Probability Methods in Civil Engineering
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Lecture No. # 05
Probability of Events

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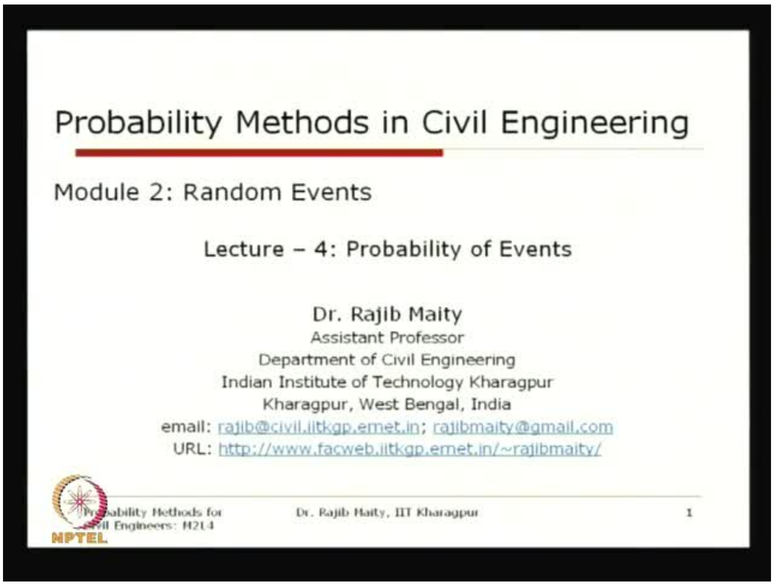

Hello and welcome to the lecture in the course Probability Methods in Civil Engineering. Today, we will cover the Probability of Events, which is very useful for the different applications in the problems related to Civil Engineering.

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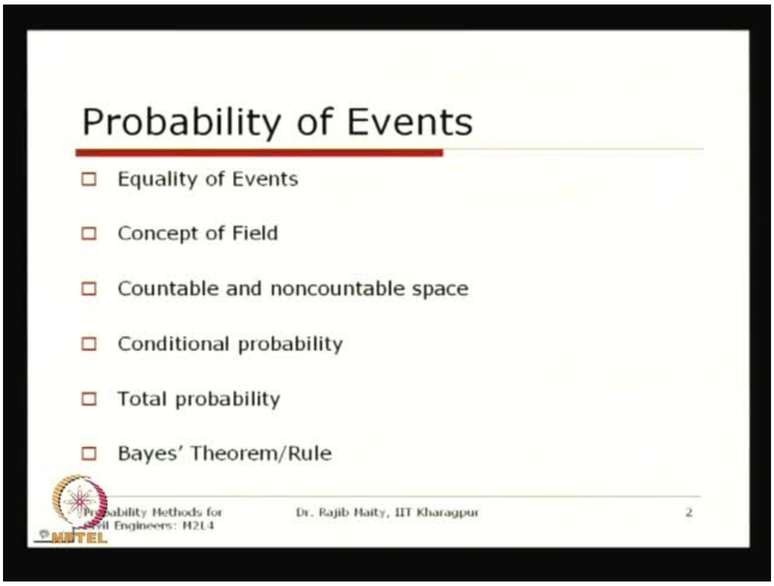

In this lecture, we will first touch a few basic concepts, that is, equality of events and concept of fields, which are useful, particularly when we deal with the most of the problems in the Civil Engineering. Then, we will touch the countable and non-countable space, followed by, we will go to the conditional probability; and with the help of this, we will try to explain the total probability and related theorem, Theorem of Total Probability; and, after that, we will cover this Bayes' theorem and rule. And finally, we will see some of the application problems for applying to this particular concept, and we will go one after another, starting from this equality of events.

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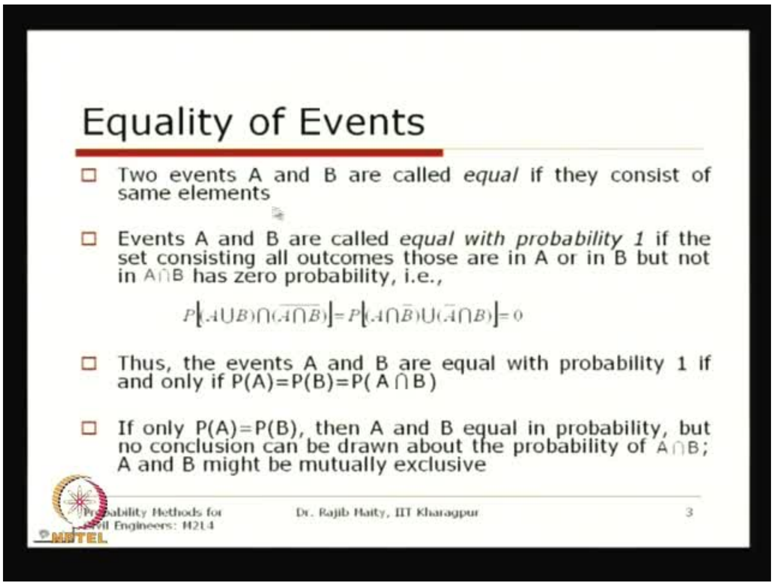

This equality of events says, that the two events A and B are called equal, if they consist of same elements; the event A and B are called equal with probability 1, this equal with probability 1 is important because, then we can say that, all these elements of both the events are same, and that is, if the set consisting all outcome, those are in A or in B, but not in A intersection B has 0 probability; that is, a probability of these which is equal to this one, as I discuss in the previous class, this probability should be equal to 0.

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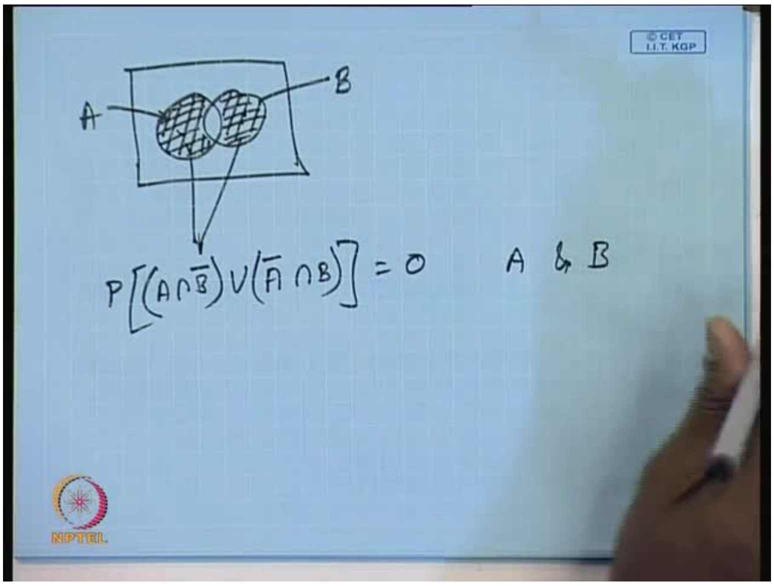

This second part, point, if I want to explain in graphically, then it looks like this. Suppose, that this is one sample space and in which there are two events, one is A; this one is your A and another one is your B. Now, what it says that, if these A and B are equal with probability 1, if the sets, if the set consist of all the outcomes; those are in A, but those are in A as well as in B, but not in the A intersection B.

So, what we mean is that, these two areas, one is this in A or in B, but not in their intersection. So, the probability of these two events, probability of this area should be equal to 0; so, this is exactly what is, what it is meant. So, these two area is nothing but your A intersection B prime, which is union with A prime, A complementary with intersection B. So, this is the, this is the area, and if we say that there is no such element in this, in this area, then that means, what we are trying to say is that, this probability of this event, if it is 0, then we can say that this A and this B, these two events' areas are equal.

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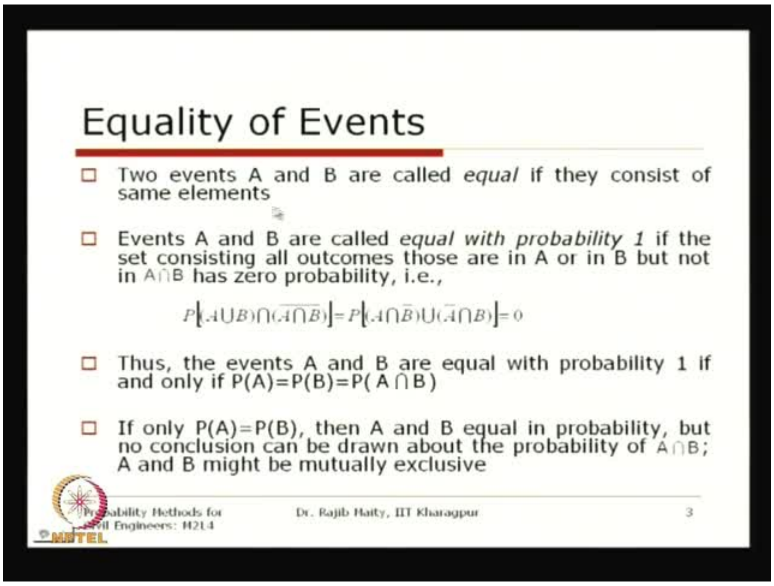

So, coming back to this point again, that this event A and B are equal with probability 1, if the set consisting all outcomes those are in A or in B but not in A intersection B, has zero probability, that is, probability of that; this is a thing I explained, which is obviously equal to… this one also; so, these two, these two events are referring to the same event, whose probability…; if this probability is 0, then we can say that this event A and B are equal. Thus, the event A and B are equal with probability 1, if and only if, the probability of A is equals to probability of B is equals to probability of A intersection B. This is important once again, if we just refer to that particular Venn diagram here.

