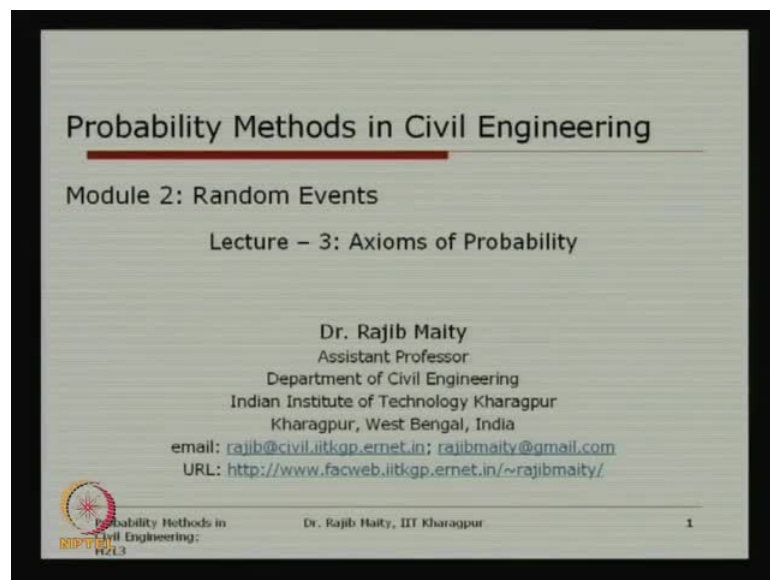


Probability Methods in Civil Engineering
Prof. Dr. Rajib Maity
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Module No.# 01
Random Events
Lecture No. # 04
Axioms of Probability

(Refer Slide Time: 00:25)



Hello there and welcome to the course Probability methods in Civil Engineering. In today's class we are going through this module 2, which is Random Events and today is the third lecture in which we will cover the part called Axioms of Probability.

So, these axioms basically are the fundamentals to the Probability theory and anything, any conclusion that we generally draw in the Theory of Probability, these are generally, directly or indirectly, somehow it is based on these axioms of probability. So, to know the Theory of Probability, **the**, to understanding these axioms is very important.

(Refer Slide Time: 01:10)

Axioms of Probability

- Probability of an event A , $P(A)$, is assigned in such a way that it satisfies certain conditions. These conditions are known as axioms of probability. There are three such axioms.
- In development of the probability theory, all conclusions are directly or indirectly based on these axioms

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So, why is it important that we should first know the probability of an event A ? In the previous classes, we have covered these concepts of event and sample space. So, here for, **to**, when we are going to assign probability to some event, we generally follow certain norms.

So, here probability of an event A , which is denoted as $P(A)$ is assigned in such a way that it satisfies certain conditions. These conditions are known as axioms of probability. There are three such axioms that will go one after another.

In the development of this probability, as we just discussed, is that, in the development of this probability theory all conclusions are directly or indirectly based on these, these axioms.

(Refer Slide Time: 02:10)

Axioms...contd.

1. For any event A belongs to the sample space, S ,

$$0 \leq P(A) \leq 1$$

S



This axiom states that probabilities are real numbers on the interval from 0 to 1

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So, the first axiom says that, for any event A , which belongs to the sample space S , as shown in this figure, which is, which should lie between the numbers, real numbers 0 to 1.

So, it should be greater than **or** equal to 0 and it should be less than **or** equal to 1.

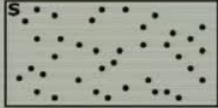
So, this axiom states that, the probability are real numbers **in** the interval from 0 to 1.

Obviously, here one point should be, should automatically be known, that if a particular event that have no, no point, if the event is a null set, then obviously the probability of that event equals to 0. On the other hand, if the event consists of the full set, which is nothing, but the axiom two and here we will go for this.

(Refer Slide Time: 03:08)

Axioms...contd.

2. Probability of full sample space (or certain event), $P(S) = 1$



This axiom states that sample space as a whole is assigned a probability 1. Since S contains all possible outcomes, the S is a certain event.

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PPT 3

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4

Axiom two is that, the probability of the full sample space or the certain event, which is denoted as S in the sample space, the probability of S equals to 1.

So, if it, if the event consists of all the sample space, that means any one of this event, if this event is a, is a certain event. So, that is why the total probability, which is probability of the full sample space, which is equals to 1.

This axiom states that, the sample space as a whole is assigned, sorry for the spelling mistake, this will be axiom, this axiom states that, sample space as a whole is assigned to a probability 1 since S contains all the possible outcomes, the S is a certain event.

So, this is the second axiom of this, of the probabilities.

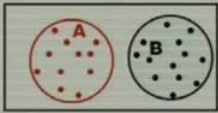
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Axioms...contd.

3. For any two mutually exclusive events A and B ($A \cap B = \{\emptyset\}$)

$$P(A \cup B) = P(A) + P(B)$$

$\mathcal{S}$



This axiom states that the probability of the union of two events is the summation of the probability of the individual events, if the events have no outcome in common.

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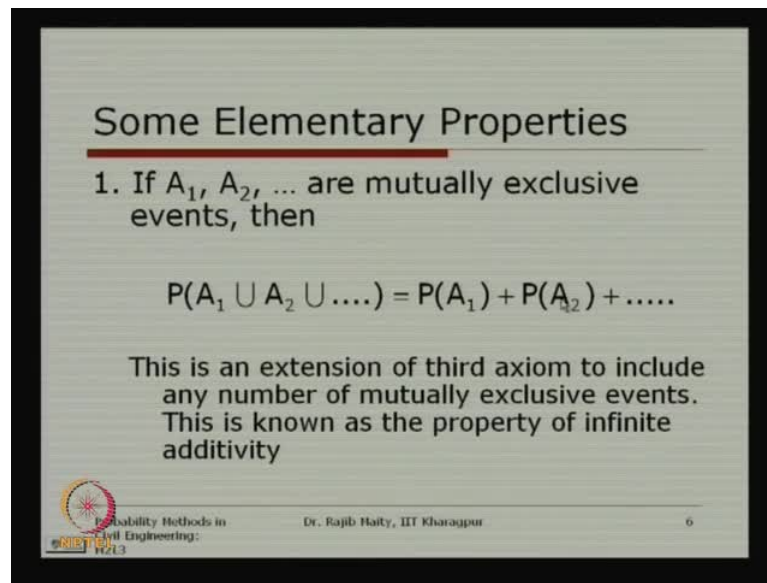
5

The third axiom states that, for any two mutually exclusive event, now, in the last class, we discussed about this mutually exclusive event. The two mutually exclusive events means the two event, if they are, there is nothing in, in common; so, then the occurrence of one event automatically implies the non-occurrence of the other event; then this two events are known as mutually exclusive event.

So, this axiom three states that, for any two mutually exclusive events A and B, that is A intersection B is a, is a null set. Then probability of A union B is the summation of, of their individual probabilities, that is probability of A plus probability of B.

So, this axiom states that, the probability of the union of two events is the summation of the probability of the individual event, if the events have no outcome in common. So, if the events have no outcome in common, that means, that there is no space, there is no overlap between these two events, as it is shown in, through this Venn diagram. That is the intersection between two events, is a null set.

