a specific value x equals to 10, which is denoted by this.

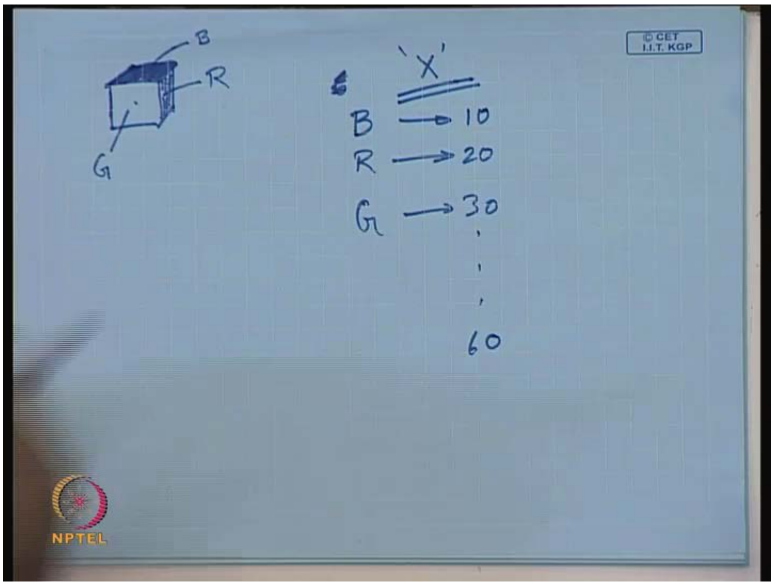
The lastone,say the strength of concrete.If, I designate is at c, so this is denoted by c and some specific value such as c equals to 25 Newton per millimetre square.It indicatesthat25 Newton per millimetre square strength was observed for a particular sample of concrete and which is expressed by c in parenthesis c equals to 5.Cis the,sorry this shall be c equals25;sorry for this mistake this should be 25.

(Refer Slide Time: 07:25)



So,a little bit formal definition of random variable.If we see, that a random variable x is a process to assign a number, which is a real numberx xi to every outcome xi of a random experiment.So, if wejust takeone,take that thing, so supposethat we are taking that example of that of tossing a tossing a dice, then we know that whatever the numbers that generally comes, if it comesa 6 on the toss, then we see,we saythat the outcome that is coming, it is my 6. Similarly,sixdifferentsurfacescan havesixdifferent numbers and that can be happened.

(Refer Slide Time: 07:51)



Now, see that in this experiment, if I just change this one, so this is quite straight forward that,whatever number of dots that we are usuallyusually used to know.Suppose that, instead of having this kind of dice, if I just have a have a dice which is having different colours.Suppose that, this is having a blue colour, this side is having may say red colour, this side is having green colour and similarly sixdifferent faces havingsixdifferent colours.Then, what I amtryingto do is that, this outcome of this is a top surface, the colour of the topsurface, if I just 1 to 1 to say that this is my outcome of this of this experiment, then what I can do is that I can take different colourcode and I can assign to some number,not necessarily 1 to 6, itcan be of any number.

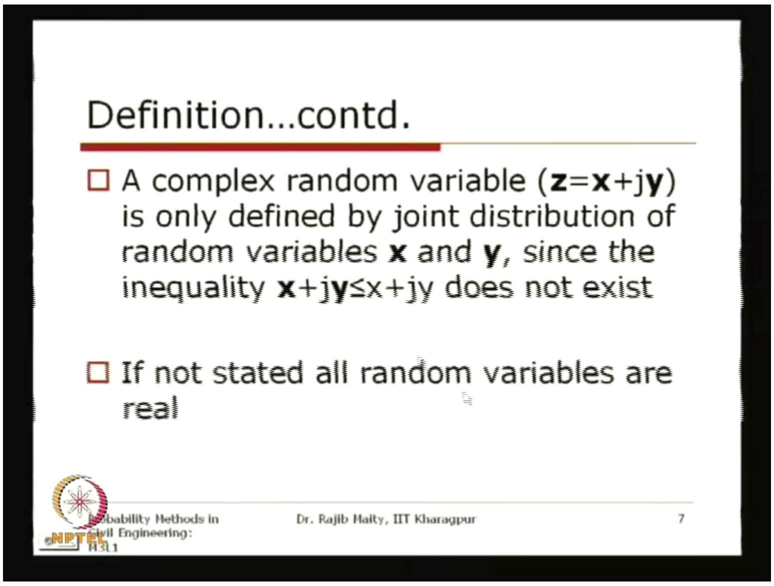
Say that b, I am giving some number 10,r giving some number 20, green giving some number 30 and this way say up to 60 this number is given. So,these mapping from this outcome to some number, this mapping is your random variable.So, we will come after this, how to assign the probabilities.Now, this is very straight forward.So each and every outcome, if this dice is fair, then the outcome of each and every of this outcome will be equal; so everything will be 1 by 6.

So, that is again another process.How we will assign the probability to that particular random variable?If it is x,so that will be, how we assign the probability.Now, when we are defining this random variable itself, so this random variable is nothing but, is a processto assign a number to each outcomeof that random experiment.This is exactly what we are calling it as a formal definition of thisrandom variable isthat, a random variable x is a process to assign a number x xi to every outcome xi of a random experiment.The resulting function must satisfy the followingtwoconditions.

Even though, we are saying that this is a this is afunction, so this function of mappingfrom thisoutcome, experimental outcome to some numbers should followtwo,suchtwoconditions.The first condition is the set x less than equals to small x. Now, you see here,this is the capital letter which is your random variable and this is the specific value.Now, this random variable, below some specific valueis an event for every x.So, whatever the x you take, so this should constitute and event for that random variable.

For that,for this x, now other things of probabilities of the events, that x equals to plus infinity plus infinity and x equals to minus infinity are equal to 0.Why this things we are just saying is that, even though means mathematically, this x can take any value on the real line but still,we say that so the extreme, that is plus infinity and minus infinity, this probabilities if I take this probabilities,this should be should be equals to 0.

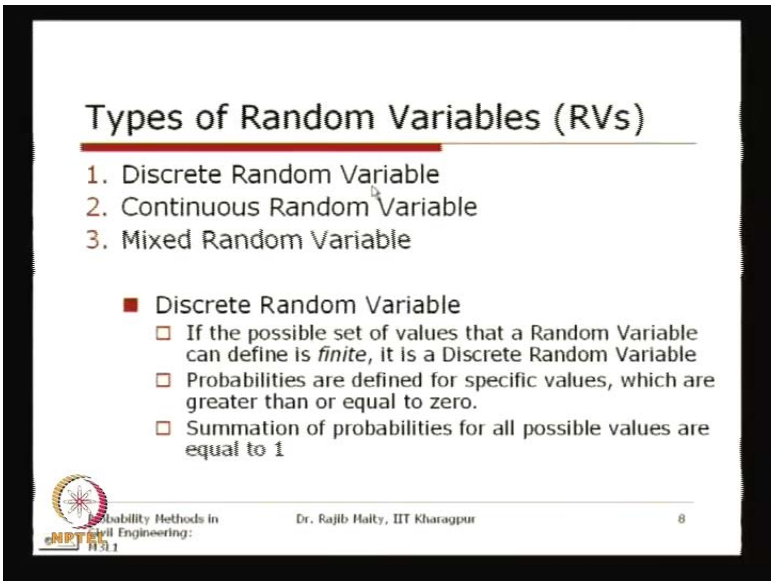
(Refer Slide Time: 11:40)



A complex random variable, so just we will to complete this definition part, if wedo not say thisone,it will not be completed.So, this is, there is another type, this is the real random variable which just we discussed,is now it can be the complex random variable as well, which is z equals tox plus j y.