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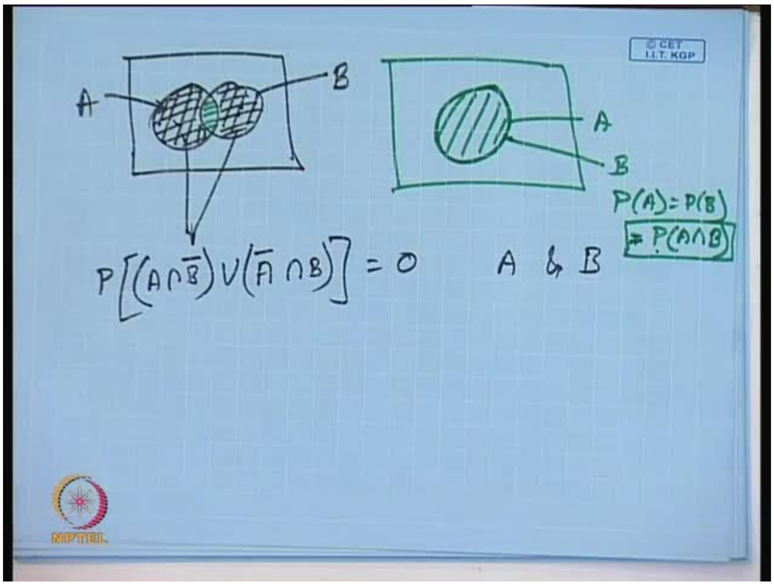

That is, this, if this probability of this A and this probability of B is equal to the probability of their of their intersection, which is nothing, but basically, we are just pulling these two events to be on the same to be on the same event, that is, this as well as this means, this is your A as well as A, this itself is your B; so then, already you can say that, probability of A is equals to, equals to probability of B equals to probability of A intersection B. This All these three elements of this equation is important because, because, if we do not consider these; if we say that probability of A equals to probability of B, that does not mean that these two events are same, so, this must be there.

For example, that if we take the, if we take the example of throwing of one dice, and if we say that the probability of getting 1 or probability of getting 2, both are same; or, if I say the probability of getting an even outcome and probability of getting an odd outcome, so, one is event A, another one is event B, the probability of these two events are same; but we cannot say that these two events are same. So, this one, this intersection, this is a last part, this probability of A intersection B is also, is important to declare, that these event A and B are equal.

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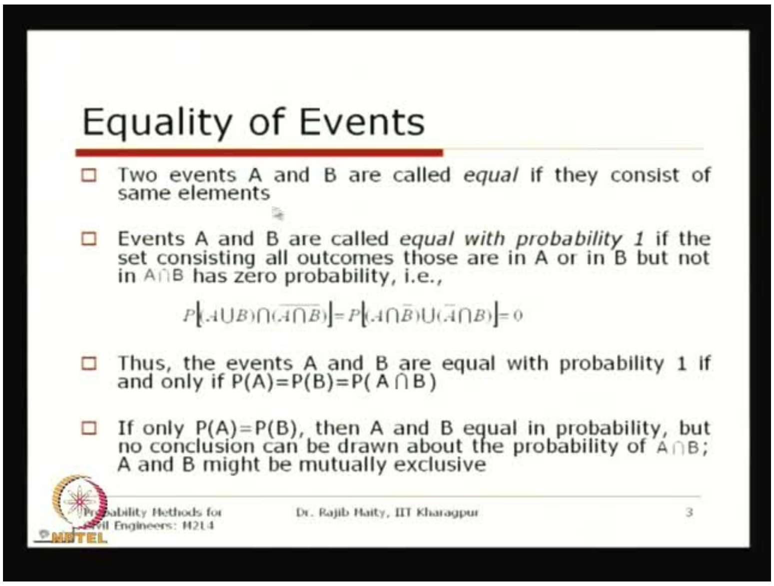

Thus, once again if we…Thus, it is stated that the event A and B are equal with probability 1, if and only if their individual probability is equal to their intersection. If only, that is what just now, we discussed; if only, we say that probability A is equals to B, then the, then A and B are equal in probability; but no conclusion can be drawn about the probability of A intersection B. The example that we are telling, that getting a dice - throwing a, throwing a dice and getting the even number and odd number.

So, these two events are equal in probability, but there, the probability of their intersection is 0. So, that is why the statement states clearly that, if only probability of A is equals to probability of B, then A and B equal in probability; but no conclusion can be drawn about their probability of, probability of A intersection B; A and B might be mutually exclusive.

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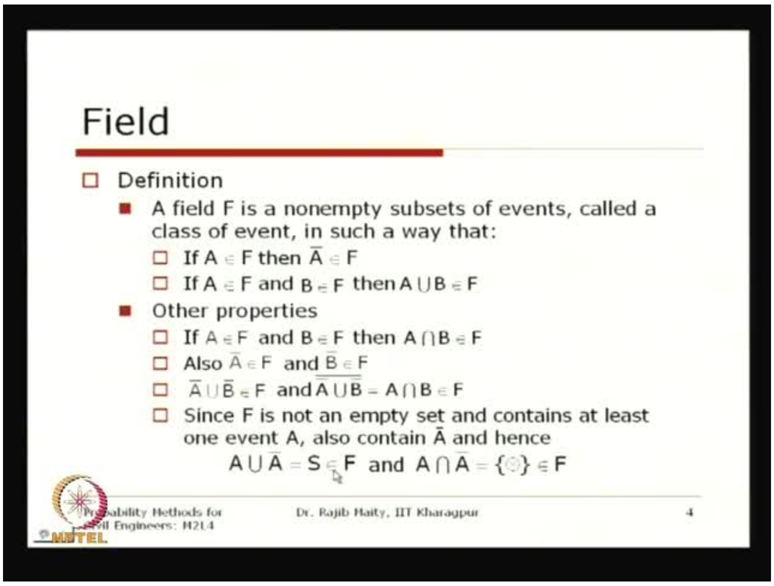

Then, a concept is important, which is known as field. The definition of fields says, that a field F is a non-empty set of events, non-empty subset of events, called a class of event. Now, a class of event is again another definition, where this class means that, we are considering, instead of considering each and every event of a sample space; we are considering only particular subset of the whole sample space; and that particular subset is generally denoted by the class of event.

So, this field, what we are now trying to understand, this field F is a non-empty subset of the event, this is called a class of event; in such a way, this F is defined in such a way that, if any event A, if that any event A belongs to F, then its complementary is also belongs to F.

If one event A belongs to F and another event B belongs to F, then their union is also belongs to F, so, these are the two…; their minimum criteria to define one field, which is a non-empty subset; so, based on these two, there are other properties, as well, which are, which can also be drawn. The other properties of this field that states that, if one event belongs to that field and another event belongs to that field, then their intersection also will be in that field.

Also, if the complementary of one event belongs to F, and the complementary of another event belongs to F, then, we can say the complementary, the union of their complementary, that is, complementary A union complementary B also will belongs to F. And, the complementary of the union of individual complementary, that is, A complementary union B complementary (full thing), their complementary, which is nothing but equal to A intersection B, is also belongs to F.

Last one, since A, since F is, F is a nonempty, sorry for this mistake, F is a nonempty set and contains at least one event A. Also, so it also contains that A complementary, that is, it contains at least one event which is denoted as A here; so, it also contains that A complementary. Thus, the A union A complementary which is nothing but the full sample space, so, that is also belongs to that field; and, A intersection A complementary, which is nothing but a non-linear event; so, that is also, that is also belongs to that F. So, this, so, this S is nothing but almost a certain event; and this is almost, this is the impossible event, that is, a null set; these two are also, these two extremes are also belongs to the field.

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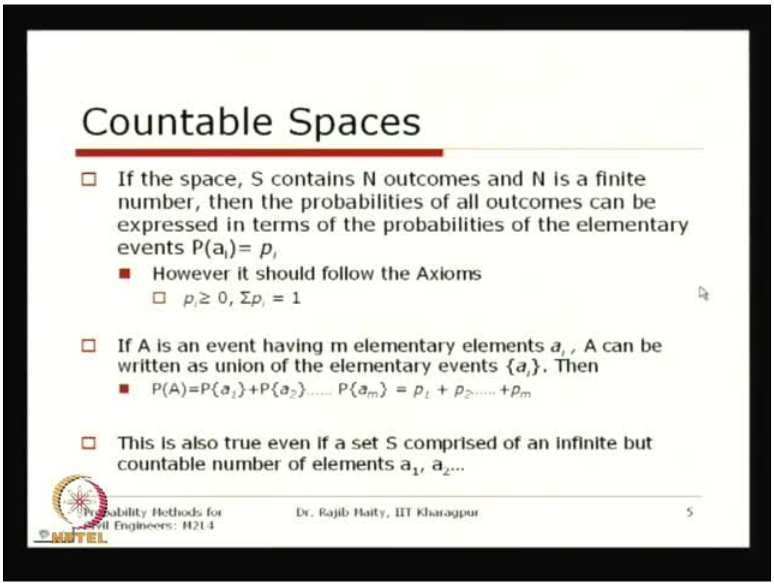

Next is countable space. Sometimes, if the space, S contains N outcomes and N is a finite number, then the probabilities of all outcomes can be expressed in terms of the probabilities of the elementary event, probability of a i equals to p i.

So, if there are N finite, N countable or countable events are there in the space, then their probability can be defined by the individual events. For example, here it is shown that probability of a i is equals to p i. However, it should follow the axioms that, this each, the probability of each, the probability of each and every events should be greater than equal to zero and their summation should be 1; which is directly following from the from the axioms of the probability that is discuss in previous classes.

If A is an event having m elementary elements, a i ; A can be written as the union of the elementary events a i. Then the probability of A is nothing but the summation of their individual probabilities, which is nothing but, p 1 plus p 2 plus up to p m. So, there are m elementary events are there; if you just add up, we will get the probability of that of that event A. This is also true; even if the set S comprised of an infinite, but countable number of elements a 1, a 2 in this, and so on. So, even though I am talking about this this countable space, if it is true, when the S comprised of an infinite but countable number of elements in such a way, that a 1, a 2 in this, and so on.