(Refer Slide Time: 05:41)



Some Elementary Properties

1. If $A_1, A_2, \dots$ are mutually exclusive events, then

$$P(A_1 \cup A_2 \cup \dots) = P(A_1) + P(A_2) + \dots$$

This is an extension of third axiom to include any number of mutually exclusive events. This is known as the property of infinite additivity

 Probability Methods in Civil Engineering: 6

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Now, based on these three axioms, there are few elementary properties that are there. Those are very useful to draw a certain conclusion from them, from these axioms that we will go one after another.

The first elementary property that we can draw from this one, is just the extension of the last axiom that we have seen. If A_1, A_2 and in this way, these are mutually exclusive events, then following the third axiom, that is probability of the union of these events is simply the summation of the probability of all such events.

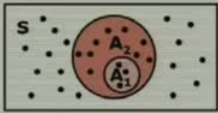
So, this is basically an extension of the third axiom, to include any number of mutually exclusive events. This is known as the property of infinite additivity.

So, for a sample space, if we see, if we can, can have the different events then and if we say that these events are not overlapping to each other, then if we want to know what is the total probability of, of occurring any of these events, then that can be achieved, that can be obtained by simply summing up the probabilities of the individual events, of is, summation of those probabilities for the individual events.

(Refer Slide Time: 07:10)

Properties...contd.

2. If $A_1 \subset A_2$ then $P(A_1) \leq P(A_2)$
and $P(A_2 - A_1) = P(A_2) - P(A_1)$



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7

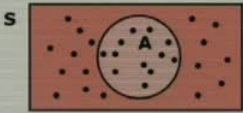
Second elementary property says, if A_1 is one event which belongs to A_2 , that means, if we refer to this particular Venn diagram, you see that, if this A_1 is a subset of this bigger set A_2 , which is denoted here as that A_1 belongs to A_2 , then the probability A_1 is less than equals to probability A_2 and probability of A_2 minus A_1 , it is equals to probability of A_2 , sorry, probability of A_2 minus probability of A_1 .

So, in this Venn diagram to represent this one, this probability of A_2 minus A_1 is nothing but, this, this dark red area which is nothing, but this probability of A_2 is the total, is the total probability as shown in this inside the bigger circle minus the probability of this small subset of this A_2 , which is quite straight forward, from this Venn diagram representation.

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Properties...contd.

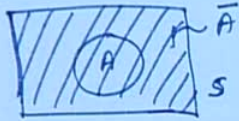
3. If $\bar{A}$ is complement of A, then
 $P(\bar{A}) = 1 - P(A)$



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8

(Refer Slide Time: 08:50)



$P(S) = 1$
 $P(A) + P(\bar{A}) = 1$
 $P(\bar{A}) = 1 - P(A)$

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The third property is, if that A complement, that is A bar, is the complement of the event A, then the probability of A complement is equals to 1 minus probability of A. Here one thing is that, the, that event A and its complement, union of this two events consist of the full set. Now, if you see here, if this is your, if, if this is your full sample space, then and if this is your event A, then the event A and the event, this area is your event A complement. That means, the union of these two event is nothing but, the full set, which is the, which is full sample space S.

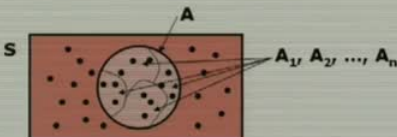
Now, from the second axiom we know that the probability of S is equals to one. So, what we can, so, this S we can break by probability of A plus probability of A complement. So, that we will get this equals to 1. So, this probability of A complement is equals to 1 minus probability of A.

So, from this second axiom, we got this particular conclusion which is shown here, that probability of A complement is equals to the total probability which is 1 minus probability of A.

(Refer Slide Time: 10:01)

Properties...contd.

4. If $A = A_1 \cup A_2 \cup \dots \cup A_n$, where $A_1, A_2, \dots, A_n$ are mutually exclusive events, then

$$P(A) = P(A_1) + P(A_2) + \dots + P(A_n)$$


Clearly, if $A=S$, then $P(A_1) + P(A_2) + \dots + P(A_n) = 1$

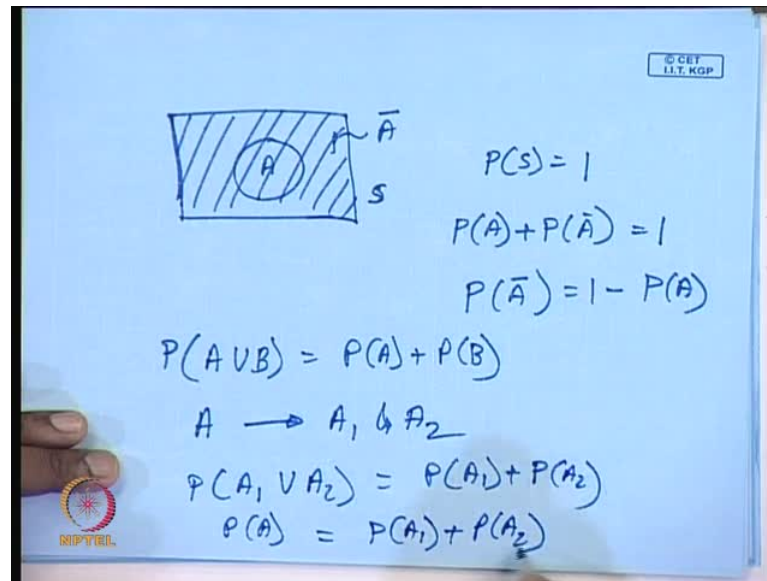
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R24.3

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9

Then, the next property says that, if we say, it is just in, in the sense, it is just opposite to the, to the last property. Here if we say that one event which is the union of different n numbers of mutually exclusive events, as it is written here, if A is the union of A 1 to A n or A 1 to A n are the mutual exclusive event, then the probability of the total event that is A is nothing, but the summation of probability of A 1, A 2 up to probability of A n,

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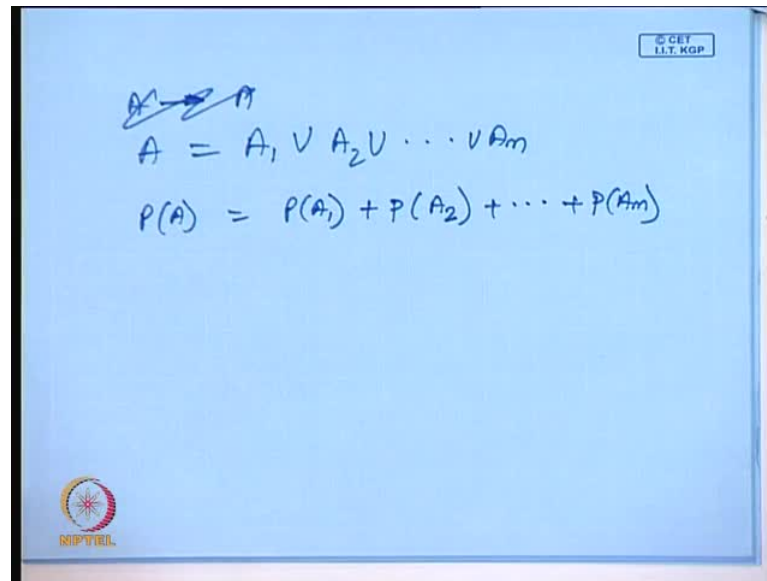


which is again the extension of those, that we just discuss, that third axiom. That in the third axiom, we have seen that, that probability of A union B is equals to your probability of A plus probability of B.