So, this random variable is only defined by their joint distribution.So, if I want to define the complex random variable z, this should be defined by their joint distribution.This joint distribution will be discussedlater in thisin this in thiscourse.So, but for the time being,you can say that this joint distribution is generally obtained from that distribution, of this xand y both.So this both,this distribution should be taken care to find the joint distribution to define the distribution of the complex random variable, of this,of the random variable, ofindividual x and y.Since, this inequality because, so this kind of inequality,say this of the of the random variable and this is the less than equal to that number does not exist.But, for this course, if it is notstated specifically, all the random variablesare real.We should follow this norm.

(Refer Slide Time: 13:15)



Now, with this concept,we will just see different types of random variables.There are basicallytwotypes of random variables;oneis the discrete random variable and otheroneis continuous random variable.But, there is, there aresome times, in particularly in the civil engineering application, we have seen there are some distributions which can be treated as themixed random variable as well.So,we will discussthese differenttypes of random variableoneafter another now. The first we take this discrete random variable.If the possible set of values that a random variable can define is finite, it is a discrete random variable.

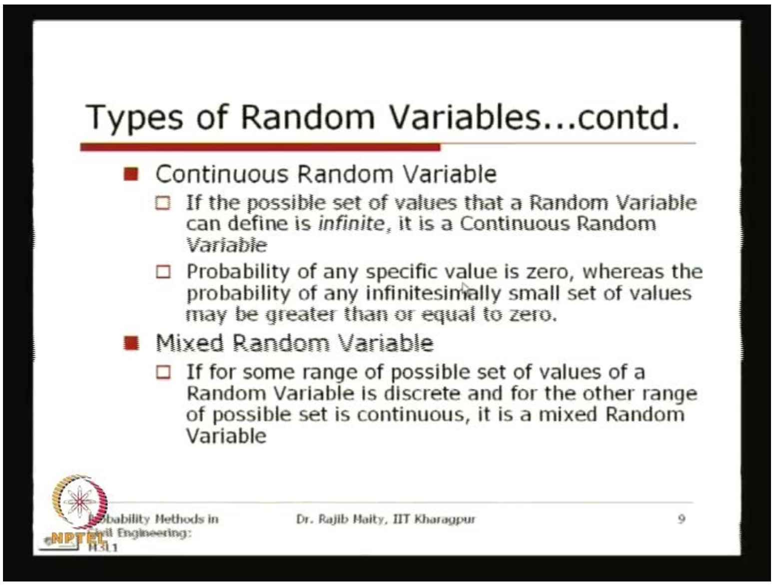
Now, so just to tell this thing here, that suppose that thisis my this is mysamplespace.Now, if I saythatthese random variable is defined for some specific specific value and rest of the part it is not defined,so it takes some specific some specific value.So, those values only it canit can define.So, in that case, what we say that this is this is a discrete case.So, in the in the previous example, if we just see here, that it can define that some specific outcome of this of this random variable. It cannot, if it is not that continuous, that a range cannot be cannot be defineddefined here.So, if it is if it is dots, then we can say that it is defined for the outcome 1,defined of the outcome 2.But, it is not defined for the outcome anything any number in between thistwo.

Similarly, if we just see from this remote sensing concept, that colour code, then we can say that it is defined for the blue for this coloured surface, it is defined for the blue, it is defined for the red but, in betweenmixed.Ithink you know this colour code.This stands for this blue, if it is the colour are r g b are the primary colours and this 1 00 is for the red.Now, you can just change this number to some other to get some in between colour.So, that is not defined there.So what we want the say for this discrete is the, these random variables are described only for some specific outcome of this experiment.It is not defined for this all.So, those those random variables which outcome who can map, who can define the outcome of the of therandom experiment forsome specific values, those are known as the discrete random variable.

Thus, if the possible set of values that a random variablecan define is finite, it is discrete random variable.The probabilities are defined for the specific values which are greater than or equal to 0.So, this we will see in detail which are greater than or equal to 0.But, what we are trying to stress here is that, the probabilities are defined for the specific value.So,it is just like that, for that point this probabilities aredefined for the adjoining point, this is not defined.

Summation of all theall the probabilities, all the possible values are equals to 1.Thesetwothings are basically coming from the actions of the probability theory that we discussed in the previous classes.

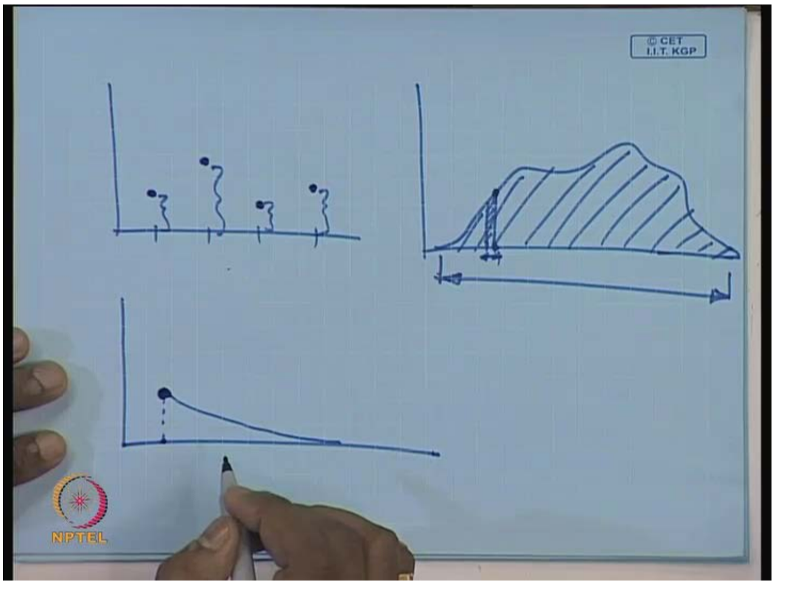
(Refer Slide Time: 16:45)



Thenext type is continuous random variable.So, on the other end, in contrast to this, to this discrete random variable, if the possible set of the values that a random variable can define is infinite,it is continuous random variable.

So over a zone,it isover a zone, over a range thisrandom variable is,if it is also defined,then this is known as the continuous random variable.The probability of any specific value is 0.Now, this is coming from this density point of view.Now,once we are saying that this probability of any specific value, if I just take, then this specific value probability is 0.This is again coming from the actions of this probability. But, even though we say that it is exactly, specific value it is 0but, whatever small set of the values if we think, that can be greater than or equal to 0.So, whatever the small range, if weif we consider,then it will be 0.I think this will be more clear from thispictorial view here.

(Refer Slide Time: 18:01)



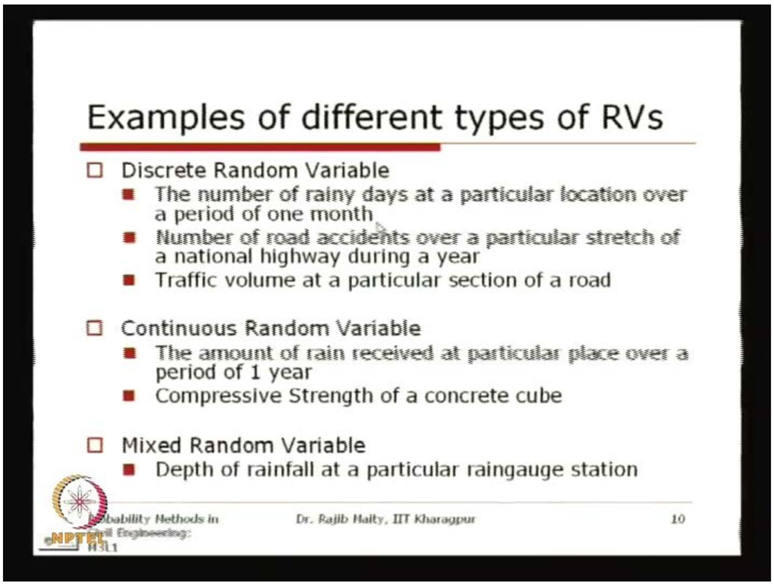
So, suppose what we are trying to say that, if there are some specific outcome and thoseprobabilities are assignedlike this.Whatever may be theprobabilities,if it is notthese are,these are like the concentrated mass for that specific outcome.