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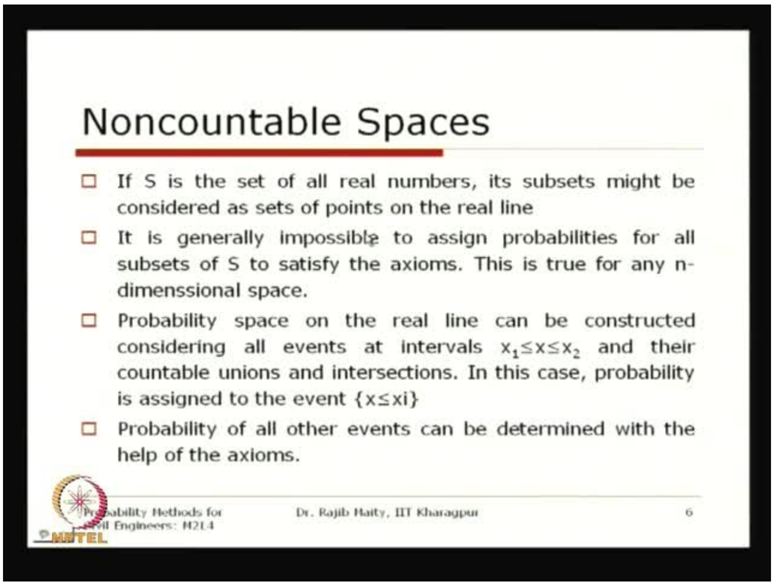

So, a contrast to these countable space, which is, which is more important in…; particularly for these applications in this Civil Engineering, where most of the cases we will see, which is the non-countable space. In many cases, we have seen that the total sample space cannot be cannot be defined just in terms of few elementary events, rather it should be expressed in terms of a non-countable set. For example, if we take the example of the real line, then whatever the number that lies on this real line, it consist of this full sample space.

Now, for such cases, how to define the probability, that is, now our, now we are going to understand. So, it is not only for the real line which is a one dimensional picture, it can be that the concept can be extended to any n dimensional space. So, it can be the two dimensional, where it refers to the areas, or three dimensional which is referring to the volume; and in this way, it can be explained that the concept can be extended to any n dimensional space. Now, here we will discuss about the one dimensional, that is, the real line. So, if the, if S is set of all real numbers, its subset might be considered as a set of points on the real line. This is generally impossible to assign the probabilities of, probabilities for all subsets of the S to satisfy the axioms. It is true for any n dimensional space, just now what I have discussed.

So, the probability space on the real line can be constructed considering all the events at any intervals, where x lies between x 1 and x 2, and x 1 and x 2 can be of any real number on the real line and their countable unions and intersection. So, in this, in this case, particularly… So, what I mean is a real line, that one dimensional case, the probability is assigned to the event, x less than equal to x i, so, x i is any number on this real line. So, if we just define the probability that, probability of x less than equals to x i, then, then this is sufficient to explain the entire set of this probability for the entire sample space. The probability of any other event, all other events can be determined with the help of the probability axioms.

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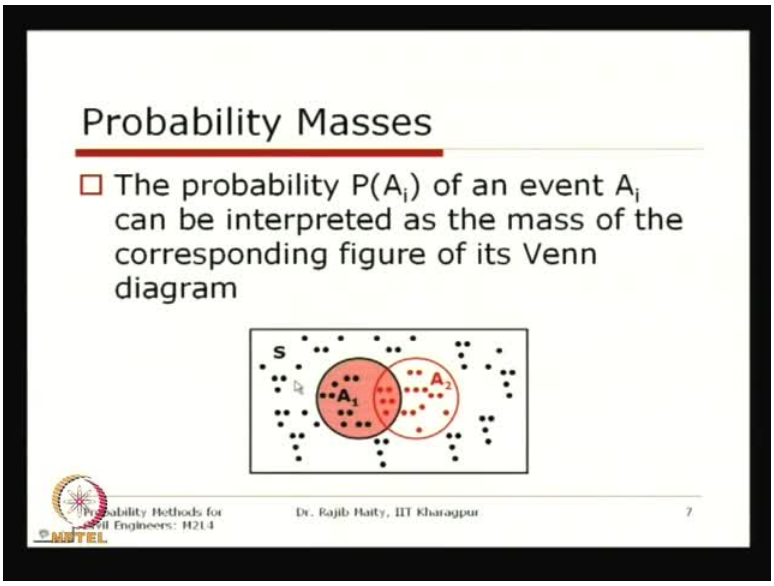

Now, probability masses. Now this, the probability P A i of an event A i, of an event A i can be interpreted as a mass of the, as the mass of the corresponding figure of its Venn diagram here; whatever you can see here, that, this is a Venn diagram that is shown. Now, if these dots are the outcome of the experiment and all these dots are consist of the sample space, now the probability can be treated as a concentrated mass to these points, to this outcome.

Now here, if I just extend this one to this, to this continuous field; suppose that, instead of being an elementary event, if the set consist of this, consists of a continuous event, then what will happen; we will just discuss in a, in a minute.

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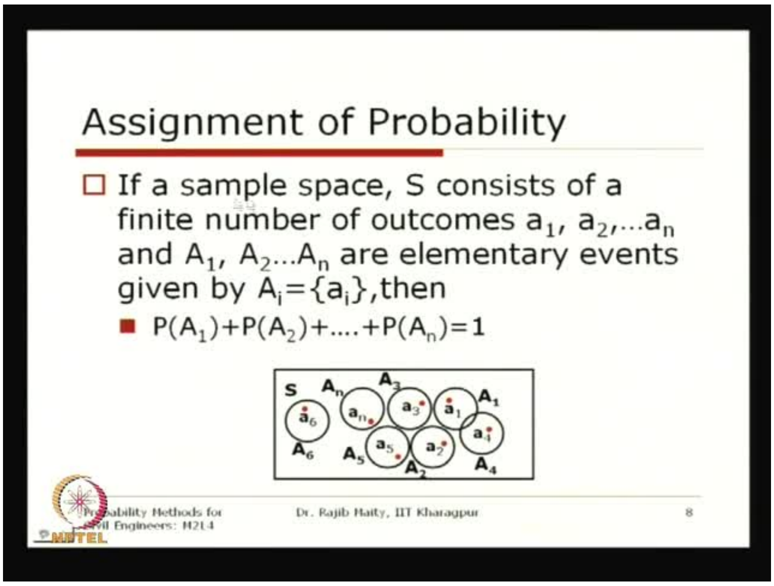

The second thing is that, for these cases, where it is an elementary A events, the if a sample space S consist of this finite number of outcomes a 1, a 2 up to a n, and this A 1, A 2… A n are the elementary events, then by that, this a i corresponds to the event A, the A i, then this probability of all these events should equal to the 1 which directly follows from the axiom of this probability.

Now, what just now I was telling was that, instead of being this discrete point, if it is a continuous point; this is important in, in the sense that, what we can…; in that case, what we can imagine that, this probability is a, is a mass in where, in this field, that is, that can be expressed in terms of the density. Now, if I take one elemental area of that particular sample space, then the total mass, that is, the total area multiplied by the density; the total mass will be give you the probability for that particular, for that particular event.

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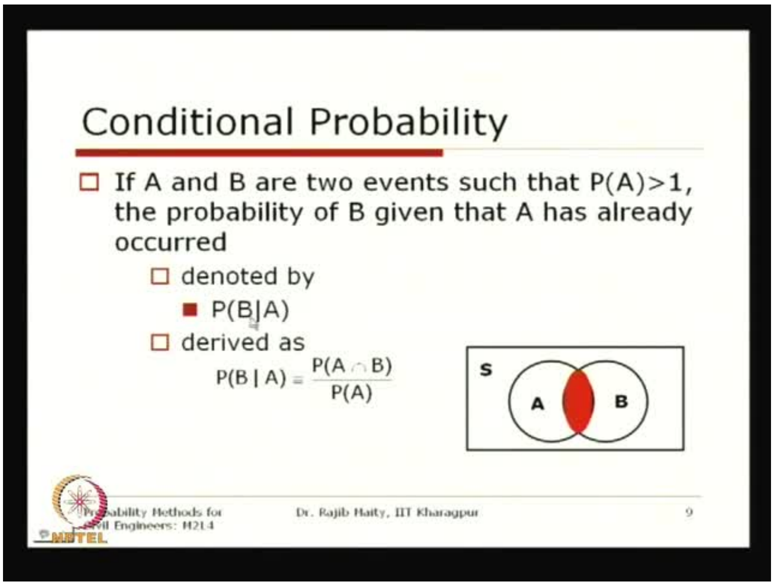

Now, in the previous class also we have discussed about the concept of this conditional probability. So, here, just to recall the fact that, if there are two events, one is A and another one is B, and these events are taken in such a way that, the probability of A is, sorry for this mistake, this should be greater than 0. So, if we say that the probability of A greater than 0 - not 1, this is greater than 0 - then, the probability of B given that A has already occurred.

So, this is expressed in terms of the probability B on condition A, so, this is known as the conditional probability; so, which is differed from this probability of this B, in the sense that, when there is no other information is available, this is simply the probability of one event B, so, probability of B.

Now, if we say that A has already occurred, now, that, so, one information is available. So, based on the available information, whether the probability of the other event may or may not change; and this is known as the conditional probability, which is denoted as like this, B on condition A, which is derived as the, as that probability A on condition A which is equal to the probability A intersection B divided by probability of A.

So, here, so, if we see, if we refer to this Venn diagram, then what if we simply say, what is the probability of B? Then, we will just concentrate to the event B which is shown by this circle. Now, if we say that A has already occurred, then we know that our sample space or our total feasible space as, as is now within this; this zone which is denoted as the event A. Now, the success of this one; so, what is the probability of B? So, the success area is highlighted in these areas, in this way, which is the intersection of these two event; that is A intersection B. That is why, this is the success, this is the area, where the success is realized to declare the probability of B.

Now, the total feasible space as we have seen that, as A has already occurred, so, the probability of A comes here; so, it says that conditional probability of B, given that A has already occurred, is equal to probability of A intersection B divided by probability of A. We will use this relationship to form our base role which is very important, so far as the application is concerned; we will refer to this one. Before that, we will try to understand how this probability of particular event can be derived in terms of the other events.

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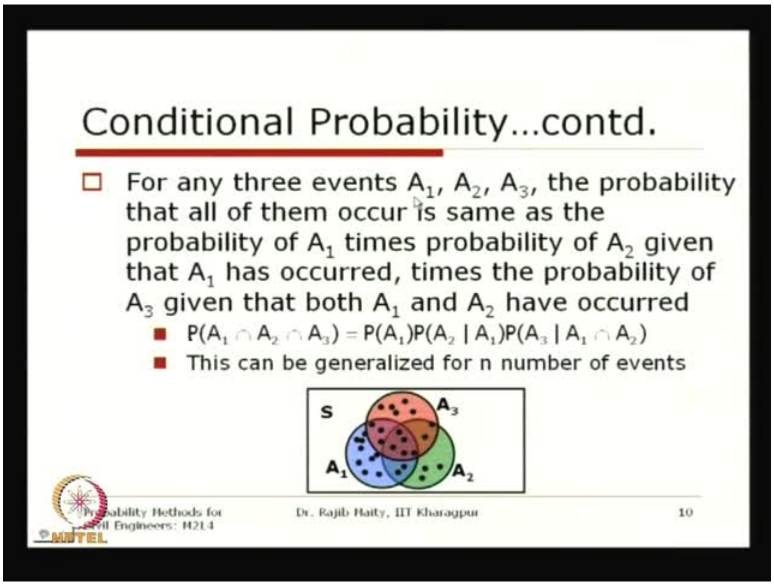

However, still, before that discussion, we will discuss about the same, that conditional probability, when we are talking about the more than two events. So, for any three events, that is A 1, A 2 and A 3, the probability that all of them occurred is the same as the probability of A 1 times probability of A 2, given A 1 has occurred times; probability of A 3 given that both A 1 and A 2 has occurred.