That means, now, if I say that, probability of A plus probability of B, now, in other side we are just saying that A is the split up of, say for example, A 1 and A, A 2, which are mutually exclusively of course, then probability of A 1 union A 2 is equals to probability of A 1 plus probability of A 2.

And this A 1 union A 2 is nothing, but probability of A which is equals to probability of A 1 plus probability of A 2.

(Refer Slide Time: 11:46)



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$$A = A_1 \cup A_2 \cup \dots \cup A_m$$
$$P(A) = P(A_1) + P(A_2) + \dots + P(A_m)$$

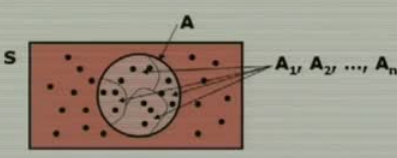
Logos: NPTEL (bottom left) and IIT KGP (top right)

Now, just by going on this induction, if I just say that, this A is the, A is the union of, so, A, also, I can write that A equals to the union of this events up to A n, then obviously, the probability of A should be equals to probability of A 1 plus probability of A 2 and in this way, it will go up to probability of A n, which is here, I have shown in this diagram, that, if these are the partition, partitioned in such a way that, these are mutually exclusive events, then the probability of A should be equals to the summation of their individual probability.

(Refer Slide Time: 10:01)

Properties...contd.

4. If $A = A_1 \cup A_2 \cup \dots \cup A_n$, where $A_1, A_2, \dots, A_n$ are mutually exclusive events, then

$$P(A) = P(A_1) + P(A_2) + \dots + P(A_n)$$


Clearly, if $A=S$, then $P(A_1) + P(A_2) + \dots + P(A_n) = 1$

Logos: NPTEL (bottom left) and IIT KGP (bottom right)

So now, if I replace this A, so if, so, this is also going to this. If I just replace this A in terms of this full set, full, full sample space and this full sample space is partitioned like this and it is collectively, if it is, if it is exhaustive, then, then we can say that, probability of A 1 plus probability of A 2 up to probability of A n is equals to probability of S, which is nothing but, equals to 1.

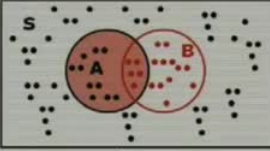
So, if for, if the full sample space is partitioned into A 1, A 2 up to A n, which are mutually exclusive and collectively exhaustive, then the summation of these probabilities are equals to 1.

Then, so, this one also we discussed, one in the last lecture also.

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Properties...contd.

5. For any events A and B,

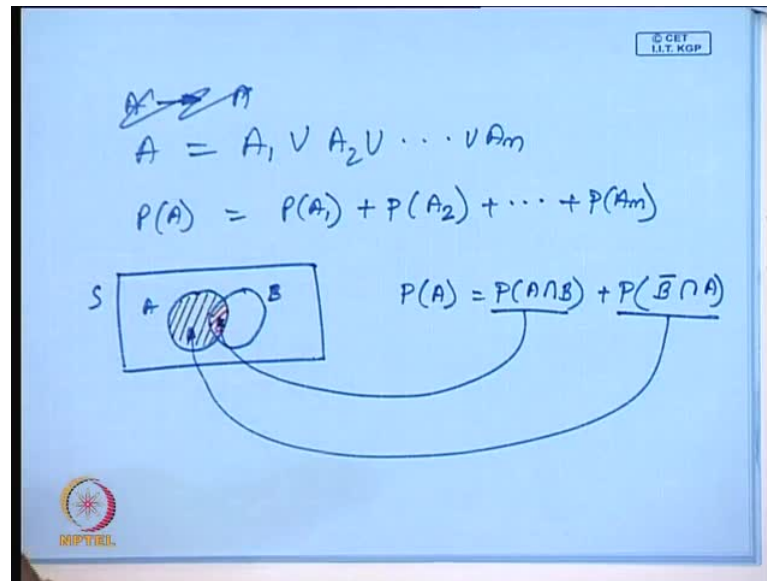
$$P(A) = P(A \cap B) + P(A \cap \bar{B})$$


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10

For any two events in the sample space, this is the fifth property now. For any two events A and B, inside this sample space which is shown in this Venn diagram, one is that black shaded circle and one is the red circle. These are two events. Now, any one event, any one event, the probability of the one event can be expressed in terms of this, this expression.

(Refer Slide Time: 14:04)



Now, if we just see here, that is, this is your full sample space. Now, we are having the two events A and B . Now, if we want to know what is the probability of A , this is equals to, what we are doing is that, this probability of A , we are just making it the summation of two zones, one is that, this red set shaded zone and another one is this particular, this black shaded zone.

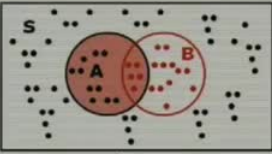
Now, what is this, that your red set shaded zone? This red shaded zone is the probability of A intersection probability of B . So, this much done.

Now, what is this black shaded zone? This black shaded join again is the intersection of two events. One is that B , B complement. So, outside this B area, the full area is the B complement intersection with A . So, which is nothing but, this black shaded area. So, which is your B complement intersection A .

(Refer Slide Time: 13:28)

Properties...contd.

5. For any events A and B,

$$P(A) = P(A \cap B) + P(A \cap \bar{B})$$


The diagram shows a rectangular sample space S containing two overlapping circles, A and B. Circle A is shaded red and contains black dots. Circle B is also shaded red and contains red dots. The intersection of A and B is shaded a darker red. The region of A that does not overlap with B is shaded a lighter red. The entire region of A is highlighted with a red border. The sample space S is represented by a rectangle containing many small black dots.

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10

So, these are the two events. So, this, this event corresponds to this red zone and this one, this probability corresponds to this, this zone. So, this is, these two summation is nothing, but your probability of A. Here, if you see that, this probability, if you see the monitor here, you see that, this probability of A is equals to probability of A intersection B, which is nothing, but this, this zone and this A intersection B complementary, which is nothing but, this area.

So, one particular, the probability of anyone event can be expressed in terms of the summation of its partition, which is in terms of this two, two events in the sample space.

(Refer Slide Time: 16:08)

Properties...contd.

6. If A_1 and A_2 are any two events, then

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$$

S

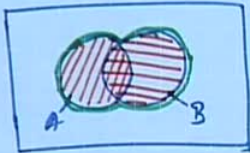


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11

The next property tells us that, if A_1 and A_2 are any two events, then the probability of A_1 union A_2 is equals to probability of A_1 plus probability of A_2 minus probability of A_1 intersection A_2 . This is one of this important in the sense, you see as you, if you compare this property with respect to the third axiom of this, of this probability, then we see that, here it is the any two events. We are not putting the constant of this mutually exclusive event here. So, here this A_1 and A_2 need not be mutually, need not be mutually exclusive. So, if these are any two events, then this is the expression that holds.