So,this is this is your discreterandom variable.Now, for this continuous random variable, what is defined is that it looks like this.So over this zone, from this to this zone, for any specific value, this is nothing but, the density of thisone. Now following the actions of thisprobability, if we just adapt with this axis, is youris yourprobability.As if, I adapt all thisvalues to what we just discussed, this will be equal to 1.But, in this case, when we are talking about, this isonecontinuousdistribution then, what happens for a specificpoint, the probability is 0because this is, basically, adensity.Now, whatever the small area that we will consider, then this area is giving you the probability for this small zone.Now, for the full zone, if I just take then the total probability, that is,total,this is yourfeasible range.So the total probability in this feasible range should be equals to 1.That comes from the actions of this probability theory.

Now, again here,the next type what comes is the mixeddistribution.So mixed distribution, sometimes what happens for specific value, it is defined some probability.Some probabilities concentrated here as a mass and for the rest of some range it can beit can be continuous and goes up to some level or up to the end of thisor up to the infinity.Now, this for this particular point, this probability mass is concentrated here and for the rest of the region, it is having some continuous things.

In such cases, we say that thesekind of random variables are mixed random variable.So, this is the third thing, what we areseen that.So,mixed random variable says, if for some range of possible set of the values of a random variable is discrete and for the other range of possible set it is continuous, it is a mixed random variable.Now,with thesethreetype of random variables, some example in the civil engineeringcivil engineeringcontext, we will see now.

(Refer Slide Time: 20:40)



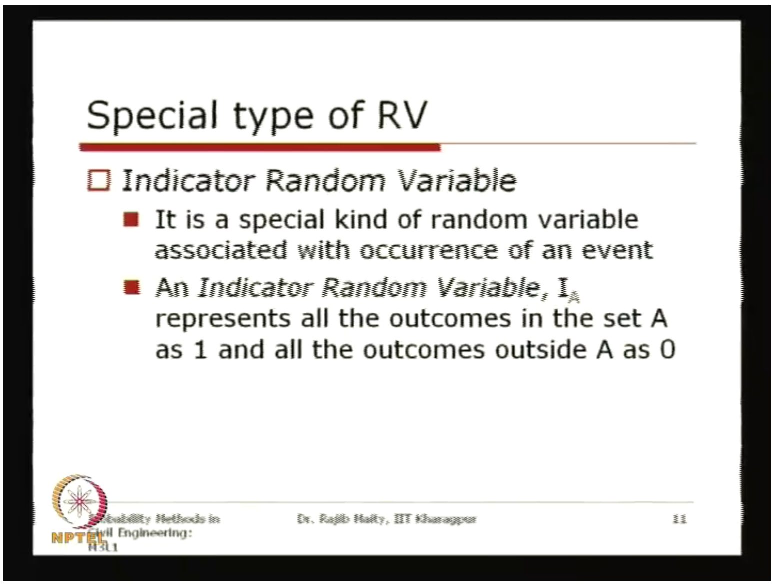
The first is that, we discuss is thediscrete random variable.Say that, this number of rainy days.Now number of rainy days at a particular location over a period ofonemonth say, so it can take,either the specially the integer value, either 1 day or 2 days 3 days, in this way up to 30or31 days, whatever may be the case.

But, it can never take any value in betweentwointegers.So, as this so as this can take only some specific values, so this is an example ofdiscrete random variable.Second is a number of road accidents over a particular stretch of a national highway during a year.So again, these road accidents can be either 1 2 3 4 in this way up to infinity but, it can never take anyvalue in betweentwointegers.So, this alsooneexample of discrete random variable.Third is the traffic volume at particular location of a road.So, this is the number of vehicle that is crossing that particular section ofthe road.Again, this can take only the integer valuesfor thatone. So, this is alsooneexample of theexample of the discrete random variable.

Now, the example of continuous random variable, on the other end, that amount of rain received at particular place over a period ofoneyear.So, annual rainfall for that session,for the, say,it is the number of rainy days, which is a discrete.Now,if I just say that total depth of the rain received atthat particular point that can vary from 0to any possible any possible number.So, thisis oneexample of the continuous random variable.Compressive strength of a concrete cube; now, there are different ways, there are different criteria,maintaining different design criteria, if we prepare the concrete cube and then, we testwhat is this compressive strength.That can actually take any value betweencertainrange.So,thisonealsooneexample of the continuous random variable.

Now, coming to the mixed random variable.So, here say that depth of rainfall at a particular rain gaugestation.So,this depth of rainfall at a particular raingauge station,which is,gives thisone, that isas a continuous random variable here.Now, if you see that, there if, there are station where most of the days, if it is the 0rainfall for the station,then how thehow it will take.So, for a 0,suppose that out of 365 days in a year, suppose that, there are 220 days are 0rainfall.So, there is a probability mass that is concentrated at a particular value of depth of rainfall 0.For the non zerovalue, it can havesome distribution, so that distribution should have shown the first part at particular value that x equal to 0,that is the depth equal to 0.Some mass is concentrated there, some probability mass is concentrated there and rest of the probability is distributed over the positive side of the axis.So, this is an example of the mixed random variable.

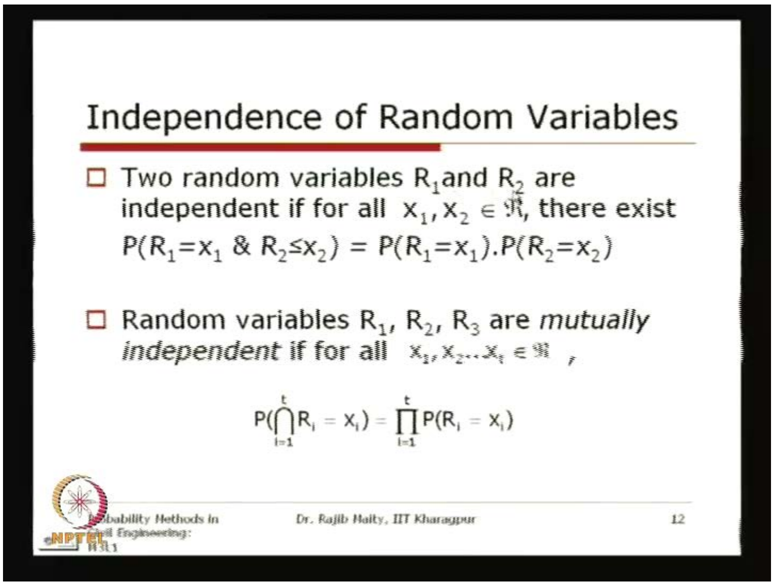
(Refer Slide Time: 24:10)



So, saying thesethreetypes of random variable, it is important to note that another special type of random variable, which is known as the indicator random variable.So, it is a special kind of random variable associated with the occurrence of an event.So, this random variable,which is, says that whether the,a particular event is occurred or not.Say, an indicated random variable,ia, is represents all the outcomes in a set A as 1 and all the outcome outside Aas 0.So, it is like that, whether yes or no.So, if I say that again, ifI take the road accident, whether there is a road accident. If there is road accident,this random variablein fixed in such a way that is, that functional correspondence that we are talkingabout again here.