So, this if you want to, if you know that there are three events A 1, A 2, A 3; if we say that - what is the probability of the simultaneous occurrence of all these three events; this can be expressed in terms of probability of A 1 multiplied by probability of A 2 on condition A 1, probability of A 3 on condition A 1 and A 2, both has occurred. This is just followed from this two event case, from this conditional probability case, that is, probability of A on condition B is nothing, but equal to probability of A multiplied by probability of B on condition A.

Now, extending the same thing, we are getting here, for these three events, that is probability of A 1, A 2, A 3. First we take the first event, that is probability of A 1 multiplied by probability of A 2 on condition A 1, probability of A 3 on condition A 1 and A 2, both has occurred.

So, extending this same thing, this can be generalized for this n numbers of events also; so, probability of A 1 intersection A 2 intersection A 3 intersection A 4 up to, if we go ahead like this, then we will say that probability of A 1 multiplied by probability of A 2 on condition A 1, probability of A 3 on condition A 1 and A 2 multiplied by probability of A 4 on condition A 1 A 2 A 3, all three has occurred; and this will go on in the same way, as it is go, as it is going on for this n numbers of different events.

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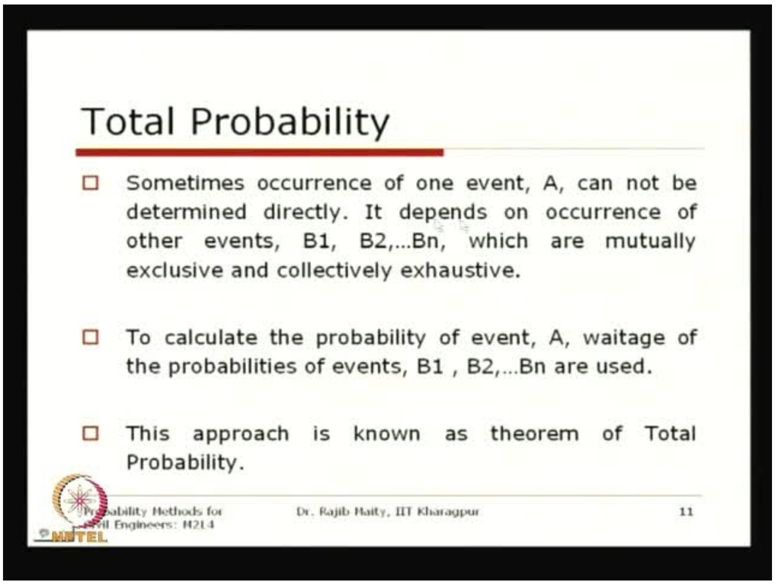

Next is the concept of the Total probability. Sometimes, the occurrence of one event A cannot be determined directly. It depends on the occurrence of the other events such as say B 1, B 2 upto B n which are mutually exclusive or collectively and collectively exhaustive.

So, now, I just spoke these condition here, which are mutually exclusive and collectively exhaustive; which is generally leading to the Theorem of Total probability. So, even I do not know the probability of a particular event, but from the experience if you know the probability of the other events, then this probability can be expressed in terms of this one. To calculate that probability of this event A, the weightage of the probabilities of the event, of the event B 1, B 2 and B n are generally, are used; and this approach is known as the Theorem of Total probability.

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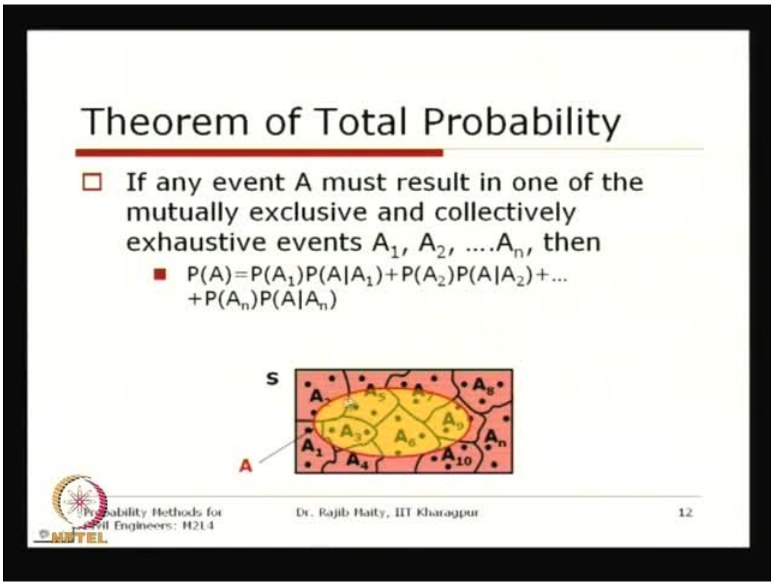

So, this Theorem of Total Probability says, that if any event A must result in one of the mutually exclusive and collectively exhaustive events A 1 to A n; I take a minute to explain one second; the mutually exclusive and collectively exhaustive - this means, the occurrence of one event, for example, this A 1 to A n, what is shown here, the occurrence of any one event implies the non-occurrence of all other events. This is meant by the mutually exclusive.

And, collectively exhaustive means, the probability of A 1 plus probability of A 2 plus, up to in this way, probability of A n should equal to 1. So, if we see here, if this full rectangle is your sample space, then this is, these are the events which are non-overlapping to each other, this A 1, A 2, A 3, A 4, up to A n. So, these events are known as mutually exclusive and collectively exhaustive. So, if these are the events, then the probability of another event A which is overlapping with, with all these events is equal.

It can be expressed as, probability of A is equals to the probability of A 1 multiplied by probability A on condition A 1 plus probability of A 2 on condition probability of A on condition A 2, in this way it will go up to probability of A n multiplied by probability of A on condition A n; so, this theorem is known as the Theorem of Total probability.

So, these, the events, this probability of A 1, probability of A 2, probability of A n generally as known from the experience, and probability of A 1 on condition A on condition A 1, probability A on condition A 2 also known from the previous experience. When both these information is known to us, then, if you want to know what is the total probability of the event A, then these probabilities, these conditional probabilities are, are weightage to the individual probability of the individual events A 1, A 2 up to A n. This is the, this is the basis of this total probability theorem.

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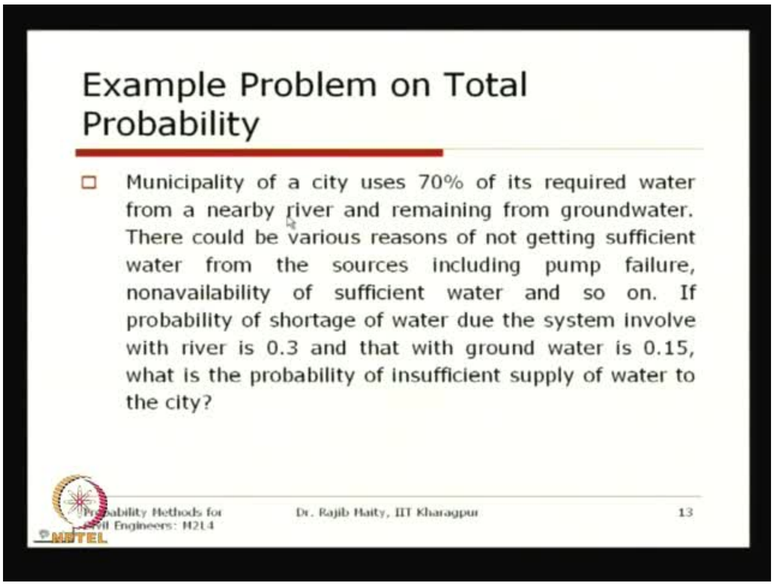

Now, if you, if you see one example problem using this total probability problem, this is a water supply problem; Municipality of a city uses 70 percent of its required water from a nearby river and remaining from the ground water, that is, 30 percent is used from this groundwater. Now, there could be various reasons for not getting sufficient water from the sources including the pump failure, non-availability of the sufficient water and so on. So, the failures can be of, I can, I am just dividing the failure of not supplying to, not supplying sufficient water into 2 parts.

One is which is related to the supply from the river, another one is related to the supply from the groundwater. So, if the probability of the shortage of water due to the system involved with the river is 0.3, and that with the ground water is 0.15, what is the probability of insufficient insufficient supply of this water to the city?

Now, here in this problem, we can see, that this probability of insufficient supply of the water is my total probability that I am looking for, which is depending on this; here it is 2, and that can depend on the many factors.

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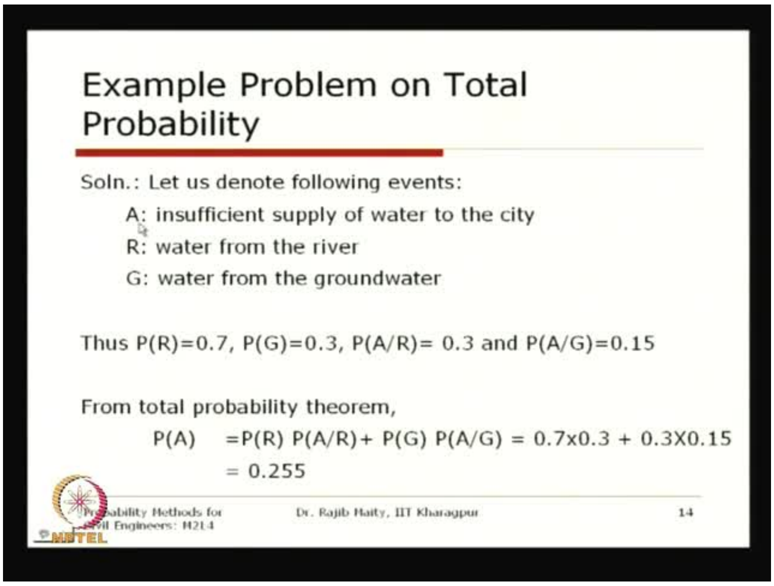

So, now, if I just define the events like this, that A event is the insufficient supply of the water to the city and R is the water from the river and G is the water from the groundwater. Now, from the problem, we have seen the probability of the the water that we get from the river is 0.7, from the groundwater it is 0.3. Now, probability of the insufficient supply in the case of the river, it is 0.3; and probability of insufficient water in case of the ground water, it is 0.15 Then, so, from the total probability theorem, that is probability of the insufficient supply of the water to the city, this equals to probability of R weighted to the probability of insufficient supply in case of river, and similarly, for this groundwater, so, which is equals to your this calculation which comes, that the total probability of insufficient supply of the water to the city is 25.5 percent.