(Refer Slide Time: 17:07)


$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

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If you now, see here, that if, in this area there are two events. Again what we are trying to get is, that is the union, if this is your A and if this is your B, then what we are trying to get is the probability of A union probability of B. So, what is this probability of A union B is this, this green area. If I just draw it in the, along the side of this circles, this union concept was given in the last class. So, this is your union of two events.

Now, this two events, so, how can I express this one? First thing, from this thing, I can write probability of A. That means I am just writing it, I am just shading it, the area, corresponding area here, in this Venn diagram, which is this. This is your probability of A. Now, I am adding it up, probability of B, which is again, I am just shading it in this, this is the area.

So, if you see, while adding up this two process, while adding up this two probabilities, we are adding this area twice. One, which is there in probability of A and also in probability of B, that is why this area has to be, has to be deducted. So, this is minus probability of A intersection probability of B.

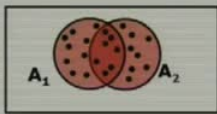
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Properties...contd.

6. If A_1 and A_2 are any two events, then

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$$

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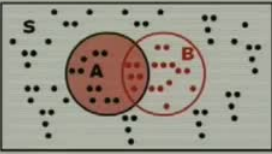
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11

(Refer Slide Time: 18:56)

Properties...contd.

5. For any events A and B,

$$P(A) = P(A \cap B) + P(A \cap \bar{B})$$


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10

(Refer Slide Time: 19:06)

Properties...contd.

□ Proof

$$\begin{aligned} P(A \cup B) &= P(A \cap B) + P(A \cap \bar{B}) + P(\bar{A} \cap B) \\ &= [P(A \cap B) + P(A \cap \bar{B})] \\ &\quad + [P(\bar{A} \cap B) + P(\bar{A} \cap \bar{B})] - P(\bar{A} \cap \bar{B}) \\ &= P(A) + P(B) - P(A \cap B) \end{aligned}$$

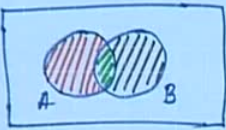
Probability Methods in
Civil Engineering:
R24.3

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12

So, this is the graphical representation of this one, this particular equation. Now, now, this equation can also be proven in terms of this, what we have just now, what you have seen, the representation of one event in terms of, in, in terms of the event, in terms of two different event in the sample space. So, if we see that proof, it states like these, that probability of A union B can be expressed in this way. That probability of A intersection B plus this two. Now again, if you refer to this diagram, that this probability of, I just take a separate sheet again.

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$$\begin{aligned}P(A \cup B) &= P(A \cap B) + P(A \cap \bar{B}) + P(\bar{A} \cap B) \\&= [P(A \cap B) + P(A \cap \bar{B})] + [P(A \cap B) + P(\bar{A} \cap B)] \\&\quad - P(A \cap B)\end{aligned}$$

So, now, this one, for this is A and this is your B. So, probability of A union probability of B, which is equals to, I am just separating it out into three different places, one is the, first one is this place, second one is this area and third one is here, this particular thing.

So, then, what we are getting is, this, the middle one, that is the intersection part, probability of A intersection B plus. So, this first area, the way that we can write is, that, this is nothing, but the intersection between the B complement that is outside B and intersection with the event A. So, this will give you this red shaded area.

So, which I can write as this probability of A intersection B complementary. So, this is, this one corresponds to this red shaded area plus. Similarly, if I want to write this black shaded area which is probability of A complementary intersection B. Now, these events can be now, these events can be written that, probability of A intersection B plus probability of A intersection B prime. I am just clubbing this two part together plus, I am adding one more part, which is probability of A intersection B plus, I am taking this term, which is probability of A complementary B intersection with B. So, I have added one part here. So, I have to delete this part to make this equality probability of A intersection B.

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Properties...contd.

□ Proof

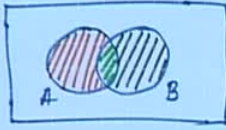
$$\begin{aligned} P(A \cup B) &= P(A \cap B) + P(A \cap \bar{B}) + P(\bar{A} \cap B) \\ &= [P(A \cap B) + P(A \cap \bar{B})] \\ &\quad + [P(A \cap B) + P(\bar{A} \cap B)] - P(A \cap B) \\ &= P(A) + P(B) - P(A \cap B) \end{aligned}$$

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12

(Refer Slide Time: 22:11)



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$$\begin{aligned} P(A \cup B) &= P(A \cap B) + P(A \cap \bar{B}) + P(\bar{A} \cap B) \\ &= [P(A \cap B) + P(A \cap \bar{B})] + [P(A \cap B) + P(\bar{A} \cap B)] \\ &\quad - P(A \cap B) \\ P(A \cup B) &= P(A) + P(B) - P(A \cap B) \end{aligned}$$

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Now, this one if you just match and this one if you just match with our fifth property that we have seen in the monitor, if you see, just, if you just see in this one, then in the fifth of property, then what we can do here, if you see it here, so, this we can write as probability of A and similarly, this we can write as probability of B and obviously, this minus probability of A intersection B which is obviously, this A union B.

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Properties...contd.

□ Proof

$$\begin{aligned}P(A \cup B) &= P(A \cap B) + P(A \cap \bar{B}) + P(\bar{A} \cap B) \\&= [P(A \cap B) + P(A \cap \bar{B})] \\&\quad + [P(\bar{A} \cap B) + P(A \cap B)] - P(A \cap B) \\&= P(A) + P(B) - P(A \cap B)\end{aligned}$$

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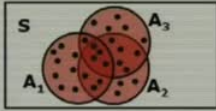
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So, this proof is given here on this slide also. These are the breakup of three, this three probabilities which is shown in the Venn diagram there and there are some algebraic calculation to get this proof.

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Properties...contd.

As an extension of the last properties, if A_1 , A_2 , A_3 are any three events,

$$\begin{aligned}P(A_1 \cup A_2 \cup A_3) &= P(A_1) + P(A_2) + P(A_3) \\&\quad - P(A_1 \cap A_2) - P(A_2 \cap A_3) - P(A_3 \cap A_1) + P(A_1 \cap A_2 \cap A_3)\end{aligned}$$


 Probability Methods in
Civil Engineering:
PZL3

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13

The next property, rather, I should say that this is the extension of, instead of having this two events, if we have more than two events, if we have the three events, then this will be simply the extension of this thing, which can be easily shown in terms of this Venn diagram.

Here, what we are saying that, that if these A_1 , A_2 and A_3 are any three events in the sample space, then what is the probability of $A_1 \cup A_2 \cup A_3$?

So, similarly, what we should do, we should add up these probability, probability of A_1 , which corresponds to this circle here, you can see, probability of A_2 , this one and probability of A_3 which is this. Now, after doing all, after adding all this probabilities, what we are doing that, we are adding individual intersection, the pair-wise intersection of these probabilities twice. So, we have to deduct that part. So, we are deducting this first pair, A_1, A_2 , minus A_2, A_3 minus A_3, A_1 .