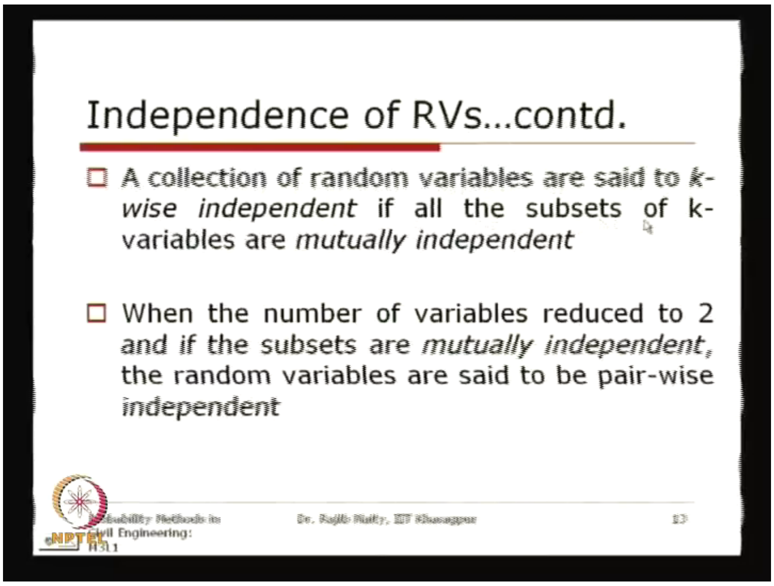
That is, it is function is that,it will just see whether the accident has taken place or not.If it isyes, it will say 1;if it is no, it will say 0.Say, the each day I am just taking this, taking this measurement and just preparing that series,so that series consist of either 1 or 0.Again for the rainy days,I will just see every day whether it is raining or not.Iam not interested in this random variable, if it is designed in such a way, that it is not interested, what is depth of the rainfall.Only, the interested is that whether it is a rainy day or not.If it is not a rainy day, then it will return 0;if it is rainy day, it will return 1.So, these types of random variable just indicate the occurrence ofoneparticular event.Then, the first case, whether the occurrence of the event is the accident, anotheroneis the occurrence of the rainfallor not.So, if it is yes, that is, it is within the outcome of the set, it indicates 1.If it is outside,then it is indicates 0.So, this kind of random variable is known as indicator random variable.

(Refer Slide Time: 26:22)



Independence of randomvariable, this is another important concept to understand.When we can say thattworandom variables are independent, thetworandom variables r 1 and r 2 are independent, iffor all x 1 x 2 belongs to this realline.There exist that probability of r 1 equals to x 1 andthe probability of r 2 less than x 2 is equals to probability r 1 equals to x 1 multiplied by probability of r 2 equals to x 2.Then,if this condition issatisfied, we say that thesetworandom variablesare independent.Now, there are other kinds of independentsare also there, which is known as the mutuallyindependent andthere arek wise or the pair wise in independence.Suppose, that there arethree,more thantworandom variables, which is the r 1 r 2 r 3,if we will saythat these are mutually independent, if the probability of this particularone,particular that is r 1 equals to x 1 and r 2 equals to x 2 r 3 equals to x 3 and all is equals to the multiplication of the individual probabilities.Then, this can be this can be told that, these random variables r 1 r 2 r 3 are mutually independent.Then, there are k wise independentindependent random variables.Now, out of this set,suppose there are n random variables are there and in that, out of that out of thatn random variables, it willsayto be k wise independent, ifall the subsets of the k variables are mutually independent.

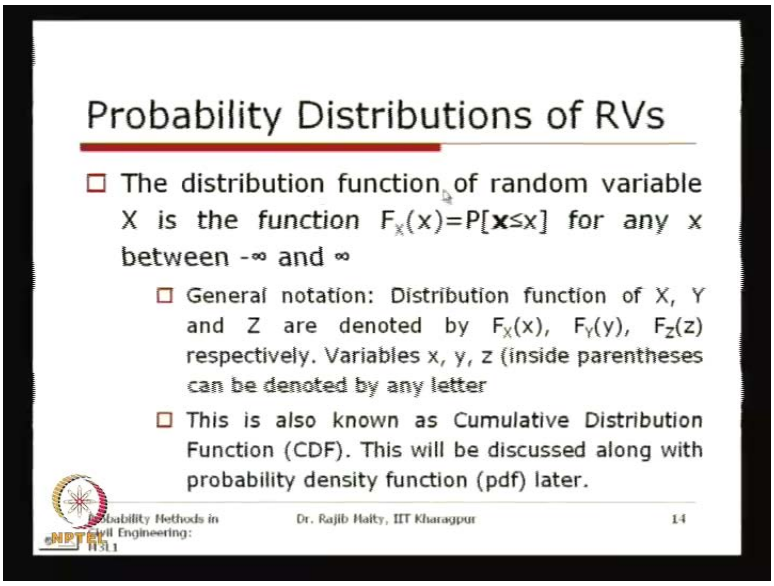
(Refer Slide Time: 28:05)



So, obviouslythe k is less than n.So,whatever the total number of random variables are there, if I now pickup randomly k random variables and if it satisfies that the mutually independence among this k random variables, then we said those n random variable are k wise independent.Now, if this k becomes 2,then when this number of the variable reveals 2 out of n, I take the 2,that if thesubsets are in mutually independent.Then, this random variable are said to be pair wise independent.So, whatis happening?Out of n random variables, Iam picking up n random; I am ppicking up onlytworandom variables.

One small correction here is that, what that r 2 is less than equals to x 2 was written;so it should be r 2 equals to x 2.Now,the themost important thing, why we just learn this random variable is that probability distribution of random variables.

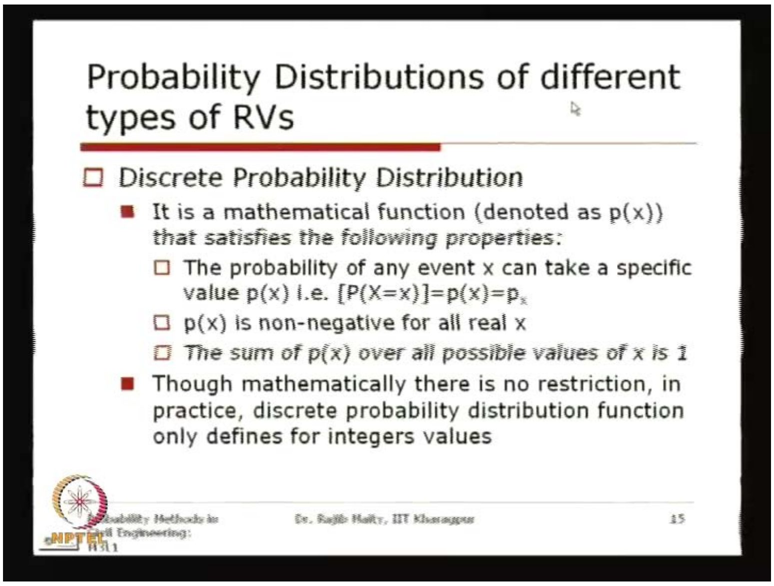
(Refer Slide Time: 29:20)



The distribution function of a random variable x is a function, which is denoted bythis capital letter f subscript.The capital letter of that random variable, if it is x, then this a capital letter of this x.With somesmaller case letter, it may or may not be x, which can which can take any letter.So, which is nothing but this, inside thisone,it is nothing but, as in few previous slides we have seen that this is nothing but, the specific value of that random variable.So, this is denoted as the,this isthe distribution function of the random variable x.

Whichis nothing but, is the probability of that random variable x, when it is less thanequals to that specific value of that random variable.So, this is the way, we define the distribution function of the random variable x, which is which is valid over the region from the minus infinity to the plus infinity, so, the entire real axis. So, here again, the general notation says that the distribution function of x y and z are generally denoted by f subscript capital x any letter lowercaseletter f x x, f y y, f z zrespectively.Thisvariable,that is a lower case letters, thatwhat I was justtelling here, is x inside y inside z inside the parentheses can be denoted by any letter, as these are nothing but, the specific value of that random variable.This is also known as the cumulative distributionfunction,when in the,most probably in the next class, we will just discuss about this cumulative distribution function.This is actually the cumulative distribution function of the,thisoneof this particular random variable; this will be discussed along with probability density function pdf later.So, what we want to tell is that, for a particular random variable, this random variable over this range, some probabilities are assigned for the specific range and that that the how it is how it is distributed over this real axis is known as the distribution function of that particular random variable.