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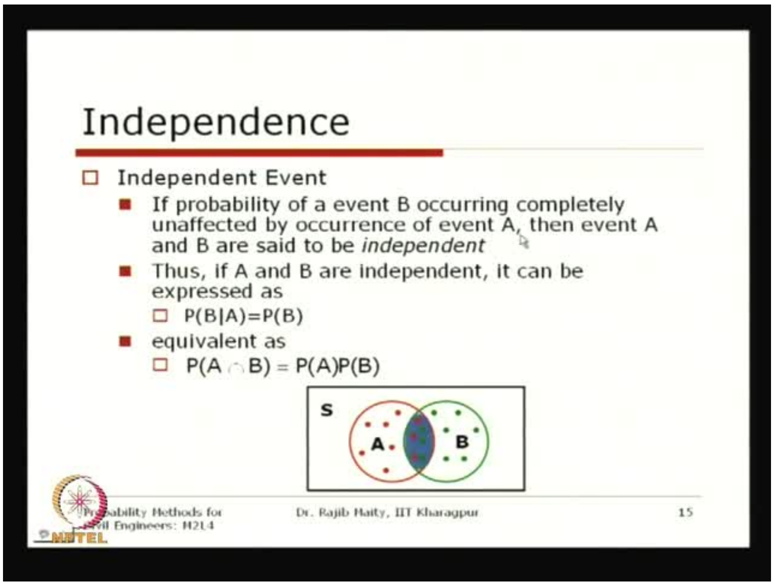

Now, another important concept is known as independence which is, which is relevant here to discuss is that, independent event, if the probability of a event B occurring completely unaffected by the occurrence or non-occurrence of the event A, then event A and B are said to be independent. Thus, if A and B are independent, then it can be expressed as probability of B on condition A is equals to probability of B.

So, whether the A has occurred or not, it does not have anything to change the probability of B, so, which is equivalent as the probability of A intersection B is equals to probability of A multiplied by p

robability of B.

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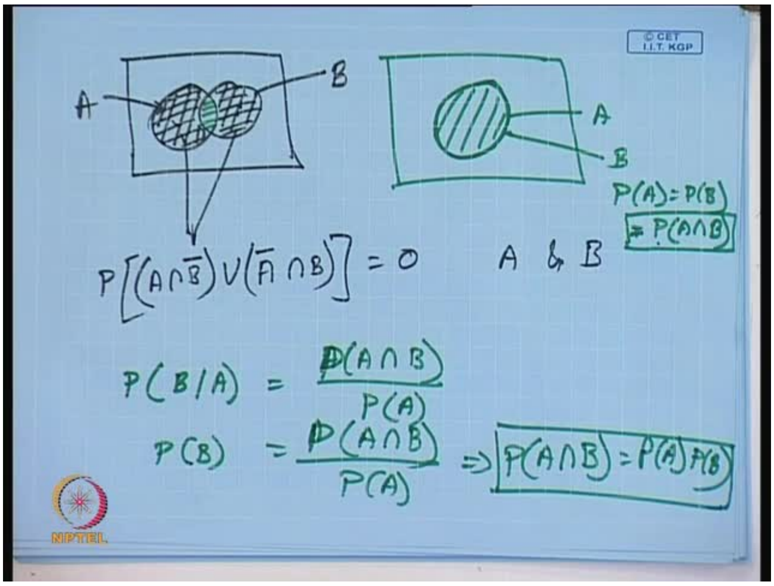

So, which is directly following from the conditional probability which is, just we have seen, the probability of B on condition A is equals to probability of A intersection B, probability of A. Now, this is probability of B on condition A; if these two are independent, then, this is nothing, but equal to probability of B. So, A intersection B divided by probability of A which… So, this is generally the basis to declare in mathematically the two, two variables, two random variables are independent which will be used in from the, from the next class onwards.

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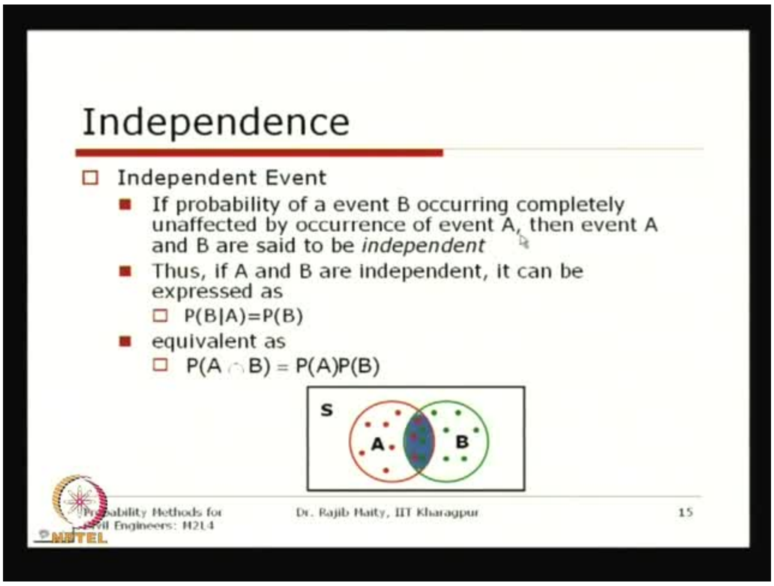

So, this is forming the mathematical basis to declare two events to be independent; it is said that, if and only if, the probability, their joint probability is equals to the, to the multiplication of their individual probability, then we say that these two events are independent.

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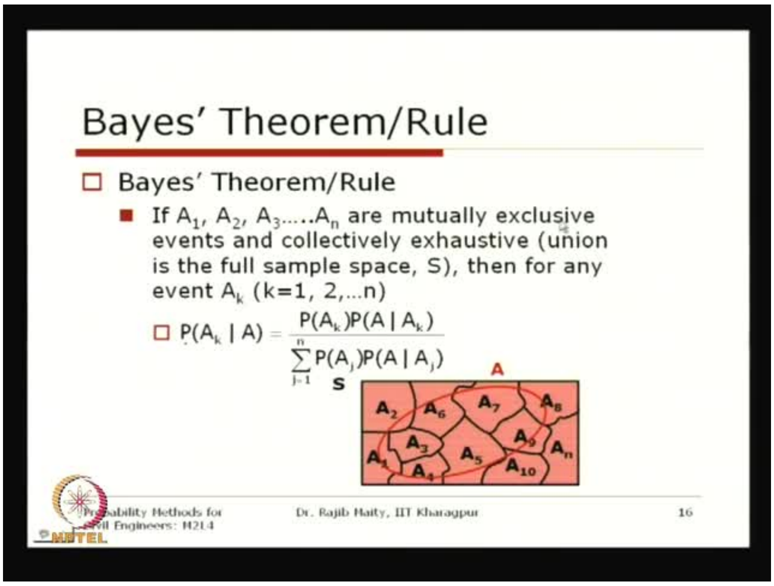

Bayes' theorem or rule. If A 1, A 2, A 3, A n are mutually exclusive events and collectively exhaustive; just now, we have discussed about this mutually exclusive and collectively exhaustive; then for any event A k, k can range from 1 to n, what we can say that this probability of A k on condition A is equals to probability of A k multiplied by probability of A on condition A k divided by summation of all these probabilities, which probability of A j multiplied by probability of A on condition A j. This is known as this Bayes' theorem which we will just see that, this comes from this total probability theorem; that is, if we know the probability of individual event and after that we know the occurrence of one, one particular event A, which is shown as a red ellipse here, then, what are the probability of these different sub-events which are mutually exclusive and collectively exhaustive, is generally obtain from this Bayes' theorem or Bayes' rule.

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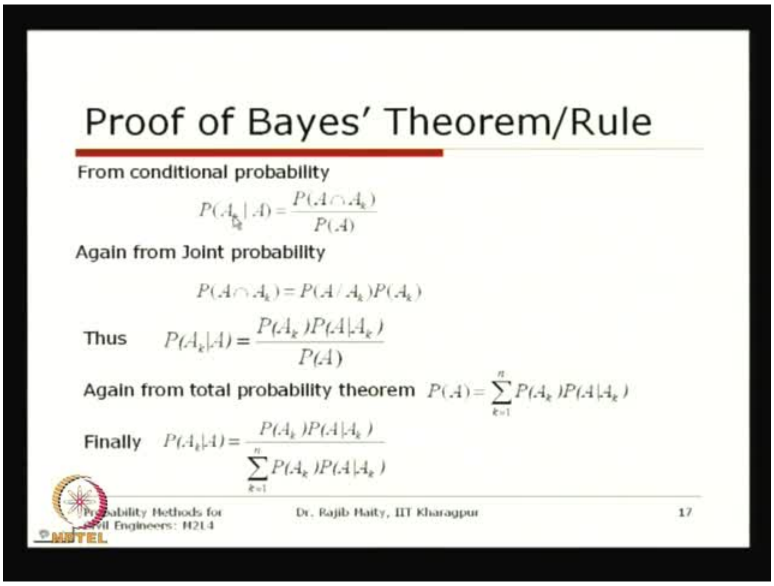

The proof of Bayes' theorem can be explained like this, that is, if I take any particular event A k, k can be from 1 to n; So, A k, a particular event on condition that A has occurred can be expressed in terms of, just now we have seen the conditional probability, so, this A intersection A k divided by probability of A. Now, again from this Joint probability, this probability A intersection A k can be expressed as probability of A on condition A k multiplied by probability of A k.