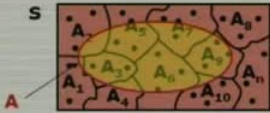
Now, while doing this deduction from this one, we can see that this particular area has been deducted once more. So, we have to add this particular area, which is the intersection of all three, all three events. So, have to add this part, that is probability of A_1, A_2, A_3 intersection.

So, this is just a simple extension of this earlier thing. If we just see, if we just take out this part and this part, you will see that, this part is, has been whatever the extra, that part has been deducted again. When, when you are taking the third pair this is, one negative area is coming here. So, that have to add here, to get this two are equal. So, this is the extension of this last property which is discussed in terms of two events, here it is in terms of any three, three events.

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Properties...contd.

6. If an event A must result in the occurrence of one of the mutually exclusive events $A_1, A_2, \dots, A_n$, then

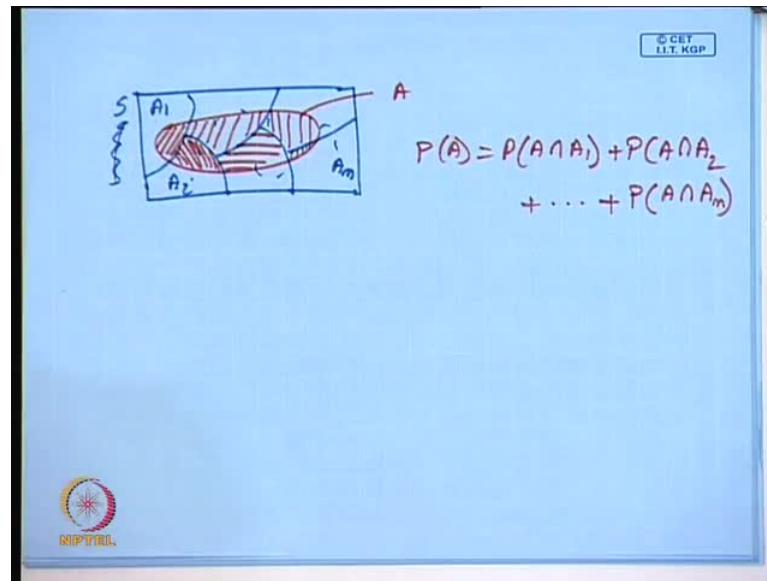
$$P(A) = P(A \cap A_1) + P(A \cap A_2) + \dots + P(A \cap A_n)$$


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14

(Refer Slide Time: 25:45)



Now, it is just a opposite sense of what we discuss in this fifth property. This is, this says that, if an event A must result in the occurrence of one of the mutually exclusive event in A_1, A_2 and up to A_n , then the probability of A can be expressed as their individual intersection. If you just see it here, that, so, if, if there is, this is your sample space and this sample space is having the partition like that mutually exclusive and collectively exhaustive partitions are there and there is another event like this.

And, if I say that, these are all, these, these partitions are this, this is total S , this is partition is A_1, A_2 and up to this, it is going and it is coming up to A_n and if I say that this event is your A , then this probability of A can be expressed as a summation of this particular area. First is this area. So, what is this area? This area is nothing, but the intersection between the event A and event A_1 . So, we just write it event A intersection event A_1 plus. I am writing now, this area, this corresponding area in this Venn diagram, which is nothing, but probability of intersection of A_2 and this event A . So, A intersection A_2 . In this way, I will go on adding up. Then this area will come, then this area will come, then this area will come and finally, the last event will come.

(Refer Slide Time: 25:08)

Properties...contd.

6. If an event A must result in the occurrence of one of the mutually exclusive events $A_1, A_2, \dots, A_n$, then

$$P(A) = P(A \cap A_1) + P(A \cap A_2) + \dots + P(A \cap A_n)$$


Probability Methods in
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NITRR
R24.3

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14

(Refer Slide Time: 27:52)

Example problems

1. If a steel section manufacturer produce a particular section and initial quality check reveals that the probability of producing a defective unit is 0.022. Further investigation reveals that the probability of producing a defective unit in terms of measurement is 0.01 and in terms of material quality is 0.017. What is the probability of producing an unit that defective in measurement as well as material quality?

Ans. Let us denote –

A – Event of production of a defective unit in terms of measurement.

B – Event of production of a defective unit in terms of material quality

Thus, $P(A) = 0.01$ and $P(B) = 0.017$ and $P(A \cup B) = 0.022$. It is required to calculate

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.017 + 0.01 - 0.022 = 0.005$$

Probability Methods in
Civil Engineering:
NITRR
R24.3

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15

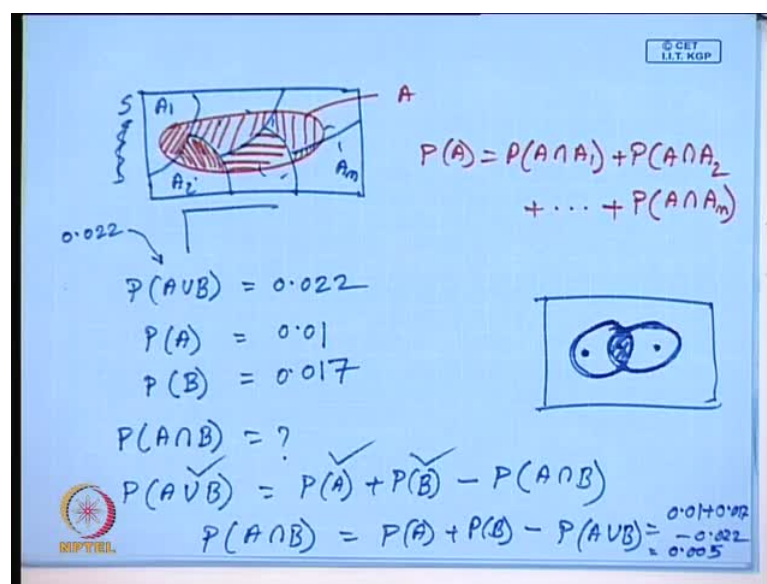
So, in this way, I will just add up to probability of A intersection A n. So, this, this is stated here in this slide, that if an event A must result in the occurrence of one of the mutually exclusive events A_1, A_2 upto A_n , then probability of A is a summation of the intersection of that event with the individual events A_1 up to A_n . So, the next property, the, before we go to that one, that is on the, on the conditional probability. That we will see in a minute.

Before that, we will just see, we will just quickly go through one small example problem which states like this. That if a steel section manufacturer produce a particular section, and the initial quality check reveals that the probability of producing a defective unit is 0.022. Further investigation reveals that the probability of producing a defective unit in terms of the measurement is 0.01 and the defective unit in terms of the material quality is 0.017, then what is the probability of producing an unit that is defective in measurement as well as in material quality? So, here what the information is given is that, what is the total probability of producing a defective unit is given.

And these two events, that is probability of the, it is defective, it can be defective in two different ways. One, one way of defective unit is that this measurement is the, the measurement was wrong and other one is that, the material quality was not satisfied. So, in two different way, the particular section can be defective. So, their individual probability that we got is 0.01 and another one the 0.017 and the total is given here, this 0.022.