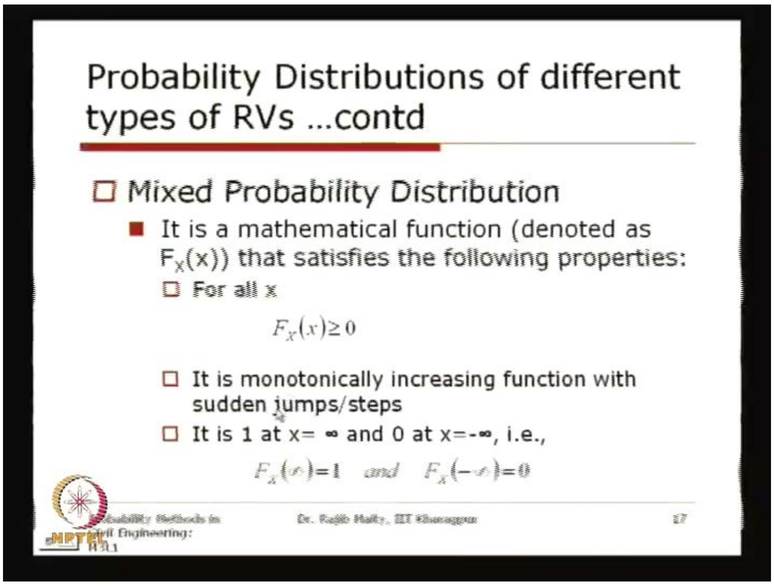
(Refer Slide Time: 31:46)



Now, probability distribution of different types of random variables.Now, we have discussed aboutthreedifferent types of random variables; the first is the discrete random variable.We will see how it is defined thatit is the mathematical function denoted by generally, for when it is discrete we denote by this p x; this is also known as the probability mass function.We will discuss this later.So, this is denoted by p x that satisfy the following properties.The firstoneis that thatthe probability of any particular event x, because this is, as it is discrete, it can any take specific value.all anyMeans, that those specific value, which is there in the feasible sample space, so that particular specificvalue is denoted by p x which is nothing but, the probability of the random variable taking the specific value x, denoted by either p inside thatspecific value x or p subscript that value, the specific value.Obviously, from theaxioms of this probability,from the axioms of this probability this p x is a nonnegative, for all the real x, it can eitherbe 0or greater than 0.

And, the summation of this all this p x over this possible values of x is 1.This, again from the axioms of this probability.Though mathematically, there is no restriction, in practice, discrete probability distribution function only defines for integer values.So, this is just, when we have also seen in previous slide,that some example of this discrete probability distribution that this isgenerally take the integer values.But, it is not specific.It is not, mathematically there is no restriction,and it can take any any specific value,the way I am way I amdefining that random variable.Second type of this random variable is the continuous randomcontinuous random variable.So, this continuous distribution function for that thing, it is a mathematical,it is again a mathematical function, which is denoted by this capital f subscript, that random variable or any lower case letter, particularly this random variable lower case letter that satisfy the following property.

(Refer Slide Time: 35:28)



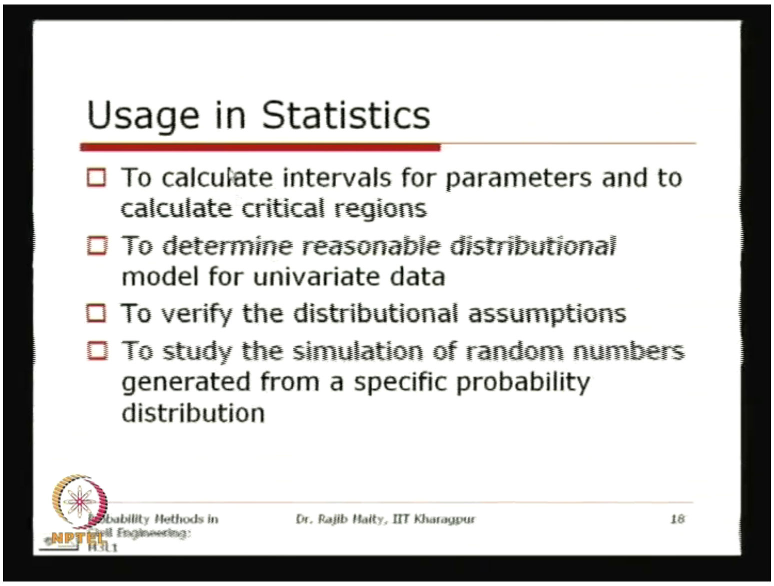
The first is that, for any specific value ofthis(()), should begreater than equal to 0.It is monotonically increasing and continuousfunction.So, here it is monotonically increasing, that means whenever it starts from and it will go and it will go onincreasing and you know that this can go to maximum.So, generally for it is defined from 0to 1.So this, it can go up, it starts from 0and go up to 1 and it increases monotonically and it is then continuousfunction, as this random variable itself iscontinuous.So, it is 1 at x is equals to infinity and 0at x is equals to minus infinity, that is this probability at thisx is 1 and probability x minus infinity is 0.

Then, the last thing is this mixed probability distribution, which is, whichcan take it,for somerange it can take thatdiscrete values and some range can take continuous value.This also denoted by f x and keeping the all other properties, all other conditions, that is the properties are same, which is obviously greater thanequal to 0.Though, with the only difference with this continuous is that, it is monotonically increasing function with sudden jumps or the steps and again the thirdoneis again same.

Now thisone,this lies the difference between this continuous probability distribution and this mixed probability distribution.Now, if you just see it here, as we arejusttelling that there are at somepoint, where the probabilityis concentrated, now, if I want to just see that how this cumulated accumulated over time and so, it starts from here and it will go and go up to maybe the way it is drawn, it should be assumed (())1.So, this is the jump that we are talking about.So, this is the jump where the probability masses are concentrated.

Here it is onlyonce, so it can be concentrated in some other range.Then, there also will beone jump.So,wherever, the probability masses concentrated for some specific value, so they are generally in this distribution function.We see that type of jump here, which is the mixed probability distribution.

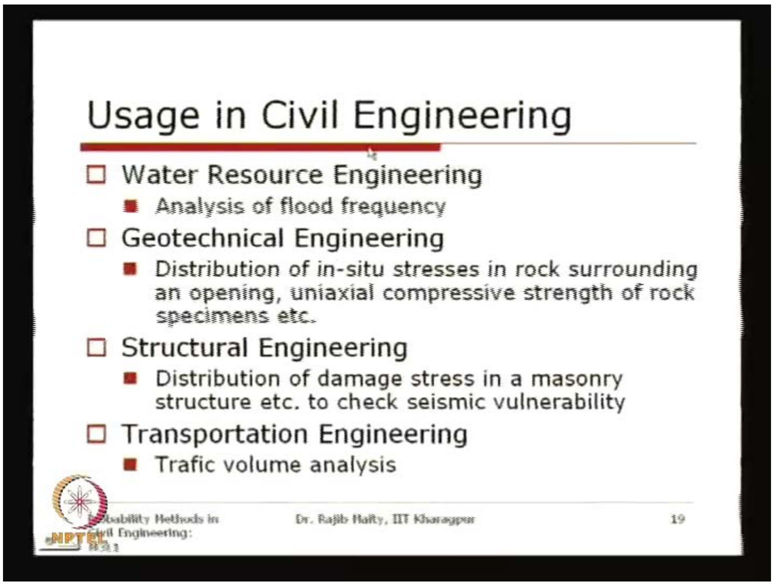
(Refer Slide Time: 36:27)



Now this, the concept, the usage of this concept of this random variable in statistics,inin it is ait is a never ending list, I should say.So,hereare just few examples are given that to calculate the intervals of the parameters and to calculate critical region.We will see this, what is critical region and this interval of parameters in thesubsequent classes to determine the reasonable distributional model for invariant data. Now, this data can be of any field of this civil engineering.