Now, from here if we just replace this one to this part, then we see that probability of A k on condition A is equals to probability of A k multiplied by probability of A on condition A k divided by probability of A. Now, this probability of A can be expressed in terms of this total probability theorem. Then, from this total probability theorem, this probability of A can be expressed as this; for all this k, it should be the summation of the probability of A k multiplied by probability of A on condition probability of A k. Now, if we replace this one here, so, probability of a particular event on condition A is equals to probability of A k multiplied by probability of A on condition A k divided by summation of for all k probability of A k multiplied by probability of A on condition A k.

Now, these probabilities, these individual probabilities have, is generally having different meaning. First of all, this probability of A k; so, what we are doing is that, if we say that, this left hand side is unknown and right hand side is known, then what we are trying to do is that, we know the probability of A k without any condition; what we are trying to understand, what we are trying to get is, of course, the probability of the same event here is A k, here is also A k; but here, the condition is that occurrence of the particular event is known. So, this one, this probability of this individual events, which are mutually exclusive and collectively exhaustive, these events are my prior knowledge.

Now, from this prior knowledge, due to the occurrence of the particular event, I want to update the knowledge of the probability of A k. So, this is known as the posterior, so, probability of A k is your prior and probability of A k on condition A is your posterior, posterior. Now, this probability of A on condition A k, that is, if we have the data available to us, then these probabilities can be calculated from the previous experience and the previous experiments. So, these are, this is known as the likelihood of that particular, of that particular event; so, this is known as the likelihood. Now, this denominator part is, is coming from this Total probability theorem, which is equals to the probability of the, probability of A. So, as compared to these probabilities, this can be treated as a constant. So, as this can be treated as a constant, if we take this out, then this equality sign will convert to proportional, proportional sign.

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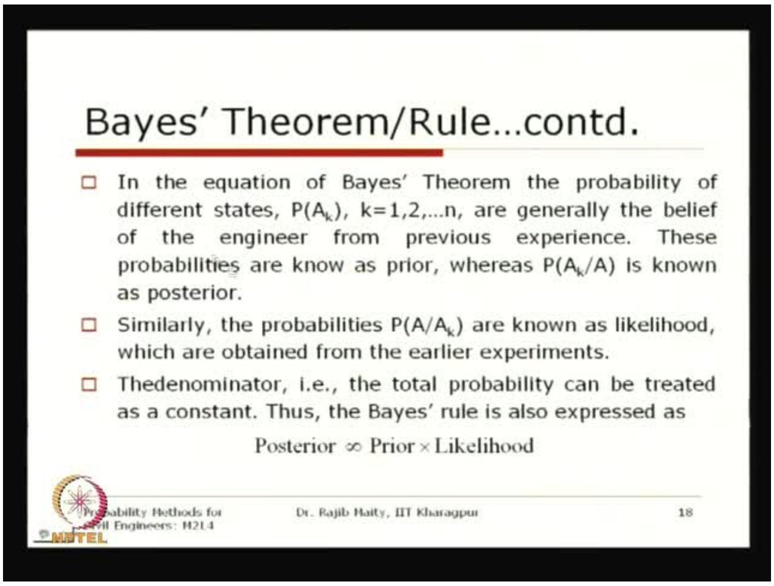

So, this, if we just want to discuss it, then in the equation of this Bayes' theorem, we have seen that this probability of different events, that is probability of A k are generally the belief of the engineer from the previous experience. These probabilities are known as prior. Whereas the probability of that particular event on the condition that one event, one particular event has already occurred, this is known as the posterior.

Similarly, the probabilities of this A on condition A k are known as a likelihood, which are obtained from the earlier experiments. In the denominator, there would be one space here; the denominator, that is the total probability can be treated as a constant. Thus, the Bayes' rule is also expressed as so, left hand side, that, that is your posterior, which is proportional to the prior multiplied by the likelihood.

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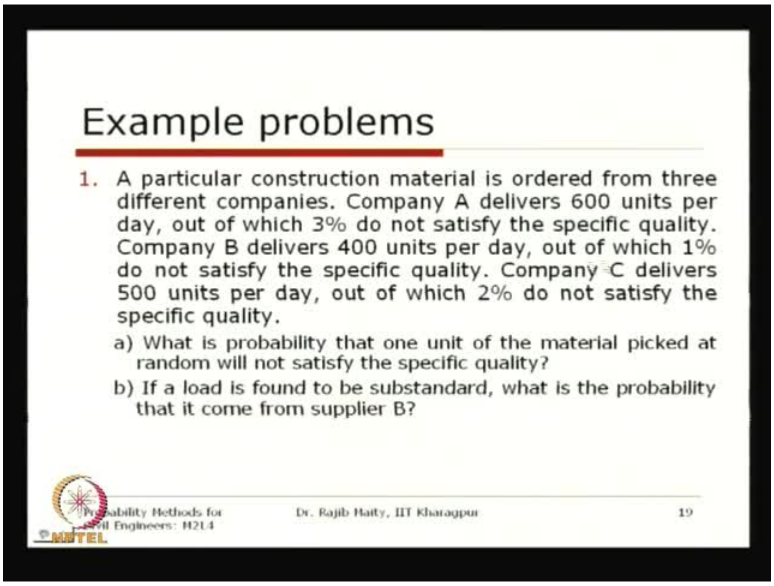

So with this, we will take one particular example where a particular construction material is ordered from 3 different companies. So, the company A delivers 600 units per day, out of which, the 3 percent do not satisfy the specific quality. Company B delivers 400 units and out of which 1 percent, 1 percent do not satisfy the specific quality. Now, the company C delivers the 500 units per day, so, out of which 2 percent do not satisfy the specific quality. So, the total units per day is being supplied is, 1500 units are being supplied by 3 different companies.

Now, we want to know at the construction site that, what is the probability that 1 unit of the material picked at random, this picked at random is important, so, I do not know; without knowing, the knowledge that which company has supplied to this one, if you do not know that information, that is why it is picked at random, will not satisfy the specific, specific quality. So, to, at this, this problem we have to use the theorem of Total probability; the total probability of getting one particular unit which is defective or not satisfying the specific quality.

Now, on the other hand, if a, if a particular unit is found to be the, to be substandard, then what is the probability that it has come from supplier B? Now, here we are giving one condition that one, one unit has found to be substandard; that is already fact. Now depending on that fact, depending on that event, what is the probability that it is being supplied by B? So, we will see this two problem. The first one will be solved by this Total probability theorem and the second one will be solved by this Bayes' rule.

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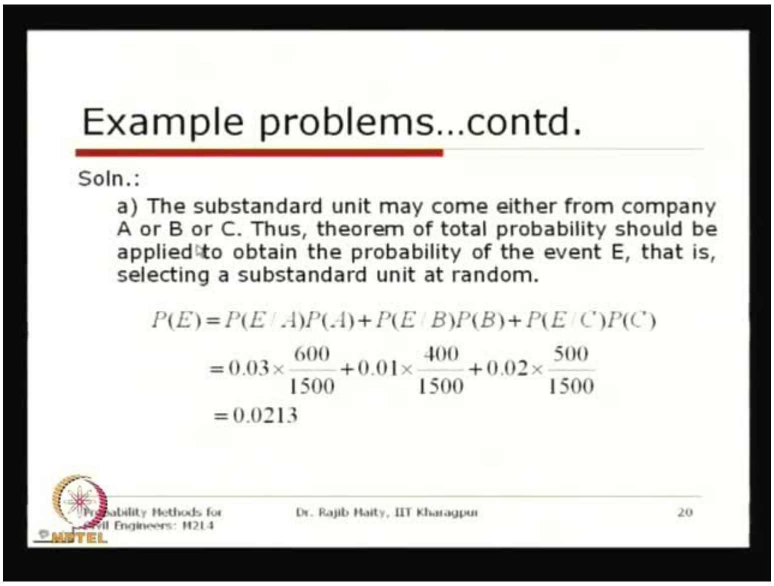

So, to answer the first one, the substandard unit may come either from the company A or B or C. Thus, the theorem of Total probability should be applied to obtain the probability of the event E, that is the selecting a substandard unit at random. So, this E is denoted as the, as that the probability of the event, that is this selecting a substandard, substandard unit. So, probability of E which should be expressed in terms of that probability of A, that is, it is supplied by what is the probability of supplying A, probability of supplying B and probability of supplying B, supplying C and what are their chances of supplying the defective units.

So, this first one is 0.03 which is, and their probability of supplying by, by event A is 600 by total units being supplied in a day is 1500. And, the second one is 0.01 and the probability it is being supplied by company B is 400 by 1500, and the probability that it is being supplied by C, company C is 500 by 1500. So, if we do this calculation, it comes that the total probability of getting a substandard unit is 0.0213. So, this is the total probability that we get. Now, if we just see the second question, if the unit is defective, then what is the probability that it has come from company B?

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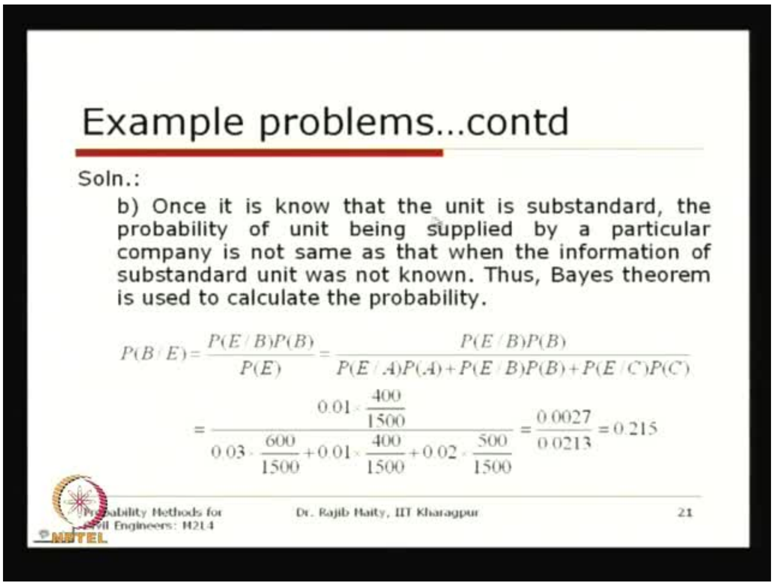

So, once it is known that the unit is substandard, the probability of the unit being supplied by a particular company is not the same as that when the information of the substandard unit was not known. So, what I, what is meant here is that, if, if this information was not available, then probability of supplying the substandard unit by with the company B is known to us, which is nothing but, here 0.01 as supplied in this, it from the earlier, earlier experiences. Company B delivers this one, out of which 1 percent do not satisfy the specific quality.