The question is that, what is the probability of producing an unit which is defective in terms of both. That is, it is defective in terms of the measurement and also in the material quality. So, we here, we can, we can assign two different, two different events. The first event is that, it says that it is event A, that event says that the event of production of A of a defective unit in terms of the measurement.

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Handwritten slide content showing probability calculations and Venn diagrams.

Diagram 1: A rectangle divided into regions $A_1, A_2, \dots, A_m$. A shaded region A is shown, representing the union of these regions. A label 0.022 points to the shaded region.

Equation 1: $P(A) = P(A \cap A_1) + P(A \cap A_2) + \dots + P(A \cap A_m)$

Equation 2: $P(A \cup B) = 0.022$

Equation 3: $P(A) = 0.01$

Equation 4: $P(B) = 0.017$

Equation 5: $P(A \cap B) = ?$

Equation 6: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Equation 7: $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.01 + 0.017 - 0.022 = 0.005$

Diagram 2: A Venn diagram showing two overlapping circles, representing sets A and B.

So, this is your first thing and the second is that B, that is event of production of the defective unit in terms of the material quality. So, from the problem, the information that is supplied is the, what is the probability of A and what is the probability of B. So, probability of A is your 0.01 and probability B is 0.017 and it is also that supplied, what is the probability A union B, that is the whatever way it is the defective unit, probability of defective unit is 0.022. So, here the idea is that, this when we are saying that, that probability of A union B, that means, if you, if we again refer to that Venn diagram, that A and this B.

So, this A union B that is when and saying that, that the, the unit, a particular section is defective it can be anywhere. It can be either, it can be defective in case of only for the reason, only for the event A, it can be defective by both or it can be defective by the second event, that is event B. So, that is why, whatever the, the defect, the probability of a defective unit that is the value is given 0.022. This should refer to this probability of A union B. So, there is the probability of A union B is given, so 0.022. Also, we have seen that probability of individual event, that is the probability of A is given as 0.01 and probability of B is given as 0.017.

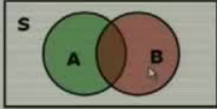
So, now, it is asked that, what is the probability that a particular unit is defective in terms of both, that is in terms of its measurement as well as the material quality? Now, so, here according to this Venn diagram, we are basically referring in to this intersection, of this two, this two event, that is the, we are, we want to know what is the probability A intersection B. Now, we know, so, do this one. Thus, one simple property that we have use. We should use this one, that is probability of A union B is equals to probability of A plus probability of B minus probability of A intersection B. Now, this is known to, this is known and this is known. Obviously, this should be known, which is probability A intersection B equals to probability of A plus probability of B minus probability of A union B, which is 0.01, 0.017, minus 0.022 equals to 0.005.

So, this, this refers to this particular area, where I can say that, this units is the defective, in terms of both measurement as well as your material, material quality. This is an simple application of that, that property that we discuss, that if there are two events which are, which are any two events, it does not, it does not mean that these two are mutually exclusive, then we can use this property to get that simple answer.

(Refer Slide Time: 34:01)

Conditional Probability

□ If A and B are two events in S and $P(A) \neq 0$, then the probability of B given that A has already occurred is denoted as $P(B|A)$ and expressed as

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$


Probability Methods in
Civil Engineering:
R21.3

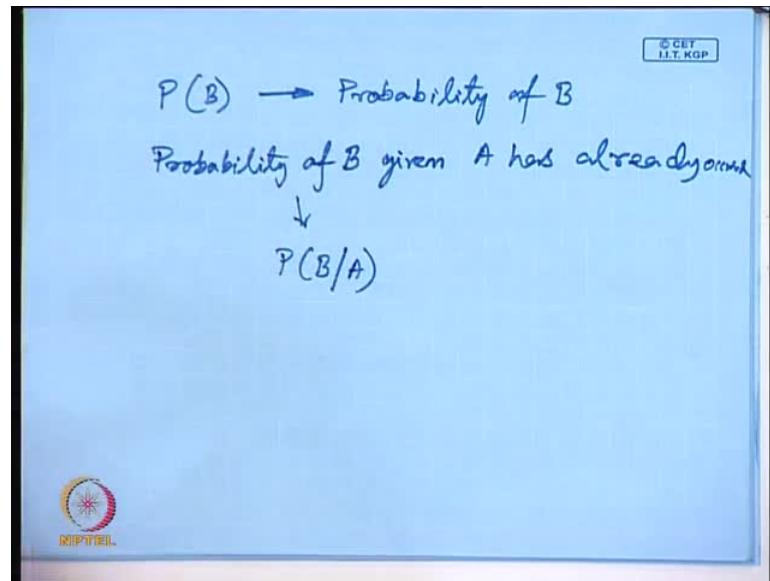
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17

The next thing which is, which is, which we are going to cover is known as the, the conditional probability. Then this conditional probability means in the sample space what happens, these two events are given and you are interested to know the probability of a particular event giving some condition, that some, that other events has already occurred. Now, if these two events are shown here, as you can see in these Venn diagram, that is, this is the, this is the area that corresponds to these event A and these red is the corresponds to this B and this overlap between A and B is nothing but, the intersection of this two.

So, now, if you want to know, what is the probability of B, this can be obtained from this, that technique, that we discuss in the previous class. Obviously, we can also know what is the, what is the probability of the, of the individual event, probability of A as well as probability of B. Now, here, that, here the question that we are quoting is, the conditional, it is conditional means that I want to know the probability of one event, any one of these two event that, probability of A, probability of B, with certain condition. Here the condition is that, I am ensuring that occurrence of the other event, the occurrence of the other event here means that, here its means that, if that A and B are two events, are to be specific, any two events in the, in the sample space S and the probability of A is not equal to 0, then the probability of B given that A has already occurred.

(Refer Slide Time: 36:10)



Now, the probability of B means, there is no condition is given here, which is, which we generally, which we simply write in terms of the probability of B. Now, what here, what we are seeing is that, this is your probability of B. Now, when we are saying that, when we are saying that, that probability, probability of B, that given A has already occurred.