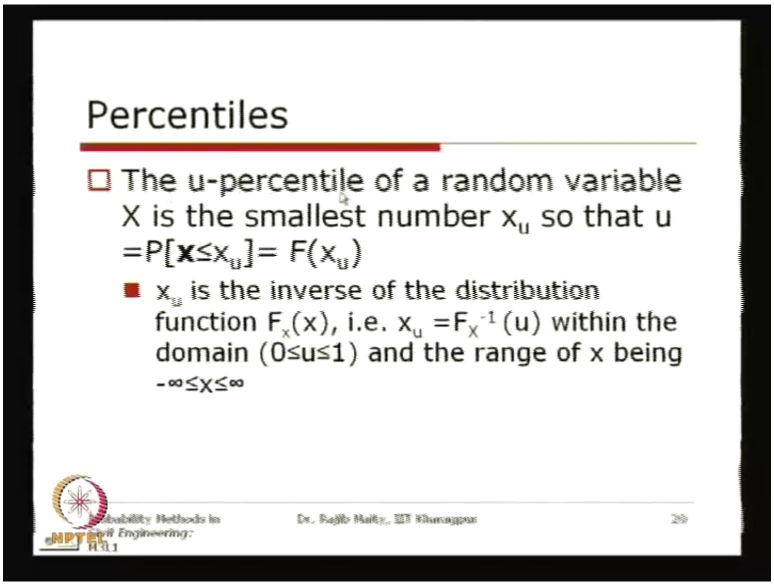
Now, for this analysis,the distributional model for those kinds of data, this is useful to verify the distributional assumptions.We generally,for any probabilistic model, wehave assumed some distribution.Now,we have toverify whether that particular distribution is followed or not to study the simulation of random numbers generated from a specific probabilitydistribution.

(Refer Slide Time: 37:25)



Coming to the specificcivil engineering, there are different facets ofcivil engineering, starting from water resource engineering, geotechnical engineering, structural engineering, transportation engineering,environmental engineering and there are many such.So, in water resource, foroneexample, the analysis of flood frequency, this concept is used.In geotechnical engineering, distribution of in situ stresses in the rock surrounding and opening or the uniaxial compressive strength of the rock specimen, etcetera.In structural engineering,distribution of the damage stress in a masonry structure,etcetera,to check the seismic vulnerability of the structure.Transportation engineering, for example, the traffic volume analysis and this kind of thing indifferentapplication of this civilengineering.

(Refer Slide Time: 38:25)

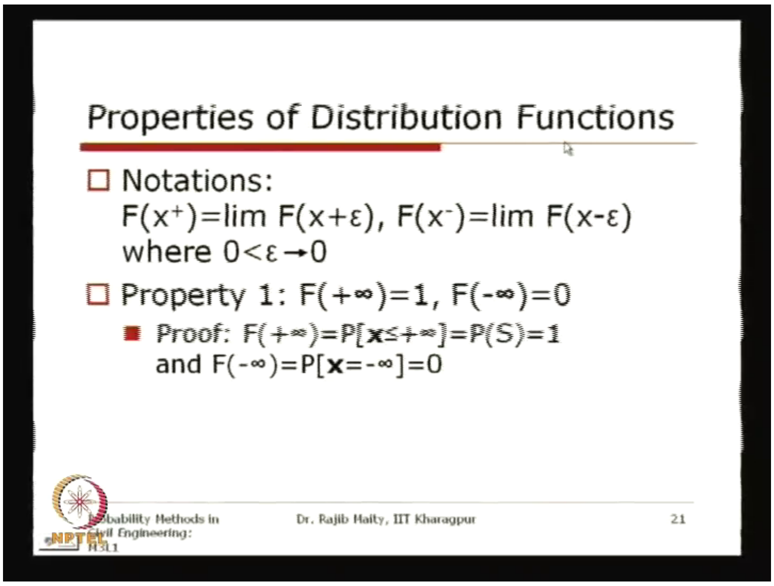


Now, we will seethe the concept of the percentilefor a random variable.The u percentile of a random variable x is the smallest number x u, so that u equals to the probability of x less than u, which is equals to probability of x u.So, to determine what is the value of this x u, the x u is generally the inverse of the distribution function f x, that is, x u is equals to the f x inverse u, within the domain 0 to u to 1.

So now, to know this thing, basically,if the graphically if I just want to see it here, now to show this is the feasible range of that particular variable,suppose this goes on here.So I,now to calculate this percentile, what some percentile if we just say, some u, so here we can say, where is the u.Now, basically, suppose this is here, so basically, we just go and see how much percentage is covered here. So, this particular value, is graphically representing this particular value,is youru percentile of that randomvariable.So, generally what happens, we generally see this particular value, calculate its cumulative probability and get thisone. So, that is what is your mapping, as this f x of any value x, that is your mapping.Now, when you are comingfrom this side, then what we are doing, we are just giving this f x of u inverse will give you some value that x u, which is your x u here.

So, this is thatinverseinversefunction of thatoneto get thatget thatpercentile.So,so the u percentile of a random variable x is the smallest number x u so,that u equals to probability of x less than equals to x u.So, this is obtained that x uis equals toinverse of thatof the distribution,of that particular number,particular random variable for that percentile u.And obviously,the u have the range from 0to 1 and so it is expressed inpercentages taken from 0to 1 range,and the range ofx being whateverwhateverthat range of particular random variable.

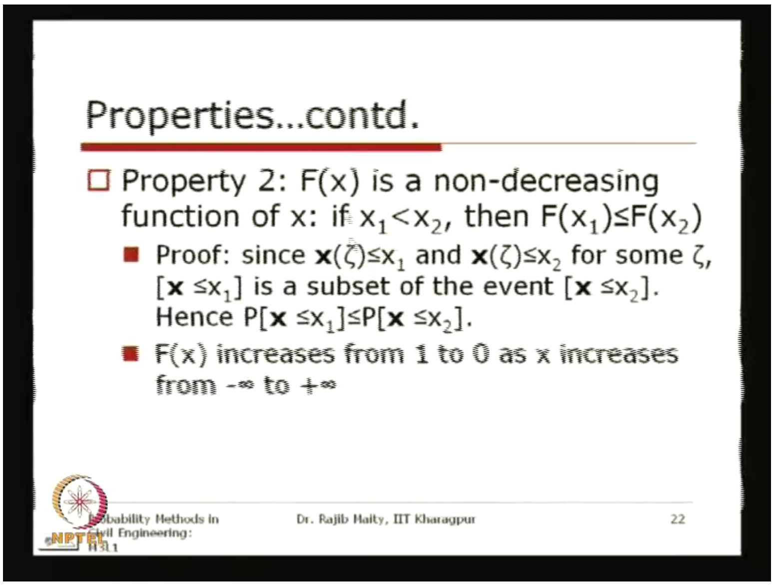
(Refer Slide Time: 40:55)



There are different properties ofthis distribution functions.So we will seeoneby another,onebyoneof theseproperties.

In this discussion, we will follow some notationthis istaken mostly fromthePapoulisbook.So, this is thatf ofx plus of this random variable x,of course, is equals to limit f that particular valuex plus epsilon andfminus means the limit of thisone,when this epsilons are greater than 0but, it is tending to 0.So very smallnumber,so this f x plus means, just right side of that x and f x minus is the just left side of that of thatx.So, propertyone,the first property that is,if I take thisdistributionfunctionfor this infinityis equals to 1,so thatthe right extreme of thisreal x andleft extreme of the real x,it is starts from 0.So, it always starts from 0ends at 1.So, to put this 1,that is f plus infinityis equals to f x less than equals toplus infinity.So,if is f x is less than equals to plus infinity, that means it is encompassing the full sample space.So this is nothing but, thatprobability of the full sample spaces and we know that full sample space from the axioms of the probability, that this is equals to 1.Similarly, so if minus infinity is nothing but, the probability of x equalstox less than equals to, basically, less than equals to minus infinity,which is a,which is basically,basically null set, so probability of the null setis equals to 0.