So, now we have to update that information, that is the probability of B on condition that one substandard unit has come. So, this is equals to probability of E on condition that probability of E on condition, it is supplied by B multiplied by the probability of B, this one; which is now this A is again expressed in terms of this total probability which is expressed in this form - Probability of E and condition A multiplied by a probability A, similarly from probability B, similarly from, from company C.

Now, if we just put this forms, then it comes that probability B is 400 by 1500. Similarly, the total probability as we have seen in this last slide that this quantity comes to 0.0213 and this is 0.0027, a ratio gives that 0.215.

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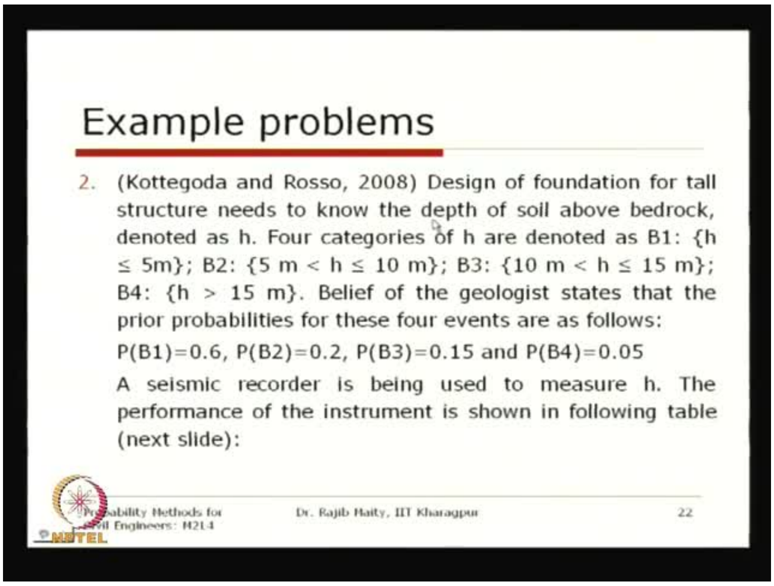

So, so, the first one, we have solved from this Total probability; and second one, we have solved from this Bayes' rule. We will take another interesting problem which is taken from Kottegoda and Rosso, 2008, from that book. Or, this is a basically a geotechnical problem, or the design of, the design of foundation for the tall structure, needs to know the depth of the soil above the bedrock which is denoted as h.

Now, four categories, four categories of h are denoted as B1; so, there are four different categories of this depth. The first one which is less than 5 meter and second one B2 is 5 meter to 10 meter; third one is 10 meter to 15 meter and B4 four is greater than 15 meter. So, belief of the geologist states that, the prior probabilities of these 4 events are as follows: the probability of B1, that is the, that is the bedrock should be within the 5 meter depth, is equals to 60 percent; probability of B2 is equals to 0.2; probability of B3 is equals to 0.15 and probability of B4 is equals to 0.05.

Now, a seismic recorder is being used to measure this h. Now, obviously any instrument that we will use; this type of, this type of measurement is generally not always perfect, this is also having some certain percentage of error. So, this is coming from this earlier experiment, where both the true depth, as well as the the record both, are available to us.

So, the performance, so, which is denoted the performance of this instrument, the performance of the instrument is shown in that table in the next slide which is here.

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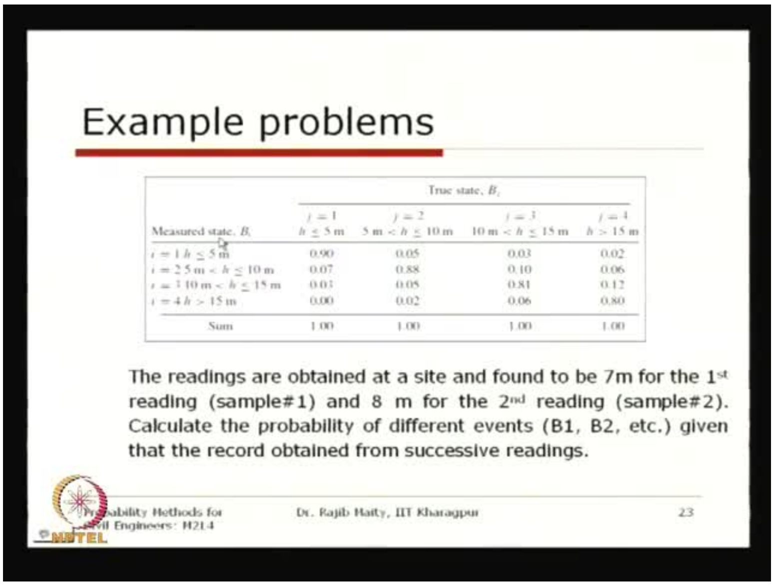

So, this is the measured state which is B i and these are the true state which is B j. So, if the measured states generally say that, it is within the 5 meter, so, there is 90 percent chance that it is actually in the first state; there is still 5 percent chance it is in the second state, 3 percent chance it is in the third state and 2 percent chance - this one. So similarly, in this way for all these cases, if it is measured to this, this particular fact and if the true data is also available, then we can complete this particular table here. See this is quite… There are two important thing that should be observed here; one is that, if the instrument is perfect, then we can say that this one, if it is measured in the, in the, in the state one, then this should be 100 percent probability and all other probability should be 0.

If it is true, this is also the true state will be true, so, this should be a perfect one, that is, 100 percent probability and other should be 0. But, as this instrument is, is not perfect, that is why we are getting this distribution of these probabilities from the earlier experiments; and obviously, this diagonal, diagonal is heavy diagonal, that means, most of the time it generally measures the true fact. Another thing, that is, if it is measured in state one or a particular state, and there are the probabilities, the, where this state will be; so, it should be exhausted. So, the state one whatever the probability is, if we just add up the probabilities in a row-wise, row-wise fashion or in a column-wise fashion, this should be equal to 1. So, this is denoting that this is collectively exhaustive.

Now, the readings are obtained at a site, so, now, the same instrument, once we know the performance of this instrument, the same instrument is being used to know the depth of the bedrock at a particular site. Now, the readings are obtained at a site and found to be 7 meter for the first, first reading which is denoted as sample 1 and 8 meter for the second reading. Now, we will have to calculate the probability of the different event, that is B1, B2, B3 given that the record obtained from the successive readings.

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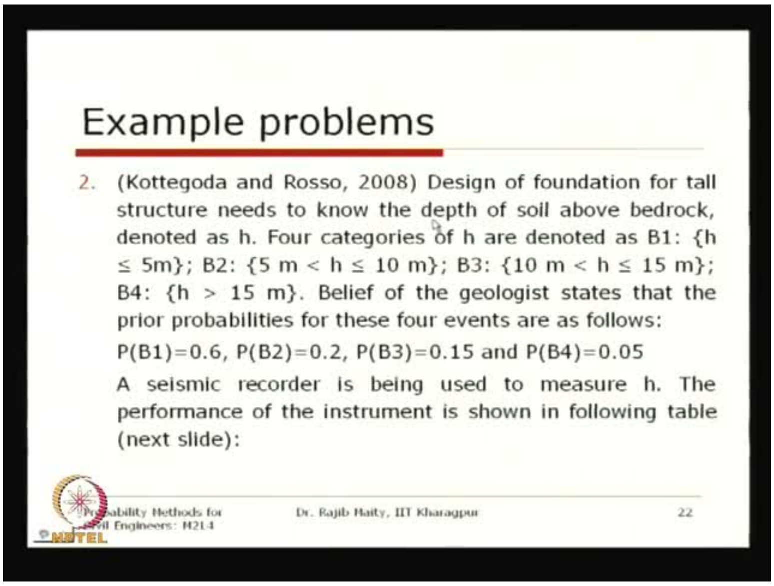

So, if the no reading is taken, then the probabilities are listed here. So, probability of being it in the B1 state, that is, below 5 meter is 6.6, 60 percent, 20 percent, 15 percent, 5 percent. Now, I got one sample which is 7 meter depth from this instrument which is obviously not perfect, erroneous instrument. So, after getting that sample#1 from that instrument, what are the, this probability, how these probabilities are being updated?

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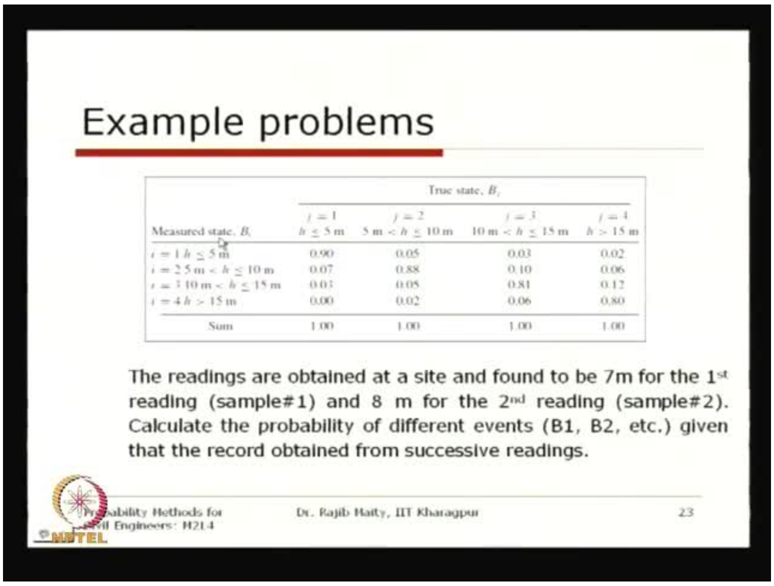

So, to know this fact, once we know this, after this sample one, then another sample has been taken which is sample#2. So, these probabilities will be updated after sample#1. Again it will be, again updated after taking this sample#2. How these probabilities are changing? We have these successive probabilities; that, we will see now in this problem.