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Conditional Probability

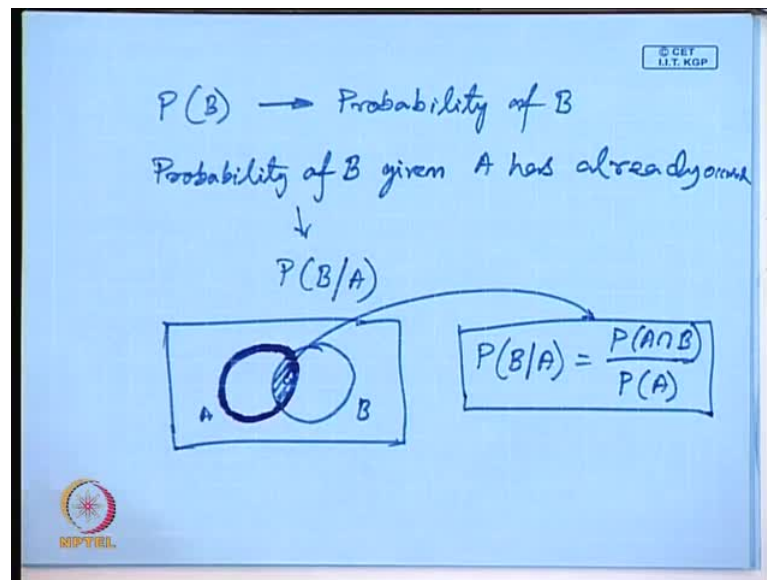
□ If A and B are two events in S and $P(A) \neq 0$, then the probability of B given that A has already occurred is denoted as $P(B|A)$ and expressed as

$$P(B | A) = \frac{P(A \cap B)}{P(A)}$$

The Venn diagram shows a rectangle labeled 'S' containing two overlapping circles. The left circle is green and labeled 'A'. The right circle is red and labeled 'B'. The intersection of the two circles is shaded brown.

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(Refer Slide Time: 37:40)



So, this we generally denote as probability of B given A. So, this is the notation for this one. So, the probability of B, given that A has occurred and this just simply the probability of B. Now, if you come back to this one, then we see that, which is denoted as probability of B, given A and this is expressed as probability of B given A is equals to, this should be equals to, not 3, 1. Sorry for this mistake. So, this is equals to probability of A intersection B divided by probability of A. Now, if you just refer to this Venn diagram, that is shown here, is that, this is your probability A and this is your probability B.

Now, from the traditional definition, in this probability, we will just see, so, we are interested to know that, probability of B on condition A. Now, there must be something in the numerator and something in the denominator. So, in this denominator what you should get, it is the total possible case that you know from this, in the previous classes we discussed. That, these denominator should have the total possible case. Now, here, what is the possible case? The possible case is that, the, that given A has already occurred.

So, now, if A has already occurred, so, I should ensure that, the whatever the outcome of the experiment, that outcome of this experiment is certainly within this A region, should corresponds to this particular area of this Venn diagram. So, this is already in this one. When we are talking about this probability of B, the total feasible or total feasible space

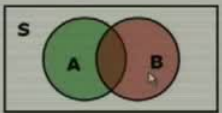
or the total sample space is the full sample space. Now, when we are giving some condition, the condition that A has already occurred, so, my, my feasible space is this total, this A.

So, obviously in the denominator, this probability of A should occur and in the numerator you know that, we should, we should put the quantity which is the favorable case. Now, we want to know what is the probability of B, condition that A has already occurred. A has already occurred, that is why I put in the denominator probability of A. Now, what is the probability of B? Now, the success area between, within this area, the success area is nothing but, this area. If it is in this area, then we can say that B has also, B has also occurred.

(Refer Slide Time: 34:01)

Conditional Probability

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Probability Methods in
Civil Engineering:
PZL3

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17

So, what is this area corresponds to? This area is nothing but probability of A intersection B. So, this is your expression for this conditional probability, which says that probability of B given A has already occurred, which is equals to probability of A intersection B divided by probability of A. So, here, so, the same thing has been shown here, which is the probability of A intersection B is this shaded area, which is orange type area and probability of A means this green area plus this orange, and this area, this intersection area. So, this is the conditional probability.

(Refer Slide Time: 40:38)

Example problems

1. On a national highway, a stretch of 10 km is declared to be accident prone. Over this stretch, probability of accident at any place is equally likely. In the middle of the stretch there is bridge of 755 m length. Given an accident has occurred within first 6 km stretch, what is the probability that it has occurred on the bridge?

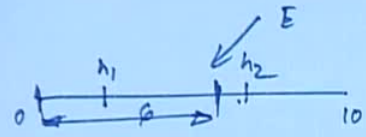
Ans. Let us denote –
A – Event of accident within first 6 km of the stretch.
B – Event of accident on the bridge
Thus, $P(A) = 6/10 = 0.6$ and $P(B) = 0.755/10 = 0.0755$
Thus,

$$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{0.0755}{0.6} = 0.126$$

 Probability Methods in Civil Engineering: Dr. Rajib Haity, IIT Kharagpur 18

Now, we will see the application of this simple, this simple conditional probability equation to some civil engineering related problem. Here, on a national highway, a stretch of 10 kilo meter is declared to be the accident prone zone. Now, over this stretch, the probability of accident, probability of accident at any place, sorry for this spelling mistake, at any place is equally likely. Now, here when, if you just recall, some of our previous class, here the equally likely means that I am just giving an ((attribute)), how to assign the probability for a particular events.

(Refer Slide Time: 41:58)



$P(E) = \frac{h_2 - h_1}{10}$

A → Event of accident within first 6 km
 $P(A) = \frac{6 - 0}{10 - 0} = 0.6$

B → Event of accident on the bridge (755 m)
 $P(B) = \frac{0.755}{10} = 0.0755$

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So, here the event means that one accident is taking place at any, over a any particular sub-stretch of this 10 kilo meter area and it is equally likely. So, in the middle of this stretch, this 10 kilometers stretch there is a bridge of 755 meter length. Now, given, an accident has occurred within the first 6 kilo meter stretch, what is the probability that it has occurred on the bridge? Now, this is interesting, in the sense that. So, my total stretch is this, starting from 0 to 10 kilometer. Now, I say that, this is equally likely accident prone area and it is equally likely. So, if given that any stretch, if I just give from 0 to or from the any kilometers range from h_1 to say h_2 , the probability of the accident between this two range, the probability, if this is some event say E, then, probability of E should be equals to h_2 minus h_1 divided by 10. So, this is, this we can get from for the any event, that we can assign as, these are equally likely. Now, for this, for this problem, if we just assign that, this is say, A is one event, that event, that, event of accident, within the first 6 kilometer, so, what is the, what is the probability of A that we will get here is nothing, but so, 6 minus 0, divided by 10 minus 0, which is 0.6. So, from, starting from here up to 6 kilometer range, the probability of the accident in between these two is your 0.6. Now, if I assign, if I, if I just denote another event, the event of accident on the bridge, here the bridge is 755 meter length.

(Refer Slide Time: 44:30)

$B \rightarrow$ Event of accident on bridge (755m)
 $P(B) = \frac{0.755}{10} = 0.0755$

Diagram: A number line from 0 to 10. A segment from 5 to 6 is marked with a bracket above it labeled 755.

$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{0.0755}{0.6} = 0.126$

$P(B) = 0.0755$
 $P(B/A) = 0.126$

So, the probability of B will be, it is equally likely all over the stretch. So, this is also equally likely. So, this is 0.755. I am just converting them into kilometer which is 0.0755. So, our condition, so, here, the, what is the probability that we want to know is

that, that the, the probability of B, that is, it should occur on the bridge. What is the probability that it has occurred on the bridge? That is the event B on condition, that it is occurred within the first 6 kilometer of the stretch. So, this is B given A, just by putting it here. The probability of, then A intersection B, divided by probability of A. Now, this probability of A intersection B can be, so, you have to see at the middle of this bridge. So, if this is the location on this bridge, then this is your 5 kilometer. So, bridge has some length and this is within this first 6 kilometer, most probably comes here. So, this is your 755 meter.