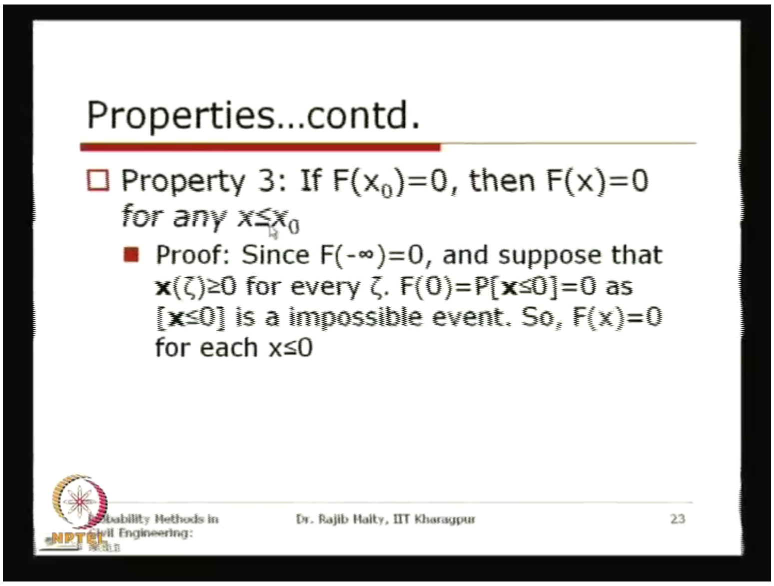
(Refer Slide Time: 42:40)



Second property, f xis a non-decreasingfunction of x.That is, this non-decreasing function or monotonically increasing function,whichis the word that weused in few slide previous.So, it says that,ifthat x 1 is less than x 2,then always this f x 1,the value of this distributiondistributionfunction at x 1 is less than equals to f x 2.So, it can never decrease.So, it will always,it willeither be same or it willit will increase. So,that is basically what is known as this monotonicallyincreasing.Proof of thisone is that, if x xi is less than equal to x 1 and x xi less than equal x 2,for some outcome of this xi, then x random variable less than equals tox 1 is a is asubset of the eventx less than equals to x 2.So what, so this x 1 is always always there within this x less than equal to x 2.If this is greater than,if this x 1 is less than x 2,so that is why if it is a subset of thisone,then obvious the probability this should be less than equals to probability of this x less thanx 2.

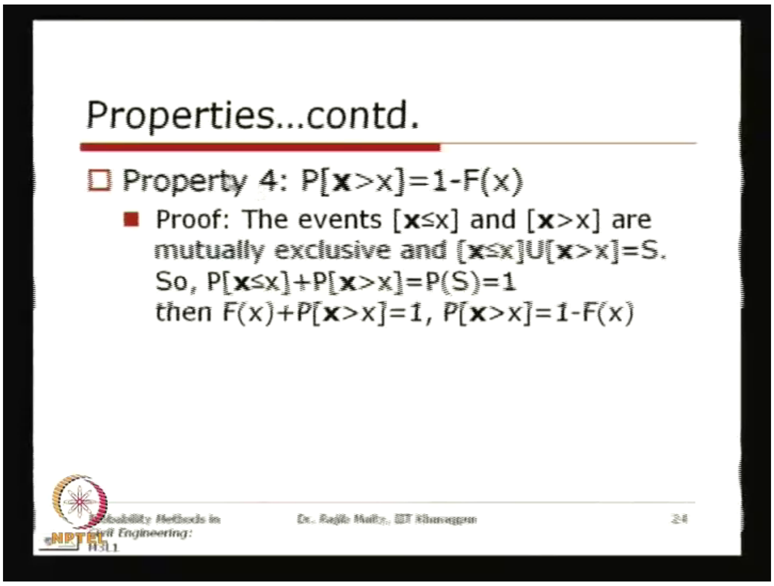
F x increases fromf x increases from0 to 1. Sorry for this mistake.F x increases from 0to 1 as x increases fromminus infinityto plus infinity.This will be 0to 1 as x increases from minus infinity to plus infinity.

(Refer Slide Time: 44:35)



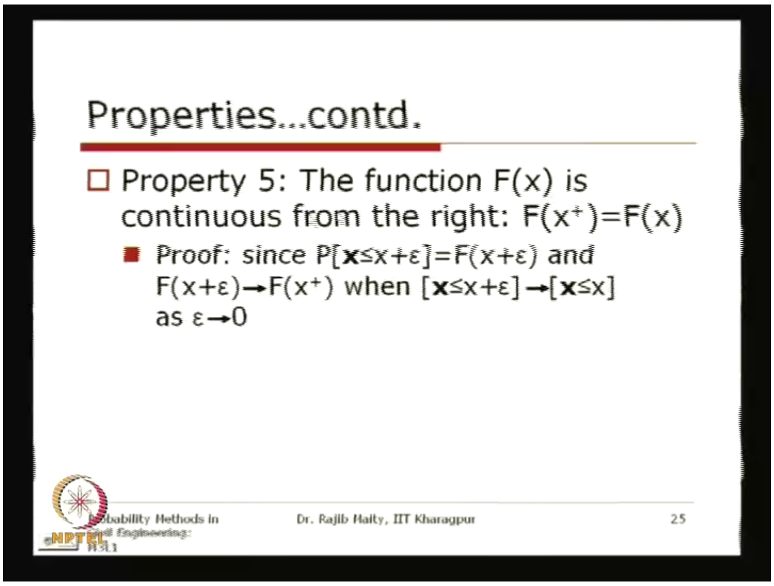
Third property says that,if f x not is equals to 0,then f x equals tof x equals to 0for any, which is which is less than this x naught.So,for a specificvalue, ifthe f x equals to 0,anything which islower than this x naughtobviously will be 0.This is basically the same concept of that it isnon-decreasing function.So, if it is some portion,if it is 0,leftside of that in the real accesscontext

(Refer Slide Time: 45:35)



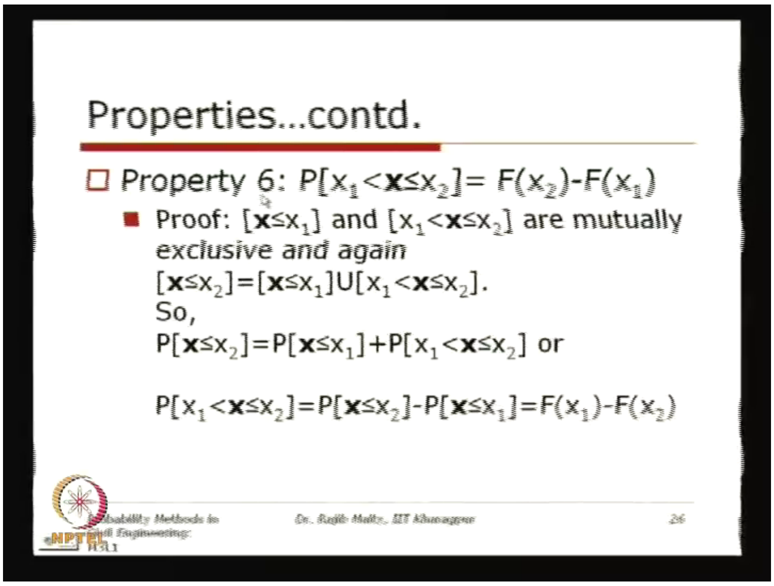
andX greater than x are mutually exclusive.So, if these two events mutuallyexclusive and collectively exhaustive, so collectivelyexhaustive meansthisx less than equals to x union x greater thanx is equals to full samplespace s.So that, probability of this less than x and greater than x, is nothing but, probability ofthe total sample space.That is, the s equals to equal to 1 and now this is denoted by f x, which is thisoneequals to 1.So probability of x greater than x results to 1 minus f x.