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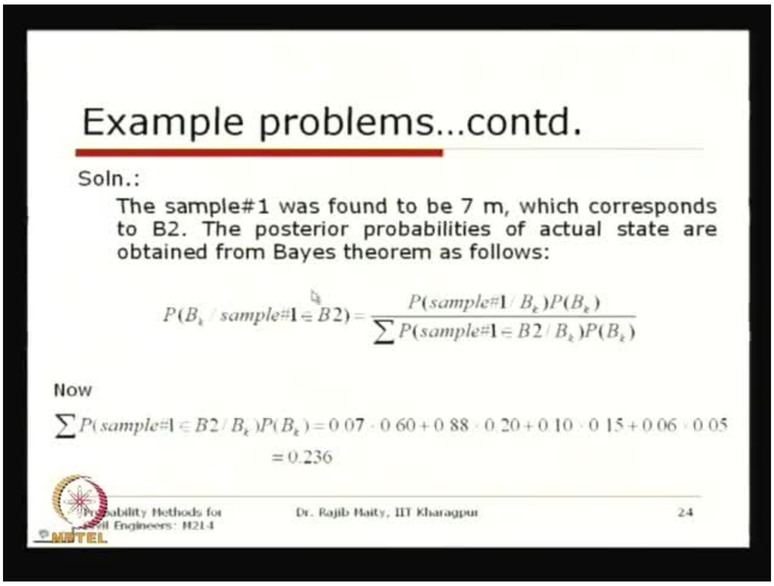

So, the sample#1 was found to be 7 meter which corresponds to the B2. Now, the posterior probabilities of the actual states are obtained from Bayes' theorem, that is, what is the probability of a particular state B k? Now, here B k means that B1, B2, B3, and B4 on condition that sample#1 is in B2. So, this is from this, derived from the Bayes' rule, we can, we can state that, this is the probability of sample#1 belonging to B2 on condition B k multiplied by B k; and the, and the total probability is the probability of sample#1 belongings to B2 on condition B k probability of B k.

Now, this total probability is calculated from this, this one, that is from, if I just take that, that sample is two, what is on condition that it is in actually in one and multiplied the probability one, which is 0.07 multiplied by 0.6. Now, if you see this slide that, this is your 0.07; it is in measure state two. So, this is in 0.07 and the probability of the prior knowledge of B1 is 0.6; and, that is why this 0.07 and 0.6 are multiplied. Similarly, taking the other probabilities from that table and that probability 0.2, in this way, if you just add up, we get the total probability is 0.236. Now, putting this one, so, we will get the updated probabilities for difference state which is as follows:

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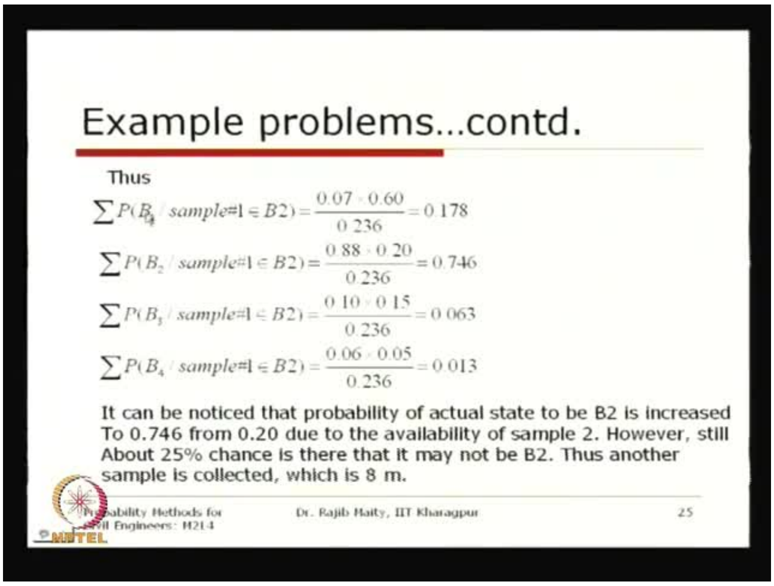

So, probability that, it is the state one is equals to 0.07 multiplied by 0.6 is divided by the total probability 0.236. Now, this 0.07 comes from that table, I had just now I showed from here, and this 0.6 comes from this prior knowledge. So, this is now after the sample#1, the prior information was 0.6; it is updated to 0.178. So, this the probability of the depth of this strata in one is, is now, is now reduced from 0.6 to 0.175.

Similarly, the state which is in the B2 is in the 0.2, which is going to be increased from this 20 percent to 74.6 percent; similarly, other probabilities also. Now, after taking the sample#1, these probabilities are modified, and this probability of being it in the second state has increase to the almost 75 percent.

But, still it is not, still there is 25 percent chance that, this depth of this bedrock may not be in these strata two, in this, in this strata two. So, the another observation is collected, where it is again saying that it is the depth is 8 meter, that is, it is again, in again in that state two, there is the second stage. So, after getting the second information again this probabilities are being updated.

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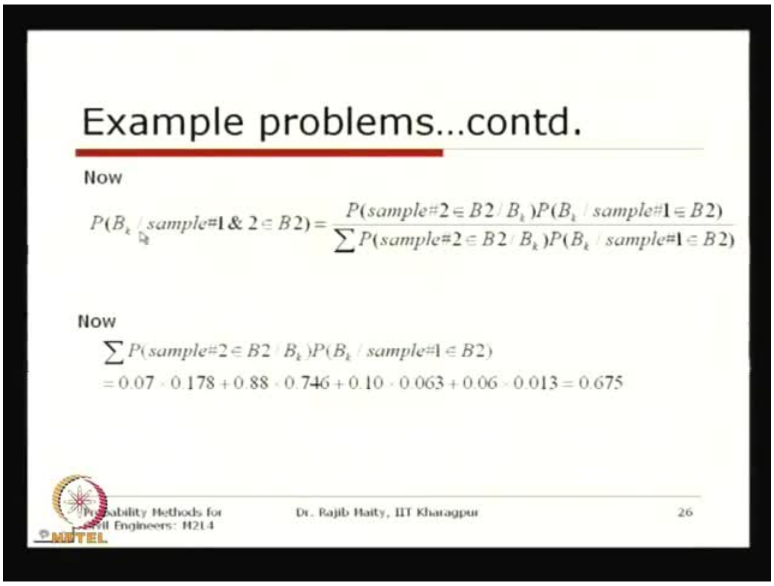

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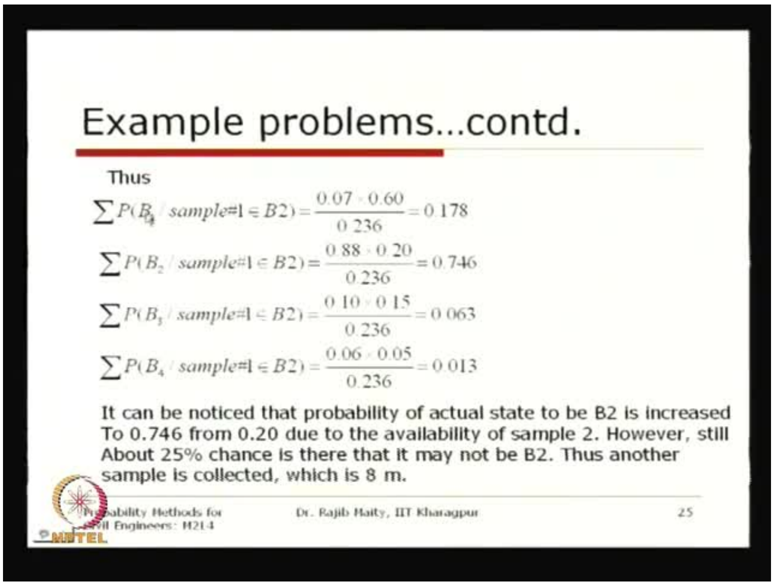

So, this is now the, what is the probability of B k? One condition the sample#1 and #2, both are in B2. So, this can be expressed in terms of this Bayes' theorem and here the total probability, when we are calculating; we are using that the performance of the, this is from that table, performance of the instrument, and this is from the updated information after getting sample#1. So, if we add up this thing it comes to be 0.675. Now, using that 0.675 we are getting different probability for the difference state. So, from 175 it is further reduced to that 1 percent, 1.8 percent, and the probability of that it is instead 2, it is increase to the 97 percent and similarity for the other states.

So, thus, it is noticed that after obtaining sample#2, the chance of true state being B2 is very high, which is 97.2 percent. Thus, with the help of the Bayes' theorem, the probability of unknown are improved with the availability of the more information.

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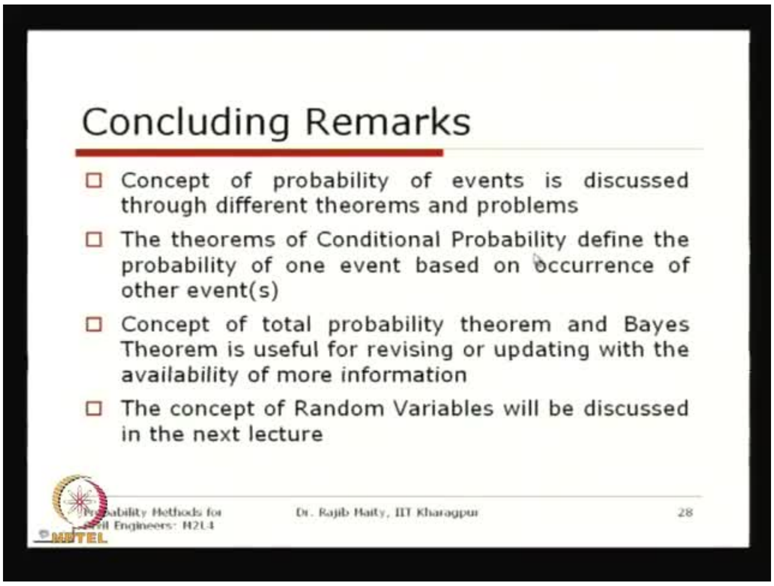

So, in this class the concept of probability events is discussed through different theorems and problems. The theorem of conditional probability is defined, the probability of one event based on the occurrence of the other events. Concept of total probability theorem and Bayes' theorem is useful for revising or updating with the availability of more information which is seen in the last example and concept of, in the next class, we will cover the concept of random, random variable and this we will see in the next class.

Thank you.