So, the full bridge is coming within this first 6 kilometer range. So, that means, thus A intersection B is nothing but, so, B is a subset of A. So, the intersection obviously will be within this region. So, which is nothing but, equal to the probability, probability of B. So, which is the probability of B, that you got there is a 0.0755 divided by probability of A is 0.6, which is 0.126, as we got in this calculation.

So, here if you just compare that, what is the probability of B is your 0.0755. So, this is the probability that you got. Now, when we are giving some condition, the, that probability information changes. This is the, this is the useful thing that, we should know that, that if we give some condition, then the probability of the same event may change, as we have seen in this particular problem as well.

(Refer Slide Time: 40:38)

Example problems

1. On a national highway, a stretch of 10 km is declared to be accident prone. Over this stretch, probability of accident at any place is equally likely. In the middle of the stretch there is bridge of 755 m length. Given an accident has occurred within first 6 km stretch, what is the probability that it has occurred on the bridge?

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Thus,

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.0755}{0.6} = 0.126$$

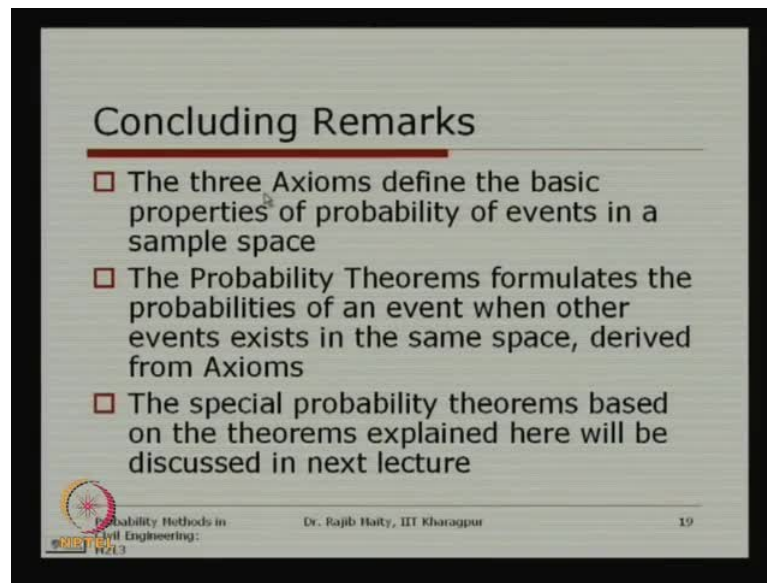


Probability Methods in
Civil Engineering

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18

(Refer Slide Time: 46:59)



Concluding Remarks

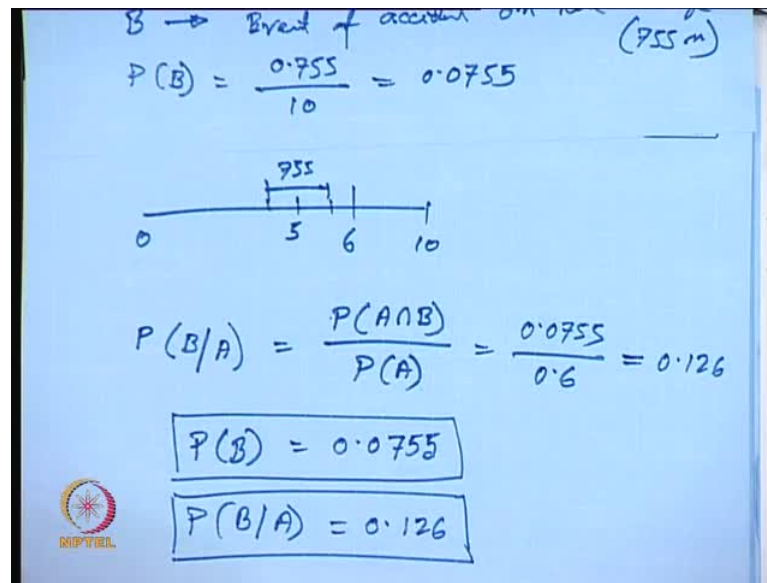
- The three Axioms define the basic properties of probability of events in a sample space
- The Probability Theorems formulates the probabilities of an event when other events exist in the same space, derived from Axioms
- The special probability theorems based on the theorems explained here will be discussed in next lecture

 Probability Methods in
Civil Engineering: 

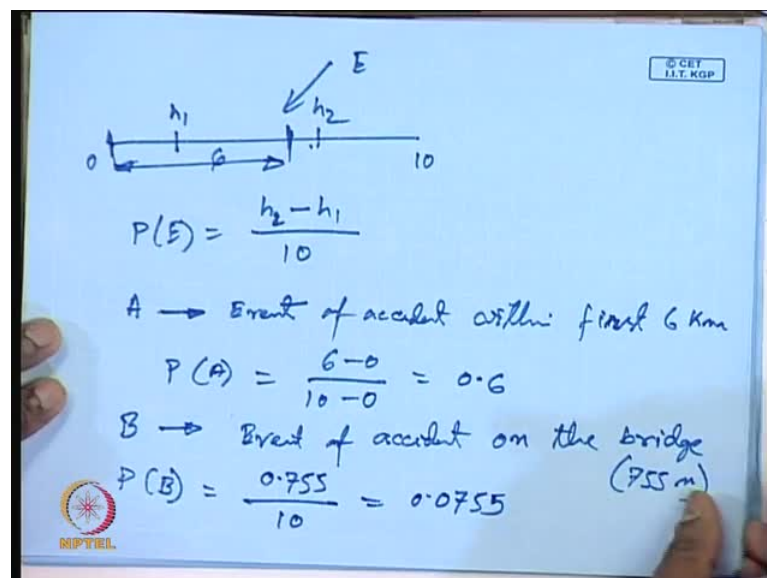
Dr. Rajib Haity, IIT Kharagpur 19

So, here you can see that, this probability of B given that A, it comes to the 0.126. So, in this class, we have seen that, there are three axioms that defines the basic properties of the probability of events in a, in a sample space and the probability theorems formulates the probabilities of an event, when other events exist in the same, in the same space derives from this axioms. The special probability theorems based on the theorems explained here will be discussed in the, in next lecture. But, before I conclude, I just want to add to one point here. When you are talking about this axioms of this probability, one thing should be kept in mind, that this axioms of probability never tells how should we, should, how should we assign the probability. It only gives some guidelines, that how the probability should be assigned. But, what should be the actual probability of a particular event, it is nothing to do with the axioms of probability.

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So, if we just see that the last, last example that, that, what we have done is the, it is that when we are, when we are talking about this probability of B and the probability of B and probability of A or whatever, we are just now, have seen, but these are coming from this, from the equally likely probable events, all these things that we discussed in the last class. Only thing the axioms states that there are some certain guideline that must be followed to get to assign this probabilities. But, what should be the actual probability for the particular event is nothing to do with the axioms of, of probability. So, with this I

conclude this class. In the next class, some more properties and the special probability theorem will be discuss in the next lecture. Thank you.