(Refer Slide Time: 46:39)



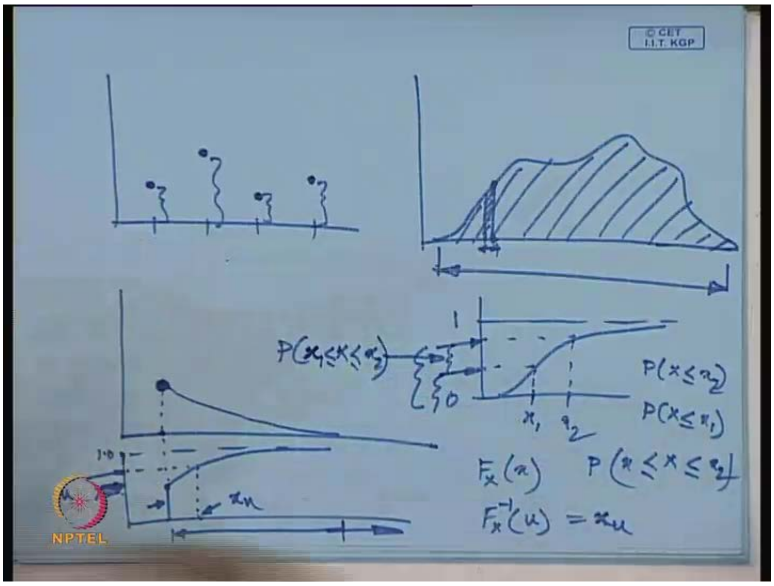
The fifth property says, the function f x is continuous from right.So, f x plus,that is thefrom right is equals to f x.Proof, since probability of x less than equals to x plus xi where this xi is nothing but veryis standing to 0is equals to f x plus xiand f x plus xi is tending to f x plus, when this f, this x is less than equals to x xiis tending to f x less than equal to x, as this xi is tending to 0.So,that is why, this is from this right handside, if we say this is continuousfrom the right hand side of any specific value x.

(Refer Slide Time: 47:31)



Thensixth property says, forfor a randomvariable, if it is bounded by this x 1 and x 2,theprobability of the of thevalue from starting from x 1 to x 2 isequals to the probability of x 2 minus x1.

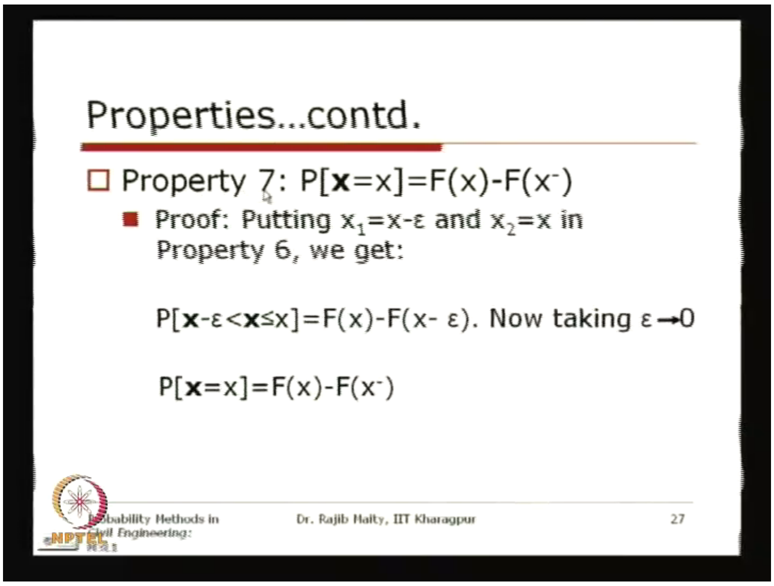
(Refer Slide Time: 47:53)



So, graphically if I just see it again here, that is, if I just taketwovalues, if this is your saystarts form 0and goes like this. So, ifI want to know that, what if this is your x 1,this is x 2, then,theseprobabilities that is, x 2,so probabilitythatf xless than equals to x 2 is nothing but, this particular value.Now, probability of x less than equals to x 1 is nothing but, this particular value.Now,if I want to know that, if this x is in between thesetwo,x 2 and x 1,then this probability is nothing but, whatever total probability this minus thisprobability.So, this is the probabilitythatwe are talking about,which is nothing but, the probability of x 1 less than equals to x less then equals to x 2.So, this is how we get the probability for for a range.

Which are which are again, this proof is proof says thatthat x less than equals to x 1 and this x 1 less than x less thanx 2 are mutually exclusive.Again, x less than equals to x 2 is equals to x less than equalsto x 1 union x 1 to x 2.theseSo, the probability x less than equals to x 2 is probability x less than equals to x 1 plus probability x in between x 1 to x 2.Or, if I just take thisonehere, then probability of x 1,this x random variable between x 1 to x 2is equals to probability of x less than x 2 minus probability of x x 1,which is again nothing but, this f x 1 f x 2.

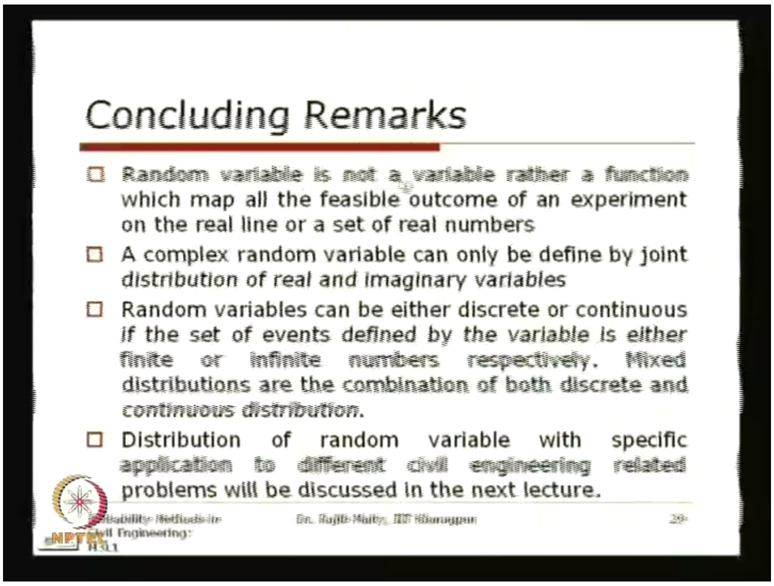
(Refer Slide Time: 49:51)



Now,the propertysevensays, that if probability of x is equals toa particular value, x is equals to probability of x minus probability of xjust left tothatone. So, at a particular point, the probability there, now thesethisproperties are in general for the discreet continuous.So, for a particular point, theprobability says, that at that particular point and just left to that, whatever the probability is there, so it will be like this; from that particularside tojust to the left of thisone. Proof, putting that x 1 equalsto x minus xi and x 2 equal to x in this propertysix,then we can say that probability of x minus xi less than x less than that x equals to probability x equals to probability x minus xi.Now, taking this xitending to 0,then we can say that probability at a particular specific value is that point, probability at that point minus immediate previous value to thatone,that that particular value.

So, if it is a discreet random variable, then justjust left to this value, this xivalue comes to 0. So, that for a discreet value at particularparticular point, the probability is equals to thatthat the functionalvalue at that particular point.

(Refer Slide Time: 51:25)



Finally, in this lecture, we have seen that random variable is not a variable rather a functionwhich map all the feasible outcome of an experiment on the real line or a set of real numbers.A complex random variable can only be defined by the joint distribution of the real and the imaginary variable, that is, if that x plusiy will be equal to that, the joint distribution of this x andx and y, both the random variables.Random variables can be either discrete or continuous if the set of events defined by this variable is either finite orinfinite numbers respectively.Mixed distributions are the combination of both discrete and continuous distribution.Distribution of random variable with specific application to this different civil engineeringrelated problemswill be discussed in the next lecture. I will meet that concept of this probability density function and cumulative distribution function.Thank you.